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An empirical test of the gravity *p*-median model

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Abstract: A customer is presumed to gravitate to a facility by the distance to it and the attractiveness of it. However regarding the location of the facility, the presumption is that the customer opts for the shortest route to the nearest facility. This paradox was recently solved by the introduction of the gravity *p*-median model. The model is yet to be implemented and tested empirically. We implemented the model in an empirical problem of locating locksmiths, vehicle inspections, and retail stores of vehicle spare-parts, and we compared the solutions with those of the *p*-median model. We found the gravity *p*-median model to be of limited use for the problem of locating facilities as it either gives solutions similar to the *p*-median model, or it gives unstable solutions due to a non-concave objective function.

Key words: distance decay, market share, network, retail, simulated annealing, travel time

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1. The background to the gravity *p*-median model

Consider a market area with already existing facilities (or service points) competing for customers. Conventionally, a model for estimating market shares is based on the gravity model presented by Huff (1964, 1966). He proposed the probability that a customer patronizes a certain facility to be a function of the facility's attractiveness and distance. The model defines for each customer a probability distribution of patronage for each facility in a market area. Thereby, the market share of a facility can be evaluated by aggregating all the customers and corresponding probabilities in the area of interest.

The same model may be used for investigating the effect of adding or removing a single facility in the market area contingent to a specific location of that facility (see Lea and Menger, 1990). Moreover, an optimal location with regard to some outcomes can be identified (Holmberg and Jornsten, 1996).

However, the general problem of allocating *P* facilities to a population geographically distributed in *Q* demand points is usually executed in a different manner. Hakimi considered the task of locating telephone switching centers and formalized what is now known as the *p*-median model. The *p*-median model addresses the problem of allocating *P* facilities to a population geographically distributed in *Q* demand points such that the population's average or total distance to its nearest facility is minimized (e.g. Hakimi 1964, Handler and Mirchandani 1979, and Mirchandani 1990). The *p*-median objective function is $\sum_{q \in N} w_q \min_{p \in P} \{d_{qp}\}$, where *N* is the number of nodes, *q* and *p* indexes the demand and the facility nodes respectively, w_q the demand at node *q*, and d_{qp} the shortest distance between the nodes *q* and *p*. Hakimi (1964) showed that the optimal solution of the *p*-median model existed at the network's nodes. After Hakimi's work, the *p*-median model has been used in a remarkable variety of location problems (see Hale and Moberg, 2003).

However, it has been argued that the *p*-median model is inappropriate the locations of competing facilities because of the assumption that customers opt for the nearest facility (see e.g. Hodgson, 1978 and Berman and Krass, 1998). Recently, Drezner and Drezner (2007) presented the gravity

p-median model that integrates the gravity rule with the *p*-median model. In their paper, they restate arguments for the gravity rule be found elsewhere: 1) the population is often spatially aggregated and approximately represented by the center of the demand point, 2) customers might act on incomplete information regarding the distance to each of the facilities, and 3) facilities vary in attractiveness to customers. There is also a fourth argument namely that the choice of facility may depend on other purposes for a trip (Carling and Håkansson, 2012).

Thus far, the computational aspects of the gravity *p*-median model have been studied with the intention of finding good solutions to the NP-hard problem (Drezner and Drezner, 2007 and Iyiguna and Ben-Israel, 2010).

The aim of this paper is to put the gravity *p*-median model to an empirical test. We consider the problem of locating 7 locksmiths, 11 vehicle inspections, and 14 retail stores of vehicle spare-parts in a Swedish region where we have detailed network data and precise geo-coding of customers. The *p*-median model ought to be appropriate in the vehicle inspection problem, whereas the gravity p-median model is presumably more suitable for the retail store problem. The problem of locating locksmiths may be regarded both a *p*-median problem and a gravity *p*-median problem.

This paper is organized as follows: section two presents the empirical setting and discusses the implementation of the gravity *p*-median model. Section three presents the results. And the fourth section concludes this paper.

2. Implementing the gravity *p*-median model

2.1 Geography

Figure 1 shows the Dalecarlia region in central Sweden, about 300 km northwest of Stockholm. The size of the region is approximately 31,000 km^2. Figure 1a illustrates the location of customers in the region[1]. As of December 2010, the Dalecarlia population numbers 277,000 residents. About 65 % of the population lives in 30 towns and villages of between 1,000 and 40,000 residents, whereas the remaining third of the population resides in small, scattered settlements.

