Analyzing an Equivalent Single Layer Shim Model to be used for Brake Squeal Reduction

Växjö May, 2011
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Title: Analyzing an Equivalent Single Layer Shim Model to be used for Brake Squeal Reduction.

Subtitle: Investigation of FEA implementation of shims modelled by single element layer approach by use of 5 parameter model.

Key Words
Multilayer shim, equivalent single layer (ESL), Finite element analysis, Abaqus/Cae, Squeal Noise, Complex Eigenvalue

Abstract
The goal in this thesis was to reduce a multilayer shim model, which was modeled from steel and polymer (isotropic materials), into an equivalent single layer shim model. The procedure was to use mathematical formulations to convert a multilayer shim into an ESL (equivalent single layer) shim. Here, a transverse isotropic model is used to prepare for future orthotropic layers. The results show that the ESL model behaves isotropically. In the 2 layer model there was no squeal noise whereas in the ESL models there is.
Acknowledgement

Thanks to Andreas Linderholt, our supervisor at Linnaeus University, for his confidence and belief in us, for his advices and for his assistance from the first day in our thesis work.

Thanks to Alwin Stikvoort, FEA & NVH research engineer at Trelleborg Automotive AB, for his support in the thesis work and special help with software Abaqus/Cae.

Last and not least, thanks to our families for their support along this thesis work and in our life.

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Table of contents

Abstract ........................................................................................................ II
Acknowledgement ........................................................................................ IV
Table of contents ......................................................................................... V

1. Introduction ........................................................................................... 1
  1.1 Background .......................................................................................... 1
  1.2 Types of Brake System ........................................................................ 2
    1.2.1 Drum Brake System ...................................................................... 2
    1.2.2 Disc Brake System ....................................................................... 3
  1.3 Purpose .................................................................................................. 5
  1.4 Hypothesis and Limitations .................................................................. 6
  1.5 Reliability, Validity and Objectivity .................................................... 6

2. Theory ...................................................................................................... 6

3. Methods .................................................................................................... 7

4. Analysis ..................................................................................................... 8
  4.1 Complex Eigenmode Solution ............................................................... 8
  4.2 Multilayer Shim Model ........................................................................ 9
  4.3 Equivalent Single Layer Shim Model .................................................. 10

5. Results ...................................................................................................... 11
  5.1 Analyzing of Multilayer Shim Model by Using Abaqus CAE11 .......... 11
  5.2 Calculating Parameters of ESL Shim Model ....................................... 13
  5.3 Analyzing the ESL Shim Model by Using Abaqus/Cae 6.10-1 .......... 14

6. Analysis of the Results ........................................................................... 15

7. Discussion .................................................................................................. 15

8. Conclusions ............................................................................................. 15

9. References ................................................................................................ 16

10. Appendix .................................................................................................. 17
    Matlab Calculations .................................................................................. 17
    Results ....................................................................................................... 19
    B. Mathematica Program ......................................................................... 24
    Results ....................................................................................................... 25
1. Introduction

In automobile industry, the brake system is one of the most important parts and a lot of improvements have been achieved in brake systems technology. In the last decades, to increase performance and efficiency, brake systems are changed from drum brake systems to disc brake systems. Brake squeal, which usually occurs in the frequency range between 1 and 16 kHz, has been one of the most difficult concerns associated with vehicle brake systems [1]. The challenge is to reduce the squeal noise in the disc brake system; this is often made by using shims. A typical brake shim or insulator consists of a multilayer of steel and polymer (visco-elastic) and this sandwich shape layout is designed for decreasing the squeal.

1.1 Background

As a historical background after mid-1890s, disc brake systems have begun developing. In 1902 the British Engineer F. William Lanchester (1868–1946) patented a disc brake which consists of a disc of sheet metal which is connected rigidly to one of the rear wheels of the vehicle, and it is used for slowing down the vehicle, the disc is pinched at its edge with a pair of jaws [2]. Also an American inventor E.A.Sperry (1860–1930) worked on a disc in which a disc magnet is placed in contact with another disc making a couple. This couple produce a torque due to friction between the discs. As a result it reduces the speed of the vehicle.

Sperry’s design is a clutch type brake system which is now used in automobile industry whereas Lanchester’s is a spot type disc brake system.

During the Second World War, and also in the twentieth century Lanchester’s and Sperry’s designs are developed and modified not only regarding the actuation method, but also regarding the material of the pads etc.
In vehicles, brakes are one of the most important components for safety, reliability, performance and quality. Development in brake systems firstly focussed on performance, and efficiency. However aesthetic design parameters, customers comfort, and vehicle acoustic became more important in recent most years.

For customers brake noise is irritant and a big part of customers also believe that the sound is related to low performance and/or bad quality. Thus, reducing noise became important in vehicle manufacturing and design. As noted in Abendroth and Wernitz paper [3], many pad material manufacturer use 50% of their engineering budgets on harshness, vibration and noise problems.

1.2 Types of Brake System

There are several types of brake systems which depend on their applications. Two general types of brake systems are disc brake and drum brake.

1.2.1 Drum Brake System

For a drum brake, shoes work against the rotating surface, which is called a Drum. Drum brakes are made of special type of cast iron, and they are usually used on the rear wheels. A drum brake has more parts than a disc brake and it is harder to service, but it has low cost in manufacturing.

