Dynamics of metallic dust particles in tokamak edge plasmas

Ladislas Vignitchouk

Master of Science Thesis
Stockholm, Sweden 2013
Dynamics of metallic dust particles in tokamak edge plasmas

LADISLAS VIGNITCHOUK

Master of Science Thesis
Stockholm, Sweden, 2013
XR-EE-SPP 2013:001
Acknowledgments

I would like to thank Prof. Svetlana Ratynskaia for her exemplary supervision and all the time she devoted to help me with this project. I am also very grateful to Jean-Philippe Banon for his everyday help and for having shared the numerous hours of computer frustration inherent to the development of a numerical code.

I wish to express my gratitude to Dr. Carmine Castaldo and Dr. Panagiotis Tolias for the fruitful discussions regarding physical models and their help in the search for relevant literature.

Prof. Henric Bergsäker, Prof. Thomas Jonsson and Igor Bykov have significantly contributed to this work by providing valuable guidance regarding experimental results and measurements.

Finally, I would like to thank the Istituto di Fisica del Plasma in Milan, and more specifically Prof. Enzo Lazzaro, Dr. Igor Proverbio and Federico Nespoli, for allowing me to continue the work they have done on the numerical code.
Abstract

The study of dust dynamics in tokamaks has been carried out by means of the DDFTU numerical code solving the coupled equations of motion, charging and heat balance for a dust grain immersed in plasmas with given profiles. The code has been updated to include (i) a non-steady state heat balance model and phase transitions, (ii) geometrical properties of the vessel such as gaps, (iii) realistic boundary conditions for dust-wall collisions. The models for secondary electron emission (SEE), thermionic emission and black body radiation have also been refined, and sensitivity of the results to the SEE strength is demonstrated.

The DDFTU code has been used for the first time to explore a large range of initial conditions (position, velocity and radius) for dust grains of various tokamak-relevant materials. This study confirmed the impact of the drag force as one of the main factors in dust dynamics and allowed to estimate average lifetimes, to locate preferred sites for dust deposition and to judge the sensitivity to initial conditions. This is a first step towards the use of the code as a predictive tool for devices of importance, such as JET and ITER.

Preliminary simulations of scenarios relevant for dust injection experiments in TEXTOR have yielded results in remarkable agreement with experimental data.

These preliminary studies allowed to identify the most crucial issues affecting dust dynamics, lifetime, deposition rate and contribution to impurities, which are to be pursued in future studies.
Contents

List of figures 8
List of tables 9
Nomenclature 10

Introduction 11

I Physical models and their implementation 13
  1 OML theory framework 14
  2 Charging processes 16
    2.1 Ambient plasma currents 16
    2.2 Secondary electron emission current 17
    2.3 Thermionic emission current 20
    2.4 Instantaneous charging hypothesis 22
  3 Heating processes 24
    3.1 Ambient plasma 25
    3.2 Secondary electron emission 27
    3.3 Thermionic emission 27
    3.4 Black body emission 27
    3.5 Phase transitions 28
      3.5.1 Melting 28
      3.5.2 Sublimation and evaporation 29
    3.6 Characteristic heating time 31
  4 Dust dynamics 34
    4.1 Coordinate systems 34
    4.2 Forces acting on the dust particle 35
      4.2.1 Collection drag 36
      4.2.2 Orbital drag 36
    4.3 Interaction with the wall 39
Contents

5 Plasma flow
  5.1 Plasma core ................................................. 41
  5.2 SOL plasma ................................................. 42

6 Numerical treatment ........................................... 43
  6.1 Differential system ....................................... 43
  6.2 Conditions for trajectory termination .................. 44

7 Wall geometry ................................................. 45
  7.1 Square gaps ................................................. 45
  7.2 Triangular gaps ............................................. 46
  7.3 Circular gaps ................................................. 47

II Simulations of FTU and TEXTOR scenarios .................. 48

8 FTU simulations ............................................... 49
  8.1 Profiles ..................................................... 49
    8.1.1 Electromagnetic field ................................ 49
    8.1.2 Plasma parameters ..................................... 51
    8.1.3 Plasma flow ........................................... 51
  8.2 Results ...................................................... 54
    8.2.1 Typical trajectories .................................. 54
    8.2.2 Preliminary study of dust size and velocity distributions and material effects .................. 57

9 TEXTOR simulations ........................................... 64
  9.1 Dust injection experiments ................................ 64
  9.2 Profiles ..................................................... 64
    9.2.1 Electromagnetic field ................................ 66
    9.2.2 Plasma parameters ..................................... 67
    9.2.3 Plasma flow ........................................... 67
  9.3 First comparison with calibrated dust injection experiments ........................................... 67

Conclusions and outlook ........................................ 70

Bibliography ..................................................... 72

Appendix .......................................................... 76

A Ambient plasma currents ...................................... 76
  A.1 Electron current ............................................ 76
  A.2 Ion current ................................................ 76
List of Figures

1.1 Collection of one plasma particle attracted by a spherical dust grain . . . . . . . 15
2.1 SEE corrective factor . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 19
2.2 Typical scaling of the charging time with the dust radius . . . . . . . . . . . . 23
3.1 Molar heat capacity and molar enthalpy . . . . . . . . . . . . . . . . . . . . . . 26
3.2 Tungsten dust emissivity . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 28
3.3 Vapour pressure . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 30
3.4 Enthalpy of gaseous phase transition . . . . . . . . . . . . . . . . . . . . . . . . 32
3.5 Relaxation towards thermal equilibrium . . . . . . . . . . . . . . . . . . . . . . . 33
4.1 The two coordinate systems. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 35
4.2 Orbital Coulomb collision between one plasma particle and the dust grain. . . . 37
4.3 Critical velocities and normal restitution coefficient models . . . . . . . . . . . . 40
6.1 Main loop in the numerical algorithm. . . . . . . . . . . . . . . . . . . . . . . . 44
7.1 The three gap shapes and their geometric parameters. . . . . . . . . . . . . . . . 46
8.1 Electromagnetic field profiles in FTU. . . . . . . . . . . . . . . . . . . . . . . . . 50
8.2 Plasma profiles in FTU. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 52
8.3 Flow profiles in FTU. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 53
8.4 Typical trajectory of vaporised dust in FTU . . . . . . . . . . . . . . . . . . . . 55
8.5 Typical trajectory of sticking dust in FTU . . . . . . . . . . . . . . . . . . . . . . 56
8.6 Influence of SEE on beryllium dust particles of initial radius 2 µm in FTU. . . 59
8.7 Influence of SEE on beryllium dust particles of initial radius 6 µm in FTU. . . 60
8.8 Influence of SEE on tungsten dust particles of initial radius 2 µm in FTU. . . 61
8.9 Influence of SEE on tungsten dust particles of initial radius 6 µm in FTU. . . 62
8.10 Average lifetime of beryllium and tungsten dust in FTU . . . . . . . . . . . . . 63
9.1 Dust collector used in TEXTOR experiments . . . . . . . . . . . . . . . . . . . . 65
9.2 Tungsten dust collected in TEXTOR . . . . . . . . . . . . . . . . . . . . . . . . 65
9.3 Electromagnetic field profiles in TEXTOR. . . . . . . . . . . . . . . . . . . . . . 66
9.4 Plasma profiles in TEXTOR. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 67
9.5 Simulated trajectories in TEXTOR. . . . . . . . . . . . . . . . . . . . . . . . . . 69
List of Tables

2.1 Material properties relative to secondary electron and thermionic emissions .... 21
3.1 General material properties ................................................. 24
3.2 Melting point material properties ........................................... 29
3.3 Enthalpy of gaseous phase transition and fit coefficients for the vapour pressure 31
### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_s$</td>
<td>Property $X$ of any plasma species $s$</td>
</tr>
<tr>
<td>$X_e$</td>
<td>Property $X$ of plasma electrons</td>
</tr>
<tr>
<td>$X_z$</td>
<td>Property $X$ of any plasma ion species $z$</td>
</tr>
<tr>
<td>$X_i$</td>
<td>Property $X$ of plasma primary ions</td>
</tr>
<tr>
<td>$X_{imp}$</td>
<td>Property $X$ of plasma impurity ions</td>
</tr>
<tr>
<td>$X_n$</td>
<td>Property $X$ of plasma neutrals</td>
</tr>
<tr>
<td>$X_d$</td>
<td>Property $X$ of dust</td>
</tr>
</tbody>
</table>
Introduction

Thermonuclear fusion is one of the most promising energy sources currently being developed for the future. Whereas today’s nuclear reactors follow the principle of nuclear fission and retrieve the energy released by breaking heavy nuclei such as uranium, thermonuclear fusion is the dual process in which the desired energy is made available by fusing two light nuclei. Although similar in principle, these two processes require very different conditions to be performed and fusion technologies remain at the experimental stage. To allow thermonuclear fusion, gases must be heated to extremely high temperatures, so that the kinetic energy of the positively charged nuclei is large enough to overcome their electrostatic repulsion. In such conditions, atoms are fully ionised and matter is in the plasma state. Future fusion reactors will have to fulfill three roles: to heat and confine a plasma well enough to ignite sustainable fusion reactions, and to retrieve the released energy.

There exist two main processes that allow to confine a plasma on Earth: magnetic confinement and inertial confinement. Magnetic confinement, where the plasma is trapped in a closed volume by magnetic fields whose magnitude can reach several tesla, currently is the most researched process. Such a confinement is realised in toroidal chambers surrounded by strong electromagnets. Depending on the characteristics of the magnetic field, these devices may be designated by different names such as tokamak, reversed field pinch or stellarator. Tokamaks currently constitute the most promising design and are the focus of the work presented here.

Experiments performed on the various existing tokamaks have shown that the presence of dust – extraneous particles whose size can range from a few nanometers to several hundreds of micrometers – in the plasma chamber is of critical importance in the operation of the discharge. Dust is inevitably produced in tokamaks because of the impossibility to achieve a perfect confinement. The tokamak wall is always in contact with hot plasma and is therefore subject to various electromagnetic and thermomechanical stresses. Dust particles released due to these processes are free to migrate in the plasma chamber, effectively modifying the physical properties of the surrounding plasma. Some experiments have shown that this modification can be strong enough to cause the plasma discharge termination. The issue of dust in tokamaks also has important implications regarding the security of the future reactors. Dust particles may indeed retain radioactive fuel elements such as tritium, which causes the overall radioactivity in the reactor to increase and might be a source of radioactive contamination of the environment, should an incident occur. The accidental release of hydrogenated dust in the air also increases the risk of explosion.
In order to improve the understanding of dust-related phenomena in tokamak plasmas, several simulation codes have been created over the last decade. The work presented here revolves around one of these codes, the DDFTU code, originally developed by the Istituto di Fisica del Plasma in Milan to help to model dust dynamics in the Frascati Tokamak Upgrade (FTU) device. The DDFTU code aims at simulating the trajectory of a spherical dust grain in a limiter tokamak.

Originally designed to treat iron dust in FTU, it has been updated to be able to handle other dust materials and limiter tokamaks. In the course of this work, five fusion-relevant metals have been selected: beryllium, iron, nickel, molybdenum and tungsten. The following sections aim at presenting the various models used to perform the simulations as well as the results obtained with conditions corresponding to FTU and TEXTOR. In addition to the computation of typical dust trajectories, the DDFTU code can be used to perform predictions such as the preferred sites of dust deposition or the average lifetime of dust particles.

A detailed description of the various physical models used in the code to predict the behaviour of dust particles in tokamaks is given in Part I. Then, Part II presents various simulations performed with the code in an environment matching FTU and TEXTOR conditions.
Part I

Physical models and their implementation
Chapter 1

OML theory framework

The first general hypothesis in the DDFTU code is that dust particles are sparse in the plasma. Consequently, it is assumed that all interactions between dust grains can be neglected and that each dust grain can be treated individually. The interaction between the dust particle and the plasma is then assumed to follow the orbital motion limited (OML) theory, in which plasma sheath effects are neglected. An initially neutral and immobile test particle immersed in a plasma will start to collect plasma particles and become electrically charged. This electric charge will in turn modify the collected flow of plasma particles and eventually lead to an equilibrium situation where the net electric current received by the particle is zero. The electric potential corresponding to this equilibrium situation is known as the floating potential and, in absence of emission processes, is typically negative with respect to the surrounding plasma potential, due to the higher mobility of the electrons with respect to ions. As a consequence, a positively charged region known as the plasma sheath will form around the dust grain and shield its negative charge to far-away particles. The electric potential near the dust grain can then be modelled by the screened Coulomb potential formula

\[ \varphi(r) = \frac{q_d}{4\pi\varepsilon_0 r} \exp \left( -\frac{r}{\lambda_D} \right), \]  

where \( r \) is the distance to the center of the dust particle, \( q_d \) is the particle electric charge and \( \lambda_D = \sqrt{\frac{\varepsilon_0 k_B T_e}{ne^2}} \) is the Debye length of the plasma, defined for the electron density \( n_e \) and temperature \( T_e \).

The leading assumption in OML theory is that the radius \( R_d \) of the dust grain and the characteristic length scale of all studied phenomena are small with respect to \( \lambda_D \), so that Eq. (1.1) can be rewritten as

\[ \varphi(r) = \frac{q_d}{4\pi\varepsilon_0} \frac{R_d}{r} \varphi_d, \]  

which is the classical Coulomb potential, with \( \varphi_d = \frac{q_d}{4\pi\varepsilon_0 R_d} \) being the dust electric potential. In typical Scrape-Off Layer (SOL) plasmas, the Debye length is of the order of several tens of micrometers, making the OML approximation suitable for micrometer-sized dust particles,
although some corrections might be needed in the case of bigger particles.

Under the OML approximation, one can easily compute the cross-section for the collection of plasma particles by the dust grain by placing oneself in the reference frame where the grain of given radius $R_d$, mass $M_d$ and potential $\varphi_d$ is immobile. One can then consider a plasma particle of mass $m_s \ll M_d$ and charge $Z_s e$ coming from infinity with an initial kinetic energy $E$ and impact parameter $b$, as shown in Fig. 1.1.

Applying Newton’s second law, one finds that the plasma particle is collected if and only if $E \geq Z_s e \varphi_d$ and $b \leq b_0 = R_d \sqrt{1 - \frac{Z_s e \varphi_d}{E}}$ so that

$$\sigma(E, \varphi_d) = \pi b_0^2 = \begin{cases} \pi R_d^2 \left(1 - \frac{Z_s e \varphi_d}{E}\right) & \text{if } E \geq Z_s e \varphi_d \\ 0 & \text{if } E \leq Z_s e \varphi_d \end{cases}$$

(1.3)

is the cross-section for the collection of plasma particles of charge $Z_s e$ and initial kinetic energy $E$.

Figure 1.1: Collection of one plasma particle attracted by a spherical dust grain
Chapter 2
Charging processes

As mentioned in Chap. 1, the incoming flow of plasma particles towards the dust grain results in a variation of its electric charge, following the equation

$$\frac{dq_d}{dt} = I_{tot} = I_e + I_i + I_{imp} + I_{SEE} + I_{TI}, \quad (2.1)$$

where $I_{tot}$ is the total electric current, due to the charged particles collected and emitted by the dust grain. Indeed, phenomena such as secondary electron emission or thermionic emission have to be taken into account in order to predict the evolution of $q_d$. Analytical expressions of these currents in immobile and flowing Maxwellian plasmas can be derived within the OML framework, as shown below.

2.1 Ambient plasma currents

Following the reasoning of Smirnov et al [1], the number $dN_s$ of particles of a given plasma species of charge $Z_se$ and mass $m_s$ with initial kinetic energy $E$ collected by the dust grain during $dt$ can be expressed using the OML cross section given by Eq. (1.3)

$$dN_s = \sigma(E(v), \varphi_d)v f_s(v)d^3v dt, \quad (2.2)$$

where $v = \sqrt{\frac{2E}{m_s}}$ is the initial velocity of the plasma particle and $f_s(v)$ is the distribution function of the plasma species $s$ in the three-dimensional velocity space, unperturbed by the dust grain and in the reference frame where the dust grain is immobile. The current due to collected particles can then be calculated as

$$I_s = Z_se \int \frac{dN_s}{dt} = Z_se \int \sigma(E(v), \varphi_d)v f_s(v)d^3v. \quad (2.3)$$

Since tokamak plasmas are flowing, the flow velocity $v_{fs}$ of the plasma species with respect to the dust must be taken into account in their distribution function, which is assumed to be a shifted Maxwellian, given by

$$f_s(v) = n_s \left( \frac{m_s}{2\pi k_BT_s} \right)^{3/2} \exp \left( -\frac{m_s|v - v_{fs}|^2}{2k_BT_s} \right), \quad (2.4)$$
2.2. Secondary electron emission current

where \( k_B \) is the Boltzmann constant, \( n_s \) is the density of plasma species and \( T_s \), its temperature. Since the thermal velocity of the electrons is much larger than their flow velocity, \( v_{fe} \) can be assumed to be zero. This, however, does not apply to the ionic species. The calculations, detailed in Appendix A, yield the following results for the ambient plasma currents, as presented in Ref. [1],

\[
I_{\chi d \leq 0} = -2\sqrt{\pi}en_e v_{Te} R_d^2 (1 - \chi_d) \tag{2.5}
\]

\[
I_{\chi d > 0} = -2\sqrt{\pi}en_e v_{Te} R_d^2 \exp(-\chi_d) \tag{2.6}
\]

\[
I_{\chi d \leq 0} = Z_s en_i v_{Te} R_d^2 \left[ \sqrt{\pi} \left( \text{erf}(u_{z+}) + \text{erf}(u_{z-}) \right) \left( 1 + 2u_z^2 \frac{Z_s \chi_d}{\tau_s} \right) \right] \tag{2.7}
\]

\[
I_{\chi d > 0} = Z_s en_i v_{Te} R_d^2 \left[ \sqrt{\pi} \text{erf}(u_z) \left( 1 + 2u_z^2 + 2 \frac{Z_s \chi_d}{\tau_s} \right) + 2u_z \exp(-u_z^2) \right] \tag{2.8}
\]

where \( v_{Te} = \sqrt{\frac{2k_B T_e}{m_e}} \) is the thermal velocity of the plasma species \( s \), \( \tau_s = \frac{T_s}{T_e} \) is the normalised species temperature with respect to the electron temperature, \( u_s = \frac{v_{fs}}{v_{Te}} \) is the normalised species flow velocity with respect to its thermal velocity, \( u_{s\pm} = u_s \pm \sqrt{\frac{Z_s \chi_d}{\tau_s}} \) and \( \chi_d \) is the normalised dust potential, defined by

\[
\chi_d = -\frac{e \varphi_d}{k_B T_e}, \tag{2.9}
\]

so that a positive value of \( \chi_d \) corresponds to the case of attracted positive charges, or a negative value of \( \varphi_d \).

