Control of Water Content and Retention in Hydropower Plant Cascades

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Abstract

Title: Control of Water Content and Retention in Hydropower Plant Cascades

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The discharge through a river hydropower plant must be controlled such that the water level at a pre-specified point close to the facility is kept within given bounds. The controllers used today have a somewhat demanding tuning and often create too much amplified, unnatural discharge variations resulting in unsatisfactory control performance. This will affect both surrounding nature and imposing problems for river navigation.

This thesis will present a new type of controller called Override Selector feedback Control that adds an estimator for the water levels and water flows in the up- and downriver for each hydropower plant on top of the old controller. The objective of the state feedback control is to keep the total variation of the water levels and the water flows as small as possible. After the linear, discrete time model of the power plant cascade in a river derived from the Saint Venant equations have been developed, the new concept is evaluated.

Both the water level sloshing and the amplification of the discharges compared to the structure used today is damped with the new control structure. Other advantages of the proposed controller is that it will be cost efficient to implement because of the add-on approach. This is seen as a very important factor while the actual benefit that can be made by improving the water level control is very limited and thereby also the will to make extensive control investments. The control structure will be easily implemented as the estimators only need the same input data as used today.
Background and Motivation

The technical investigation and analyze that lays as ground for this thesis is the following:

- The discharge controllers used in the greater part of the hydropower plants of today often create too much amplified, unnatural discharge variations. This will affect both surrounding nature and imposing problems for river navigation.

- The actual benefit that can be made by improving the water level control is very limited and thereby also the will to make extensive control investments. This small benefit from investment comes from the almost constant generated electrical energy in a non water storing hydropower plant.

- The motivation for improved controllers comes mainly from threshold value, i.e. bounding regulations for water levels and water flows, imposed by public authorities. These regulations give rise to a ”just enough” need of control performance.

- The ownership landscape of Swiss hydropower plants is very diverse. There are very few plant cascades that belongs to the same company. This will complicate the information exchange between different plants and will contribute to a difficult implementation for every controller that depends on such a structure.

With these facts and assumptions as background the following project approach and project limitations are chosen:

- The current controller will be improved by a state feedback estimator for both the incoming upriver disturbances and the imposed downriver disturbances.

- The estimation horizon is limited to the next upper and lower hydropower plant to avoid any additional information exchange beside the one used today.

- The controller will be evaluated by a general one dimensional river simulation.

- The controller will be developed with an add-on approach. This will be cost efficient to implement while the current operational know how on the actual hydropower plant will be reused together with the current control structure. The system will thereby be very redundant and failsafe.
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Chapter 1

Introduction

Hydropower plants are hydroelectric and are built into the course of a river. They produce, with few exceptions, as much energy as possible during the whole year, i.e. they are base load power plants. A certain amount of water is always retained and dammed by the power plant constructions, but the intention is not to store water for later use.

1.1 General topology and structure

The hydropower house is the main part of a hydropower plant. It contains the turbines and the generators which produce electrical energy. The main control of the water discharge is also situated in the hydropower. The water flow through the turbines can be regulated by adjusting the inflow to the turbines and by varying the angle of the turbine blades. If the water flow exceeds the maximal flow for which the turbines are set there are always alternative paths for the water flow besides going through the turbines. These paths, that can consist of man made channels or just a division of the river flow as such, have weirs or flaps that will regulate this side flow. The controller will therefore either control the flow through this side flow or the flow through the turbines. But never both at the same time. In this thesis the paths will always be considered to be directly adjaent to the powerhouse and no further constructions are considered. The calculated hydropower plant discharge that we will talk about later is a joint flow control for the turbines and the side flow.

Figure 1.1: Adjoint Hydropower plant located in the Swiss city of Schaffhausen. The turbines and generators to the left and side flow to the right.
Chapter 2
Mathematical River model

2.1 The Saint Venant equations

The model of the fluid flow, or as called below, the river model, is made to describe the water flow in between the hydropower plants. The model rudiment used in this thesis is mainly based on Chapuis [1]. The following assumptions are made:

- Flow velocity and change of water depth are in every section constant.
- The curvature of the flow line is small. The acceleration of water masses in vertical direction is therefore neglectable.
- The friction against the waterbed can be modelled by empirical stationary conditions.
- The slope of the waterbed is small.
- The river cross section can be approximated as rectangular without any significant losses in river dynamics.

The river model is based on two equations presented for the first time by De Saint Venant 1871. The two equations are the equation of continuity and the equation of momentum.

### 2.1.1 Equation of continuity

The equation of continuity can be described in many alternative but equally valid forms. To derive the form presented in this thesis we start by defining the flow $Q(x,t)$ present in the cross volume $dx$ (fig. 2.1). Under the time $\delta t$ (from $t_0$ to $t_0+\delta t$) will the inflow to this water volume be

$$ V' = Q(x_0, t_0) \delta t \quad (2.1) $$

In the same time $\delta t$ will the outflow from the same water volume be

$$ V' = Q(x_0+\delta x, t_0)\delta t = \left( Q(x_0, t_0) + \frac{\partial Q}{\partial x} \bigg|_{x_0, t_0} \delta x \right) \delta t \quad (2.2) $$
Where we assume the step distance $\delta x$ as infinitesimal small. Further on can the volume change be expressed by the first order approximation of the change in the cross section $A(x_0, t_0)\delta x$ over the time $\delta t$

$$\Delta V = [A(x_0, t_0 + \delta t_0) - A(x_0, t_0)]\delta x = \frac{\partial A}{\partial t} \bigg|_{x_0, t_0} \delta t \delta x$$ \hspace{1cm} (2.3)

These three conclusions will then be combined by the fact that

$$\Delta V = V' - V$$ \hspace{1cm} (2.4)

Using the equations above for a infinitesimal small volume gives us the first of the two river equations, namely the equation of continuity

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0$$ \hspace{1cm} (2.5)

### 2.1.2 Equation of momentum

The equation of momentum describes the relationship between different forces acting on the water in the river, namely: pressure, gravity, acceleration and friction (fig. 2.2).

