Thesis for the degree of Licentiate of Technology
Östersund 2013

OPTIMIZED PACING STRATEGIES IN CROSS-COUNTRY SKIING
AND TIME-TRIAL ROAD CYCLING

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ISSN 1652-8948
Mid Sweden University Licentiate Thesis 95
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Printed by Kopieringen Mid Sweden University, Sundsvall, Sweden, 2013
ABSTRACT

This thesis is devoted to the analysis and optimization of pacing strategies in cross-country skiing and time-trial road cycling. In locomotive sports, it is well known that variable pacing strategies using changes in the distribution of power output are beneficial when external forces vary along the way. However, there is a lack of research that more in detail investigates the magnitude of power output alteration necessary to optimize performance.

A numerical program has been developed in the MATLAB software to simulate cross-country skiing and time-trial road cycling, as well as pacing strategy optimization in these two locomotive sports. The simulations in this thesis are performed by solving equations of motion, where all the main forces acting on the athlete are considered. The motion equations also depend on the course profile, which is expressed as a connected chain of cubical splines.

The simulation process is linked to an optimization routine called the Method of Moving Asymptotes (MMA), which strives to minimize the finishing time while altering the power output along the course. To mimic the human energetic system, the optimization is restricted by behavioural and side constraints.

Simple constraints like maximum average power output are used for cross-country skiing in Papers I and II. In Paper III a more sophisticated and realistic constraint is used for the power output in time-trial road cycling. It is named the concept of critical power for intermittent exercise and combines the aerobic and anaerobic contributions to power output.

In conclusion, this thesis has demonstrated the feasibility of using numerical simulation and optimization in order to optimize pacing strategies in two locomotive sports. The results are clearly showing that these optimized pacing strategies are more beneficial to performance than an even distribution of power output.

Keywords: Optimization, numerical simulation, equations of motion, MMA, cross-country skiing, cycling
SAMMANDRAG

Denna avhandling är dedikerad att analysera och optimera farthållningsstrategier i längdskidåkning och tempocykling på landsväg. I idrotter som bygger på kontinuerlig framåtdrivning är det väl känt att farthållningsstrategier med variabel effekt är fördelaktiga om de yttre krafterna varierar längs banan. Ändå saknas forskning som mer i detalj utreder hur mycket effekten ska variera för att optimera prestationen.

Ett numeriskt program har utvecklats i programvaran MATLAB för att simulera längdskidåkning och tempocykling samt optimera farthållningsstrategin i dessa idrotter. Simuleringarna använder sig av rörelseekvationer som består av de huvudsakliga krafter som verkar på idrottsutövaren under färd. Rörelseekvationerna påverkas också av banprofilen, som är uppbyggd av en sammankopplad kedja av tredjegradspolynom.

Simuleringsprogrammet är kopplat till en optimeringsalgoritm med namnet Method of Moving Asyptotes (MMA), som strävar efter att minimera tiden mellan start och mål genom att ändra effekten längs med banan. Optimeringen begränsas av bivillkor i ett försök att efterlikna den mänskliga kroppens fysiologiska begränsningar.

Enkla begränsningar såsom maximal medeleffekt används för längdskidåkningen i artikel I och II. I artikel III används mer sofistikerade och realistiska bivillkor för att begränsa uteffekten vid landsvägscykling. Här används modellen för kritisk effekt vid intervallträning, som kombinerar aerobt och anaerobt arbete.

Sammanfattningsvis har denna avhandling visat på möjligheterna med att använda numerisk simulering och optimering för att optimera farthållningsstrategin i två idrotter. Resultaten visar tydligt att dessa optimerade farthållningsstrategier med varierande effekt är mer fördelaktiga för prestationen jämfört med en farthållningsstrategi med helt jämn effektfördelning.

Nyckelord: Optimering, numerisk simulering, rörelseekvationer, MMA, längdskidåkning, cykling
PREFACE

The work presented in this thesis has been carried out at the Department of Engineering and Sustainable Development, Mid Sweden University, Östersund, Sweden. Financial support for this work was provided by the European Union’s structural funds and the Mid Sweden University.

A number of people have been involved in my work with this thesis. Firstly, I would like to thank my brother and my parents for their firm support and understanding. My brother Martin has given me great support and encouragement in fulfilling this thesis. I thank my mother Iren and father Rune for raising me in an impeccable way, for giving me support through thick and thin and for giving me the opportunity to discover my own way of life, both professionally and socially.

Furthermore I would like to thank my supervisor, Professor Mats Tinnsten, and co-supervisor, Professor Peter Carlsson, for their co-authorship and for the fruitful guidance and discussions that has been more than sufficient for a proper mentorship in my research studies.

I would like to acknowledge the stimulating research environment of the Sportstech research group at Mid Sweden University in Östersund.

Special thanks also go to Dr. Fredrik Ståhl for his co-authorship and the rewarding cooperation and discussion.

Finally, I would like to thank all the students I have taught on the Bachelor’s degree program in Sports Technology at Mid Sweden University in Östersund. Thanks for enjoyable discussions and for making me learn more than I would otherwise have done.

Östersund, January 2013

David Sundström
LIST OF PAPERS

This thesis is mainly based on the following three papers, herein referred to by their Roman numerals:


