Advanced Physics - Project Course (1FA565)
Comparison of track reconstruction algorithms for the Moon
Shadow Analysis in IceCube

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1 Introduction

ICECube is a neutrino telescope placed at the geographic South Pole and its intended purpose is to detect and analyze astrophysical neutrino sources. It is a Cherenkov detector using the arctic ice as detection medium. The detection devices called DOM (Digital Optical Modules) are held by 86 vertical strings and cover an area of roughly 1 km\(^3\) between 1450 m and 2450 m depth below the surface. In total there are 5160 DOMs deployed. On its actual state it is running since December 2010. Before the experiment was running with fewer DOMs and the data of this study is based on the 79-string configuration. The DOMs are basically photomultiplier tubes placed within a protective glass sphere and are sensitive to single photons. Since neutrinos are not charged they do not emit Cherenkov light. However neutrinos can nevertheless be detected indirectly if they interact with the ice and charged leptons are induced.

Since ICECube not only registers neutrino induced leptons but all other high energetic charged particles emitting Cherenkov light during their propagation through the ice and since the interaction rate of neutrinos in the ice is very low one has to deal with a large amount of background events. Especially events induced by atmospheric muons outnumber the neutrino events by many orders of magnitude. However these background events which are disturbing for neutrino detection can be used for other purposes, e.g. the Moon Shadow Analysis. Atmospheric muons are produced among many other particles when cosmic rays enter the atmosphere and interact with particles in there. A large fraction of the incident cosmic particle momentum is carried by one or several muons and therefore the track of the muons points nearly in the same direction as the track of the incident cosmic particle.

Contrary to particle accelerator experiments for ICECube it is not possible to gauge and verify the outcoming signals by exposing it to well known events. Furthermore it is not possible to look at stable candles as it is done for telescopes since there are no well known high energy neutrino point sources. A solution for this problem is to look at the Moon to check the quality and accuracy of the provided signals. The Moon is nearby and its position and shape are well known. However the Moon is not a cosmic ray source such that ICECube does not directly detect it. Instead it shields incoming cosmic rays and since the Moon is very near to Earth the cosmic ray shadow of the Moon can be detected. By analyzing the size, shape and position of this shadow the quality of the track reconstructions provided by the ICECube detector can be estimated.

For the purposes of the Moon Shadow Analysis it is necessary to find an appropriate method to reconstruct tracks of downgoing muons which are low energetic compared to neutrinos that are typically considered in point source searches. The considered reconstruction methods are based on the log-likelihood method. The likelihoods are evaluated from probability density functions of arrival times of photons at the DOMs. However the likelihood depends on how many and which photo electrons are considered.
Furthermore there are different methods to describe the probability density function. So far the Pandel reconstruction [1] has been a preferably used tool for the Moon Shadow Analysis. However the Spline reconstruction is predicted to consider the heterogeneity of the ice better and therefore to be a more suitable reconstruction method [2]. It is the major goal of this study to analyze the performance of the Spline reconstruction method compared to the Pandel reconstruction method. The log-likelihood analysis according to both reconstruction methods will be presented further in section 2.

In this study, the performance of Pandel and Spline reconstruction has been analyzed for SPE and MPE log-likelihood descriptions, respectively, using the CORSIKA (Cosmic Ray Simulation for Kascade) Monte Carlo simulation data, stored in the dataset 7444 (IC 79). CORSIKA simulates cosmic particles and secondary particles produced by interactions between those and atmospheric particles. To reduce unnecessary background signals the Moon Shadow Filter has been applied, that is only events have been selected which have been within a $10^\circ$ zenith band around the Moon position.

From the incoming secondary particle data, the propagation of the secondary muons and their energy loss and accordingly the photon production within the detector is simulated by MMC (Muon Monte Carlo) [3]. Subsequently by the PHOTONICS Package, the propagation of the photons within the detector ice is simulated. Thereby the properties of the ice is simulated by the SPICE-MIE model. The development of this model has been done rigorously and can be expected to be good [5]. Finally also the response of the DOMs are simulated and the output signals correspond to the signals yielded by the actual experiment. On these reconstructed events the same filter algorithms are applied as it is done for the experimental data.

In order to select predominantly well-reconstructed events further quality cuts to the simulated events are applied. To assign adequate quality cut conditions the reconstruction qualities are reviewed relative to some properties of the events. These issues will be dealt with in section 3. In section 4 then the outcomes of the reconstruction analysis are presented and a comparison between the different reconstruction methods will be done.

## 2 Track Reconstruction

### 2.1 Log-Likelihood Analysis

Due to the scattering during the propagation of the photons and also due to other minor fluctuating effects, e.g. jitter in the response of the DOMs, it is not applicable to reconstruct the muon tracks in deterministic manner. Instead for different hypotheses for the muon tracks the likelihood is estimated based on observations and the hypothesis with the highest likelihood is selected.
The reconstruction of an event can be generalized to the problem of estimating a set of unknown parameters \( \{a\} \), e.g., track parameters, given as to experimentally measured values \( \{x\} \). The parameters, \( \{a\} \), are determined by maximizing the likelihood function \( L(x|a) \) which for independent components \( x_i \) of \( x \) reduces to

\[
L(x|a) = \prod_i p(x_i|a),
\]

(2)

where \( p(x_i|a) \) is the probability density function (p.d.f.) of observing the measured value \( x_i \) for given values of the parameters \( \{a\} \).