![Figure 1: Automotive-type Drum Brake](image)
1.2.2 Disc Brake System

Disk brakes are more common in modern vehicles because of its advantages like;

- It dissipates heat better than a drum brake,
- It is more efficient,
- It has lower squeal noise,
- It is better in wet weather conditions, and
- It takes longer time to loss its effectiveness.

The mechanism of a disc brake system works by applying hydraulic pressure to a caliper piston; by increasing the pressure, the caliper moves and causes the outside pads to contact the disc. The generated braking force is created by the friction between the pads and the disc rotor. More pressure causes more friction due to compression against a rotating surface, and more heat is generated due to friction.

Figure.2 Disc Brake System [4]
1.2.2.1 Components of Disc Brake System

A disc brake system includes three major components:

a) **Disc rotor** rotates with the wheel. It is manufactured from cast-iron.

b) **Caliper assembly** attached to the steering knuckle.

c) **Brake pads** (friction materials) which are connected to the caliper assembly.

d) **Brake Shims** that are connected to the back side of the pad.

a) Disc Rotor

The disc rotor is generally made of gray cast iron, and there are two types of rotors which are known as ventilated rotor and solid rotor. The ventilated type has a wider disc with fins cast in the middle which ensures good cooling. The shape of the fins can be different, i.e. spiral fins provide better cooling, and allow more air flow, also they prevent fading which means a longer pad life. On the front of the new models of vehicles ventilated rotors are used.

The second type of rotor is called a solid rotor. Solid rotors were used on the front wheels in earlier vehicles and they are used on the rear wheels of brake system in new vehicles. There are no fins in the middle of a solid rotor.

b) Caliper

The second part in the disc brake system is the caliper, which is mounted to the torque plate and houses one to four pistons. There are two different types of caliper; floating caliper and fixed caliper.

Generally the floating caliper is more common than the fixed caliper because it has more advantages; it is more economical, lighter in weight, and requires only a few parts to connect. The floating caliper needs one or two pistons, depending on the application.
c) Brake Pads

The type of pad depends on the using application of the brake and therefore, different kinds of friction material are needed. Several parameters should be taken into consideration in pad design, i.e. it should have a constant coefficient of friction over a wide range of temperature, it should not wear out rapidly, it should not wear the rotor and it should carry the highest temperatures without fading. The whole system (pads together with shims) should be able to do all this without generating any squeal noise.

d) Brake Shims

Brake systems vibrate which can cause squeal noise. To prevent this squeal noise, brake shims are used. The brake shims are placed on the backside of the disc brakes thereby emitting the vibration and preventing vibration to transform to other parts of the automobile. The materials used for brake shims are steel and polymer (visco-elastic) layers stuck together by some adhesives. There is disadvantage of having a visco-elastic material in a brake shim, because visco-elastic material properties can undergo a transition as the temperature increases. The transition starts at a specific temperature which is called the glass transition temperature, \( T_g \), and the modulus of the material starts to decrease above \( T_g \).

In the middle of the transition region at a reference temperature \( T_0 \) the loss in the modulus of the material reaches its peak point [5].

1.3 Purpose

The purpose of this thesis work is to establish an equivalent single layer (ESL) model of a shim instead of a multilayer model. An ESL model will reduce the steps of calculations and it will also simplify squeal noise analyses of brake system.
1.4 Hypothesis and Limitations

The goal here is to achieve a better relation between test and simulation using an ESL model; that is a multilayer insulator model simplified to an ESL model. This will lead to a better understanding of the system. Some assumptions are needed. Here the assumptions are that the ESL acts like a thin plate of homogeneous material, that the mid surface is the neutral surface, that a line straight and normal to the mid surface remains straight under deformation and that the shear deformation is accounted for to the extent of capturing the dynamic modes of the pad assembly.

1.5 Reliability, Validity and Objectivity

Preventing brake squeal noise is a difficult process; some authors assume that the system is linear. Difficulty is to cover the non linear behavior in the linear. In this paper, instead of assuming the system as a whole linear system we will take the problem into two steps. The first is a nonlinear step in which the load is applied and the second step is a linear perturbation extracting the complex eigenmodes. To make the calculations easier an ESL will be used.

2. Theory

By using finite element analysis, different shim designs are included in the disc brake assemblies to find the effect of brake squeal.

Using an ESL model instead of multilayer model for the pad assembly is required due to time and physical clearness. “For that conversion a transversely isotropic material is used in our search, because it allows a more number of parameters to be used in the optimization” [5].
A transversely isotropic material model which is given for the equivalent single layer can be defined with an elastic coefficient matrix $[C_{ij}]$.