ABBREVIATIONS

A projected frontal area
\(A_w\) incremental drag area associated with wheel spoke rotation
AWC available anaerobic work capacity
\(AWC_{max}\) maximal available anaerobic work capacity
\(a, \ddot{s}\) athlete’s acceleration in the course direction
B shape parameter for \(\varphi\)
C constant
\(C_D\) drag coefficient
\(C_D A_{SSP}\) drag area of a skier in the semi-squatting posture
\(C_D A_{URP}\) drag area of a skier in the upright posture
\(C_D A_{\varphi}\) continuous function of drag area with respect to speed
CP critical power
\(C_{RR}\) coefficient of rolling resistance
\(\dot{E}\) rate of energy expenditure
F net force
\(F_{BR}\) bearing friction force
\(F_D\) drag force (air resistance)
\(F_g\) gravity force
\(F_{RR}\) rolling resistance
\(F_S\) athlete’s propulsive force
\(f_\mu\) frictional force
\(f(x)\) equation for the course section, i.e. \(y = f(x)\)
\(g\) acceleration of gravity
h altitude above sea level
I moment of inertia
L temperature lapse rate
M molar mass of dry air
\(m\) total mass (\(m_b + m_{eq}\))
\(m_b\) athlete’s body mass
\(m_{eq}\) equipment mass including bicycle or skis and poles
N normal force between bicycle and road or ski and snow
\(n\) local coordinate normal to course direction
\(P\) power output
\(\bar{P}\) average power output
\(P_{min}\) global minimum power output
\(P_{max}\) global maximum power output
\(p_0\) standard atmospheric pressure at sea level
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>course curvature radius</td>
</tr>
<tr>
<td>$R_d$</td>
<td>specific gas constant for dry air</td>
</tr>
<tr>
<td>$R_e$</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>$R_g$</td>
<td>ideal gas constant</td>
</tr>
<tr>
<td>$R_v$</td>
<td>specific gas constant for water vapor</td>
</tr>
<tr>
<td>$r$</td>
<td>wheel radius</td>
</tr>
<tr>
<td>STA</td>
<td>seat tube angle</td>
</tr>
<tr>
<td>$s$</td>
<td>local coordinate tangential to course direction</td>
</tr>
<tr>
<td>$T$</td>
<td>finishing time</td>
</tr>
<tr>
<td>$T_0$</td>
<td>sea level standard temperature</td>
</tr>
<tr>
<td>$T_a$</td>
<td>absolute temperature</td>
</tr>
<tr>
<td>TA</td>
<td>torso angle</td>
</tr>
<tr>
<td>$t$</td>
<td>current time</td>
</tr>
<tr>
<td>$t', t''$</td>
<td>derivatives of time with respect to the $x$-coordinate</td>
</tr>
<tr>
<td>$u$</td>
<td>resulting wind speed</td>
</tr>
<tr>
<td>$v, \dot{s}$</td>
<td>athlete’s ground speed in the course direction</td>
</tr>
<tr>
<td>$v_{lim}$</td>
<td>limit speed where efficiency is reduced to the half of $\eta$</td>
</tr>
<tr>
<td>$w$</td>
<td>environmental wind speed</td>
</tr>
<tr>
<td>$x$</td>
<td>global horizontal coordinate</td>
</tr>
<tr>
<td>$\dot{x}, \ddot{x}$</td>
<td>derivatives of the horizontal coordinate with respect to time</td>
</tr>
<tr>
<td>$y$</td>
<td>global vertical coordinate</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>course incline</td>
</tr>
<tr>
<td>$\beta$</td>
<td>environmental wind angle</td>
</tr>
<tr>
<td>$\eta$</td>
<td>base value of mechanical efficiency</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Lagrange multiplier</td>
</tr>
<tr>
<td>$\mu$</td>
<td>friction coefficient</td>
</tr>
<tr>
<td>$\rho$</td>
<td>air density</td>
</tr>
<tr>
<td>$\phi$</td>
<td>reducing function for mechanical efficiency</td>
</tr>
<tr>
<td>$\phi$</td>
<td>relative humidity</td>
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</table>
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1. INTRODUCTION

In locomotive sports like road cycling and cross-country skiing, the main goal for the athlete is to travel the distance faster than the other competitors (i.e. complete the course in less time than the opponents). Therefore, the ultimate performance measure is the time from the start to the finish line; this is called the finishing time. Factors that influence the finishing time can be divided into internal factors, related to the athlete, and external factors, related to the surroundings. Internal factors include metabolic rate, gross efficiency and projected frontal area, while external factors can be course inclination, environmental wind and air density. There are also factors in a grey zone, like the rolling resistance or the snow glide friction that is dependent on both the properties of the road/snow, as well as the properties of the tire/ ski.

A pacing strategy is a way of varying (or not varying) the travelling speed along the course distance. In reality, this is done by altering the power output. That is why pacing strategy and power distribution are sometimes used interchangeably. However, the strict meanings of the two constructs are not the same. There is a clear connection between power output and speed, making it possible to control the pacing strategy through the corresponding distribution of power output. This connection can be expressed mathematically with the equations of motion (section 2.3).

When it comes to performance enhancement, an optimal pacing strategy varies the travelling speed in order to minimize the finishing time. Accordingly, an optimal power distribution varies the power output in order to minimize the finishing time. And, as there is always a restriction on the power output that an athlete might generate, the optimization of pacing strategies will always be a constrained optimization problem.

When the athlete is travelling at a constant velocity, the propulsive force is opposed by and equal in magnitude to the external forces. These external forces are aerodynamic drag, gravitational force, friction and, in cycling, rolling resistance. However, in the case of acceleration or deceleration the external and propulsive forces are not equal. On level roads with constant or no wind, this acceleration of inertia will have a minor influence, but on hilly courses and in variable wind conditions it may have a significant impact on the optimal pacing strategy.

The benefit of even pacing was first hypothesized over 100 years ago by Kennelly (1906) who made mathematical comparisons of speed records in quadrupeds and bipeds. This has later been corroborated by a number of studies (Atkinson, Davison, Jeukendrup, & Passfield, 2003; Foster et al., 1993; Robinson, Robinson, Mountjoy, & Bullard, 1958). However, this is correct for constant external factors or for unconstrained power output. In reality (in road cycling and
cross-country skiing), external conditions change continuously and the power output of the athlete must be controlled to avoid fatigue. Therefore, variable pacing strategies are beneficial in most real-life circumstances. This has been shown in studies of time-trial road cycling (Atkinson & Brunskill, 2000; Atkinson, Peacock, & Passfield, 2007; Swain, 1997). Gordon (2005) showed the potential for a mathematical model using analytical differentiation, to optimize the distribution of power output. He concluded that a variable pacing strategy in time-trial road cycling is beneficial for courses with varying inclinations, but he could not confirm that the same is true for variable wind conditions. Cangley, Passfield, Carter & Bailey (2011) showed that a 2.9% gain in finishing time could be achieved by utilizing an optimized pacing strategy rather than a more or less even power distribution. This study considered a 4 km hilly course with almost constant wind conditions and all subjects’ power output averaged about 255 W.

In cross-country skiing there has been no specific research into pacing strategies. However, some studies treat the mechanics of motion and discuss the fundamental properties of pacing theory (Carlsson, Tinnsten, & Ainegren, 2011; Moxnes & Hausken, 2008).

The aims of this thesis were to (i) develop numerical models for optimization of the pacing strategy in time-trial road cycling and cross-country skiing and to (ii) compare the results with previous studies of pacing strategies.
2. SIMULATION MODEL FOR LOCOMOTIVE SPORTS

Each of the three appended papers includes one simulation model. For simplicity, each model is named with the Arabic numeral that corresponds to the appended paper (i.e. model I corresponds to paper I). All models were programmed into a computer software called MATLAB. Several modeling functionalities are presented in this section, and some of them are shared for all models. However, for functionalities that are unique to one or two of the models, this will be stated in the text. Table 1 presents the main features of the models.

