Test turbine measurements and comparison with mean-line and throughflow calculations

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Abstract

This thesis is a collaboration between Siemens Industrial Turbomachinery (SIT) and Royal Institute of Technology (KTH). It is aimed to study and compare the outputs of two different computational approaches in axial gas turbine design procedure with the data obtained from experimental work on a test turbine. The main focus during this research is to extend the available test databank and to further understand and investigate the turbine stage efficiency, mass flow parameters and reaction degree under different working conditions. Meanwhile the concept and effect of different loss mechanisms and models will be briefly studied.

The experimental part was performed at Heat and Power Technology department on a single stage test turbine in its full admission mode. Three different pressure ratios were tested. For the medium pressure ratio a constant temperature anemometry (CTA) method was deployed in two cases, with and without turbulence grid, to determine the effect of free-stream turbulence intensity on the investigated parameters. During the test campaign the raw gathered data was processed with online tools and also they served as boundary condition for the computational codes later.

The computational scope includes a one-dimensional design approach known as mean-line calculation and also a two-dimensional method known as throughflow calculation. An in-house SIT software, CATO, generated the stage geometry (vane, blade and the channel) and then two other internal computational codes, MAC1 and BETA2, were employed for the one-dimensional and two-dimensional computations respectively. It was observed that to obtain more accurate mass flow predictions a certain level of channel blockage should be implemented to represent the boundary layer development and secondary flow which is typically around 2%. The codes are also equipped with two options to predict the friction loss: One is a more empirical correlation named as the Old approach in SIT manuals and the other works based on allocation of boundary layer transition point, named as BL in the present thesis. Simulations were done by use of both approaches and it turned out that the latter works more accurately if it is provided with appropriate transition point and blockage estimation.

The measured data also suggests the idea that the transition point of the vane and blade is not affected by a change in turbulence intensity at least up to 6% in the tested Reynolds numbers, $\sim 5 \times 10^5$. Amongst
different solutions the one which used BL approach and constant transition point (while the turbulence intensity changed) managed to predict this behavior. Also it was investigated and revealed that the codes inherently predict poor results in off-design loadings which is mainly due to positive incidence angle in addition to high spanwise gradient of the flow parameters.
Acknowledgments

Immediately I would like to thank my very dedicated supervisors Dr. Jens Fridh, KTH, and Lars Hedlund at Siemens Industrial Turbomachinery, SIT. I had their great support during this study and they led the project to be also a compact gas turbine course for me via a perfect plan.

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## Nomenclature

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<thead>
<tr>
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<tbody>
<tr>
<td>Λ</td>
<td>Flow channel area</td>
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<tr>
<td>b</td>
<td>Axial chord length</td>
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<tr>
<td>C</td>
<td>Compressor</td>
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<tr>
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<td>Absolute velocity</td>
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### Greek letters

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<td>Flow coefficient</td>
</tr>
<tr>
<td>ψ</td>
<td>Stage loading</td>
</tr>
<tr>
<td>ω</td>
<td>Angular velocity</td>
</tr>
<tr>
<td>Λ</td>
<td>Degree of Reaction</td>
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### Subscripts

<table>
<thead>
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<th>Symbol</th>
<th>Description</th>
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<tr>
<td>0</td>
<td>Total quantity</td>
</tr>
<tr>
<td>1, 2, 3</td>
<td>Stations</td>
</tr>
<tr>
<td>l</td>
<td>Quasi-orthogonal component</td>
</tr>
<tr>
<td>m</td>
<td>Meridional component</td>
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<td>n</td>
<td>Normal component in meridional system</td>
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<td>Pressure</td>
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<td>Relative</td>
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<td>Isentropic process</td>
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<tr>
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<td>Stator</td>
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<tr>
<td>ss</td>
<td>Static-to-static, Fully isentropic process</td>
</tr>
<tr>
<td>th</td>
<td>Thermodynamic</td>
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### Abbreviations

- **CFD**: Computational fluid dynamics
- **CTA**: Constant temperature anemometry
- **KTH**: Royal Institute of Technology
- **SIT**: Siemens Industrial Turbomachinery
1 Introduction

1.1 Background

Siemens Industrial Turbomachinery AB (SIT) runs an air driven test turbine at the Department of Energy Technology of the Royal Institute of Technology (KTH). The turbine is used to test turbine blading and for validation of calculation tools.

Two of the in-house calculation programs used at SIT are MAC1 and BETA2. The former is a 1D mean-line code that simplifies the calculations by applying empirical loss models to the flow at mid span to simulate the real 3D flow. The latter is a 2D radial equilibrium throughflow code that in addition can predict the radial distribution of the flow parameters. In retrospect, the test turbine had been modeled before e.g. by use of another Siemens code, AXIAL, which is a meanline code and the results together with the measurement data was reported by Hedlund (2004). The test turbine turbulence level though has not been investigated thoroughly as a case study before.

