Effect of freestream turbulence on roughness-induced crossflow instability

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The effect of freestream turbulence on generation of crossflow disturbances over swept wings is investigated through direct numerical simulations. The setup follows the experiments performed by Downs et al. in their TAMU experiment. In this experiment the authors use ASU(67)-0315 wing geometry which promotes growth of crossflow disturbances. Distributed roughness elements are locally placed near the leading edge with a span-wise wavenumber, to excite the corresponding crossflow vortices. The response of boundary layer to external disturbances such as roughness heights, span-wise wavenumbers, Reynolds numbers and freestream turbulence characteristics are studied. It must be noted that the experiments were conducted at a very low level of freestream turbulence intensity ($T_u$). In this study, we fully reproduce the freestream isotropic homogeneous turbulence through a DNS code using detailed freestream spectrum data provided by the experiment. The generated freestream fields are then applied as the inflow boundary condition for direct numerical simulation of the wing. The geometrical setup is the same as the experiment along with application of distributed roughness elements near the leading edge to precipitate stationary crossflow disturbances. The effects of the generated freestream turbulence are then studied on the initial amplitudes and growth of the boundary layer perturbations. It appears that the freestream turbulence damps out the dominant stationary crossflow vortices.

Swept-wing boundary layer, surface roughness, receptivity, freestream turbulence, crossflow instability

1. Introduction

A conventional three-dimensional boundary layer on a wing bears a certain number of instabilities, such as crossflow instability. It normally arises due to an inflection point in the boundary layer which creates imbalanced momentum and pressure forces. A negative pressure gradient can play a destabilising role leading to the growth of such disturbances on the upper side of the wing, particularly in a negative angle of attack. The growth rate is mainly dictated by the flow configuration, while the excited initial amplitude is dependent on
a multitude of factors. For instance, freestream turbulence, surface roughness characteristics, and acoustic waves along with the receptivity characteristics of the flow determine the initial perturbation amplitude.

Bippes (1999) gives an overview of the fundamentals of the receptivity theory and the possible influential factors. It was concluded that crossflow disturbances exhibit a strong dependency on external disturbances. Effect of freestream turbulence was observed to alter the dominance between stationary and non-stationary crossflow disturbances. Saric et al. (2002) provide an extensive review on the boundary layer receptivity to freestream perturbations. A couple of experiments have been discussed including the experiments by Kendall (1998) where the effect of freestream perturbation on the initial amplitude of Tollmien-Schlichting waves was studied. Three distinctive responses were observed, a streaky high amplitude structure inside the boundary layer, an outer layer oscillation connected to the continuous spectrum of the Orr-Sommerfeld equations, and the classical T-S waves exhibiting higher growth rates. Previously Kendall (1991) established the nonlinear correlation between the freestream perturbations characteristics and the excited T-S wave amplitude.

Numerous experiments have also been conducted to study the problem of boundary layer receptivity to freestream turbulence. In the experiments by Matsubara & Alfredsson (2001), the authors further correlated the streaky structures with transient growth which is augmented by an increase in freestream turbulence levels. The presence of surface roughness can promote such effects. Moreover there exists a conjecture that a continuous receptivity process plays an important role in feeding the boundary layer perturbations in the streamwise direction. Jonáš et al. (2000) investigated the effect of freestream turbulence length scale on a flat-plate bypass transition. Larger integral length scales showed to advance the transition location while keeping the level of turbulent intensity. Fransson et al. (2005) studied the transition caused by freestream turbulence on a flat plate boundary layer. They introduce a transitional Reynolds number inversely proportional to $Tu^2$. Kurian et al. (2011) conducted experiments on a swept flat plate with a leading edge studying the receptivity of the three-dimensional boundary layer to freestream turbulence. They also confirm the previous observations that higher turbulence environments give way to dominance of travelling crossflow waves. Nevertheless, they showed that a linear mechanism prevails in the range of studied freestream turbulence intensities. Moreover, it was observed that above a certain threshold, increasing the turbulence intensities has no tangible effect on the growth of the travelling crossflow modes. Moreover Shahinfar (2013) studied the effect of varying turbulence intensities on boundary layers and correlate the Reynolds number of the transition location with the turbulence intensity.

