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Glow corona generation and streamer inception at the tip of grounded objects during thunderstorms: revisited

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Abstract
The initiation of streamers prior to a lightning strike can be reportedly inhibited by glow corona discharges generated from tall objects. In contrast to previous studies based on a simplified one-dimensional model of glow corona, a two-dimensional evaluation of the corona ion drift from tall objects is used here to analyse this effect quantitatively. Proper estimates for the corona space charge distribution generated during both the charging process of a thundercloud and the descent of the downward stepped leader are thus calculated. It is found that the shielding effect of the corona space charge on the streamer inception is not as severe as previously reported. Estimations of the effective height of the downward leader tip at which streamer inception takes place are presented and discussed for lightning rods and dissipation array systems.

1. Introduction
During the formation of a negatively charged thundercloud, the background electric field close to the ground slowly increases from the fair weather field (about 100 V m$^{-1}$), up to a few tens of kilovolts per metre [1]. As the background electric field intensifies, point corona discharges are initiated from the tips of sharp objects on the ground. It is known from laboratory experiments that corona discharges from sharp objects occur in two different forms: as a thin luminous glow layer and as bright, filamentary streamer channels [2]. The former, known as a glow corona, appears as a thin bright plasma, with a continuous current. The latter, called a streamer corona, is characterized by sequences of filamentary ionized channels, which produce a pulsating current. While glow coronas are generally formed when slowly changing electric fields are applied to a sharp electrode, streamers are usually associated with rapidly changing electric fields [2].

During thunderstorms glow corona discharges readily emanate from the tips of tall, slender, grounded objects. However, corona discharges can also be produced at irregularities on the ground surface such as grass, bushes, etc [1]. These discharges generate positive ions that move upwards under the influence of the background thundercloud electric field. If sufficiently abundant, these ions may create a thick space charge layer around the object, which can significantly shield the background electric field [1]. This shielding effect is produced by the electric field of opposite polarity produced from the corona space charge layer, which superimposes onto the background electric field.

Under certain conditions, a discharge that initiates a cloud-to-ground lightning flash, called a stepped leader, can form inside a negatively charged thundercloud [4]. As the stepped leader propagates downwards from the cloud base, it induces an increasing electric field close to the ground, which superimposes onto the thundercloud electric field. If the total background electric field produced by these two fields is sufficiently strong, streamer corona discharges may emanate from the tips of tall, nearby objects. In some cases, in a sufficiently non-uniform electric field, streamers may be launched from the already existing glow corona [2]. Once an initial burst of streamers is formed, new streamers can be...
subsequently produced when the electric field conditions are restored as the downward leader continues approaching the ground.

A streamer-to-leader transition occurs when the charge of a streamer corona burst is sufficiently large to thermalize its stem. When this occurs the newly created leader channel propagates upwards, powered by streamers generated at its tip. This upward positive leader discharge advances steadily towards the descending, stepped leader. If the upward leader discharge successfully intercepts the stepped leader, a conductive path is created through which the high lightning current is drained to the ground: a lightning strike occurs.

Since the lightning attachment process may be preceded by glow corona discharges, it is of practical interest to evaluate their effect on the temporal evolution of the subsequent discharges (i.e. streamers and leaders). Extensive theoretical research on glow corona discharges from tall objects, and their effect on the initiation of streamers and upward leaders, has been published in the literature in recent years [5–13]. However, in all of these studies the analysis has been based on very simple models of the glow corona and the leader discharges.

According to the above-cited studies, the space charge produced by a glow corona redistributes the electric field near the tips of tall objects. Consequently, the production of streamers is delayed, which can inhibit the inception and propagation of upward positive leaders. Based on this conclusion, it was suggested that the glow corona might significantly reduce the efficiency of tall objects to intercept lightning flashes. Consequently, it was suggested that glow corona discharges might be practically utilized in the design of an unconventional lightning protection system [6, 11–13]. Such a system, usually known as a dissipation array, uses a uniform, umbrella-shaped array of needles, capable of generating a massive corona space charge [6, 13]. Recently, another device capable of generating a corona space charge was introduced [14], which was claimed to provide superior lightning protection, based on an analytical approach similar to that in [5], together with results from high-voltage laboratory tests.

Since the efficacy of a glow corona for controlling lightning strikes is still under debate, an independent and thorough analysis of this phenomenon is required. Due to continuous improvements in computer power and numerical algorithms, it is now possible to model corona production from tall objects without some of the constraints implemented in previous studies. In addition, the conditions required for the initiation and propagation of upward positive leaders are now better understood [15]. Therefore, it is now possible to assess the effect of glow coronas on the complete lightning attachment process much more robustly.

This paper intends to carefully revisit the theoretical analysis of glow corona production from tall objects and to evaluate its effect on the initiation of streamers. For this, a two-dimensional analysis of the corona space charge is performed with a commercial finite element method (FEM) multiphysics software [16]. In order to validate the calculations, the model is first implemented to evaluate corona generation in a laboratory long air gap. Next, the model is applied to evaluate glow corona generation from grounded lightning rods and a dissipation array system (DAS), taking into account both the charging of a simulated thundercloud and the approach of a downward stepped leader. Estimates obtained from the simplified models used in the literature [5–13] and the present model are presented, compared and discussed, focusing particularly on the effect of the injected corona space charge on streamer initiation. The influence of the glow corona on the initiation and propagation of subsequent upward leader discharges will be addressed in a future paper.

