A Finite Element Model of Bi-Stable Woven Composite Tape-Springs

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Abstract

The recent development of CubeSat nano-satellites shows that it is an effective way to send a payload onto orbit, as it is a relatively inexpensive and quick access to space. If the small size of these satellites is their main advantage, it is also the principal source of problems when it comes to designing it. The control electronics, electric power system and the payload are limited in mass and have to fit in a tiny ten-centimeter cube. The necessity of a compact deployable structure to hold the payload once the satellite reached its orbit is one of the principal subject of study for the design of a CubeSat. In the CubeSat program SWIM (Space Weather using Ion Spectrometers and Magnetometers) that KTH takes part in, the deployable structure developed consists of bi-stable semi-tubular booms made by a woven-composite fabric. Preliminary tests show that this structure is very compact and stable in the packaged configuration while being sufficiently long and stiff in the deployed configuration. However little is known about the deployment phase, the physical model of the booms is very inaccurate in determining the deployment force and speed, because of the complexity of the material mechanics behind it. Modeling a woven composite material in a finite element analysis software is a difficult task due the structure of the material itself. The fiber yarns interlace each other like in textile material, and they are impregnated in a soft matrix resin. Although in-plane properties of these materials can be calculated accurately using the classic lamination theory (CLT), the corresponding out-of-plane properties lack any accuracy for one-ply woven composites. Solutions are found through micromechanical approaches but these models are difficult to implement and are computationally expensive. The solution to this problem is to decline the CLT model of the material in two versions, each with a its own purpose. This paper presents first a CLT model of the woven composite which aim is to predict in-plane properties accurately and giving a good estimation of the out-of-plane properties. The second version of the CLT model is developed with the aim of predicting accurately the amount of strain energy stored and the stable radius of the rolled-up configuration. The purpose of this version is to be used in deployment analysis. This paper also presents the main lines of a fully parameterized finite element model of the deployment analysis for future use.
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Chapter 1

Introduction

1.1 The CubeSat project

1.1.1 Overview

The CubeSat Project was developed by California Polytechnic State University (CalPoly), San Luis Obispo and Stanford University’s Space Systems Development Lab. The CubeSat program creates launch opportunities for universities previously unable to access space. The term CubeSat denotes nano-satellites that adhere to the standards described in the CubeSat design specification.

With their relatively small size, using commercial off-the-shelf parts and material are possible. The price tag is therefore far lower than most satellite launches, and the CubeSat is a viable option for schools and universities around the world. Nowadays a large number of universities, some companies and government organizations across the world are developing CubeSats. It is estimated that between 40 and 50 universities were developing CubeSats in 2004, according to CalPoly. However the success rate of the CubeSat missions was only about 54% in 2009 (23 satellites out of 42 successfully established contact), which is very low compared to conventional satellite programs.

CubeSat forms a cost-effective independent means of getting a payload into orbit. Most CubeSats carry one or two scientific instruments as their primary mission payload.

1.1.2 Design of a CubeSat

The standard $0.1 \times 0.1 \times 0.1 \text{ m}^3$ basic CubeSat is often called a ”1U” CubeSat meaning one unit. CubeSats are scalable in 1U increments and larger. CubeSats such as a 2U CubeSat ($0.2 \times 0.1 \times 0.1 \text{ m}^3$) and a 3U CubeSat
1.1 The CubeSat project

(0.3 × 0.1 × 0.1 m³) have been both built and launched. A 1U CubeSat typically weighs less than 1kg.

![A typical 1U CubeSat, (www.stensat.org)](image)

Since CubeSats have all the same 0.1 × 0.1 m² cross-section, they can all be launched and deployed using a common deployment system. CubeSats are typically launched and deployed from a mechanism called a PolyPicoSatellite Orbital Deployer (P-POD), also developed and built by CalPoly. P-PODs are mounted to a launch vehicle and carry CubeSats into orbit and deploy them once the proper signal is received from the launch vehicle. The P-POD Mk III has capacity for three 1U CubeSats however, since three 1U CubeSats are exactly the same size as one 3U CubeSat, and two 1U CubeSats are the same size as one 2U CubeSat, the P-POD can deploy 1U, 2U, or 3U CubeSats in any combination up to a maximum volume of 3U.

![The P-POD MKIII deployer [1]](image)
1.2 The SWIM project

The SWIM project is a CubeSat project led by the University of Puerto Rico (IUPR), University of Florida (UF), Virginia Polytechnic Institute and State University (VT) and the Royal Institute of Technology, Stockholm (KTH), with the aim of bringing a CubeSat from prototype to flight stage.

The actors of this project each specialize in a separate domain. IUPR develops the embedded electronic hardware and software while UF, VT and KTH, together with the United States Air Force Research Laboratory (AFRL) develop the attitude control and deployable structural booms destined to carry the payload developed in KTH.

1.2.1 Mission and requirements

The mission of the SWIM CubeSat is to deploy onto a 600 km circular polar orbit its payload, among which a SMILE magnetometer sensor developed at KTH, which will measure the magnetic field of the Earth. The requirements of the mission are that the magnetometer is deployed in a position which is well determined (less than 5° error), in order to get relevant measures, and that the electro-magnetic interferences produced by the satellite be negligible compared to the Earth electro magnetic field.

For this to be possible, the magnetometer has to be relatively far from the electronics and metallic equipment of the satellite. The minimum distance is estimated to be one meter for reliable measurements. Therefore a deployable structure or boom one meter long is needed to carry the sensor, and this structure should be made of a non-metallic material.

The second important point is that the magnetometer must be in a well determined position or orientation. It means the structure carrying the magnetometer has to be stiff enough to avoid too large deformations during the orbit cycle, and that the orientation of the sensor is known after deployment of the structure.

Design of the SWIM CubeSat

The SWIM CubeSat is a 3U Satellite ($0.1 \times 0.1 \times 0.3 \, \text{m}^3$), would have a mass of approximately 3.64 kg and will deploy a 0.22 m$^2$ solar array. The payload of the satellite is comprised of an Energetic Electron Spectrometer (EES), developed at Boston University, a Wind Ion Neutral Composite Suite (WINCS) developed at the National Aeronautics and Space Administration (NASA), and a Small Magnetometer in Low mass Experiment (SMILE) developed by Ivchenko at KTH [2].
1.2 The SWIM project

Figure 1.3: The SWIM satellite concept, with the magnetometer far from the satellite body

Figure 1.4: SMILE sensor
1.2 The SWIM project

1.2.2 The deployable structure architecture

The need for the payload to be located one meter from the satellite body is one of the main concerns in the project development. The need of deployable structures in space applications is not new though. If a satellite has to be compact during launch for a cost and volume effective mission it can however be as large as desired when deployed in space. Antennas and solar panels are two examples of large structures which are deployed from a compact configuration. The current solutions employed to deploy structures in space are very different and are in constant development, the objective being to have increasingly larger structures for the same packaging volume. Mikulas defined three performance criteria for deployable space structures: stiffness, volume and mass [3].

In the CubeSat project, considering the dimensions of the satellite, the driving criteria for developing the deployable boom carrying the sensor is the volume efficiency: the packaged boom must fit inside the 10 cm cubic satellite.

Existing technologies

A vast number of solutions for deployable booms exist and have been reviewed in detail in [4]. Among them are telescopic booms, folding beams, cable systems, coilable trusses, thin walled tubular booms like the Storable Tubular Extendible Member (STEM) which is of great interest in the project. All the other solutions are difficult to scale down to match requirements on packaging and mass efficiency, which are the primary criteria for the nano satellite.

![Figure 1.5: Foldable beam concept](image)

STEM structure design

In STEM tubes, the compact configuration consists of a thin-walled shell material that can be elastically collapsed and reeled around a cylindrical hub or drum. In this configuration the material is in an unstable high strain...
1.2 The SWIM project

Figure 1.6: Astromast coilable truss [4]

Figure 1.7: STEM types - (a) tubular boom, (b-d) different cross-sections (e) deployment cassette for double STEM [4]
1.2 The SWIM project

energy state, therefore substantial mechanisms are needed to maintain the tape spring coiled around the drum before the deployment phase starts. In the deployment phase, the tape spring goes back to a practically unstressed configuration, taking a high stiffness tubular shape, typically overlapping to increase the shear stiffness of what would be an otherwise open section. Although the strain energy release of the tape spring drives the deployment, the STEM booms need to be deployed using an electric motor controlling the rotation of the hub. If the deployment is not controlled, STEM booms will deploy in a highly dynamic, uncontrolled and difficult to predict, manner. Once the tape spring is entirely deployed, a locking system maintains the deployed state and the whole structure becomes rigid. Materials commonly used for the shells are Beryllium Copper, Stainless Steel and Carbon Fiber Reinforced Plastics (CFRP).

The advantage of the STEM is that it is well developed, it has been used in numbers of satellites (Hubble), the deployment is easy to control, the size of the drum can be very small, with diameters from 10 to 130 mm, and the deployed length can reach more than 20 m for the largest ones. Several deployment configurations are also available: drum, tip drum and Jack-in-a-box, which allows for more flexible design.

![Figure 1.8: STEM deployment configurations - (a) drum, (b) tip-drum (c) jack-in-a-box](image)

However this solution cannot be retained because of two main drawbacks. A substantial amount of strain energy is stored in the tubular shell when reeled in an unstable state around the drum. The mechanisms maintaining the coiled state and controlling the deployment path and the motor driving the deployment rate of the shell consume space, energy and add mass to the
1.2 The SWIM project

system, three critical factors in the design of nano-statellited. Moreover, it is risky to embed a high potential energy structure in a small and light satellite like CubeSat, and to a general extent in any space structure. If the structure fails, the quick and high energy release may heavily damage the other instruments nearby, beside damaging the structure of the satellite itself.

The SIMPLE boom concept

AFRL has developed a concept for the deployable boom prior to the initiation of the SWIM project. The Self-contained Linear Meter-class Deployable (SIMPLE) boom is similar to STEM tubes but without its inconveniences.

The concept developed uses bi-stable plain-weave composite tape-springs as thin-walled shells instead of the usual materials used in STEM structures. The bi-stable tape-spring advantage is that it can be reeled around a hub without the material being in high energy strain state, the material is stable in the coiled configuration. This results in a highly compact boom. Indeed there is no need of mechanisms maintaining the tape spring reeled around the hub, only lightweight guides are used to control the path of deployment.

Another advantage of this structure is that it can self-deploy only from the stored energy between the coiled and deployed configurations, no need for a motor actuating the hub. The forces are small, but given the relative "freedom" of the shell compared to STEM tubes (no guides friction, and no motor) they are sufficient to deploy the whole structure.

The absence of motor and bulky guides keeping a high strain energy tape in shape allows the design to be very light and compact. In their first prototype, AFRL arranged two pairs of bi-stable tape-springs rolled

![General dimensions of the SIMPLE boom prototype.](image)
1.3 Master thesis project

around two hub cylinders connected to a shaft with bearings. The tape-springs self deploy due to their stored strain energy and cause the spools to counter rotate. Once fully deployed, the tape-springs lock out to form a structure one meter long. To sum up, the SIMPLE boom is an answer to the requirements of volume, mass and non-metallic material for the deployable structure. There is also good expectations for this design to fulfil the stiffness and dynamic behavior required for the mission.

1.3 Master thesis project

In the SWIM project frame, this master thesis work is mainly focused on establishing a model of the bi-stable plain-weave composite tape-spring material through numerical calculations. Stiffness and deformation characteristics of the material are investigated, along with the influence of a number of parameters specific to the plain-weave composite material. The objective of this work is to propose a parameterized material model for this plain-weave composite which will serve as a basis for further analysis.

In the first part the main concepts behind the development of the model are explained. Then, the model is applied to a plain-weave material similar to the one at KTH, and the preliminary results of the model and the experiments are compared in term of elastic properties, to assess the limits of the model.

The second part of this report is focused on the study of bi-stability on the shell, through a number of parametric analyses. The objective is to propose a base study which would serve as a tool to model correctly the bi-stable behaviour of the shell in ABAQUS later (for coiling and deployment analyses mainly).

The third part of the report proposes the foundations of a model for the coiling and deployment phase of the boom. These simulations are also carried out using ABAQUS, using both quasi-static and explicit dynamics analysis.
Chapter 2

The material model

Organization of the chapter

The layout of this chapter will consist first in presenting the main features of the analytical model of cylindrical shells developed by Iqbal and Pellegrino together with the analytical model of woven composites derived by Naik [17]. In a second step, a finite element model of the woven-composite material, based on Naik’s findings, is developed. The behavior of the model is compared to the available commercial tests results and the tests carried at KTH. In a third step, this model is modified so that it can display bistability. The parameters of the model and their influence on the bistable behavior is analyzed. The objective is to create a simple material model which yields good estimation of the real material, so that it can be used in subsequent analyses of the deploying or deployed complete boom. The model is modified in the next chapter so that it can predicts accurately the bi-stability behavior and strain energy levels, though at the expense of accuracy for in-plane properties.
2.1 Bi-stability

Bi-stability characterizes a structure that has two stable configurations. From the stored strain energy point of view, a stable configuration corresponds to a local minimum. The two configurations of a bi-stable structure can be of any strain energy state, as long as it is a local minimum. Usually, one of the configurations stores less strain energy than the other and the structure is more stable in this state. One can trigger the transition between the less stable state to the more stable state by adding a substantial amount of energy to the system (activation energy), by mechanic loading for example, and the stored strain energy will be released into kinetic energy.

In order to return the structure to its less stable state, high strain energy needs to be added to the structure (mechanical loading again).

The behavior of a bi-stable structure is heavily dependent of how it stores strain energy. Some bi-stable structures display a highly dynamic and uncontrolled transition when there is a substantial difference in strain energy stored, while others feature a fully controlled and quasi-static transition behavior. This is the case for neutrally stable structures, where the structure has no preferred configuration.