For a small cross section of water will the hydrostatic pressure difference between the back and forth side of the body in a first order approximation be expressed by

$$\Delta F_p(x_0, t_0) = F_p(x_0 + \delta x, t_0) - F_p(x_0, t_0) = \frac{\partial F_p}{\partial x} \bigg|_{x_0, t_0} \delta x$$ \hspace{1cm} (2.6)
2.1 The Saint Venant equations

Taking use of the information about the river width $B(z,x)$ and the depth of the water $H(x,t)$ (fig. 2.2) it is possible to express the hydrostatic pressure at position $x_0$ at the time $t_0$ by

$$F_p(x_0,t_0) = \rho g \int_0^{H(x,t)} B(z,x_0) [H(x_0) - z] \, dz$$  \hspace{1cm} (2.7)

Where $\rho = 1000 \text{ kg/m}^3$ is the water density and $g = 9.81 \text{ m/s}^2$ is the gravity. If we now form the derivative of the pressure as expressed to the right in equation 2.6 we will get

$$\frac{\partial F_p}{\partial x} = \rho g \frac{\partial}{\partial x} \left\{ \int_0^{H(x,t)} B(z,x) [H(x_0) - z] \, dz \right\}$$  \hspace{1cm} (2.8)

We will now regard the change of the river width as very small, $\partial B/\partial x \approx 0$, that reduces our problem to

$$\frac{\partial F_p}{\partial x} = \rho g \int_0^{H(x,t)} B(z,x) \frac{\partial H}{\partial x} \, dz$$  \hspace{1cm} (2.9)

$$= \rho g \int_0^{H(x,t)} B(z,x) \, dz$$  \hspace{1cm} (2.10)

$$= \rho g \frac{\partial H}{\partial x} A(x,t)$$  \hspace{1cm} (2.11)

We take finally use of (2.11) to express the hydrostatic pressure difference (2.6)

$$\Delta F_p(x_0,t_0) = \left. \frac{\partial F_p}{\partial x} \right|_{x_0,t_0} \delta x$$  \hspace{1cm} (2.12)
Under the conditions of a waterbed slope of $S_0 = \tan \theta(x)$ and a friction gradient of $S_f(x,t)$ will the gravitational force $F_g$ acting on the water mass yield

$$F_g(x_0, t_0) = \rho g A(x_0, t_0) \delta x \sin \theta(x_0) \approx \rho g A(x_0, t_0) S_0(x_0) \delta x$$ (2.13)

It now remains to show the friction force $F_r$ where we take use of a generally proven approach [2]

$$F_r(x_0, t_0) = \rho g A(x_0, t_0) S_f(x_0, t_0) \delta x \frac{\partial U}{\partial t}$$ (2.14)

We are now ready to establish the sum of all forces that acts on the water. They can be expressed explicitly by the product of the water mass and water acceleration ($F=ma$) where we use the derivation of the water velocity $U=U(x,t)$ to form the latter

$$F_g - \Delta F_p - F_r = \rho A(x_0, t_0) \delta x \frac{\partial U}{\partial t} \bigg|_{x_0,t_0}$$

$$= \rho A(x_0, t_0) \delta x \left( \frac{\partial U}{\partial x} \bigg|_{x_0,t_0} U(x_0, t_0) + \frac{\partial U}{\partial t} \bigg|_{x_0,t_0} \right)$$ (2.15)

We are now in position to substitute all the forces on the left side in the equation above and after some straightforward simplification and elimination of the velocity through $U(x,t)=Q(x,t)/A(x,t)$ we get the form of the second river equation as it is used in the simulation, namely the equation of momentum

$$\frac{1}{g} \frac{\partial}{\partial t} \left( \frac{Q}{A} \right) + \frac{1}{2g} \frac{\partial}{\partial x} \left( \frac{Q^2}{A^2} \right) + \frac{\partial H}{\partial x} + S_f - S_0 = 0$$ (2.17)

**2.2 Solving the Saint Venant equations**

To solve the two nonlinear partial Saint Venant equations (2.5 and 2.17) we have to take use of a numerical method. There are assumptions that can be made that will result in a linear equation [1], but that is of no use for this model. There are different numerical methods that can be applied. In this thesis the very well proven method spatial discretization is used [3]. But before the discretization is made we make a linearization our equations to, later in the process, be able to use linear control theory. To prepare for the linearization of the De Saint Venant equations we first take use of the assumption that the length of the river bed $P(x)$ in a cross section can be approximated as rectangular (fig 2.3).

$$P(x, t) = 2B(x) + H(x, t) \iff A(x, t) = B(x)H(x, t)$$ (2.18)
2.2 Solving the Saint Venant equations

Further on we define the hydraulic radius as

\[ R(x, t) = \frac{A(x, t)}{P(x, t)} \quad (2.19) \]

We are now in position to define the friction variable \( S_f(x, t) \) in the equation of momentum (2.17). This is done by the Strickler approach [2]

\[ S_f(x, t) = \frac{U^2(x, t)}{k_{str}(x) R^{4/3}(x, t)} \quad (2.20) \]

Where we decide the Strickler coefficient \( k_{str} \) by calibrating its value in the complete river model until the static water lever is levelled with the water bed (see section 3.3.1 on page 15).

2.2.1 Linearization of the De Saint Venant equations

If we sum up the De Saint Venant equations and manipulate them so they become functions of the water flow \( Q(x, t) \), the water depth \( H(x, t) \) and the length of the cross section river bed \( P(x, t) \) we get

\[ \frac{1}{B_0} \frac{\partial Q}{\partial x} + \frac{\partial H}{\partial t} = 0 \quad (2.21) \]

\[ \frac{1}{g} \frac{\partial}{\partial t} \left( \frac{Q}{B_0H} \right) + \frac{1}{2g} \frac{\partial}{\partial x} \left( \frac{Q^2}{B_0^2H^2} \right) + \frac{\partial H}{\partial x} + S_f - S_0 = 0 \quad (2.22) \]

Where, in the same manner, the friction variable \( S_f(x, t) \) becomes

\[ S_f = \frac{1}{k_{str}^2} \left( \frac{P}{B_0H} \right)^{4/3} \left( \frac{Q}{B_0H} \right)^2 \quad (2.23) \]
We are now ready to linearize the equations around the stationary state \( Q_0, H_0(x) \) and \( P_0(x) \) with small linear variation \( \Delta Q(x,t), \Delta H(x,t) \) and \( \Delta P(x,t) \)

\[
Q(x,t) = Q_0 + \Delta Q(x,t) \quad (2.24)
\]
\[
H(x,t) = H_0 + \Delta H(x,t) \quad (2.25)
\]
\[
P(x,t) = P_0 + \Delta P(x,t) \quad (2.26)
\]

Observe that the last variable can be expressed as \( \Delta P(x,t) = 2\Delta H(x,t) \) due to our rectangular river approximation. We now continue to make a first order Taylor approximation of the momentum equation (2.22) [1] (neglecting the static terms \( \delta B_0/\delta x \) and \( \delta H_0/\delta x \)). With help of the stationary loss coefficient

\[
S_f = \frac{1}{k_{str}^2} \left( \frac{P}{B_0 H} \right)^{4/3} \left( \frac{Q}{B_0 H} \right)^2 \quad (2.27)
\]

we can finally form the partial differential equations for the river model

\[
\frac{1}{B_0} \frac{\partial \Delta Q}{\partial x} + \frac{\partial \Delta H}{\partial t} = 0 \quad (2.28)
\]

\[
\frac{\partial \Delta Q}{\partial t} + \left( gB_0H_0 - \frac{Q_0^2}{B_0 H_0^2} \right) \frac{\partial \Delta H}{\partial x} + \frac{2Q_0}{B_0 H_0} \frac{\partial \Delta Q}{\partial x} + \frac{2g\Gamma_0 Q_0^2}{B_0 H_0^2} \left( \frac{4H_0}{3P_0} - \frac{5}{3} \right) \Delta H + \frac{2g\Gamma_0 Q_0^2}{B_0 H_0} \Delta Q = 0 \quad (2.29)
\]