On the computational side, a new boundary layer based friction loss model has recently been incorporated in MAC1 and BETA2. To see if this new model predicts the loss better than the old correlation based model, comparisons between predictions by these models and existing test turbine data is necessary. Also SIT is developing a new blade generation software, CATO, and the rather simple geometry of the test turbine blades helps to comment on its strengths and deficiencies. The present thesis is the first use of CATO for the test turbine geometry simulation. Aggregation of experimental data of a few different working conditions with 1D and 2D simulations in a unit task can highlight the demands that should be met during the codes development procedure.

1.2 Objective

Given the test outcomes of this thesis there is an opportunity for SIT to extend the already available experimental data of the test turbine in KTH. Along with that, this project was set to partly evaluate the functionality and to validate the results of the currently used prediction codes in comparison to measured values.

By performing tests and simulations in three pressure ratios, observing the effect on four turbine parameters is of interest. Flow capacity and turbine constant as two representatives of mass flow rate in addition to the stage total to static efficiency and degree of reaction are the main investigated parameters. Furthermore, the effect of the free stream turbulence intensity is going to be studied both experimentally and computationally.

In computation phase, the results from the meanline and the throughflow codes are constantly compared to each other to provide an idea of their differences and supremacies. Also since a new friction loss model has been inserted in the codes, qualifying and evaluating the outputs of this approach is a major goal of the work.

Inherently, in all cases finding the effect of various stage loadings on turbine performance and the codes prediction is pursued. Particularly, the calculation results in off-design condition are qualitatively explored. Note that by the extensive use of “results” through the text, it actually refers to the four abovementioned parameters.

Also it will be tried to suggest a way to obtain a rough estimation for the channel blockage based on measurement data and the Stodola’s ellipse formula. It is highly necessary to state that fitting the calculated curves to those of experiments by any possible means (i.e. considering solution of the test
turbine as a total objective) is not aimed but to the deviations and their causes and effects are set as a target.

### 1.3 Method

The task is approached using both simulation tools and test facilities. An experimental campaign was set on the test turbine at KTH where three different pressure ratios and two turbulence levels were implemented. Test data was acquired by pressure and temperature probes and was accumulated by a real-time Labview program, TURBAQ, and was processed by another online tool, TURBAN. In case of turbulence intensity measurement the constant temperature anemometry method by use of a single hotwire probe was employed. Hotwire was mounted on a traverse system at the stator inlet station. The probe calibration, data acquisition and post process was done by an IFA-300 unit.

During the simulation part CATO, MAC1 and BETA2 served as geometry modeler, meanline and throughflow solvers. The main structure of solvers input file immediately established by help of previous experimental data where evaluation of results led to obtain a decent channel blockage percentage and more robust input files. A large number of calculation cases (for newly run tests) and the outputs post processing were both conducted by MATLAB programs.

A comparative viewpoint between different applied cases (either distinct boundary conditions or different solver settings) has been followed throughout the computational section and the results quality is tried to be also justified rather than a pure reportage.

### 1.4 Limitations

It must be known that the software which will be deployed throughout this project are mainly used by the gas turbine aerodynamics group at SIT for real working conditions and they involve many correlations which are founded experimentally by use of common gas turbine blade profiles. Whereas, the test facility at KTH has been designed based on steam turbine blading and it works under room temperature and low Mach numbers and Reynolds numbers. All these together impose difficulties and limitations into the computational section of the present work.

In addition, measurements of lower stage loadings could not be done because of the bearings temperature limitation especially at the higher pressure ratio. Also turbulence intensity measurement was just limited to two levels according to the time required to manufacture a new turbulence grid (however it was designed by Dr. Jens Fridh).
2 Turbine Theory

2.1 Gas Turbine Outline

Nowadays a major part of power which runs industries, transportation systems and daily life is supplied by turbomachines. The growth of population and constantly increasing demands for energy in addition to other limitations such as fuel price and emission regulations have led all the providers over the world to extensively search for more efficient and reliable design of their products. Herein, turbines as a not so newly born technology but always developing play a main role to supply the need of power. Turbines can be categorized from different aspects such as their working fluid, the pressure levels they encounter, shrouding types, subsonic or supersonic conditions, etc.

From thermodynamic point of view, it is rather easy to depict the main process of power generation in a gas turbine by use of a simple Brayton cycle which is shown in a temperature-entropy diagram, Figure 2-1.