(Found by the stress strain relationship: $\{\sigma_i\} = [C_{ij}].\{\epsilon_j\}$) [5]

Independent Coefficients of $[C_{ij}]$

\[
\begin{align*}
C_{11} &= E_1 \left(1 - \nu_{23}^2\right) / \Delta \\
C_{33} &= E_3 \left(1 - \nu_{12}^2\right) / \Delta \\
C_{12} &= E_1 \left(\nu_{12} + \nu_{23}^2\right) / \Delta \\
C_{13} &= E_1 \left(\nu_{23} + \nu_{12} \cdot \nu_{23}\right) / \Delta \\
C_{55} &= \text{Shear Modulus } G_{23} \text{ or } G_{13}
\end{align*}
\]

where $E$ refers to the Young Modulus, $\nu$ refers to the Poisson’s ratio, and,

\[
\Delta = 1 - \nu_{12}^2 - 2 \cdot \nu_{23}^2 - 2 \nu_{12} \cdot \nu_{23}^2
\]

3. Methods

Our study is to reduce squeal noise depends on two working areas;

a) Finite element analysis (Abaqus/ Cae).

b) An experimental method.

a) Finite element analysis (Abaqus/ Cae)

Finite element analysis is an approximate method to solve complicated problems which are not solved by numerical classical method. By this method the material is divided to finite elements which are interconnected at points called nodes. Each element’s behavior is studied and determined and through the nodes we connect between elements and make an approximation about the entire region of the material [6]. The software (ABAQUS/CAE) is used in visualizing the finite element analysis result that we get from our calculations. In
Abaqus software we create a file about the shim (insulator) that we study, and upload all results. After uploading all results Abaqus models the shim and gives us a whole visualization about the shim.

**b) An experimental method**

The brake shim that is used in our study will be tested at Trelleborg Automotive AB, in automobile brake system.

### 4. Analysis

The squeal phenomenon is studied by measurements and numerical analyses. The analyses are based on complex eigenmode solution of the complete brake system.

#### 4.1 Complex Eigenmode Solution

A complex eigenvalue analysis will be used, because that allows all unstable eigenvalues to be found in one simulation [7]. By applying forces to the brakes, the eigenmodes of the system are excited. The subspace projection method will be used to solve the complex eigenproblem. Thus, in order to determine the projection subspace a natural frequency extraction must be performed first. The equation of motion of the multiple degree of freedom system is:

\[
[M] \ddot{X} + [C] \dot{X} + [K] X = \{f\} \tag{1}
\]

where \(M\) is the mass matrix, \(C\) is the damping matrix, which includes friction-induced contributions, and \(K\) is the stiffness matrix, which is unsymmetric due to friction [1].
In order to determine natural modes of the system we consider the case where there is no excitation and assume an equation which has the complex form:

\[ \{x\} = \Phi e^{i\mu t} \]  

(2)

where \( \mu \) is the eigenvalue, \( \Phi \) is the corresponding eigenvector and \( t \) is time.

By deriving equation (2), \( \dot{X} \) and \( \ddot{X} \) can be obtained. By substituting \( \dot{X} \) and \( \ddot{X} \) into equation (1) a new equation of motion for free vibration of multidegree of freedom system will be:

\[ (-\mu^2 M + i\mu C + K) \Phi = 0 \]  

(3)

4.2 Multilayer Shim Model

A shim consists of many layers which are stuck together by adhesives like glue. In Figure 2, the layout of a typical brake shim is shown. It consists of many layers; alternate steel and polymer (visco-elastic) bonded to the pad backing plate by using an adhesive layer.

Figure.3 A Typical Shim Construction [5]
The steel layers thickness is of the order of hundreds of micrometers and the thickness of the core (polymer layer/visco-elastic adhesive) is of the order of ten micrometers. Depending on the desired performance, the outer layers can be thicker. The thickness of the whole insulator does usually not exceed 1.2 mm [5].

4.3 Equivalent Single Layer Shim Model

In general applications multilayer shim models are currently used in dynamic systems. In such a model, problem arises from having temperature and frequency dependences for each layer. A multilayer sandwich shape makes the problem more complex, because more calculations have to be taken into consideration which are also more expensive and time consuming, therefore, in our case we will use an equivalent single layer (ESL) to simplify the model in stability analysis. Figure 3 shows the reduction of the multilayer model into the ESL model [5].

![Figure 4 Multilayer to Equivalent Single Layer](image)
5. Results

5.1 Analyzing of Multilayer Shim Model by Using Abaqus CAE

The first step in analysis of the brake shim was modeling the multilayer shim by using Abaqus CAE 6.10-1. The model which was used has three layers. It consists of a glue layer between two steel layers and both the glue and the steel are isotropic materials.

First steel layer has a length of 200 mm, a width of 60 mm and a thickness of 0.5 mm. The glue between the layers has a length of 180 mm, a width of 60 mm and a thickness of 0.12 mm. The upper part of the model is steel and it has a length of 180 mm, a width of 60 mm and a thickness of 0.5 mm.

<table>
<thead>
<tr>
<th></th>
<th>Modulus of Elasticity (E, Mpa)</th>
<th>Poisson’s Ratio (v)</th>
<th>Density (ρ, kg/mm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>STEEL</td>
<td>206000</td>
<td>0.3</td>
<td>7.849E-9</td>
</tr>
<tr>
<td>GLUE</td>
<td>20</td>
<td>0.4975</td>
<td>1.0E-9</td>
</tr>
</tbody>
</table>

Table 1 Mechanical Properties of Multilayer Shim Materials

Our aim was to understand the behavior of multilayer shim model by using Abaqus Cae simulation program. The eigenmodes and frequencies were observed in visualization part of the program.
After simulating the three layers model we reduce it to two layers by omitting the first steel layer which has a length of 200 mm. Dynamic analyses was used to get the eigenmodes and frequencies of the two layers shim model. By running the model the second bending mode was observed. The results of two layers model was used to compare with the results of equivalent single layer model.