### 2.2.2 Normalization and Transformation

To make our system even simpler to implement into the simulation we first normalize all the variables and transform the time and space vectors. To do so we start to rewrite our system using two help variables \( \epsilon(x), \eta(x) \), and the Froude Number [2] \( Fr(x) \)

\[
\epsilon(x) = \frac{5}{3} - \frac{4H_0}{3P_0(x)} \quad (2.30)
\]

\[
\eta = \frac{Fr^2(x)}{1 - Fr^2(x)} \quad (2.31)
\]

\[
Fr(x) = \frac{Q_0}{B_0(x)H_0(x)\sqrt{gH_0(x)}} \quad (2.32)
\]

The Froude number has an important physical meaning. It is the ratio between the flow- and the wave velocity. If \( Fr > 1 \) is the water in a so called supercritical state where the water conditions at any point cannot be influenced by the flow anywhere downstream of that point. If \( Fr < 1 \) the water is in a subcritical state and any point within the subcritical region is affected from both the upstream and downstream
directions. In this thesis will the water always be assumed to be in the subcritical state without any significant losses in dynamics [12].

We continue to simplify our system by normalizing the variables

\[
q(x, t) = \frac{\Delta Q(x, t)}{Q_0}, \quad h(x, t) = \frac{\Delta H(x, t)}{H_0}.
\]

\[
b_0 = \frac{B_0(x)}{B_N}, \quad \gamma_0(x) = \frac{\Gamma_0(x)}{\Gamma_N},
\]

and making a time and space transformation

\[
\tau = T_N t = \frac{g \Gamma_N Q_0}{B_N H_0}, \quad \chi = g \Gamma_N x.
\]

After implementation of (2.30)-(2.34) is our partial differential system expressed in Matrix form and it will thereby take the expression that later will be implemented in the simulated model

\[
\begin{bmatrix}
\frac{\partial h}{\partial \tau} \\
\frac{\partial q}{\partial \tau}
\end{bmatrix} =
\begin{bmatrix}
0 & -\frac{1}{b_0} & 0 & 0 \\
\frac{1}{b_0 h_0^2} & -\frac{2}{b_0 h_0} & 2 \gamma_0 & -\frac{2 \gamma_0}{b_0 h_0}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial h}{\partial \chi} \\
\frac{\partial q}{\partial \chi} \\
h \\
q
\end{bmatrix}
\]

To solve the River model equation (2.35) we assume static initial conditions \( q(\chi, 0) = h(\chi, 0) = 0 \). As a last step, we now need to calculate the boundary condition, which could be either the variable \( H(\chi, 0) \), the variable \( Q(\chi, 0) \) or a combination of the two. For simplicity we choose to back calculate \( H(\chi, 0) \) and we therefore need the water depth at the very end of river \( H(L_c, 0) \) as input, where \( L_c \) is the length of the river. The back calculation is described more in detail in section 3.1.1 Static initial conditions.

### 2.2.3 Spatial Discretization

To prepare our model for the numerical discretization we divide our river into \( n+1 \) small cross sections along the direction of the river (fig. 2.4). The length of every section (except the first and last that will be half as long) will be \( l_s = \frac{g \Gamma_N L_c}{n} \) where \( L_c \) is the total river length (fig. 2.4). To avoid unnecessary stiffness in our algorithm we overlap the crossings of the different variables. The variables \( q \) is calculated at the crossing of each section and the variable \( h \) is calculated in the middle of every section. The discretization is made by the finite difference method [22]. To approximate out derivatives we use the first term in the Taylor’s series expansions of \( q \) and \( h \) around any point \( \chi_i \). This is shown for \( \frac{\partial q}{\partial x_{2i+1}} \) below, \( \frac{\partial h}{\partial x_{2i}} \) is constructed in i similar manner. Observe that the equations must be adjusted in a straightforward manner for the first and last section as they are only \( l_s/2 \) long.
Chapter 2 Mathematical River model

\[ q(\chi_{2i+2}) \approx q(\chi_{2i+1}) + \frac{\partial q}{\partial \chi_{2i+1}}(\chi_{2i+2} - \chi_{2i+1}) \]  \hspace{1cm} (2.36)

\[ q(\chi_{2i}) \approx q(\chi_{2i+1}) - \frac{\partial q}{\partial \chi_{2i+1}}(\chi_{2i+1} - \chi_{2i}) \]  \hspace{1cm} (2.37)

\[ \chi_{2i+2} - \chi_{2i+1} = \chi_{2i+1} - \chi_{2i} \]  \hspace{1cm} (2.38)

or

\[ \frac{\partial q}{\partial \chi_{2i+1}} = \frac{q(\chi_{2i}) - q(\chi_{2i+2})}{2(\chi_{2i+2} - \chi_{2i})} \]  \hspace{1cm} (2.39)

Using this approximation means that within every section \( \chi_1 - \chi_2, \chi_{2i} - \chi_{2i+2} \) and \( \chi_{2n} - \chi_{2n+1} \) will the variable \( \frac{\partial q}{\partial \chi} \) be defined and within every section \( \chi_{2i-1} - \chi_{2i+1} \) will the variable \( \frac{\partial h}{\partial \chi} \) be defined (see example of calculation fig. 2.4). When the cross sections are made small enough (small \( l_s \)) the model can be assumed to be accurate.

Figure 2.4: Discretization of the river with example of calculation
Chapter 3
Matlab simulation of the river model

3.1 Implementing the river model in Matlab

3.1.1 Static initial conditions

Before we can implement the river model equation into Simulink Matlab we have to calculate the boundary condition as already mentioned at the end of section 2.2.2. This is done by constructing a Matlab algorithm which, starting from the water depth \( H(L_c, 0) \) going backwards, calculates the static water depth. For every two meter a new iteration is started and the new water depth will be calculated. After \( j = \frac{L_c}{2} \) iterations is the now calculated water depth vector \( H_0(x_j, 0) \) reduced into the normalized and transformed vector \( h_0(\chi_i, 0) \) and all conditions needed for a simulation are at hand. Boundary and initial conditions are summarized below

\[
q(\chi, 0) = h(\chi, 0) = 0 \quad q_0(\chi, 0) \quad h_0(\chi, 0)
\]  \hspace{1cm} (3.1)

**Boundary and initial conditions**

3.1.2 Simulink model

The discretization in Simulink (fig. 3.1) is made with \( n + 1 = 7 + 1 = 8 \) number of sections. Considering our River cross sectioning (fig. 2.4) this means that we will have a total of \( 2n + 1 = 15 \) different height \( h_0(\chi_i, \tau) \) and \( n = 7 \) flow \( q_0(\chi_i, \tau) \) states. The subsystems in the Simulink model (fig. 3.2) is an realization of the equation system 2.35.