![Figure 2-1. Brayton power generation cycle](image)

In this simple cycle the compressor and turbine are assumed to be as one unit where the inlet air enters the compressor at state 1 and it will be ideally compressed up to state 2'. But, it must be considered that the real compression process involves losses and hence the entropy will increase and the air leaves the compressor at state 2 to the combustion chamber. Thereafter fuel is injected into the combustion chamber and the mixture will be burnt ideally at constant pressure which raises the mixture temperature to state 3. Then the working fluid enters the turbine and expands (typically to the ambient pressure) through its stages and generates power. A part of this power will be consumed to drive the compressor shaft and the rest will be the useful output work. Hence, one can calculate the net work out of the turbine unit in terms of total temperatures as:

$$W_{net} = W_T - W_C = c_{p,gas} \left(T_{03} - T_{04}\right) - c_{p,air} \left(T_{02} - T_{01}\right)$$  \hspace{1cm} (Eq.2.1)

where ideal gas is assumed. Here, the thermodynamic efficiency can also be defined as the ratio of the net output work to the heat added to the system during combustion process:

$$\eta_{th} = \frac{W_{net}}{Q} = \frac{W_{net}}{c_{p,gas} \left(T_{03} - T_{02}\right)}$$  \hspace{1cm} (Eq.2.2)

And using isentropic relations it can be re-written as:
\[ \eta_{th} = 1 - \frac{1}{\pi_{tt}^{(\gamma-1)/\gamma}} \]  
(Eq. 2.3)

where \( \gamma \) the specific heat ratio and \( \pi_{tt} \) is a widely used designing parameter, total-to-total pressure ratio:

\[ \pi_{tt} = \frac{p_{02}}{p_{01}} \]  
(Eq. 2.4)

Another concept which is used in turbine design procedure, the specific work output, is defined as the ratio of the net output work to the available energy of the inlet air which can be expressed as:

\[ \frac{W_{net}}{c_p T_{01}} = t \left( 1 - \frac{1}{\pi_{tt}^{(\gamma-1)/\gamma}} \right) - \left( \pi_{tt}^{(\gamma-1)/\gamma} - 1 \right) \]  
(Eq. 2.5)

where \( t = T_{02}/T_{01} \) is the total temperature ratio of the turbine.

From the above equations it is obvious that the thermodynamic efficiency of the cycle is not affected by the temperature ratio while the specific work output is a function of both temperature and pressure ratios and hence optimizing these ratios simultaneously is a key point at the first steps of turbine designing procedure. It is also simple to see from the trend of the equations that both ratios tend to higher values which suggests that other parameters like the mechanical stresses and life of the turbine should also be taken into account and compromised.

It is necessary to state that the abovementioned cycle is a basic and simple case and in practice it usually involves other components such as heat exchanger, reheating circuits, etc. which is out of the scope of this thesis but can be easily found in many turbomachinery text books e.g. Saravanamuttoo (2009).

### 2.2 Generation of Work in a Turbine Stage

To be more focused on the main subject area of this task, the fundamentals of work generation via expansion of the working fluid (combustion chamber products) in turbine section will be briefly discussed. A turbine cascade includes at least one row of static vanes, stator, followed by a row of rotating blades, rotor. Figure 2-2 shows a fluid particle moving through a row of blades.

![Fluid particle velocity components through a row of blades](Image)

**Figure 2-2. Fluid particle velocity components through a row of blades. (Moustapha, 2003)**

-4-
The velocity vector of the particle, \( \mathbf{c}_1 \), consists of three components \( c_x, c_r \) and \( c_\theta \) in the axial, radial and circumferential directions respectively. The tangential component is the one that contributes in the turbine energy transfer and from the Newton’s 2nd Law of Motion the torque can be expressed as the rate of change of angular momentum through each row of blades:

\[
\tau = m(r_2 c_\theta + r_1 c_{\theta 1})
\]

( Eq.2.6 )

Where \( m \) is the mass flux (since the direction of tangential velocities are opposite, their magnitude have added to each other in the scalar form). Knowing \( \omega \) (angular velocity) and hence the blade velocity \( U = r_\omega \), the specific work output is:

\[
W_{\text{spec}} = \frac{\tau \omega}{m} = (U_2 c_\theta + U_1 c_{\theta 1})
\]

( Eq.2.7 )

this is the so-called Euler’s turbomachine equation.

A blade terminology is shown in Figure 2-3. Different angles and distances often used in the literature can be recognized there. It is important to know that the flow inlet and exit angles are different from those of the blade since the flow barely follows the blade (also known as “metal”) angles.