The track of a muon has five geometrical degrees of freedom, which can be represented by three coordinates assigning any point on the track and two coordinates, e.g., angular coordinates, assigning the direction. Since the muons are highly relativistic the velocities of all muons are assumed to be equal to the speed of light. Furthermore for this study it is irrelevant at which time a muon is detected. Therefore the velocity of the muons and time of detection are not considered as degrees of freedom. Thus any track of the muons can be represented by the set of variables

\[
\{a\} = (r_0, \hat{p}),
\]

(1)

whereby \( r_0 \), also called vertex, is the position of the muon at an arbitrarily chosen time \( t_0 \) and \( \hat{p} \) its direction. Figure 1 illustrates these variables.

For a set of measured variables \( \{x\} \) then a probability can be estimated depending on the track parameter set \( \{a\} \) and the likelihood \( L(x|a) \) can be obtained for each observation \( \{x\} \). By maximizing this likelihood function the best fitting parameter set \( \{a\} \) is determined for the observation \( \{x\} \). However in order to find the maximum it is more convenient to deal with the negative log-likelihood defined as

\[
-\log L = \sum_i -\log p(x_i|a)
\]

(2)

Figure 1: Geometry of the muon and photon propagation in the ice [1].
and to minimize it. Since the logarithmic function is strictly monotonic the maximum of $L$ corresponds to the minimum of $-\log L$.

For this study $\{x\}$ is the set of residual time $t_{res,i}$ whereby $i$ indexes hits, that is registered photons in the context of IceCube. The time residual $t_{res}$ thereby is defined as

$$t_{res} = t_{hit} - t_{geo}$$

with

$$t_{geo} = t_0 + \hat{p} \cdot (r_i - r_0) + d \tan \theta_c$$

whereby $t_{hit}$ is the time of the hit, $c$ is the vacuum speed of light neglecting the effect that photons propagate with lower velocities in the ice and the other variables are according to figure 1. Thus $t_{res,i}$ is the deviation of the time at which the hit DOM actually registers the photon $i$ to the geometrically proposed time of detection if no scattering or other disturbing effects occurred. For simplicity and due to limitations in electrical and optical signals only one hit per DOM is considered, namely the first hit. After calculating probability density functions $p(t_{res,i}|a)$ (see section 2.2) a simple approach to estimate the likelihood is

$$L_{SPE} = \prod_{j=1}^{N} p(t_{res,j}|a)$$

whereby $N$ represents the number of hit DOMs indexed by $j$ within an event. Furthermore $t_{res,j}$ then corresponds to the residual time of the first hit of DOM $j$. This simple approach corresponds to the SPE (Single-Photo-Electron) method.

Another approach is the MPE (Multi-Photo-Electron) method in which also the information is used that only the residual time of the first hits are considered. This approach leads to the likelihood

$$L_{MPE} = \prod_{j=1}^{N} \left(n_j \cdot p(t_{res,j}|a) \cdot (1 - P(t_{res,j}))^{(n_j-1)}\right).$$

whereby $N$ again refers to the number of hit DOMs and $n_j$ to the number of hits registered by the DOM $j$. Furthermore $P(t)$ is the cumulative distribution of $p(t|a)$, that is

$$\int_{t_{res}}^{\infty} p(t|a) dt = 1 - P(t_{res}).$$

This approach is assumed to improve the reconstruction if a large amount of charge is deposited in the DOMs, that is if many photons are registered by the same DOM.
Figure 16: Values of the effective scattering coefficient $b_{e}(400)$ and absorption coefficient $a_{e}(400)$ vs. depth for a converged solution are shown with a solid line. The range of values allows to estimate uncertainties is indicated with a grey band around this line. The updated model of [4] (AHA) is shown with a dashed line. The uncertainties of the AHA model at the AMANDA depths of $1730 \pm 225$ m are roughly 5% in $b$ and roughly 14% in $a$. The scale and numbers to the right of each plot indicate the corresponding effective scattering length and absorption lengths in [m].

2.2 Pandel and Spline Reconstruction Methods

Two methods to describe the probability density function $p(t_{res}, i | a)$ are the Pandel and the Spline reconstruction. The Pandel reconstruction method has been originally applied for the BAikal experiment [6] and adopted for low energetic muon track reconstructions in AMANDA, the predecessor experiment of ICECUBE. Since then the Pandel reconstruction has been approved and has become a popular reconstruction method for AMANDA and for ICECUBE as well. However the Pandel reconstruction does not account for characteristics and inhomogeneities of the ice with respect to photon propagation. The properties of the ice has been analyzed rigorously and the resulting model shows significant variations of the effective scattering as well as absorption coefficients depending on the depth of the ice as illustrated in figure 2. And indeed for larger $t_{res}$ the probability density function of the Pandel reconstruction method does deviate significantly from...
distributions obtained by detailed simulations [1]. The Spline reconstruction method is an attempt to consider also the specific ice characteristics of ICECUBE. However this method has just recently been considered and still needs to be approved. The Pandel and Spline reconstruction methods will be dealt with only shortly here and for a more detailed description the reader is referred to [1, 2]. The probability density function obtained by the Pandel reconstruction method is a modified Gamma distribution and can be expressed as

\[ p(t_{res}|a) = e^{d_{eff}/\lambda_a} \left( 1 + \frac{\tau \cdot c_m}{\lambda_a} \right)^{d_{eff}/\lambda_a} \frac{\tau^{d_{eff}/\lambda} \Gamma(d_{eff}/\lambda)}{\Gamma(1)} e^{-(t_{res} \cdot (1/\tau + c_m/\lambda_a) + d_{eff}/\lambda_a)} \]  

whereby \( \tau \) and \( \lambda \) are free parameters needed to be chosen empirically, \( d_{eff} \) is the so called effective distance depending on \( d \) and \( \eta \) from figure 1, \( \lambda_a \) is the absorption length and \( c_m \) the light propagation speed in the ice. The parameter set \{a\} goes in by the dependence of \( \tau \), \( \lambda \) and \( d_{eff} \) of it. The Pandel function does not allow for negative time residuals which can occur due to jitter in the detection devices. To overcome this problem the Pandel function is convoluted afterwards with a Gaussian. This procedure is described in detail in [8].