3.2 Model verification

To verify that our model is accurate we put it up for two test cases consisting of a \( \pm 10\% \) step change in the outflow \( Q_{out} \) (3.3 and 3.4). In both cases will we examine the time it takes for a surge wave to propagate through the whole river length, reflect and come back again.

\(^1\)the even numbered water height states are simply a linearization of the two adjacent odd water height states.
Chapter 3 Matlab simulation of the river model

Figure 3.1: Part of Sectioning in Matlab Simulink

Figure 3.2: Example of river sector 3 in Matlab Simulink
3.2 Model verification

3.2.1 Theoretical model dynamics

The theoretical speed of the surge wave \( v_{tW} \) is given by the Froud number, the flow \( q(x, t) \) and the flow direction

\[
v_{tW} = \begin{cases} \frac{Q_0}{A} \frac{1}{Fr} = \sqrt{gH} \\ \frac{Q_0}{A} (\frac{1}{Fr} - 1) = \sqrt{gH} - \frac{Q_0}{A} \end{cases}
\] (3.2)

The theoretical propagation time \( t_{tp} \) therefore becomes

\[
t_{tp} = \frac{L_c}{v_{tW}} + \frac{L_c}{v_{tW}} = \frac{L_c}{\sqrt{gH}} + \frac{L_c}{\sqrt{gH} - \frac{Q_0}{A}}
\] (3.3)

To continue our verification we will also do an estimation of the water volume change \( \Delta V \) in the river under the same test cycle as above. The theoretical volume change is given by

\[
\Delta V_t = Q_{out} \Delta t
\] (3.4)

3.2.2 Case 1: medium flow conditions

The first test case is constructed under conditions of a medium initial water flow \( Q_0 = 160 \text{ m}^3/\text{s} \) and steep river bed \( S_0 = 1/2000 \). This will result in fast surge waves. In figure 3.3 is an average derivative calculated for the water height \( h33 \) called \( h33\text{linear} \) up to the time of outflow change, 45 minutes, to approximate the water volume change. It is possible to measure the propagation time \( t_{c1P} \approx 13 \text{ min 8sec} \). If we now calculate the theoretical propagation time \( t_{t1P} \) in equation 3.3 for \( Q_0 = 160 \text{ m}^3/\text{s} \) we will get \( t_{t1P} = 13\text{min 27sec} \). Finally we use figure 3.3 to make a rough approximation of the volume change \( \Delta V_{c1} \). After a linearization of the data for the first step we'll get \( \Delta V_{c1} \approx 44200 \text{ m}^3 \). This should be compared to the theoretical value \( \Delta V_{t1} \approx 44600 \text{ m}^3 \).

3.2.3 Case 2: Low flow conditions

The second test case is constructed in the same manner as the first test case with the difference that the initial water flow is low, \( Q_0 = 40 \text{ m}^3/\text{s} \), and the river bed is flat, \( S_0 = 1/12000 \). This will result in slow surge waves. The results we will obtain are \( t_{c2P} \approx 14 \text{ min 8sec} , t_{t2P} = 14\text{min 37sec} , \Delta V_{c2} \approx 10400 \text{ m}^3 \) and \( \Delta V_{t2} \approx 10800 \text{ m}^3 \) (see fig. 3.4). Observe that \( h33\text{linear} \) is an average derivative calculated for the water height \( h33 \) up to the time of outflow change, 45 minutes, to approximate the water volume change.

3.2.4 Summarize model verification

To summarize our two test cases we have to keep in mind that our modelled water volume change \( \Delta V_c \) is very roughly calculated. Despite this uncertainty factor will the results, in all test cases, show a maximal deviation of \(< 4\% \) between theoretical and calculated values. This is considered as a sign that our model is accurate enough for our task.
Chapter 3 Matlab simulation of the river model

Figure 3.3: Water level response from $\pm 10\%$ changes in the outflow $Q_{out}$ with $Q_0 = 160 m^3/s$ and $S_0 = 1/2000$

Figure 3.4: Water level response from $\pm 10\%$ changes in the outflow $Q_{out}$ with $Q_0 = 40 m^3/s$ and $S_0 = 1/12000$
3.3 Geographical Parameters

As the general river model now is created we move on into defining our control scenario. This is done by fixing all the geographical variables in the river model. The values are deliberately chosen, in cooperation with ABB Utility Automation Baden and Automatic control lab ETH Zurich, for a narrow low water river with a low nominal water flow. This is considered as a typical complicated setup to control and will strengthen the complications that are included in a River Water level control.

3.3.1 Determination of Geographical Parameters

As seen in table 3.2, is the model parameters chosen so that the river always is assumed to have the same width \(B_0 = 22\text{m}\). The river length \(L_0\) is chosen to be 2000 m. The nominal flow at steady state \(Q_0\) is chosen to be 40 m\(^3\)s\(^{-1}\), which could be assumed to be a reasonable value for such geographical river geometry \([4]\). As already mentioned in section 2.2 on page 7 the initial water depth \(H_0(x)\) is calculated with help of the Strickler coefficient \(k_{str}\) and a back calculation. By trial and error is the Strickler coefficient \(k_{str}\) changed until the friction variable \(S_f(x)\) is levelled with the calculated water depth \(H_0(x)\) (see equation 2.20). These river data will explicitly be chosen for all the following river simulations.

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t_{cP})</td>
<td>13min 8sec</td>
</tr>
<tr>
<td>(t_{tP})</td>
<td>44200 m(^3)</td>
</tr>
<tr>
<td>(\Delta V_c)</td>
<td>(&lt; 3%)</td>
</tr>
<tr>
<td>(\Delta V_t)</td>
<td>(&lt; 4%)</td>
</tr>
</tbody>
</table>

Table 3.1: Results from the model verification

<table>
<thead>
<tr>
<th>Chosen Geographical parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(L_c) [m]</td>
<td>2000 m</td>
</tr>
<tr>
<td>(Q_0) (\frac{m^3}{s})</td>
<td>40 (\frac{m^3}{s})</td>
</tr>
<tr>
<td>(H_0(L_c)) [m]</td>
<td>2.5 m</td>
</tr>
<tr>
<td>(B_0) [m]</td>
<td>22 m</td>
</tr>
<tr>
<td>(k_{str})</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table 3.2: Chosen values for the restriction parameters
Chapter 3 Matlab simulation of the river model
Chapter 4

Single Hydropower Plant Control

4.1 Introduction to river water level Control

This chapter discusses the general features of building a River water level control system. The objective for all river water level control systems is to adjust the outflow $q_{\text{out}}$ from the power plant and thereby achieve the sought water level $h(x)$ and overall flow $q(x)$. This is, as it shall be explained in the following chapters, not a trivial task. There are several aspects of the water transportation in a river that has to be taken into account before a functioning river water level control can be developed. Two of the most important of these aspects are the so called Backwater effect and the closely related Retention.