5.2 Calculating Parameters of ESL Shim Model

The idea is to reduce a two layers shim which is modelled by using isotropic materials into an equivalent single layer by using a transversely isotropic material. The ESL model will have the same mechanical properties as two layers shim.

The two layers model was converted to an equivalent single layer by using formulations given below:
Here \( S \) is the material flexibility matrix depending on the material properties, i.e. Young's Modulus (E), Poisson's Ratio (\( v \)) and Shear Modulus (G), \( \sigma \) is stress and \( \varepsilon \) is the strain [8]. For our calculations Matlab 2010 is used and the codes are given in appendix.

The reducing will be done by finding \( S \) matrix from the formulation which is using for isotropic material:

\[
S = \begin{bmatrix}
\frac{1}{E} & -\frac{V}{E} & -\frac{V}{E} & 0 & 0 & 0 \\
-\frac{V}{E} & \frac{1}{E} & -\frac{V}{E} & 0 & 0 & 0 \\
-\frac{V}{E} & -\frac{V}{E} & \frac{1}{E} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G}
\end{bmatrix}
\]

After that the stiffness matrix \( Q \) was calculated as the inverse of the \( S \) matrix.

\[
Q = S^{-1} ; \quad S = Q^{-1}
\]

Some constants for the multilayer shim model are calculated which is expected to be same with the ESL shim model. These parameters are \( A, B \) and \( D \) which are given below:

\[
A = \sum_k Q_k h_k \\
B = \sum_k Q_k h_k \bar{\sigma}_k \\
D = \sum_k Q_k h_k \left( \sigma^2_k + \frac{h_k^2}{12} \right)
\]
$h_k$ is the thickness of the materials and $z_k$ is the distance between the neutral plane of the multilayer model and the mass center of the materials [8].

The neutral plane of the multilayer shim model is assumed in the middle of the whole thickness in $z$ direction.

After finding $A, B$ and $D$ values for multilayer shim model calculated two stiffness matrices $Q$ for $A$ and $D$ constants and $B$ value is zero, because of overlapping the neutral plane with the mass center of the model. The value of $z_k$ for the ESL will be zero since they are both in the middle of the whole thickness.

For the ESL shim model by taking the inverse of $Q$ matrices we have two different $S$ matrices for $A$ and $D$ constants.

The results show that the new material which is expected to be transversely isotropic is actually an isotropic material.

Since the parameters in the $S$ matrix (Young’s Modulus, Poisson’s Ratio and Shear Modulus) are symmetric the new material behaves like an isotropic material.

### 5.3 Analyzing the ESL Shim Model by Using Abaqus/Cae 6.10-1

The next step is to model and run an equivalent single layer shim by using Abaqus CAE 6.10-1. The results of $S$ matrices for both $A$ and $D$ values are compared with the multilayer model’s result statically and dynamically. The results which are close to the two layers shim values are chosen.

The last step is to run the whole brake system with the new shim (an equivalent single layer) by Abaqus Cae with specific condition of friction and angular frequency.

### 6. Analysis of the Results

After running the three models (two layers shim model, ESL_A shim model and ESL_D shim model) by Abaqus/Cae, the results
(natural frequencies) showed that the ESL_D shim model has close results to the two layers shim model comparing with the ESL_A shim model.

As shown in the table (appendix, Table-2) the next step was to run the three models separately with the whole brake system to check the squeal noise phenomenon.

In the two layers shim model there was no squeal noise as shown in the real part of the complex eigenvalues, because parameters were either negative (no squeal noise) or positive but so small which means that they could be neglectable.

Results of the ESL_A and ESL_D shim models showed that there is squeal noise in one mode of each model. In ESL_A model, the real part of complex eigenmode number 17 has zero frequency. In ESL_D model, the real part of complex eigenmode mode number 47 has also zero frequency.

7. Discussion

The results show that there is a squeal noise in the ESL shim models (A and D) comparing with the two layers shim model, but it does not mean that the ESL shim models are not satisfied, because the analysis and calculations of both ESL shim models capture just between 6-10% of the whole project.

8. Conclusions

This thesis work has come up with a conclusion that the ESL shim model can be used in automobiles in the future instead of multilayer shim model if the other calculations of ESL shim model; shear deformation, tension analysis and the other details are same or close to two layers shim model.
9. References

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10. Appendix

Matlab Calculations
Matlab has used to convert the two layers shim model in to ESL shim model.