With the Spline reconstruction method the probability density function is not estimated by an analytic ansatz but by simulations. Photon propagations in the ice are simulated according to the PHOTONICS package and tabulated for various parameter sets \{a\} and out of these tables probability density functions are constructed numerically. However the problem arises that in order to enhance the quality of the probability density function the number of table entries has to be enlarged which can lead to the need of enormous computational capacities. The Spline fit solves this problem by reducing the number of parameters to be memorized even though keeping the necessary information to obtain smooth and accurate probability density functions by using interpolation techniques. Also the probability density function obtained by Spline reconstruction is convoluted by a Gaussian to consider effects caused by the DOMs.

### 2.3 Seeding of the Reconstructions and Preparation of the Data

Since the log-likelihood analysis is an optimization technique by running iterations there must be an initial track chosen to start with. Although ideally the choice of the first track should not matter it happens that an iteration gets stuck in local minima since highly complex structures of \(- \log(L)\) can occur. Therefore it can happen that tracks are severely misreconstructed and to reduce this effect as much as possible the first track is supposed to be as nearest to the global minimum as possible. The delivery of first track assumptions for the log-likelihood analysis is called seeding. A popular first guess method is the Line-Fit [9, 1] method. This method completely neglects scattering effects in the ice and utilizes the \( \chi^2 \) method which is fairly sufficient for a first guess.
Thus the track is reconstructed by minimizing

\[ \chi^2 = \sum_{i=1}^{N_{\text{hit}}} (r_i - r_0 - v \cdot t_{\text{hit},i})^2 \]  

(9)

whereby the variables are according to section 2.1 and \( v \) corresponds to the velocity of the muon. The Line-Fit method can be improved by considering that according to the underlying model only one photon can hit each DOM at most and therefore additional hits are discarded.

For this study track reconstruction data according to the methods Pandel SPE, Pandel MPE, Spline SPE and Spline MPE has to be prepared. The Pandel SPE analysis data is prepared first and seeded by the improved Line-Fit first guess method. For Pandel SPE after the first minimization the arbitrarily selectable parameter \( t_0 \) is set such that the vertex of the track \( r_0 \) is nearest to the center of gravity (cf. section 2.4) and subsequently another minimization is done. Analogously two further such minimizations are done and from the four minimizations the minimum with lowest negative log-likelihood is chosen to be the best fit. The best fit according to Pandel SPE then is seeded for the Pandel MPE and Spline SPE reconstruction analyses. For these analyses only one minimization is run. Finally the best fit obtained by Pandel MPE is seeded for the reconstruction analysis of Spline MPE. Also for Spline MPE only one minimization is run.

### 2.4 Track Properties and Fit Parameters

The simulated data values for each event are provided by the CORSIKA dataset 7444 (IC79).

Some track properties as well as reconstruction parameters which are of interest in this study will be presented briefly in the following:

**Zenith and azimuth angles:** The direction of the muon track can be represented by two angles, the zenith angle \( \theta \) and the azimuth angle \( \phi \) whereby in this report these angles will without exception refer to the local coordinates of IceCube. Note that the direction vector given by the local coordinates point from the detector away while the muons fly towards the detector. Thus the direction vector has to be inverted such that e.g. for vertically down-going tracks it holds \( \theta = 0^\circ \). The range of \( \theta \) is \([0, \pi] \) and the range of \( \phi \) is \([0, 2\pi] \).

**Angular Deviation:** The space angle difference between reconstructed track and the Monte Carlo simulated track will be called simply angular deviation throughout this report. Given \( \theta_r \) and \( \phi_r \) the reconstructed track direction and \( \theta_{MC} \) and \( \phi_{MC} \) the simulated track direction the angular deviation \( \psi \) is obtained by

\[ \psi = \arccos \left( \sin \theta_r \sin \theta_{MC} \sin \phi_r \sin \phi_{MC} + \cos \phi_r \cos \phi_{MC} \right) + \cos \theta_r \cos \theta_{MC} \]  

(10)
The median of the angular deviation is a quantity indicating the resolution of the detector.

**Paraboloid error**: The paraboloid error is referred to as \( pb_{err} \) and comes about by the negative log-likelihood structure in the \((\theta, \phi)\)-plane. The ellipse in figure 3 marks the confidence area and contains space points at which the negative log-likelihood value has increased less than \( \Delta(-\log L) = 0.5 \) compared to the minimum. By assuming that the negative log-likelihood in the 1 dimensional \( \theta \) and \( \phi \) space, respectively, is according to a Gaussian it can be shown that the confidence area is indeed an ellipse and that the true track direction lies within this area with a probability of 39.4%. \( \alpha \) then accounts for the covariance between \( \theta \) and \( \phi \) while \( \sigma_1 \) and \( \sigma_2 \) account for the magnitude of the error. The paraboloid error is defined as

\[
pb_{err} = \sqrt{\frac{\sigma_1^2 + \sigma_2^2}{2}}. 
\]  

(11)

Although the track is represented by five parameters the error consideration can be reduced to 2 dimensions since the covariance between the error of \( r_0 \) and \( \theta \) and \( \phi \) is already included in \( \sigma_\theta \) and \( \sigma_\phi \) [10] and one is not interested in \( r_0 \) itself.