<table>
<thead>
<tr>
<th>Version</th>
<th>Sport</th>
<th>Course length</th>
<th>Steps</th>
<th>Wind</th>
<th>Variable number</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model I</td>
<td>XC skiing</td>
<td>1425 m</td>
<td>Time</td>
<td>No</td>
<td>35</td>
<td>Min, mean, max power, 3 variables max</td>
</tr>
<tr>
<td>Model II</td>
<td>XC skiing</td>
<td>1425 m</td>
<td>Distance</td>
<td>No</td>
<td>75</td>
<td>Min, mean and max power</td>
</tr>
<tr>
<td>Model III</td>
<td>Road cycling</td>
<td>42500 m</td>
<td>Distance</td>
<td>Varying direction</td>
<td>80</td>
<td>Min, max and critical power</td>
</tr>
</tbody>
</table>

Vectors are written in bold typeface. Prime denotes differentiation by the coordinate $x$ and dot denotes differentiation by time.

2.1 Classical mechanics

Classical mechanics is a subfield in the science of mechanics. It constitutes of a set of physical laws describing the motion of bodies under action of forces. Some early efforts in classical mechanics were carried out by Galileo Galilei (1564 – 1642), Tycho Brahe (1546 – 1601) and Johannes Kepler (1571 – 1630). However, the largest contribution was made by Isaac Newton (1643 – 1727) in his work *Philosophiae Naturalis Principia Mathematica*, where he postulated the law of gravity and the three laws of motion.

In contrast to quantum mechanics, classical mechanics assumes Galilean relativity, meaning time is considered an absolute. Classical mechanics also assumes Euclidean geometry, which is a good approximation to the properties of physical space when the gravitational field is weak. These principals are the foundation of the simulation models presented in this thesis.
2.1.1 Kinematics

In all the models, the course is a straightened out 2-D course consisting of a connected chain of cubical splines formed by 2-D coordinates \((x, y)\), see Figure 1. The advantage of cubical splines is the fact that both the inclination \(\alpha\) and curvature radius \(R\) will be available for all points from start to finish. During a simulation, the athlete’s center of mass is set to follow the splines.

![Figure 1. Arbitrary course section for cross-country skiing (left) and road cycling (right) with local and global coordinates, as well as the forces acting on the athlete.](image)

2.1.2 Kinetics

According to Newton’s second law, the acceleration of a body is parallel and proportional to the force acting on the body and is inversely proportional to the mass of the body. Mathematically, it is expressed as:

\[
F = m \cdot a
\]

where \(F\) is the net force acting on the body, \(m\) is the mass of the body and \(a\) is the linear acceleration of the center of mass. If only linear acceleration is permitted, this is a correct description. However, in model III, the angular acceleration of the spinning wheels is permitted and thus there is an extension to equation (1). Hence, the equation can be rewritten as:

\[
F = (m + \frac{I}{r^2})a
\]
where $I$ is the total moment of inertia of the wheels and $r$ is the wheel radius. The net force is the sum of the propulsive force and all the resistive forces acting to restrict motion. The propulsive force $F_s$ is expressed as:

$$F_s = \frac{P}{v}$$

where $P$ is the propulsive power output and $v$ is the ground speed.

The gravitational force is parallel and proportional to the acceleration of gravity. The acceleration of gravity is considered to be constant for the area of interest in all simulation models and it is always acting down in a vertical direction. The gravitational force can be expressed as:

$$F_g = mg$$

where $g$ is the acceleration of gravity. When the gradient of the course changes, the normal force between the track and the means of locomotion will alter with the variation of centripetal forces. The normal force $N$ (Figure 1, left) can be expressed as:

$$N = m\left(g \cos \alpha - \frac{v^2}{R}\right)$$

### 2.1.3 Aerodynamics

A good approximation of the aerodynamic drag of a blunt body travelling in a fluid can be calculated by the equation:

$$F_D = \frac{1}{2} C_D A \rho v^2$$

where $F_D$ is the drag force, $C_D$ is the drag coefficient, $A$ is the projected frontal area and $\rho$ is the air density. The accuracy of this equation fails when speeds are low, i.e. at low Raynolds numbers ($R_e < \sim1000$). At the starting speed (minimal speed) of all three models, Raynolds numbers are about $R_e \approx 300 000$.

For consideration of variable air density, environmental winds and drag area, see sections 2.2.2, 2.2.3 and 2.2.5 respectively. Drafting is not considered in any of the models presented in this thesis.

### 2.1.4 Coulumb friction and rolling resistance

For models I and II, the glide friction between snow and skies is considered as Coulumb friction. This depends both on the normal force and the friction coefficient. Usually, the friction coefficient varies between 0.02-0.10 (Colbeck, 1994)
for different snow and ski properties. The expression for the friction force $F_\mu$ (Figure 1, left) is formulated as:

$$F_\mu = \mu \cdot N = \mu \cdot m \left( g \cos \alpha - \frac{v^2}{R} \right)$$

(7)

where $\mu$ is the kinetic friction coefficient. For model III, the rolling resistance between the road and the tires is also set to be proportional to the normal force. The coefficient of rolling resistance is usually between 0.002 (Kyle, 1996) and 0.008 (Pugh, 1974) on smooth asphalt roads, depending on the tire construction and material as well as the inflation pressure (Grappe et al., 1999). The rolling resistance (Figure 1, right) can be expressed as:

$$F_{RR} = C_{RR} \cdot N = C_{RR} \cdot m \left( g \cos \alpha + \frac{v^2}{R} \right)$$

(8)

where $C_{RR}$ is the coefficient of rolling resistance.