% Isotropic part properties, steel 6x6 matrix

E=2.06E11;
poi=0.3;
G=E/(2*(1+poi));
h_steel=0.5E-3;
z_steel=-0.06E-3;
S_steel=[1/E -poi/E -poi/E 0 0 0;-poi/E 1/E -poi/E 0 0 0;-poi/E -poi/E 1/E 0 0 0;0 0 0 1/G 0 0;0 0 0 0 1/G 0;0 0 0 0 0 1/G];
Q_steel=inv(S_steel);

% Isotropic part properties, glue 6x6 matrix
E=20E6;
poi=0.4975;
G=E/(2*(1+poi));

h_glue=0.12E-3;
z_glue=0.25E-3;
S_glue=[1/E -poi/E -poi/E 0 0 0;-poi/E 1/E -poi/E 0 0 0;-poi/E -poi/E 1/E 0 0 0;0 0 0 1/G 0 0;0 0 0 0 1/G 0;0 0 0 0 0 1/G];
Q_glue=inv(S_glue);

% calculating matrices A, B and D for isotropic materials
A_ML=Q_glue*h_glue + Q_steel*h_steel
B_ML=Q_glue*h_glue* z_glue + Q_steel*h_steel*z_steel
D_ML=Q_glue*h_glue*(z_glue^2+((h_glue^2)/12))+Q_steel*h_steel*(z_steel^2+((h_steel^2)/12))

% ESL part properties
h_ESL=0.62E-3;

%calculating matrices A, B and D for an ESL material
Q_ESL_A=A_ML/h_ESL
S_ESL_A=inv(Q_ESL_A)
Q_ESL_D=12*D_ML/(h_ESL)^3
S_ESL_D=inv(Q_ESL_D)

%Calculation of parameters for ESL
Et_ESL_A=1/S_ESL_A(1,1)
Et_ESL_D=1/S_ESL_D(1,1)
$$V_{t\_ESL\_A} = -E_{t\_ESL\_A} \cdot (S_{ESL\_A}(1,2))$$
$$V_{t\_ESL\_D} = -E_{t\_ESL\_D} \cdot (S_{ESL\_D}(1,2))$$

$$E_{z\_ESL\_A} = 1 / S_{ESL\_A}(3,3)$$
$$E_{z\_ESL\_D} = 1 / S_{ESL\_D}(3,3)$$

$$V_{zx\_ESL\_A} = -E_{z\_ESL\_A} \cdot (S_{ESL\_A}(1,3))$$
$$V_{zx\_ESL\_D} = -E_{z\_ESL\_D} \cdot (S_{ESL\_D}(1,3))$$

$$V_{xz\_ESL\_A} = -E_{t\_ESL\_A} \cdot (S_{ESL\_A}(3,1))$$
$$V_{xz\_ESL\_D} = -E_{t\_ESL\_D} \cdot (S_{ESL\_D}(3,1))$$

$$G_{yz\_ESL\_A} = 1 / S_{ESL\_A}(4,4)$$
$$G_{yz\_ESL\_D} = 1 / S_{ESL\_D}(4,4)$$

$$G_{t\_ESL\_A} = 1 / S_{ESL\_A}(6,6)$$
$$G_{t\_ESL\_D} = 1 / S_{ESL\_D}(6,6)$$

**Results**

$$A_{ML} =$$

```
1.0e+008 *

1.3881 0.5958 0.5958 0 0 0
0.5958 1.3881 0.5958 0 0 0
0.5958 0.5958 1.3881 0 0 0
0 0 0 0.3962 0 0
0 0 0 0 0.3962 0
0 0 0 0 0 0.3962
```
B_ML =

1.0e+003 *

-8.2790  -3.5255  -3.5255  0  0  0
-3.5255  -8.2790  -3.5255  0  0  0
-3.5255  -3.5255  -8.2790  0  0  0
0  0  0  -2.3767  0  0
0  0  0  0  -2.3767  0
0  0  0  0  0  -2.3767

D_ML =

3.3980  1.4621  1.4621  0  0  0
1.4621  3.3980  1.4621  0  0  0
1.4621  1.4621  3.3980  0  0  0
0  0  0  0.9680  0  0
0  0  0  0  0.9680  0
0  0  0  0  0  0.9680

Q_ESL_A =

1.0e+011 *

2.2390  0.9610  0.9610  0  0  0
0.9610  2.2390  0.9610  0  0  0
0.9610  0.9610  2.2390  0  0  0
0  0  0  0.6390  0  0
0  0  0  0  0.6390  0
0  0  0  0  0  0.6390

S_ESL_A =
1.0e-010 *

\[
\begin{align*}
0.0602 & & -0.0181 & & -0.0181 & & 0 & & 0 & & 0 \\
-0.0181 & & 0.0602 & & -0.0181 & & 0 & & 0 & & 0 \\
-0.0181 & & -0.0181 & & 0.0602 & & 0 & & 0 & & 0 \\
& & 0 & & 0 & & 0.1565 & & 0 & & 0 \\
& & 0 & & 0 & & 0 & & 0.1565 & & 0 \\
& & 0 & & 0 & & 0 & & 0 & & 0.1565 \\
\end{align*}
\]

\[Q_{ESL\_D} =\]

1.0e+011 *

\[
\begin{align*}
1.7109 & & 0.7362 & & 0.7362 & & 0 & & 0 & & 0 \\
0.7362 & & 1.7109 & & 0.7362 & & 0 & & 0 & & 0 \\
0.7362 & & 0.7362 & & 1.7109 & & 0 & & 0 & & 0 \\
& & 0 & & 0 & & 0.4874 & & 0 & & 0 \\
& & 0 & & 0 & & 0 & & 0.4874 & & 0 \\
& & 0 & & 0 & & 0 & & 0 & & 0.4874 \\
\end{align*}
\]

\[S_{ESL\_D} =\]