The problem of swept wing transition has long been under investigation mainly through two large campaigns in Arizona State University and DLR Göttingen. An extensive range of data has been produced addressing different
contributing factors, in receptivity, growth, and breakdown of perturbation in such boundary layers. Recently, Hunt (2011) conducted experiments regarding crossflow instability on swept wing including detailed measurements of surface roughness quality, and freestream turbulence characteristics, such as frequency spectrum. In a continuation of this work, Downs (2012) include additional freestream information comprising of Taylor micro scales and integral length scales. In the study by Hunt (2011), the effect of freestream turbulence at very low levels of freestream turbulence is examined. One counter intuitive observation was the transition delay by slightly increasing turbulence density at that low level range. Downs (2012) covers a wider range in his study in terms of freestream turbulence intensities and length scales. The inclusion of detailed measurements of freestream turbulence length scales and spectrum provides invaluable information to properly quantify the receptivity characteristics of such boundary layers exclusively for numerical reproduction of the experimental conditions.

On the numerical side the problem of freestream turbulence has occupied the minds of researchers for many decades from different aspects. Rogler & Reshotko (1975) and Rogler (1978) studied the role of freestream turbulence on laminar turbulent transition both numerically and experimentally. The freestream turbulence was represented by a low-density array of vortices. Jacobs & Durbin (2001) for the first time followed the methodology proposed by Grosch & Salwen (1978) to synthesize freestream turbulence. Their method allowed to skip over the simulation of the far-field and the leading edge. Later, Brandt et al. (2004) followed a similar method to produce the synthetic turbulence as an inflow boundary condition to study the transition process in a boundary layer. They varied the energy spectrum of the generated synthetic field. Increasing the integral length scale moved the transition location to lower Reynolds numbers. Moreover, two mechanisms were found playing a major role in exciting the perturbations inside the boundary layer. A linear mechanism, the so called lift up effect, is dominant if low-frequency modes diffuse into the boundary layer. On the contrary, if the freestream perturbations are mainly located above the boundary layer a nonlinear process takes over and generates streamwise vortices inside the boundary layer. This method was further applied to a swept flat plate in the simulations by Schrader et al. (2010), where stationary crossflow vortices are generated through roughness elements. They also confirm the results of experiments where a higher turbulent intensity promotes the dominance of travelling crossflow modes. The initial amplitude of the perturbations scales linearly with the level of turbulent intensity. Nevertheless, larger turbulent intensities amplifies the effect of non-linearities.

Ovchinnikov et al. (2008) included the leading edge of a flat plate in their study of receptivity of boundary layers to freestream turbulence perturbations. In their simulation the box was extended to the upstream. They generate the freestream perturbations similar to Jacobs & Durbin (2001) but through including the Fourier modes instead of the Orr-Sommerfeld modes. They notice
In this study the experimental set up by Downs (2012) is considered in order to numerically investigate the effect of freestream turbulence on crossflow dominated flows. Direct numerical simulations are initially performed to extract the characteristics of stationary crossflow vortices. Furthermore, two test cases from the experiment having different levels of freestream turbulence are selected. The perturbation field are generated via direct numerical simulations and then are fed in on top up of the inflow boundary condition and convected downstream with the meanflow. At this stage the method of generating freestream turbulence and the validity of direct numerical simulations are to be examined.