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[1] The population data used in this study comes from Statistics Sweden, and is from 2002 (www.scb.se). The residents are registered at points 250 meters apart in four directions (north, west, south, and east) implying a maximum error of 175 meters in the geo-coding of the customers. There are 15,729 points that contain at least one resident in the region.
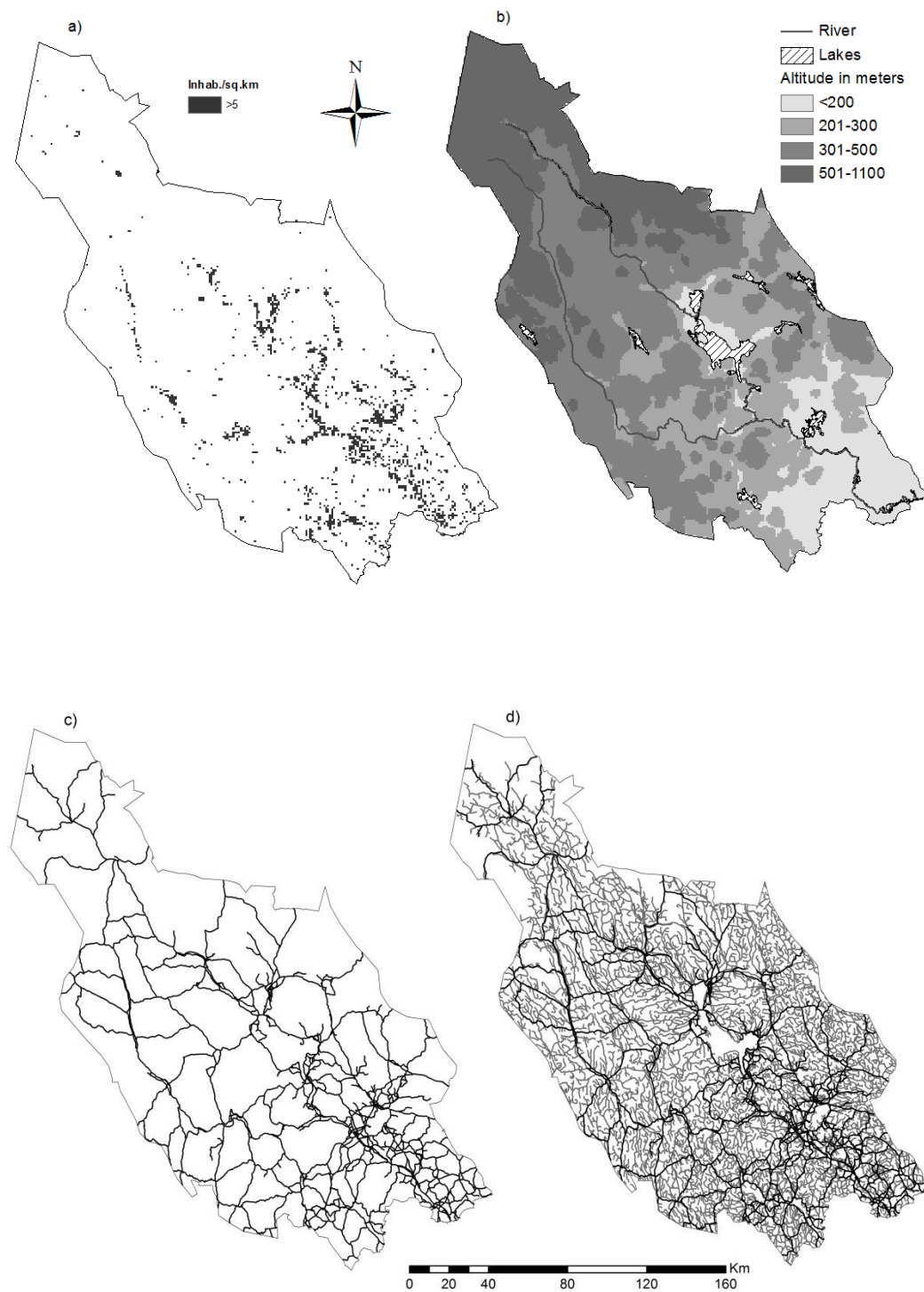
Figure 1: Map of the Dalecarlia region showing (a) one-by-one kilometer cells where the population exceeds 5 inhabitants, (b) landscape, (c) national road system, and (d) national road system with local streets and subsidized private roads.

Figure 1b shows the landscape and it gives a perception of the geographical distribution of the population. The altitude of the region varies substantially; for instance in the western areas, the

altitude exceeds 1,000 meters above sea level whereas the altitude is less than 100 meters in the southeast corner. Altitude variations, the rivers' extensions, and the locations of the lakes provide many natural barriers to where people could settle and how a road network could be constructed in the region. The majority of residents live in the southeast corner while the remaining residents are located primarily along the two rivers and around Lake Siljan in the middle of the region. The region constitutes a secluded market area as it is surrounded by extensive forest and mountain areas which are very sparsely populated. Hence, in the following we ignore potential influence of customers and facilities outside the region.

2.2 Distance measure

Carling, Han, and Håkansson (2012) found the Euclidian distance measure to perform poorly for the *p*-median problem in this rural area. In the empirical analysis we have tested the Euclidean measure but because of its shortcomings we focus on what follows from the travel-time distance. To obtain the travel-time, we assumed that the attained velocity corresponded to the speed limit on the road network.

The Swedish road system is divided into national roads and local streets which are public as well as subsidized and non-subsidized private roads. In Dalecarlia, the total length of the road system in the region is 39,452 km (see Figure 1d).[2] Han, Håkansson, and Rebreyend (2012) used the *p*-median model on this road network, and they noted that for *P* small the national road network was sufficient. Therefore, we only use the national roads in this study.

Figure 1c shows the national road network in the region. The national road system in the region totals 5,437 km with roads of varying quality which are in practice distinguished by a speed limit. The speed limit of 70 km/h is default and the national roads usually have a speed limit of 70 km/h or more.

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[2] The road network is provided by the NVDB (The National Road Data Base). The NVDB was formed in 1996 on behalf of the government and is now operated by the Swedish Transport Agency. NVDB is divided into national roads, local roads and streets. The national roads are owned by the national public authorities, and their construction is funded by a state tax. The local roads or streets are built and owned by private persons, companies, or by the municipalities. Data was extracted in spring 2011 and represents the network of winter 2011. The computer model is built up by about 1.5 million nodes and 1,964,801 road segments.