The advantage of using bi-stable materials for deployable structures is clear. The need of a containment/deployment mechanism to prevent the structure from releasing its stored elastic energy in an uncontrolled way can be avoided, or much reduced.

2.1.1 Origins

Bi-stability in structural mechanics is a long known phenomenon, and is used in a number of applications. In the case of cylindrical shell structures, the bi-stability behavior originates from the fact that the transformation of a surface of zero Gaussian curvature (the Gaussian curvature is the product of the two principal curvatures) into other surfaces of zero gaussian curvatures requires only bending energy, as the transformation of the mid-surfaces is isometric. Therefore, if the shell has two preferred directions of bending, it only requires a sufficient amount of activation energy for the shell to flip into each configuration with zero Gaussian curvature.

The preferred directions of bending are usually achieved by pre-stressing an isotropic material, stainless steel for example. Residual stresses are introduced either by heat treatment or by deforming the material beyond the yield stress. The latter is used in the steel tapes inside “slap bracelets” that can be found in many stores. In the deployed configuration (with a zero
2.1 Bi-stability

The material model

Gaussian curvature), the residual stresses along the shell are non zero and are sources of bending in the direction of coiling. Across the shell, however, the residual stresses are zero and the shell is curved. In the coiled state, also with a zero Gaussian curvature, the stresses along the shell are reduced to almost zero while the stresses across the shell rise up. The stress state in the tape is source of preferred bending directions between two geometrical surfaces of zero Gaussian curvature, therefore the tape is bi-stable.

The other way to obtain preferred bending directions without introducing residual stresses is by using an orthotropic material. The straightforward fabrication method is to use composite materials. Bi-stable composite shells were discovered by Daton-Lovett in 1996 [6]. His findings have been studied and developed further by Iqbal and Pellegrino [7], and more recently by Guest [8], in attempts to obtain a better understanding of the structural mechanics of these structures, as well as analytical and computational models to predict their behavior.

2.1.2 The composite tape-spring used in the SWIM CubeSat

The tape-spring used in the concept presented by AFRL is made of a particular composite material. It is not a regular laminate (a combination of plies stacked on each other) but a plain-weave composite material. The composite is made of interlacing perpendicular fibers strips, exactly like the arrangement of fibers in a textile material. The whole fibers arrangement is impregnated by a polymer matrix which maintains the integrity of the structure and distributes external loads to the fibers. These types of composite materials are called woven-composite, in relation to the wavy shape of the fibers interlacing each other. The most successful attempts to derive an analytical model for this type of material are credited to Naik [17].
2.2 Bi-stable cylindrical shells

The approach adopted to model the cylindrical shell is the same as for a plate, where the main axes coincide with the longitudinal and transverse directions of the shell.

2.2.1 Constitutive equations

The constitutive equations for a thin plate, in terms of stress and strain resultants are

$$
\begin{pmatrix}
N_x \\
N_y \\
N_{xy} \\
M_x \\
M_y \\
M_{xy}
\end{pmatrix}
= 
\begin{bmatrix}
A & B \\
\cdots & \cdots \\
B & D
\end{bmatrix}
\begin{pmatrix}
\epsilon_x \\
\epsilon_y \\
\gamma_{xy} \\
\kappa_x \\
\kappa_y \\
\kappa_{xy}
\end{pmatrix}
$$

(2.1)

where the plate coordinates can be described as follow:

![Figure 2.1: Sign convention for force and moment resultants](image)

The stiffness matrix relating the stress and strain resultants is called the ABD matrix [10]. The A matrix is named the extensional stiffness matrix, B the coupling stiffness matrix and D the bending stiffness matrix. In the next section the ABD matrix of a bi-stable antisymmetric laminate composite will be derived, in order to expose the conditions of bi-stability in a composite shell.
2.2 Bi-stable cylindrical shells

2.2.2 ABD stiffness matrix of a laminate composite

The ABD matrix of a laminate composite plate (material formed of several layers or plies arranged in a different orientation) is derived from the classic laminate plate theory, which shares the same assumptions as in the classical plate theory, also known as Kirchhoff or thin plate theory:

- The deformation of the plate is uniquely determined by the displacement of the middle surface, plane stress is assumed.

- Points on a normal to the undeformed middle surface after deformation form a normal to the deformed middle surface.

- The effect of shear deformation is neglected as in the simplest technical theory of beams.

- The membrane forces do not affect bending.

In addition, perfect bonding between layers is assumed:

- The bonding itself is infinitesimally small (there is no flaw or gap between layers).

- The bonding is non-shear-deformable (no lamina can slip relative to another).

- The strength of bonding is as strong as it needs to be (the laminate acts as a single lamina with special integrated properties).

The approach of classical laminate theory is to first determine the stiffness of each lamina in the laminate coordinate system. Then these reduced stiffness components are assembled along the thickness of the laminate to obtain the laminate stiffness matrix [11].
2.2 Bi-stable cylindrical shells

The material model

Lamina constitutive equations

First, we define a local coordinate system \((x_1, x_2, x_3)\) for each lamina. For convenience we adopt the following notation for stress and strain components in the lamina.

\[
\begin{align*}
\sigma_{11} &= \sigma_1, & \epsilon_{11} &= \epsilon_1 \\
\sigma_{22} &= \sigma_2, & \epsilon_{22} &= \epsilon_2 \\
\sigma_{33} &= \sigma_3, & \epsilon_{33} &= \epsilon_3 \\
\sigma_{12} &= \sigma_4, & \gamma_{12} &= 2\epsilon_{12} = \epsilon_4 \\
\sigma_{13} &= \sigma_5, & \gamma_{13} &= 2\epsilon_{13} = \epsilon_5 \\
\sigma_{23} &= \sigma_6, & \gamma_{23} &= 2\epsilon_{23} = \epsilon_6
\end{align*}
\] (2.2)

The constitutive equations for a lamina can then be written through the \(6 \times 6\) stiffness matrix \([C]\) or \(6 \times 6\) compliance matrix \([S]\).

\[
\sigma_i = \sum_{j=1}^{6} C_{ij}\epsilon_j, \quad i = 1 \sim 6 \tag{2.3}
\]

or

\[
\sigma = [C]\epsilon,
\]

\[
\epsilon_i = \sum_{j=1}^{6} S_{ij}\sigma_j, \quad i = 1 \sim 6 \tag{2.4}
\]

or

\[
\epsilon = [S]\sigma.
\]

Since both the \(C_{ij}\) and \(S_{ij}\) matrices are symmetric, there are at most 21 independent elastic constants:

\[
[C] = \begin{bmatrix}
C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\
C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\
C_{33} & C_{34} & C_{35} & C_{36} \\
sym. & C_{44} & C_{45} & C_{46} \\
& C_{55} & C_{56} \\
& & C_{66}
\end{bmatrix} \tag{2.5}
\]

\[
[S] = \begin{bmatrix}
S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\
S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\
S_{33} & S_{34} & S_{35} & S_{36} \\
sym. & S_{44} & S_{45} & S_{46} \\
& S_{55} & S_{56} \\
& & S_{66}
\end{bmatrix} \tag{2.6}
\]
Simplification of the equations

Orthotropy  The unidirectional lamina is considered to be an orthotropic material along the main axes $x_1, x_2, x_3$. These symmetries in the material allow us to remove several terms coupling certain shear, tensile and bending deformations together. As seen in Eq. (2.5), these terms are $C_{ij}$, $(i \neq j, j = 4, 5)$. Same goes for the $[S]$ matrix elements.

Transverse isotropy  Furthermore, in the lamina the continuous fibers are aligned parallel to the $x_1$ axis and randomly packed in the $x_2x_3$ plane, showing transverse isotropy in this plane. Then

$$C_{22} = C_{33}, \quad C_{12} = C_{13}, \quad C_{44} = C_{55}, \quad C_{66} = 1/2(C_{22} - C_{23})$$

Plane stress  In our case, the composite plate studied is very thin (thickness $h \simeq 0.24$ mm) compared to the in-plane smallest dimension $a$ (order of 20 mm). In agreement with thin-plate theory, the transverse stress $\sigma_3$ is assumed to be 0 everywhere through the thickness except in very thin boundary layers near the surface where it is equal to the surface pressures. Assuming $\sigma_3 = 0$, the normal strain $\epsilon_3$ can be expressed in terms of in-plane normal stresses $\sigma_1$ and $\sigma_2$. Therefore the elements $C_{ij}$ and $S_{ij}$, $(j = 3$ or $i = 3)$ can be eliminated from the constitutive equations as they are no longer independent.

Classic plate theory  Two main theories are used for characterizing plates behavior. The classic plate theory (CPT) assumes the transverse shear is zero everywhere, and the Mindlin or shear-deformable theory assumes that transverse shear cannot be neglected. Several papers treat the difference and the applicable domain of each theory. Wetherhold [11] has shown that for composite plates the CPT is as accurate as Mindlin theory when the ratio $a/h$ (in-plane dimension over thickness) is superior to 30, and Mansfield (1989) [12] demonstrated that the disagreement between the two theories lies near the free edges of the plate at a distance of approximately $1.5h$, where the shear flow distribution differs.

In our case, the ratio $a/h \simeq 42$ and $1.5h \simeq 0.36$ mm (less than 2% of the in-plane dimensions in our case). Hence, the decision to use CPT instead of Mindlin plate theory is straightforward. CPT assumes that transverse shear deformation components $\gamma_{xz}, \gamma_{yz}$ are zero. We can then eliminate the stiffness matrix elements $C_{55}$ and $C_{66}$.

The constitutive equations system, Eq. (2.3) and (2.4), can finally be reduced to
2.2 Bi-stable cylindrical shells

The material model

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_4 \\
\epsilon_1 \\
\epsilon_2 \\
\epsilon_4
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & 0 \\
C_{12} & C_{22} & 0 \\
0 & 0 & C_{44}
\end{bmatrix}
\begin{bmatrix}
\epsilon_1 \\
\epsilon_2 \\
\epsilon_4
\end{bmatrix}
\quad (2.7)
\]

\[
\begin{bmatrix}
\epsilon_1 \\
\epsilon_2 \\
\epsilon_4
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} & 0 \\
S_{12} & S_{22} & 0 \\
0 & 0 & S_{44}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_4
\end{bmatrix}
\quad (2.8)
\]

Usually, the notation \([Q]\) is adopted for the reduced form of the stiffness matrix \(Q\).

**Engineering constants**

Using the definition of engineering constants and using Eq. (2.8), we can relate the \(S_{ij}\) to engineering constants. The resulting values of the compliances are summarized as:

\[
S_{ij} = \begin{cases} 
1/E_i & i = j, \\
-\nu_{ij}/E_i & i \neq j, 
\end{cases} \quad i, j = 1, 2
\]

\[
S_{44} = 1/G_{12}
\]

(2.9)

Since, by symmetry, \(S_{12} = S_{21}\), we have the reciprocal relation

\[
\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}
\]

(2.10)

It is important to note that the requirements on the values of orthotropic compliances \(S_{ij}\) are not the traditional requirements of isotropic materials. It may be possible, for example, to have a Poisson’s ratio that exceeds 0.5, which is thermodynamically forbidden for isotropic materials. The conditions that are satisfied are based on the positive definiteness of both \([C]\) and \([S]\), since the strain energy density must remain positive. These requirements can be found in Jones [13].

**Transformation of the constitutive equations from the lamina local coordinate system to the laminate coordinate system**

The lamina can be arbitrarily oriented in the \(x_3\) direction in order to obtain the desired mechanical properties for the laminate. It is therefore necessary to transform the stiffness and compliance matrices from the lamina’s local \((x_1, x_2, x_3)\) coordinate system to the global \((x, y, z)\) laminate coordinate system. The components of stress \((\sigma_{ij})\), strain \((\epsilon_{ij})\), reduced stiffness \((Q_{ij})\),
2.2 Bi-stable cylindrical shells

and compliance ($S_{ij}$) are all tensors. The transformation of these coefficients into another coordinate system is achieved through a series of matrix multiplications.

We define a transformation matrix $[T_\sigma]$ for the stress components according to Eq. (2.11) for a rotation angle $\phi$ around the $x_3$ axis.

$$\sigma' = [T_\sigma] \sigma \quad (2.11)$$

The stress transformation matrix is given by

$$[T_\sigma(\phi)] = \begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & (c^2 - s^2) \end{bmatrix} \quad (2.12)$$

where $c = \cos \phi$; $s = \sin \phi$

Because we use a convenient notation for strain in Eq. (2.2), different from the tensorial notation of stress ($\epsilon_4 = \gamma_{12}$ here, whereas the tensorial notation should be $\epsilon_4 = \frac{1}{2} \gamma_{12}$), there is a different transformation $[T_\epsilon]$ matrix for strain.

$$\epsilon' = [T_\epsilon] \epsilon \quad (2.13)$$

The strain transformation matrix is given by

$$[T_\epsilon(\phi)] = \begin{bmatrix} c^2 & s^2 & cs \\ s^2 & c^2 & -cs \\ -2cs & 2cs & (c^2 - s^2) \end{bmatrix} \quad (2.14)$$

There is a special relationship between $[T_\epsilon]$ and $[T_\sigma]$, the inverse of one is the transpose of the other.
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The material model

\[
[T_\sigma]^{-1} = [T_\varepsilon]^T, \quad [T_\varepsilon]^{-1} = [T_\sigma]^T
\]  

(2.15)

Using Eq. (2.15) in Eqs. (2.11) and (2.13) yields the reduced stiffness matrix in the transformed coordinate system, \([Q']\).

\[
\sigma' = [Q']\varepsilon'
\]

(2.16)

where

\[
[Q'] = [T_\sigma][Q][T_\sigma]^T
\]

(2.17)

Similarly,

\[
\varepsilon' = [S']\sigma'
\]

(2.18)

where

\[
[S'] = [T_\varepsilon][S][T_\varepsilon]^T
\]

(2.19)

From Eqs. (2.16) and (2.17), the constitutive equations of an arbitrarily oriented lamina can be written in the \((x, y, z)\) laminate coordinate system.