2. Flow configuration

In the experiment a wing model is mounted vertically in the wind tunnel. The incoming flow hits the leading edge at an angle of attack equal to $\alpha = -2.9^\circ$. The wing is mounted at a sweep angle of $\phi_\infty = 45^\circ$ while keeping an infinite span condition. The negative angle of attack along with the design of the wing...
profile ASU(67)-0315 favors the growth of crossflow instabilities. The chord Reynolds number is $Re_C = Q_\infty C/\nu = 2.8 \times 10^6$ which is based on the long swept chord, i.e. $C = c/cos(\phi) = 1.83m$, while $c$, the unswept short chord, is defined in figure 1. The coordinate system for the DNS is chosen to be along the short chord, covering the upper side of the wing where measurements have been performed. The nose radius $r_n$ has been used in order to normalize lengths, and the velocities are normalized by the chordwise velocity of the freestream velocity, $U_\infty$. The dictating Reynolds number is then:

$$Re_{r_n} = \frac{U_\infty r_n}{\nu},$$

(1)

with $\nu$ being the kinematic viscosity of the fluid. The short chord Reynolds number in the experiment is $Re_c = 1.4 \times 10^6$, and in terms of the nose radius is equal to $Re_{r_n} = 14676.1$.

The freestream turbulence measurements have been documented. Note, that low levels of freestream turbulence is under consideration here in order to resemble the free flight conditions. Noticeable discrepancies have been observed in the experiments by Reibert (1996), Hunt (2011), and free flight test of Carpenter et al. (2010), conducted in different environmental conditions. This mainly points to the responsibility of freestream turbulence. To further analyze this problem, the freestream turbulence field is generated using direct numerical simulations (DNS).

The roughness elements were also placed at $x/c = 0.029$ similar to the experiment with a spacing $L_z = 12mm$, where $x$ denotes the chordwise direction (see figure 1). The wavelength of the naturally most unstable stationary cross-flow mode is $\lambda_z = 12mm$. The height and diameter of the elements were chosen as $\varepsilon_r = 12\mu m$ and $d_r = 3mm$ respectively. Three simulations are performed, one without the freestream turbulence, and two including the freestream turbulence with different turbulence length scales and intensities.

3. Direct numerical simulations

Direct numerical simulations were performed using the incompressible Navier-Stokes solver ‘Nek5000’ by Fischer et al. (2008), which uses the spectral element method proposed by Patera (1984). Enabling geometrical flexibility using finite element methods combined with the accuracy provided by spectral methods are the main advantages of using such codes. The spatial discretisation is obtained by decomposing the physical domain into spectral elements. The solution to the Navier-Stokes equations is approximated element-wise as a sum of Lagrange interpolants defined by an orthogonal basis of Legendre polynomials up to degree $N$. The following results have been obtained using $N = 8$. The present SEM code is optimised for MPI based usage on supercomputers with thousands of processors (Tufo & Fischer 2001). Here, we have performed parallel computations on 8196 processors.
3.1. Baseflow

The portion of the wing that is simulated is chosen such that it includes all the interesting phenomena occurring on the wing, as depicted in figure 1. This entails the receptivity mechanism, initial perturbations growth, and the transition to turbulence. The upstream inflow is placed in such a way as to rule out any numerical contamination near the leading edge following the studies by Tempelmann et al. (2012a,b). The upper bound also follows the same recommendations by the mentioned studies. The downstream positions for the outflow is set at $x/c > 0.5$ a bit further than the observed transition location ($x/c > 0.4$) in the experiment at the lowest turbulent intensity. A mesh generator called gridgen-c developed by Sakov (2011) is used to generate the mesh. The upper and lower bounds conform to the streamlines extracted from complementary Reynolds Averaged Navier-Stokes (RANS) computations. A grid refinement is performed between two streamlines regarding the higher receptivity near the leading edge as depicted by figure 2. An optimisation has been performed in order to find a good compromise between the number of elements and element order. A higher element order naturally results in a tougher restriction on the time step. In terms of obtaining the proper resolution, element distribution is more efficient, compared to increasing the element order where the latter increases the resolution in non-essential areas.