2.3 Objective function and parameters

The objective function for the gravity *p*-median model is similar to the objective function of the *p*-median model with the addition of a term specified the probability that a customer located at node *q* will visit a facility at node *p*. Drezner and Drezner (2007) specify the probability term as $\frac{A_p e^{-\lambda d_{qp}}}{\sum_{p \in P} A_p e^{-\lambda d_{qp}}}$, where A_p is the attractiveness of the facility and λ is the parameter of the exponential distance decay function[3]. As a consequence, the gravity *p*-median objective function is

$$\min_{p \in P} \left\{ \sum_{q \in N} \left[w_q \frac{\sum_{p \in P} d_{qp} A_p e^{-\lambda d_{qp}}}{\sum_{p \in P} A_p e^{-\lambda d_{qp}}} \right] \right\}.$$

As noted above, we use travel-time as the distance measure which means that the quickest path between *q* and *p* needs to be identified. We implemented the Dijkstra algorithm (Dijkstra 1959) and retrieved the shortest travel time from the facilities to residents in each evaluation of the objective function. We impose that facilities are located at the nodes of the network even though the Hakimi-property does not generally apply to the gravity *p*-median model (Drezner and Drezner, 2007). The reason for this choice is to enable a fair comparison with the *p*-median solutions which will be at the nodes.

The attractiveness parameter, A_p, is discussed under subsection 2.5 but it is varied for only one of the businesses.

The value of lambda[4] is decisive on how far a customer is likely to travel for patronize a facility. For λ=0, all (equally attractive) facilities are equally likely to be patronized by the customer, irrespective of the customer's distance to them. The larger the value of lambda, the more attached the customer is to the nearest facility. Drezner (2006) derived λ=0.245 for shopping malls in California whereas Huff (1964, 1966) reported, albeit using the inverse distance function, on larger values for grocery and clothing stores. We use Drezner's value converted from Euclidean distance and English miles into the corresponding value for the network distance and in kilometres. By

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[3] The exponential function and the inverse distance function dominate in the literature as discussed by Drezner (2006).
[4] The solutions to the location models are obtained in the travel time network. To conform to the existing literature, we discuss lambda in terms of a parameter for a road network. In the algorithm we adjust lambda to the corresponding value in the travel time network.

assuming the network distance[5] to be 1.3 times the Euclidean distance we have λ=0.11.

Table 1: Swedes self-estimated network distance for purchases of durable goods.

	Travel distance (km)						
	<2.5	2.5-5	5-25	25-50	50-125	125-250	>250
Proportion (%)	14	22	32	17	9	4	2

A value of lambda specific for the applications here is λ=0.035. We obtained this value as the maximum likelihood estimate of the parameter based on grouped data from the Swedish Trade Federation (Svensk Handel). The data values are shown in Table 1. In the empirical part, we only consider goods and services requiring infrequent trips which ought to be like durables.

2.4 Implementation of simulated annealing

The *p*-median problem is NP-hard (Kariv and Hakimi, 1979) and so is the gravity *p*-median problem. Han et al (2012) discussed and examined solutions to the *p*-median problem for the region's network. They advocated the simulated annealing algorithm which is used here and also used for the gravity *p*-median model.[6] This randomized algorithm is chosen due to its ease of implementation and the quality of results regarding complex problems. Most important in our case is that the cost of evaluating a solution is high and therefore we prefer an algorithm which keeps the number of evaluated solutions low. This excludes for example algorithms like Genetic Algorithm and some extended Branch and Bound. Moreover, we have good starting points obtained from pre-computed trials. Therefore a good candidate is simulated annealing (Kirkpatrick, Gelatt, and Vecchi, 1983).

The simulated annealing (SA) is a simple and well described meta-heuristic. Al-khedhairi (2008) describes the general SA heuristic procedures. SA starts with a random initial solution *s*, the initial temperature T_0, and the temperature counter $t = 0$. The next step is to improve the initial solution. The counter $n = 0$ is set and the operation is repeated until $n = L$. A neighbourhood solution s' is evaluated by randomly exchanging one facility in the current solution to the one not in the current solution. The difference, Δ, of the two values of the objective function is evaluated. We replace *s* by s' if $\Delta < 0$, otherwise a random variable $X \sim U(0,1)$ is generated. If $X < e^{(\Delta/T)}$, we still replace

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[5] This relationship has been observed in the literature and Carling et al (2012) found this relationship also for this network.
[6] Drezner and Drezner (2007) discuss alternative heuristic algorithms.

s by s'. The counter $n = n + 1$ is set whenever the replacement does not occur. Once n reaches L, $t = t + 1$ is set and T is a decreasing function of t. The procedure stops when the stopping condition for t is reached.

The main drawback of the SA is the algorithm's sensitivity to the parameter settings. To overcome the difficulty of setting efficient values for parameters such as temperature, an adaptive mechanism is used to detect frozen states and if warranted re-heat the system.[7] In all experiments, the initial temperature was set at 400 and the algorithm stopped after 10,000 iterations. Each experiment was computed twice with different random starting points to reduce the risk of local solutions. Among the two trials, we selected the solution with the lowest value of the objective function.