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_{xy}
\end{bmatrix} =
\begin{bmatrix}
Q'_{11} & Q'_{12} & Q'_{14} \\
Q'_{12} & Q'_{22} & Q'_{24} \\
Q'_{14} & Q'_{24} & Q'_{44}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\]

(2.20)

The laminate plate constitutive equations

The force and moment resultants are defined in the following standard way for plates.

\[
(N_x, N_y, N_{xy}) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \sigma_{xy}) \, dz
\]

\[
(M_x, M_y, M_{xy}) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \sigma_{xy}) \, zdz
\]

(2.21)

These definitions used in combination with the constitutive equations (2.20) yields the resultants in terms of derivatives of the displacement functions.
2.2 Bi-stable cylindrical shells

The material model

\[ \begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{14} & : & B_{11} & B_{12} & B_{14} \\ A_{12} & A_{22} & A_{24} & : & B_{12} & B_{22} & B_{24} \\ A_{14} & A_{24} & A_{44} & : & B_{14} & B_{24} & B_{44} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ B_{11} & B_{12} & B_{14} & : & D_{11} & D_{12} & D_{14} \\ B_{12} & B_{22} & B_{24} & : & D_{12} & D_{22} & D_{24} \\ B_{14} & B_{24} & B_{44} & : & D_{14} & D_{24} & D_{44} \end{bmatrix} \begin{bmatrix} \epsilon_0^x \\ \epsilon_0^y \\ \gamma_{0}^{xy} \\ \kappa_x \\ \kappa_y \end{bmatrix} \]

or in brief form

\[ \begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \epsilon^0 \\ \kappa \end{bmatrix} \]

(2.22)

where the terms of the ABD matrix are defined by

\[ A_{ij} = \sum_{k=1}^{N} Q_{ij}^{(k)} (z_k - z_{k-1}) \]

\[ B_{ij} = \frac{1}{2} \sum_{k=1}^{N} Q_{ij}^{(k)} (z_{k}^2 - z_{k-1}^2) \]

\[ D_{ij} = \frac{1}{3} \sum_{k=1}^{N} Q_{ij}^{(k)} (z_{k}^3 - z_{k-1}^3) \]

(2.23)

The \( z_k \) values are signed distances varying from \( z_0 = -h/2 \) to \( z_N = h/2 \) in a laminate of \( N \) layers. The angle definition for the transformed stiffness \( Q_{ij}^{(k)} \) for lamina \((k)\) is illustrated in Fig. 2.2. The laminate stacking sequence is the description of the angle \( \phi \) for each lamina from the bottom to the top. The notation adopted in this report is \([\phi_1/\phi_2/.../\phi_N]\) where \( \phi_k \) is the rotation angle of lamina \((k)\).

The elements of the ABD matrix each have a different influence on the behavior of the plate under loading. Their role is described in Table 2.1.
Table 2.1: Role of the stiffness matrix elements

<table>
<thead>
<tr>
<th>Matrix elements</th>
<th>Load-deformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{11}, A_{12}, A_{22}$</td>
<td>Normal-normal</td>
</tr>
<tr>
<td>$A_{44}$</td>
<td>Shear-shear</td>
</tr>
<tr>
<td>$A_{14}, A_{24}$</td>
<td>Normal-shear</td>
</tr>
<tr>
<td>$B_{11}, B_{12}, B_{22}$</td>
<td>Normal-bend</td>
</tr>
<tr>
<td>$B_{44}$</td>
<td>Shear-twist</td>
</tr>
<tr>
<td>$B_{14}, B_{24}$</td>
<td>Shear-bend, normal-twist</td>
</tr>
<tr>
<td>$D_{11}, D_{12}, D_{22}$</td>
<td>Bend-bend</td>
</tr>
<tr>
<td>$D_{44}$</td>
<td>Twist-twist</td>
</tr>
<tr>
<td>$D_{14}, D_{24}$</td>
<td>Bend-twist</td>
</tr>
</tbody>
</table>

2.2.3 Laminate layup configurations

Laminates usually fall into the symmetric or antisymmetric categories, and laminae are oriented so that two consecutive plies have opposite angle (e.g.: $\phi_1 = 45^\circ$, $\phi_2 = -45^\circ$). In most of the cases laminae have the same thickness. From Eq. (2.23) and Table 2.1, we can relate the ABD matrix elements to the configuration of the laminate itself.

Symmetric laminates

Consider a symmetric laminate $[+45^\circ/-45^\circ/0^\circ/-45^\circ/+45^\circ]$. From Eq. (2.12) and (2.17), we have the relation

\[
Q_{ij}^+ = \begin{bmatrix} Q_{11}' & Q_{12}' & Q_{14}' \\ Q_{12}' & Q_{22}' & Q_{24}' \\ Q_{14}' & Q_{24}' & Q_{44}' \end{bmatrix}
\]

\[
Q_{ij}^- = \begin{bmatrix} Q_{11}' & Q_{12}' & -Q_{14}' \\ Q_{12}' & Q_{22}' & -Q_{24}' \\ -Q_{14}' & -Q_{24}' & Q_{44}' \end{bmatrix}
\]

and

\[
Q_{ij}^0 = \begin{bmatrix} Q_{11}' & Q_{12}' & 0 \\ Q_{12}' & Q_{22}' & 0 \\ 0 & 0 & Q_{44}' \end{bmatrix}
\]

Considering all the plies have the same thickness, the term $z_k - z_{k-1}$ is equal to $\pm h_{ply}$. When calculating $A_{ij}$, the terms $ij = 14, 24$ will vanish in the sum. The matrix $A$ takes the form:

\[
A_{ij} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{44} \end{bmatrix}
\]
If we carry the same analysis for $B_{ij}$, all the terms in the sum will vanish and we obtain

$$B_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (2.27)$$

However for the matrix $D_{ij}$, none of the elements disappear

$$D_{ij} = \begin{bmatrix} D_{11} & D_{12} & D_{14} \\ D_{12} & D_{22} & D_{24} \\ D_{14} & D_{24} & D_{44} \end{bmatrix} \quad (2.28)$$

The general form of the ABD matrix for a symmetric laminate is

$$\begin{bmatrix} A & B \\ B & D \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 \\ A_{12} & A_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{24} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{11} & D_{12} & D_{14} \\ 0 & 0 & 0 & D_{12} & D_{22} & D_{24} \\ 0 & 0 & 0 & D_{14} & D_{24} & D_{44} \end{bmatrix} \quad (2.29)$$

Refering to Table 2.1, in symmetric laminates there is no coupling between stretching and bending ($B = 0$), just as in plates made of isotropic materials. Furthermore, the $D$ matrix is full, so bending and twisting are coupled. This means that a flat plate subject to pure bending moments ($M_x, M_y \neq 0, M_{xy} = 0$) along its edges will both bend and twist ($\kappa_x, \kappa_y, \kappa_{xy} \neq 0$). In a cylindrical shell like our case, this coupling results in a rolled up configuration being twisted like a helix, as the longitudinal and transverse directions for the shell are not principal directions of curvature in the folded state (Fig. 2.3).

This coupling between bending and twisting has to be eliminated in order to model our bistable cylindrical shell. The terms $D_{14}, D_{24}$ must be equal to 0, so that the longitudinal direction is the principal direction of curvature in the rolled up state.

**Antisymmetric laminates**

From Eq.(2.23) and the expression of $D_{ij}$, we can see that if there is a $+\phi$ ply at $-z$ and a $-\phi$ ply at $+z$, the terms $D_{14}$ and $D_{24}$ will effectively vanish in the sum. This corresponds to an antisymmetric laminate layup, for example $[+45^\circ/ -45^\circ/ 0^\circ/ +45^\circ/ -45^\circ]$.

Carrying out the same analysis than for the symmetric laminate, the general form of the ABD matrix for an antisymmetric laminate is
2.2 Bi-stable cylindrical shells

The material model

\[
\begin{bmatrix}
A & B \\
B & D
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} & 0 & 0 & 0 & B_{11} \\
A_{12} & A_{22} & 0 & 0 & 0 & B_{21} \\
0 & 0 & A_{24} & B_{14} & B_{24} & 0 \\
0 & 0 & B_{14} & D_{11} & D_{12} & 0 \\
0 & 0 & B_{24} & D_{12} & D_{22} & 0 \\
B_{14} & B_{24} & 0 & 0 & 0 & D_{44}
\end{bmatrix}
\] (2.30)

We can see that bending and twisting are now independent. However, there is now coupling between bending and stretching, as \( B \neq 0 \). This coupling is not critical for the bi-stability of a shell, it has only a weak effect [7]. The crucial aspect is that now the longitudinal and transverse directions are the principal directions of curvature in the rolled up configuration. Therefore, as explained in section 2.1, a surface of zero gaussian curvature like a cylindrical shell will flip into another cylindrical shell of zero gaussian curvature, except that here the cylinders are perpendicular. In practice, the slender cylindrical shell can be rolled up on itself.

![Figure 2.3: Antisymmetric (left) and symmetric laminates rolled up configurations](http://www-civ.eng.cam.ac.uk/dsl/research/ki206/bistable.html).

2.2.4 Strain energy

In order to characterize bi-stability in a structure, one has to analyze the strain energy of this structure when loaded. As said before, a bi-stable structure has two stable states. Usually one state is a zero strain energy configuration, and the other a configuration with a local minimum of strain energy amongst other configurations.

In an isotropic plate, the strain energy is defined by the sum of energies due to bending and due to stretching in the middle surface. However in a
2.2 Bi-stable cylindrical shells

The material model

multi-layer anisotropic plate, there is coupling between moments and planar strains, and the total strain energy per unit area \( U' \) may be obtained by integrating the strain energy density through the thickness.

From Eq. (2.21) we obtain

\[
U' = \frac{1}{2} \int_{-h/2}^{h/2} \sigma^T \epsilon dz
\]

\[
= \frac{1}{2} \int_{-h/2}^{h/2} \sigma^T (\epsilon^0 + z\kappa) dz
\]

\[
= \frac{1}{2} (N^T \epsilon^0 + M^T \kappa)
\]

(2.31)

Combining this result with Eq. (2.22) yields

\[
U' = \frac{1}{2} (\epsilon^{0,T} A \epsilon^0 + 2\kappa^T B \epsilon^0 + \kappa^T D \kappa)
\]

(2.32)

\[ \text{(a) Original configuration} \quad \text{(b) Change of transverse curvature} \quad \text{(c) Change of longitudinal curvature} \]

Figure 2.4: Two-step deformation of the cylindrical shell [7].

Where the first term is the strain energy due to stretching the middle surface, the second term that due to the coupling between bending and stretching and the last term the bending strain energy. Iqbal and Pellegrino [7] derived the bending energy \( U_b \) and stretching energy \( U_s \) for the cylindrical shell described in Fig.2.4(a). For simplification purposes, the strain energy due to coupling between bending and stretching will be neglected. The simple model proposed for bi-stability of the shell structure assumes that it is subjected to uniform curvature changes \( \kappa_x \) and \( \kappa_y = 1/R \). The laminate being antisymmetric, it is also assumed that \( \kappa_{xy} = 0 \) (no twisting).

This model is very simple but it cannot model accurately a partially rolled shell where the curvature is not uniform along the length. The deformation of the shell is divided in two phases as described in Fig. 2.4. The first phase consists in bending the tube to make it flat, and the second step is stretching and bending it into the stable alternative configuration.
2.2 Bi-stable cylindrical shells

The material model

Figure 2.5: Cross section of the cylindrical shell. In our case, $\beta = 180^\circ$ [7].

Bending strain energy

The bending strain energy per unit area is written as

$$U_b = \frac{1}{2} \begin{bmatrix} \kappa_x & \kappa_y & \kappa_{xy} \end{bmatrix} D \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

(2.33)

Since $\kappa_{xy} = 0$, and because the plate has an initial curvature $\kappa_y = 1/R$, we have

$$U_b = \frac{1}{2} \left[ D_{11} + 2D_{12}\kappa_x(\kappa_y - \frac{1}{R}) + D_{22}(\kappa_y - \frac{1}{R})^2 \right]$$

(2.34)

The bending strain energy is uniform in the shell, so for a 180° circular arc cross-sectional tube, the bending strain energy per unit length $u_b$ can be expressed by multiplying Eq. (2.34) by the cross section arc-length $\pi R$:

$$u_b = \frac{1}{2} \pi R \left[ D_{11} + 2D_{12}\kappa_x(\kappa_y - \frac{1}{R}) + D_{22}(\kappa_y - \frac{1}{R})^2 \right]$$

(2.35)

Stretching energy

The bending strain energy per unit area is written

$$U_s = \frac{1}{2} \begin{bmatrix} \epsilon_x & \epsilon_y & \gamma_{xy} \end{bmatrix} A \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}$$

(2.36)

Stretching of the tube only occurs in the second deformation phase, when the tube is made flat. The centroid of the (deformed) semicircular arc cross-section lies on the neutral axis of the shell, at a distance $d = \frac{2\sin(\pi R\kappa_y/2)}{\pi R\kappa_y^2}$ from the center of curvature.