2.2 Secondary electron emission current

Incoming electrons with a high enough kinetic energy will induce the emission of so-called secondary electrons from the dust grain. The resulting incoming positive current has to be taken into account in the charging equation. The secondary electron emission (SEE) can be modelled through the use of the SEE yield \( \delta(E, \alpha) \) which corresponds to the number of secondary electrons the dust material emits when it is hit by an electron of kinetic energy \( E \) with an incidence angle \( \alpha \). Coming back to reasoning developed in Chap. 1, if a collected electron has an impact parameter equal to \( b \), it hits the dust grain with an incidence angle \( \alpha \) given by \( \sin \alpha = \frac{b}{m} \). Thus, the number \( dN_e \) of electrons of initial energy between \( E \) and \( E + dE \) that hit the dust surface with an incidence angle between \( \alpha \) and \( \alpha + d\alpha \) during \( dt \) is given by

\[
dN_e = f_e(E) 2\pi b v dE dt = f_e(E) 2\pi b \sqrt{\frac{2E}{m_e}} dB dE dt = f_e(E) 2\pi b_0^2 \sin \alpha \cos \alpha \sqrt{\frac{2E}{m_e}} d\alpha dB dE dt, \tag{2.10}
\]
where \( f_e(E) \) is the electron energy distribution in the plasma. The SEE current \( I_{\text{SEE}} \) can then be computed by integrating \( e \, dN_e/dt \) over the SEE yield (angles and energies). However, when the dust is positively charged, some secondary electrons will be pulled back and recollected by the dust. Since such electrons will not contribute to the net SEE current, one must take into account a corrective factor \( \rho \) which depends \textit{a priori} on \( \chi_d, E \) and \( \alpha \), and is less than 1 when \( \chi_d \leq 0 \). Recalling that incoming electrons of initial kinetic energy \( E \) hit the dust grain with a kinetic energy \( E - \chi_d k_B T_e \), the general expression for SEE current under the OML assumption is

\[
I_{\text{SEE}} = 2 \pi R_d^2 e \sqrt{\frac{2}{m_e}} \int_{E_0}^{\infty} dE \int_{0}^{\pi/2} d\alpha f_e(E) \left( 1 - \frac{\chi_d k_B T_e}{E} \right) \rho \sin \alpha \cos \alpha \sqrt{E \delta(E - \chi_d k_B T_e, \alpha)},
\]

(2.11)

where \( E_0 = \max(0, \chi_d k_B T_e) \). Assuming that the SEE is isotropic, if the energy distribution \( f_{\text{SEE}}(E) \) of secondary electrons is known, then the SEE current escaping the particle is given by

\[
I_{\text{esc}}^{\text{SEE}} = 4 \pi R_d^2 e \int_{E_{\text{crit}}}^{\infty} \sqrt{\frac{2E}{m_e}} f_{\text{SEE}}(E) dE,
\]

(2.12)

where \( E_{\text{crit}} = \max(0, -\chi_d k_B T_e) \) is the critical energy the SEE electrons must possess to overcome the potential barrier and escape. Following Chung and Everhart \[2\] the energy distribution function for secondary emitted electrons from a metal – measured from the critical energy necessary for the electrons to leave the metal itself – does not depend on the energy and incidence angle of primary electrons as long as the secondary electrons have a relatively low energy – which is assumed here – and is of the form

\[
f_{\text{SEE}}(E) \propto \frac{E}{(E + W)^4},
\]

(2.13)

where \( W \) is the work function of the metal. Assuming that the SEE is isotropic, one can then write

\[
\rho = \frac{I_{\text{esc}}^{\text{SEE}}}{I_{\text{SEE}}} = \frac{4 \pi R_d^2 e \int_{E_{\text{crit}}}^{\infty} f_{\text{SEE}}(E) v(E) dE}{4 \pi R_d^2 e \int_{0}^{\infty} f_{\text{SEE}}(E) v(E) dE} = \frac{\int_{E_{\text{crit}}}^{\infty} \frac{E \sqrt{E}}{(E + W)^4} dE}{\int_{0}^{\infty} \frac{E \sqrt{E}}{(E + W)^4} dE},
\]

(2.14)

which can be computed analytically for \( \chi_d \leq 0 \), leading to

\[
\rho = 1 - \frac{2}{\pi} \arctan \left( \sqrt{-\zeta} \right) + \frac{2}{3 \pi (1 - \zeta)} \sqrt{-\zeta} (3 - 8 \zeta - 3 \zeta^2),
\]

(2.15)

where

\[
\zeta = \frac{\chi_d k_B T_e}{W}.
\]

(2.16)

Prokopenko and Laframboise \[3\] derived a different expression \( \rho_{PL} = \left( 1 - \frac{\chi_d T_e}{T_{\text{SEE}}} \right) \exp \left( \frac{\chi_d T_e}{T_{\text{SEE}}} \right) \) assuming a Maxwellian energy distribution of temperature \( T_{\text{SEE}} = T_d \) for the secondary electrons. Smirnov \textit{et al} \[1\] have adopted this Maxwellian expression, but with the more realistic
2.2. Secondary electron emission current

Figure 2.1: The SEE corrective factor $\rho(\chi_d)$ depending on the energy distribution of secondary electrons, with $T_e = 10$ eV, $W = 4.7$ eV and $k_B T_{\text{SEE}} = 2W$.

The energy part of the SEE yield is fairly well modelled by Kollath’s semi-empirical formula \[ \delta_E(E) = \delta_m \frac{E}{E_m} \exp \left( 2 - 2 \sqrt{\frac{E}{E_m}} \right) , \] (2.18)
where $\delta_m$ and $E_m$ refer to the maximum SEE yield at normal incidence and the incident energy at which this maximum occurs, respectively. Neglecting the angular dependence of the SEE yield corresponds to taking $\delta_\alpha(\alpha) = 1$, which leads to the formula derived for instance by Meyer-Vernet \[6\]. Here, the angular part of the SEE yield is instead modelled by \[ \delta_\alpha(\alpha) = (\cos \alpha)^{-\beta} , \] (2.19)
where $0 \leq \beta < 2$ depends on the dust material [5]. Then, assuming a Maxwellian distribution for the electrons in the plasma, that is \( f_e(E) = \frac{2}{\sqrt{\pi}} n_e (k_B T_e)^{-3/2} \sqrt{E} \exp \left( -\frac{E}{k_B T_e} \right) \), calculations yield

\[
I_{\text{SEE}} = \frac{4\sqrt{\pi} R_d^2}{2 - \beta} \rho e \sqrt{\frac{2}{m_e}} n_e (k_B T_e)^{-3/2} \frac{\delta_m}{E_m} \int_{E_m}^{\infty} (E - \chi_d k_B T_e)^2 \exp \left( 2 - \frac{E}{k_B T_e} - 2 \sqrt{\frac{E - \chi_d k_B T_e}{E_m}} \right) dE,
\]

which becomes, after substituting \( \varepsilon = s + \sqrt{\frac{E - \chi_d k_B T_e}{k_B T_e}} \) and \( s = \sqrt{\frac{k_B T_e}{E_m}} \),

\[
I_{\text{SEE}} = \frac{8\sqrt{\pi} R_d^2}{2 - \beta} \rho e \sqrt{\frac{2}{m_e}} n_e (k_B T_e)^{3/2} \exp(2) \frac{\delta_m}{E_m} \exp(s^2 - \chi_d) \int_{s + s'}^{\infty} \exp(-\varepsilon^2) d\varepsilon,
\]

where \( s' = \sqrt{\text{max}(0, -\chi_d)} \). The Gaussian integral can then be performed analytically to get

\[
I_{\text{SEE}}^{\chi_d \leq 0} = e n_e v_{te} R_d^2 \frac{\delta_m \sqrt{\pi}}{2 - \beta} \rho \exp(2 - \chi_d) \left[ \exp(\chi_d - 2s\sqrt{-\chi_d}) \left( 4s^2(\chi_d^2 - 2\chi_d + 2) - 2s^3\sqrt{-\chi_d}(7 - 2\chi_d) + 2s^4(9 - 2\chi_d) - 4s^5\sqrt{-\chi_d} + 4s^6 \right) - \sqrt{\pi} \exp(s^2) \left( 1 - \text{erf}(s + \sqrt{-\chi_d}) \right) \left( 15s^3 + 20s^5 + 4s^7 \right) \right]
\]

\[
I_{\text{SEE}}^{\chi_d \geq 0} = e n_e v_{te} R_d^2 \frac{\delta_m \sqrt{\pi}}{2 - \beta} \rho \exp(2 - \chi_d) \left[ 8s^2 + 18s^4 + 4s^6 - \sqrt{\pi} \exp(s^2) \left( 1 - \text{erf}(s) \right) \left( 15s^3 + 20s^5 + 4s^7 \right) \right].
\]

The values of \( \delta_m, E_m \) and \( W \) for the materials considered in this work are reported in Table 2.1. The value of \( \beta \) is more problematic as some experimental results, such as those presented in Refs. [7] and [8], appear to contradict the values predicted by the theory. Since a clear consensus does not seem to exist, the choice is made here to let \( \beta \) as a parameter, whose value can range from 0.4 – that is the value measured for carbon dust by Pedgley and McCracken [8] – to the theoretical value presented in Ref. [5]. Future comparisons with experimental data (for example videos where the termination of dust trajectories due to evaporation is visible) should allow to help with the choice of appropriate values of \( \beta \).

2.3 Thermionic emission current

As the temperature of the dust grain increases due to the various heat fluxes from the surrounding plasma, thermionic emission – that is the heat-induced emission of electrons – starts playing
a role in the grain’s charging process. The resulting incoming positive thermionic current is typically well described by Richardson’s formula [9]

$$I_{\text{Richardson}} = \lambda_R \frac{16\pi^2 R_d^2 k_B^2 T_d^2 m_e e}{h^3} \exp \left( -\frac{W}{k_B T_d} \right), \quad (2.24)$$

where $h$ is the Planck constant and $\lambda_R$ is a material-dependent factor of the order of 0.5 whose value is reported in Table 2.1. However, two additional effects must be taken into account depending on the sign of $\chi_d$. When $\chi_d \leq 0$, some of the emitted electrons will be pulled back to the dust grain. As in Sec. 2.2, a corrective factor can be calculated if the energy distribution of the emitted electrons is known. Following Ref. [10], the emitted electrons can be assumed to be Maxwellian with the dust temperature

$$f_{\text{TI}}(E) \propto \sqrt{E} \exp \left( -\frac{E}{k_B T_d} \right), \quad (2.25)$$

so that the thermionic current in case of negative normalised dust potential is

$$I_{\text{TI}}^{\chi_d \leq 0} = \lambda_R \frac{16\pi^2 R_d^2 k_B^2 T_d^2 m_e e}{h^3} (1 - \xi) \exp \left( -\frac{W}{k_B T_d} + \xi \right), \quad (2.26)$$

where

$$\xi = \frac{\chi_d T_e}{T_d}. \quad (2.27)$$

If $\chi_d \geq 0$, the negative charge of the dust grain will enhance the thermionic emission through what is known as the Schottky effect [10]. This effect appears as a reduction of the work function of the dust material by

$$\sqrt{\frac{e^2 |\chi_d|}{4\pi\varepsilon_0 R_d}} = e \sqrt{\frac{\chi_d k_B T_e}{4\pi\varepsilon_0 R_d}},$$

leading to

$$I_{\text{TI}}^{\chi_d \geq 0} = \lambda_R \frac{16\pi^2 R_d^2 k_B^2 T_d^2 m_e e}{h^3} \exp \left( -\frac{W - e \sqrt{\frac{\chi_d k_B T_e}{4\pi\varepsilon_0 R_d}}}{k_B T_d} \right). \quad (2.28)$$

<table>
<thead>
<tr>
<th>Dust material</th>
<th>$\delta_m$</th>
<th>$E_m$</th>
<th>$\beta$</th>
<th>$W$</th>
<th>$\lambda_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beryllium</td>
<td>0.50</td>
<td>200</td>
<td>0.4-1.3</td>
<td>4.98</td>
<td>0.26</td>
</tr>
<tr>
<td>Iron</td>
<td>1.30</td>
<td>400</td>
<td>0.4-1.0</td>
<td>4.74</td>
<td>0.22</td>
</tr>
<tr>
<td>Nickel</td>
<td>1.35</td>
<td>550</td>
<td>0.4-1.0</td>
<td>5.20</td>
<td>0.25</td>
</tr>
<tr>
<td>Molybdenum</td>
<td>1.25</td>
<td>375</td>
<td>0.4-1.0</td>
<td>4.57</td>
<td>0.46</td>
</tr>
<tr>
<td>Tungsten</td>
<td>1.40</td>
<td>650</td>
<td>0.4-1.0</td>
<td>4.61</td>
<td>0.50-0.83</td>
</tr>
</tbody>
</table>

Table 2.1: Material properties relative to secondary electron and thermionic emissions. $E_m$ and $W$ are given in eV.
2.4 Instantaneous charging hypothesis

As long as the plasma parameters do not change, the evolution of $q_d$ governed by Eq. (2.1) leads to an equilibrium situation where the total current received by the dust grain is zero. The characteristic time for the establishment of this equilibrium is a question of great interest when it comes to numerical simulations. Although an exact calculation is problematic, one can easily derive the order of magnitude of this charging time by neglecting flow and impurity effects, as well as SEE and thermionic current. In these conditions, the total current $I_{\text{tot}} = I_e + I_i$ has a simple analytical expression and the equilibrium charge is almost always negative, which leads to the simplified form of the charging equation

$$\frac{dq_d}{dt} = I_{\text{tot}} = 2\sqrt{\pi} e R_d^2 \left[ Z_i n_i v_{Ti} \left( 1 + \frac{Z_i \chi_d}{\tau_i} \right) - n_e v_{Te} \exp(-\chi_d) \right]$$  \hspace{1cm} (2.29)

and allows to compute the equilibrium value $\chi_d^{eq}$ of the normalised potential as the one for which the right-hand side of Eq. (2.29) vanishes. The charging time $t_{\text{charg}}$ can then be estimated by \cite{16}

$$\frac{1}{t_{\text{charg}}} = \left| \frac{dI_{\text{tot}}}{dq_d} \right|_{q_d=\chi_d^{eq}} = \frac{e}{4\pi \epsilon_0 R_d k_B T_e} \left| \frac{dI_{\text{tot}}}{d\chi_d} \right|_{\chi_d=\chi_d^{eq}},$$ \hspace{1cm} (2.30)

that is

$$t_{\text{charg}} = \frac{2\sqrt{\pi} \epsilon_0 k_B T_i}{e^2 R_d n_i v_{Ti} Z_i (\tau_i + Z_i (1 + \chi_d^{eq}))}.$$ \hspace{1cm} (2.31)

It is noteworthy that the expression of $t_{\text{charg}}$ depends mainly on the ion parameters, since the ions are slower to react than the electrons. In typical scrape-off layer plasmas with deuterium ions, $n_e = n_i \sim 10^{17}$ m$^{-3}$, $T_e \sim T_i \sim 10$ eV and $\chi_d^{eq} \sim 3$ regardless of the dust radius. This leads to the characteristic curve for $t_{\text{charg}}$ presented in Fig. 2.2. In the code, the time step for the calculations is of the order of 0.5 $\mu$s. It is therefore legitimate to assume that micrometer-sized dust grains instantaneously acquire their equilibrium charge in the plasma. Thus, instead of numerically solving Eq. (2.1), the code solves the steady-state equation

$$I_{\text{tot}}(\chi_d) = 0.$$ \hspace{1cm} (2.32)

Since the calculations presented above only allow to give the order of magnitude of the charging time, the code regularly estimates the charging time and compares it with the time step to check the validity of instantaneous charging.
Figure 2.2: Typical scaling of the charging time with the dust radius in scrape-off layer plasmas.
Chapter 3

Heating processes

The fluxes of plasma species collected by the dust grain play a major role in the variation of its temperature. This phenomenon is of great importance since it governs the possible phase transitions undergone by the dust grain, especially vaporisation, through which dust can be viewed as a source of heavy impurity ions. The main equation describing the heating process is

\[
\frac{dH_d}{dt} = Q_{\text{tot}} = Q_e + Q_i + Q_{\text{imp}} + Q_n + Q_{\text{SEE}} + Q_{\text{TI}} + Q_{\text{rad}} + Q_{\text{gas}},
\]

(3.1)

where \(H_d\) is the dust enthalpy and \(Q_{\text{tot}}\) is the total incoming heating power to the dust grain. The enthalpy \(H_d\) varies as a function of the grain’s mass \(M_d\) and temperature \(T_d\)

\[
H_d = M_d h_d = M_d \int_{T_{\text{ref}}}^{T_d} c_{pd}(T) dT,
\]

(3.2)

where \(h_d\) and \(c_{pd}\) respectively refer to the specific enthalpy and the heat capacity at constant pressure of the dust material. The room temperature \(T_{\text{ref}} = 298.15\, \text{K}\) is chosen as a reference so that the enthalpy is zero in the standard conditions. The models used for \(h_d\) and \(c_{pd}\) for the five metals of interest, as well as some of their general properties, are presented in Table 3.1 and Fig. 3.1. It should be noted that, for practical reasons, most of the specific thermodynamical properties appearing in tables and figures are given per mole, not per unit of mass.