1.0e-010 *

\[
\begin{align*}
0.0789 & & -0.0237 & & -0.0237 & & 0 & & 0 & & 0 \\
-0.0237 & & 0.0789 & & -0.0237 & & 0 & & 0 & & 0 \\
-0.0237 & & -0.0237 & & 0.0789 & & 0 & & 0 & & 0 \\
& & 0 & & 0 & & 0.2052 & & 0 & & 0 \\
& & 0 & & 0 & & 0 & & 0.2052 & & 0 \\
& & 0 & & 0 & & 0 & & 0 & & 0.2052 \\
\end{align*}
\]

\[Et_{ESL\_A} = 1.6617e+011\]
\[ Et_{ESL \_D} = 1.2680e+011 \]

\[ Vt_{ESL \_A} = 0.3003 \]

\[ Vt_{ESL \_D} = 0.3008 \]

\[ Vz_{x\_ESL \_A} = 0.3003 \]

\[ Vz_{x\_ESL \_D} = 0.3008 \]

\[ Ez_{ESL \_A} = 1.6617e+011 \]

\[ Ez_{ESL \_D} = 1.2680e+011 \]

\[ Gz_{y\_ESL \_A} = 6.3897e+010 \]

\[ Gz_{y\_ESL \_D} = 4.8739e+010 \]

\[ Gt_{ESL \_A} = 6.3897e+010 \]

\[ Gt_{ESL \_D} = 4.8739e+010 \]

---

**Table 2** Frequencies of the three models at free boundary conditions

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>2 Layers Model Frequencies</th>
<th>ESL_A Model Frequencies</th>
<th>ESL_D Model Frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.19367E-3</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>2</td>
<td>8.41768E-3</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>3</td>
<td>1.19903E-2</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>4</td>
<td>3.62316E-2</td>
<td>6.62841E-3</td>
<td>0.0000</td>
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<tr>
<td>5</td>
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<td>9.89417E-3</td>
<td>0.0000</td>
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<tr>
<td></td>
<td>3.87237E-2</td>
<td>1.02917E-2</td>
<td>0.0000</td>
</tr>
<tr>
<td>---</td>
<td>------------</td>
<td>------------</td>
<td>--------</td>
</tr>
<tr>
<td>7</td>
<td>80.327</td>
<td>99.702</td>
<td>87.095</td>
</tr>
<tr>
<td>8</td>
<td>147.25</td>
<td>182.62</td>
<td>159.5</td>
</tr>
<tr>
<td>9</td>
<td>223.16</td>
<td>276.91</td>
<td>241.9</td>
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<td>312.45</td>
<td>386.87</td>
<td>337.90</td>
</tr>
<tr>
<td>11</td>
<td>439.48</td>
<td>545.19</td>
<td>476.27</td>
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<td>12</td>
<td>512.29</td>
<td>632.92</td>
<td>552.82</td>
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<td>718.46</td>
<td>890.81</td>
<td>778.18</td>
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<td>741.87</td>
<td>920.65</td>
<td>804.34</td>
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<td>761.85</td>
<td>939.04</td>
<td>820.23</td>
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<tr>
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<td>816.92</td>
<td>1012.5</td>
<td>884.58</td>
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<tr>
<td>17</td>
<td>967.36</td>
<td>1194.6</td>
<td>1043.6</td>
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<td>1073</td>
<td>1319.8</td>
<td>1152.9</td>
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<tr>
<td>19</td>
<td>1113.5</td>
<td>1380.6</td>
<td>1206.2</td>
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<tr>
<td>20</td>
<td>1229.8</td>
<td>1512.2</td>
<td>1321.0</td>
</tr>
</tbody>
</table>
Diagram 1. Squeal noise at modes 7 & 42

(for the brake system without any change on brake shim)

Figure 6 squeal phenomenon at mode 7 shows by Abaqus/Cae
Figure 7 squeal phenomenon at mode at 17 shows by Abaqus/Cae

(for ESL_A)
**B. Mathematica Program**

Mathematica program was used to check which parameters our model will depend on.

In[1]=

\[ Q_{\text{steel}} = \text{Inverse}[[\{1/E_s, -poi_s/E_s, -poi_s/E_s, 0, 0, 0\}, \{-poi_s/E_s, 1/E_s, -poi_s/E_s, 0, 0, 0\}, \{-poi_s/E_s, -poi_s/E_s, 1/E_s, 0, 0, 0\}, \{0, 0, 0, 2(1+poi_s)/E_s, 0, 0\}, \{0, 0, 0, 0, 2(1+poi_s)/E_s\}]] \]

\[ Q_{\text{Glue}} = \text{Inverse}[[\{1/E_g, -poi_g/E_g, -poi_g/E_g, 0, 0, 0\}, \{-poi_g/E_g, 1/E_g, -poi_g/E_g, 0, 0, 0\}, \{-poi_g/E_g, -poi_g/E_g, 1/E_g, 0, 0, 0\}, \{0, 0, 0, 2(1+poi_g)/E_g, 0, 0\}, \{0, 0, 0, 0, 2(1+poi_g)/E_g\}]] \]