Figure 3: Paraboloid error ellipse in the \((\theta, \phi)\) parameter space [10]
**Pull:** The pull is defined as the relation between angular deviation and $pbferr$, that is

$$\text{pull} = \frac{\psi}{pbferr}. \quad (12)$$

The pull distribution of an event set indicates how much the error predicted by the negative log-likelihood method correspond to the actual error of the reconstruction.

**Fit status:** The fit status is provided by the data for each event and is assigned a number whereby each number corresponds to a specific warning during the reconstruction. If there has been no warnings during the reconstruction the fit status is assigned the value 0. Warnings appear e.g. if the number of pulses is below the number of degrees of freedom or if there are indications for that the minimization fits do converge. During this whole study events with defective fit status, i.e. fit status not equal to 0, will be abolished. The fraction of defective events are around 2-4% depending on the reconstruction method. The MPE reconstructions have typically higher fractions of defective events.

**Reduced log-likelihood:** The reduced log-likelihood is referred to as $rlogl$ and is defined as the negative log-likelihood value at the global minimum divided by the number of degrees of freedom. Furthermore the number of degrees of freedom is the number of photon hits, i.e. number of measurements, subtracted by 5 since five parameters describe the track. Low reduced log-likelihood values indicate high quality of the reconstruction.

**Direct hits:** Direct Hits refer to photon hits with time residual $t_{res}$ between -15 ns and +75 ns. An event with only a few or no direct hits is highly probable to be not well reconstructed. The number of direct hits in an event are referred to as $n_{dir}$. Another parameter in association with direct hits is the direct track length. It is defined as the maximal distance between two distinct directly hit DOMs within the same event projected onto the reconstructed track. Comparable with ”lever arms” statistical errors have less impact for events with larger direct hits. Direct track lengths are referred to as $l_{dir}$.

**Center of gravity:** To be able to assign a position to an event the center of gravity of the hit DOMs weighted by their deposited charge is determined for each event. The dataset provides the center of gravity by using the cartesian coordinates $cog-x$, $cog-y$ and $cog-z$ whereby the origin of this coordinate system corresponds to the center of ICECUBE. Particularly $cog-z$, representing the depth is of interest since the ice properties change with $cog-z$ as well as the number of events. Since the shape of ICECUBE is rather cylindrical than cubic $cog-x$ and $cog-y$ are converted into the euclidean distance $cog-\rho = \sqrt{(cog-x)^2 + (cog-y)^2}$. The polar coordinate then is irrelevant for this study.
3 Event Selection

3.1 Distribution of Reconstruction Parameters

In the following the angular deviation, the paraboloid error and the pull distributions of the simulated events are inspected for the four reconstruction methods Pandel SPE, Pandel MPE, Spline SPE and Pandel MPE. These quantities indicate the quality of the reconstructions and will be of major interest for this study. The distributions are depicted in figure 4 whereby events with defective fit status are discarded. The x-axes are plotted logarithmically in order to reveal the structure of the distributions also at larger scales.

Some properties of the distributions are presented in table 1. The different numbers of entries occur because the fit status can depend on the reconstruction method. Before going in detailed interpretation of these values it might be judicious to improve the data sample by applying further quality cuts, i.e. discarding unuseful events.

The angular deviation distributions in figure 4 indicate clearly by the "bumps" appearing above around $\psi = 30^\circ$ that there is a significant amount of events with unreasonably large angular deviations. These events will be called "bump" events in the following. In the paraboloid error distributions on the other hand such "bumps" do not show up which indicates that these "bump" events are not only completely misreconstructed but also that their error estimates do not reflect the uncertainties properly. However events with such large angular deviations even if the error estimates were correct are not needed to be kept for the Moon Shadow Analysis. Therefore it is a

![Figure 4: Reconstruction parameter distribution of each reconstruction before any quality cuts. The color code is the same in all three graphs and are labeled in the box of the pull distribution plot. The medians of all distributions are indicated by the straight vertical lines.](image-url)
reasonable procedure to enhance the data sample by discarding those "bump" events. Since for the experimental Moon Shadow Analysis the angular deviation is not known opposed to studies with simulation data, it is not applicable just to discard events with angular deviations above $30^\circ$. Therefore some other properties of the events must be found which hint for misreconstruction. Then based on these properties quality cuts are applied.

### 3.2 Quality Cut Conditions

In order to find appropriate quality cut conditions the dependence of the angular deviation distributions on some event properties are analyzed. Therefore the event sample is partitioned according to a specific event property, e.g. cog-$z$, and for each partition an angular deviation distribution is obtained. Such an analysis is well represented by a 2-dimensional histogram plot. The portion of events contributing to the "bump" in figure 4 (also cf. table 3) is around 10%. Therefore particularly the 0.9-quantiles are of interest.

In figure 5 the angular deviation distributions partitioned by cog-$z$ is shown. The events going into the plots are the same as in figure 4. The y-axis represents the angular deviation and the x-axis the analyzed event property which is in this case cog-$z$. As in figure 4 also here the angular deviation axis is logarithmically scaled to reveal the "bump". The bluish colors indicate the number of events in each bin referred to as $n_{bin}$ whereby it is to point out that the scale is logarithmic to uncover the structure of the "bump". Furthermore the median and 0.9-quantile of the angular deviation distribution for each bin range of cog-$z$ is illustrated by the red and yellow graphs. The horizontal black straight line marks the angular deviation $\psi = 30^\circ$. This angle has not any specific physical meaning but is used as a guideline to separate "bump" and well-reconstructed events.