We have the following in this second phase: $\epsilon_x = z\kappa_x$ where $z$ is the distance
from point $P$ to the neutral axis. Using the polar coordinates in Fig. 2.5, this distance is

$$z = d - \frac{\cos(\theta)}{\kappa_y}$$

(2.37)

We also have $\gamma_{xy} = 0$ and it is assumed that $\epsilon_y \simeq 0$ in the second deformation phase. Equation (2.36) becomes

$$U_s = \frac{1}{2} A_{11} \epsilon_x = \frac{A_{11}}{2} \left[ \frac{2 \sin(\pi R \kappa_y/2)}{\pi R \kappa_y^2} - \frac{\cos(\theta)}{\kappa_y} \right]$$

(2.38)

Integrating this equation along the whole cross-section requires integration with $\theta$ ranging from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$. This yields the stretching energy per unit length:

$$u_s = \frac{A_{11}}{2} \left[ \frac{\pi R \kappa_x^2}{2 \kappa_y^2} + \frac{\sin(\pi R \kappa_y)}{2 \kappa_y^3} - \frac{4 \sin^2(\pi R \kappa_y/2) \kappa_y^2}{\pi R \kappa_y^4} \right]$$

(2.39)

**Total strain energy**

The total strain energy per unit length reduces then to

$$u' = u_b + u_s$$

(2.40)

Iqbal and Pellegrino [7] plotted the expression for the total strain energy in function of $\kappa_x$ and $\kappa_y$ for a 5-ply laminate [+45/−45/0/+45/−45] made of polypropylene (PP)/Glass unidirectional laminas. The radius of the shell they used was $R = 25$ mm and the subtended angle was $160^\circ$ (in our case we have a $180^\circ$ angle). The result is shown in Fig. 2.6. We can see that there is the absolute minimum $u' = 0$ for $\kappa_y = 1/R = 0.04$ mm$^{-1}$ on the left part of the plot, and that another local minimum exists on the right part of the plot, with $\kappa_y \simeq 0$ and $\kappa_x \simeq 0.028 = 1/36$ mm$^{-1}$. This local minimum corresponds to the rolled up configuration of the shell.
2.3 Woven composite model

We saw in the previous section a simplified model for bi-stable shells, and that only a carefully chosen arrangement of fibers in a laminate leads to bi-stability. In our case however, we do not have a laminate composite, but a woven composite material.

Thin woven composites have been popular for space structures due to their symmetrical and balanced properties, and the integrated nature of these fabrics which enables easier manufacturing and complex curvatures conforming.

A woven composite is a fabric produced by the process of weaving, in which the fabric is formed by interlacing warp and fill strands. Several interlacing patterns exist, the simplest one being the plain-weave where the warp and fill strands interlace each other in a regular sequence of one under and one over. Another pattern of interest in the SWIM project is the twill, where the interlacing sequence is two under and two over (Fig. 2.7). A simple and accurate model of the in-plane properties of a plain weave material was presented by Naik and Ganesh [9], which models the woven composite by an equivalent cross-ply laminate composite.

Figure 2.6: Strain energy plot for a [+45/-45/0/+45/-45] cylindrical shell, \( R = 25 \) mm.
2.3 Woven composite model

2.3.1 Naik model of a single-ply plain-weave composite

This model was developed through an analytical method, which computes the equivalent elastic properties of the undulated strand considering the micromechanics of a one-quarter, symmetric and uniformly distributed weave unit cell (Fig. 2.8). This representative unit cell is modeled as an asymmetric cross-ply laminate composed of four layers, two pure matrix layers and two unidirectional laminae (Fig 2.8(b)). Each equivalent unidirectional lamina of the cross-ply laminate represent one strand in the interlacing region.

In order to determine the properties of the woven laminate, it is sufficient to determine the effective elastic properties of this idealized cross-ply laminate cell.

**Figure 2.8:** The modeling approach of the plain-weave fabric [9]

**Modeling approach**

First of all, the geometrical parameters of the plain-weave fabric must be determined. These are the thickness of the fiber yarns, the thickness of the fabric and the size of the gaps at the cross-point of four yarns. In the unit cell considered, the strand cross-section is assumed to be quasi-elliptical and the fibers in the strand are undulated in the longitudinal direction, following
a sinusoidal shape function. Secondly, the elastic properties of the straight strands (not undulated) must be determined, as in the idealized cross-ply laminate, the unidirectional laminae use these properties. In the fabric, the strands are made of fibers impregnated in epoxy resin. From the fiber volume content and with the fiber and epoxy elastic properties, one can calculate the elastic properties of the straight strands. In a third step, the undulated strand elastic properties are computed by integrating the straight strand elastic properties along the sinusoidal shape function. The thickness of the unidirectional laminae in the equivalent laminate lay-up is then computed by scaling the original thickness of the strand by a coefficient, the thickness scalar. Finally, the CLT is then used to compute the elastic properties of the equivalent laminate lay-up.

The model proposed by Naik and Ganesh [9] proved to be in good agreement with experiments for various types of single-ply plain weave composites. It is simple enough and only with the geometrical parameters of the fabric, the in-plane properties of the material are well predicted.

Let us first consider the representative unit-cell of the plain-weave fabric, Fig. 2.9. We assume that the dimensions of the warp and fill strands are the same. In a first approach we also assume that the thickness of the matrix layer at the interlacing point is zero \((h_{\text{m}} = 0)\), so that he have \(H_L = h_w + h_f = 2h_s\), the total thickness of the fabric is equal to the sum of the thicknesses of two strands. The subscript \(L\) refers to the laminate, \(w\) and \(f\) to warp and fill respectively, and \(s\) to strand.

**The gap ratio and the thickness scalar**

The gap ratio \(C = g/a\) is a parameter in the first analysis, ranging from 0 to 0.25, from no gap to a gap equal 25% of yarn width. This upper bound was chosen according to the real plain-weave fabric cured at KTH. From this gap
2.3 Woven composite model

The material model
	ratio we can calculate the thickness scalar \( k_{th} \) which yields the thickness of the equivalent UD laminae in the cross-ply model.

\[
k_{th} = \frac{2}{\pi} \frac{1 + C}{1 + 2C} \sin \left\{ \frac{\pi}{2(1 + C)} \right\}
\] (2.41)

This equation is the result of assuming a sinusoidal undulation for the yarn [9].

![Figure 2.10: Thickness scalar as function of the gap ratio](image)

We can see in Fig. 2.10 that the thickness scalar variation is fairly linear for the gap ratio range chosen.

The thickness of each layers in the equivalent cross-ply laminate model is then determined by

\[
\bar{h}_s = k_{th} h_s \\
\bar{h}_m = H_L - \bar{h}_s
\] (2.42)

The total thickness of the laminate is fixed at \( H_L = 0.24 \) mm, a common value in real fabrics.

The fiber volume content and uni-directional tensile modulus

Another parameter to take into account when modeling composites is the fiber volume content, which has great influence on the mechanical properties of the fabric. The overall fiber volumic content \( V_f^o \) where \( f \) stands for fiber and \( o \) for overall in a laminate fabric is highly dependent on the curing process. Depending on the temperature applied, and the time of application, more or less matrix material will be sucked out from the fabric, leading to a different fiber volume content in the resulting material. The same goes
for woven composite fabrics as the thickness of the matrix layer left on the surface of the fabric can vary with different curing processes. In order to have an accurate value for the surface matrix thickness, micromeasurements on the cross section of the fabric need to be performed with special equipment, which can be tedious. Therefore, we made the assumption that there is no surface matrix layer at the yarn cross-points of the fabric, so that the thickness of the fabric is equal to the sum of the thicknesses of two interlacing strands.

**Uni-directional lamina properties**  In the model, a strand is idealized as an equivalent uni-directional (UD) lamina. Therefore the elastic properties of the straight strand are to be calculated at the strand fiber volume fraction $V^s_f$, where $s$ stands for strand. The equivalent UD lamina tensile modulus along the fiber direction $E^s_1$ is calculated with the rule of mixtures

$$E^s_1 = V^s_f E^f_1 + V^s_m E_m$$  \hspace{1cm} (2.43)

where $E^f_1$ is the tensile modulus of the fiber material in the longitudinal direction, $E_m$ the tensile modulus of the matrix material and $V^s_m$ the strand matrix volume ratio. There is the relation $V^s_m = 1 - V^s_f$.

Note that the Poisson ratio of the strand $\nu^s_{12}$ can also be calculated with the rule of mixtures

$$\nu^s_{12} = V^s_f \nu^f_{12} + V^s_m \nu_m$$  \hspace{1cm} (2.44)

where $\nu^f_{12}$ and $\nu_m$ are the Poisson ratio of the fiber material and of the matrix, respectively.

The equivalent UD lamina transverse modulus $E^s_2$ and shear modulus $G^s_{12}$ is calculated with the Tsai—Hahn equation, as it proved to be more suitable than the rule of mixtures [14].

$$\frac{1}{E^s_2} = \frac{1}{V^s_f + \eta^s_{12} V^s_m} \left\{ \frac{V^s_f}{E^f_2} + \frac{\eta^s_{12} V^s_m}{E_m} \right\}$$  \hspace{1cm} (2.45)

$$\frac{1}{G^s_{12}} = \frac{1}{V^s_f + \eta^s_{12} V^s_m} \left\{ \frac{V^s_f}{G^f_{12}} + \frac{\eta^s_{12} V^s_m}{G_m} \right\}$$  \hspace{1cm} (2.46)

where $\eta^s_2$ and $\eta^s_{12}$ are the stress partitioning parameters. A value of 0.5 for this parameter was found suitable for both the transverse modulus and the in-plane shear modulus. For refined analyses, Tsai and Hahn [15] provided equations for estimating the stress partitioning parameter from fiber and matrix properties.
2.3 Woven composite model  The material model

Fiber volume content  In order to measure the overall fiber content, the techniques commonly used involve removal of the matrix material by evaporation or chemical digestion, and measurement of the weight of the remaining fibers. To obtain the strand fiber content $V_f^s$ value accurately, either single strand tensile tests or microphotography analysis of one strand cross section area can be performed [16]. As these methods were unavailable to us at that time, the decision was made to assume standard values for $V_f^s$. From these values, $V_f^o$ is calculated using Eq.(2.47).

\[ V_f^o = k_{th}V_f^s \]  

Note that this equation uses the thickness scalar $k_{th}$, calculated previously for a plain-weave fabric. Therefore the $V_f^o$ value obtained may prove to be inaccurate for a twill fabric. Many models in the literature assign the values $(0.7-0.85)$ to $V_f^s$, [19]. Fig.2.11 shows the values of the overall fiber content $V_f^o$ calculated for different values of the gap ratio, for the standard lower and upper bounds of $V_f^s$.

![Figure 2.11: Overall fiber volume ratio as function of the gap ratio, for the upper and lower bounds of the strand fiber volume ratio](image)

It is interesting to notice that for a single-ply plain-weave fabric, the overall fiber volume ratio is capped at 54%. The datasheets from Prospector:Composites database, with several plain-weave fabrics often provide mechanical properties data normalized to certain value of the overall fiber content, which can be superior to 54%. However these tests are performed on multi-ply laminates (up to 12 layers for tensile tests), where the fiber content can be higher according to how the plies are stacked (the more layers, the less penetration of the resin), [17].

Due to the difficulty of determining the actual fiber volume content in the
fabrics used in the manufacturers tests, the elastic properties given by the model were derived for the lower and upper bound of $V_f^s$, for the gap ratio range $(0 - 0.25)$. Then the properties of each commercial fabric were added in the resulting graphs, for easier comparison.

### Material properties

The SIMPLE boom material employs T300 carbon fibers, so their properties were considered in the analyses. In T300 fiber manufacturer datasheets, the only elastic property given is the tensile modulus $E_f^1$. The rest of the properties were taken from Soykasap, [18]. They are listed in Table 2.2.

**Table 2.2: T300 fiber properties**

<table>
<thead>
<tr>
<th>$E_f^1$</th>
<th>230.0 GPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_f^2$</td>
<td>14.0 GPa</td>
</tr>
<tr>
<td>$G_{12}, G_{13}$</td>
<td>9.0 GPa</td>
</tr>
<tr>
<td>$G_{23}$</td>
<td>5.36 GPa</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1.76 g/cm$^3$</td>
</tr>
</tbody>
</table>

The matrix used in the SIMPLE boom is a 350°F toughened epoxy resin. The epoxy resin is modeled as an isotropic material. However none of the elastic properties for this specific epoxy resin were available, so generic epoxy resin properties were used, [9].

**Table 2.3: Generic epoxy resin properties**

<table>
<thead>
<tr>
<th>$E_m$</th>
<th>3.5 GPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{1m}$</td>
<td>1.3 GPa</td>
</tr>
<tr>
<td>$\nu_m$</td>
<td>0.35 GPa</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1.17 g/cm$^3$</td>
</tr>
</tbody>
</table>

These properties were used in Eqs. (2.43)–(2.46) to obtain the strand elastic properties in Table 2.4.

### 2.3.2 Results of the model

The mechanical properties of the equivalent cross-ply laminate model were obtained through finite element analysis with ABAQUS. The protocols of the tests are described in Appendix A. The aims of the tests are to first compare the results to the available commercial materials, and secondly to assess the influence of the gap ratio $g/a$ and strand fiber volume content $V_f^s$ on the material model properties.
2.3 Woven composite model

Table 2.4: Material properties of the equivalent cross-ply laminate model

<table>
<thead>
<tr>
<th>Total thickness</th>
<th>0.24 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix layer</td>
<td></td>
</tr>
<tr>
<td>$E$</td>
<td>3.5 GPa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.35</td>
</tr>
<tr>
<td>UD Lamina (strand) $V_f = 0.70$</td>
<td></td>
</tr>
<tr>
<td>$E_1$</td>
<td>162 GPa</td>
</tr>
<tr>
<td>$E_2$</td>
<td>9.15 GPa</td>
</tr>
<tr>
<td>$G_{12} , G_{13}$</td>
<td>4.4 GPa</td>
</tr>
<tr>
<td>$G_{23}$</td>
<td>3.46 GPa</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.25</td>
</tr>
<tr>
<td>UD Lamina (strand) $V_f = 0.85$</td>
<td></td>
</tr>
<tr>
<td>$E_1$</td>
<td>196 GPa</td>
</tr>
<tr>
<td>$E_2$</td>
<td>11.26 GPa</td>
</tr>
<tr>
<td>$G_{12} , G_{13}$</td>
<td>6.08 GPa</td>
</tr>
<tr>
<td>$G_{23}$</td>
<td>4.28 GPa</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Properties along the strand direction

First the tensile modulus tests along the strand direction are performed, as this data is available in all the manufacturers data sheets.