<table>
<thead>
<tr>
<th>Dust material</th>
<th>(M_{\text{at}})</th>
<th>(\rho_{\text{sol}})</th>
<th>(\rho_{\text{liq}})</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nickel</td>
<td>58.69</td>
<td>8.91</td>
<td>7.81</td>
<td>[11]</td>
</tr>
<tr>
<td>Molybdenum</td>
<td>95.94</td>
<td>10.3</td>
<td>9.33</td>
<td>[11]</td>
</tr>
<tr>
<td>Tungsten</td>
<td>183.9</td>
<td>19.3</td>
<td>17.6</td>
<td>[11]</td>
</tr>
</tbody>
</table>

Table 3.1: General material properties. Atomic mass is given in g mol\(^{-1}\) and density is given in g cm\(^{-3}\).
3.1 Ambient plasma

The following sections describe the various heating powers that contribute to \(Q_{\text{tot}}\) and address the issue of phase transitions.

### 3.1 Ambient plasma

The power \(dQ_s\) received by the dust grain when it collects particles of a given plasma species of charge \(Z_s e\) and mass \(m_s\) with initial kinetic energy \(E\) during \(dt\) is

\[
dQ_s = (E + Z_s \chi_d k_B T_e) \frac{dN_s}{dt} = (E + Z_s \chi_d k_B T_e) \sigma(E, \chi_d) v(E) f_s(v) d^3v, \tag{3.3}
\]

where \(\sigma\) is the OML cross-section defined by Eq. (1.3) and \(dN_s\) is given in Eq. (2.2). The same reasoning as in Sec. 2.1 yields the expression for the total heating power \(Q_s\) received from a given species in flowing Maxwellian plasmas

\[
Q_s = \int_{v_0} \frac{n_s R_3^2}{v_n T_n k_B T_s} \left( \frac{1}{2} m_s v^2 + Z_s \chi_d k_B T_e \right)^2 \exp \left( -\frac{m_s |v - v_n|}{2 k_B T_s} \right) d^3v, \tag{3.4}
\]

where \(v_0 = v_n \sqrt{\max(0, -\frac{Z_s \chi_d}{\tau_z})}\) is the minimal velocity the plasma particles must have with respect to the dust to be collected. Calculations, detailed in Appendix B, yield the following analytical expressions – as presented in Ref. [1] – with the notations used in Sec. 2.1

\[
Q_{\chi_d \leq 0}^{e} = \frac{\chi_d k_B T_e}{e} I_{\chi_d \leq 0}^{e} + 2 \sqrt{\pi} R_3^2 n_e v_n k_B T_e \left( 2 - \chi_d \right) \tag{3.5}
\]

\[
Q_{\chi_d \geq 0}^{e} = \frac{\chi_d k_B T_e}{e} I_{\chi_d \geq 0}^{e} + 2 \sqrt{\pi} R_3^2 n_e v_n k_B T_e \left( 2 + \chi_d \right) \exp(-\chi_d) \tag{3.6}
\]

\[
Q_{\chi_d \leq 0}^{z} = \frac{\chi_d k_B T_e}{e} I_{\chi_d \leq 0}^{z} + 1 \frac{\pi R_3^2 n_z v_n k_B T_z}{2} \left[ \left( 5 + 2 u_z^2 - \frac{3 + 2 u_z^2}{u_z} \sqrt{-\frac{Z_s \chi_d}{\tau_z}} \right) \exp(-u_{z+}^2) \right] + \frac{1}{u_z} \left[ 3 + 12 u_z^2 + 4 u_z^4 + 2 \frac{Z_s \chi_d}{\tau_z} \right] \left[ \text{erf}(u_{z+}) + \text{erf}(u_{z-}) \right] \tag{3.7}
\]

\[
Q_{\chi_d \geq 0}^{z} = \frac{\chi_d k_B T_e}{e} I_{\chi_d \geq 0}^{z} + \frac{1}{4} \pi R_3^2 n_z v_n k_B T_z \left[ \frac{2}{\sqrt{\pi}} \left[ 5 + 2 \left( u_z^2 + \frac{Z_s \chi_d}{\tau_z} \right) \right] \exp(-u_z^2) \right] + \frac{1}{u_z} \left[ 3 + 12 u_z^2 + 4 u_z^4 + 2 \frac{Z_s \chi_d}{\tau_z} \right] \text{erf}(u_z) \tag{3.8}
\]

\[
Q_n = \frac{1}{4} \pi R_3^2 n_n v_n k_B T_n \left[ \frac{2}{\sqrt{\pi}} (5 + 2 u_n^2) \exp(-u_n^2) + \frac{1}{u_n} (3 + 12 u_n^2 + 4 u_n^4) \text{erf}(u_n) \right]. \tag{3.9}
\]
Figure 3.1: Molar heat capacity and molar enthalpy as functions of temperature, re-plotted after Ref. [17]. The peaks in the heat capacity of iron and nickel are due to ferromagnetic properties. The discontinuities in the molar enthalpy occur at the melting point.
3.2 Secondary electron emission

Secondary electron emission takes part in the heating – or rather, cooling – process of the dust grain due to the loss of kinetic energy of the emitted electrons. The emission of an electron with a kinetic energy \( E \) at the grain’s surface results in the loss of an energy \( E + W \), since \( W \) is the energy necessary to extract the electron. When \( \chi_d \leq 0 \), the electrons with initial kinetic energy \( E \leq -\chi_d k_B T_e \) will be recaptured by the dust grain and do not contribute to the cooling process. Assuming that the secondary electron distribution function is the same as in Eq. (2.13), the probability to find an electron of energy \( E \geq -\chi_d k_B T_e \) among the electrons that escape the dust grain is

\[
P_{\text{SEE}}^{\chi_d \leq 0}(E) = \int_{-\chi_d k_B T_e}^{E} \frac{E}{(E + W)^{1/2}} d\varepsilon = \frac{6W^2 (1 - \zeta)^3}{(E + W)^4} \frac{E}{1 - 3\zeta},
\]

where \( \zeta \) is defined by Eq. (2.16). The corresponding negative power input is

\[
Q_{\text{SEE}}^{\chi_d \leq 0} = -\frac{I_{\text{SEE}}}{e} \int_{-\chi_d k_B T_e}^{E} \varepsilon \left[ \frac{E}{(E + W)^{1/2}} \right] d\varepsilon = -3W \frac{(1 - \zeta)(1 - 2\zeta) I_{\text{SEE}}}{1 - 3\zeta}.
\]

When \( \chi_d \geq 0 \), the same reasoning leads to

\[
Q_{\text{SEE}}^{\chi_d \geq 0} = -\frac{I_{\text{SEE}}}{e} \int_{0}^{\infty} \frac{E(E + W)}{(E + W)^{1/2}} d\varepsilon = -3W \frac{I_{\text{SEE}}}{e}.
\]

3.3 Thermionic emission

The thermionic electrons can be treated the same way as the secondary emitted electrons. The only difference is that their probability distribution is assumed to be Maxwellian with the dust temperature.

\[
Q_{\text{Thi}}^{\chi_d \leq 0} = -\frac{\sqrt{-\pi} \xi}{\sqrt{1 - \text{erf}(\sqrt{1 - \xi})}} \left( W + \frac{3}{2} k_B T_d \right) \frac{2\xi \exp(\xi) (W + (\xi - \frac{3}{2}) k_B T_d) I_{\text{Thi}}^{\chi_d \leq 0}}{\sqrt{-\pi} \xi (1 - \text{erf}(\sqrt{1 - \xi})) - 2\xi \exp(\xi)}.
\]

\[
Q_{\text{Thi}}^{\chi_d \geq 0} = -\left( W + \frac{3}{2} k_B T_d \right) \frac{I_{\text{Thi}}^{\chi_d \geq 0}}{e},
\]

where \( \xi \) is defined by Eq. (2.27).

3.4 Black body emission

Black body emission is the dominant cooling process when the dust temperature is above the melting point. The corresponding negative power input can be estimated by the Stefan-Boltzmann law
Chapter 3. Heating processes

\[ Q_{\text{rad}} = -4\pi R_d^2 \varepsilon \sigma_{SB} \left( T_d^4 - T_{\text{wall}}^4 \right), \]  

where \( \sigma_{SB} \) refers to the Stefan-Boltzmann constant and \( T_{\text{wall}} \) is the temperature of the tokamak wall. The emissivity \( \varepsilon \) of the dust grain is a function of \( R_d \) and \( T_d \). Its modelling is based on Mie theory and the Drude approximation \([18, 19, 20]\) and is detailed in Ref. \([21]\). As an example, some emissivity curves for tungsten are shown in Fig. 3.2.

3.5 Phase transitions

Given that the temperature of tokamak plasmas typically exceeds 10 eV inside the Last Closed Magnetic Surface (LCMS), the immersed dust grains undergo phase transitions rather quickly once they are in the vicinity of the LCMS. Whereas sublimation is the only phenomenon of interest for carbon dust, metallic dust is also subject to melting and evaporation. Since phase transitions are mathematically translated into discontinuities of some thermodynamical properties – namely \( c_{pd} \) and \( h_d \) – dedicated models must be used.

3.5.1 Melting

When no phase transitions are occurring, Eq. (3.1) can be rewritten in terms of dust temperature and specific enthalpy

\[ M_d c_{pd} \frac{dT_d}{dt} + h_d \frac{dM_d}{dt} = Q_{\text{tot}}. \]  

Figure 3.2: Tungsten dust emissivity as a function of dust radius and temperature, adopted from Ref. \([21]\).
Dust material & $T_{\text{melt}}$ & $c_{\text{pd, sol}}(T_{\text{melt}})$ & $c_{\text{pd, liq}}(T_{\text{melt}})$ & $h_{\text{sol}}(T_{\text{melt}})$ & $h_{\text{liq}}(T_{\text{melt}})$ \\
Beryllium & 1560 & 32.2 & 28.8 & 33048 & 47725 \\
Iron & 1811 & 41.3 & 46.0 & 58660 & 72467 \\
Nickel & 1728 & 38.8 & 43.1 & 47449 & 64927 \\
Molybdenum & 2896 & 53.2 & 42.6 & 89540 & 127020 \\
Tungsten & 3695 & 53.7 & 54.0 & 116780 & 169094 \\

Table 3.2: Melting point material properties. $T_{\text{melt}}$ is given in K, heat capacity is given in J mol$^{-1}$ K$^{-1}$ and specific enthalpy is given in J mol$^{-1}$. The specific enthalpy of fusion is given by $\Delta h_{\text{melt}} = h_{\text{liq}}(T_{\text{melt}}) - h_{\text{sol}}(T_{\text{melt}})$.

However, when the dust grain is heated to its melting point, its temperature stops evolving as long as the phase transition is not complete. This can be modelled by the introduction of the specific molten fraction $\psi_d$ of the dust grain. Eq. (3.16) can then be replaced by

$$\frac{d}{dt}(M_d\psi_d) = \frac{Q_{\text{tot}}}{\Delta h_{\text{melt}}}$$

and $T_d = T_{\text{melt}}$, \hspace{1cm} (3.17)

where $T_{\text{melt}}$ is the melting temperature and $\Delta h_{\text{melt}}$ is the specific enthalpy of melting. Numerical values for these constants are reported in Table 3.2. The values of $T_{\text{melt}}$ correspond to the crystallised state, although it is known that carbon dust deposited on tokamak walls can be in an amorphous state. Since amorphous metals have a significantly lower melting temperature than crystallised ones, this could have an effect on the dust heat balance [22, 23]. The choice is made here to assume that the metal stays in crystallised state. The code could be used in further studies to compare the characteristic solidification time of dust grains in tokamaks with their characteristic crystallisation time.

### 3.5.2 Sublimation and evaporation

Since the partial pressure of impurity ions is extremely small in tokamak plasmas, some atoms at the surface of the dust grain continuously undergo a gaseous phase transition – sublimation if the grain solid, evaporation if it is liquid. This mechanism is the only source of mass loss considered in the code and has a strong impact when the dust temperature reaches values comparable with the melting temperature.

The mass loss due to gaseous phase transition can be estimated by the Hertz-Knudsen formula [24]

$$\frac{dM_d}{dt} = -4\pi R_d^2 \sqrt{\frac{M_{\text{at}}}{2\pi k_B T_d N_A}} P_{\text{vap}}(T_d),$$

where $M_{\text{at}}$ is the atomic mass of the dust material, $N_A$ is the Avogadro constant and $P_{\text{vap}}$ is the vapour pressure of the dust material. To model the variations of $P_{\text{vap}}$ with the dust temperature, one can use a common analytical fit [11].
Figure 3.3: Vapour pressure as a function of temperature. The fits used in the simulations (see Table 3.3) are plotted in solid lines. Squares are values taken from Ref. [11], triangles are values taken from Ref. [25] for iron, from Ref. [26] for molybdenum and from Ref. [27] for tungsten.

\[
\log_{10} \left( \frac{P_{\text{vap}}(T_d)}{P_{\text{atm}}} \right) = A + \frac{B}{T_d} + C \log_{10} \left( \frac{T_d}{T_0} \right),
\]

(3.19)

where \( P_{\text{atm}} = 1013 \) hPa is the standard atmospheric pressure and \( T_0 = 1 \) K. The fits for the five metals of interest are presented in Table 3.3 and Fig. 3.3. The mass loss also corresponds to a loss of enthalpy that can be modelled by a negative input power \( Q_{\text{gas}} \), which can be estimated by considering that if the dust grain receives an energy \( Q_{\text{ext}} dt = \left( Q_e + Q_i + Q_{\text{imp}} + Q_n + Q_{\text{SEE}} + Q_{\text{TI}} + Q_{\text{rad}} \right) dt \) during \( dt \), then the cloud of evaporated matter takes away the energy it had as a part of the dust particle and the energy that was necessary to evaporate its mass:

\[
Q_{\text{gas}} dt = \frac{M_d(t) - M_d(t + dt)}{M_d(t)} H_d(t) + \Delta h_{\text{gas}} \left( M_d(t) - M_d(t + dt) \right),
\]

(3.20)

where \( \Delta h_{\text{gas}} \) is the specific enthalpy of gaseous phase transition. In the end

\[
Q_{\text{gas}} = \frac{dM_d}{dt} (\Delta h_{\text{gas}} + h_d).
\]

(3.21)

The values of \( \Delta h_{\text{gas}} \) in the relevant temperature ranges are reported in Table 3.3 and Fig. 3.4. They are assumed to be constant for liquid and solid state and have been modelled using Refs. [25, 26, 27, 28]. The values that are not available in the literature were estimated with the Clausius-Clapeyron relation and two sets of values \((T_1, P_{\text{vap1}})\) and \((T_2, P_{\text{vap2}})\) from the vapour pressure fits.
3.6 Characteristic heating time

\[
\Delta h_{\text{gas}} = \frac{N_A k_B T_1 T_2}{M_{\text{at}} (T_2 - T_1)} \ln \left( \frac{P_{\text{vap}2}}{P_{\text{vap}1}} \right). \tag{3.22}
\]

<table>
<thead>
<tr>
<th>Dust material</th>
<th>Phase</th>
<th>(\Delta h_{\text{gas}})</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beryllium</td>
<td>solid</td>
<td>323200</td>
<td>8.042</td>
<td>-17020</td>
<td>-0.444</td>
</tr>
<tr>
<td></td>
<td>liquid</td>
<td>308500</td>
<td>5.786</td>
<td>-15731</td>
<td>0</td>
</tr>
<tr>
<td>Iron</td>
<td>solid</td>
<td>397400</td>
<td>-57.96</td>
<td>-7333</td>
<td>17.67</td>
</tr>
<tr>
<td></td>
<td>liquid</td>
<td>375500</td>
<td>6.347</td>
<td>-19574</td>
<td>0</td>
</tr>
<tr>
<td>Nickel</td>
<td>solid</td>
<td>415000</td>
<td>10.56</td>
<td>-22606</td>
<td>-0.872</td>
</tr>
<tr>
<td></td>
<td>liquid</td>
<td>397500</td>
<td>6.666</td>
<td>-20765</td>
<td>0</td>
</tr>
<tr>
<td>Molybdenium</td>
<td>solid</td>
<td>636200</td>
<td>11.53</td>
<td>-34626</td>
<td>-1.133</td>
</tr>
<tr>
<td></td>
<td>liquid</td>
<td>585000</td>
<td>5.622</td>
<td>-28681</td>
<td>0</td>
</tr>
<tr>
<td>Tungsten</td>
<td>solid</td>
<td>858600</td>
<td>149.6</td>
<td>-98507</td>
<td>-35.66</td>
</tr>
<tr>
<td></td>
<td>liquid</td>
<td>806300</td>
<td>7.355</td>
<td>-42862</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.3: Enthalpy of gaseous phase transition and fit coefficients for the vapour pressure (see Eq. (3.19)). \(\Delta h_{\text{gas}}\) is given in J mol\(^{-1}\) and B in K.

3.6 Characteristic heating time

Similarly to the charging time issue addressed in Sec. 2.4, it is important to investigate the characteristic time scale at which thermal equilibrium is established. Considering the simplified case of a clean, non-flowing, thermalised and fully ionised plasma where the only cooling mechanism is ideal black body emission, the equilibrium temperature of a negatively charged dust grain is given by the solution of

\[
Q_{\text{tot}} = 2\sqrt{\pi} R_d^2 n_e k_B T_e (2 + \chi_d) v_{Te} \left( \exp \left( -\chi_d \right) + \sqrt{\frac{m_e}{m_i}} \right) - 4\pi R_d^2 \sigma_{SB} T_d^4 = 0, \tag{3.23}
\]

that is

\[
T_d^{\text{eq}} = \left( \frac{n_e k_B T_e (2 + \chi_d) v_{Te} \left( \exp \left( -\chi_d \right) + \sqrt{\frac{m_e}{m_i}} \right)}{2\sqrt{\pi} \sigma_{SB}} \right)^{1/4}. \tag{3.24}
\]

The characteristic heating time \(t_{\text{heat}}\) can the be estimated by

\[
t_{\text{heat}} = M_d c_{\text{pd}} \left| \frac{dT_d}{dQ_{\text{tot}}} \right|_{\text{eq}} = \frac{M_d c_{\text{pd}} (T_d^{\text{eq}})}{16\pi R_d^2 \sigma_{SB} (T_d^{\text{eq}})^3}. \tag{3.25}
\]

For typical plasma parameters in scrape-off layer plasmas, one finds that \(t_{\text{heat}}\) is of the order of 10 ms, that is much longer than the time step used for the simulations. This result is confirmed by actually simulating the evolution of the temperature of an immobile iron dust particle with
a radius of 2 µm placed 1 cm away from the wall of FTU, with two different initial temperature conditions; Far from equilibrium temperature and close to equilibrium temperature. The characteristic time for the well-known exponential-shaped relaxation curve is the same in both cases, as shown in Fig. 3.5.