The appearance of the "bump" is evident in figure 5. However the 0.9-quantile curve lies quite

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<table>
<thead>
<tr>
<th></th>
<th>Pandel SPE</th>
<th>Pandel MPE</th>
<th>Spline SPE</th>
<th>Spline MPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entries</td>
<td>177263</td>
<td>172813</td>
<td>174040</td>
<td>172421</td>
</tr>
<tr>
<td>Angular deviation Median</td>
<td>1.714</td>
<td>2.126</td>
<td>1.765</td>
<td>2.044</td>
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<tr>
<td></td>
<td>1st quartile</td>
<td>0.899</td>
<td>1.167</td>
<td>0.974</td>
</tr>
<tr>
<td></td>
<td>3rd quartile</td>
<td>4.654</td>
<td>5.453</td>
<td>4.514</td>
</tr>
<tr>
<td>Paraboloid error Median</td>
<td>1.487</td>
<td>1.158</td>
<td>1.159</td>
<td>1.101</td>
</tr>
<tr>
<td></td>
<td>1st quartile</td>
<td>1.104</td>
<td>0.858</td>
<td>0.883</td>
</tr>
<tr>
<td></td>
<td>3rd quartile</td>
<td>2.286</td>
<td>1.831</td>
<td>1.734</td>
</tr>
<tr>
<td>Pull Median</td>
<td>1.173</td>
<td>1.836</td>
<td>1.537</td>
<td>1.883</td>
</tr>
<tr>
<td></td>
<td>1st quartile</td>
<td>0.696</td>
<td>1.089</td>
<td>0.917</td>
</tr>
<tr>
<td></td>
<td>3rd quartile</td>
<td>2.093</td>
<td>3.232</td>
<td>2.717</td>
</tr>
</tbody>
</table>

Table 1: Statistical properties of the distributions in figure 4.
between the region of the "bump" and the region of well-reconstructed events over the whole cog-z range and therefore no clear accumulation of misreconstructed events for a certain range of cog-z can be ascertained.

On the other hand the median curve shows that the angular deviation at the edges get worse and it is indeed rather plausible that events at the edges are worse reconstructed. Therefore it is a common procedure to cut the events at the edges away and also for this study only events with cog-z within the interval \([-400 \, \text{m}, 400 \, \text{m}]\) are kept for all four reconstruction methods. For the same reasons also the events lying outside the euclidean distance \(\text{cog-} \rho = 400 \, \text{m}\) will be discarded.

In figure 6 the corresponding plots depending on number of direct hits \(n_{\text{dir}}\) are presented. As predicted already in the earlier chapter the graphs show clearly that events with larger numbers of direct hits are better reconstructed. The event distributions of the different reconstruction methods look similar as well as the median and 0.9-quantile curves. The 0.9-quantile curves lie for events with \(n_{\text{dir}} < 5\) in the "bump" region which means that in this range more than 10% of events contribute to it. Therefore events lying in this range are discarded for each reconstruction method.

Next the event distributions partitioned by direct track length is examined. The corresponding plots are presented in figure 7. Also for the direct track length the plots show a correlation with the reconstruction performance. Furthermore also here the event distributions and the 0.9-quantile curves are similar for all reconstruction methods such that the same cut condition can be applied.
Figure 6: Angular deviation event distribution partitioned by number of direct hits.

Figure 7: Angular deviation event distribution partitioned by direct track length.
Events with $l_{\text{dir}} < 250$ m contribute with more than 10% to the "bump". Therefore these events are discarded.

Finally the correlation between the angular deviation distribution and reduced log-likelihood $r\logl$ is analyzed. Figure 8 shows the corresponding plots. Compared to the other event properties in this case the shapes of the distributions tend to differ, namely for Pandel SPE the distribution is shifted slightly to larger $r\logl$. However the 0.9-quantile curves cut the 30° guideline quite at $r\logl = 8.5$ for each reconstruction method. Therefore also here the same quality cut is applied for all reconstruction methods and events with $r\logl > 8.5$ are discarded.

The chosen quality cut conditions are collected in table 2. For all reconstruction methods the same conditions are applied which is not a simplification but is due to the similar shape of the event

<table>
<thead>
<tr>
<th>Event property</th>
<th>Cut condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$cog-z$</td>
<td>below -400 m and above 400 m</td>
</tr>
<tr>
<td>$cog-\rho$</td>
<td>larger than 400 m</td>
</tr>
<tr>
<td>$n_{\text{dir}}$</td>
<td>smaller than 5</td>
</tr>
<tr>
<td>$l_{\text{dir}}$</td>
<td>larger than 250 m</td>
</tr>
<tr>
<td>$r\logl$</td>
<td>larger than 8.5</td>
</tr>
</tbody>
</table>

Table 2: Assigned quality cut conditions which is applied on all reconstruction methods.
distributions. Compared to other studies with comparable data the cut conditions chosen here can be considered as rather mild cuts. Furthermore also other properties, e.g. the number of direct pulses, the charge deposited by direct pulses, number of direct hit strings and paraboloid error, has been examined. These properties show up indeed biases to the angular distribution but are also strongly correlated to the already analyzed properties above and quality cuts on these do not affect the selected data sample very much. Therefore additional conditions to table 2 are dropped.