2. Previous theoretical studies

The first theoretical studies on the glow corona emitted from tall objects were performed using a simple analytical model [5], which assumes that the corona space charge generated from a tall object could be studied with an equivalent concentric spherical shell configuration. The corona generated at the tips of tall objects thus retains a hemispherical shape as the ions drift in the background electric field, as shown in figure 1. This assumption (hereinafter referred to as the 1D approximation) reduces the analysis of the corona ion drift to one dimension, allowing the derivation of simplified analytical expressions. The corona discharge process was modelled as the injection of monopolar space charge by a hemispherically capped rod. The amount of charge injected was considered to be such that the electric field on the rod cap surface remains equal to the corona onset electric field \( E_{\text{cor}} \) (i.e. Kaptzov’s assumption was employed [3]).
The analytical analysis introduced in [5] was later complemented with a numerical simulation [6–13], which allowed the inclusion of a large ion drift, resulting from the attachment of small ions to aerosol neutrals. However, the 1D approximation was still maintained, due to the considerably lower effort required to analyse glow corona production. As the numerical simulation of the corona space charge must be performed in at least two dimensions, it was argued that such a calculation is computationally expensive and therefore impractical compared with the simpler 1D analysis [5]. Therefore, it is important to consider the following conditions under which the 1D approximation is valid:

(a) The electric field normal to the cap surface of a tall object is constant and defined by the electric field at the cap tip (A in figure 1). Therefore, a tall object emits corona ions with the same intensity over the entire cap surface.

(b) The electric field at some point in front of the object’s tip depends only on the distance from the centre of its hemispherical cap. This means that the direction of the electric field vector at any point in the air gap is parallel to the radius vector drawn from the cap’s centre.

(c) The drift velocity of the corona ions can be defined as an average value computed from the electric field along the axis of symmetry.

(d) The effect of the electric charges distributed over the body of a tall object is negligible. That is, the object being analysed is considered to be equivalent to a sphere connected to ground through a negligibly thin wire.

(e) Streamers can only be generated in front of the object’s tip (A in figure 1). No streamers can be generated from the cap or body of the object.

Although conditions (a), (b) and (c) are only valid for a concentric spherical shell geometry, it was explicitly assumed that violations of those conditions in regions further from the object’s tip are of minor importance [5]. Condition (d) was assumed due to the simplicity of representing a tall object by a single point charge and a set of line charges [5–13]. Directly following from this rough approximation, the inception of streamers could only be evaluated in front of the object’s tip—condition (e). The 1D approximation presented in [5–13] was verified with analytical expressions obtained for a coaxial spherical shell configuration [5]. However, it has not been validated for configurations relevant to the problem of glow corona generation from grounded objects under thunderstorms.

The results of the simulations presented in [5–13] suggest that the glow corona space charge cloud generated around the tip of a tall object under a typical thundercloud electric field can reach a radius of dozens of metres in about 10 s. As a result, the computed hemispherical space charge cloud was found to smoothen the spatial distribution of the electric field sufficiently to prevent the initiation of streamers during the slow charging process of the thundercloud. Therefore, the creation of streamers was considered to be possible only if the background electric field rapidly increased [5].

3. A two-dimensional corona ion drift model

Here, the one-dimensional model proposed by Aleksandrov et al [5–13] is extended to two dimensions. To do so, three convection/diffusion modules and an ac/dc module of COMSOL Multiphysics [16] are used to solve the continuity equations for small ions \( n_+ \), large aerosol ions \( N_+ \) and aerosol neutrals \( N_0 \):

\[
\frac{\partial n_+}{\partial t} = D \cdot \nabla^2 n_+ - \nabla \cdot (n_+ \cdot \mu_{n_+} \cdot \vec{E}) - k_{nN} \cdot n_+ \cdot N_0 \quad (1)
\]

\[
\frac{\partial N_+}{\partial t} = D \cdot \nabla^2 N_+ - \nabla \cdot (N_+ \cdot \mu_{N_+} \cdot \vec{E}) + k_{aN} \cdot n_+ \cdot N_0 \quad (2)
\]

\[
\frac{\partial N_0}{\partial t} = D \cdot \nabla^2 N_0 - k_{nN} \cdot n_+ \cdot N_0 \quad (3)
\]

together with the Poisson equation for electric field \( \vec{E} \) and potential \( \Phi \):

\[
\nabla \cdot \vec{E} = -\nabla^2 \Phi = \frac{e \cdot (n_+ + N_+)}{\varepsilon_0} \quad (4)
\]

where \( \mu_{n_+} \) and \( \mu_{N_+} \) are the mobilities for small ions and large aerosol ions, respectively, \( k_{nN} \) is the attachment coefficient of small ions to aerosol particles, \( D \) is the diffusion coefficient, \( e \) is the elementary charge and \( \varepsilon_0 \) is the vacuum permittivity.

Although the corona space charge drift in a thunderstorm is in fact a three-dimensional problem, the calculations can be reduced to two dimensions if asymmetric horizontal convection of the generated ions is neglected. This simplification is valid when there is no air convection due to wind and the horizontal driving force due to the electric field (produced by a thundercloud and a downward stepped leader directly above the tall object) is rotationally symmetric. Under these conditions, the problem can be reduced to a two-dimensional, axial-symmetric coordinate system. Thus, equations (1)–(3) can be rewritten according to the generic form of the convection/diffusion equation in COMSOL Multiphysics [2], as follows:

\[
\frac{\partial n_+}{\partial t} + \nabla (-D \nabla n_+) = \left(-n_+ \cdot \mu_{n_+} \cdot \frac{e \cdot (n_+ + N_+)}{\varepsilon_0} - k_{nN} \cdot n_+ \cdot N_0 \right) - (\mu_{n_+} \cdot \vec{E}) \cdot \nabla n_+ \quad (5)
\]

\[
\frac{\partial N_+}{\partial t} + \nabla (-D \nabla N_+) = \left(-N_+ \cdot \mu_{N_+} \cdot \frac{e \cdot (n_+ + N_+)}{\varepsilon_0} + k_{aN} \cdot n_+ \cdot N_0 \right) - (\mu_{N_+} \cdot \vec{E}) \cdot \nabla N_+ \quad (6)
\]

\[
\frac{\partial N_0}{\partial t} + \nabla (-D \nabla N_0) = (-k_{nN} \cdot n_+ \cdot N_0) \quad (7)
\]