The tensile modulus of the different types of commercial plain-weave fabrics is comprised between the lower and upper bounds of $V_f$, except for the T300/977-6 fabric. However the analysis cannot tell much more as the real elastic properties of the epoxy resin are needed along with an accurate value for $V_f$. Then the results could be compared directly.

As expected, the more gaps in the fabric, the lower the tensile modulus, following a linear decrease. For a gap ratio of 0.25, the tensile modulus decreases by 19.4\% for $V_f = 0.85$ and 19.2\% for $V_f = 0.70$ Fig. 2.12. The upper bound $V_f = 0.85$ gives a tensile modulus value which is 16.8\% higher than the lower bound $V_f = 0.70$ in average.

For the following test results, the T300/E765 fabric was used as the reference, as it is the only fabric where all the in-plane material properties are available in the data sheets.

The in-plane Poisson ratio is very small for this type of material, as the fiber material only supports small strains. Therefore the less fibers in the fabric, the higher the Poisson ratio is. That is what we observe in Fig. 2.13, where increased gaps or low values of $V_f$ give higher Poisson ratio values. We can observe here that the T300/E765 fabric Poisson ratio is out of the
2.3 Woven composite model

The material model

Figure 2.12: Tensile modulus along the strand direction as function of the gap ratio.

Figure 2.13: Poisson ratio along the strand direction as function of the gap ratio.
2.3 Woven composite model

The material model

Table 2.5: Naik’s results for the Poisson ratio of plain weave as function of $g/a$ ($V_f^s = 0.70$), [17].

<table>
<thead>
<tr>
<th>$g/a$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.088</td>
</tr>
<tr>
<td>0.10</td>
<td>0.097</td>
</tr>
<tr>
<td>0.15</td>
<td>0.106</td>
</tr>
<tr>
<td>0.25</td>
<td>0.121</td>
</tr>
</tbody>
</table>

bounds of the model. Using a laminate layup greatly underestimates the actual value in a woven composite in our case ($-68\%$). These results are opposed to those of Naik for a T300/epoxy laminate model with the same $V_f^s = 0.70$, Table 2.5, which overestimate the actual value ($+33\%$). The only difference in the model by Naik is the transverse elastic properties of the fibers ($E_{2f}^I = 40$ GPa, $G_{12}^I = 24$ GPa, $G_{23}^I = 14.3$ GPa, to compare to the values in Table 2.2). It is therefore of great importance to know the exact values for these properties in order to get a comparable value for the Poisson ratio. It would have been of interest to use the values provided by Naik for the T300 fibers and check the difference in results, however the lack of time decided otherwise.

Figure 2.14: Shear modulus along the strand direction as function of the gap ratio

The shear modulus also diminishes as the gap ratio increases, with a $12\%$ difference between $g/a = 0$ and $g/a = 0.25$, and an average $23.7\%$ difference between the lower and upper bound of the model. The shear modulus of the T300/E765 fabric is comparable to the values predicted by the model.
The last test concerns the out-of-plane property, the flexural modulus $E_b$. The higher the flexural modulus, the stiffer the material is in bending.

To the contrary of the in-plane properties, the flexural modulus does not vary linearly with the gap ratio, but is rather inversely proportional, with a decrease of approximately 35% between $g/a = 0$ and $g/a = 0.25$. The average difference between the upper and lower bound is of 12%. The data given by the manufacturers is quite unclear on the type of sample tested. The values lie around 50 GPa, which suggests that they use a laminate fabric made of several woven layers. However the number of layers and the thickness of the fabric is unknown, so the data is not comparable.

**Properties at 45° from the strand direction**

The properties of the fabric at $45°$ are important to obtain, as in the SIMPLE boom the strands are oriented at $45°$ from the longitudinal and transverse direction $(x,y)$. These in-plane properties are not listed in the datasheets, however they can be calculated from the properties at $0°$. First the $S$ matrix is build by inserting the datasheet properties of the fabric, as defined in Eq.(2.9). Then using in Eq.(2.19) the transformation matrix $T_\phi$ defined in Eq.(2.14) for $\phi = 45°$, we obtained the compliance matrix $S'_\phi=45°$ in the desired direction. The elastic properties at $45°$ are then obtained through Eq. (2.9).
2.3 Woven composite model

Figure 2.16: Tensile modulus at 45° from the strand direction as function of the gap ratio

Figure 2.17: Poisson ratio at 45° from the strand direction as function of the gap ratio
2.3 Woven composite model

The elastic properties at 45° are very different than those along the strands. The tensile modulus is almost 4 times lower, whereas the shear modulus is 7 times greater, and the Poisson ratio almost 20 times higher. The tensile and shear modulus and Poisson ratio results confirm that the model estimates pretty well the in-plane properties of the T300/E765 plain-weave fabric, as all the results are well comprised between the bounds of the model.

It is interesting to note that the variation of the Poisson ratio at 45° with the gap ratio is the opposite of that at 0°. Concerning the flexural modulus at 45°, the variation is also inversely proportional to the gap ratio. The bending stiffness of the equivalent laminate layup is almost two times weaker at 45° than along the fibers direction.

Configurations of interest

Two configurations of interest can be noted from the previous section, which best model the T300/E765 fabric. The first configuration is for $V_f^s = 0.70$ and $g/a = 0$, the second $V_f^s = 0.85$ and $g/a = 0.21$. The predictions and errors of these two configurations are listed in Table 2.7.

The second configuration, with $V_f^s = 0.85$ and $g/a = 0.21$ is the best fit for the T300/E765 fabric. The error in the prediction of the Poisson ratio at 0° is high, but it is the only elastic property that this model cannot predict well. It is not known how critical this value is regarding the bistability of the shell, but it has been demonstrated that the rolled-up dimensions of the boom highly depends on the Poisson ratio at 45°, well predicted by this

Figure 2.18: Shear modulus at 45° from the strand direction as function of the gap ratio
2.3 Woven composite model

The material model

Figure 2.19: Flexural modulus at 45° from the strand direction as function of the gap ratio

Table 2.6: Data sheet properties of the T300/E765 fabric, data from Prospector:Composites

<table>
<thead>
<tr>
<th>T300/E765 fabric</th>
<th>Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_1$</td>
<td>56.5 GPa</td>
</tr>
<tr>
<td></td>
<td>$G_{12}$</td>
<td>3.86 GPa</td>
</tr>
<tr>
<td></td>
<td>$\nu_{12}$</td>
<td>0.059</td>
</tr>
<tr>
<td></td>
<td>$E_x$</td>
<td>13.7 GPa</td>
</tr>
<tr>
<td></td>
<td>$G_{xy}$</td>
<td>26.68 GPa</td>
</tr>
<tr>
<td></td>
<td>$\nu_{xy}$</td>
<td>0.772</td>
</tr>
</tbody>
</table>

model.

Remark  Prior to developing this model, a similar laminate layup was studied, based on much more approximate assumptions for determining the UD lamina properties. However, that layup predicts very well the in-plane properties above, including the $0^\circ$ Poisson ratio. But that model requires the shear modulus of the plain weave material as a parameter, hence is based on test results, which are often not available. Furthermore, this leads to unrealistic shear strain and stresses in the material, as no difference is made between fiber and epoxy material. That model is described in Appendix B. The current model is only based on the micromechanics and geometry of the weave for determining the UD lamina properties, and allows fine tuning of the geometrical parameters to model the real material.
2.3 Woven composite model

Table 2.7: Model predictions for the two configurations of interest

<table>
<thead>
<tr>
<th>$V_s f = 0.70, g/a = 0$</th>
<th>Properties</th>
<th>Model prediction</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$</td>
<td>56.0 GPa</td>
<td>−0.89</td>
<td></td>
</tr>
<tr>
<td>$G_{12}$</td>
<td>3.25 GPa</td>
<td>−18.7</td>
<td></td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.035</td>
<td>−68.6</td>
<td></td>
</tr>
<tr>
<td>$E_x$</td>
<td>11.75 GPa</td>
<td>−16.6</td>
<td></td>
</tr>
<tr>
<td>$G_{xy}$</td>
<td>27.0 GPa</td>
<td>+1.19</td>
<td></td>
</tr>
<tr>
<td>$\nu_{xy}$</td>
<td>0.794</td>
<td>+2.77</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$V_s f = 0.85, g/a = 0.21$</th>
<th>Properties</th>
<th>Model prediction</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$</td>
<td>56.5 GPa</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$G_{12}$</td>
<td>3.8 GPa</td>
<td>−1.58</td>
<td></td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.035</td>
<td>−68.6</td>
<td></td>
</tr>
<tr>
<td>$E_x$</td>
<td>13.5 GPa</td>
<td>−1.48</td>
<td></td>
</tr>
<tr>
<td>$G_{xy}$</td>
<td>27.5 GPa</td>
<td>+2.98</td>
<td></td>
</tr>
<tr>
<td>$\nu_{xy}$</td>
<td>0.765</td>
<td>−0.92</td>
<td></td>
</tr>
</tbody>
</table>

Conclusion

Naik’s approach of the plain-weave fabric, modeled by an equivalent laminate composite proves to be a good estimator of the in-plane properties of the material, although the accuracy of the model can only be assessed if one provides accurate input of the elastic properties of the epoxy matrix and carbon fibers, along with precise geometric parameters of the weave (gap ratio, thickness of the surface epoxy layer and strand fiber volume ratio).

The advantage of this model is that it is easy to implement in finite element analyses, through a “composite layup definition”. However, the results for the flexural modulus are to be taken with great care, as the model has been designed for predicting in-plane elastic properties, therefore lacks accuracy for the out-of-plane properties.

Few finite element models have been developed for predicting both in-plane and out-of-plane properties, based on the micromechanics of the woven fabric, and proved to be accurate [18]. But the implementation of these models is a difficult task, mainly because of the micromechanical geometry of the model of which it is tedious to modify the shape and requires very long computations when it comes to simulating large structures.

In the next section, the objective is to combine the equivalent laminate model with the bistable laminate structure developed in the first section, so that we obtain a model of a bi-stable woven composite.
2.4 Bi-stable woven composite model

In the first section we derived the main features of a laminate model of bistable cylindrical shells. Necessary conditions are that the composite layup should be anti-symmetric and balanced. The \([45^\circ/-45^\circ/0^\circ/45^\circ/-45^\circ]\) layup of PP/Glass laminae is a good example of a bistable fabric [7]. In the second section a second laminate model was derived for plain-weave composites. This model features two cross-ply laminae between two layers of isotropic matrix, \([0^\circ/45^\circ/-45^\circ/0^\circ]\). In order to apply the plain-weave model to the bistable cylindrical shell, the laminate layup used should have the same features than the two described just before.

The method is to divide each strand lamina into two orthogonal strand laminae, each half as thick as the original, so that we obtain a \([0^\circ/45^\circ/-45^\circ/45^\circ/-45^\circ/0^\circ]\) structure. This structure is antisymmetric, balanced, and possesses the same in-plane characteristics as the original (the tests in ABAQUS showed a difference less than 0.5\% for each properties), except for the out-of-plane properties.

Figure 2.20: Method to apply the equivalent plain weave model to bistable cylindrical shell layup
2.4 Bi-stable woven composite model

2.4.1 Results

Flexural modulus

![Graph showing flexural modulus as a function of gap ratio for different structures and bias angles.](image)

Figure 2.21: Flexural modulus at 45° from the strand direction as function of the gap ratio, for the two structures of interest.

The change in the layup increases by 12% the flexural modulus at 45° of the laminate. The tests results for the 45° bias are also plotted on the graph. The plain-weave material is much stiffer in bending than the twill. The model underestimates the flexural modulus of the plain weave and tends to overestimate the flexural modulus of the twill fabric (the latter is well predicted only for a high gap ratio). However one must remember that the micromechanical behavior and the determination of the equivalent UD lamina properties are based on a plain-weave geometry. The adaptation and tuning of the model parameters to match the twill fabric properties is questionable.

Adjusting the flexural modulus

It is interesting to see that this laminate model underestimates the flexural modulus at 45° of the plain-weave fabric, as it is the opposite for the flexural modulus at 0° [18]. Fortunately it is simple to increase the flexural modulus given by this model without affecting the in-plane properties. It is however not possible to reduce it. The method is to adjust the z-position of the UD laminae in the laminate layup, while keeping the total epoxy and fiber thickness constant. Then the in-plane properties are not affected. The further the UD laminae from the mid-plane, the higher the flexural modulus. The position of the UD layers
2.4 Bi-stable woven composite model

The material model

is quantified by a parameter we call the Core Matrix Ratio (CMR). This parameter is equal to the volume percentage of epoxy matrix in the mid-layer, and ranges from 0 to 1. CMR = 0 for the original configuration and CMR = 1 for a configuration where the UD laminae are at the surface of the laminate (the matrix is located only in the mid-layer). The laminate thickness and the UD laminae thickness are not modified. The flexural modulus at

![Diagram](image)

Figure 2.22: left: Original layup. right: Modified layup with UD layers further from the mid-plane

is increasing non-linearly with the CMR. Figure 2.23 shows that for the case $V_f^{s} = 0.85$, a CMR value of 0.17 would match the tests results.

**Consequences on the flexural modulus at 0°** The laminate structure model overestimates by nature the flexural modulus of the plain-weave fabric, even though the in-plane properties match, [18]. The variation of the flexural modulus at 0° with the CMR is much more important than at 45°.
2.4 Bi-stable woven composite model

The flexural modulus value increases by more than 100% from CMR = 0 to CMR = 0.6. If we consider the case $V_f^s = 0.85$ and CMR = 0.17, the flexural modulus at 45° is accurate but the flexural modulus at 0° is overestimated by 135%.