2.5 Businesses under study

The problem of locating vehicle inspections appears frequently in the literature on the *p*-median model (see e.g. Francis and Lowe, 1992). In Sweden, vehicle inspection was a state monopoly until 2009 when the market was deregulated. A state monopoly may be clearly regarded as a central planner and we therefore expect current locations of the inspections to resemble the *p*-median solution.

As of October 2012 there are eleven vehicle inspections operated by two companies in Dalecarlia. The inspections perform vehicle safety checks of vehicles according to EU protocol; hence there is no reason to expect the inspections to vary in attractiveness. Furthermore, the owner of a vehicle is required to regularly have the vehicle inspected. Older vehicles are subject to annual inspections whereas newer ones, inspections are triennial. Thus, a trip to the vehicle inspection is an infrequent patronage.

There are seven locksmiths in the region. These are small business without any central control. The virtue of the business makes it far-fetched that locksmiths differ much in attractiveness. Putting these two facts together, it is difficult to decide whether to expect locksmiths to follow a *p*-median or a gravity *p*-median location pattern.

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[7] Our adaptive scheme to dynamically adjust temperature works as follow: after n=10 iterations with no improvement, the temperature is increased according to newtemp=temp*3^β, where β starts at 0.5 and is increased by 0.5 each time the system is reheated. As a result, the SA will never be in a frozen state for long. The temperature is decreased each iteration with a factor of 0.95. The settings above are a result of substantial preliminary testing on this data and problem. In fact, some of the solutions were compared to those obtained by alternative heuristics.

The third business is retail stores of vehicle spare-parts. There are two competitors in the region. One has 12 facilities in the region and the other has 2 facilities. However, the stores of the latter competitor are large and offer an ample selection of spare-parts as well as many complementing products. We expect these two stores to be quite more attractive. We consider two assumptions. The first is the case where the two stores are twice as attractive as the competitor's stores. The second is the case where the two stores are assumed to be five times as attractive.

3. Results

Figure 2 shows the current location of the 11 vehicle inspections (Figure 2a) and the 7 locksmiths (Figure 2b) in the region. Imposed on the map in the figure is the solution to the p-median model (hereafter PM) for the two businesses. As expected, the current location of the vehicle inspections is quite near to the PM solution where ten out of eleven facilities coincide. The current locations of the seven locksmiths differ from the PM solution, but not by much.
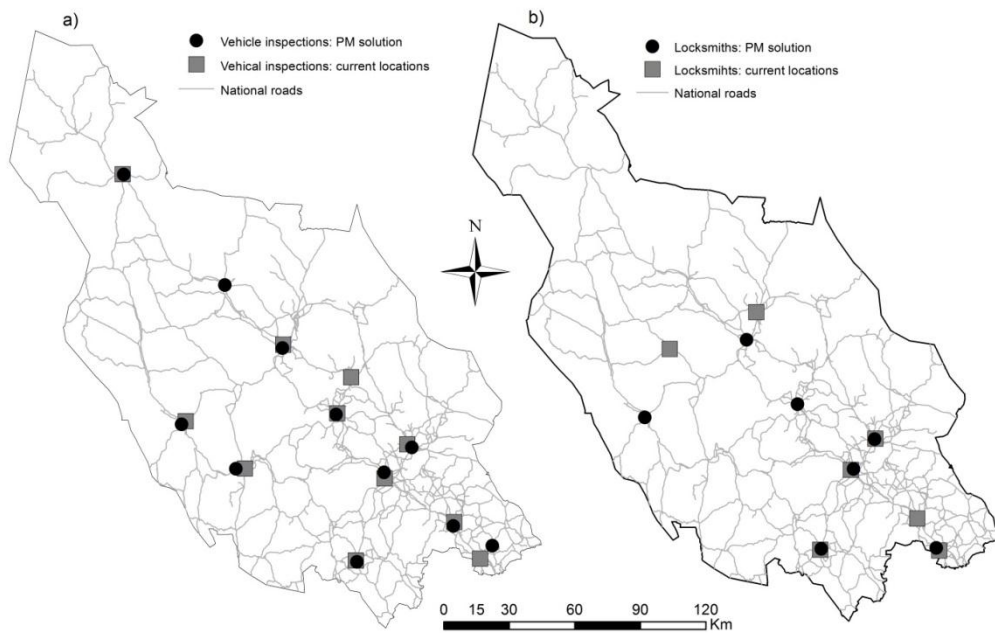

Figure 2: Map of the Dalecarlia region showing the current locations and the p-median (PM) solution for (a) vehicle inspections and (b) locksmiths.

We now turn to the gravity p-median model (hereafter referred to as GPM followed by λ used) and how it compares to PM. Figure 3 shows that the GPM(0.11) solution is similar to the PM solution;