The mobilities are taken as \( \mu_{n_+} = 1.5 \times 10^{-4} \text{ m}^2 \text{ s}^{-1} \text{ V}^{-1} \) for small positive ions and \( \mu_{N_+} = 1.5 \times 10^{-6} \text{ m}^2 \text{ s}^{-1} \text{ V}^{-1} \) for large positive ions. The rate of conversion of small ions into aerosols \( k_{nN} \) is assumed to be equal to \( 2.9 \times 10^{-12} \text{ m}^3 \text{ s}^{-1} \), while the
The diffusion coefficient $D$ is chosen as $1\, \text{m}^2\, \text{s}^{-1}$. The values used for these variables are taken from [17].

Figure 2 shows an example of the geometry considered for the 2D axisymmetric model. Note that a subdomain $A$ is used to describe the region where space charge drifts within the simulation time interval, while the remaining geometry is defined by a second subdomain $B$. By symmetry, only half the geometry needs to be considered. To reduce the number of mesh points while maintaining the accuracy of the calculations, a different meshing is used for the two subdomains. Subdomain $A$ has a mapped mesh distributed exponentially along the longitudinal edges. The mesh element size increases from values of the order of $10^{-5}$ m close to the rod’s tip, up to a maximum of 0.05 m far from the tip. A uniform linear mapping with 30 edge elements is used for the transverse edges of this subdomain. A free meshing of normal predefined size is used for subdomain $B$. A maximum edge element size of 0.006 m is used for the upper 2 m of the rod’s body.

Direct comparison of the electric fields calculated within the 1D approximation [5–13] with those of the 2D model presented here is not possible, as the former are based on a very rough electric field calculation (as described in section 2). For this reason, a second model is implemented based on the 1D approximation, utilizing a more accurate FEM electric field calculation. For this, the 2D model described above is modified such that the expanding space charge cloud is forced to retain a hemispherical shape as it drifts in the gap, as assumed in the 1D approximation (figure 1). In this situation then, subdomain $A$ is considered to be a hemisphere around the rod’s tip. To reproduce the radial space charge movement, the electric field along the axis of symmetry is projected radially into the entire subdomain $A$. For this purpose, a projection coupling variable is instantiated within the subdomain, which is used in the ion velocity evaluation. The sole driving force in the calculation is then the electric field along the axis of symmetry, as is the case in the 1D approximation [5–13].

The boundary conditions used are as follows. For the electrostatic module, a potential $V_{\text{plane}} = E_{\text{back}} \cdot H$ is applied to the upper plane boundary, where $E_{\text{back}}$ is the total, time-dependent background electric field and $H$ is the height of the plane. The left vertical boundary is set as the axis of symmetry while the right vertical boundary is defined as an electric insulator. The other boundaries corresponding to the ground and the rod surface are set to the ground potential.

For the convection/diffusion module, the upper horizontal boundary is set as a convection edge and the remaining boundaries (except for the rod surface boundary) are considered as zero flux boundaries. The surface of the rod where the local electric field is equal to or larger than the onset corona field $E_{\text{cor}}$ is defined as a concentration boundary, generating a corona according to Kaptzov’s assumption. Thus, the thickness of the ionization layer is neglected, and the density of small corona ions at this boundary $n_{e}^{\text{rod}}$ is defined such that the electric field on the surface $E_{\text{rod}}$ remains constant and equal to $E_{\text{cor}}$ [2, 3, 5–13]. Note that even though a rigorous analysis of the glow corona requires detailed simulation of the charged particles in the anode fall zone (as in [18], for instance), it is not practical (or even possible) to perform such a quantitative analysis with current computer technology due to the prohibitively large time scales involved in lightning studies. Instead, Kaptzov’s assumption is used here, which has support from both simulations [18] and experiments [2]. Moreover, as shown in the following sections, the use of Kaptzov’s assumption allows us to obtain useful, quantitative estimates of the glow corona for the problem addressed, with only moderate computational effort. Thus, Kaptzov’s approximation is implemented with a global constraint equation, where the unknown quantity $n_{e}^{\text{rod}}$ is added to the system such that

$$E_{\text{rod}}(t) - E_{\text{cor}} = 0$$

is satisfied. For hemispherically capped lightning rods, the electric field for the onset of glow corona $E_{\text{cor}}$ is defined as a function of the tip radius $r_{\text{tip}}$, following Peek’s law [19]:

$$E_{\text{cor}} = E_{0} \cdot \delta \cdot \left(1 + \frac{c_{1}}{\sqrt{r_{\text{tip}}}}\right)$$

where $\delta$ is the relative air density, and $E_{0}$ and $c_{1}$ are empirical constants equal to $2.7 \times 10^{6}$ V m$^{-1}$ and 0.054 m$^{0.5}$, respectively. Due to the time dependence of the background electric field (resulting from the thundercloud charging and the downward leader’s propagation), the extension of the corona-emitting area on the rod’s surface is updated periodically, in order to maintain its electric field within the range $E_{\text{cor}} \pm 5\%$ (according to (8)). As for the initial conditions, a constant concentration of aerosol neutrals $N_{e}$ equal to $10^{11}$ particles m$^{-3}$ is assumed in all subdomains [6]. The initial concentration of small ions is assumed to be zero. The direct UMFPACK iterative solver is used.