2.2 Other modeling functionalities

2.2.1 Wheel bearing friction

The friction emerging from the wheel bearings creates a motion restricting force. It has been shown that this force is nearly independent of the normal force acting on the bearing, but depends on the rotational speed and the type of bearing and lubricant (Dahn, Mai, Poland, & Jenkins, 1991). According to the measurements by Dahn et al. (1991), the friction in greased cartridge bearings with a normal load of 27 kg on each wheel can be expressed as

$$F_{BR} = \frac{(89 + 8.4 \cdot v)}{1000}. \quad (9)$$

2.2.2 Altitude, temperature and humidity compensated air density

In model III, the air density is set to depend on altitude $h$, relative humidity $\phi$ and absolute temperature $T_a$. The following equation was thus used to calculate air density

$$\rho = p_0 \left( 1 - \frac{L h}{T_0} \right)^{\frac{g - M}{(R_g L)}} \left( \frac{R_d \cdot T_a}{R_v \cdot T_a} \right) + 0.061078 \cdot \phi \left[ \frac{1}{(R_v \cdot T_a)} - 1 \left( \frac{R_d \cdot T_a}{(R_v \cdot T_a)} \right) \right] 10^{(7.5 \cdot T_a - 2.048625) / (T_a - 35.85^\circ)}$$

(10)

where $p_0$ is the standard atmospheric pressure at sea level ($p_0 = 101,325$ kPa), $L$ is the temperature lapse rate ($L = 0.0065 \text{ K} \cdot \text{m}^{-1}$), $T_0$ is the sea level standard temperature ($T_0 = 288.15$ K), $g$ is the acceleration of gravity at sea level ($g = 9.81 \text{ m/s}^2$).
9.80665 m·s⁻²), \( M \) is the molar mass of dry air (\( M = 0.0289644 \text{ kg·mol}^{-1} \)), \( R_g \) is the ideal gas constant (\( R_g = 8.31447 \text{ J·mol}^{-1}	ext{·K}^{-1} \)), \( R_d \) is the specific gas constant for dry air (\( R_d = 287.058 \text{ J·kg}^{-1}	ext{·K}^{-1} \)) and \( R_v \) is the specific gas constant for water vapor (\( R_v = 461.495 \text{ J·kg}^{-1}	ext{·K}^{-1} \)).

### 2.2.3 Environmental wind compensated air resistance

While allowing environmental wind (model III) to influence the athlete, the aerodynamic drag force in the direction of travel can be expressed as:

\[
F_D = \frac{1}{2}(C_D A + A_w) \rho [(v + w \cos \beta)^2 + (w \sin \beta)^2]^\frac{1}{2} (v + w \cos \beta) = \frac{1}{2}(C_D A + A_w) \rho \cdot u(v + w \cos \beta) \tag{11}
\]

where \( A_w \) is the incremental drag area associated with wheel spoke rotation, \( w \) is the environmental wind speed and \( \beta \) is the environmental wind angle, see Figure 2.

![Figure 2.](image) The directions of ground speed (\( v \)), wind speed (\( w \)) and resulting wind speed (\( u \)).

In model III, only the direction of the environmental wind is set to change in the simulations. However, equation (11) is also useful when the variable magnitude of the wind velocity is considered.

### 2.2.4 Speed compensated efficiency

The athlete’s ability to generate power output will decrease at high speeds. This is because of the equivalent increase in muscle contraction speed, which dramatically decreases the muscle force exerted by the athlete. Muscle efficiency is thus decreased at high speeds in model II. In cycling, the speed at which efficiency falls is significantly higher than in cross-country skiing. This is due to the use of chain wheel gearing in cycling that effectively enables a wide range of gears even at relatively high speeds. Therefore, no decrease in efficiency was programmed in model III. In model II, the propulsive power in cross-country skiing is expressed as:
\[ P = \eta \varphi \dot{E} \]  

(12)

where \( \eta \) is the base value of mechanical efficiency, \( \dot{E} \) is the rate of energy expenditure and the reducing function \( \varphi \) is calculated as:

\[ \varphi = \frac{1}{2} - \frac{1}{\pi} \tan^{-1}(B(v - v_{lim})) \]  

(13)

where, \( B \) is a parameter that controls the shape of the function, \( v \) is the ground speed and \( v_{lim} \) is the limit speed where the efficiency is reduced to half of \( \eta \). Varying mechanical efficiencies associated with different skiing techniques (classic or freestyle) and different gears (e.g. double poling and diagonal stride) (Sidossis et al. 1992; Sandbakk et al. 2010) are not considered in any model.

2.2.5 Speed compensated drag area

In cross-country skiing, it is common to crouch, or semi-squat, at high speeds (Figure 3). This lowers the drag area but restricts the athlete’s ability to generate power output. In model II, this is done by reducing the drag area, using the same reducing function \( \varphi \) (equation 13). The effective drag area is calculated as:

\[ C_{DA}\varphi = C_{DURP} \cdot \varphi + C_{DSSP} (1 - \varphi) \]  

(14)

where \( C_{DURP} \) is the athlete’s drag area in the upright posture and \( C_{DSSP} \) is the drag area in the semi-squatting posture. The reduced functions in equation (12) and (14) are synchronized in speed.

Figure 3. Upright (left) and semi-squatting (right) postures in cross-country skiing.
In model I, the drag area is set to decrease linearly from 100% of the original drag area at $\alpha = 0$ to 60% at $\alpha = -6^\circ$, due to crouching (Carlsson, Tinnsten, & Ainegren, 2011).

In time-trial road cycling, it is hard to decrease drag area at high speeds. On the other hand, the athlete has to assume a more stable but less aerodynamic posture at high speeds. By moving the hands to the base-bar, the athlete is able to control the bicycle even when small cross-winds disturb the balance. This position is close to the traditional aero position that was used before the aero-bar was introduced to time-trial road cycling. In model III, the transition from a modern aero position to a traditional aero position (Figure 4) is set to take place at a speed of $v_{lim} = 19.44 \text{ m}\cdot\text{s}^{-1}$ (70 km·h$^{-1}$) using equations (13) and (14). In this case, $A_{URP}$ is the frontal area of the modern aero position ($A_{URP} = 0.00433 \cdot \text{STA}^{0.172} \cdot \text{TA}^{0.0965} \cdot m_b^{0.762} + 0.066$) (Heil, 2001) and $A_{SSP}$ is the frontal area of the traditional aero position ($A_{SSP} = 0.03608 \cdot m_b^{0.589}$) (Heil, 2002).