Figure 2-4 shows what happens for the velocity of the fluid particle through a turbine stage. Now it can be inferred that \( \beta_2 \) and \( \beta_3 \) are angles defined by use of velocities relative to the rotor. In fact they are the aforementioned flow inlet and exit angles of the rotor blades respectively if one assumes a stationary blade.

---

Figure 2-3. Blade terminology (Moustapha, 2003)

Figure 2-4. (a)Mollier enthalpy-entropy diagram , (b)Velocity vectors over a stage. Adopted from (Dixon, 2010)
An enthalpy-entropy diagram, known as Mollier diagram, is a simple way of understanding the thermodynamics of flow through a single stage. \(c_p\) and \(\gamma\) are basically temperature dependant although considering constant \(c_p\) one may use temperature-entropy diagram instead which seems to be less accurate at least in the case of high temperature combustion exhaust gases as the working fluid where chemical reactions take place and slightly change the gas composition and hence the specific heat value. Figure 2-4 also shows the Mollier diagram for one stage where the subscripts \(\theta\) and \(s\) indicate the total value and isentropic process respectively.

Note that in absence of coolant air the total enthalpy remains constant over the stator blades since the work is just generated via the rotor blades and energy equation can be reduced to:

\[
h_{01} = h_{02} \quad \text{ (Eq.2.8)}
\]

Evidently, the specific work output of the stage can be expressed as the change in the total enthalpy over the blades:

\[
W_{\text{spec.}} = h_{01} - h_{03} = h_{02} - h_{03} = (h_2 - h_3) + \frac{1}{2}(c_2^2 - c_3^2) \quad \text{ (Eq.2.9)}
\]

Implementation of constant rothalpy across the rotor blades gives:

\[
I = \text{Rothalpy} = h_{0,\text{rel}} - \frac{1}{2} U^2 \quad \text{ (Eq.2.10)}
\]

\[
h_{0,\text{rel}} = h + \frac{1}{2} w^2 \quad \text{ (Eq.2.11)}
\]

Then:

\[
h_2 - h_3 = \frac{1}{2}\left((U_2^2 - U_3^2) - (w_2^2 - w_3^2)\right) \quad \text{ (Eq.2.12)}
\]

Thus (Eq.2.9) can be written as:

\[
W_{\text{spec.}} = \frac{1}{2}\left((U_2^2 - U_3^2) - (w_2^2 - w_3^2) + (c_2^2 - c_3^2)\right) \quad \text{ (Eq.2.13)}
\]

Considering a constant blade speed, \(U\), which means also a constant radius it gives:

\[
W_{\text{spec.}} = \frac{1}{2}\left((w_3^2 - w_2^2) + (c_2^2 - c_3^2)\right) \quad \text{ (Eq.2.14)}
\]

### 2.3 Turbine Parameters

It is important to define a couple of non-dimensional parameters since they make the design procedure easier and provide explicit values which let the designers to study, optimize and compare the effects of changing different factors on turbines operation. Amongst these parameters, those which are frequently used in this text are described.

#### 2.3.1 Total-to-total and Total-to-static Efficiencies

Looking to the Mollier diagram of the turbine stage, Figure 2-4, two isentropic efficiencies are usually defined as the ratio of the actual work of the turbine to the work that can be extracted through an isentropic process. In total-to-total efficiency an expansion from \(P_{01}\) to \(P_{03s}\) is considered as the isentropic part and in the total-to-static efficiency it is an expansion from \(P_{01}\) to \(P_{3s}\).
Thus:

\[ \eta_{tt} = \text{Total to total eff.} = \frac{h_{01} - h_{03}}{h_{01} - h_{3ss}} \]  \hspace{1cm} (Eq.2.15)

\[ \eta_{ts} = \text{Total to static eff.} = \frac{h_{01} - h_{03}}{h_{01} - h_{3ss}} \]  \hspace{1cm} (Eq.2.16)

Note that the former is more commonly used - but not always – and it is relevant for a multi-stage turbine since the remaining kinetic energy, \( \frac{c_2^2}{2} \), is captured by the following stage. It is always important to exactly know what kind of efficiency has been used to compare two turbines since their values are rather different as can be seen from their formulas and the Mollier diagram.