for the vehicle inspections problem, the results of the models coincides almost completely. The similarity is also apparent in the case of locksmiths.
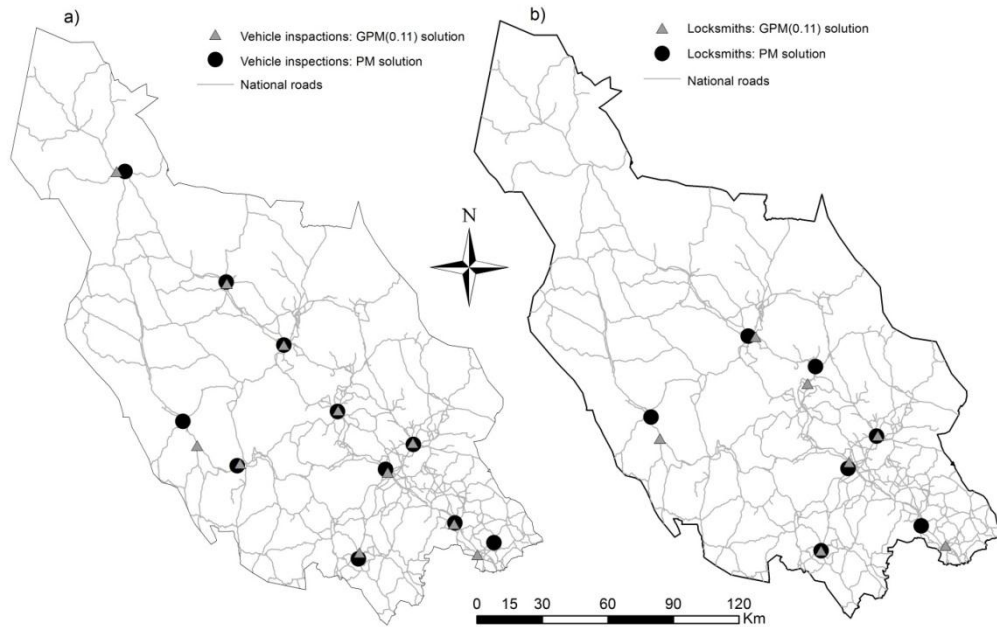

Figure 3: Map of the Dalecarlia region showing the *p*-median (PM) solution and the gravity p-median (GPM) solution with $\lambda = 0.11$ for (a) vehicle inspections and (b) locksmiths.

To understand the practical difference between the solutions of the PM and the GPM(0.11) models, we compute the travel-time to the nearest facility for customers in the region. Table 2 shows the average travel-time to the current locations, the PM, and the GPM solutions. The GPM(0.11) gives solutions that imply some two per cent longer travel time to the nearest vehicle inspection or locksmiths compared to the PM solutions.

Table 2: The customers' average travel-time (seconds) to the nearest facility for current locations and *p*-median (PM) as well as gravity *p*-median (GPM) solutions.

	Location model			
Business	Current	PM	GPM (λ=0.11)	GPM (λ=0.035)
Vehicle Insp.	612.65	611.09	629.59	863.77
Locksmiths	1014.36	798.45	815.92	1188.09
Spare-parts	789.94	545.80	551.97	808.19
- twofold A_p	n a	n a	588.29	823.73
- fivefold A_p	n a	n a	583.83	897.11

Table 2 also gives the average travel-time for the GPM(0.035) solutions. Recall that this model is

the best estimate of how Swedish customers patronize facilities of durable goods and services. The GPM(0.035) solutions differ substantially from the PM where the GPM(0.035) solutions imply some 50 per cent longer trips to the nearest facility on average.

Following up on the findings in Table 2, Figure 4 contrasts the GPM(0.035) solutions to the PM solution for vehicle inspections (Figure 4a) and locksmiths (Figure 4b). The models provide distinctively different geographical configuration of locations. For the GPM(0.035), facilities tend to be clustered in some towns, and we stress that it is not because the algorithm entered local minima as we have tested several starting values and the clustering pattern repeated itself.
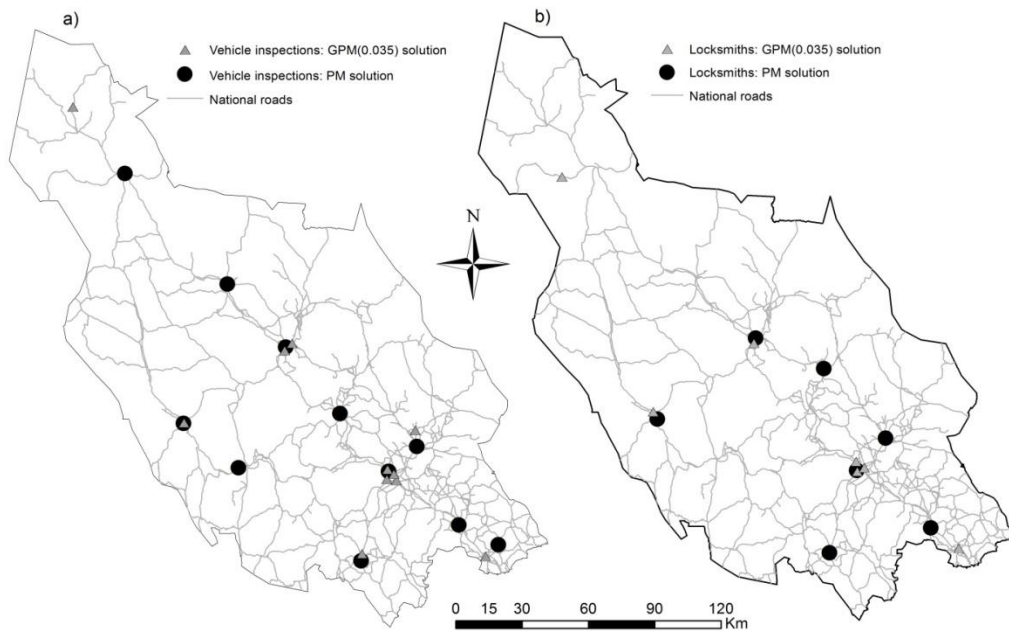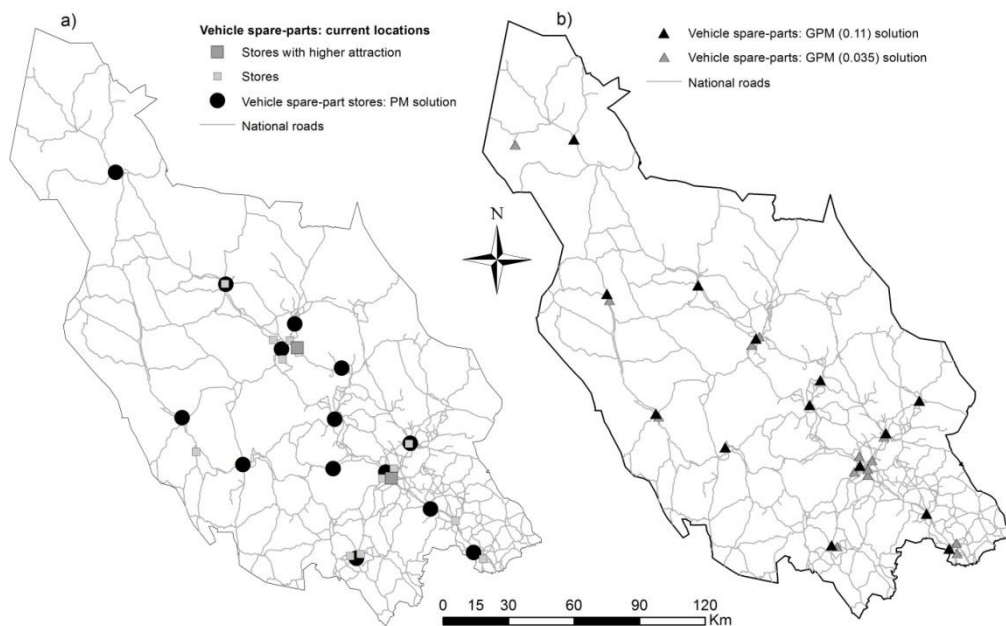