![Figure 4. Modern aero (left) and traditional aero (right) positions in time-trial road cycling.](image)

### 2.3 Equations of motion

The motions described in all models are the planar motion of a particle. This means that the center of mass is travelling the course profile and the only moment of inertia to be considered is the rotating wheels (only model III). Firstly, the equations of motion are formulated in the local directions, normally and tangentially to the direction of travel. Then, they are transformed into global $x$- and $y$-coordinates (equation 15), using the course profile splines (equation 16):

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & -\cos \alpha \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

(15)

$$\alpha = \tan^{-1}(y')$$

(16)
where $s$ and $n$ are the tangential and normal coordinates of motion (Figure 1) and $\alpha$ is the course inclination. Expressions for speed (equation 17) and curvature (equation 18) can also be stated using the geometrics of the cubical splines:

\[
\dot{s} = \sqrt{\dot{x}^2 + \dot{y}^2} \tag{17}
\]

\[
\frac{1}{R} = \frac{y''}{\left(1 + (y')^2\right)^{3/2}} \tag{18}
\]

The equation system in equation (15) can be utilized, along with the force expressions in section 2.1-2.2, to form the equations of motion. Equation (19) is an example of how the motion equations can be formulated for models I and II.

\[
\begin{bmatrix}
\ddot{x} \\
\ddot{y}
\end{bmatrix} = \begin{bmatrix}
\cos \alpha & \sin \alpha \\
\sin \alpha & -\cos \alpha
\end{bmatrix} \begin{bmatrix}
\dot{s} \\
\dot{n}
\end{bmatrix} = \begin{bmatrix}
\cos \alpha & \sin \alpha \\
\sin \alpha & -\cos \alpha
\end{bmatrix} \begin{bmatrix}
(F_s - F_D - F_\mu)/m \\
-N/m
\end{bmatrix} - \begin{bmatrix}
0 \\
F_g/m
\end{bmatrix} \tag{19}
\]

2.3.1 Transformation of motion equations

In models II and III the equations of motion are transformed in order to be distance dependent instead of being time dependent. This is done by using the relationship between time, distance and course profile:

\[
\ddot{x} = -t''/(t')^3 \tag{20}
\]

\[
\dot{s} = 1 / (t' \cos \alpha) \tag{21}
\]

The benefit of this transformation is that exact distances and splits can be simulated and adaptive step-size determination can be used to reduce the process time. Further information about this transformation is presented in Paper II.

Finally, to make the equations of motion solvable, they are transformed into a system of first order ordinary differential equations, with the method of introducing a new variable.

2.4 Solving ordinary differential equations numerically

A differential equation is an equation for an unknown function of one or several variables that relates to the values of the function itself and its derivatives. Ordinary differential equations (ODE) are differential equations containing a function of one variable and its derivatives. In many practical applications, differential equations describe a course of events. The motion equations derived in section 2.3 describe the event of an athlete travelling a two-dimensional course.
However, to actually move along the course, the equations of motion have to be solved in an iterative manner for discrete values of either time (model I) or distance (models II and III). In every iteration, the acceleration is calculated for the current time and input speed by solving the equations of motion. By taking a predetermined time step, the displacement of motion and the speed at the new position is calculated. The displacement and speed gives a new distance where the equations are solved. The inverse is true for models II and III, thus taking a distance step. An ODE for the models presented in this thesis can be expressed as:

$$\dot{x} = f(t, x(t)), \quad x(t_0) = x_0$$  \hspace{1cm} (22)

where $t$ is time, $x$ is the horizontal coordinate and $f$ is the equation of motion. $t_0$ and $x_0$ is the initial values of time and distance.

In some cases, but not all, ODEs have analytical solutions. The most basic numerical method for solving an initial value ODE is the Euler method. It effectively uses a striding process (taking steps) by using the finite difference approximation of the derivative to solve the ODE. In the case of higher order derivatives (2nd or higher), the method of introducing a new variable gives a system of first order ODEs. To derive the Euler method, we start with the finite difference method:

$$\dot{x} = f(t, x(t)) \approx \frac{x(t+\Delta t)-x(t)}{\Delta t}$$  \hspace{1cm} (23)

where $\Delta t$ is the time step. Solving for $x(t+\Delta t)$ gives the expression for taking one step with the Euler method:

$$x(t + \Delta t) \approx x(t) + \Delta t \cdot f(t, x(t))$$  \hspace{1cm} (24)

Another, far more sophisticated method for solving ODEs, is the Runge-Kutta-Fehlberg method (RKF) (Fehlberg, 1969). This method is used in all three models to simulate the athlete’s locomotion and is a combination of the 4th and 5th order Runge-Kutta methods, making comparisons of the error in those formulas. In those Runge-Kutta formulas there are several approximations of derivatives, between $t$ and $t + \Delta t$, which in combination estimate the function value.
3. OPTIMIZATION

Making a good design is not the same thing as making the best design. Optimization is all about finding the best solution possible in a predetermined set of circumstances. This area of research grew in the 1960s, along with advances in the area of transistor-based computers, which enabled high-speed numerical computing. Optimization methods can be applied to almost every kind of engineering problem; even the area of economics assimilates the benefits of optimization. Mathematical optimization can be divided into analytical and numerical optimization. Analytical optimization (section 3.2.1) is constrained to simpler problems, while numerical optimization (section 3.2.2) can be applied to a broad range of different kinds of problems.

3.1 Formulation of a general optimization problem

An optimization problem is formulated to minimize a certain quantity while varying some other quantities. Mathematically, a general constrained optimization problem can be formulated as:

\[
\begin{align*}
\text{Minimize} & \quad f(x) \\
\text{Subject to} & \quad h(x) = 0 \\
& \quad g(x) \leq 0
\end{align*}
\]

where \( x \) is a vector containing the variables, \( f(x) \) is the objective function and \( h(x) \) and \( g(x) \) are vectors with equality and inequality constraints respectively. Problems that do not contain any constraints are called unconstrained problems. Many optimization problems have the same fundamental structure and can therefore be solved with several different optimization methods.

3.1.1 Design variables

In order to optimize a structure or design, some parameters must be able to change. These parameters are usually called design variables in the optimization problem and there must be a finite number of them. A set of design variables represents a design point in the design space. Each design point can be either a possible or impossible solution to the optimization problem, depending on the constraints. In this section (3), all variables will be named \( x \).

3.1.2 Constraints

As stated in the foregoing section, any set of design variables represents a design point. However, not all design points are practically possible (feasible). Hence,
constraints are constructed to restrict the design variables. Constraints that only restrict each design variable are most commonly named geometrical or side constraints. Restrictions that limit the performance or behavior of the design are named functional or behavioral constraints. All constraints must be functions of the design variables to make sense of the problem. If a design point does not satisfy the constraints, it is an infeasible design.

3.1.3 Objective function

The objective function or cost function describes the quantity to be optimized and has to be expressed as a function of the design variables. In the majority of optimization problems, the objective function is set to be minimized. In some cases the purpose of the optimization is to maximize a function. However, the same methods can be used as for minimization if the negative equivalent of the function is used as the objective function.