The characteristic relaxation time is – as always – driven by the slowest process, in this case radiative cooling, which happens on much larger time scales than plasma-related heating. Therefore, moving dust grains in a tokamak discharge will not have the time to reach thermal equilibrium and Eq. (3.1) cannot be replaced by a steady-state equation.

Figure 3.4: Enthalpy of gaseous phase transition. The constant values assumed in the simulations (see Table 3.3) are plotted in solid lines. Circles are values obtained with the fits presented in Ref. [28], triangles are values taken from Ref. [25] for iron, from Ref. [26] for molybdenum and from Ref. [27] for tungsten.
Figure 3.5: Simulated relaxation towards thermal equilibrium for an iron dust particle in FTU.
Chapter 4

Dust dynamics

The motion of dust is governed by Newton’s second law

\[ M_d \frac{d\mathbf{v}_d}{dt} = \mathbf{F}_g + \mathbf{F}_E + \mathbf{F}_{v \times \mathbf{B}} + \nabla B + \mathbf{F}_{\text{drag}}, \]  

where \( \mathbf{v}_d \) is the velocity of the dust particle, \( \mathbf{F}_g \) is the gravitational force, \( \mathbf{F}_E + \mathbf{F}_{v \times \mathbf{B}} \) is the Lorentz force, \( \nabla B \) is the magnetic dipole force – relevant only for ferromagnetic materials such as iron and nickel – and \( \mathbf{F}_{\text{drag}} \) is the drag force due to plasma species. It is assumed that the mass ablation due to evaporation is spherically symmetric, so that there is no rocket force term.

4.1 Coordinate systems

The code focuses on limiter tokamaks, whose shape can be considered in first approximation as an ideal torus, defined only by its major radius \( R_{\text{maj}} \) and minor radius \( R_{\text{min}} \). Such a geometry is conveniently described in the cylindrical and toroidal coordinate systems. In the cylindrical system, the coordinates of a point in space are denoted by \( (R, Z, \phi) \) with the \( Z \)-axis being taken along the vertical axis of the tokamak and \( (R, \phi) \) being the polar coordinates in the horizontal plane. However, some quantities such as the poloidal magnetic field or the plasma density usually are easier to describe in the toroidal coordinate system \( (r, \theta, \phi) \), where \( (r, \theta) \) are the polar coordinates in the meridian half-plane relatively to the centre of the poloidal cross-section of the tokamak. Eq. (4.2) and Fig. 4.1 provide a visualisation of these two coordinate systems and the conversion formulas from one to the other.

\[
\begin{align*}
R &= R_{\text{maj}} + r \cos \theta \\
Z &= r \sin \theta \\
\phi &= \phi
\end{align*}
\]  

The cylindrical coordinates are the ones used by the code to solve the equation of motion. In this system, the dust velocity and acceleration are expressed as follows,

\[ \mathbf{v}_d = \dot{R} \hat{e}_R + \dot{Z} \hat{e}_Z + R \dot{\phi} \hat{e}_\phi \]  

\[ \frac{d\mathbf{v}_d}{dt} = \left( \ddot{R} - R \ddot{\phi}^2 \right) \hat{e}_R + \ddot{Z} \hat{e}_Z + \left( \ddot{\phi} + 2 \dot{R} \dot{\phi} \right) \hat{e}_\phi, \]  

respectively.
where the dot(s) denotes the first (second) time derivative. It is important to note that the positive orientation is defined by the vector system \((\mathbf{e}_R, \mathbf{e}_Z, \mathbf{e}_\phi)\), not \((\mathbf{e}_R, \mathbf{e}_\phi, \mathbf{e}_Z)\).

\[ \vec{F} = (\mathbf{u}_d, \nabla) \mathbf{B}. \]  

Figure 4.1: The two coordinate systems.

### 4.2 Forces acting on the dust particle

Among the forces taken into account by the code, the Lorentz force is a function of the electromagnetic field profiles, which are treated as input by the code. Therefore, this section will only focus on the magnetic dipole and plasma drag effects.

The dipole force due to the interaction between the magnetic moment \(\mu_d\) of the dust grain and the surrounding magnetic field is given by

\[ \mathbf{F}_{\nabla B} = (\mu_d, \nabla) \mathbf{B}. \]  

Since the magnetic field in a tokamak is much larger than the coercive field of most metals, it is assumed that \(\mu_d = \mu_d B_B\) and that \(\mu_d\) is equal to the saturated magnetic moment, which depends only on dust temperature and can typically be approximated by

\[ \mu_d = \mu_{d0} \left(1 - \frac{T_d^2}{T_c^2}\right) \gamma, \]  

where \(\mu_{d0}\) is the magnetic moment of the dust grain at 0 K, \(T_c\) is the Curie temperature of the dust material and the value of \(\gamma\) is to be fitted with experimental data [21].

The drag force due to plasma species can be separated in two terms, one due to the collection of plasma species by the dust grain and the other one due to small-angle Coulomb collisions with
orbiting particles. Because of their small mass, the electrons are neglected in the momentum transfer process, so that the drag is assumed to be exclusively caused by ambient plasma and impurity ions.

### 4.2.1 Collection drag

Like previously, the calculations are done in the reference frame where the dust grain is immobile. When a plasma particle is collected by the grain, it transfers all of its momentum to it. Thus, the drag force due to the ionic species $z$ is given by

$$ F_{z,\text{coll}} = \int \frac{dN_z}{dt} m_z v = \int \sigma(E(v), \chi_d) m_z v \nu_f z(v) d^3 v, \quad (4.7) $$

where $\sigma$ and $dN_z$ are respectively defined by Eqs. (1.3) and (2.2). In the case of flowing Maxwellian plasmas, Eq. (4.7) becomes

$$ F_{z,\text{coll}} = \int_{v \geq v_0} \pi R_d^2 m_z n_z \left( \frac{m_z}{2 \pi k_B T_z} \right)^{3/2} \left( 1 + \frac{2Z_z \chi_d k_B T_z}{m_z v^2} \right) v \exp \left( -\frac{m_z |v - \nu_f z|^2}{2k_B T_z} \right) v d^3 v, \quad (4.8) $$

where $v_0 = v_{T_z} \sqrt{\max(0, -Z_z \chi_d \tau_z)}$. The calculations detailed in Appendix C.1 allow to derive the formulas given in Ref. [1]

$$ F_{z,\text{coll}}^{\chi_d \leq \nu_0} = \pi R_d^2 m_z n_z v_{T_z} \nu_f z \left\{ \frac{1}{4u_z^2} \left[ \left( 1 + 2w_{z+} \right) \exp \left( -u_{z+} \right) \right] ight\} $$

$$ + u_z \left[ 1 + 2w_{z+} - \frac{1}{2u_z^2} (1 - 2w_{z-}) \right] \left[ \text{erf} (u_{z+}) + \text{erf} (u_{z-}) \right] \quad (4.9) $$

$$ F_{z,\text{coll}}^{\chi_d \geq \nu_0} = \pi R_d^2 m_z n_z v_{T_z} \nu_f z \left\{ \frac{1}{2u_z^2} \left[ \left( 1 + 2w_{z+} \right) \exp \left( -u_{z+} \right) \right] \right\} $$

$$ + u_z \left[ 1 + 2w_{z+} - \frac{1}{2u_z^2} (1 - 2w_{z-}) \right] \text{erf} (u_{z}) \quad (4.10) $$

where $w_{z\pm} = u_z^2 \pm \frac{Z_z \chi_d}{\tau_z}$, and $u_z$ and $u_{z\pm}$ are defined as in Sec. 2.1.

### 4.2.2 Orbital drag

The momentum transferred to the dust grains by orbiting plasma species is non negligible, due to the rather long interaction range of electric forces. Assuming that the interactions in the vicinity of the dust grain are governed by an unscreened Coulomb potential, one can analytically compute the momentum transfer due to one ionic particle of species $z$, in the situation
4.2. Forces acting on the dust particle

depicted in Fig. 4.2.

Figure 4.2: Orbital Coulomb collision between one plasma particle and the dust grain.

It can be shown (see Appendix C.2) that the deviation angle $\psi$ of the plasma particle after the collision is given by

$$\tan \frac{\psi}{2} = -\frac{Z_s R_d \chi_d k_B T_e}{m_z b v^2}, \quad (4.11)$$

so that the momentum gained by the dust grain is

$$\delta p = m_z (v - v') = m_z v \left( (1 - \cos \psi) \hat{e}_x - \sin \psi \hat{e}_y \right). \quad (4.12)$$

For a given $v$, the contribution along $\hat{e}_y$ is always cancelled by that of another particle colliding symmetrically. Thus the effective transferred momentum is

$$\delta p_{\text{eff}} = m_z (1 - \cos \psi) v = 2 m_z \frac{\tan^2 \frac{\psi}{2}}{1 + \tan^2 \frac{\psi}{2}} v \quad (4.13)$$

and the orbital drag force can be expressed by

$$F_{z,\text{orb}} = \int \frac{dN_{z,\text{orb}}}{dt} \delta p_{\text{eff}} = 4\pi m_z \int d^3 v \int_{b_0}^{b_{\text{max}}} db \frac{bv}{1 + \frac{m_z^2 b^2 v^4}{Z_s^2 R_d^2 \chi_d^2 k_B^2 T_e^2}} f_z(v) v, \quad (4.14)$$

where $dN_{z,\text{orb}} = 2\pi b v f_z(v) d^3 v db dt$ is the number of plasma particles of species $z$ Coulomb-scattered with initial velocity $v$ and impact parameter $b$ during $dt$. The impact parameter for grazing collision $b_0 = R_d \sqrt{1 + \frac{2Z_s \chi_d k_B T_e}{m_z v^2}}$ is defined as in Chap. 1 and a maximal impact parameter $b_{\text{max}}$ has been introduced to prevent the integration from diverging and to take into account the fact that electric charges are always screened in reality. The value to be taken for $b_{\text{max}}$ is still an open issue [29], though it is generally agreed that it should be of the order of the Debye length. The integration over $b$ yields

$$F_{z,\text{orb}} = \frac{2\pi Z_s^2 R_d^2 \chi_d^2 k_B^2 T_e^2}{m_z} \int_{0}^{1} \frac{1}{v^3} \ln \left( 1 + \frac{m_z^2 v^4}{Z_s^2 R_d^2 \chi_d^2 k_B^2 T_e^2 b_{\text{max}}^2} \right) f_z(v) v d^3 v. \quad (4.15)$$
Then, denoting $b_{90} = \left| \frac{Z_e R_d \chi_{kn T_e}}{m_z v^2} \right|$ as the 90° impact parameter for particles coming with a velocity $v$, it is possible to define the Coulomb logarithm as

$$\ln \Lambda_z = \frac{1}{2} \int \frac{1}{v^3} \ln \left( \frac{b_{90}^2 + b_{\text{max}}^2}{b_{90}^2 + b_0^2} \right) f_z(v) v d^3v \times \left| \int \frac{1}{v^3} f_z(v) v d^3v \right|^{-1}, \quad (4.16)$$

so that

$$F_{z, \text{orb}} = \frac{4\pi Z_z^2 R_d^2 \lambda_d^2 k_B T_e^2}{m_z} \ln \Lambda_z \int \frac{1}{v^3} f_z(v) v d^3v. \quad (4.17)$$

Eq. (4.16) gives a generalisation of the classical Coulomb logarithm. Indeed, in the case of a particle beam of velocity $v_{\text{beam}}$, $f_z(v) = n_z \delta(v - v_{\text{beam}})$ and

$$\ln \Lambda_z = \frac{1}{2} \ln \left( \frac{b_{90}(v_{\text{beam}}) + b_{\text{max}}}{b_{90}(v_{\text{beam}}) + b_0} \right). \quad (4.18)$$

For flowing Maxwellian plasmas, Eq. (4.17) becomes (see Appendix C.3 for more details)

$$F_{z, \text{orb}} = 2\pi R_d^2 m_z n_z v_{\text{T_z}} v_{\text{fz}} \left( \frac{Z_z \chi_d}{T_z} \right)^2 G\left( \frac{u_z}{u_{\text{z}}} \right) u_{\text{z}} \ln \Lambda_z, \quad (4.19)$$

where $G(u_z) = \frac{\sqrt{\pi} \text{erf}(u_z) - 2u_z \exp(-u_z^2)}{2\sqrt{\pi} u_z^2}$ is known as the Chandrasekhar function. Nonetheless, the difficulty to compute an exact value for $\ln \Lambda_z$ as defined in Eq. (4.16) is an important practical issue. Moreover, the simplified OML theory approach adopted here is known to show rather poor numerical agreement with simulations of the ion drag force for small values of $\ln \Lambda_z$.

Consequently, in the code, the following expression for the Coulomb logarithm, proposed by Hutchinson [29] and based from the work of Khrapak et al [30], is adopted

$$\ln \Lambda_z = \ln \left( \frac{b_{90}(v_{\text{z,eff}}) + \lambda_s}{b_{90}(v_{\text{z,eff}}) + R_d} \right). \quad (4.20)$$

Here the screening length $\lambda_s$ and the effective velocity $v_{\text{z,eff}}$ are defined from the electron Debye length $\lambda_{D_e} = \sqrt{\frac{e(k_B T_e)}{n_e e^2}}$

$$\lambda_s = \sqrt{R_d^2 + \frac{\lambda_{D_e}^2}{1 + \frac{2Z_k e T_e}{m_z v_{\text{z,eff}}}}} \quad (4.21)$$

$$v_{\text{z,eff}} = \sqrt{\frac{v_{\text{T_z}}^2 + v_{\text{fz}}^2}{1 + \left( \frac{v_{\text{fz}}}{Z_z k_B T_e} \right)^2}} \left[ 0.6 + 0.05 \ln \left( \frac{m_z}{Z_k T_{\text{proton}}} \right) + \frac{\lambda_{D_e}}{5R_d} \left( \frac{\sqrt{Z_z} - 0.1}{1 + \frac{Z_z}{n_z}} \right) \right]. \quad (4.22)$$
4.3 Interaction with the wall

The last issue to be addressed is the modelling of boundary conditions, namely the treatment of the collisions between dust grains and the tokamak wall or Plasma Facing Components (PFC). The simplest model to treat this situation would be to consider purely specular reflections, without any loss of momentum from the dust grain and a completely determined reflection angle. A more realistic model would be one in which the dust grains loses a fraction of its normal velocity as it rebounds on the wall. This effect is governed by the normal restitution coefficient $R^\perp$ which can be a function of the radius and incident velocity of the dust particle. In particular, a dust grain coming to the wall with a velocity $v$ rebounds with a velocity $v'$ given by

$$v' = v - (1 + R^\perp) \langle v, \hat{n} \rangle \hat{n},$$

where $\hat{n}$ is the normal unit vector to the wall at the collision point. The following sections aim to describe more thoroughly the model used for $R^\perp$.

Several restitution coefficient models have been developed in the field of continuum mechanics. The model used in the code is an analytical model proposed by Thornton and Ning [31]. It involves two critical velocities which depends on the materials in contact and the particle radius, namely the sticking velocity $v_s$, below which the incident particle does not rebound, and the yield velocity $v_y$, above which the particle is expected to undergo plastic deformations. The restitution can then be expressed as a function of the normal incident velocity $v_n$ as follows

$$R^\perp = \begin{cases} 0 & \text{if } v_n \leq v_s \\ \sqrt{1 - \left(\frac{v_n}{v_s}\right)^2} & \text{if } v_s \leq v_n \leq v_y \\ \sqrt{\frac{6\sqrt{3}}{5} \left[1 - \frac{1}{6} \left(\frac{v_n}{v_y}\right)^2\right] \sqrt{\frac{v_y}{v_n} + \frac{\sqrt{6}}{5} \frac{v_n}{v_y} - \left(\frac{v_n}{v_y}\right)^2} - \left(\frac{v_n}{v_y}\right)^2} & \text{if } v_n \geq v_y \end{cases}$$

(4.24)

The issue of critical velocities is problematic and one faces two choices; the use of (i) analytical expressions based on the mechanical parameters of the materials involved [31, 32], (ii) empirical input. The problem with the first solution is that such models assume perfectly spherical particles and are not available for tokamak-relevant materials. The latter solution was realised in the dedicated experiments of Ref. [33], where projectiles and targets made of relevant materials have been used. While critical velocities were evaluated, their accuracy in under question due to the fact that dense dust clouds, rather than single particles, were employed. Hence, in the code, the values of the critical velocities have been chosen in agreement with the theoretical scaling – that is $v_s \propto R_d^{-5/6}$ and $v_y \propto R_d^{-1}$ – but with coefficients intended to reproduce the experimental results from Ref. [33]. The resulting critical velocities and restitution coefficient curves are shown in Fig. 4.3.
Figure 4.3: Critical velocities and normal restitution coefficient models depending on dust radius and normal incident velocity.
Chapter 5

Plasma flow

The flow velocity of the plasma species is one of the most important parameters to assess, since it is essentially responsible for the drag force, which is often the dominant effect in dust dynamics.

5.1 Plasma core

In a tokamak, the plasma core – that is the part of the plasma volume which is well confined inside the LCMS – is generally hot enough \((T_e, T_i \gtrsim 50 \text{ eV})\) to vaporise dust grains. However, some relatively large dust grains may be able to cross the last closed magnetic surface and be dragged back towards the wall before being completely vaporised. It is therefore important to be able to model realistic plasma flow velocities in this region.