Figure 4 Map of the Dalecarlia region showing the *p*-median solution (PM) and the gravity *p*-median solution (GPM) with $\lambda = 0.035$ for (a) vehicle inspections and (b) locksmiths.

The clustering pattern indicates a difficulty to identify potential locations which give a unique market area for a facility. Consider that λ=0.035 implies that a customer's expected travel distance is about 30 kilometers, and consequently facilities cover vast market areas leaving no or only remote areas uncovered in this spatially saturated market. And in a spatially saturated market, market shares will not be found in uncovered areas but in large market areas with relatively few competing facilities; thus the clustering pattern of facilities.

Consider now the more challenging business of spare-parts for vehicles. Figure 5 shows the geographical configuration of locations for the three models and current locations. In Figure 5a the current locations of spare-parts stores is contrasted with the PM solution of 14 facilities showing a substantial difference between them. In Figure 5b configuration of GPM(0.11) and GPM(0.035) are contrasted. Again, the two values of λ lead to substantially different configurations where the clustering pattern of GPM(0.035) is pronounced. By comparing Figure 5a with 5b, there is a notable similarity between the PM and GPM(0.11) solutions on the one hand whilst on the other hand a similarity between GPM(0.035) and current location of the stores of vehicle spare-parts.

As noted above, there are two existing facilities in the region which are substantially more attractive than the competitor's twelve stores. We postulate that the difference in attractiveness is either twofold or fivefold. Figures 5c-d give the configuration of stores for the GPM solutions as well as indicate the two more attractive stores. In spite of introducing heterogeneity in attractiveness, GPM(0.11) continues to produce a solution similar to the PM. The GPM(0.035) solution gives a strong clustering with a remarkable location of facilities in the north-west of the region. This aberrant solution points at an instability of the model because of a spatially saturated market.
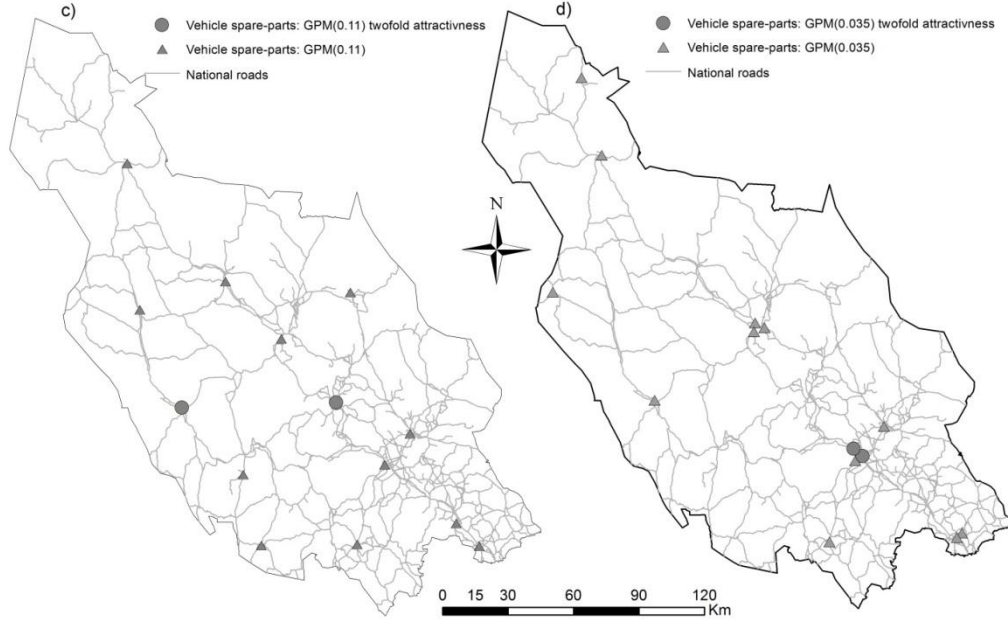
Figure 5: Map of the Dalecarlia region showing (a) the current location and the p-median solution, (b) the gravity p-median solution with $\lambda = 0.11$ and $\lambda = 0.035$ and $A_p = 1$, (c) twofold attractiveness and $\lambda = 0.11$, and (d) twofold attractiveness and $\lambda = 0.035$ for retail stores of vehicle spare-parts.