In order to validate the computations of the baseflow, in addition to obtaining an optimum mesh resolution, a semi-three-dimensional simulation has been performed where the spanwise velocity component is computed via a passive scalar equation. Dirichlet boundary conditions are introduced at the inflow and the upper and lower bounds of the domain. The corresponding velocities are extracted from the steady baseflow computed by RANS which is obtained by manually prescribing the transition location on the upper side at $x/c \approx 0.7$ and
$x/c \approx 0.01$ on the lower-side. The specification of transition location, enables the RANS to avoid formation of separation bubbles near the pressure minimum locations. Further, we choose zero-slip conditions at the wall while zero-stress boundary conditions are imposed at the outflow of the domain ($x/c = 0.5$). Periodic boundary conditions are also prescribed at the lateral boundaries. A total element number of 30000 elements are used for with a polynomial order of $N = 8$.

3.2. Roughness induced crossflow vortices

The most unstable mode based on the linear stability theory as is reported in the experiment has a spanwise periodicity of 12mm which is excited by an array of cylindrical roughness element. Numerically such periodicity can be achieved by imposing periodic boundary conditions on lateral boundaries of the domain spaced at that specific periodic wavelength. The method has been successfully tested in the studies by Tempelmann et al. (2012b). The roughness shape is formed by displacement of the Gauss Lobatto Legendre (GLL) points normal to the surface, illustrated in figure 3. In the experiment by Downs (2012), the effect of turbulent intensity on the stationary crossflow vortices has been investigated for this configuration along with a number of different levels of turbulent intensity. A sponge region is also used along with the outflow boundary condition ramping the local velocity field to the DNS computed from the semi-three-dimensional DNS. The usage of a sponge, region facilitates evading back-flows near the outflow region. Equation 2 dictates the forcing term present in the sponge region:

$$F(x, t) = A_{\text{max}} \lambda(x)[U_f(x) - U(X, t)],$$

(2)

where $A_{\text{max}}$ determines the maximum strength of sponge region, and $\lambda_f$ is a step function (cf. Tempelmann et al. 2012b). $U_f$ denotes the velocity field extracted from the semi-three-dimensional simulation. In other words, $F(x, t)$ is proportional to the instantaneous local velocity. In our set up a maximum strength of 2 has proven to be sufficient.

The parabolised stability equations introduced by Simen (1992) and Herbert (1997) are common tools in stability analysis of convective unstable flows. An initial condition needs to be provided for such methods to predict the evolution of perturbations. A detailed derivation of the linear and nonlinear method can be found in Bertolotti et al. (1992), Hanifi et al. (1994), and Herbert (1997). In this study the initial condition, form DNS, together with the baseflow is provided to the NOLOT code (cf. Hein et al. 2000), to validate the evolution of the perturbations in order to establish an adequate mesh resolution.

3.3. Freestream turbulence

Different methods have long existed for generating freestream isotropic turbulence. One typical tool of generating freestream turbulence is through the
Fourier periodic codes. One important aspect of using such codes is the continuous injection of energy into the periodic box. This maintains the spectrum as long as the energy injection rate and dissipation rate cancel out. Shlatter (2005) gives an overview of such methods, and present relevant results for different cases. A very low level of freestream turbulence as is reported in the experiment, results in a very low Taylor’s micro-scale Reynolds number \( Re_\lambda \). In other words the spectrum can be very viscous leaving a very narrow band for energy injection. For studies regarding such spectrums refer to (Kerr 1985; Mansour & Wray 1994; Burattini et al. 2006; Ishihara et al. 2009).

Initially in our study, a periodic Fourier code developed by Shlatter (2005) was used. Figure 4a shows isosurfaces of a velocity component within the periodic box. The presence of large structures is apparent in the figure indicating the fact that the transfer of energy in this viscous spectrum occurs very fast. Evolution of such structures is barely noticeable in time. In other words, the frozen modes are forced in the box and overall a very slow evolving flow remains which is somewhat unphysical. Figure 4 depicts the energy and \( Re_\lambda \) for a sample simulation. The energy level and \( Re_\lambda \) have a good convergence. This has been achieved by a periodic box of \( 128^3 \) points. Note that a simple forcing method has been used, whereby a certain number of modes stay frozen while the energy is transferred to the smaller scales.