3.3 Comparison of the Distributions with and without Quality Cuts

Figure 9 depicts the angular deviation, the paraboloid error and the pull distributions for each reconstruction method without and with quality cut respectively. The corresponding curves without and with quality cuts are superimposed whereby the distributions without quality cut is labeled red and the distributions with quality cut are labeled blue.

It turns out that even with quality cuts the "bump" above $\psi = 30^\circ$ may be clearly reduced but still remains. Even by considering other quality cut values and other event properties a cut condition to get rid of the "bump" completely has not been found. It is suspected that this remaining "bump" is caused by events in which two (or more) atmospheric muons appear nearly coincidently. Then if only one of these enters the detector and the reconstruction is build based on the direction of this muon it can happen that the $\theta_{MC}$ and $\phi_{MC}$ in order to determine $\psi$ (cf. section 2.4) are taken from the other muon such that $\psi$ becomes large while $pbferr$ is reasonable. Thus these "bump" events would not be misreconstructed events but the angular deviations were falsely constructed and for the experimental Moon Shadow Analysis it would not be a problem at all since from the experimental data the angular deviation cannot be obtained anyway. It still would remain to check whether indeed the coincident events cause the "bump" but this task is seen to be out of scope for

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<thead>
<tr>
<th>Reconstruction Method</th>
<th>Total number of events</th>
<th>Events with $\psi &gt; 30^\circ$</th>
<th>Fraction of &quot;bump&quot; events</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pandel SPE</td>
<td>without cut</td>
<td>177263</td>
<td>20327</td>
</tr>
<tr>
<td></td>
<td>with cut</td>
<td>86396</td>
<td>4781</td>
</tr>
<tr>
<td>Pandel MPE</td>
<td>without cut</td>
<td>172813</td>
<td>19718</td>
</tr>
<tr>
<td></td>
<td>with cut</td>
<td>93651</td>
<td>5227</td>
</tr>
<tr>
<td>Spline SPE</td>
<td>without cut</td>
<td>174040</td>
<td>18611</td>
</tr>
<tr>
<td></td>
<td>with cut</td>
<td>92977</td>
<td>5088</td>
</tr>
<tr>
<td>Spline MPE</td>
<td>without cut</td>
<td>172421</td>
<td>18676</td>
</tr>
<tr>
<td></td>
<td>with cut</td>
<td>93625</td>
<td>5114</td>
</tr>
</tbody>
</table>

Table 3: Number of events and "bump" events.
Figure 9: Reconstruction parameter distributions without and with quality cuts.
this study.
Table 3 lists the absolute numbers of events and "bump" events as well as their ratio. It is shown that the fraction of "bump" events has been lowered from around 10% to around 5%. For the purposes of the Moon Shadow Analysis the amount 5% of misreconstructed events is acceptable and therefore a more sophisticated cut condition analysis is not necessary here.

4 Analysis of the Reconstruction Methods

4.1 Distribution of the Reconstruction Parameters

The angular deviation, the paraboloid error and the pull distributions after quality cuts are presented in figure 10. These are the same distributions as in figure 9 but here the distributions of different reconstruction methods are superimposed in the same graph in order to compare between them. The corresponding statistical properties are listed in table 4. Note that after quality cuts the quantiles are lowered significantly (cf. table 1).

It is rather difficult to point out differences between the distributions in figure 10 which clearly shows their similarity. This is surprising reconsidering that the heterogeneity of the ice is not negligible (cf. figure 2). Solely the Pandel SPE distributions tend to contrast with the other distributions a little. Also taking table 4 into account it turns out that the angular deviations get larger with Spline compared to Pandel for the SPE distributions a little, i.e. the angular resolution gets worse.

![Figure 10](image-url)

Figure 10: Reconstruction parameter distribution of each reconstruction after quality cuts. The color code is the same in all three graphs and are labeled in the box of the pull distribution plot. The medians of all distributions are indicated by the straight vertical lines.
On the other hand it seems to improve slightly for MPE. Furthermore SPE reconstructions have significantly better angular resolutions than MPE reconstructions. Despite the higher angular resolution with SPE Pandel the paraboloid errors are estimated more distrustfully and the quantiles of the SPE Pandel distribution differs significantly from SPE Spline distribution. For MPE the paraboloid error distributions and their quantiles are very similar whereby Spline MPE estimates are slightly lower. Consistently the pulls for SPE Pandel are significantly lower than for SPE Spline and for MPE there is not much difference between Pandel and Spline. The pull distributions will be analyzed more detailed in the next section.

### Table 4: Statistical properties of the distributions in figure 10.

<table>
<thead>
<tr>
<th></th>
<th>Entries</th>
<th>Pandel SPE</th>
<th>Pandel MPE</th>
<th>Spline SPE</th>
<th>Spline MPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angular deviation</td>
<td>Median</td>
<td>1.106</td>
<td>1.507</td>
<td>1.251</td>
<td>1.448</td>
</tr>
<tr>
<td></td>
<td>1st quartile</td>
<td>0.663</td>
<td>0.920</td>
<td>0.757</td>
<td>0.880</td>
</tr>
<tr>
<td></td>
<td>3rd quartile</td>
<td>1.850</td>
<td>2.489</td>
<td>2.025</td>
<td>2.397</td>
</tr>
<tr>
<td>Paraboloid error</td>
<td>Median</td>
<td>1.128</td>
<td>0.936</td>
<td>0.943</td>
<td>0.896</td>
</tr>
<tr>
<td></td>
<td>1st quartile</td>
<td>0.942</td>
<td>0.754</td>
<td>0.777</td>
<td>0.726</td>
</tr>
<tr>
<td></td>
<td>3rd quartile</td>
<td>1.379</td>
<td>1.189</td>
<td>1.158</td>
<td>1.119</td>
</tr>
<tr>
<td>Pull</td>
<td>Median</td>
<td>0.990</td>
<td>1.616</td>
<td>1.333</td>
<td>1.636</td>
</tr>
<tr>
<td></td>
<td>1st quartile</td>
<td>0.614</td>
<td>1.001</td>
<td>0.829</td>
<td>1.016</td>
</tr>
<tr>
<td></td>
<td>3rd quartile</td>
<td>1.528</td>
<td>2.512</td>
<td>2.051</td>
<td>2.534</td>
</tr>
</tbody>
</table>