It should be noted that the implementation of equations (5)–(8) for both the 2D calculation and the 1D approximation was first crosschecked for the coaxial spherical...
Comparison of the stationary corona current of a 1 m tall pencil-shaped rod computed with the 2D model and measured in [20].

Unsurprisingly, the calculated corona space charge density, and the gap electric field, agreed with the analytical expressions derived in [5]. On the other hand, it is important to bear in mind that only the glow corona is simulated in the present model. Although pre-onset streamers under slowly charging electric fields are produced first, once the ionization threshold is reached [3], they are quickly displaced by the glow corona in electrodes (as those studied here) with a large divergence of the electric field at the tip surface [2]. For this reason, the presence of pre-onset streamers is disregarded.

4. Validation of the corona ion drift model

In this section, we compare results obtained from the 2D corona drift model with published data [20], obtained from experiments using a 1 m tall pencil-shaped rod with a diameter of 25 mm, a conical tip length of 70 mm and a hemispherical cap (1 mm radius). The rod was placed in a quasi-uniform electric field, produced by an overhead electrode energized with a dc voltage and a conductive ground. The corona current generated from the rod was then measured as a function of the effective background electric field. This effective background electric field, defined by the ratio between the voltage applied to the upper plate and the distance between the plate and ground, ranged between 20 and 60 kV m$^{-1}$ [20]. For comparison with these results, properties of the corona generated from a rod in a 5 m long air gap (i.e. 5 m between the rod tip and the upper plate) were calculated in the steady state, using the 2D model described in the previous section.

Figure 3 displays the computed and measured stationary corona currents as a function of the background electric field. Fairly good agreement between the measured and calculated corona currents is obtained. At this point, it should be noted that there is a significant difference between the glow corona produced in the laboratory under dc voltages, and that produced during a thunderstorm. The corona discharges in the laboratory quickly reach the stationary condition, as opposed to corona discharges occurring under natural conditions [9]. Since the ions generated from a rod in the laboratory can rapidly cover the complete gap between the electrode and the upper plate, the corona current reaches a steady-state condition soon after the electric field is applied. This is not the case for corona ions generated from a tall object during a thunderstorm. In this case, the discharge is non-stationary, as the ions can cover only a small fraction of the air gap between the object tip and the thundercloud [9]. Nevertheless, the fundamental physics of corona production and ion drift is identical in both cases. Therefore, comparison with laboratory measurements [20] is appropriate for assessing the adequacy of the 2D model for representing corona production from slender objects in an electric field.

5. Glow corona generation and streamer inception under natural conditions: a case study

In this section, the corona generated from a 60 m tall rod with a cap radius of 0.02 m is analysed, to identify the most important features of glow corona production and its effect on the initiation of streamers during a thunderstorm. Due to the differences between the rates of change of the background electric field during the charging process of a thundercloud and the descent of a downward stepped leader, the analysis is divided into two stages. Furthermore, the calculations are performed with the 1D approximation as well as with the 2D model, in order to highlight the differences between these two approaches.

5.1. The thundercloud charging process

The background electric field produced during the formation of charged regions inside a thundercloud is believed to increase from the fair weather field (of the order of hundreds of volts per metre) to several tens of kilovolts per metre in about 10–30 s [13]. After this, the thundercloud electric field is approximately constant for several minutes, until a lightning discharge occurs.

In this paper, the background electric field is assumed to increase linearly up to 20 kV m$^{-1}$ in 10 s, and then remains constant. Figure 4 shows the estimated corona current under...
Figure 5. Contour plots of the density of small ions (per cubic metre) produced by the corona from a 60 m tall rod computed (a) with the 2D model and (b) with the 1D approximation. The results correspond to the electric field shown in figure 5 at time $t = 10\, \text{s}$.

Figure 6. Computed percentage of the cap area of a 60 m tall rod, with 0.02 m tip radius, that generates a corona during the thundercloud charging process and descent of a downward stepped leader, as a function of time.

the influence of the thundercloud electric field. While the thundercloud electric field increases, the corona currents computed with the 1D approximation reach considerably larger values than those estimated with the 2D model. However, the corona current computed with the 1D approximation decreases rapidly when the background electric field remains constant, reaching considerably lower values than those computed with the 2D model.

Details of the spatial distribution of the corona space charge for both the 1D and 2D calculations are shown in figure 5, as contour plots of the density of small ions when the background electric field reaches its maximum value at $t = 10\, \text{s}$. Interestingly, the charge density profiles predicted with both models along the axis of symmetry are fairly similar. This indicates that the space charge conditions and electric field along the axis of symmetry (i.e. in front of the rod tip) are not significantly affected by the assumptions underlying the 1D approximation.

However, significant differences between the 1D and 2D approximations appear away from the symmetry axis. Firstly, the corona space charge does not naturally propagate in the radial direction as much in the latter as that enforced within the 1D approximation. In the considered case, the radial displacement of small ions estimated with the 2D model does not reach more than 26 m (while the assumptions of the 1D approximation would force it to reach 46 m). Secondly, the effective generation of the glow corona, estimated with the 2D model, does not take place over the entire surface of the rod’s cap, contrary to assumption (a) (section 2) of the 1D approximation. As shown in figure 6, the section of the cap generating the corona varies in time as the thundercloud field increases, reaching a maximum value of 65% in the particular case considered here, at 10 s. Note that the remaining areas of the cap surface do not generate a space charge, since the electric field does not reach the corona inception field $E_{\text{cor}}$.