\[ A_{\text{ML}} = \text{Inverse}[[\{1/E_g, -poi_g/E_g, -poi_g/E_g, 0, 0, 0\}, \{-poi_g/E_g, 1/E_g, -poi_g/E_g, 0, 0, 0\}, \{-poi_g/E_g, -poi_g/E_g, 1/E_g, 0, 0, 0\}, \{0, 0, 0, 2(1+poi_g), 0, 0\}, \{0, 0, 0, 0, 2(1+poi_g)\}]]h_{\text{glue}} + \text{Inverse}[[\{1/E_s, -poi_s/E_s, -poi_s/E_s, 0, 0, 0\}, \{-poi_s/E_s, 1/E_s, -poi_s/E_s, 0, 0, 0\}, \{-poi_s/E_s, -poi_s/E_s, 1/E_s, 0, 0, 0\}, \{0, 0, 0, 2(1+poi_s)/E_s, 0, 0\}, \{0, 0, 0, 0, 2(1+poi_s)/E_s\}]]h_{\text{steel}} \]

\[ Q_{\text{ESL}} = \text{Inverse}[[\{1/E_g, -poi_g/E_g, -poi_g/E_g, 0, 0, 0\}, \{-poi_g/E_g, 1/E_g, -poi_g/E_g, 0, 0, 0\}, \{-poi_g/E_g, -poi_g/E_g, 1/E_g, 0, 0, 0\}, \{0, 0, 0, 2(1+poi_g), 0, 0\}, \{0, 0, 0, 0, 2(1+poi_g)\}]]h_{\text{glue}} + \text{Inverse}[[\{1/E_s, -poi_s/E_s, -poi_s/E_s, 0, 0, 0\}, \{-poi_s/E_s, 1/E_s, -poi_s/E_s, 0, 0, 0\}, \{-poi_s/E_s, -poi_s/E_s, 1/E_s, 0, 0, 0\}, \{0, 0, 0, 2(1+poi_s)/E_s, 0, 0\}, \{0, 0, 0, 0, 2(1+poi_s)/E_s\}]]h_{\text{steel}} \]
Results

Out[1]=

{{(8/@s5+(24 poi_s)/@s5+(16 poi_s2)/@s5-(16 poi_s3)/@s5-(24
poi_s4)/@s5-(8 poi_s5)/@s5)/(8/@s6+(24 poi_s)/@s6-(20
poi_s3)/@s6-(120 poi_s4)/@s6-(72 poi_s5)/@s6-(16
poi_s6)/@s6),(8 poi_s)/@s5+(32 poi_s2)/@s5+(48 poi_s3)/@s5+(32
poi_s4)/@s5+(8 poi_s5)/@s5)/(8/@s6+(24 poi_s)/@s6-(80
poi_s3)/@s6-(120 poi_s4)/@s6-(72 poi_s5)/@s6-(16
poi_s6)/@s6),(8 poi_s)/@s5+(32 poi_s2)/@s5+(48 poi_s3)/@s5+(32
poi_s4)/@s5+(8 poi_s5)/@s5)/(8/@s6+(24 poi_s)/@s6-(80
poi_s3)/@s6-(120 poi_s4)/@s6-(72 poi_s5)/@s6-(16
poi_s6)/@s6),0,0,0),{(8 poi_s)/@s5+(32 poi_s2)/@s5+(48
poi_s3)/@s5+(32 poi_s4)/@s5+(8 poi_s5)/@s5)/(8/@s6+(24
poi_s)/@s6-(80 poi_s3)/@s6-(120 poi_s4)/@s6-(72 poi_s5)/@s6-(16
poi_s6)/@s6),(8 poi_s)/@s5+(24 poi_s)/@s5+(16 poi_s2)/@s5+(16
poi_s3)/@s5-(8 poi_s5)/@s5)/(8/@s6+(24
poi_s)/@s6-(16
poi_s6)/@s6),(8 poi_s)/@s5+(32 poi_s2)/@s5+(48 poi_s3)/@s5+(32
poi_s4)/@s5+(8 poi_s5)/@s5)/(8/@s6+(24 poi_s)/@s6-(80
poi_s3)/@s6-(120 poi_s4)/@s6-(72 poi_s5)/@s6-(16
poi_s6)/@s6),0,0,0),{(8 poi_s)/@s5+(32 poi_s2)/@s5+(48
poi_s3)/@s5+(32 poi_s4)/@s5+(8 poi_s5)/@s5)/(8/@s6+(24
poi_s)/@s6-(80 poi_s3)/@s6-(120 poi_s4)/@s6-(72 poi_s5)/@s6-(16
poi_s6)/@s6),(8 poi_s)/@s5+(24 poi_s)/@s5+(16 poi_s2)/@s5+(16
poi_s3)/@s5-(8 poi_s5)/@s5)/(8/@s6+(24
poi_s)/@s6-(80 poi_s3)/@s6-(120 poi_s4)/@s6-(72 poi_s5)/@s6-(16
poi_s6)/@s6),0,0,0),{0,0,0,(4/@s5+(8 poi_s)/@s5-(8 poi_s2)/@s5-(32
poi_s3)/@s5-(24 poi_s4)/@s5-(8 poi_s5)/@s5)/(8/@s6+(24
poi_s)/@s6-(80 poi_s3)/@s6-(120 poi_s4)/@s6-(72 poi_s5)/@s6-(16
poi_s6)/@s6),0,0},0,0,0,4/@s5+(8 poi_s)/@s5-(8 poi_s2)/@s5-(8 poi_s3)/@s5-(8 poi_s2)/@s5-}
\[(32 \text{ poi } s_3)/\text{ s}_5-(28 \text{ poi } s_4)/\text{ s}_5-(8 \text{ poi } s_5)/\text{ s}_5)/(8/\text{ s}_6+(24 \text{ poi } s)/\text{ s}_6-(80 \text{ poi } s_3)/\text{ s}_6-(120 \text{ poi } s_4)/\text{ s}_6-(72 \text{ poi } s_5)/\text{ s}_6-(16 \text{ poi } s_6)/\text{ s}_6),0\},(0,0,0,0,0,(4/\text{ s}_5+(8 \text{ poi } s)/\text{ s}_5-(8 \text{ poi } s_2)/\text{ s}_5-(32 \text{ poi } s_3)/\text{ s}_5-(28 \text{ poi } s_4)/\text{ s}_5-(8 \text{ poi } s_5)/\text{ s}_5)/(8/\text{ s}_6+(24 \text{ poi } s)/\text{ s}_6-(80 \text{ poi } s_3)/\text{ s}_6-(120 \text{ poi } s_4)/\text{ s}_6-(72 \text{ poi } s_5)/\text{ s}_6-(16 \text{ poi } s_6)/\text{ s}_6))\]
Out[3]=