This is precisely the problem when it comes to modeling plain-weave with a laminate layup. The micromechanics of the two structures are different, and predicting accurately the elastic properties multidirectionally is not possible.

**Bi-stability**

The material properties are the same as in the previous section. The bi-stability of the structure is assessed for the upper and lower bound of the model in terms of strand fiber volume ratio $V_f^s$, and also as function of the gap ratio. The bi-stability tests were performed in ABAQUS, the protocol is described in Appendix C. The parameters extracted from the analyses are the strain energy value at the transition between the two states, the strain energy of the second stable state if it exists and the radius of the shell in the coiled state.

**Strain energy value at the transition (Maximum strain energy)**

The strain energy value at the transition is an indication on how difficult it is to switch from one configuration to another. The higher this value,
the higher the force needed. For the deployable boom, a higher transition energy means a faster deployment.

**Strain energy stored in the second state (local minimum strain energy)**

The strain energy value in the second state represents how stable this state is. For a fixed value of the strain energy at the transition, a higher strain energy value in the second state characterizes a less stable configuration. The difference \( \Delta U = U_{\text{transition}} - U_{\text{second state}} \) is a criteria for assessing the stability of the second configuration.

**Radius in the second state**

The radius in the second state \( R_c \), where the shell curvature flips in the longitudinal direction, is important to measure as it gives an idea on how compact the rolled-up configuration is. The smaller the radius, the more compact the rolled-up tape-spring.

---

Figure 2.25: Strain energy interesting points and coiling radius as function of the gap ratio

The results in Fig. 2.25 are interesting. First, the strain energy in the tape spring is higher for a higher concentration of fibers in one strand. From 70% to 85% of fiber volume ratio, the strain energy stored increases by 16%, which is approximately linear with \( V_f^s \). Secondly, the strain energy decreases as the gap ratio increases. The fact that mainly the fibers store the strain energy is well captured by the model. For this analysis, it turns out that the
configuration $V_s^f = 0.85$ is bistable in the (0–0.25) gap ratio range. However, this is not true for $V_s^f = 0.70$, where the shell is not bistable for $g/a = 0.25$, even if a maximum value of stress was passed as the shell was deforming. Hence, the local minimum strain energy value recorded in the plot is not a local minimum of strain energy regarding the deformation of the shell. More specifically, it means that a different deformation path exists, in which the strain energy is gradually decreasing, until the shell recovers its original shape. For a bi-stable shell, there is no such deformation path: from the second stable configuration, any deformation of the shell increases the strain energy stored. Concerning the radius in the rolled-up configuration, we can see that it increases as the gap ratio increases. It is interesting to see that the value of $V_s^f$ does not have much influence on the coiling radius. The radius is even the same for both lower and upper bounds of $V_s^f$ when $g/a = 0.08$.

### 2.4.2 Conclusion

The results show that the model is able to capture the main features of the deployable boom material. The in-plane properties can be accurately predicted with a fine tuning of the model parameters. The out-of-plane properties are more problematic, as the micromechanics in a laminate (model) and a plain-weave are very different across the thickness of the fabric. The bending stiffness of the fabric at 45° from the fiber direction can be adjusted in the model, to match the real value. But the bending stiffness in the direction of the fibers is greatly overestimated by the model, and cannot be matched without affecting the accuracy of the in-plane properties predictions.

The material model therefore can be divided into two versions for the same fabric. The first version of the model would be designed so that the in-plane properties are accurate and represent the real fabric. This version would not be used for deployment analysis, but for the fully deployed straight booms (modal analysis, transient thermal analysis and boom stiffness characterization as function of the geometry for example). The second version of the model would be designed so that the bi-stable behavior predictions are accurate. This includes the stored strain energy levels, the radius of coiling and the damping within the material.

In the next section, the main features of modeling bi-stability with the FEM are developed. The influence of a selection of parameters on the bi-stable behavior prediction is analyzed in order to lay the basis of a more refined model of the real bi-stable composite tape-spring.
Chapter 3

Bi-stability characterization

Organization of the chapter

This chapter is focused on how the bi-stable behavior of a composite plain-weave tape-spring can be accurately predicted. First the main parameters characterizing bi-stability are exposed, and the method to extract them from the FEM model is described. In a second section, the model is subject to a design of experiment. A sensitivity analysis is carried out for several parameters of the CLT model, regarding its bi-stability response. The last part develops the concept of deformation path, which is crucial to understand how bi-stability can be better assessed. The accuracy in modeling bi-stability is very important for developing a good model of the boom deployment phase, as the dynamics involved are essentially related to the bi-stability of the tape-spring.
3.1 Finite element model

Reminder

A structure is stable in a certain configuration if it remains in the same configuration without any external energy input. Energy input covers mechanical loading, temperature increase or chemical interaction. A bi-stable structure possesses two such configurations, generally one with zero energy stored and the other with a certain amount of energy stored. In order to characterize bi-stability, the analysis of the energy stored in the structure is straightforward. In the case of the tape-spring, the first configuration is the straight semi-tubular shape, the second configuration is the rolled-up shape. This second configuration is characterized by the radius of coiling.

3.1 Finite element model

A 25 mm long section of the tape-spring shell is modeled and loaded so that it adopts its second configuration. Then the loads are progressively removed until the shell is left free. The final configuration of the shell is either rolled-up in case of bi-stability, or straight otherwise (non bi-stable).

During the analysis, the load and strain energy values and the vertical displacement of one corner node are recorded. The monitoring of the y-displacement of a corner node is a simple means to observe the switch in configurations. The load and strain energy record is used for characterizing the stability of the second configuration. A more detailed description of the analyses is available in Appendix C.
3.1 Finite element model  

3.1.1 Two different cases

Non bi-stable case

The tape-spring shell is either bi-stable or non bi-stable. The characteristics of a typical non bi-stable shell model are described in the following.

Several points of interest characterize a non bi-stable shell in Fig. 3.2. This plot is progressively drawn as ABAQUS performs the analysis. On the bottom of the figure the three steps are illustrated. In the first step the value of the loads (shell edge moments) is ramped until its maximum value at $t = 1$ s. In the second step the loads are ramped back until zero at $t = 2$ s. A third step without any loading is performed in order to ensure that the shell at $t = 2$ s is stable.
3.1 Finite element model  

Bi-stability characterization

The shell in this case is non bi-stable. Although the shell adopted a rolled-up configuration after a certain load value, it did not remain in the rolled-up configuration as the loads were progressively removed, but switched back into the straight configuration.

The second plot of interest to characterize non bi-stability is the strain energy stored in the shell as function of the load, Fig. 3.3. This plot gives information on the critical load values, where the switches in configuration occur. The first switch (into the rolled-up configuration) occurs for a maximum value of the load. The second switch (back to the straight configuration) occurs for a minimum value of the load. This plot gives also the strain energy value levels in the rolled-up and straight configurations.

The shell is not bi-stable if these two plots display two switches in the shell configuration, illustrated by a sharp vertical variation in the $y$-displacement and strain energy.
3.1 Finite element model  

Bi-stability characterization

Figure 3.3: Strain energy stored in the shell as function of the load, for a non bi-stable shell.

Bi-stable case

Figure 3.4: Monitoring of the y-displacement of a corner node during the analysis in a bi-stable shell

In the case of bi-stability, the monitor plot is similar to Fig. 3.4. A single switch in configuration is observed. The shell stays in the rolled-up configuration until the end of the analysis. The strain energy as function of load plot, Fig. 3.5, is the main tool used to characterize bi-stability, as it gives all the crucial energy parameters. The difference with a non bi-
3.1 Finite element model  

Bi-stability characterization

Figure 3.5: Strain energy stored in the shell as function of the load, for a bi-stable shell

stable shell on this plot is that there is a second value of the strain energy for a zero load. This is the strain energy stored by the shell in the stable rolled-up configuration (local minimum). It is also interesting to note that the transition zone (switch of configuration) occurs at a much higher energy level than for the non bi-stable case. The difference between the peak value of strain energy (maximum) and the strain energy value at the last stable point of the rolled-up configuration is higher in a bi-stable shell than in the non bi-stable shell. This difference of strain energy is denoted $\Delta SE$ in the following. The plot Fig. 3.5 can be extended to the negative loads, by applying opposite moments on the deformed shell. This would give an estimation of how stable the second configuration is (the higher the moment applied before the switch occurs, the more stable the configuration). The shell is bi-stable if these two plots display only one switch of configuration, and if there are two different value of the strain energy for a zero load.

3.1.2 Radius in the rolled-up configuration

The radius in the rolled-up configuration is only measured in bi-stable cases. The shell has to be fully unloaded for the radius to be measured.

First a nodal path is created in ABAQUS, passing through all the nodes in the middle of the shell, in the longitudinal direction (undeformed shell). In the deformed configuration, the coordinates of the path nodes are recorded and processed in MATLAB. The coordinates are input of a circle fitting script (Appendix D), in order to determine the best fitting circle for the path nodes. The radius of that circle is equal to the radius in the rolled-up
configuration or coiling radius.

Figure 3.6: The nodal path used to measure the coiling radius. The shell is in the second stable configuration.

3.1.3 Model used in the analyses

At the time of the bi-stability analysis, the model presented in the previous chapter was not entirely developed. The material properties used then were similar though more approximate (The transverse properties were calculated from a fixed shear modulus \( G = 3.86 \text{ GPa} \)). However it is important to note that the conclusions of the following parametric analyses are valid for both models.

Table 3.1: Material properties of the model

<table>
<thead>
<tr>
<th>Material</th>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total thickness</td>
<td></td>
<td>0.24 mm</td>
</tr>
<tr>
<td>Gap ratio ( g/a )</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Matrix layer</td>
<td>( E )</td>
<td>10.04 \text{ GPa}</td>
</tr>
<tr>
<td></td>
<td>( \nu )</td>
<td>0.3</td>
</tr>
<tr>
<td>UD Lamina</td>
<td>( E_1 )</td>
<td>162.05 \text{ GPa}</td>
</tr>
<tr>
<td></td>
<td>( E_2 )</td>
<td>10.04 \text{ GPa}</td>
</tr>
<tr>
<td></td>
<td>( G_{12}, G_{13}, G_{23} )</td>
<td>3.86 \text{ GPa}</td>
</tr>
<tr>
<td></td>
<td>( \nu_{12} )</td>
<td>0.3</td>
</tr>
</tbody>
</table>
3.2 Influence of some parameters on bi-stability

3.2.1 Effect of the CMR

As explained in the paragraph on the origins of bi-stability, the shell is only subjected to bending energy when it switches from one configuration to another, the transformation is isometric. The CMR parameter defined in the previous chapter is the first parameter analyzed, as it only affects the out-of-plane properties of the material, the bending stiffness notably (Figs. 2.23 and 2.24).

![Figure 3.7: Influence of the CMR on the strain energy vs load curve.](image)

From the results in Fig. 3.7, the CMR has a strong influence on the strain energy behavior. The higher the CMR (the further the UD laminae from the mid-plane of the composite layup) the higher the characteristic values of the strain energy. The peak value of strain energy almost doubles between CMR = 0 and CMR = 1, and so does the critical load (from 2.15 N to 4.1 N). Comparatively, the stored strain energy in the second stable configuration does not increase much (from 80 Nmm to 95 Nmm). These variations induce an increase in ΔSE as the CMR increases. It means that the second configuration is more stable as the CMR increases. Although it has not been verified, we can predict that the higher the CMR, the higher the load needed to switch back the shell in the original configuration.

In the context of the deployable boom, we can distinguish 3 different states of the shell, Fig. 3.8, present during the deployment phase. The energy density in each zone almost does not vary during deployment. The first
3.2 Influence of some parameters on bi-stability

Bi-stability characterization

Figure 3.8: The three energy zones of the tape spring during deployment.

zone, stable configuration 1, does not store any energy. The transition zone stores the most strain energy, though its distribution is not uniform in the zone. There is a peak of energy density somewhere in the transition zone, exactly like there is a peak of strain energy in Fig. 3.5. This peak of strain energy remains until the end of deployment, as there is always an attachment transition between flat and cylindrical cross-sections. The third zone, stable configuration 2, is stable and stores strain energy. The energy density is lower than in the transition zone. It is shown in Fig. 3.7 that the energy stored does not vary much compared to the peak energy. The difference in strain energy $\Delta SE$ between the energy density in this zone and the transition zone defines how stable the configuration is. The stored energy in this zone drives the deployment of the boom. The higher the energy stored, the higher the deployment force [5].

Regarding Figure 3.9, we can see that the radius in the rolled-up configuration is also influenced by the CMR. The higher the CMR, the lower the radius. As in Figure 3.7, we can see that the shell at CMR = 0 and 0.1 is not bi-stable. The analytical model curve corresponds to the findings of Iqbal and Pellegrino [7], who demonstrated from the strain energy equation that the radius in the rolled-up configuration $R_c$ is related to the radius in the original configuration $R$ through the following equation:

$$R_c = R \frac{D_{11}}{D_{12}} \quad (3.1)$$
3.2 Influence of some parameters on bi-stability

Bi-stability characterization

Figure 3.9: Coiling radius as function of the CMR.

where $D_{11}$ and $D_{12}$ are elements of the laminate stiffness matrix. The approximated Naik model is the model described in the Table 3.1, and the Naik model is the one developed in the previous chapter (Table 2.4). First we can see that the coiling radius is much less sensitive to the CMR parameter in the Naik model than in the approximated model (8% variation compared to 25% for analytical results). The model applied into ABAQUS yields higher and more sensitive results. Bi-stability is not observed for the first two cases, so radius cannot be measured. It is interesting to see that the radius at CMR = 0 is almost the same for every curve (6% variation).