In the code, the plasma flow is assumed to be due to the classical \(E \times B\) drift and neoclassical transport effects, as presented in Refs. [34, 35]. In toroidal coordinates \((r, \theta, \phi)\), the neoclassical poloidal velocity for primary and impurity ions is given by

\[
v_{fi}.\hat{e}_{\theta} = \frac{k_B B_\phi}{e B^2} K_1 \nabla T_i, \tag{5.1}
\]

\[
v_{\text{imp}}.\hat{e}_{\theta} = \frac{k_B T_i B_\phi}{Z_i e B^2} \left[ \left( K_1 + \frac{3}{2} K_2 \right) \frac{\nabla T_i}{T_i} - \frac{\nabla p_i}{p_i} + \frac{Z_i T_{\text{imp}} \nabla p_{\text{imp}}}{Z_{\text{imp}} T_i p_{\text{imp}}} \right], \tag{5.2}
\]

where \(p_z\) is the pressure of species \(z\), and \(K_1\) and \(K_2\) are transport coefficients approximately equal to 0.5 [34]. Then, the radial projection of the force balance equation

\[
n_z Z_z e (E + v_{lz} \times B) = \nabla p_z \tag{5.3}
\]

allows to derive the toroidal flow velocity

\[
v_{lz}.\hat{e}_\phi = \frac{E_r}{B_\theta} + \frac{B_\phi}{B_\theta} v_{lz}.\hat{e}_\theta - \frac{\nabla p_z}{n_z Z_z e B_\theta}. \tag{5.4}
\]

The radial flow velocity, which is usually negligible, is assumed to be due only to the \(E \times B\) drift.
\[ \mathbf{v}_{fz} \hat{\mathbf{e}}_z = -\frac{V_{\text{loop}} B_\theta}{2\pi R B^2}, \]  

where \( V_{\text{loop}} \) is the toroidal loop voltage, typically of the order of a few volts.

### 5.2 SOL plasma

In the SOL, magnetic field lines are open, allowing losses of plasma species to the wall, or any PFC, along the direction of the magnetic field. The aforementioned model is not valid anymore and parallel flow velocities for primary plasma ions usually are of the order of the speed of sound \( c_s = \sqrt{\frac{Z_k n_i T_e m_i}{m_i}} \). The standard theory for parallel flow in the scrape-off layer is outlined in Ref. [36] and neglects the effects of viscosity and curvature of magnetic field lines. Assuming a constant source of plasma species from the core region, the parallel flow is always directed towards the closest limiter surface along the magnetic field line and its Mach number \( M_\parallel = \frac{\mathbf{v}_f}{c_s} \) is a solution of

\[ \frac{x}{L} = \frac{2 M_\parallel}{1 + M_\parallel^2}, \]  

where \( L \) is the length of the field line and \( x \) is the curvilinear coordinate along the field line, chosen so that \( x = 0 \) corresponds to the middle of the field line, at equal distance from the two surfaces on which the line ends. The Mach number \( M_\parallel \) is chosen to be positive if the flow is in the direction of increasing \( x \). The solution of Eq. (5.6) yields

\[ M_\parallel = \frac{L}{x} \left( 1 - \sqrt{1 - \frac{x^2}{L^2}} \right). \]  

The other components of the plasma flow are assumed to be driven only by the \( \mathbf{E} \times \mathbf{B} \) drift, so that the toroidal and poloidal flow velocities can be computed by a simple geometrical projection

\[ \mathbf{v}_{\parallel} \hat{\mathbf{e}}_\phi = \frac{B_\phi}{B} M_{\parallel} c_s, \]  

\[ \mathbf{v}_{\parallel} \hat{\mathbf{e}}_\theta = \frac{B_\theta}{B} M_{\parallel} c_s - \frac{E_r B_\phi}{B^2}, \]  

and the radial flow velocity is assumed to remain as given by Eq. (5.5).
Chapter 6
Numerical treatment

6.1 Differential system

The main equations involve ten dust parameters: position \((R, Z, \phi)\), velocity \((v_R, v_Z, v_\phi)\), mass \(M_d\), specific enthalpy \(h_d\), temperature \(T_d\) and specific molten fraction \(\psi_d\).

The equations for position and velocity are

\[
\frac{dR}{dt} = v_R, \quad \frac{dZ}{dt} = v_Z, \quad \frac{d\phi}{dt} = \frac{v_\phi}{R},
\]

\[
\frac{dv_R}{dt} = \frac{F_{\text{tot}} \cdot \hat{e}_R}{M_d} + \frac{v_\phi^2}{R}, \quad \frac{dv_Z}{dt} = \frac{F_{\text{tot}} \cdot \hat{e}_Z}{M_d}, \quad \frac{dv_\phi}{dt} = \frac{F_{\text{tot}} \cdot \hat{e}_\phi}{M_d} - \frac{v_R v_\phi}{R},
\]

where \(F_{\text{tot}} = F_g + F_E + F_{\nabla \times B} + F_{\nabla B} + F_{\text{drag}}\) is the total force acting on the dust particle. The equations for the dust mass and specific enthalpy are

\[
\frac{dM_d}{dt} = -\frac{4\pi R_d^2}{2\pi k_B T_d N_A} P_{\text{vap}}(T_d), \quad \frac{dh_d}{dt} = \frac{Q_{\text{tot}}}{M_d} - \frac{h_d}{M_d} \frac{dM_d}{dt},
\]

where \(Q_{\text{tot}} = Q_e + Q_i + Q_{\text{imp}} + Q_n + Q_{\text{SEE}} + Q_{\text{TI}} + Q_{\text{rad}} + Q_{\text{gas}}\) is the total heating power received by the dust grain. The dust temperature and specific molten fraction obey different equations depending on the state of the dust grain. If the grain is completely solid or liquid, then

\[
\frac{dT_d}{dt} = \frac{Q_{\text{tot}}}{M_d c_{pd}} - \frac{h_d}{M_d c_{pd}} \frac{dM_d}{dt} \quad \frac{d\psi_d}{dt} = 0.
\]

If the dust is melting, then

\[
\frac{dT_d}{dt} = 0 \quad \frac{d\psi_d}{dt} = \frac{Q_{\text{tot}}}{M_d \Delta h_{\text{melt}}} - \frac{\psi_d}{M_d} \frac{dM_d}{dt}.
\]

This differential system is solved numerically by the code using a Runge-Kutta method. The source terms – mainly electric currents and heating powers – are computed at every time step from the ten main parameters. The reflections against the wall are handled by checking at every time step whether the next predicted position of the particle is outside the tokamak chamber.
6.2 Conditions for trajectory termination

Regarding the termination of the solving algorithm, thorough conditions must be specified. There are four of them in the current version of the code:

1. the grain is vaporised
2. the grain sticks to the tokamak wall or PFC
3. the grain undergoes a given number of reflections against the wall
4. the lifetime of the grain reaches a given value

The choice of an adequate condition for vaporisation is the most problematic. Since several effects, e.g. field emission, not modelled in the code are expected to play a role when the radius of the dust particle falls below 100 nm, this value is currently adopted as the critical radius for the termination of the trajectory.

The solving algorithm can be summarised by the flowchart pictured in Fig. 6.1.
Chapter 7

Wall geometry

In reality, the geometry of the inner wall of a tokamak is made quite complex by the presence of various objects such as limiters, probes or ports. Although they have little influence on the confined plasma, the scrape-off layer environment can be strongly modified by their presence. Moreover, there are many macroscopic gaps between the tiles that form the wall itself. These gaps are of large interest in this work since they might act as traps for dust particles. For this reason, the DDFTU code was updated to allow the possibility to include the presence of such macroscopic gaps in the tokamak wall. Currently, it is possible to simulate three different gap shapes, which are sketched in Fig. 7.1. The geometry is still considered to be toroidally symmetric, so that all the gaps are modelled as toroidal belts all around the tokamak wall.

Each toroidally symmetric gap can be entirely defined by four parameters: the shape (square, triangular or circular), the poloidal angular position \( \theta_0 \) of the middle of the gap, the poloidal angular half-width \( \varepsilon \) and the maximum depth \( \eta \). These geometric parameters are described in Fig. 7.1 and used as input in the code. The interaction between dust particles and this new wall surface is mathematically described, as before, by Eq. (4.23). Therefore, the main concern is to compute the normal vector to the wall and to be able to test whether the dust particle is in contact with the wall. When no gaps are present, the test for reflection is performed by checking the sign of \( r - R_{\text{min}} \), and the normal vector to the wall is \( \hat{e}_r \).

7.1 Square gaps

The square gaps are defined by two radial walls and an orhordradial wall, so that the normal vectors respectively are \( \hat{e}_{\theta} \) and \( \hat{e}_r \). To test whether a reflection against the orhordradial wall must be handled, the code checks the sign of \( r - (R_{\text{min}} + \eta) \). Similarly, to test whether a reflection against the radial walls is to occur, the code checks whether the \( \theta \) coordinate of the dust particle remains between \( \theta_0 - \varepsilon \) and \( \theta_0 + \varepsilon \). The usual numerical precautions must be taken in case the gap is located close to the inboard side \( \theta = \pi \), usually chosen as the discontinuity point in angle values by programming languages.
7.2 Triangular gaps

Triangular gaps are defined by two non-radial straight lines labeled 1 and 2 in the following. The line 1 covers the poloidal region $\theta_0 - \varepsilon \leq \theta \leq \theta_0$ and the line 2 covers the poloidal region $\theta_0 \leq \theta \leq \theta_0 + \varepsilon$. The reflection test can be performed by checking the sign of the quantity

$$G = \frac{r_k}{\cos(\theta - \theta_k)} - r,$$

(7.1)

where $k$ is the line number corresponding to $\theta$ and the parameters $r_k$ and $\theta_k$ are given by

$$r_k = \frac{|c_k|}{\sqrt{a_k^2 + b_k^2}} \cos \theta_k = \frac{a_k r_k}{c_k} \quad \sin \theta_k = \frac{b_k r_k}{c_k},$$

(7.2)

where

$$
\begin{align*}
a_1 &= a \sin \varepsilon \cos \theta_0 + \sin \theta_0 [(1 - \cos \varepsilon)a + \eta] \\
a_2 &= -a \sin \varepsilon \cos \theta_0 + \sin \theta_0 [(1 - \cos \varepsilon)a + \eta] \\
b_1 &= a \sin \varepsilon \sin \theta_0 - \cos \theta_0 [(1 - \cos \varepsilon)a + \eta] \\
b_2 &= -a \sin \varepsilon \sin \theta_0 - \cos \theta_0 [(1 - \cos \varepsilon)a + \eta] \\
c_1 &= a(a + \eta) \sin \varepsilon \\
c_2 &= -a(a + \eta) \sin \varepsilon
\end{align*}
$$

(7.3)

The normal vector to the wall number $k$ can be expressed in cylindrical coordinates by

$$\hat{n}_k = \frac{1}{\sqrt{a_k^2 + b_k^2}} (a_k \hat{e}_R + b_k \hat{e}_Z).$$

(7.4)
7.3 Circular gaps

The circular gaps are defined by an outward circular arc. The corresponding quantity $G$ whose sign must be checked to decide whether a reflection is to occur is

$$G = r_0 \cos(\theta - \theta_0) - r + \sqrt{r_c^2 - r_0^2 \sin^2(\theta - \theta_0)},$$

(7.5)

where $r_0$ is the radial position of the center of the gap and $r_c$ is the radius of the gap itself. These parameters can be computed, using

$$r_0 = \frac{\eta(2a + \eta)}{2a(1 - \cos \varepsilon) + 2\eta}, \quad r_c = a + \eta - r_0.$$  

(7.6)

The normal vector can then be expressed in cylindrical coordinates by

$$\hat{n} = \frac{1}{\sqrt{(R - r_0 \cos \theta_0)^2 + (Z - r_0 \sin \theta_0)^2}} [(R - r_0 \cos \theta_0) \hat{e}_R + (Z - r_0 \sin \theta_0) \hat{e}_Z].$$

(7.7)
Part II

Simulations of FTU and TEXTOR scenarios
Chapter 8

FTU simulations

8.1 Profiles

The major and minor radius of FTU respectively are $R_{\text{maj}} = 93.5$ cm and $R_{\text{min}} = 33$ cm. The profiles used in the code are adopted from Ref. [37].

8.1.1 Electromagnetic field

The magnetic field profile is given via the toroidal magnetic field $B_\phi$ and the safety factor $q$, modelled with the following analytical fits

$$B_\phi = \frac{B_0 R_{\text{maj}}}{R}, \quad q = q_0 + \left(q_{\text{LCMS}} - q_0\right) \frac{r^2}{r_{\text{LCMS}}^2} ,$$

where $B_0 = 5$ T is the magnetic field at the tokamak major radius, $q_0 = 1$ is the safety factor in the center of the poloidal cross-section, $q_{\text{LCMS}} = 5$ is the safety factor at the last closed magnetic surface and $r_{\text{LCMS}} = 30$ cm is the radial position of the last closed magnetic surface. The poloidal magnetic field $B_\theta$ can then be computed, using

$$B_\theta = \frac{r B_\phi}{R q} .$$

The electric field $E$ is assumed to be radial and its value is given by

$$E = E_r = \begin{cases} E_1 \alpha_E \left(\frac{\alpha_E + 1}{\alpha_E x_0}\right)^{\alpha_E + 1} (x - x_0) x^{\alpha_E} & \text{for } x \leq x_0, \\ E_2 \frac{4}{(1 - x_0)^2} (x - 1)(x_0 - x) & \text{for } x \geq x_0, \end{cases}$$

where $x = \frac{r}{R_{\text{min}}}$ is the reduced minor radius coordinate, $r_0 = 29.5$ cm is the minor radius coordinate of the annulation point of the electric field, $x_0 = \frac{r_{\text{LCMS}}}{R_{\text{min}}}$, $\alpha_E = 3$, $E_1 = 12$ kV m$^{-1}$ is the peak value inside the LCMS and $E_2 = 6$ kV m$^{-1}$ is the peak value in the scrape-off layer. The electromagnetic field profiles for FTU are plotted in Fig. 8.1.
Figure 8.1: Electromagnetic field profiles in FTU.
8.1.2 Plasma parameters

The density and temperature profiles correspond to the plateau stage of the discharge. The analytical expressions for plasma density \( n = n_e = n_i \), plasma temperature \( T = T_e = T_i \), neutral density \( n_n \), and neutral temperature \( T_n \) are

\[
\begin{align*}
  n &= \begin{cases} 
  n_0 + P_1(\rho) & \text{if } \rho \leq 1 \\
  n_{\text{LCMS}} \exp \left( \frac{1-\rho}{\lambda_n} \right) & \text{if } \rho \geq 1 
  \end{cases} \\
  T &= \begin{cases} 
  T_0 + P_2(\rho) & \text{if } \rho \leq 1 \\
  T_{\text{lim}} \exp \left( \frac{1-\rho+\varepsilon}{\lambda_T} \right)^2 & \text{if } \rho \geq 1 
  \end{cases} \\
  n_n &= n_{n_{\text{wall}}} \exp \left( \frac{\rho - \rho_{\text{wall}}}{\lambda_n} \right) \\
  T_n &= \begin{cases} 
  T_0 (1 - \rho^2)^4 + T_{\text{lim}} & \text{if } \rho \leq 1 \\
  T_{\text{lim}} \exp \left( \frac{1-\rho}{\lambda_T} \right) & \text{if } \rho \geq 1 
  \end{cases}
\end{align*}
\]

(8.4)

where \( \rho = \frac{r}{r_{\text{LCMS}}} \), \( \rho_{\text{wall}} = \frac{R_{\text{min}}}{r_{\text{LCMS}}} \), \( n_0 = 12 \times 10^{19} \text{ m}^{-3} \), \( T_0 = 1400 \text{ eV} \), \( \lambda_n = 0.02 \), \( \lambda_T = 0.05 \), \( \varepsilon = \frac{\lambda_n^2}{2T} \left| \frac{d\varepsilon}{d\rho} \right|_{\rho=1} \), \( n_{n_{\text{wall}}} = 2 \times 10^{18} \text{ m}^{-3} \), and \( P_1 \) and \( P_2 \) respectively are a third-order and eighth-order polynomial. The other parameters are chosen to ensure the continuity of the profiles. It is moreover assumed that two impurity ions are present, namely oxygen and iron. Their density is modelled to satisfy

\[
n_{\text{imp}} = K n_{\text{LCMS}} \sqrt{\frac{n}{n_{\text{LCMS}}}},
\]

(8.5)

where \( K = 4.2 \times 10^{-2} \) for oxygen and \( K = 5.46 \times 10^{-4} \) for iron [38]. Fig. 8.2 summarises all these plasma profiles, as well as the model for the impurity charge number \( Z_{\text{imp}} \), which also results from an analytical fit but will not be detailed here.

8.1.3 Plasma flow

The FTU profiles for plasma flow were chosen to follow Eqs. (5.1) to (5.5) in the whole plasma volume, including the SOL, although the use of these equations is, as discussed in Sect. 5.2, not appropriate in the region with open magnetic field lines. However, as Fig. 8.3c shows, the flow velocity of deuterium ions is of the order of the sound speed near the wall, thus not contradicting the values predicted by aforementioned SOL flow models.

Impurity ions have a toroidal flow velocity similar to that of deuterium ions, and a poloidal velocity which basically differs by a factor \( \frac{1}{Z_{\text{imp}}} \).
Chapter 8. FTU simulations

Figure 8.2: Plasma profiles in FTU.
Figure 8.3: Flow profiles in FTU.
8.2 Results

8.2.1 Typical trajectories

Simulated trajectories of iron dust grains in FTU allow to identify the main effects involved in scrape-off layer dust dynamics. These trajectories can roughly be separated in two main classes: those that lead to the vaporisation of the dust grain and those that end with the dust grain sticking to the wall. Vaporisation can only happen if the dust grain travels too far towards the core plasma, whereas sticking usually occurs after several reflections against the wall that slow down the dust grain below its critical sticking velocity. Aside from initial conditions, the toroidal ion flow appears to be the determining factor in the outcome of the trajectory. Indeed, simulations show that ion drag is the dominant mechanism driving the toroidal acceleration of the dust grain. The resulting toroidal velocity then contributes to accelerate the dust grain towards the outboard side of the tokamak through inertial (centrifugal force) effects. Thus, a dust particle in the inboard side of the plasma chamber will be dragged towards the core and burn more easily, whereas a dust particle in the outboard side will be dragged towards the outboard wall and be more likely to stick. Since the drag acceleration scales as $1/R_d$, small dust grains are the most influenced by the plasma flow.