4.1.1 Backwater effect

The section in the river where the water flow without any influence by a dam is called uniform flow section. In this section will the water flow unaffectedly without any influence from dams or other obstacles and therefore will the water level have the same height (measured as height over ocean) over the whole section length. In the section close to the hydro power plant will the water be retained by the dam and therefore will the water level in this section be affected. This later section is called the backwater section (see figure 4.1).

4.1.2 Retention

If a hydropower plant is introduced into a river will it always cause Retention [7]. The effects of the retention is easiest shown by an example. We start by presuming that there is an initial steady state condition with the constant flow $q_1$ over the whole river and also as discharge and a constant river water height $h_1$ (see figure 4.2). We further presume that there is an ideal headwater level control (see figure Retention1.eps) just before the outflow power plant (the measured value is often related to as the concession level). An overall flow change from $q_1$ to $q_2$ will impose a change in the water level

\[ \Delta h = \frac{q_2 - q_1}{2g} \]

\[ \text{in this thesis thanks to the state overlap} \]

\[ ^{1}\text{In reality is the concession level situated a little bit further upstream compared to the headwater level. This is to avoid the swiftly-flowing water close to the hydropower plant. This will impose a small variation in the head water level as the retention level changes. But the distance between the headwater and concession level is always small compared to the total length of the river. This distance is conveniently approximated as} \]

\[ \frac{L}{2} \]
from $h_1(x)$ to $h_2(x)$. In other words, an increase in the overall flow will increase the water volume that is stored in the river. This volume is called Retention volume (see figure 4.3). This is a very important factor to consider if a river Water level control is going to be constructed. As shown in figure 4.3 will the end of the backwater also be changed with the overall flow change from $q_1$ to $q_2$. Having introduced the concept of backwater effect and retention volume we are now ready to sketch some simple demands for our controller.

4.1.3 Retention Demands for River Water level Control

Simply explained is the change of the river discharge our control mechanism and the fixed headwater level our goal. That means that given, from steady state initial conditions, a change in the river inflow $q_{in}$ by $\Delta q$, the outflow discharge $q_{out}$ has to be adjusted in the same manner after the time $T_R$ to keep a constant headwater level. In the same time is the water stored in the river section changed with the retention
4.1 Introduction to river water level Control

Figure 4.3: Retention volume imposed by inflow change from $q_1$ to $q_2$

The retention volume $V_R$

$$T_R = \frac{V_R}{\Delta q}$$  (4.1)

For a decreasing (negative) disturbance inflow step will the discharge $q_{out}$ control slowly adjust to the same discharge as the inflow $q_{in}$, and thereby automatically releases the retention volume. For an increasing (positive) disturbance inflow step will the slow discharge thereby automatically store the retention volume, just as we want it to do. This is the reason why we will only need to chose a positive step disturbances in the later control scenarios. Nevertheless, this is a very important conclusion when we talk about control of a cascaded hydro powerplants. Its consequence will be:

To keep a constant headwater level we are forced to inform the next down-river hydropower plant about increases and decreases in $q_{in}$ so the retention volume wont be exaggerated. This demands an information exchange between, at least, the neighborhood powerplants.

To achieve a damped behavior of the propagating flow wave over the cascaded hydropower plants the theoretical main task for our controller will be to try to start to adjust for the retention volume before the retention time has passed. This will lead to a smooth discharge adjustment without overshoot and thereby will an amplification of the discharges along the cascade be avoided.

Note that we until now always assumed steady state conditions in the initial as well as in the final hydraulic state of the river. In a more realistic case would the water level control have to consider other dynamics beside the strict compensation of the retention and therefore include more of a tradeoff optimization of the retention.
Figure 4.4: Retention time $T_R$ imposed by a change $\Delta q$ in the inflow $q_{in}$

4.2 PI Control with feed-forward

4.2.1 Theory PI Control with feed-forward

The control that is applied to the most of the current power plants is a PI controller with a feed-forward term. The manipulated variable in this setup is the discharge $q_{out}$ and the variable that is being tracked and controlled is the concession level $h_c$. The objective is to keep the concession level $h_c$ between certain limits that in its simplest form is being expressed by a single reference value $h_r$. The information from the upriver Power plant, i.e. the inflow $q_{in}$, is filtered by a term $q_{FF}(t)$. Combining this with the proportional gain $K_P$ and the Integral term $K_I$ of a classic PI controller gives us the following expression for the discharge $q_{out}$

$$q_{out}(t) = K_P \delta h(t) + K_I \int \delta h(t) dt + q_{in} \cdot q_{FF}(t)$$  \hspace{1cm} (4.2)

where

$$\delta h(t) = h_c - h_r$$  \hspace{1cm} (4.3)

The feed-forward term $q_{FF}$ will in this setup represent an anticipation of the disturbances in the river and if it was ideal it would more or less make the PI controller useless. As the $q_{FF}(t)$ in reality often is represented by a low pass filter to account for the wave damping and a time delay to account for the retention this is hardly the case. As we have seen above the retention volume will change with the discharge and this dynamic will be impossible to capture with such a filtering. Therefore will the PI-controller have to account for the error in the $q_{FF}(t)$ feed forward estimation by locally watch over the concession level $h_c$ and adjust the outflow $q_{out}$ as the concession level differ from the reference value $h_r$. 
We are now ready to add a PI Control with feed-forward to our model according to figure 4.5. In the same figure is there an element added after the PI controller called restrictions1. This is added to simulate the restrictions given by the Power Plant turbines and gates. The limitations are supposed to be of two kinds. The first one sets the maximum speed of change in the discharge $q_{out}$. This is decided by the parameter $RT$ in figure 4.6 which gives the minimum time in minutes to go from discharge 0 to $Q_0$. The second limitation deals with the gate sensitivity. The parameter $DZ$ sets the minimum error (difference between the headwater level $h_{out}$ and reference value $h_r$) in the discharge to impose a discharge change. The restrictions are summarized in table 4.1.

Figure 4.5: Example of a PI controller implemented into a river model. Observe that the picture show a river system with $n=32$ states
4.2.3 Tuning of the PI Control with feed-forward

To tune the PI controller, i.e. to decide the proportional gain $K_P$ and the Integral term $\frac{1}{T_i}$ in figure 4.7, we take use of classical pole placement theory. Motivated by the river section design in figure 3.2, we approximate our river model as a first order pure integral system, i.e. it could be approximated with the following transfer function

$$G_m(s) = k_s \frac{1}{sT_v}$$  \hspace{1cm} (4.4)

If we now place the close loop system poles at a position $\Omega$, it is possible to express the two controller parameters $K_P$ and $\frac{1}{T_i}$ as a function of $k_s$, $T_v$ and $\Omega$ \[5\]

$$\frac{1}{T_i} = \frac{1}{k_s} T_v \Omega^2$$  \hspace{1cm} (4.5)

$$K_P = \frac{1}{k_s} 2T_v \Omega$$  \hspace{1cm} (4.6)