The GPM(0.035) has given unstable solutions in several of the problems as indicated by multiple locations at the same node and several facilities being located close to the region's border. To examine the problem of a spatially saturated market we conduct an experiment. Figure 6 gives the attained value of the objective function for the three models when locating two to twenty facilities in steps of two. It shows that the attained value of the objective function consistently decreases for the PM solutions when the number of facilities is increased. For GPM(0.035) the objective function decreases slowly initially and then flattens out at about 8 facilities. Hence, in the location of 8 or more facilities the objective function lacks a distinct minimum because of its non-concave form. The practical interpretation of this is in a spatially saturated market there is no geographical location that will make a facility successful from offering an improved accessibility to the customers.
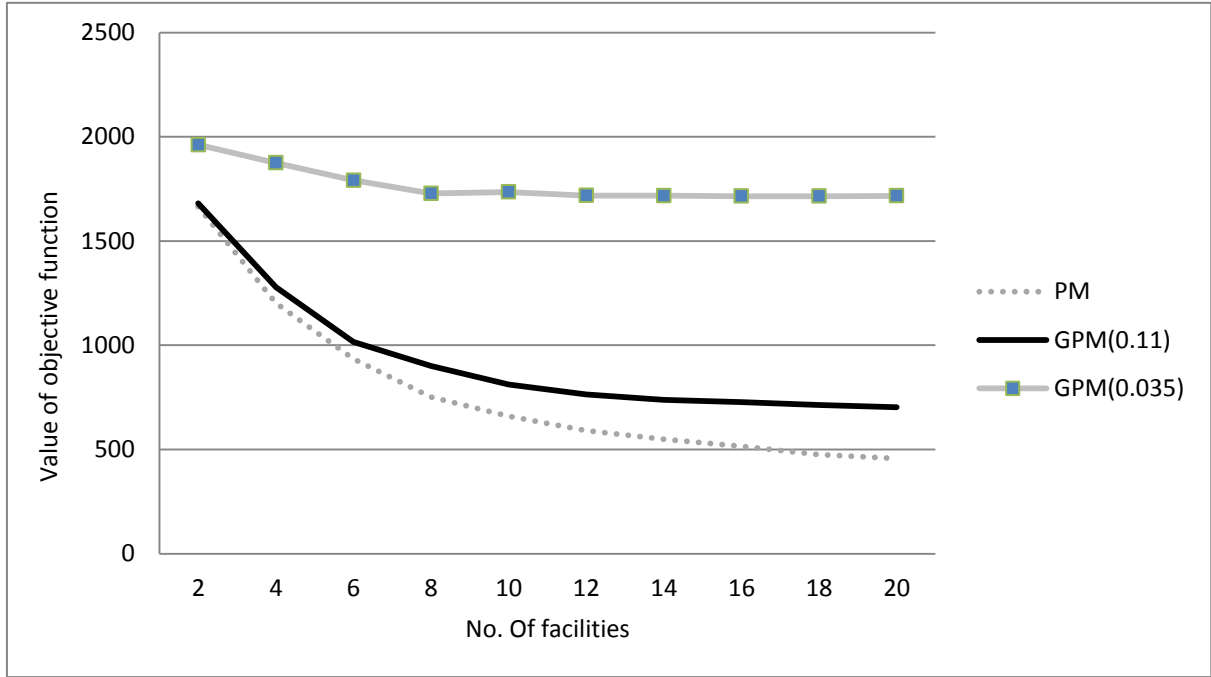
Figure 6: The attained value of the objective functions for the different location models in an experiment with locating 2 to 20 facilities in steps of 2.

Table 3: The market share for seven locksmiths in the region.

Facility	Location model		
	Current	PM	GPM (λ=0.11)
1	16.30%	12.45%	13.23%
2	14.21%	14.33%	13.96%
3	27.46%	23.85%	24.08%
4	21.76%	19.93%	19.84%
5	13.37%	13.53%	13.10%
6	=0	9.89%	9.56%
7	6.90%	6.02%	6.23%