Figs. 8.4 and 8.5 show a comparison between the two aforementioned types of trajectories. The initial position is denoted by a green circle and the final position by a red circle. The initial conditions are the same for both trajectories, except for the initial dust radius which is 3 $\mu$m in Fig. 8.4 and 1 $\mu$m in Fig. 8.5. It can be seen on the temperature and radius plots in Fig. 8.4 that the dust grain reaches its melting point – characterised by a plateau in the temperature plot and an increase in radius since metals have a lower density in the liquid state than in the solid state – after 3 ms and starts vaporising soon after. The heating power is mainly provided by plasma electrons and the vaporisation phase is identified by the rapid decrease of the dust radius and the corresponding large cooling power due to mass loss. The latter results in a slower growth of dust temperature.

The same phenomena appear in the results shown in Fig. 8.5, but the centrifugal acceleration is strong enough to allow the dust grain to escape from the hot plasma after approximately 4 ms, before the radius of the particle reaches the termination value of 100 nm. From that moment, the particle cools down to solid state, loses most of its velocity by bouncing on the wall and ends up sticking to the wall after another collision.
Figure 8.4: Typical trajectory of vaporised dust in FTU
Figure 8.5: Typical trajectory of sticking dust in FTU
8.2. Results

8.2.2 Preliminary study of dust size and velocity distributions and material effects

Although the study of single trajectories allows to identify the main mechanisms at work in dust dynamics, it is strongly influenced by the choice of initial conditions on position and velocity. For a better understanding of the behaviour of dust grains in tokamak plasmas, it is necessary to study a large number of trajectories, corresponding to a broad range of initial conditions. Such a study allows for example to judge the relative amount of dust which is vaporised and becomes a source of impurity ions, the preferred sites for deposition on the wall and the expected lifetime of the dust grains. Moreover, the simulated results can be compared with experiments in which dust in often collected over the course of several discharges.

Since the trajectories appear to be driven by a competition between the heating power due to electrons, which enhances vaporisation, and the ion drag, which often results in sticking (at least on the outboard side of the plasma chamber), the main dust parameters that play a role are radius, mass and electric charge. Radius and mass influence the acceleration due to the ion drag, while radius and charge regulate the heating power due to electrons. Since the electric charge is in continuous evolution as the dust grain travels in the plasma, its influence can be tested by changing the $\beta$ parameter in the secondary electron emission model. It is to be expected that higher values of $\beta$ lead to higher SEE current and a more positive dust charge, which in turn increases the heating power due to electrons and enhances vaporisation. The effect of mass and radius can be tested by exploring a large range of initial radii and by changing the dust material.

Figs. 8.6 to 8.9 present the simulated results of 612 trajectories for a given dust material, initial radius and $\beta$ parameter. Each trajectory is simulated with a different initial condition on position and velocity. The initial positions are spread every 10 degrees in the poloidal cross-section and for each initial position, the launch angle is chosen among 17 different values. All the dust particles have the same initial velocity of $20 \text{ m s}^{-1}$. The plots show the end points of the trajectories as well as a visualisation of the preferred sites for sticking in the poloidal cross-section, and the distribution of lifetime.

It can be noticed from these results that in the case of beryllium dust, high SEE tends to suppress the asymmetries between the inboard and outboard sides of the plasma chamber. This can be easily explained since this asymmetry is most likely due to the ion drag that tends to sweep dust particles towards the outboard side. When high SEE is involved, the dust particles are heated faster and cannot survive long enough to allow the drag to play a role. This is confirmed by the fact that the peak value of the lifetime is decreased when $\beta$ increases. The sticking sites, while concentrated around the outboard midplane for small particles with $\beta = 0.4$, are spread more evenly as the initial radius and $\beta$ increase. This, again, can be explained by the influence of the drag force. Small particles are quickly dragged towards the outboard midplane were they stick easily, whereas large particles will stick closer to their initial position. When $\beta$ is increased, both small and large particles are less likely to stick far from their initial positions as this requires them to travel through hot plasma. However, due to the implementation of the trajectory termination condition involving vaporisation, large particles are allowed to lose
more mass before being considered as vaporised. This explains why not a single particle is vaporised around the outboard midplane in Fig. 8.7a: the radius of the dust grains entering the hot outboard plasma decreases as they undergo vaporisation, which allows the drag force to sweep them back to wall before they are completely vaporised.

The effects are less pronounced in the case of tungsten dust. The asymmetry between the inboard and outboard sides is barely visible in Fig. 8.8a, SEE does not appear to modify the trajectories or the lifetime distribution, and sticking sites are evenly spread in the poloidal cross-section. This is probably imputable to the high mass of tungsten, which tends to suppress the effects of the drag force.

Fig. 8.10 shows the evolution of the average lifetime for beryllium and tungsten dust particles in FTU as a function of initial radius for the total population of dust grains as well as for the sub-population of stuck and vaporised grains. The small influence of $\beta$ on tungsten particles is confirmed and, as expected, larger dust particles tend to survive longer, but the impact of size is much larger on the population of stuck particles than on the two other populations. This appears to be due to the low values of the critical sticking velocity for large particles. The fact that sticking beryllium particles live longer when $\beta = 1.3$ is quite surprising at first, but is explained by the fact that only particles which remain very close to the wall avoid vaporisation when the SEE is strong. The trajectories of these particles form a small angle with the wall, which allows them to survive long before sticking. The average lifetime of the whole population is still decreased when $\beta$ is increased, because the fraction of vaporised particles is closer to 100%.
Figure 8.6: Influence of SEE on beryllium dust particles of initial radius 2 µm in FTU.
Figure 8.7: Influence of SEE on beryllium dust particles of initial radius 6 µm in FTU.
8.2. Results

Figure 8.8: Influence of SEE on tungsten dust particles of initial radius 2 \( \mu \text{m} \) in FTU.
Figure 8.9: Influence of SEE on tungsten dust particles of initial radius 6 \( \mu \text{m} \) in FTU.
8.2. Results

![Graphs showing the average lifetime of dust in FTU for different materials and emission strengths.](image)

Figure 8.10: Average lifetime of beryllium and tungsten dust in FTU depending on the strength of secondary electron emission.
Chapter 9

TEXTOR simulations

9.1 Dust injection experiments

The most effective way to gain insight on the dynamics of dust and confidence in the predictive power of numerical simulations is to conduct experiments with artificially introduced calibrated dust. In such campaigns, dust with known characteristics (material, size and shape) is injected in the plasma volume by various means. In the first experiments in DIII–D and TEXTOR, dust was introduced in an open holder and released upon contact with the plasma when the discharge was ignited [39].

The use of a dispenser or a dust dropper allows for more control of the most important parameter, namely the initial velocity of dust. This method was successfully employed in the latest dust-dedicated campaigns on TEXTOR in fall 2012. The experimental setup and results of this campaign serve as a motivation and a reference scenario for the TEXTOR simulations presented below.

The experiment is mimicked as follows. Tungsten dust was dropped from the top of the poloidal cross-section and entered the plasma volume with a velocity of $1 \text{ m s}^{-1}$. The metal powder loaded in the dropper consisted of grains whose radius ranged from sub-micrometer to several micrometers. Moreover, part of the released dust formed clusters and agglomerates of dimensions above $10 \mu\text{m}$. Dust was collected by Si substrates introduced in the slits of the cylindrical probe head shown in Fig. 9.1. The probe was located in the outboard midplane, near the wall and $120^\circ$ away from the injection point in the toroidal direction. The preliminary analysis revealed that a fraction of the injected dust did reach the collector [40]. Some of the dust grains were partially covered with carbon flakes (left panel) and some others were partially molten (right panel), as shown in Fig. 9.2.

9.2 Profiles

The profiles used for TEXTOR are reconstructed from modelling input and data of Refs. [41, 42, 43]. Its major and minor radius respectively are $R_{\text{maj}} = 175 \text{ cm}$ and $R_{\text{min}} = 55 \text{ cm}$. 
Figure 9.1: Dust collector used in TEXTOR experiments (courtesy of Igor Bykov).

Figure 9.2: Tungsten dust collected in TEXTOR (courtesy of Igor Bykov).
9.2.1 Electromagnetic field

The electromagnetic field profiles used in the code are plotted in Fig. 9.3. The toroidal magnetic field is assumed to follow Eq. (8.1) with $B_0 = 2.23$ T but the safety factor $q$ is defined through a fourth-order polynomial fit

$$ q = q_0 + (q_{LCMS} - q_0) \frac{r^4}{r_{LCMS}^4}, $$  

(9.1)

with $q_0 = 0.75$, $q_{LCMS} = 3.5$ and $r_{LCMS} = 46$ cm. The electric field is evaluated as before by Eq. (8.3) with $r_0 = r_{LCMS}$, $\alpha_E = 3$, $E_1 = 7.5$ kV m$^{-1}$ and $E_2 = 2.5$ kV m$^{-1}$, except that it remains constant for $r \geq 50.5$ cm.
9.2.2 Plasma parameters

The plasma density and ion temperature in TEXTOR are approximated by a third-order polynomial fit inside the LCMS and decay exponentially in the SOL. The fitting coefficients are computed to match the profiles of Ref. [41]. The electron and ion temperatures are assumed to scale the same way, their values being adjusted by $T_e/T_i = 1200/1500$ (corresponding to the ratio of core temperatures) [41], with a decay length of 5 cm [44]. The effects of neutral species and impurity ions are not taken into account in these simulations.

9.2.3 Plasma flow

The plasma flow in the core region of TEXTOR is modelled after Eqs. (5.1–5.5). In the SOL, the flow is assumed to be mainly toroidal with a constant velocity equal to the sound speed at the LCMS. This simplified model is likely to result in an overestimation of the effects of the drag force and is to be refined in future simulations where the profile of parallel Mach number described by Eq. (5.7) will be taken into account.

9.3 First comparison with calibrated dust injection experiments

The code was used to simulate the trajectories of 20 tungsten dust grains with initial conditions corresponding to the dust injection experiment performed in TEXTOR. The initial radius of the dust grains ranges from 0.5 µm to 10 µm. The results plotted in Fig. 9.5 show three different classes of trajectories. Particles whose radius is below 3 µm are immediately swept towards the wall by the plasma flow and stick without being heated, whereas particles whose radius is above 6 µm are vaporised as they enter hot plasma regions. Particles in the intermediate range get heated above their melting point and are dragged towards the wall before being completely

![Figure 9.4: Plasma profiles in TEXTOR.](image)
vaporised. They are then cooled down to solid state and collide several times while migrating quite a large distance toroidally before sticking.

These preliminary simulations are encouraging as being able to explain the main results of the experiment described in Sec. 9.1; (i) the particles travelled a toroidal angle of $120^\circ$ between the injection point (dispenser) and the collector (ii) some of them were partially molten (iii) some of them were covered with carbon flakes, implying collisions with the wall. This agreement, however, must be taken cautiously as the current TEXTOR profiles used as input in the code need to be refined.
Figure 9.5: Simulated trajectories in TEXTOR.
Conclusions and outlook

The DDFTU code has been updated to include a non-steady state heat balance model and phase transitions, and the self-consistency of models such as the instantaneous charging has been given a greater importance in the simulations. The models for secondary electron emission, thermionic emission and black body radiation have been refined, and realistic boundary conditions have been introduced via the use of a restitution coefficient and the possibility to take into account the presence of gaps in the tokamak wall.

The code has been used for the first time to explore a large range of initial conditions (position, velocity and radius) for dust grains of various tokamak-relevant materials. The strength of secondary electron emission, affecting not only charging, and hence dynamics, but also lifetime, has been kept as an input parameter as well. This study confirmed the impact of the drag force as one of the main factors in dust dynamics and allowed to estimate average lifetimes, to locate preferred sites for dust deposition and to judge the sensitivity to initial conditions. This is a first step towards the use of the code as a predictive tool for devices of importance, such as JET and ITER.

Preliminary simulations of relevant scenarios for dust injection in TEXTOR have produced results that are in remarkable agreement with experimental data. However, the simulations are strongly dependent on the electromagnetic field and plasma flow profiles used to model the scrape-off layer environment.

These preliminary studies allowed to identify the most crucial issues affecting dust dynamics, lifetime, deposition rate and contribution to impurities. These questions will be pursued in future studies and particular attention will be payed to

- the acceptable limits of accuracy of input plasma profiles
- the implementation of a detailed vessel geometry including ports, gaps, limiters, etc
- the combination of theoretical and experimental efforts to allow a more realistic treatment of the collisions between dust particles and the wall or PFC
- the improvement of the secondary electron emission models

Existing empirical data on TEXTOR and JET will be used to benchmark the code. Moreover, linear devices simulating scrape-off layer conditions relevant for future reactors should be taken
these machines allow control on both injected dust and plasma parameters, providing for example the possibility to identify the conditions of dust vaporisation.
Bibliography


Appendix A

Ambient plasma currents

A.1 Electron current

Plugging in a Maxwellian electron distribution into Eq. (2.3) yields

\[ I_e = -\frac{1}{\sqrt{\pi}} e n_e v_{Te}^{-3} R_d^2 \int_{v \geq v_0} \left( 1 - \chi_d \frac{v^2}{v_{Te}^2} \right) v \exp \left( -\frac{v^2}{v_{Te}^2} \right) d^3v, \]

where \( v_{Te} = \sqrt{\frac{2 k_B T_e}{m_e}} \) and \( v_0 = v_{Te} \sqrt{\max(0, \chi_d)}. \) A substitution into spherical velocity coordinates and letting \( u = \frac{v}{v_{Te}} \) then yields

\[ I_e = -4 \sqrt{\pi} e n_e v_{Te} R_d^2 \int_{u_0}^{\infty} (u^2 - \chi_d) u \exp(-u^2) du, \]

where \( u_0 = \frac{v_0}{v_{Te}}. \) The use of the classical integrals reported in Appendix D allows to conclude.

A.2 Ion current

Plugging in a shifted Maxwellian ion distribution into Eq. (2.3) yields

\[ I_z = \frac{1}{\sqrt{\pi}} Z_z e n_z v_{Tz}^{-3} R_d^2 \int_{v \geq v_0} \left( 1 + \frac{Z_z \chi_d v_{Tz}^2}{\tau_z v^2} \right) v \exp \left( -\frac{|v - v_{fz}|^2}{v_{Tz}^2} \right) d^3v, \]

where \( \tau_z = \frac{T_z}{T_e}, \) \( v_{Tz} = \sqrt{\frac{2 k_B T_z}{m_z}} \) and \( v_0 = v_{Tz} \sqrt{\max(0, -\frac{Z_z \chi_d}{\tau_z})}. \) A substitution into spherical velocity coordinates \((v, \theta, \phi)\) favouring \( v_{fz} \) as being along the main axis then allows to rewrite the integral as

\[ \int_{v \geq v_0} = \int_{v_0}^{\infty} dv \int_0^{2\pi} \int_0^{\pi} d\theta d\phi \left( 1 + \frac{Z_z \chi_d v_{Tz}^2}{\tau_z v^2} \right) v^3 \sin \theta \exp \left( -\frac{v^2}{v_{Tz}^2} + \frac{2 vv_{fz} \cos \theta}{v_{Tz}^2} - \frac{v_{fz}^2}{v_{Tz}^2} \right), \]

which becomes, after integrating in \( \theta \) and \( \phi \)
\begin{align*}
\int_{v \geq v_0} &= 2\pi \exp\left(-\frac{v_z^2}{v_{T_z}^2}\right) \int_{v_0}^\infty \left(v^2 + \frac{Z_z\chi_d v_z^2}{\tau_z} \right) v \exp\left(-\frac{v^2}{v_{T_z}^2}\right) \frac{v_z^2}{v v_{T_z}} \sinh\left(\frac{2vv_{T_z}}{v_z^2}\right) dv \\
&= \pi \frac{v_z^2}{v_{T_z}} \exp\left(-\frac{v_z^2}{v_{T_z}^2}\right) (I_+ - I_-),
\end{align*}

where

\[ I_\pm = \int_{v_0}^\infty \left(v^2 + \frac{Z_z\chi_d v_z^2}{\tau_z} \right) \exp\left(-\frac{v^2}{v_{T_z}^2} \pm \frac{2vv_{T_z}}{v_z^2}\right) dv. \]

Substituting \( u = \frac{v \pm v_{T_z}}{v_{T_z}} \) yields

\[ I_\pm = \exp\left(u_z^2\right) v_{T_z}^3 \int_{u_0 \mp u_z}^\infty \left((u \pm u_z)^2 + \frac{Z_z\chi_d}{\tau_z}\right) \exp\left(-u^2\right) du, \]

where \( u_z = \frac{v_z}{v_{T_z}} \) and \( u_0 = \frac{v_0}{v_{T_z}} \). The integration can then be performed analytically using formulas from Appendix D.
Appendix B

Ambient plasma heat fluxes

B.1 Electron heat flux

Setting $v_{fe} = 0$, Eq. (3.4) can be rewritten as

$$Q_e = \frac{1}{\sqrt{\pi}} R_3^2 n_e v_{Te}^{-5} k_B T_e \int_{v \geq v_0} \frac{1}{v} \left( v^2 - \chi_d v_{Te}^2 \right)^2 \exp \left( -\frac{v^2}{v_{Te}^2} \right) d^3v .$$

A substitution into spherical velocity coordinates and letting $u = \frac{v}{v_{Te}}$ then yields

$$Q_e = 4\sqrt{\pi} R_3^2 n_e v_{Te} k_B T_e \int_{u_0}^{\infty} (u^2 - \chi_d)^2 u \exp (-u^2) du$$

$$= \frac{\chi_d k_B T_e}{e} I_e + 4\sqrt{\pi} R_3^2 n_e v_{Te} k_B T_e \int_{u_0}^{\infty} (u^2 - \chi_d) u^3 \exp (-u^2) du .$$

The use of the classical integrals reported in Appendix D allows to conclude.