3.2 Solving an optimization problem

3.2.1 Analytical optimization

The most fundamental optimization example is to find the minimum of an ordinary analytical function without constraints. If it is possible to analytically differentiate the function, the solution is easy to find. For problems where there is more than one variable and non-linear constraints are considered, the solution is not that obvious. For such a problem, it is a good idea to use the method of Lagrange multipliers. Consider an optimization problem with only inequality constraints:

\[
\begin{align*}
\text{Minimize} & \quad f(x) \\
\text{Subject to} & \quad g(x) \leq 0
\end{align*}
\]  

With the use of the Lagrange multiplier vector \( \lambda = (\lambda_1, \lambda_2, \ldots, \lambda_p) \), the Lagrangian function can be expressed as:

\[
L(x, \lambda) = f(x) + \lambda \cdot g(x)
\]  

It can be shown that a stationary point of equation (27) can be the solution of equation (26). The gradient of the Lagrangian function, along with some other conditions, creates the Karush-Kuhn-Tucker conditions for an inequality constrained problem:

\[
\nabla f(x) + \lambda \cdot \nabla g(x) = 0
\]
\[ \lambda \cdot g(x) = 0 \quad (29) \]
\[ g(x) \leq 0, \lambda \geq 0 \quad (30) \]

By rewriting equation (28) as \( \nabla f(x) = -\lambda \cdot \nabla g(x) \), one can see that the gradient of the objective function must be a linear combination of the active constraint gradients.

### 3.2.2 Numerical optimization

For complex non-convex functions, the usage of analytical methods is not preferred. Instead, there are a variety of numerical optimization routines that can estimate the optimal solution with good accuracy. These methods are built up by iterative numerical computations that ultimate leading to a minimum. A general description of a numerical optimization routine can be expressed as:

**Step I.** Guess initial design \( x^{(0)} \) and calculate the solution.

**Step II.** For \( k = 0, 1, \ldots, N \)
- If \( x^{(k)} \) is optimal, stop.

**Step III.** Calculate an improved estimate of the solution
\[ x^{(k+1)} = x^{(k)} + \alpha_k \cdot d^{(k)}. \]

**Step IV.** Repeat from Step II.

Here, \( d^{(k)} \) is the search direction and \( \alpha_k \) is the step length in that direction. Usually the optimality is determined by some kind of convergence criterion. However, for some problems it is hard to create an adequate convergence criterion, and thus a maximal number of iterations \( N \) must be stated in order to receive a solution.

### 3.2.3 Approximated subproblems

There are numerical methods for constrained optimization that use approximated subproblems \( \bar{P} \) to estimate the real problem \( P \). The subproblems are created between Step II and Step III in the algorithm in the previous section. The approximations are obtained through linear Taylor series expansions for the objective function and constraints (termed \( \bar{f}(x) \) and \( \bar{g}(x) \)). The problem in equation (26) can be approximated with the following linearized subproblem \( \bar{P}^{(k)} \) using Taylor expansion around \( x^{(k)} \):

\[ \bar{P}^{(k)}: \]
\[ \text{Minimize} \quad \bar{f}(x) = f(x^{(k)}) + \nabla f(x^{(k)}) \cdot (x - x^{(k)}) \]
\[ \text{Subject to} \quad \bar{g}(x) = g(x^{(k)}) + \nabla g(x^{(k)}) \cdot (x - x^{(k)}) \leq 0 \]
The above problem (equation 31) can easily be solved with linear programming methods such as the Simplex method. The original problem in equation (26) is solved as a sequence of linearized subproblems, where the linearization is carried out in every iteration. However, the change of the design may not be too large as the Taylor expansion is only valid near the original design point. Hence, a concept called move limits is utilized. These limits are set to constrain the change of the design within each iteration and the limits are usually corrected between iterations. The move limits may be expressed as:

\[ \underline{x} \leq x \leq \bar{x} \]  (32)

where \( \underline{x} \) is the lower limit of the design \( x \) and \( \bar{x} \) is the upper limit.

### 3.2.4 Method of Moving Asymptotes (MMA)

The Method of Moving Asymptotes (Svanberg, 1987, 1993) is a method based on approximated subproblems. However, the approximated objective function \( \tilde{f}(x) \) and the constraints \( \tilde{g}(x) \) of MMA are obtained through linearization in variables of the type \( 1/(x_j - l_j) \) and \( 1/(u_j - x_j) \), where \( l_j \) and \( u_j \) are the lower and upper asymptotes of \( x_j \), thus \( l_j \leq x_j \leq u_j \). Consequently, the approximated MMA subproblem \( \tilde{p}^{(k)} \) can be expressed as:

\[
\begin{align*}
\tilde{p}^{(k)}: \\
\text{Minimize} & \quad \tilde{f}^{(k)}(x) = f^{(k)}(x) + \sum_{j=1}^{J} \left( \frac{p_{0j}^{(k)}}{u_j^{(k)} - x_j} + \frac{q_{0j}^{(k)}}{x_j - l_j} \right) - \left( \frac{p_{ij}^{(k)}}{u_j^{(k)} - x_j} + \frac{q_{ij}^{(k)}}{x_j - l_j} \right) \\
\text{Subject to} & \quad \tilde{g}_i^{(k)}(x) = g_i^{(k)}(x) + \sum_{j=1}^{J} \left( \frac{p_{ij}^{(k)}}{u_j^{(k)} - x_j} + \frac{q_{ij}^{(k)}}{x_j - l_j} \right) - \left( \frac{p_{ij}^{(k)}}{u_j^{(k)} - x_j} + \frac{q_{ij}^{(k)}}{x_j - l_j} \right) \leq 0
\end{align*}
\]

If \( \frac{\partial f}{\partial x_j} > 0 \) then \( p_{0j}^{(k)} = (u_j^{(k)} - x_j^{(k)})^2 \cdot \frac{\partial f}{\partial x_j} \) and \( q_{0j}^{(k)} = 0 \)

If \( \frac{\partial g_i}{\partial x_j} > 0 \) then \( p_{ij}^{(k)} = (u_j^{(k)} - x_j^{(k)})^2 \cdot \frac{\partial g_i}{\partial x_j} \) and \( q_{ij}^{(k)} = 0 \)

If \( \frac{\partial f}{\partial x_j} < 0 \) then \( p_{0j}^{(k)} = 0 \) and \( q_{0j}^{(k)} = -(x_j^{(k)} - l_j^{(k)})^2 \cdot \frac{\partial f}{\partial x_j} \)