{{(8 h_glue(1+poi_g)3(1/@_g2-poi_g2/@_g2))/(@_g3+(24poi_g)/@_g3-(8 poi_g)/@_g3-8 poi_g3)/@_g3-(120 poi_g4)/@_g3-(72 poi_g5)/@_g3-(16 poi_g6)/@_g3),0,0,0),(0,0,0,(4/@_g5+(8 poi_g)/@_g5-(8 poi_g2)/@_g5-(32 poi_g3)/@_g5-(28 poi_g4)/@_g5-(8 poi_g5)/@_g5)/(@_g6+(24 poi_g)/@_g6-(8 poi_g3)/@_g6-(120 poi_g4)/@_g6-(72 poi_g5)/@_g6-(16 poi_g6)/@_g6),0,0),(0,0,0,0,(4/@_g5+(8 poi_g)/@_g5-(8 poi_g2)/@_g5-(32 poi_g3)/@_g5-(28 poi_g4)/@_g5-(8 poi_g5)/@_g5)/(@_g6+(24 poi_g)/@_g6-(8 poi_g3)/@_g6-(120 poi_g4)/@_g6-(72 poi_g5)/@_g6-(16 poi_g6)/@_g6),0,0),(0,0,0,0,0,(4/@_g5+(8 poi_g)/@_g5-(8 poi_g2)/@_g5-(32 poi_g3)/@_g5-(28 poi_g4)/@_g5-(8 poi_g5)/@_g5)/(@_g6+(24 poi_g)/@_g6-(8 poi_g3)/@_g6-(120 poi_g4)/@_g6-(72 poi_g5)/@_g6-(16 poi_g6)/@_g6)}}
Out[4]=

{{{{(8 \_g3+(8 poi_s3)/\_s6-(120 poi_s4)/\_s6-(72 poi_s5)/\_s6-(16
poi_s6)/\_s6),0,0,0),(0,0,0,poi_g2)/\_g3-(8 poi_g)/\_g3-(8
poi_g2)/\_g3-(32 poi_g3)/\_g3-(28 poi_g4)/\_g3-(8
poi_g5)/\_g3))/\_g3+(24 poi_g)/\_g3-(80 poi_g3)/\_g3-(120
poi_g4)/\_g3-(72 poi_g5)/\_g3-(16 poi_g6)/\_g3)+(h_steel (4/\_s5+(8
poi_s)/\_s5-(8 poi_s2)/\_s5-(32 poi_s3)/\_s5-(28 poi_s4)/\_s5-(8
poi_s5)/\_s5))/\_s6+(24 poi_s)/\_s6-(80 poi_s3)/\_s6-(120
poi_s4)/\_s6-(72 poi_s5)/\_s6-(16
poi_s6)/\_s6),0),(0,0,0,0,poi_s3)/\_s3+(8 poi_s4)/\_s5+(8 poi_s5)/\_s6-(16 poi_s6)/\_s6-(16 poi_s6)/\_s6,(0,0,0,0,0,poi_g2)/\_g3-(32 poi_g3)/\_g3-(28 poi_g4)/\_g3-(8
poi_g5)/\_g3))/\_g3+(24 poi_g)/\_g3-(80 poi_g3)/\_g3-(120
poi_g4)/\_g3-(72 poi_g5)/\_g3-(16 poi_g6)/\_g3)+(h_steel (4/\_s5+(8
poi_s)/\_s5-(8 poi_s2)/\_s5-(32 poi_s3)/\_s5-(28 poi_s4)/\_s5-(8
poi_s5)/\_s5))/\_s6+(24 poi_s)/\_s6-(80 poi_s3)/\_s6-(120
poi_s4)/\_s6-(72 poi_s5)/\_s6-(16 poi_s6)/\_s6))}}

30