Before concluding that the PM and GPM(0.11) solutions are interchangeable, we need to verify that they give a similar market share and market area of the facilities. In doing so we take locksmiths as an example simply because it is easy to match PM-facilities to GPM(0.11)-facilities in this case. Table 3 gives the expected proportion of customers patronizing the seven locksmiths. In calculating the expected proportion, we stipulate that the customers patronize the facilities in accordance with the probability $\frac{e^{-0.11d_{qp}}}{\sum_{p\in P} e^{-0.11d_{qp}}}$, i.e. the gravity model with $\lambda = 0.11$. The table shows that the PM solution and GPM(0.11) solution matches. In the table the market shares for the current locksmiths is also shown, setting the market share at zero for the sixth facility as found in the PM and GPM solutions but not in reality.
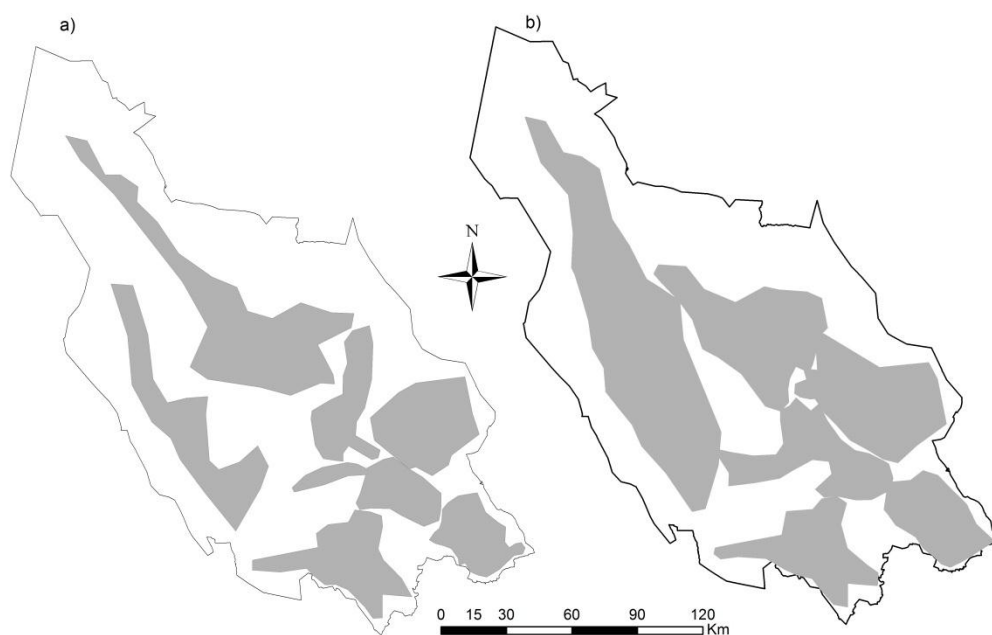
Figure 6: Map of the Dalecarlia region showing the market areas for the locksmiths; (a) areas for current location of locksmiths, (b) in areas for PM location of locksmiths.

Figure 7: Map of the Dalecarlia region showing the market areas for the locksmiths; (a) areas for PM location of locksmiths, (b) areas for GPM ($\lambda = 0.11$) location of locksmiths.

The similarity in the geographical extension of the market areas for the locksmiths is illustrated in Figures 6-7. The figures show the market areas for the locksmiths including only dedicated customers i.e. those who have at least 50 per cent probability of patronizing the facility. In figure 6 the current market areas is compared with market areas of the PM solution. The PM solution suggests a market area in the middle of the region which partly contributes to making the market areas quite different even though the location of facilities is similar between current and the PM solution (see Figure 2).

Figure 7 illustrates the similarity in market areas for the PM and the GPM(0.11) solutions. In summary, the PM and the GPM(0.11) solutions are found to give similar location of facilities, similar market shares, and also similar market areas. Hence, they appear interchangeable as location models.

4. Concluding discussion

The p-median model is used when optimal locations are sought for facilities. It is assumed that customers travel to the nearest facility along the shortest route. In a competitive environment, such as the retail sector, this is not necessarily realistic. To address the location problem more realistically, the gravity p-median model has recently been suggested as a tool to for seeking location of multiple facilities in competitive environments. This model is not yet tested empirically. In this study we implemented the gravity p-median model in an empirical problem of locating locksmiths, vehicle inspections, and retail stores of vehicle spare-parts. In doing so, we contrasted the solutions of gravity p-median model to those of the p-median model.

We find that the p-median model gives solutions similar to the current location of vehicle inspections as expected and fairly similar to the current location of locksmiths. The current location of retail stores of vehicle spare-parts does not match the solution of the p-median model which indicates that the model is unrealistic in this case.

The gravity p-median model requires a parameter defining the reach of the facility to customers. We examined two values. The first is $\lambda = 0.11$ which is a derived value for shopping malls in California implying that the expected travel length in the road network is about 9 km (Drezner, 2006). The

second value, $\lambda = 0.035,$ was obtained from a Swedish survey with an implied while expected travel length in the road network of about 30 km. For $\lambda = 0.11,$ the gravity p-median model gives solutions that coincide with the p-median solutions irrespective of heterogeneity in attractiveness of the facilities. Note, however, that we introduced heterogeneity in attractiveness only in the case of stores of vehicle spare-parts where such heterogeneity was realistic.

For the most realistic value of $\lambda = 0.035,$ we find the model to produce unstable solutions for at least the cases of vehicle inspections and stores of vehicle spare-parts. The instability results from a spatially saturated market in which no improvement in the objective function can be made from adding facilities. We illustrate that the market here is saturated for P at around 6-8 facilities. Given a small value of lambda, the competitive edge of a facility in a spatially saturated market is not given by its location, but by its attractiveness. In summary, we find the gravity p-median model to be of limited use for location problems.

Acknowledgements

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