4.2 Pull Distribution partitioned by Event Properties

Particularly the pull distribution gives an ultimate estimate of a reconstruction method’s performance since the pull is a relation of the actual angular error to the predicted error and therefore indicates the reliability of the reconstructions. In order to analyze the pull distributions further they will be partitioned by event properties analogous to the angular deviation analysis in section 3.2. The true track direction is expected to be within the paraboloid ellipse with the confidence level of 39.4%. For the special case that the ellipse is a circle, i.e. that \( \sigma_1 \) and \( \sigma_2 \) (cf. section 2.4) are equal it holds that the angular deviation is smaller than \( pbferr \) with the confidence level of 39.4%. It can be shown then that by using a factor correction of \( w = 1.177 \) the angular deviation is smaller than \( M = 1.177 \cdot pbferr \) with the confidence level of 50% which is derived in detail in [10]. Consequently for the special case of \( \sigma_1 = \sigma_2 \) the median of the angular deviation distribution is expected to be equal to \( M \) which is equivalent to expect the median of the pull to be \( w = 1.177 \). However the factor \( w \) changes for different eccentricities \( \epsilon = \sigma_1 / \sigma_2 \) according to figure 11. Note
that in [10] $pbferr$ is defined as the geometrical mean opposed to the definition used in this study. However for the case that $\epsilon = 1$ both definitions coincide and the behavior of $w$ in the $\epsilon$ region $[0.5, 2]$ is assumed to be similar and according to figure 11 approximately constant with $w = 1.177$. Figure 12 shows that for most of the events it holds that $\epsilon$ is smaller than 2 whereby the minimum is $\epsilon = 1$ because in the used data $\sigma_1$ is by definition larger than $\sigma_2$. Therefore 1.177 is supposed to be a good reference value to compare the median of the pull distributions with.

Figure 12 shows that for most of the events it holds that $\epsilon$ is smaller than 2 whereby the minimum is $\epsilon = 1$ because in the used data $\sigma_1$ is by definition larger than $\sigma_2$. Therefore 1.177 is supposed to be a good reference value to compare the median of the pull distributions with.

The pull distribution partitioned by cog-z is presented in figure 3. As in section 3.2 the bluish color indicates the number of events contained in each bin. The yellow, red and magenta curves mark the
Figure 13: Pull event distribution partitioned by cog-z after quality cuts except the quality condition for cog-z.

lower quartiles, medians and upper quartiles of the pull distribution at each partition. Furthermore as a guide the black horizontal line marks the predicted pull at 1.177. Opposed to section 3.2 the scaling of the y-axis and the color axis are linear. The events going into the plots are selected according to table 2. Only the cut condition for cog-z is left out since due to the partitioning it is redundant. Furthermore it is also interesting to see how the pull distribution behaves outside the selected event range to affirm or argue the chosen cut condition. However as already mentioned the quality cut conditions chosen for this study are rather mild therefore some structures are expected to be visible.

It is greatly desirable for the Moon Shadow Analysis that the pull distribution remains nearly constant. Otherwise additional correction algorithms become necessary which can be complex due to the dependency of the events on other variables and therefore can bring about systematic errors. The quantile curves in figure 13 show that the distributions are indeed rather constant in the event selection interval [-400 m, 400 m] while outside this area the quantile curves jump randomly. Therefore these plots indeed affirm that cut conditions are applied on cog-z.

Furthermore the plots show as already pointed out in the previous section that by comparing the Pandel and Spline reconstructions there is not much difference for MPE but for SPE the pull is significantly lower. Note that for Pandel SPE the median is slightly below 1.177 and for Spline SPE slightly above.
Another remarkable difference is that the MPE distributions are more spread away than the SPE distributions considering the displacement between the upper and lower quartiles. Thereby the Pandel SPE distributions are even narrower than the SPE Spline distributions. Furthermore except for Pandel SPE there can be seen some structure at around $cog-z = -100$ m just below the dust layer.

In figure 14 the pull distribution partitioned by the number of direct hits $n_{dir}$ is presented. Also here the events are selected according to table 2 except the cut condition for $n_{dir}$. The plots show clearly that the reconstructions get bad for low $n_{dir}$. However for higher $n_{dir}$ the plots show that there is no visible correlation between the median of the pull distribution and $n_{dir}$. The fluctuations of the quantile curves for $n_{dir} > 18$ is assumed to be due to low statistics. Also in these plots the difference between the reconstruction methods pointed out for figure 13 are visible.

The pull distribution partitioned by the direct track length $l_{dir}$ is presented in figure 15. Also here the events are selected according to table 2 except for $l_{dir}$ cut condition. The plots show some structure of the quantile curves for $l_{dir}$ below 700 m and show clearly that the distributions get more stable for larger $l_{dir}$. Furthermore the quantile curves for Spline SPE tends to fall slightly with larger $l_{dir}$. No further differences between Pandel and Spline reconstructions than the features already pointed out are obtained from figure 15.