Our task is now to decide the system parameters $K_s$ and $T_v$. To do so we assume the time constant $T_v$ to be a function of the River width $B(x)$ and River length $L_c$. The gain parameter $K_s$ is set to one

$$T_v = \int_0^{L_c} \frac{A_0(x)}{Q_0} dx = \frac{L_c B_0(x) H_0(x)}{Q_0}$$  \hspace{1cm} K_s = 1$$  \hspace{1cm} (4.7)

Finally, we implement these numbers into the equations 4.5 and 4.6 followed by a trial and error procedure for a couple of reasonable values for the pole placement $\Omega$. This leads to an acceptable step answer for $\Omega = 0.12$ and by that are all the control parameters decided and we are ready to test the behavior of our PI control.
4.2.4 Results from PI Control with feed-forward

Applying a disturbance step of $+4\Delta q^3$ (i.e. 10% of the value for $Q_0$) to the inflow $q_{in}$ with the discussed PI Control attached will lead to water level and flow changes as shown in figure 4.8. The result is quiet satisfactory if we look at the tracking of the reference value $h_r$. But a concerning factor is the used gain in the discharge $q_{out}$ related to the inflow $q_{in}$. As we shall see later on when we connect several powerplants in a cascade, the use of a moderate discharge $q_{out}$ is one of the most important and difficult goals to obtain in building a River Water level Control. The large gain used in $q_{out}$ comes mainly from the fact that the error reduction of the feed forward estimation, i.e. our PI controller, has a very short field of vision in terms of time and space. This leads to unnecessary short-term adjustments of the discharge and we will get a "stiff" controller that puts far to much focus on the local changes in the concession level $h_c$ and neglects the more important long-term damping factors of the disturbances. In our example is this shown by the initial rise of the discharge in the lower graph in figure 4.8 at approximately the time 5 minutes. This implements an initial drop in the headwater level $h_{out}$ that immediately is corrected by a the PI control in form of a decrease in the discharge. But as the disturbance flow reaches the headwater level at about two minutes later the PI controller have to work in the opposite direction by increasing the discharge. Its easy to see that the first correction was unnecessary seen in terms of damping and actually just contributed to an unnecessary large discharge in the later correction. This is a well known problem in reality and have led to several extensions to our example in forms of local tuning, fixes and adoptions to overcome the disturbance enlargement. The most easiest way to approach the problem, and which is broadly accepted in reality [17], is simply to tune the PI-controller to be very slow.
Figure 4.8: Result from applying a disturbance of $+4 \frac{m^3}{s}$ to the inflow $q_{in}$
Chapter 5
Cascaded Hydropower Plant Control

The developed Hydro Water control systems that are presented in this thesis is mainly influenced by the work of Linke and Arnold [17] and presents a partial state feedback control with either a fast up basin or slow down basin control. The chosen name for this controller is Override Selector feedback Control. The idea here is to, as long as possible, choose a damping slow controller that puts much effort into keeping the water level and flow sloshing in the downriver under control. If, but only if, the headwater level error exceeds a certain level called DeltaL (approximately somewhere between 2cm-5cm), will attention be paid to the upriver until the system is yet again below this certain error level.

5.1 PI Control with feed-forward

5.1.1 Results PI Control with feed-forward

As a reference we attach the PI Control with feed-forward that we developed in section 4.2 to a four hydropower cascade. To make our implementation as realistic as possible we try to make the PI controller less stiff compared to the choice we had for a single Hydropower plant according to the discussion in 4.2.4. The setup seen in table 5.1 have been chosen to evaluate the system. The result for a step disturbance is seen in figure 5.1. We can in the lower four plots clearly see how the propagating wave is increased along the way, despite the fact that the single objective for this controller is to keep the water level at the reference. The reason for this behavior is the same as we have seen for the single hydropower plant, i.e. because of the increased water flow, is there a specific retention volume that has to be stored in between the hydropower

<table>
<thead>
<tr>
<th>Step Simulation Time [min]</th>
<th>90min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step disturbance [m³/s]</td>
<td>4 m³/s</td>
</tr>
<tr>
<td>Sinus Simulation Time [min]</td>
<td>440min</td>
</tr>
<tr>
<td>Sinus Amplitude disturbance [m³/s]</td>
<td>2 m³/s</td>
</tr>
</tbody>
</table>

Table 5.1: Simulation and Control setup for the step disturbance
plants. After this volume has been stored should the discharge increase to the same amount as the inflow according to the retention theory (to impose a damping behavior, even a little bit earlier). But the controller will not be able to increase the discharge in time as the PI part holds it back until the propagating wave reaches the concession level. This typical behavior stores to much water in each section. This water must then be released at a later time. This is the reason why the four upper plots, the ones over the local water flow, also has an non damping behavior. The result from a sinus disturbance is seen in figure 5.2 (please note the different time scale compared to the step disturbance). Here is another shortcoming of the PI controller obvious. The inflow will in this case assist the discharge by, after the initial increased inflow, deliver less water and thereby avoid the complication with to much stored water as in the case for the step disturbance. This damps the water level very well. But as the flow changes also propagates down the river together with the water wave will the controller always add more discharge to an already increased water flow. Remember that we don’t have any reference tracking of the water discharge, so the controller is not aware of the big amplifications it delivers to the discharge.

5.2 Override Selector State Feedback Control

The more specific idea behind the Override Selector feedback Control approach is to feed the controller with, on one hand the approaching flow and water levels from the upriver, and on the other hand the effects the controller has on the flow and water level in the downriver. This information is of course not at hand, that would demand an extensive amount of sensors and measure points down the river, so the theory of State feedback control is implemented to estimate these values [14].

5.2.1 Theory State feedback Control

The general concept behind a State feedback control is to build an estimator parallel to the real system where the variables that are normally not at hand can be collected (see figure 5.3). These variables are then forwarded to the controller which can use this information to make a better prediction, in our case a better prediction of the optimal discharge. The two estimators, one for the upriver and one for the downriver, have the same objectives. That is to keep the water level and the water flow as stable as possible with small differences between the collected variables. The difference between the two estimators, beside the tuning, is that they concern two different parts of the river. That will lead to the situation where the two estimators objectives often will contradict each other. Very simply explained will the objectives for one of the river sections surrounding a hydropower plant be made worse when we change the discharge to make the objectives for the other surrounding river section hydro power plant better. This calls for a trade off between the two estimators. The trade off presented in this thesis is made by simply implement a selector between the two estimators so only one of them will affect the discharge at the same time. Given the right conditions for the selector, will lead this to a satisfactory solution for the operational discharge control.
5.2 Override Selector State Feedback Control

Figure 5.1: Step Disturbance applied to a four hydropower plant cascade controlled with a PI Control with feed-forward. The four river sections between the hydro power plants, in the figure called River A, B, C and D, are organized with the inflow in the top left corner and hydro power plant control to the right of the plots. The outflow from the upper river section is inflow to the lower.
Figure 5.2: Sinus Disturbance applied to a four hydropower plant cascade controlled with a PI Control with feed-forward. The four river sections between the hydro power plants, in the figure called River A, B, C and D, are organized with the inflow in the top left corner and hydro power plant control to the right of the plots. The outflow from the upper river section is inflow to the lower.
5.2 Override Selector State Feedback Control