If \( \frac{\partial g_i}{\partial x_j} < 0 \) then \( p_{ij}^{(k)} = 0 \) and \( q_{ij}^{(k)} = -(x_j^{(k)} - l_j^{(k)})^2 \cdot \frac{\partial g_i}{\partial x_j} \)

If \( \frac{\partial f}{\partial x_j} = 0 \) then \( p_{0j}^{(k)} = 0 \) and \( q_{0j}^{(k)} = 0 \)

If \( \frac{\partial g_i}{\partial x_j} = 0 \) then \( p_{ij}^{(k)} = 0 \) and \( q_{ij}^{(k)} = 0 \)
where $j = 1, \ldots, J$ (number of variables), $i = 1, \ldots, I$ (number of constraints) and all derivatives $\frac{\partial f}{\partial x_j}$ and $\frac{\partial g_i}{\partial x_j}$ are evaluated at $x^{(k)}$. 
4. THE PACING STRATEGY OPTIMIZATION PROBLEM

All models (I-III) in this thesis use MMA to optimize the pacing strategy. The gradients of the objective function $\partial f / \partial x_j$ as well as the constraints $\partial g_i / \partial x_j$ are calculated numerically using the finite difference approximation. The finite difference can be seen as a perturbation ($\Delta x$) of the variable ($x$). A schematic description of the optimization process for pacing strategies can be seen in Figure 5. The change in variables proposed by MMA is $x_{new}$ and $x_{opt}$ for the improved and optimum solutions, respectively. The initial estimate of the variable values is set to $x_{ini}$ (Figure 5).

![Figure 5. Schematic description of the process for optimizing the pacing strategy.](image)

4.1 Formulation of the pacing strategy optimization problem

The objective function of all three models is the finishing time. However, save for the side constraint in equation (37), different constraints are used in the different models. As behavior constraint, the Critical Power constraint stated in equation (38) is the most sophisticated. It is based on the model of critical power for intermittent exercise (Morton & Billat, 2004), which considers both aerobic and anaerobic metabolism. The optimization problem for optimizing the pacing strategy is formulated as:
Minimize:

\[ T = \sum_{i=1}^{K} \Delta t_i \]  
(34)

Subject to the constraints:

Model I:

\[ (P_j + P_{j+1} + P_{j+2}) \leq 2 \cdot P_{\text{max}} + C \cdot \bar{P} \quad j = 1, 2, \ldots, (J - 2) \]  
(35)

Model I, II:

\[ \frac{1}{T} \int_0^T P(t) dt \leq \bar{P} \]  
(36)

Model I, II, III:

\[ P_{\text{min}} \leq P_j \leq P_{\text{max}} \quad j = 1, 2, \ldots, J \]  
(37)

Model III:

\[ 0 \leq \text{AWC}_q \leq \text{AWC}_{\text{max}} \quad q = 1, 2, \ldots, J + Q \]  
(38)

where \( T \) is the total race time, \( \Delta t_i \) is the time segment during iteration \( i \), \( K \) is the total number of time segments during the actual simulation, \( P_j \) is the power output variable at \( j \), \( J \) is the number of power output variables, \( P_{\text{min}} \) is the minimum power output, \( P_{\text{max}} \) is the maximum attainable power output, \( C \) is a constant, \( \bar{P} \) is the average power output limit, \( \text{AWC}_q \) is the available anaerobic work capacity at \( q \), \( \text{AWC}_{\text{max}} \) is the maximal level of available anaerobic work capacity, \( Q \) is the number of intersections between the critical power (CP) and the linear interpolation of power output (\( P \)). In order to stabilize the numerical process, all constraints are normalized during the optimization.
5. OPTIMIZED PACING STRATEGIES IN LOCOMOTIVE SPORTS

The optimization problem for pacing strategies, as described in section 4, was implemented into the MATLAB software. This section reports the most important results appearing in the appended papers as well as some unpublished results.

5.1 Optimized pacing strategy in cross-country skiing

Model I was used to perform the optimization for two athletes of different body mass on a sprint course for cross-country skiing, using the freestyle technique. One athlete had a body mass of \( m_b = 60 \text{ kg} \) and the other had a body mass of \( m_b = 75 \text{ kg} \). Their power output was scaled by body mass to the 0.94th power and their projected frontal area was set to scale homogenously like body mass to the 2/3rd power. The simulation contained no wind and a constant glide friction coefficient of \( \mu = 0.03 \). 15 iterations were completed and the optimized distributions of power output can be seen in Figure 6 and 7.

![Figure 6](image)

Figure 6. Course profile and optimized power output distribution for a 60 kg skier generating 281 W.

The optimized power distribution for the 60 kg skier generated a finishing time of 206.9 s which represents a time gain of 3.3 s (1.6%) compared to an even power distribution equal to the average power output (Figure 6).
Figure 7. Course profile and optimized power output distribution for a 75 kg skier generating 347 W.

The optimized power distribution for the 75 kg skier generated a finishing time of 204.55 s which represents a time gain of 3.35 s (1.6%) compared to an even power distribution equal to the average power output (Figure 7). This is also 2.35 s (1.1%) faster than the 60 kg skier.

Figure 8. Course profile, speed, optimized power output distribution and average power output for a world class male skier of 78 kg using model II.
In model II, the maximum power output was raised compared to model I, and the simulated athlete had an average power output of 376 W and a body mass of 78 kg. However, power output was reduced at high speeds, making the average power output somewhat lower. The same course as in model I was used. 20 iterations were completed and the optimized distribution of power output is presented in Figure 8.

The optimization and simulation of the 78 kg skier using model II resulted in a finishing time of 187.4 s. This is 13.0 s (6.5%) faster than an even power distribution equal to the average power output, subsequently reduced with the reducing function \( \varphi \) in Paper II. This optimized pacing strategy gives a finishing time that is 19.5 s (9.4%) and 17.55 s (8.4%) faster than the 60 kg and 75 kg skiers in model I, respectively.

### 5.2 Optimized pacing strategy in time-trial road cycling

The optimization and simulation of time-trial road cycling was performed using model III on a course profile, replicating the 20th stage of the 2011 Tour de France. Environmental wind was considered in the model (III) which, together with other model input, aimed to replicate the actual circumstances of the competition. The simulated athlete was supposed to resemble the race winner. The athlete’s aerobic and anaerobic sources of energy were approximated with the critical power model for intermittent exercise (Morton & Billat, 2004), see section 4.1 and Paper III. The average power output depends on the finishing time of the athlete, but it reached approximately 493 W, in the crank spindle.