Finally figure 16 shows the pull distributions partitioned by the reduced log-likelihood. Also here

Figure 14: Pull event distribution partitioned by number of direct hits $n_{dir}$ after quality cuts except the quality condition for $n_{dir}$.
Figure 15: Pull event distribution partitioned by direct track length $l_{dir}$ after quality cuts except the quality condition for $l_{dir}$.

Figure 16: Pull event distribution partitioned by $rlogl$ after quality cuts except the quality condition for $rlogl$. 
the events are selected according to table 2 except for $rlogl$. The quantile curves show that the distributions are rather stable in the event selection range. However as for $l_{dir}$ also here a slight correlation between the median of the pull distribution and $rlogl$ seems to exist in the range with $rlogl$ between 7 and 8 for Spline SPE which is not visible for Pandel SPE.

4.3 Evaluation of the Outcomes and Conclusions

To summarize the major outcomes of the previous two sections, for MPE there are no significant differences between Pandel and Spline reconstructions while for SPE the Pandel track reconstructions tend to have lower angular deviations and lower pulls than the Spline track reconstructions. Thereby the median of the Pandel SPE pull distribution is slightly below the value 1.177 while the median of the Spline SPE is slightly above.

That the angle deviation is lower for Pandel SPE than for Spline SPE speaks in favor for the Pandel reconstruction opposed to what was proposed before. Also the Spline SPE distribution has shown slight correlations to $l_{dir}$ and $rlogl$ opposed to the Pandel SPE distribution which also indicates that Pandel reconstruction is more advantageous. However the correlations are very low and hardly noticeable and the disadvantage of having lower angular resolution, if not too severe, can be compensated by large amount of statistics which is available for the Moon Shadow Analysis. Wrong estimation of the error on the other hand can lead to wrong results which is to be avoided. Therefore the pull distributions ultimately indicates the performance of the reconstruction methods.

Assuming that the expected median of the pull is 1.177 it can be concluded that Pandel SPE overestimates the error while Spline SPE underestimates it. However one has to reconsider some simplifications and defects accepted for this study. First despite the quality cuts there was still a fraction of around 5% of events left which were completely misreconstructed. These events evidently increase the quantiles of the distributions systematically. Furthermore the expected value of 1.177 only holds for the special case that the eccentricity of the paraboloid error ellipse is $\epsilon = 1$. Of course this is not the case for all the selected events such that the true expected value is supposed to be slightly higher than 1.177. Thus taking theses artifacts into account the Pandel SPE is supposed to overestimate the errors even more than it seems according to the plots of the previous sections while the Spline SPE underestimates the errors less. Unfortunately within the limits of this study it is not possible to analyze the systematic errors mentioned above further such that the impact of those cannot be estimated properly.

Altogether an improvement of Spline reconstruction compared to Pandel reconstruction could not be proved by this study. However there are hints that at least for SPE the Spline reconstruction estimates the error more properly and therefore can be an improvement compared to Pandel SPE.
5 Suggestions and Outlook

The Pandel reconstruction has been an approved and popular reconstruction method for IceCube and the predecessor experiment AMANDA for years. The more sophisticated Spline reconstruction has been considered very recently and this study is one of the early attempts to prove the Spline reconstruction. Of course further studies also using other approaches need to be done to get a better understanding of the Spline reconstruction in IceCube. Some suggestions how to improve the analysis done in this study will be presented in the following.

In the previous section it has been already mentioned that the results are flawed by some systematic shifts due to the "bump". It is suggested to discard coincident events which is possible while working with simulation data. However up to now it is just an assumption that the bump consists mostly of coincident events. If the "bump" even after discarding coincident events does not disappear the origin of this "bump" has to be traced further. Anyhow it is not a good idea just to cut the bump away by cutting on angular deviation since this is not possible for the experimental Moon Shadow Analysis.

Furthermore it was mentioned in the previous section that also the expected value insists a systematic shift. This problem can be solved by correcting the pull by the $\epsilon$-depending factor $w$ (cf. figure 11) as it is done in [10]. It has to be considered that in [10] the corrections are calculated assuming that $pbferr$ is the geometric mean of $\sigma_1$ and $\sigma_2$ while in this study the $pbferr$ is defined to be the root mean square of those. Thus the results of [10] cannot be simply adopted but the calculation must be done analogously for the root mean square defined $pbferr$. The median of the corrected pull distribution then is expected to be 1.177.

To enhance the validity of the pull distribution analysis further it is also suggested not only to check the median to the expected value but also other distinctive quantiles. The corresponding expected values can be calculated analogously as it is done for the median. In a more sophisticated manner the whole pull distribution can be predicted numerically and subsequently compared.

As the last point the seeding strategy is discussed (cf. section 2.3). For this study the same seeding strategy is chosen as it has been done by Kai Schatto in his study about using Spline in point source analysis (within the frame of a PhD thesis at the University in Mainz, not yet published). However in order to compare Pandel and Spline it might be better to use same seeding to treat them on equal footing. For the same reason it is also suggested to run the same number of iterations for all reconstructions. However it is assumed that the seeding does not affect the results very much. Otherwise the structure of the angular deviation distribution at higher values is supposed to differ significantly between the reconstruction methods. What is observed instead is that the "bump" after quality cuts look quite alike for all reconstruction methods.
References


[9] C. Stenger: *Track fitting for DUMAND-II Octagon Array*, University of Hawai‘i at Manoa, 1990