Since the 1D approximation enforces the presence of space charge in areas where ions do not drift naturally, and, at the same time, overestimates the effective corona-generating surface, it overestimates the total space charge in the air gap. In the case considered, the total corona charge computed with the 1D approximation is 1.2 mC; 40% larger than the 0.85 mC computed with the 2D model.

The effect of this charge excess estimated with the 1D approximation on the computed corona current depends strongly on the rate of change of the background electric field (as can be seen in figure 4). When the background electric field increases with time, the larger corona-generating area assumed by the 1D approximation leads to a higher estimated current than the 2D model. Nevertheless, if the background electric field rate of change is small, or zero, the corona current injection estimated by the 1D approximation rapidly decreases, due to the larger shielding effect of the overestimated corona space charge.
The spatial distribution of the space charge computed with the 2D model also shows that in the presence of the complete corona space charge streamers cannot be produced under the slowly varying electric fields typically generated during the thundercloud charging process. Due to the continuous generation of glow corona, the electric field around the rod cap is smoothed and maintained at values lower than the inception field $E_{\text{inc}}$. Since streamers require local electric fields significantly stronger than those required to maintain a glow discharge [3], it is confirmed that streamers cannot emanate from a tall structure under the thundercloud electric field alone, when (as the 1D approximation predicts) the space charge generated by the glow corona remains (i.e. it is not displaced by wind or other means). For this reason, it has been suggested that the inception of streamers from a tall object can occur only if the background electric field increases rapidly [5], for instance via neutralization of a thundercloud charge packet by a remote lightning discharge (i.e. a distant cloud-to-ground flash or an intra-cloud discharge) [4], or by the approach of a nearby downward stepped leader.

Interestingly, recent measurements have shown that positive leaders (and therefore streamers) can also emanate from corona-generating tall objects (e.g. towers and windmills) under thunderclouds without any nearby preceding activity [21, 22]. This indicates that the space charge generated by the glow corona can be diminished under thunderclouds (e.g. by wind or the rotation of a wind turbine blade), allowing the production of streamers. Note that the small ions that have drifted a few metres away from the rod tip have a vertical velocity of less than 8 m s$^{-1}$, comparable to the horizontal velocity of moderate winds, or with the tip speed of a standard wind turbine blade. This suggests that the space charge in areas affected by such perturbations can be efficiently displaced laterally, thus distorting the symmetry. Hence, the effective shielding of these charges at the rod tip may depend strongly on externally induced variations of the lateral convection.

Similar effects have been observed in classical rocket-triggered lightning experiments [5], in which streamers are produced by the tip of an ascending rocket towing a grounded wire under a thundercloud. If the rocket has an upward velocity greater than about 100 m s$^{-1}$ [23], the glow corona space charge trails behind it, so that streamer production is uninhibited. Therefore, experimental observations clearly show that estimates of the effect of the space charge generated by the glow corona presented in this paper should be regarded as upper bounds. The reason for this is that the 2D model presented in section 3 does not consider the convection of ions (e.g. by wind or movement of the object). On the other hand, the lower bound corresponds to a case where the corona space charge is efficiently removed from the object, and thus has a negligible effect on the lightning attachment process.

5.2. Approach of the descending stepped leader

As already mentioned, an approaching downward stepped leader induces rapid variation of the background electric field. In order to estimate the contribution to the total background electric field of the descending stepped leader, its channel is modelled by a non-uniform vertical line charge. The downward leader charge distribution is calculated as a function of both the prospective return stroke current peak (taken to be 30 kA) and the height of the downward leader tip above the ground $z_{\text{down}}$ [15]. The stepped leader is assumed to propagate from the cloud base ($z_{\text{down}} = 4000$ m) towards the ground, with an average velocity of $2 \times 10^5$ m s$^{-1}$ [4]. The initial conditions for the corona calculation are taken from the previous stage, when the thundercloud electric field reached 20 kV m$^{-1}$ (at $t = 10$ s, as shown in figure 4).

Figure 7 shows the corona current and the average background electric field produced by an approaching stepped leader. Unsurprisingly, the corona current rapidly increases as the background electric field increases. Note that the corona current increases by more than an order of magnitude, even when the background electric field produced by the downward leader is of the same order of magnitude as the thundercloud electric field. This substantial increase in the corona current is primarily driven by the rapid rate of change of the total background electric field, caused by the downward propagating leader.

Due to the short duration of this phase, the corona ions that are injected from the tip cannot drift far during the rapid approach of the stepped leader, as can be seen in figure 8. For instance, the density of small ions along the axis of symmetry varies significantly only within 0.2 m of the rod tip when $z_{\text{down}} = 1140$ m (compared with the initial case when the downward leader starts at the cloud base $z_{\text{down}} = 4000$ m). About 3 ms later, when the downward leader tip is approximately 530 m above the ground, the corona ions drift further into the gap, although not more than 0.6 m from the rod tip. However, the density of small ions at the rod surface (i.e. at $z = 0$) increases to as much as an order of magnitude greater than the density in the initial case. Consequently, a considerable amount of space charge accumulates in the close proximity to the rod tip as the downward leader approaches the ground. This dense accumulation of charge strongly shields the spatial electric field distribution in front of the rod tip, as previously reported [5–13].
Figure 8. Small ion concentration estimated with the 2D model along the axis of symmetry at the start of the downward leader propagation and when the downward leader tip reaches 1460 and 530 m above the ground.