5.2.2 Development of State feedback Control: upriver

The estimators are constructed with a classic parallel State feedback method (see [5] pp. 231-252). The structure for the estimator is shown in figure 5.4. Note that the lower part with the gains $k_P s$ and $\frac{1}{T_i}$ has the same structure as the headwater PI control in section 4.2.3. The time constants for the estimator is constructed with help of the Froud number (see equation (3.2))

\[
T_1 = T_3 = T_f = \frac{L_c}{2v_{W}} \approx 202s
\]

\[
T_2 = T_w = \frac{L_c Q_0}{AH_0g} \approx 59s
\]
Out of these time constants can we construct the system \( \Omega_r \) defined as

\[
\Omega_r = \sqrt{\frac{2}{T_f T_w}} \approx 0.013 \text{s}^{-1}
\]  

(5.3)

To check if this value is reasonable as a system time constant can we compare it with the echo wave propagation time

\[
T_{echo} = \frac{2L_c}{v_{W} + v_{W}} \approx 473 \text{s}
\]  

(5.4)

\[
T_P = \frac{2\pi}{\Omega_r} \approx 483 \text{s}
\]  

(5.5)

As the two time constants are of the same order will this indicate that \( \Omega_r \) is a good measurement for the system behaviors. We will later use it to place our poles for the closed system.

Our task is now to decide the gains \( k_2s, k_3s, k_Ps \) and \( \frac{1}{T_I} \) for the upriver control in figure 5.4. This is made in a similar matter as the pole placement for the single PI control in section 4.2.3. But as we now have four unknown variables and therefor have to place four poles at the same time we have to construct the state space matrix and solve the system for the four unknown gains. To do this we first define the four states \( x_{h1}, x_{q2}, x_{h3} \) and \( x_I \) with reference to the figure 5.4 where the last state \( x_I \) refers to the PI Integral. The state matrix will then be

\[
\begin{bmatrix}
T_1 s & 0 & 0 & -1 \\
0 & T_1 s & 1 & 0 \\
0 & -1 & T_2 s & 1 \\
0 & k_3 & (-1 + k_2)(T_1 s + k_3 + k_P)
\end{bmatrix}
\begin{bmatrix}
x_I \\
x_{h1} \\
x_{q2} \\
x_{h3}
\end{bmatrix}
\]  

(5.6)

To place the poles of the closed system we choose two different placements, two equal slower poles \( \Omega_1 \) for the water level states, i.e. \( x_{h1} \) and \( x_{h3} \), and two faster \( \Omega_2 \) for the flow states, i.e. \( x_{q2} \) and \( x_I \)

\[
\begin{bmatrix}
T_1 s & 0 & 0 & -1 \\
0 & T_1 s & 1 & 0 \\
0 & -1 & T_2 s & 1 \\
0 & k_3 & (-1 + k_2)(T_1 s + k_3 + k_P)
\end{bmatrix}
= (\Omega_1 + s)^2(\Omega_2 + s)^2 = 0
\]  

(5.7)

The poles \( \Omega_1 \) and \( \Omega_2 \) will in a first approximation be chosen as

\[
\Omega_1 = 0.4\Omega_r
\]  

(5.8)
5.2 Override Selector State Feedback Control

\[ \Omega_2 = 0.7\Omega_r \]  \hspace{1cm} (5.9)

These values will then be slightly changed, showed by the index s in figure 5.4, as the parameters for the variables \( k_2 \), \( k_3 \), \( k_P \) and \( \frac{1}{T} \) will be adjusted around their original values, given from equation 5.7, to trim the behavior of the controller. Doing this will make the poles travel away into the imaginary plane. The rule of a thumb that we use to avoid creating an unstable system (because of a too aggressive imaginary part) is that the real part always should be greater than the imaginary.

![Figure 5.5: Downriver State feedback Estimator](image)

**5.2.3 Development of State feedback Control: downriver**

The downriver estimator is constructed in the same manner as the upriver estimator. But as the controller now is situated at the inflow of the river section will the setup look a little bit different compared to the upriver estimator. The complication is that the headwater level that the PI-controller regulates is situated in the upriver. Therefore it is not possible to only concern the downriver in the setup. One solution would be to construct a system of the whole upriver and the whole downriver together with the controller. But as this system would be expressed as an 8x8 matrix will it be too complicated to use for classical pole placement. The solution used in this thesis is to only consider the last state in the upriver, namely \( x_{k_3} \) (see figure 5.5). This will lead to a system state matrix of the size 5x5 which will be considerably much more easy to solve.
Figure 5.6: Sought control procedure. When the headwater level error exceeds $\delta L$, will the stiffer upriver control take over to ensure that the maximum headwater error is $\leq \delta L$.

Our task is now to decide the gains $k_4 s$, $k_5 s$, $k_2 P s$ and $\frac{1}{T_2 I}$ for the downriver control in figure 5.5. We use the Maple function Solve, implemented into Matlab, to extract the gains out of the 5x5 state matrix. This enables us to define a separate pole for every state, i.e. $\Omega_1$, $\Omega_2$, $\Omega_3$, $\Omega_4$ and $\Omega_5$. As the downriver control should be a slow controller we choose the first approximation of the PI-controller poles to be at half the size of the Stiffer upriver control. We then adjust them in the same manner as described for the upriver control.

5.2.4 Combining upriver and downriver control

In the final control structure, as briefly explained in chapter 5.2, we combine the two controllers. We first define the headwater level error as the variable that decides which of the two controllers, i.e. upriver or downriver, that affects the discharge in the hydro power plant. The slower of the two controller, the downriver controller, will control the discharge as long as the headwater level error is under the level $\Delta L$. If the error exceeds this level, the stiffer upriver controller will take over the control of the discharge until the error level $\Delta L$ is no longer exceeded. The sought control procedure is shown in figure 5.6. Building this control setup, we have to pay attention to the fact that the non active controller will not have the same output as the active controller in the most simple combining structure. To deal with this problem we chooses to implement two calibrated anti windups that will add an extra gain to the non active $I$-controller that correlates with the error mentioned above [20]. The final Controller will then look as shown in figure 5.7. Notice the input indexes for the upriver- and downriver-estimator. When we later attach the controller to the estimators (with $n = 7$ flow states and $2\times n + 1 = 15$ water level states) these indexes will correlate to the three state structure that we had building and tuning the two
controller. The estimators and controllers implemented into the river simulation can be seen, for a two hydropower plant system, in figure 5.8.

Figure 5.7: Combined Upriver and Downriver control, the term $\Delta L$ is the headwater level error where the controllers change who’s in charge.