B.2 Ion heat flux

Eq. (3.4) can be rewritten as

$$Q_z = \frac{1}{\sqrt{\pi}} R_3^2 n_z v_{Tz}^{-5} k_B T_z \int_{v \geq v_0} \frac{1}{v} \left( v^2 + \frac{Z_z \chi_d}{\tau_z} v_{Tz}^2 \right)^2 \exp \left( -\frac{|v - v_{Tz}|^2}{v_{Tz}^2} \right) d^3v .$$

A substitution into spherical velocity coordinates $(v, \theta, \phi)$ favouring $v_{Tz}$ as being along the main axis then yields

$$\int_{v \geq v_0}^{\infty} dv \int d\theta d\phi \left( v^2 + \frac{Z_z \chi_d}{\tau_z} v_{Tz}^2 \right)^2 v \sin \theta \exp \left( -\frac{v^2}{v_{Tz}^2} + \frac{2v v_{Tz} \cos \theta}{v_{Tz}^2} \right) ,$$

which becomes, after integrating in $\theta$ and $\phi$,
\[ \int_{v \geq v_0} = 2\pi \exp \left( -\frac{v^2}{v_{T_z}^2} \right) \int_{v_0}^{\infty} \left( v^2 + \frac{Z_z \chi_d}{\tau_z} \right)^2 v \exp \left( -\frac{v^2}{v_{T_z}^2} \right) \frac{v^2}{v_{T_z}^2} \sinh \left( \frac{2\nu v_{T_z}}{v_{T_z}^2} \right) dv \]
\[ = \pi \frac{v^2}{v_{T_z}} \exp \left( -\frac{v^2}{v_{T_z}^2} \right) (J_+ - J_-) , \]

where

\[ J_\pm = \int_{v_0}^{\infty} \left( v^2 + \frac{Z_z \chi_d}{\tau_z} \right)^2 v \exp \left( -\frac{v^2}{v_{T_z}^2} \pm \frac{2\nu v_{T_z}}{v_{T_z}^2} \right) dv \]
\[ = \frac{Z_z \chi_d}{\tau_z} v_{T_z} I_\pm + \int_{v_0}^{\infty} \left( v^2 + \frac{Z_z \chi_d}{\tau_z} \right)^2 v^2 \exp \left( -\frac{v^2}{v_{T_z}^2} \pm \frac{2\nu v_{T_z}}{v_{T_z}^2} \right) \frac{K_\pm}{v_{T_z}^2} dv \]

and \( I_\pm \) is defined in Appendix A.2. Plugging this back in the expression of \( Q_z \) yields

\[ Q_z = \frac{\chi_d k_B T_e}{e} I_z + \sqrt{\pi} R d n_z v_{T_z}^{-4} k_B T_z \exp \left( -\frac{u_z^2}{u_z^2} \right) \frac{u_z}{u_z} \left( K_+ - K_- \right) . \]

Substituting \( u = \frac{v_{T_z} \nu}{v_{T_z}} \) allows to write

\[ K_\pm = \exp \left( u_z^2 \right) v_{T_z}^5 \int_{u_0 \mp u_z}^{\infty} \left( u \pm u_z \right)^2 \left( u \pm u_z \right)^2 \exp \left( -u^2 \right) du . \]

The integration can then be performed analytically using formulas from Appendix D and the neutral power can be computed from the ion power by setting \( Z_z = 0 \) and \( I_z = 0 \).
Appendix C

Ion drag

C.1 Collection drag

Eq. (4.8) can be rewritten as

\[ F_{z,\text{coll}} = \frac{1}{\sqrt{\pi}} R_d^2 m_z n_z v_{T_z}^{-3} \int_{v \geq v_0} \frac{1}{v} \left( v^2 + \frac{Z_z \chi_d}{\tau_z} v_{T_z}^2 \right) \exp \left( -\frac{|v - v_{T_z}|^2}{v_{T_z}^2} \right) \, v \, d^3 v. \]

A substitution into spherical velocity coordinates \((v, \theta, \phi)\) favouring \(v_{T_z}\) as being along the main axis then yields

\[ \int_{v \geq v_0} = \int_{v_0}^{\infty} dv \int_{4\pi} d\theta d\phi \left( v^2 + \frac{Z_z \chi_d}{\tau_z} v_{T_z}^2 \right) v \sin \theta \exp \left( -\frac{v^2}{v_{T_z}^2} + \frac{2vv_{T_z} \cos \theta}{v_{T_z}^2} - \frac{v_{T_z}^2}{v_{T_z}^2} \right) \, v. \]

Since \(v\) can be written as

\[ v = \frac{v}{v_{T_z}} \cos \theta v_{T_z} + v \sin \theta (\cos \phi \hat{v}_x + \sin \phi \hat{v}_y), \]

it is clear that only the first term provides a nonzero contribution after the integration in \(\phi\) is performed. Thus,

\[ \int_{v \geq v_0} = 2\pi \exp \left( -\frac{v_{T_z}^2}{v_{T_z}^2} \right) v_{T_z} \int_{v_0}^{\infty} \left( v^2 + \frac{Z_z \chi_d}{\tau_z} v_{T_z}^2 \right) \frac{v^2}{v_{T_z}^2} \exp \left( -\frac{v^2}{v_{T_z}^2} \right) L(v) \, dv, \]

where

\[ L(v) = \int_0^\pi \sin \theta \cos \theta \exp \left( \frac{2vv_{T_z} \cos \theta}{v_{T_z}^2} \right) d\theta \]

can be integrated by parts, yielding

\[ L(v) = \frac{v_{T_z}^2}{vv_{T_z}} \cosh \left( \frac{2vv_{T_z}}{v_{T_z}^2} \right) - \frac{v_{T_z}^4}{2vv_{T_z}^2} \sinh \left( \frac{2vv_{T_z}}{v_{T_z}^2} \right) \]

so that
\[
\int_{v \geq v_0} = \pi \exp \left( -\frac{v_{iz}^2}{v_{Tz}^2} \right) \frac{v_{Tz}^2}{v_{iz}^2} v_{iz} \left[ L_+ + L_- - \frac{v_{iz}^2}{2} (I_+ - I_-) \right],
\]
where \( I_\pm \) is defined in Appendix A.2 and
\[
L_\pm = \int_{v_0}^{\infty} \left( v^2 + \frac{Z_z \chi_d}{\tau_z} v_{Tz}^2 \right) v \exp \left( -\frac{v^2}{v_{Tz}^2} \pm \frac{2 \nu v_{iz}}{v_{Tz}^2} \right) dv.
\]
Substituting \( u = \frac{v \pm v_{iz}}{v_{Tz}} \) allows to write
\[
L_\pm = \exp \left( \frac{u_{iz}^2}{v_{Tz}^4} \right) v_{Tz}^4 \int_{u_0 \mp u_z}^{\infty} \left( u \pm u_z \right)^2 \exp \left( -u^2 \right) du.
\]
The integration can then be performed analytically using formulas from Appendix D.

### C.2 Deviation angle for orbital Coulomb collisions

Considering the situation depicted in Fig. 4.2, it is quite straightforward to show that the angular momentum \( L = m_z \mathbf{r}(t) \times \mathbf{v}(t) \) of the plasma particle is constant and
\[
L = -m_z b \mathbf{v}_z.
\]
It follows from Newton’s second law that the Lenz vector
\[
\mathbf{A} = m_z \mathbf{v}(t) \times L - Z_z m_z R_d \chi_d k_B T_e \mathbf{r}(t)
\]
is also constant. Evaluating \( \mathbf{A} \) at both ends of the trajectory yields
\[
m_z^2 b^2 v^2 = m_z^2 b^2 v^2 \cos \psi - Z_z m_z R_d \chi_d k_B T_e \sin \psi,
\]
hence
\[
\tan \frac{\psi}{2} = -\frac{Z_z R_d \chi_d k_B T_e}{m_z b v^2}.
\]

### C.3 Orbital drag

Plugging a shifted Maxwellian ion distribution in Eq. (4.17) yields
\[
F_{z,orb} = \frac{1}{\sqrt{\pi}} \frac{R_d^2 m_z n_z v_{Tz} \left( Z_z \chi_d \right)^2}{\tau_z} \ln \Lambda \int \frac{1}{v^3} \exp \left( -\frac{|v - v_{iz}|^2}{v_{Tz}^2} \right) v d^3 v.
\]
Following the same reasoning as in Appendix C.1, only the contribution along \( v_{iz} \) is taken into account. This yields, after substitution into spherical velocity coordinates and integration in \( \phi \),
\[
F_{z,orb} = 2\sqrt{\pi} R_d^2 m_z n_z v_{Tz} \left( Z_z \chi_d \right)^2 \ln \Lambda \exp \left( -u_{iz}^2 \right) \frac{v_{iz}}{v_{Tz}} \int_0^{\infty} \exp \left( -\frac{u^2}{v_{Tz}^2} \right) L(v) dv,
\]
where $L(v)$ is defined in Appendix C.1. Expanding $L(v)$ and substituting $u = \frac{v}{v_{Tz}}$ then allows to rewrite the integral as

$$\int_{0}^{\infty} = \frac{v_{Tz}}{2u_{z}} \int_{0}^{\infty} \frac{\exp(-u^2)}{u} \left[ \left(1 - \frac{1}{2u_{z}u}\right) \exp(2u_{z}u) + \left(1 + \frac{1}{2u_{z}u}\right) \exp(-2u_{z}u) \right] du$$

$$= \frac{v_{Tz}}{2u_{z}} \sqrt{\pi} \exp(u_{z}^{2}) \text{erf}(u_{z}) - 2u_{z}$$

$$= \sqrt{\pi} v_{Tz} \exp(u_{z}^{2}) \mathcal{G}(u_{z})$$

which allows to conclude.
Appendix D

Gaussian integrals

The following integrals are useful in the various calculations involved in this work.

\[
\begin{align*}
\int_a^\infty e^{-x^2} \, dx &= \frac{\sqrt{\pi}}{2} \left(1 - \text{erf}(a)\right) \\
\int_a^\infty xe^{-x^2} \, dx &= \frac{1}{2} e^{-a^2} \\
\int_a^\infty x^2 e^{-x^2} \, dx &= \frac{a}{2} e^{-a^2} + \frac{\sqrt{\pi}}{4} \left(1 - \text{erf}(a)\right) \\
\int_a^\infty x^3 e^{-x^2} \, dx &= \frac{a^2 + 1}{2} e^{-a^2} \\
\int_a^\infty x^4 e^{-x^2} \, dx &= \frac{2a^3 + 3a}{4} e^{-a^2} + \frac{3\sqrt{\pi}}{8} \left(1 - \text{erf}(a)\right) \\
\int_a^\infty x^5 e^{-x^2} \, dx &= \frac{a^4 + 2a^2 + 2}{2} e^{-a^2}
\end{align*}
\]

\[
\begin{align*}
\int_{-a}^\infty (x + a) e^{-x^2} \, dx &= \frac{1}{2} e^{-a^2} + \frac{\sqrt{\pi}}{2} a \left(1 + \text{erf}(a)\right) \\
\int_{-a}^\infty (x + a)^2 e^{-x^2} \, dx &= \frac{a}{2} e^{-a^2} + \frac{\sqrt{\pi}}{4} \left(2a^2 + 1\right) \left(1 + \text{erf}(a)\right) \\
\int_{-a}^\infty (x + a)^3 e^{-x^2} \, dx &= \frac{a^2 + 1}{2} e^{-a^2} + \frac{\sqrt{\pi}}{4} \left(2a^3 + 3a\right) \left(1 + \text{erf}(a)\right) \\
\int_{-a}^\infty (x + a)^4 e^{-x^2} \, dx &= \frac{2a^3 + 5a}{4} e^{-a^2} + \frac{\sqrt{\pi}}{8} \left(4a^4 + 12a^2 + 3\right) \left(1 + \text{erf}(a)\right)
\end{align*}
\]
Publication
Transport and effects of ferromagnetic dust in a tokamak with a metallic vessel

E Lazzaro1, I Proverbio1, F Nespoli1, S Ratynskaia2, C Castaldo3, U deAngelis2, M DeAngeli1, J-P Banon2 and L Vignitchouk2

1 Istituto di Fisica del Plasma C.N.R., Euratom-ENEA-CNR Association for Fusion Via R.Cozzi 53, 20125 Milan, Italy
2 Royal Institute of Technology, Stockholm, Sweden
3 Associazione Euratom-ENEA per la Fusione, C.R. Frascati, Rome, Italy
4 Universita’ di Napoli ‘Federico II’, INFN Section of Naples, Italy

E-mail: lazzaro@ifp.cnr.it

Received 26 July 2012, in final form 2 October 2012
Published 21 November 2012
Online at stacks.iop.org/PPCF/54/124043

Abstract

Important physics effects in contemporary and future devices for magnetic fusion experiments depend on the interface with a ‘composite’ plasma, consisting of multiple ion species and heterogeneous dust with variable charge. A selection of processes related to dust and occurring in existing tokamaks is presented, focusing on new results on the physics of isolated micrometric ferromagnetic dust particles in the SOL of a tokamak with a metallic vessel of circular meridian cross section. Such particles in particular, in addition to usual forces, are subjected to magnetic dipole interaction with the ambient magnetic field and to strong evaporation effects at high surface temperatures. Moreover, preliminary results of inclusion of gaps in the vessel geometry suggest the possibility of dust trapping. Also reported are the effects of nanometre dust on plasma when the dust is to be considered as a plasma component.

(Some figures may appear in colour only in the online journal)

1. Introduction

As the development of tokamak devices approaches more closely and reliably the technical conditions required for confinement of a burning plasma, it appears clearly that optimization of the plasma performance requires improved understanding and ultimately better control of the interface conditions between the plasma and the first material wall. The idealized boundary conditions of the magnetohydrodynamic (MHD) equilibrium configuration and the more detailed conditions on transport fluxes in the relevant collisionality regimes should be extended to actually consider the diffuse interface with a ‘composite’ plasma, consisting of multiple ion species and heterogeneous meso-particles with variable charge and mass, as more realistic ‘boundary conditions’.

The latter element, is a ‘dust’ component, involved in additional processes, which are important both for the heat and impurity transport as well as for safety issues, for instance the tritium retention of dust trapped in the tile gaps of divertors, limiters or in the first wall gaps [1]. In a tokamak environment dust is produced mainly by thermal overload of the plasma-facing surfaces leading to brittle destruction of carbon surfaces, melt layer from metal surfaces, and disintegration of co-deposited layers; arcing and current disruption events are other important sources of dust [2]. Another source is chemical agglomeration of sputtered C_n clusters and debris from vessel electro-mechanical stress during the transient tokamak breakdown–start-up phase as well as in the current termination phase [3], and during the normal plasma phase in the SOL [2, 4, 5]. Comprehensive recent reviews on physics of dust–plasma interaction and in situ dust diagnostics are given in [4–8].

The type of interaction of the dust particles with the plasma depends on their size and density as well as on the plasma density and temperature. A typical SOL plasma has temperature \( T_i < T_e < 10–100 \text{ eV} \), and rather high density, \( n_e < 10^{11–12} \text{ cm}^{-3} \), is highly ionized and contains multiple ion species due to recycling. The dust particles have a temperature \( T_d \), which varies along the trajectory and can reach the boiling point [9–11] and typically the average density \( n_d \) of dust grains with an average radius \( \langle a_d \rangle \sim 1 \mu m \) is \( n_d < 0.1 \text{ cm}^{-3} \) [4]. Therefore, such dust grains can be
considered (noninteracting, because of large intergrain distance) and studied with a ‘test particle’ approach. In contrast for the dust population with \( (a_d) \sim 10 \text{ nm} \) the density can be expected to be in the range \( 10^5 \text{–} 10^7 \text{ cm}^{-3} \) [4] and, if actually present in the SOL should be treated as a plasma component with possible influence on plasma collective effects.

The values of dust, size and number densities made available in the literature [2, 5–8] are obtained from data ‘averaged’ with different procedures, often concerning dust accumulated over a long time and widely different experiments. Therefore, these data are not uniform in reliability and are not necessarily representative of the amount of dust present in a given experimental situation. However, for modelling purposes the order of magnitudes are comfortably close to allow their use as ‘typical’ of present-day devices.

Here, we discuss recent results of observation and modelling concerning both the micrometre and nanometre dust components. In section 2, we treat in some detail the problems of isolated particle dynamics in a tokamak, applicable to the very low density dust species of a few \( \mu \text{m} \) size, and the particular behaviour of ferromagnetic metallic dust is singled out. These particles are subjected to additional acceleration mechanisms, due to the action of the ambient magnetic field and of evaporation of the metal, which can be in the liquid state. In section 3, we discuss the possible role of the nanoparticles in tokamak plasma discharges. Section 4 is devoted to the conclusions.

2. The test particle model for metallic \( \mu \text{m} \) dust in the SOL

A test particle description is appropriate for plasmas with low dust density, implying that dust does not affect the plasma quasi-neutrality condition and that intergrain distance \( \Delta \sim (3/4\pi n_d) \) exceeds the Debye screening length, \( \lambda_D \). A quantitative figure of merit is the Havnes parameter \( P = n_d Z_d / n_i \); collective effects are expected to be not important for \( P \lambda_D < 1 \) [4]. Such a description can be applied to the modelling of dynamics of dust particles having a typical density \( n_d \ll 1 \) cm\(^{-3}\) in the SOL plasmas of fusion devices, also including realistic elements of the interaction of dust with the wall. The role of the geometry of the vessel wall and the properties of the materials is considered an important issue [1, 2, 4, 9] in predicting dust behaviour and interpreting its observed effects in fusion devices.

The interaction of these particles with the plasma and with the surrounding vessel structure is dominated by the applied external forces, mainly the ion drag and the interactions with plasma facing components (PFC). The dust dynamics might be important for plasma contamination and the integrity of the device PFCs resulting from dust particles’ hypervelocity impacts on the wall, which might eject masses of material far exceeding that of the projectile [12, 13]. This releases fresh impurities as well as neutral gas and plasma, and the contaminating mass might even grow unless additional loss processes (e.g. melting, sputtering, etc) reduce the secondary dust population considerably or unless acceleration to the hypervelocity impact regime is made impossible or sufficiently rare by suitable plasma discharge operation and suitable design of the vessel wall.

The mobilization and acceleration of metallic (and in particular ferromagnetic) dust in a tokamak with limiter (such as FTU) is an important problem that has been investigated using an improved model of the drag forces due to plasma ions, with the inclusion also of one or more impurities consistent with \( Z_{def} \) and taking into account the most relevant effects of thermionic and secondary electron emission. Realistic impurity density profiles have been evaluated using the Zagorski–Romanelli model [14] and the charge of the impurity population has been evaluated using the average ion method [15], where a single ‘equivalent’ species is considered with an average charge, which depends on the plasma temperature.