Figure 9. Electric field computed with the 2D model along the axis of symmetry at the start of the downward leader propagation and when the downward leader tip reaches 1460 and 530 m above the ground.

The spatial variation of the electric field along the axis of symmetry at three steps during the downward leader approach is illustrated in Figure 9. Since the electric field strength at the rod tip is maintained at the value of the corona onset electric field $E_{\text{cor}}$ (about 3.5 MV m$^{-1}$ in this case), the electric field in front of the tip increases as the total background electric field rises rapidly and space charge accumulates. As the downward leader continues its approach, there is a critical height of its tip ($z_{\text{down}} = 530$ m, in this case) at which the spatial distribution of the axial electric field becomes non-monotonic (Figure 9). Hence, the electric field maximum along the axis of symmetry shifts away from the rod surface, into the gap, and exceeds $E_{\text{cor}}$. This condition has been interpreted as the glow-to-streamer transition (hereafter referred to as the axial streamer inception criterion) [5–13]. For the case considered here, the 1D approximation indicates that this condition occurs when the downward leader tip is 440 m above the ground; significantly lower than the height estimated with the 2D model.

As previous studies [5–13] have exclusively analysed the conditions required for streamer inception along the axis of symmetry, it is also relevant to consider here the spatial variation of the electric field across the body of the rod. Figure 10 shows the electric field along the rod profile at indicated heights of the downward leader tip $z_{\text{down}}$, computed with the 2D model. Before the glow corona is produced (in the absence of the downward leader), the Laplacian electric field along the rod profile rapidly decreases. Once the glow corona is initiated, the electric field produced by the background electric field along the rod profile rapidly decreases. While the electric field on the corona-generating areas of the tip is reduced and maintained at the constant value $E_{\text{cor}}$, the corona shielding effect rapidly decreases along the rod profile, as the distance to the space charge drifting region increases. During the approach of the stepped leader, the electric field over the entire surface of the rod body (where the glow corona is not produced) increases due to the increasing total background electric field. The surface electric field on the lower part of the rod body (i.e. about 1 m below its tip) can then exceed the glow corona inception electric field $E_{\text{cor}}$, even at moderate heights of the downward leader tip ($z_{\text{down}} = 1140$ m).

Since the background electric fields at this time rise rapidly, it is assumed that streamers readily emanate from the rod body when an electron avalanche reaches a critical size $N_{\text{crit}}$, large enough to sustain the discharge. This condition is defined by the well-known streamer criterion:

$$e^{l_{1}^{2} (\alpha - \eta) \frac{dl}{d\ell}} \geq N_{\text{crit}}$$  \hspace{1cm} (10)

where $\alpha$ is the first ionization coefficient, $\eta$ is the attachment coefficient, $l_{1}$ is defined at the rod surface, $l_{2}$ is the point along the avalanche path where the local electric field is $2.6 \times 10^{6}$ V m$^{-1}$, and $N_{\text{crit}}$ is equal to $10^{8}$ [24]. This condition is henceforth referred to as the surface streamer inception condition.

Note that streamer initiation in the laboratory also requires that the applied voltage is raised faster than a critical value [18, 25]. Experiments indicate that the critical rate of rise necessary for streamer initiation from thin wires decreases as a function of radius [25]. The critical rate of rise is
roughly $10^9$ V s$^{-1}$ for a $5 \times 10^{-4}$ m radius wire and decreases to $10^8$ V s$^{-1}$ for a 0.005 m radius wire. Although there is no further experimental information available, critical values lower than $10^8$ V s$^{-1}$ would be expected for larger radii. In the case of lightning rods, the variation of the voltage induced by the descending stepped leader on the rod body would then define whether streamers are incepted (instead of the glow corona) or not. For the considered 0.02 m radius rod, the variation of the induced voltage (neglecting the fast electric fields generated during the formation of its steps [24]) is $9.3 \times 10^7$ V s$^{-1}$ at $z_{\text{down}} = 1140$ m. Since this voltage rise is faster than the critical value expected for a 0.02 m radius wire in the laboratory, it is justified to assume that streamers are initiated from the rod body (instead of the glow corona) once the streamer criterion is fulfilled.

Assuming that the body of the rod is perfectly cylindrical (i.e. does not have any protrusions), and considering initiation of streamers only on its upper 0.25 m, surface streamer inception occurs when the downward leader tip is as high as 1140 m above the ground. Hence, the production of streamers on the rod body (i.e. below the cap) occurs considerably earlier (i.e. when $z_{\text{down}} = 1140$ m), than along the axis of symmetry, as the axial streamer inception condition predicts (i.e. when $z_{\text{down}} = 530$ m). Therefore, streamer inception along the body of the rod requires a considerably lower total background electric field than the initiation of streamers along the axis of symmetry. Note that the axial streamer inception condition neglects the fact that the electric field on the side of the rod’s cap can reach considerably larger values than $E_{\text{cor}}$ significantly earlier (as high as 4 MV m$^{-1}$ under the conditions illustrated in figure 10).

These results clearly indicate that the effect of the glow corona on streamer inception has been vastly overestimated in the literature [5–13]. As a consequence of the assumptions inherent in the 1D approximation (section 2), the streamer inception criterion used in [5–13] disregards the possibility of streamer initiation on the body of the rod. By assuming that the complete cap surface generates a corona with the same intensity (conditions (a) and (b) in section 2), the 1D approximation predicts streamer inception only at the location of maximum space charge shielding (i.e. along the axis of symmetry). Therefore, the effect of glow corona on the initiation of streamers is exaggerated. Instead, the present results show that the space charge shielding rapidly decays below the rod tip, allowing streamers to be readily initiated from the rod body itself. This conclusion is consistent with several photographic records of lightning strikes that occurred below the tips of tall structures [26], where the successful upward connecting leader (which follows after streamer initiation) emanated from the side of the structures rather than their tips.