Figure 9 shows the optimized power distribution with no environmental wind, and Figure 10 presents the corresponding iteration history. The finishing time of this simulation was 3206 s, representing an average speed of 13.26 m·s\(^{-1}\). This is 46 s (1.4%) faster than the finishing time for an even power distribution equal to average power output at 493 W.
Figure 9. Optimized pacing strategy from simulating the winner of the 20th stage of the 2011 Tour de France without environmental wind.

Figure 10. Iteration history for the optimization of pacing strategy without environmental wind.

Figure 11 shows the optimized power distribution, considering environmental wind and Figure 12 presents the corresponding iteration history. The finishing time for this simulation was 3381 s, representing an average speed of 12.57 m·s⁻¹. This is 53 s (1.5%) faster than the finishing time for an even power distribution equal to average power output at 493 W.
Figure 11.  Optimized pacing strategy from simulating the winner of the 20th stage of the 2011 Tour de France with a constant environmental wind of 7.5 m·s⁻¹.

Figure 12.  Iteration history for the optimization of pacing strategy considering environmental wind.

Two simulations were carried out, including the optimization of the power distribution and environmental wind, in order to investigate the influence of the coefficient of rolling resistance to time-trial road cycling performance. Rolling resistance coefficients of $C_{RR} = 0.0020$ (Kyle, 1996) and $C_{RR} = 0.0080$ (Pugh, 1974) were chosen because these quantities seem to represent the limit of low and high
rolling resistance on a tarmac road. Course profile, course inclination, optimized power distributions, available anaerobic work capacities, pacing strategies and environmental winds for the different rolling resistance coefficients are presented in Figures 13 and 14.

When optimizing the pacing strategy with a rolling resistance coefficient of $C_{RR} = 0.0020$, the finishing time was 3206 s, representing an average speed of 13.27 m·s$^{-1}$.

![Figure 13. Optimized pacing strategy from simulating the winner of the 20th stage of the 2011 Tour de France with a constant environmental wind of 7.5 m·s$^{-1}$ and a rolling resistance coefficient of $C_{RR} = 0.0020$.](image)

When optimizing the pacing strategy with a rolling resistance coefficient of $C_{RR} = 0.0080$, the finishing time was 3503 s, representing an average speed of 12.13 m·s$^{-1}$. This means that there is an advantage of 297 s (8.5%) with a rolling resistance coefficient of 0.0020 compared to 0.0080.
Figure 14. Optimized pacing strategy from simulating the winner of the 20th stage of the 2011 Tour de France with a constant environmental wind of 7.5 m·s⁻¹ and a rolling resistance coefficient of $C_{RR} = 0.0080$.

The effect of aerodynamics was also investigated, but with minor changes to the input. The finishing time was compared for the original optimization including environmental wind (Figure 11) and an optimization where the projected frontal area was kept constant. The frontal area was set to the value representing the traditional aero position, with arms separated on the base-bar (Figure 4, right). That means that the frontal area is 21.8% larger in all cases except when travelling faster than 19.44 m·s⁻¹. The optimized pacing strategy for the traditional aero position is presented in Figure 15.
Figure 15. Optimized pacing strategy from simulating the winner of the 20th stage of the 2011 Tour de France with a constant environmental wind of 7.5 m·s\(^{-1}\) and a projected frontal area of \(A = 0.459\) m\(^2\).

The traditional aero position proved to be 203 s (5.7\%) slower than the modern aero position in the optimization simulations.
6. DISCUSSION AND CONCLUSIONS

The results of this thesis confirm that a variable pacing strategy is beneficial in changeable external conditions (Atkinson & Brunskill, 2000; Atkinson, et al., 2007; Cangley, et al., 2011; Gordon, 2005; Swain, 1997). The time savings from utilizing a variable pacing strategy compared to an even distribution of power were 1.4 to 1.5% for model III. Cangley et al. (2011) reported time savings from field tests of about 2.9%, also in time-trial road cycling. The roughly double gain of time in that study seems reasonable, considering the nearly halved average power output generated compared to the simulations presented in the foregoing section. It is well known that athletes who generate low power outputs will have a relatively larger gain from any mechanical or physiological improvement than the more trained athlete, who generates a greater power output. This is due to the exponential behavior of the aerodynamic drag, making time gains more costly at high speeds.

The accuracy of the simulations can never exceed the machine precision. In fact, the accuracy also depends on the number and order of operations but also the formulation of equations in the program. Furthermore, the accuracy of the input data is also essential to the accuracy of the solution. In some cases, this data is estimated or guessed and that may have a significant effect on the results.

The number and the intervals between optimization variables are also a major factor when it comes to simulation accuracy. The power output between variables is calculated as the linear interpolation of the two surrounding variables. Thus, there are minor possibilities for the optimization algorithm to react to external force changes that appear between two optimization variables. To solve this problem, one can increase the number of variables where there are known changes in external forces. However this will affect the process time (CPU time) which was about 30 min/iteration for model III on a modern lap-top computer. Another way would be to use cubical splines instead of linear interpolations between power output variables. However, initial simulations have not generated sufficiently good results. Small changes in the variables can make large alterations in the shape of the splines, which may inhibit the stability of the optimization progress.
7. FUTURE DEVELOPMENT

Future work will involve the development of more sophisticated constraints, concerning the human energetic system. Modeling aerobic and anaerobic energy expenditure as well as internal work would likely add validity. It would also be of great interest to incorporate gears into the model. This might be of greatest importance when it comes to cross-country skiing, where there are only a few. This enables the optimization of shifting between gears, which can be very useful to athletes seeking to optimize their racing strategy and technique.

The model presented in this thesis is constrained to a two-dimensional course. That means no inertial forces in the lateral direction or any braking forces in sharp turns are considered. In cross-country skiing, this is a minor shortcoming but in road cycling where the speed is higher and many roads include several 90 degree turns, this can be a major drawback of the model. To allow the athlete in the model to brake should be possible to program in future versions. However, there must be a connection between the maximum potential speed through the turn and the braking, to make it acceptable.

As discussed in the preceding section, the interval between variables is critical when it comes to accuracy. By doing case-specific simulations where only a part of the course is studied, one might realize a more detailed pacing strategy for that certain course section. In this kind of study, the variables can be spaced tighter to reach more accurate results.

Probably the most important contribution to the model would be a validating study that incorporates field testing. It is of course important to know if the model is valid to its area of application.
8. REFERENCES