An alternative approach is to use a box with an inflow superposed by random noise accompanied by periodic boundary conditions on the side walls with an outflow. The amplitude and Reynolds number are set such that the
required Taylor’s micro-scale and Reynolds number is achieved. Taylor’s micro-scale is computed by using:

$$\lambda = \sqrt[5]{\frac{E_{tot}}{E_{ns}}}$$

(3)

where $E_{tot}$ is the calculated integral energy at each streamwise plane, while $E_{ns}$ is the equivalent enstrophy value at those planes. The Taylor’s Reynolds number is defined by:

$$Re_\lambda = \sqrt{\frac{2}{3E_{tot}}} \lambda Re,$$

(4)

with $Re$ denoting the Reynolds number of the simulation. The turbulent intensity is defined based on the r.m.s value of the vertical velocity component. The resulting perturbation field is then scaled onto the wing mesh, and interpolated in time using a third order Lagrange interpolant.

4. Results

4.1. Baseflow

Figure 5a shows a comparison between the pressure coefficients computed from the RANS and DNS. A good agreement can be seen between the two solutions, nevertheless a small difference can be observed particularly further downstream. This can be explained through the RANS set up where as mentioned earlier a transition location is prescribed in order to avoid formation of separation bubbles. The absence of this assumption can slightly elevate the pressure coefficient in DNS. Similar observation was made in the numerical study by Tempelmann et al. (2012b). Moreover, the authors in that study report on the low level of
sensitivity of receptivity coefficient to the pressure coefficient. A complementary boundary layer computation using the code 'bl3D', has been performed to examine the sufficient mesh resolution in the boundary layer. Extracted velocity profiles from the boundary layer equation solver are shown in figure 5b indicating a well resolved boundary layer.

4.2. Stationary perturbations

Figure 6 illustrates the isosurfaces of spanwise velocity component. The signature of crossflow vortices manifesting themselves in the form of growing vortices can be seen in red. Figures 7a,b show the amplitudes of the stationary modes $\beta_0$ and $2\beta_0$ where the amplitude is defined as:

$$A_u = \max_{\eta} \frac{|\hat{u}_\xi|}{\bar{U}_{\xi,e}},$$  \hspace{1cm} (5)

where a hat denotes a spatial r.m.s. amplitude with respect to $z$ of the individual modal disturbances and $\bar{U}_{\xi,e}$ represents the boundary layer edge velocity. A comparison with PSE is also presented in the figure which shows a good agreement for this mode. Additionally, the good agreement ensures a sufficient resolution of the mesh for stationary crossflow vortices.

5. Unsteady disturbances

As mentioned earlier three levels of freestream turbulence intensities have been investigated in the experiment. In our set up we aim at finding the effect of freestream turbulence considering two of the cases. Turbulence intensities of 0.05% and 0.019% are chosen. There exist a difference in the Taylor Reynolds number which are selected as 1 and 10 respectively. This was due to the trial
Figure 6. Visualization of stationary crossflow vortices. The isosurfaces represent the spanwise velocity ($W$). Four spanwise periods are shown for visualization purposes. The location of the roughness elements can be seen near the leading edge.

The box has a total number of 90000 elements with an element order of ($N = 12$). A higher element order ($N = 13$) is also used in order to check the grid dependency. Figure 9a shows the comparison of the maximum amplitude of the streamwise perturbation velocity component for the cases with different polynomial orders.
Figure 7. Amplitude $A_u$ of the stationary modes in the absence of unsteady perturbations for (a) fundamental mode $\beta_0$ and (b) $2\beta_0$.