It should be noted that the 1D approximation not only leads to overestimation of the effect of glow corona on streamer production, but also significantly exaggerates the shielding effect of the space charge along the path where upward connecting leaders will later propagate. To illustrate this, contours of the shielding potential of the injected corona space charge at the surface streamer initiation time, obtained using both calculation approaches, are shown in figure 11. The shielding equipotential contours computed with the 1D approximation cover significantly larger areas than the corresponding contours computed with the 2D model, especially in the radial direction. For instance, the shielding potential estimated with the 1D approximation at a radial distance of 25 m is about 400 kV; almost double the shielding potential computed with the 2D model (about 200 kV). Moreover, it can be seen that the space charge shielding potential along the radial direction (along the r-axis) decreases significantly faster than that in front of the rod (i.e. along the z-axis). This implies that the presence of glow corona space charge hinders the inception and propagation of upward connecting leaders moving laterally considerably less than for vertically propagating upward leaders. Since upward connecting leaders moving laterally determine the lightning ‘attractiveness’ of a lightning rod, the effect of glow corona...
on the overall lightning attachment process is likely to be less significant than in the literature [5–13]. This will be addressed in more depth in a future paper.

On the other hand, note that the calculations presented in this paper assume that the thundercloud electric field increases up to 20 kV m$^{-1}$, even though the measured background electric fields increase with height above ground and can reach values of several tens of kilovolts per metre [1]. Hence, under a thundercloud rods taller than 20 m, particularly in polluted urban areas, can be exposed to background electric fields substantially exceeding 20 kV m$^{-1}$ [27]. Nevertheless, calculations performed for 60 m tall rods (not presented in this paper) indicate that the conditions required for surface streamer inception are only slightly reduced (by less than 5%) for thundercloud electric field strengths as high as 50 kV m$^{-1}$.

6. General effect of glow corona on the inception of streamers from tall objects

6.1. Lightning rods

To study the effects of glow corona on the production of streamers from a general perspective, the specific analysis presented in the previous section is augmented with analyses of lightning rods with varying heights (10–60 m), and tip radii between 0.002 and 0.05 m. In all cases, it is assumed that the thundercloud electric field increases linearly, over a period of 10 s, up to a strength of 20 kV m$^{-1}$, that the stepped leader descends with an average speed of $2 \times 10^5$ m s$^{-1}$, and its charge density is estimated for a prospective return stroke peak current of 30 kA.

Figure 12 shows the height of the downward leader tip $z_{down}$ at which the axial and surface streamer inception conditions are satisfied for the various rod heights. The estimated axial streamer inception conditions calculated within the 1D approximation are also shown for comparison. The results are similar to those for the case considered in previous sections. In the presence of the entire space charge generated by the glow corona, streamers emanate from the bodies of the rods considered considerably earlier than from along the axis of symmetry. Hence, the axial streamer inception condition leads to overestimation of the shielding effect of the glow corona on streamer production, by predicting that the downward leader tip must be significantly closer to the rod in order to initiate streamers than in the more rigorous analysis applied here. For the range of rod heights considered, the axial streamer inception condition is satisfied at heights $z_{down}$ that are 55–85% shorter than those estimated for the surface streamer inception.

Unsurprisingly, the surface streamer inception height monotonically increases with rod height. However, a larger variation of downward leader height at streamer inception is found for rods shorter than 20 m. The surface streamer inception height curve flattens out for larger rods, since the height $z_{down}$ varies by less than 25% in these cases (ranging between 1300 and 1700 m above the ground). As shown in figure 13, variations in rod radius also significantly affect the effective (surface) production of streamers. For instance, the downward leader tip height $z_{down}$ at surface streamer inception varies by more than 45% for 60 m tall rods, with cap radii ranging between 0.002 and 0.05 m.

As shown in figures 12 and 13, the axial streamer inception condition computed with the 1D approximation (as reported in [5–13]) predicts a very strong effect of the glow corona shielding compared with the more realistic case of the surface streamer inception considered here. Furthermore, it underestimates the height $z_{down}$ at streamer inception, compared with the same criterion computed within the 2D model.

6.2. Dissipation array systems

The 1D approximation has been used to provide theoretical foundations for the superior lightning protection claimed for DAS [9, 12]. Therefore, it is important to evaluate the effect of glow corona on streamer production from them. In contrast to a conventional lightning rod, a DAS is equipped with a needle array, which reduces the electric field at which the glow corona is produced. For manufactured devices, the average field on the surface of the array at corona initiation is maintained at 150 to 200 kV m$^{-1}$ [12]. In this section, the
Figure 14. Contour plot of the small ion density per cubic metre produced by the corona from a 60 m tall DAS with a radius of 2 m. The results correspond to the thundercloud electric field shown in figure 5 at time $t = 10$ s.

Geometry of a DAS is simplified as in [9, 12], by assuming that there are sufficient needles for a uniform corona layer to form over the entire surface of the device. Hence, the array is represented by a hemispherical section of radius 2 m, whose surface electric field at corona inception $E_{cor}$ is as low as $157 \text{ kV m}^{-1}$ [12]. Note that this field $E_{cor}$ on the surface of the array is significantly intensified at each needle tip, so that the glow corona is initiated under local electric fields as high as $8 \text{ MV m}^{-1}$ [12]. For the analysis presented here, an array with a total height of 60 m, mounted on a pole with a radius of 0.3 m, is considered.