Once the adhesion force holding the dust grain on the wall is overcome by any suitable force, the dust motion is governed by several forces:

\[
m_d \frac{d\mathbf{v}}{dt} = \sum_i m_i n_i \pi a_d^2 v_i \zeta_i (u_i, \chi) (v_i - \mathbf{v}) + \mu \nabla B + q_d \left( \frac{\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}}{c} \right) + u_i \frac{d m_d}{dt} + m_d g + \Theta \text{wall.}
\]

The terms on the rhs of Newton equation (1) are the drag force due to plasma flow, the force due to magnetic dipole interaction, the electric and the Lorentz force, a possible ‘rocket force’ appearing in the case of a generic asymmetric mass ejection with a nonzero relative velocity \( u \) of the sublimated/vaporized cloud with respect to that of the principal fragment, and finally the gravity force and a term representing reflections on the tokamak wall.

Here \( m_d, a_d \) and \( \mu \) are the dust grain mass, radius and magnetic dipole moment; \( m_i, n_i \) and \( v_i \) are the mass, number density and thermal velocity of the \( i \)th plasma species, \( v_i \) is the flow velocity of the plasma species and \( \zeta_i (u_i, \chi) \), defined in [4, 9], is a function of both the normalized relative velocity \( u_i = |v_i - \mathbf{v}| / v_i \) and the dust surface potential \( \chi = -e\phi_0 / T_c \) (normalized on the electron temperature \( T_c \)).

The most important active force is the drag force [4, 9, 16] due to the friction of the dust particles with all the species composing the plasma; the main contribution is given by plasma ions, which in an axisymmetric system possess an ordered parallel surface flow speed, in a neoclassical description, with a toroidal component

\[
V_\phi = \frac{c E_r}{B_0} - \frac{c_T}{en_l Z B_\theta} \frac{d p}{d r} - K_1 n_i \left( \frac{B_\theta}{B} \right)^2 \frac{d T_i}{d r} \sim \frac{c E_r}{B_0}
\]

much larger than the poloidal component \( V_\theta = (c/eZB) K_1 \times (d T_i / d r) \), where the coefficient \( K_1 \) depends on \( n_i Z_i^2 / n_i Z_l^2 \), but is \( O(1) \) [17]. The toroidal flow speed \( V_\phi \) can span the range 5–50 km s\(^{-1}\), at the periphery of the confinement volume in a tokamak like FTU [18]. Here \( B_\theta, B_0 \) are the poloidal and toroidal magnetic field components and \( E_r \) is the radial electric field in the edge region, taken from experimental information. The effect of electrons and neutrals can be neglected because of the small mass and the low density and temperature, respectively. However, the impurity drag force \( F_1 \) can be important since, relative to the ion drag force \( F_\theta \), it scales as \( (F_1 / F_\theta) \sim (n_i m_i v_i^2 Z_1^2 / n_i m_i v_i^2) \sim (n_i Z_1^2 / n_i) \) and
Figure 1. Profiles versus minor radius \( r \) of the ion-drag force (dashed line), the impurity drag force (dotted line) and the sum of the two are shown (solid line). The impurities used are iron (left) and oxygen (right), shifting the peak of the total force.

broadens the region around the peak of the drag force. Figure 1 shows the profile of the drag force, sampled by a test particle with a 2 \( \mu \)m nominal radius, projected along the minor radius of an FTU-like equilibrium configuration. The presence of 1% of O or 0.37% of Fe significantly increases the total drag force on a Fe dust particle.

Other forces, strongly dependent on the material composition of the dust grains, can be important, respectively, when the dust temperature is low or high. In the first case, the magnetic dipole interaction with the ambient \( B \) field is significant, \( F_B = \mu(B, T_d) \nabla B \), where the dipole moment of the dust is of the type \( \mu(B, T_d) = \mu_{Fe}(N_A m_d/A_{Fe}) \chi(B, T_d/\text{TCurie}) \) with the particle nonlinear magnetic susceptibility modelled by the implicit relation \( \chi = \tanh(\mu_{Fe} B/\kappa T_d) + \chi(\text{TCurie}/T_d) \) [18]. Here for the sake of example the formula is written for iron, with \( \mu_{Fe} \) the elementary magnetic moment for an iron atom, \( A_{Fe} \) the iron mass number, \( N_A \) the Avogadro number and \( \text{TCurie} \) the Curie temperature (1043 K for Fe). In a tokamak configuration, a dust particle can travel with a complex trajectory sampling regions with different magnetic field intensity, which may change its magnetization and must be calculated numerically along the path. An important aspect is the temperature dependence of the magnetization. In fact, at a fixed magnetic field, this generalized paramagnetic model exhibits a fast decay of the magnetization as a function of temperature.

For a high dust surface temperature a process of evaporation/sublimation is possible and will cause an ejection of mass at a rate \( dm_d/\text{dt} \approx 4\pi a_d^2 m_1 \Gamma_{\text{sub}} \), where \( m_1 \) is the mass of the ejected particles and \( \Gamma_{\text{sub}} \) is the evaporation/sublimation flux [19]. It should be remarked that at the temperature where this mechanism of mass loss and related effects can be expected, a metallic (Fe) dust particle is in the liquid state, while a C particle is not. The dynamic model is completed by a procedure of simultaneous calculation of the dust charge and temperature, solving the system of equations representing an instantaneous equilibrium:

\[
I_{\text{tot}} \left( -e\Phi/T_e, T_d \right) = 0, \quad W_{\text{tot}} \left( -e\Phi/T_e, T_d \right) = 0. \tag{2}
\]

In system (2) the total current and energy \( I_{\text{tot}} \) and \( W_{\text{tot}} \) also includes secondary electron emission and thermionic emission, which reduce the dust negative charge as compared with the balance of ambient currents only. The subscripts \( i, e, \text{th} \) and see indicate the ion, electron, thermionic and secondary electron emission contributions to the total current \( I_{\text{tot}} \). The second equation of system (2) represents the total energy flux balance on the test particle [9], where \( W_{\text{tot}} = W_i + W_e - (W_{\text{th}} + W_{\text{sub}} + W_{\text{see}}) \) is the difference between the heating due to the incoming energy flux due to the plasma ions and electrons (i.e) and the energy fluxes associated with secondary electron emission (see) and thermionic emission (th), as well as the blackbody (bb) emission and sublimation (sub) fluxes.

Here a few typical examples of the solution of equation (1) are described for a test particle assumed to have a mass \( m_d \), but no geometric shape details except for a nominal size in the range 2–10 \( \mu \)m. The motion of a ferromagnetic (Fe) dust particle is studied first in the case of an FTU-like tokamak configuration in two scenarios; with a smooth metallic wall and with a vessel contour with dents and recesses.

In the first instance, the test particle is subjected to the combination of the ion-drag acceleration mechanism in a layer within the SOL and exhibits a ‘sling shot’ motion to the wall, appearing as a centrifugal acceleration in a frame rotating with the ion flow [18, 20]. The impact with the wall is considered quasi-elastic with a constant restitution coefficient \( R \approx 0.8 \) and a ‘brushing’ mass loss of the same order of magnitude, and the particle is re-injected in the acceleration layer. This is a periodic action that, for a large number \( N_{\text{reff}} \) of reflections, allows one to reach quasi-stochastically a peak toroidal velocity scaling as \( V_a \propto (N_{\text{reff}}(R\Delta_{\text{SOL}})^{1/2})^{1/2} \), where \( \Delta_{\text{SOL}} \) is the radial SOL width [20]. The typical trajectories of a 2 \( \mu \)m dust particle are shown in figure 2. The additional acceleration due to the impurity drag is not negligible: for given initial conditions and number of reflections with the wall, the final velocity can be a few 10% higher with respect to the case without impurities, depending on the impurity relative density \( n_i/n_e \) and atomic species.

An intriguing effect is caused by a complex contour of the vessel cross section, with recesses, portholes and gaps of varying width and depth: if the reflections with the wall are assumed to be quasi-elastic, the complex contour can both increase the number of reflections necessary to reach a certain velocity, or decrease it since some reflections with such complex contour can result in further entering of the particle in the ‘friction zone’, as shown in figures 3 and 4.

There are mainly three types of trajectories: (a) the particle breaks through the forbidden boundary of boiling temperature (red dashed circle in figures 2(a)–5) and is lost,
Figure 2. Trajectory of a 2 µm dust particle accelerated by the friction force with the toroidal plasma flow. (a) the toroidal (left) and poloidal (right) projection of the trajectory, (b) and (c) are, respectively, the velocity as a function of time and of the toroidal revolutions. The red dashed circle is the Fe boiling temperature boundary and the outer blue dashed circle is the LCMS.

Figure 3. Trajectory of a 2 µm particle being trapped in a gap in the vessel after several inelastic reflections with the wall. The red dashed circle is the Fe boiling temperature boundary and the blue dashed circle is the LCMS. The green and red crosses are the start and end points of the trajectory, respectively.

(b) the particle is confined in the outer SOL near the wall, beyond the LCMS (outer blue dashed circle in figures 2(a)–5), (c) the particle is accelerated in the friction zone outside the boiling temperature boundary. The trajectories, displayed in these figures, start from a given initial position marked by a green cross with a given injection angle θ. The end point is marked by a red cross. On the other hand, if the reflections with the wall are assumed to be inelastic, reducing the normal component of the incoming velocity upon each impact, the dust particles tend to be focused deeper and deeper in the recess, resulting in the capture of the particle; such trajectories are shown in figure 4. This could be used to design means of extraction of the dust both in the ramp-down phase of the discharge and in the ramp-up phase. Furthermore, it gives support to the possibility to limit the damage and the plasma contamination process. Given that the experimental data on impact of micrometric iron dust impinging at a high velocity on a stainless steel wall are currently not sufficient [21], a conservative restitution coefficient $R = v_{\text{refl}}/v_{\text{inc}}$ ratio of reflected and incident velocities, for the case of inelastic collisions, has been modelled on experimental data for mm-sized particles for the high- and low-velocity ranges of impacts and applied in the simulation code to the normal velocity at each impact. In the case of inelastic collisions, the normal velocity decreases with the number of reflections as $R^{N_{\text{refl}}}$, with $R \leq 0.7$, in our simulations. This results in the quenching of the particle motion on the wall because the particle is no longer efficiently re-injected in the friction layer.
When at high dust surface temperatures evaporation/sublimation occurs it will cause an ejection of mass. In the case of asymmetric mass ejection with a nonzero relative velocity $u$ of the sublimated cloud with respect to that of the principal fragment an additional 'rocket' force term becomes important in equation (1). We have considered, as an example, a relative velocity between the dust and the ejected cloud $u = -\alpha \sqrt{2T_d/m_\text{d}v_{\text{d}}/|v_{\text{d}}|}$, where $\alpha < 1$ is a factor accounting for the asymmetric ejection. In this case, some trajectories still reach high velocities in the order of km s$^{-1}$ with a small number of reflections even in the case of inelastic reflection (figure 5).

Finally, the interesting start-up scenario [3] is shown in figures 6(a) and (b). Figure 6(b) displays the start-up waveforms of the tokamak toroidal field rising to its peak value before plasma breakdown, and the subsequent rise of the plasma current. During the first stage in the ‘empty’ vacuum vessel, ferromagnetic dust particles sticking on the wall may be ‘mobilized’ at a velocity of ~tens of m s$^{-1}$ toward the high-field side by the magnetic field-dipole force $F_B = \mu(B, T_d)\nabla B$. Reflections from the inner wall and inertial motion swing the particle across the vessel, until the build-up of a plasma and edge plasma rotation start dragging the charged dust in the way already described ending the trajectory on the outer wall, as shown in the poloidal and toroidal projections of figure 6(b). The table in the figure gives the values chosen for the particle nominal radius $a$, the peak radial electric field $E_0$ and its peak value in the SOL $E_{\text{lim}}$, the initial injection angle $\theta_0$, the total number of reflections at the wall $N_{\text{ref}}$ and the final velocity $V_f$. Also this scenario could suggest procedures of pre-cleaning a tokamak vessel from ferromagnetic dust before operations.

3. Nanosized dust in tokamak SOL environment

The fusion performance of a tokamak reactor depends significantly on heat and particle fluxes across the last closed magnetic surface (LCMS) into the SOL, up to the wall. The SOL width ($\Delta_\text{SOL}$) characterizes the impacted target area and the power loadings onto the ‘first wall’, and defines the space scale of profile decay in the low (L) regime and the ‘pedestal’ width, which characterizes high (H) energy confinement regimes. Its scaling with plasma properties depends on the effective, collisional or turbulent, perpendicular heat diffusivity $\chi_\perp [22]$:

$$\Delta_\text{SOL} \propto \sqrt{L_\parallel n/T^{5/2}} \quad v_+ \geq 1 \quad v_+ \ll 1 \quad v_+ = V_{\text{thi}}/qRv_{\text{i}}.$$  

(3)

Here $L_\parallel$ is the connection length along B and $v_+$ is the dimensionless collisionality, $v_i$ is the ion collisional frequency, $n$ and $T$ are the SOL average plasma density and temperature. What the influence may be of the presence of dust on such transport quantity is an open question, but not negligible if one considers that the accumulation of nm dust with a density $n_d$ in the range $10^4$–$10^7$ cm$^{-3}$ in certain regions of the tokamak may give rise to true dusty-plasma phenomena, interacting with...
Figure 5. Particular trajectories for the case of asymmetric mass ejection for a particle of initial size 2 µm. In these four trajectories, the restitution coefficient for inelastic collisions is accounted for. The red dashed circle is the Fe boiling temperature boundary and the outer blue circle is the LCMS. The green and red crosses are the start and end points of the trajectory, respectively. The relative velocity between the dust and the ejected cloud used here is \( u = -\alpha \sqrt{2T_d/m} v_d/|v_d| \) where \( \alpha = 0.3 \) is a factor accounting for the asymmetric ejection. In trajectory B the presence of a 1 × 1 cm\(^2\) gap is considered.

Figure 6. Start-up plasma scenario: (a) shows the time rise of the toroidal magnetic field, the plasma current and temperature; (b) shows a trajectory starting with the mobilization of the ferromagnetic dust particle via the dipole interaction during the magnetic field growth phase. The table gives the values of the particle radius \( a \), the peak radial electric field \( E_0 \) and its peak value in the SOL \( E_{lim} \), the initial injection angle \( \theta_0 \), the total number of reflections at the wall \( N_{refl} \) and the final velocity \( v_f \).

<table>
<thead>
<tr>
<th>( a )</th>
<th>( E_0 )</th>
<th>( E_{lim} )</th>
<th>( \theta_0 )</th>
<th>( \theta_{v,0} )</th>
<th>( N_{refl} )</th>
<th>( v_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 µm</td>
<td>0.4:0.2</td>
<td>statV/cm</td>
<td>4.28°</td>
<td>/</td>
<td>43</td>
<td>1.5 km/s</td>
</tr>
</tbody>
</table>
plasma turbulence and affecting various transport phenomena in SOL, including $\chi_\perp$.

For nanosized dust it has been shown that loss of plasma particles and their momenta on the grain surfaces can play a crucial role in dissipation of plasma structures [23–27]. An interesting problem concerns the radiative MARFE instability [28] at the plasma cold edge. A recent work has shown [27] that plasma and nm dust parameters consistent with observations can be determined by realizing the balance of the nm dust-impurity drag with the thermophoretic force, which tends to localize highly radiating impurities in the colder layer of the SOL, producing the MARFE instability.

Another important problem concerns a mechanism of plasma contamination. A process of plasma contamination with impurities can be expected by nm dust penetrating the hotter SOL layer. It has been argued that this may favor a deeper penetration of impurities that can affect the edge plasma parameters and cause divertor detachment [4].

A model process of ‘diffusion of a heavy gas in a light gas’ has been proposed for nm dust in the SOL [29], under the assumption that the forces on dust particles near the wall are balanced by the friction force due to collisions with ions. Such an assumption might be violated near the wall where, depending on the dust size and velocity, the adhesion forces [30, 31] can cause particle sticking. However, under conditions where the force balance hold, the nanometre dust cloud radius can increase diffusively up to crossing the SOL, as seen in figure 7, thereby contaminating the plasma.

In figure 8 a simulation of carbon contamination carried out for DIII-D with the DUSTT code [4] shows a similar phenomenology. When the role of charge fluctuations in the SOL cannot be neglected it has been predicted that an instability may develop, which can lead to stochastic heating of the nanosized particles [29]. Associated with this kind of volumetric dust effect there is the important safety problem of explosions and of retention of radioactive tritium on which there is a vast specialized literature, e.g. [32]. It has also been observed that the start-up of a dust contaminated machine is more difficult than expected [29]. Furthermore, the MHD stability of current-driven modes has also been shown to be modified by dust [33]. Finally, the difficulty in observing and measuring nm dust in tokamaks is motivating intense research on novel in situ and ex situ diagnostics [34–36].

4. Conclusions

The dynamics of micrometre-sized ferromagnetic dust has been discussed in the framework of a single particle model. It has been shown that the presence of recesses in the vessel might prevent acceleration to the hypervelocity regime and provide a method for the capture of dust particles. The dynamics of ferromagnetic particles during the magnetic field start-up phase, in the absence of a plasma has also been considered. Future work shall be dedicated to study procedures for dust pre-cleaning during such a phase.

Further improvements are needed of the description of plasma–dust interactions and dust dynamics accounting for generic asymmetric grain shape and grain rotation and acceleration mechanisms. Metallic dust in fusion devices reaches very high temperatures and can melt. The theory of grain dynamics should be extended to the liquid phase of metallic dust, with consistent shaping of the particles. The role played by nanoparticles in the transport of impurity and on the possible stochastic acceleration of micrometre-sized dust has been reviewed.

Acknowledgment

This work, partially supported by the European Communities under the contract of EURATOM-ENEA-CNR Association, was carried out within the framework of EFDA. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

© Euratom 2012.
References

[21] Li X, Dunn P F and Brach R M 1999 J. Aerosol Sci. 30 439
[26] Ratynskaia S 2012 Nukleonika 57 307–12
[34] Ratynskaia S et al 2008 Plasma Phys. Control. Fusion 50 124046