Figure 5.8: The Override Selector State Feedback Control implemented in a two hydropower plant simulation
Figure 5.9: Step disturbance comparison between the estimation and the river simulation (real in figure)
5.2.5 Verification of the State feedback Controller

The verification of the estimators is made by feeding the river simulation and the estimator with the same step disturbance and compare the calculated flow signals from the two systems (see figure 5.3). Naturally, we want the estimator to copy the behavior of the river simulation in the highest degree possible. In figure 5.9 is two system responses to a step disturbance plotted. We can see that the water level is very well tracked by the estimator in the upper plot but it still lacks, some of the dynamics at the gradient changes. This is especially obvious at the top of the very first oscillation. Looking at the lower plot in figure 5.9, we can see that the estimator has a greater problem to track the flow. This is not surprising when we compare the estimator design in figure 5.5 with the river sector design in figure 3.2. There is a small time delay between the two systems and the estimator understates the strength in the oscillation. looking closely at the plot we can suspect that this behavior is due to the second "push" present just before the top of every oscillation where the flow at first starting to weaken. This dynamic is not present in the estimator and makes its oscillation too weak. This shown behavior of the estimators is however just what we can expect when we try to map a higher order system with a lower one and it’s nevertheless convincing to see that the overall behavior in the river simulation is captured by the estimators.

5.2.6 Results of State feedback Control

Implementing our estimator and our Override Selector Controller into the river simulation we are now ready to examen the disturbance result. The setup seen in table 5.1 have been chosen to evaluate this system and the term \(\Delta L\) (see chapter 5) have been set to \(\Delta L = 0.035m\). The result from the step disturbance can be seen in figure 5.10. Notice the damping of the water level disturbance as the disturbance propagates downriver. This a a clear improvement compared to the common PI controller. The effect of the Override selector can be seen in the plot for the River B water level at the approximate time 18 minutes. Here’s the more stiffer upriver controller selected to take control over the fast increasing level hB15. This results in some magnified oscillation that is later very well reduced by the lower downriver controller for river section C and D. Overall is the fast high frequency oscillations very well damped and the slower low frequency oscillations are less damped. This is just what we could expect from our estimator setup. Looking more closely at the graph for the local water flow we’ll see that we still produce an increasing discharge amplitude as the wave propagated downriver. This is due to the fact that we don’t use our upriver estimation in other cases than when the \(\Delta L\) headwater level is exceeded. Therefor will the river section store too much water before the discharge reacts on the propagating wave just as in the case with a feedforward PI controller. The Sinus disturbance for the Override Selector State feedback Control is shown in figure 5.11. The benefits of the downriver control is here very obvious. The magnification of the discharge as seen for the PI controller with feedforward is very well reduced. In the same time do we have a total damping behavior over both the river water level and the local water flow. Noticeable is also the fast tracking of the head water level reference compared to the
Figure 5.10: Step Disturbance applied to a four hydropower plant cascade controlled with a Override Selector State feedback Control. The four river sections between the hydro power plants, in the figure called River A, B, C and D, are organized with the inflow in the top left corner and hydro power plant control to the right of the plots. The outflow from the upper river section is inflow to the lower.
PI controller with feedforward term.
Figure 5.11: Sinus Disturbance applied to a four hydropower plant cascade controlled with a Override Selector State feedback Control. The four river sections between the hydro power plants, in the figure called River A, B, C and D, are organized with the inflow in the top left corner and hydro power plant control to the right of the plots. The outflow from the upper river section is inflow to the lower.
Chapter 6
Further possible Considerations

This chapter is mainly made to conserve the knowledge and knowhow about the further consideration that could be of interest, i.e. loose ends, that every author possess at the end of a project as this one.

6.1 Defining the type of Hydropower Plant

The simulation in this thesis doesn’t consider any different types of hydropower plants than the type where the weirs and the power house are adjoin. There are several other, some more common, structures of the side flow that could be of interest to consider. Further more, a consideration what would happened for instance at an emergency case at the power plant facilities or other specific extreme power plant situations should be of interest.

6.2 Simulation according to specific data

The geographical and river data that are used in this simulation, as also mentioned in chapter 3.3, was a general case derived in discussion with the ABB Utility Automation in Baden and the Automatic control laboratory at the ETH Zurich. For a further consideration would real specific data be of interest to get a better connection between the real dynamics and the simulation. The river to simulate should then be so chosen that the advantages of a local estimation will be as large as possible. It would also be of interest to on one hand use real measured data as input and on the other compare the retrieved results with ones retrieved in reality.

6.3 Different Operating Points

As we have seen above is the Saint Venant model derived from a linearization around an operating point and a discretization in time and space. If we are forced to change this operating point, for instance if we wanted to simulate a different river, we naturally have to change our model as such. But even for the same simulation it could be interesting to change these values, as the flow vary depending on the season, the rainfall etc. A simple procedure to change these values would therefore be appropriate. For a very long river, the case of different operating point could as well be considered.
6.4 Reduction of estimation states

An interesting approach would be to try to reduce the number of states in the estimators so they will fall below the number of states in the simulation. Thereafter could an investigation be made what kind of influence this has on the tracking of the estimators as well as the final result.

6.5 Need for Proactive Disturbance Behavior

Starting from stable initial conditions, the developed Override Selector State Feedback Control will only estimate the downriver behaviors until the $\Delta L$ level is exceeded by the headwater level. But as the $\Delta L$ level is exceeded, the stored retention volume is almost always too large. This calls for another structure of the Override selector, where the behavior of the upriver is constantly supervised. This would lead to an earlier discharge answer and would according to the discussion in section 4.1.3 have a damping influence on the step disturbance discharges in figure 5.10. The idea to improve this behavior by creating a greater estimation horizon that reaches out over adjacent hydropower plants must here be commented. This would most likely improve our controller performance. But we would then lose some of the head objective advantage with this type of controller, i.e. the bounded need for information exchange between the hydropower plants. Further on, if we wanted to construct a total supervising structure for cascaded hydropower plants there are better approaches than this type of estimator. Especially the Model Predictive Control approach have been shown to be very successful for a very wide estimation horizon [21]. This area is surely one of the most critical aspects that has to be considered before this controller concept will reach its main objectives with an overall damping for both water flow and water level for all types of realistic disturbances.
Chapter 7

Conclusion

This thesis shows that the complications with too much amplified, unnatural discharge variations in the most common hydropower plant control structure of today can be greatly improved by the add-on of the suggested Override Selector feedback Control. This result is obtained by applying step and sinus disturbances to a well proven realistic open water model, i.e. a model based on the Saint Venant equations.

An analysis of the hydropower plant control market in Switzerland is also made and it suggest that low investment costs and independent information collection are key elements for a successful market approach. The suggested control structure achieves this by building a local estimation feedback controller on top of the current control structure. This will be both cost efficient and redundant at the same time as it takes use of the know how about PI-controlling at the local hydropower plants.

There are several possible improvements suggested for the new controller. Among these are an improved override selector and further estimation verification. Besides these controller improvements would it be of interest to complement the general simulation that is developed in this thesis with real disturbances and then be compared the simulated behavior with the behavior of a real system to establish the result further.
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