Figure 14 shows the spatial distribution of the corona space charge computed with the 2D model, at the point when the background thundercloud field reaches its maximum value during charging (i.e. at 10 s). The spatial distribution of the charge away from the tip of the array is similar to that of a lightning rod with the same height, as shown in figure 5(a). However, the charge density in close proximity to the DAS is not as high as on the surface of the rod. The ion density in front of the array surface does not reach more than $10^{12}$ ions $\text{ m}^{-3}$, while at the tip of a sharp rod of the same height it can exceed $10^{15}$ ions $\text{ m}^{-3}$.

As a downward stepped leader approaches the ground, the electric field in front of the DAS increases, as shown in figure 15. Similarly to the case of lightning rods (section 5b), the electric field increases until a critical condition is met at which the electric field distribution in front of the array becomes non-monotonic. For the DAS, this critical condition is reached when the maximum electric field along the axis of symmetry becomes larger than the glow corona inception field $E_{cor}$ (taken as $157 \text{ kV m}^{-1}$ from [12]), at which point the height of the downward leader tip $z_{down}$ is up to 1600 m above the ground. However, this maximum axial electric field (which is displaced several centimetres from the surface) is considerably lower than the minimum electric field required for the ionization of air $E_{ion}$ (about $2.6 \text{ MV m}^{-1}$ [3]). Only when the downward leader is very close to the ground (200 m in this case), does the maximum electric field in front of the DAS reach values comparable at least to the ionization field $E_{ion}$ (figure 15). Under these conditions, the shielding effect of the space charge generated by a DAS can clearly strongly inhibit the streamer inception along the axis of symmetry, as predicted in [9, 12].

However, to assess the possibility that streamers may be initiated from lower surfaces of the DAS instead, the spatial variation of the electric field around the DAS must also be evaluated. Figure 16 shows an example of the electric field contour lines when the downward leader tip is 1000 m above the ground. Note that even for such a moderate value of the height $z_{down}$, the electric fields on the edge of the DAS (point B) can readily reach values (about $0.2 \text{ MV m}^{-1}$) exceeding that of the tip ($0.157 \text{ MV m}^{-1}$). Interestingly, the electric field on the surface of the column supporting the DAS reaches even higher values (about $0.5 \text{ MV m}^{-1}$). Since streamer inception can occur if the lower needles, or protrusions on the pole, locally intensify the electric field, streamers are very likely to be initially produced on the edge or pole of the DAS, rather than at its tip or in front of it. However, it is not possible to accurately estimate the streamer inception condition on the edge or pole of the DAS, as neither the needles nor the actual shape of a practical device is considered in the geometries modelled here. Note that the needle locations in a real DAS are far from uniform, and the supporting pole is not a smooth cylinder with no edges or protrusions.

It is also important to bear in mind that the shielding effect of a DAS on streamer inception may be even less important than previously supposed, as downward leaders may approach from a position laterally displaced from the array. In such a case, the tips of the array’s needles are not exposed to the same symmetrical electric field. Moreover, the displacement of
space charge due to wind and raindrops during a thunderstorm will destabilize any uniform corona layer (if present). In addition, streamers can probably be initiated from edges of the DAS, the supporting pole, or even the ‘protected’ object, as the electric fields below the device may become very intense. Hence, although inception of streamers in front (along the axis of symmetry) of a DAS requires the downward leader tip to be very close to the device (200 m above the ground in this case), streamers could readily emanate from its lower parts, such as edges, poles or even the ‘protected’ object.

7. Conclusions

The presented analysis indicates that the effect of the space charge generated by a glow corona from tall objects on streamer inception during a thunderstorm is less significant than previously reported [5–13]. Although the results qualitatively confirm most of the general conclusions previously drawn [5–13], they quantitatively differ in some important respects. The discrepancies are due to the assumptions considered in the previous theoretical analyses, which lead to considerable quantitative errors in estimates of the conditions required for streamer initiation. Notably, it has been demonstrated in this paper that calculations based on the 1D approximation overestimate the amount of injected space charge, exaggerate the shielding effect of the generated ions and miscalculate the conditions for the inception of streamers.

The presented results show that streamer inception on upper parts of tall objects’ bodies requires a considerably lower background electric field than initiation of streamers along the axis of symmetry. Therefore, the generation of streamers from objects under a glow corona occurs significantly earlier than previously predicted [5–13]. Since the 1D approximation only considers the axial streamer inception, and overestimates the space charge shielding, it considerably underestimates the height of the downward leader tip at which streamer inception from an object occurs. For example, according to the 1D approximation, streamer inception on a 60 m tall rod with a cap radius of 0.02 m occurs when the downward leader tip is 440 m above the ground. A more realistic estimate, based on the 2D model, is that streamers from such a rod can be initiated, from the upper body surface, when the downward leader tip is about 1160 m above the ground. The same conclusion also applies to DAS.

Furthermore, it is important to note that the results presented in this paper correspond to the upper limit of the effect of glow corona on streamer inception. Since the effect of lateral convection of the ions has been neglected, the results presented here correspond to an idealized scenario, where all the space charge generated remains in front of the rod, and its shielding effect is thus maximized. The opposite extreme corresponds to cases where the glow corona space charge is completely removed, for example by wind, so it has no effect on streamer inception at all. Therefore, the density of the space charge created by the glow corona, and its shielding effect on streamer inception under natural conditions, is bound within those limits.

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