More generally, the order of magnitude of the radius in the rolled-up configuration is between 1 and 2 times the original radius. In relation with Fig. 3.7, we can say that the larger the coiling radius, the less stable the second configuration. During deployment, the rolled-up radius will decrease as the tape-spring unwraps, resulting in an increasing deployment force. Indeed, the density of the strain energy stored in the different coils increases as the radius decreases.

The CMR is an interesting parameter which allows us to see the effect of increasing only the bending stiffness of the laminate layup. This parameter can be adjusted to match a particular coiling radius, without affecting much the stored strain energy in the rolled-up configuration.
3.2 Influence of some parameters on bi-stability

3.2.2 Effect of the fiber orientation

The fiber orientation angle $\phi$ in the model is a parameter that also can be modified in the model. The angle $\phi$ is measured from the longitudinal direction of the cylindrical shell. First the effect of this parameter on the in-plane properties is analyzed. The shear modulus is almost unchanged, and as expected the tensile moduli are higher when the fibers form a smaller angle to the direction of measurement. The flexural modulus has the same variation as the tensile modulus in the same direction. The bi-stability behavior can be expected to be seriously influenced by the fiber angle. We can observe in Fig. 3.11 that the fiber angle has a strong influence on the stored strain energy. The rolled-up configuration stores more energy when the fibers tend toward the transverse direction. With a 10° offset, the stored strain energy is multiplied by 4 from 35° to 45°, and by 2 from 45° to 55°. The radius of coiling is greatly affected by the orientation of fibers also. The rolled-up configuration is 20% more compact with fibers at 55° than at 45°. For the deploying boom, these curves indicate that if fibers tend toward the transverse direction, the deployment will be faster (more strain energy stored in the second configuration), together with a more compact rolled-up configuration.

Triaxial weaves (Fig. 3.13) may be a good material for the deployable boom. It consists of a plain-weave structure with fiber yarns at 60°, interlacing with 0° yarns. Although the yarns at 0° increase the rolled-up radius and have a negative effect on bi-stability, the bi-stability behavior induced by the yarns at 60° may be sufficiently strong to obtain a more compact rolled-up
3.2 Influence of some parameters on bi-stability

Bi-stability characterization

![Graph showing influence of fiber angle on strain energy vs load curve.](image)

Figure 3.11: Influence of the fiber angle on the strain energy vs load curve.

![Graph showing coiling radius as function of fiber angle.](image)

Figure 3.12: Coiling radius as function of the fiber angle.

configuration and a higher deployment force than with a plain-weave fabric.

3.2.3 Effect of the gap ratio

The gap ratio is another parameter that was studied for bi-stability. The influence of the gap ratio on the in-plane properties are well described in the previous chapter. Also some bi-stability parameters are described in the last part of the previous chapter, in relation to the gap ratio.
3.2 Influence of some parameters on bi-stability

Bi-stability characterization

Figure 3.13: Triaxial weaving pattern

Note that these analyses are carried on the model developed in the previous chapter, Table 2.4 for $V_f = 0.85$, not the approximate model, Table 3.1. The values of loads and strain energy vary from the previous curves.

In Fig. 3.14, we can see that the higher the gap ratio, the less strain energy stored in the shell, and that if this value is too high, the shell is not bi-stable anymore. The radius of coiling also increases with the gap ratio until a maximum value after which it is not bi-stable.

Figure 3.14: Influence of the gap ratio on the strain energy vs load curve.
3.3 Bi-stability and deformation path

During the deformation of the shell, the strain energy stored goes through a peak value as the loads are ramped up and down. This fact occurs even when the shell is not bi-stable. One can wonder then why the structure does not remain in the second state, because the strain energy value in the rolled-up configuration is a local minimum on the strain energy vs load curve. This is due to the deformation path chosen for the analysis. In the analysis, we chose to apply symmetric and uniform moments on the edges of the shell, forcing a symmetric transition zone between the two configurations. The strain energy when the shell is in this transition state is higher than what it would be in the natural transition state (transition shape that stores a minimum of energy). This difference in strain energy stored between the forced transition shape and the natural transition shape is important if one wants to carry out a refined bi-stability analysis and obtain more explicit strain energy curves.

The problem is that it is difficult to load the structure so that it follows its natural deformation path. This deformation path can be observed when unloading the structure.

The difference between the two paths is easy to grasp when looking at Figs. 3.16 and 3.17. When loading, the two loaded edges remain fairly straight until they progressively adopt the rolled-up curvature. When unloading however, the edges directly buckle in their middle point, the curvature does not decrease progressively. The warping of the cross-section in the transition state is also more substantial during unloading, Fig. 3.18.
In a bi-stable structure, buckling of the edges increases the stored energy, however in non bi-stable structures, the stored energy is released when the edges buckle. It is a difficult task to obtain a good representation of the stored strain energy as function of the shape of the structure. The shape of the structure can be represented by its two principal curvatures, but it is still very approximate: the curvature is not uniform anywhere in the shell except in the two stable configurations and there is warping of the cross section during deformation.

### 3.4 Conclusions

Although a shell is characterized as bi-stable or non bi-stable, bi-stability is a complex phenomenon to characterize. A bi-stable structure can be “more
or less” bi-stable. As we saw, the bi-stability strength is closely related to the second stable geometric configuration. A higher coiling radius means a less stable structure and vice-versa. The purpose of this chapter was to give more insight on which parameters can be tuned in order to model the bi-stability characteristics of a real material. In order to do so, one must first assess how stable the rolled-up configuration is in the real material, that is how much mechanical energy must be added to the material for it to switch back into the original configuration. This may prove difficult to measure though. The second characteristic to determine is the stored energy in the rolled-up configuration, by measuring the deployment force (directly related to the stored energy). The last characteristic to measure is the natural radius in the rolled-up configuration, which is easy to determine in the real material.

It is difficult to set up an accurate method for modeling these three characteristics accurately with the FEM model. The important thing to note however is that the model can be tuned to capture these properties.

It seemed also that the approximated Naik model is more sensitive to the change in parameters, notably the coiling radius. It is therefore recommended to use this model, which is more adaptable.
Chapter 4

Model of the deployable boom

Organization of the chapter

This chapter presents a preliminary finite element model of the deployable booms. The model purpose is to carry out coiling and deployment of the booms. A simplified assembly which consists of two tape springs attached to one spool is modeled. In the first section, the modeling of the different parts is described. In a second section the analysis steps and boundary conditions are presented. The difficulties, problems encountered and their solutions are explained. Finally, the main lines of the future work on this model are developed.

4.1 About the model

The model is developed using Input Files (*.inp), which contain the analysis instructions written in ABAQUS language. Using input files is the best way to follow when it comes to modeling complex assemblies or analysis steps, though it is much less user-friendly and requires a good knowledge of how ABAQUS reads the instructions. In a first attempt, a fully parameterized model (geometry, meshing, material properties, interactions, boundary conditions, step duration, etc...) was developed and used to test the capacities and limitations of ABAQUS. A fully parameterizable model is a really useful tool for the analyst, as any modification can be quickly done. However it is extremely time consuming to develop this kind of model in ABAQUS language. In a second attempt a clearer and more refined model was developed, this time not parameterized (geometry and mesh are fixed), in order to provide a simple tool for carrying coiling and deployment analyses. The following sections are focused on this second model.
4.2 Modeling the parts

4.2.1 Tape-springs

Geometry

There are two identical tape-spring parts in the model. The part is modeled as a shell. The cross-section is a semi circular arc, the subtended angle is $180^\circ$ and the radius 6.35 mm. The extruded length of the shell is 500 mm, Fig 4.1.

Mesh

The shell element type used is S4R (Shell with 4 nodes and Reduced integration), preliminary analysis showed that it is more robust than S8R and as accurate. The mesh is not uniform throughout the part, Fig 4.2. A 10 mm zone at one end of the tape-spring has a refined mesh. We call it the Refined Zone. This is the zone where the tape-spring is attached to the spool. The offset for the shell thickness is defined as SPOS (positive offset).

<table>
<thead>
<tr>
<th>Refined Zone</th>
<th>Elements along the length</th>
<th>Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Elements along the cross-section</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bias</td>
<td></td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

This ensures that the thickness is distributed toward the outer radius, not toward the center of the circular-arc. This is important when it comes to the

![Figure 4.1: Geometry of one Tape-spring part](image)
4.2 Modeling the parts

Model of the deployable boom

assembly. 5 node sets are created in this part, 4 corresponding to the edges of the tape, and one corresponding the the mid axis in the length direction, Fig. 4.3. The node sets are important for defining boundary conditions and interactions between the different parts.

Material

The analyses were first conducted with an isotropic material model, for simplicity purposes. If one wants to carry deployment analyses, the material model developed in the previous chapters has to be inserted in the model, through the *shell section, composite keyword.

4.2.2 Spool

Geometry

The spool is modeled as an opened cylindrical surface, the depth is 25 mm and the radius 8.25 mm, Fig. 4.4. These values were chosen after developing an approximate material model, which yielded a 8.25 mm coiling radius. The depth of the spool has to be larger than the flattened tape-spring width to avoid contact problems.

Mesh

The cylindrical surface is modeled as a rigid body, with R3D4 elements (Rigid 3 Dimensional 4 nodes). The mesh size is 24 elements along the depth and 50 elements along the circumference. The spool motion is defined by the motion of one node only, the reference node. This node is the center of the cylinder. The 2 node sets lines defined are used for positioning the tape-springs in the assembly. These are called the joint-lines.

4.2.3 Guides

In the first model, conceptual guides were modeled, with fully parameterized geometry and mesh. These guides are not yet implemented in the second model. In Fig. 4.5, the conceptual geometry and mesh of one guide is displayed. It is important to note that the guides are only important for the deployment phase of the analysis.

4.2.4 Assembly

The different parts position and orientation must be defined prior to the analysis. The two tape-spring parts are in symmetric with respect to the center of the spool. The tape springs are adjusted in the y-direction so that the y-coordinate of the cross-section mid nodes is the same than the spool joint-lines nodes y-coordinates. In the x-direction the tape-springs are at
a distance of 10 mm from the center of the spool. In the z-direction the tape-springs are positioned at the center of the spool. As mentioned before, the thickness of the shell is distributed toward the outer direction, so it will not interfere when attaching the tapes to the spool.
4.3 Analysis steps

The analysis is divided in three steps. First the tape-springs have to be attached to the spool. Secondly, a certain length of the tape-springs must be rolled-up around the spool, modeling the packaged boom. In the final step the tape-spring is released, to model the deployment of the boom.

4.3.1 First step: attaching the tape-springs

General description

In order to model the strains the tape-spring ends are subjected to, the attachment of the tape-springs has to be modeled. From the assembly original configuration, the tapes are subject to a 10 mm translational motion in the $x$-direction, so that each tape moves toward the spool. Then a pressure load is applied on the attachment zone of the tapes in order to prepare the assembly for the second step, the coiling of the tapes.

Interactions

This step involves contact deformation of the tape-springs with the spool. The contact pairs defined in ABAQUS are the Refined Zone inner surface of each tape together with the Spool outer surface. There is no need to define contact for the whole tape-springs, as it increases the complexity of the analysis.

Boundary conditions

The spool is constrained in every direction, it is fixed in space. Only one node in each tape is constrained, to keep the system as flexible as possible and to save computational time. In Fig. 4.8 the translational boundary conditions are represented by orange arrows and the rotational boundary conditions by the blue arrows.

Loads

Distributed loads are applied only after the motion of the tapes is finished. A value of 0.5 for the pressure was found suitable, as it gives a good compromise between computational time and flatness of the attachment surface.

Analysis parameters

The analysis is carried with ABAQUS Standard, through a Static step. The large deformation algorithm is used, through the option NLGEOM. The static step is stabilized, as the tape-spring ends show instability after a certain level of deformation, as buckling appears on the edges. The factor
4.3 Analysis steps

Model of the deployable boom

used to stabilize the analysis is 0.0001. This factor can be lowered if one wants greater accuracy, but this severely increases the computational time and does not give noticeable difference in end result. The arc-length control method, theoretically well suited for this type of problem was tried but proved to be too tedious to configure in terms of increment size and total arc-length to give quick results.

Result of the step

The tape-spring end edges slide on the surface of the spool until they become perfectly flat, Fig. 4.9. After pressure is applied, a small distance of the tape spring is flattened and follows the curvature of the spool, Fig. 4.10.
4.3 Analysis steps

Model of the deployable boom

4.3.2 Second step: rolling-up the tape-springs

General description

The first step is relatively simple and no major difficulties were encountered. This step is more complicated as we will see in the following. Once the tapes are attached and in the position of Fig. 4.10, we must roll them up around the spool. The straightforward method is to first link the movement of the attached tapes to the movement of the spool. Then the spool is rotated so that the tape-springs coil around it.

Interactions

The contact definition is modified in this step. Now the whole tape surface is paired with the spool surface, as contact is expected between the two. Contact pairs are also created between the tape surfaces, as during the step the tapes coil onto each other. The tapes motion must also be linked to the spool motion. To do so, two connector elements CONN3D2 (CONNector 3-Dimensional 2 nodes) are created, which connect the reference point of the spool to a node at the middle of the end edge of the tapes. The connector type is BEAM, which simulates a beam connection between the nodes (translation and rotation degrees of freedom are shared).

Boundary conditions

The spool is now subject to rotation around its axis but is still fixed in the other directions. The mid edge of the tape springs defined in Fig. 4.3 is constrained in the transverse direction so that the tape-springs do not slide along the spool length. The end edges of the tape, far from the spool, are constrained so that their cross-section shape is fixed. The rotation of the spool is defined as angular velocity (this is the normal way to do so). The length of coiled tape is approximately defined by:

\[ L_c = R_s \omega t \]  \hspace{1cm} (4.1)

where \( L_c \) is the coiled length, \( R_s \) the radius of the spool and \( t \) the time span of the step, by default equal to 1 in a static step.