Figure 8. Isosurfaces of vertical velocity perturbations $v'$, drawn at $\pm 0.0001|v'|$.

A good level of agreement can be observed between the maximum perturbation amplitude. The same trend can be seen between the energy levels as depicted in figure 9b ensuring sufficient resolution.
Table 1. Selected cases with different turbulent characteristics.

<table>
<thead>
<tr>
<th>Case Name</th>
<th>Turbulent intensity</th>
<th>Integral length scale</th>
<th>Taylor’s micro scale</th>
<th>$Re_\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tu5</td>
<td>0.05%</td>
<td>48mm</td>
<td>1.6mm</td>
<td>1</td>
</tr>
<tr>
<td>Tu19</td>
<td>0.19%</td>
<td>36mm</td>
<td>4.5mm</td>
<td>10</td>
</tr>
</tbody>
</table>

Figure 9. (a) Maximum instantaneous streamwise perturbation ($u_{max}$) along the streamwise direction (b) Computed energy along the streamwise direction for two mesh resolutions.

Figure 10. (a) Instantaneous components of velocity perturbation within the periodic box (b) Computed energy, enstrophy from streamwise perturbation.

To determine the level of isotropy in the generated turbulence field three velocity components are depicted in figure 10 for the set up case Tu5. A good level of isotropy can be seen among the different velocity components. A quick look at the velocity isosurfaces reveals the way the perturbations and vorticity decay along the streamwise direction. A linear trend can be seen
which is similar to what is observed in the experiments. Figure 10b shows the computed instantaneous enstrophy and energy at each plane along the streamwise direction. The computed values are then used in order to compute Taylor’s Reynolds number, $Re_\lambda$, see figure 11. The computed flow is now scaled onto the grid of the wing geometry. Note the spanwise length is chosen such as to accommodate the integral length scale of the considered case. Furthermore a third order Lagrange interpolation in time is performed among the previously saved freestream perturbations.

Figure 12 depicts the streamwise velocity perturbation field belonging to the two studied cases. The dominance of the crossflow vortices can be seen near the outflow. No transition has been observed for the T5 case. Effect of different turbulent perturbations on amplitudes of the stationary crossflow vortices has been compared in figure 13. An apparent damping of the stationary crossflow vortices can be seen with introduction of freestream turbulence for Tu5 case for
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Figure 13. Amplitude evolution of steady disturbances depicted for the three studied case (a) fundamental mode $\beta_0$ and (b) $2\beta_0$.

the primary $\beta_0(12mm)$ mode. It must also be noted that in none of the cases the excited initial amplitudes by the roughness element shows any tangible difference. For the stationary $2\beta_0$ the difference in amplitudes is not as strong as the $\beta_0$ mode.

6. Conclusions & Outlook

Direct numerical simulations (DNS) have been performed in order to investigate the role of freestream perturbations at a very low turbulence level on crossflow instability. The studied cases follow the experiments conducted by Downs et al. in Texas A&M University. In their experiment the authors document the freestream perturbations to a great detail, reporting freestream turbulence length scales, intensity, spectrum, etc. This enables the numerical studies to fully reproduce the freestream perturbations in such analysis. The experiment used ASU(67)-0315 wing geometry designed to promote crossflow instability. In our study we approach the reported values in generating the low intensity freestream turbulence. A DNS code (nek5000) has been used in order to generate the perturbation field. The perturbations are then scaled and interpolated onto the wing mesh.

Two different set of freestream turbulence characteristics have been chosen. So far in the simulations, it has been observed that the growth rate of the stationary crossflow modes assumes a lower value in proportion to the freestream turbulence intensity level. However the results are so far preliminary and part of an ongoing work and further convergence of the simulated cases are required. Additional parameters must also be looked at, such as the spectrum of the perturbations, secondary instabilities, etc. It must be noted that transition has not been observed in any of the case with freestream turbulence.

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