Loads

The pressure load defined in the previous step is kept constant in this step.

Analysis parameters

Again the analysis is carried out with ABAQUS Standard, with the same options as the previous step.
4.3 Analysis steps  Model of the deployable boom

Result
The result is not satisfying, as ABAQUS stops the analysis before it completes, Fig. 4.12. The error is due to contact formulations.

Difficulties encountered
The contact algorithm used in ABAQUS Standard is well suited for small deformations and well separated surfaces. In our case, contact surfaces that are well separated at the beginning of the analysis (the tape surfaces) become close to each other when the spool rotates half a turn. Then the contact algorithm does not manage to converge, because it considers the contact surfaces as over-closed, due to the fact that the thickness of the tape springs, which separates contact pairs from each other is too small compared to the general dimensions of the tape-spring element. The problem is best described in Fig. 4.13.

Possible solutions
The first possible solution is to redefine the contact pairs, so that there will not be any overclosed surfaces when the tapes roll-up. To do so, the tapes must be divided in several surfaces each with a length approximately equal to half the spool circumference, so that each contact pair does not undergo overclosure. This solution works but is very tricky and tedious to implement, as the exact position of the surfaces must be determined in advance.

The second possible solution is to get rid of contacts between the tapes and run the deployment analysis from a configuration where the tape-springs are coiled at the same radius. However we saw that the stored strain energy varies with the radius of coiling, so this solution will not be accurate for a deployment analysis. However it can give a lower bound for the deployment force and speed.

The third solution and the one tried with success is to run the coiling in a dynamic explicit step. The Explicit version of ABAQUS is suited for highly dynamic or very complex contact problems (solved with the General Contact algorithm, only available in Explicit). The problem is that it is difficult to configure ABAQUS Explicit in order to get acceptable results, and I did not have any knowledge of this type of analysis. However the first analyses run with Explicit showed that the contact problem is solved, and the tapes coil perfectly on each other, though a small traction force at the end of the tape-springs is needed to maintain the tape-springs in contact. In Figure 4.14, we can see that the thickness of the tapes is taken into account in the coiled state.
4.4 Future work

Model of the deployable boom

The strains and stresses obtained are not developed here, as the model was tested with an isotropic material model, which is completely unrelated to the strains one can obtain with the plain-weave material model.

4.3.3 Third step: deployment of the boom

General description

For the third step to give useful results, the second step must first be as accurate in its results as possible. This was not the case here, hence the following paragraphs intention is to draw the main lines for the deployment analysis, as no result was obtained.

The deployment of the boom is likely to be analyzed in ABAQUS Explicit due to the complex contact problems involved. However if one decides to not define contacts between the tapes, but only between the tapes and spool, the Standard dynamic version can be used.

The guiding of the deployable booms can be performed either with boundary conditions, most likely constraining the end of the tape-springs to a translational motion, or one can model rigid guides similar to Fig. 4.5. There is no need to define contact pairs, as the Explicit General Contact algorithm considers any surface of the model as a contact surface.

4.4 Future work

There is a lot of work to do on the model.

A good material model must be adopted for the tape-springs. The main lines of such a model are developed in Chapter 2 and 3.

If computational power is made available, the micromechanical model presented by Soykasap [18] should be tried.

An analysis which does not consider contact between the tape-springs for simplification should be carried with ABAQUS Standard, for coiling and deployment steps. This model would give a lower bound for the deployment force and speed.

A refined analysis in Explicit should be carried. This requires a good knowledge of the software in order to configure the analysis. Parameters like Mass Scaling can be used to modify the stability of the analyses. Note however that this parameter should not be used for realistic dynamic analyses (deployment of the tape-springs). However it can be safely used for coiling the
tape-springs around the spool.

Obtaining an accurate model of the whole process of attaching the tapes, rolling them up and deploying them is possible, but it is a real challenge, and undoubtedly needs expert FEM skills and a lot of time.
4.4 Future work

Model of the deployable boom

Figure 4.2: Mesh of one tape-spring part.

Figure 4.3: Three edges and mid axis node sets of one tape-spring part.
4.4 Future work

Model of the deployable boom

Figure 4.4: Mesh and geometry of the spool

Figure 4.5: Mesh and geometry of one guide
4.4 Future work

Model of the deployable boom

Figure 4.6: Assembly: $xy$ plane

Figure 4.7: Assembly: $yz$ plane
4.4 Future work

Model of the deployable boom

Figure 4.8: Boundary conditions for the first step

Figure 4.9: Result before pressure is applied
4.4 Future work

Model of the deployable boom

Figure 4.10: Result after applying pressure

Figure 4.11: The connector elements in blue

Figure 4.12: The analysis encounters an error after this point
4.4 Future work

Model of the deployable boom

Figure 4.13: The contact problem during this step

Figure 4.14: The rolled-up tape-springs, using a dynamic Explicit step
Chapter 5

Conclusions

The model captures well the main features of the deployable boom material. The in-plane properties can be accurately modeled after a fine tuning of the model parameters. The out-of-plane properties are however more problematic and cannot be well predicted without modifying the in-plane properties at the same time.

Therefore the model can be declined in two versions, one modeling the in-plane properties accurately, for the fully deployed boom in modal or thermal analyses. The other model would focus on predicting the bending properties of the material and energy levels accurately, as bending is the main deformation mode present during the deployment of the tape-spring.

The higher the flexural modulus, the more stable the second configuration (rolled-up) and the more compact (smaller radius of coiling). However the strain energy stored is also higher. This leads to higher deployment force and speed, which might prove difficult to control.

If in the model, tweaking the flexural modulus does not yield enough accuracy on the deployment speed and force, then a different model needs to be developed. The micromechanical model developed by Soykasap, consisting of interlaced beam elements representing the fiber strands would be an interesting path to explore.
Appendices
Appendix A

Elastic properties test protocol in ABAQUS

The analyses are carried out with ABAQUS/CAE 6.9-1. Two different test parts were modeled. The first one (Part 1) is a rectangular plate from which the tensile modulus, the flexural modulus and the Poisson ratio are calculated. The second part (Part 2) is a square plate from which the in-plane shear modulus is calculated.

Plate 1 dimensions are $500 \times 25 \times 0.24$ mm$^3$. Plate 2 dimensions are $25 \times 25 \times 0.24$ mm$^3$.

Tensile tests

The tensile test results give the tensile modulus and the Poisson ratio of the material. The plate is constrained as in Fig. A.1, with a shell edge load of 0.001 N per unit area. The results extracted is first the longitudinal displacement $U_3$ of the loaded edge, which is averaged between the edge nodes. The tensile modulus can then be calculated:

$$E_x = \frac{\sigma_x}{\epsilon_x} = 0.001 \frac{hL}{U_3}$$

(A.1)

where $h$ is the thickness (0.24 mm) and $L$ the length of the plate (500 mm).

The second result extracted is the average $\epsilon_y$, by averaging the difference of transversal displacement $\Delta U_1$ of the two long edges nodes along the length. The Poisson ratio can then be calculated:

$$\nu_{xy} = - \frac{\epsilon_y}{\epsilon_x} = - \frac{\Delta U_1 hL}{U_3 w}$$

(A.2)

where $w$ is the width of the plate (25 mm).
Bending tests

Plate 1 is loaded by two equal and opposite shell edge moments of 0.001 Nm per unit area, Fig. A.2. The maximum z-displacement U2 at the center of the plate is measured. The flexural modulus $E_h$ can then be calculated:

$$E_h = 0.001 \frac{wL^2}{8U2} \frac{12}{wh^3}$$  \hspace{1cm} (A.3)

Shear tests

For shear tests, Plate 2 is used, Fig A.3. The plate is loaded in shear through a shell edge load. The result extracted is the average shear stress SSAVG3 and the average shear strain SE3 for the nodes on a path passing through the middle of the plate, Fig. A.4. The shear modulus is then calculated by:

$$G_{xy} = \frac{SSAVG3}{SE3}$$  \hspace{1cm} (A.4)
Elastic properties test protocol in ABAQUS

Figure A.1: Boundary conditions and loads for the tensile test.

Figure A.2: Boundary conditions and loads for the bending test.
Figure A.3: Boundary conditions and loads for the shear test.

Figure A.4: The path where the nodal results are extracted.
Appendix B

Approximated Naik material model

Composite layup parameters

This model was the first developed, Fig. B.1, and takes the fiber volume ratio into account through the thickness scalar $k_{th}$, not through the laminae elastic properties. The in-plane results are remarkably close to the data sheet.

Table B.1: Layup parameters

<table>
<thead>
<tr>
<th>Layup dimensions (mm)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness</td>
<td>0.24</td>
</tr>
<tr>
<td>$k_{th}$</td>
<td>0.4</td>
</tr>
<tr>
<td>Surface matrix ply thickness</td>
<td>0.0139</td>
</tr>
<tr>
<td>Core matrix ply thickness</td>
<td>0.1161</td>
</tr>
<tr>
<td>Fiber ply thickness</td>
<td>0.024</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fiber layer properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$</td>
<td>230.0 GPa</td>
</tr>
<tr>
<td>$E_2$</td>
<td>10.42 GPa</td>
</tr>
<tr>
<td>$G_{12}, G_{13}, G_{23}$</td>
<td>3.86 GPa</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Matrix layer properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>10.42 GPa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

properties for the T300/E765 fabric. However the out-of-plane properties do not have any accuracy.
Table B.2: Model elastic properties

<table>
<thead>
<tr>
<th>Properties at 0°</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_x, E_y$</td>
<td>54.62 GPa</td>
</tr>
<tr>
<td>$G_{xy}$</td>
<td>3.86 GPa</td>
</tr>
<tr>
<td>$\nu_{xy}$</td>
<td>0.058</td>
</tr>
<tr>
<td>$E_b$</td>
<td>72.98 GPa</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Properties at 45°</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_x, E_y$</td>
<td>13.62 GPa</td>
</tr>
<tr>
<td>$G_{xy}$</td>
<td>25.61 GPa</td>
</tr>
<tr>
<td>$\nu_{xy}$</td>
<td>0.765</td>
</tr>
<tr>
<td>$E_b$</td>
<td>14.14 GPa</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bi-stability</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_c$</td>
<td>8.29 mm</td>
</tr>
</tbody>
</table>

Figure B.1: The approximated Naik model layup
Appendix C

Bi-stability test protocol

Part model

<table>
<thead>
<tr>
<th>Table C.1: Test part definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions</td>
</tr>
<tr>
<td>Radius</td>
</tr>
<tr>
<td>Subtended angle</td>
</tr>
<tr>
<td>Depth</td>
</tr>
<tr>
<td>Mesh</td>
</tr>
<tr>
<td>Elements along the arc</td>
</tr>
<tr>
<td>Elements along the length</td>
</tr>
</tbody>
</table>

The part used for the bi-stability tests is one half of a cylinder, Fig. C.1, loaded so that it adopts a different geometrical configuration. The part is then unloaded. The evolution of the geometry, strain energy stored and load applied is recorded.

Boundary conditions

The center node of the part is constrained in its 6 degrees of freedom. Actually any node could have been chosen, as the loads are balanced. However, it turned out that constraining this node was not sufficient. The results obtained with ABAQUS displayed a singularity in strain and stresses at this node. While deforming, the whole part was rigidly rotating around the y-axis except at this node. The solution to this problem is to avoid rigid body rotation of the part through a second boundary condition on the neighbouring nodes. The two nodes close to the center node have their z-translation constrained.
Monitoring

The response of the part is monitored by the $y$-displacement of a node at one corner edge of the part.

Loads

The part is loaded by two counter-acting shell edge moments, on its straight edges, with the follower option. The purpose of loading the part this way is to bring it close to a flat configuration and trigger the change in geometric configuration. Another possibility is to apply counter-acting moments on the curved edges. The value of the moments to apply is determined by the response of the part. First a small value is applied, 2 N. The response of the part to this load value is then monitored, and according to this response the load is either increased or reduced.

Fig. C.3 shows a typical case of a too low force, as the switch of configuration did not occur during the analysis. This plot can be compared to Figs. 3.2 and 3.4.

Steps

There are three steps. The first step consists in ramping the loads until the value defined. This is done through a static analysis, stabilized with a factor of $10^{-6}$. In the second step, the moments are ramped down until zero. This step is also stabilized by a factor of $10^{-6}$. The last step is a static step. No loads are applied on the shell, and the step is not stabilized. It serves as a verification that the structure is stable.

Extracting results

The strain energy stored is recorded in the variable ALLSE in ABAQUS. In order to obtain the coiling radius, a node path is created (Figure 3.6), and the coordinates of the node on this path are input in a MATLAB circle fitting script (Appendix D). The result of the script is the best fit circle radius.
Bi-stability test protocol

Figure C.1: The mesh of the test part

Figure C.2: Boundary conditions and loads applied to the part
Figure C.3: A typical case of a too low load value
Appendix D

Circle Fitting script

function [xc,yc,R,a] = circfit(x,y)

% [xc yx R] = circfit(x,y)
% fits a circle in x,y plane in a more accurate
% (less prone to ill condition )
% procedure than circfit2 but using more memory
% x,y are column vector where (x(i),y(i)) is a measured point
% result is center point (yc,xc) and radius R
% an optional output is the vector of coefficient a
% describing the circle’s equation
% x^2+y^2+a(1)x+a(2)y+a(3)=0
% By: Izhak bucher 25/oct /1991,
% x=x(:); y=y(:);
a=[x y ones(size(x))]\[-(x.^2+y.^2)];
xc = -.5*a(1);
yc = -.5*a(2);
R = sqrt((a(1)^2+a(2)^2)/4-a(3));
Bibliography


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