### 2.3.2 Degree of Reaction

A widely used parameter in turbines is the degree of reaction and it implies the fraction of gas expansion in the rotor. In fact it determines how the gas expansion process is split between the rotor and stator in one stage and larger values of degree of reaction means the gas acceleration in the rotor is larger than the acceleration in the stator. There are different definitions based on static pressure, static temperature and isentropic enthalpy change for this quantity. Here these different definitions are expressed and also through the text it will be mentioned which definition is used.

\[ \Lambda_{press} = \frac{p_2 - p_3}{p_1 - p_3} \]  \hspace{1cm} (Eq.2.17)

\[ \Lambda_{temp} = \frac{T_2 - T_3}{T_1 - T_3} \]  \hspace{1cm} (Eq.2.18)

\[ \Lambda_{ent.} = \frac{\Delta h_{s,R}}{\Delta h_{s,s} + \Delta h_{s,R}} = \frac{h_2 - h_{3s}}{(h_1 - h_{2s}) + (h_2 - h_{3s})} \]  \hspace{1cm} (Eq.2.19)

The enthalpy based definition is mostly used in SIT aerodynamics group and it will be also used in this thesis. This is in fact the ratio of isentropic enthalpy drop through the rotor and sum of isentropic enthalpy drops in rotor and stator separately. In addition the definition applied in the Siemens in-house codes will also be expressed in the next chapters when applied.

It is clear that negative values of degree of reaction which already means the flow deceleration in rotor is not favorable.

### 2.3.3 Stage Loading Coefficient

This parameter is defined as the ratio of the stagnation enthalpy drop through the turbine stage, \( \Delta h_0 \) to the square of the rotor blade speed, \( U \).

\[ \psi = \Delta h_0 / U^2 \]  \hspace{1cm} (Eq.2.20)

\( \Delta h_0 \) is actually a value which indicates the specific work extraction out of a stage if one assumes the adiabatic condition. Now looking back to the Eq.2.7, the Euler’s work equation, and under assumption of a completely axial flow and a constant radius it can be written as:

\[ \Delta h_0 = U \Delta c_0 \]  \hspace{1cm} (Eq.2.21)
Where $\Delta c_\theta$ is the change in the tangential component of the absolute velocity over the rotor blades. Combining these two equations simply gives:

$$\psi = \frac{\Delta c_\theta}{U} \quad (\text{Eq. } 2.22)$$

It is then clear that a high stage loading demands a high flow turning through the blades. Finding out that the stage loading is interpreted as a non-dimensional scale of the extracted work out of a stage it is expected that a high value of this parameter might be favorable to reduce the number of stages, however it has been shown that those high loading cases cause a drop in the turbine efficiency and should be optimized in the designing procedure.

2.3.4 Flow Coefficient

Flow coefficient in a purely axial turbine is defined as the ratio of the axial component of the flow velocity to the blade speed:

$$\phi = \frac{c_x}{U} \quad (\text{Eq. } 2.23)$$

Considering the velocity triangles diagram it can be inferred that a lower value of flow coefficient in fact means the relative velocities are closer to the tangential status or actually a highly staggered blade. Assuming a constant blade rotational speed, the mass flow will increase as the flow coefficient increases.

2.3.5 Velocity Ratio

Velocity ratio can be simply realized as a non-dimensional quantity which implies the same concept as the stage loading but in a reverse ratio definition. The static-to-static velocity ratio is defined as:

$$v_{ss} = \frac{U}{\sqrt{2\Delta h_{ss}}} \quad (\text{Eq. } 2.24)$$

where $U$ is the blade speed as before and $\Delta h_{ss}$ is the fully isentropic entropy drop over the turbine stage. This definition is used in SIT steam turbine department. Comparison with the stage loading correlation Eq.2.20 reveals that high stage loads essentially means low values of velocity ratio and vice versa.

2.3.6 Flow Capacity

Flow capacity is an important quantity which indeed normalizes the mass flow rate into the turbine according to the prevailing total temperature and pressure. Use of flow capacity (known in many textbooks as “swallowing capacity”) gives an idea to compare the mass flow rate under different conditions. Flow capacity is defined as:

$$\text{Flow Capacity} = \dot{m} \sqrt{T_0}/p_0 \quad (\text{Eq. } 2.25)$$

where $\dot{m}$ is the mass flow rate (kg/sec). This relation actually originates from the fact that in the sonic flow (which may occur in the vanes throat) Mach number is close to one and the term $\dot{m} \sqrt{T_0}/(A_p_0)$ where $A$ is the throat area is a function of gas constant, $R$, and the isentropic exponent $\gamma = c_p/c_v$. Thus, assuming that the gas properties are determined it is just enough to measure the throat area to gain $\dot{m} \sqrt{T_0}/p_0$ which is representing the flow capacity into the turbine.
2.3.7 Turbine Constant

Turbine constant, $C_T$, is a mass flow indicator however it is not widely used in turbine design departments and literature. It can be conceptualized the same as flow capacity (which is more common in gas turbine industry). Turbine constant is defined as:

$$C_T = \frac{\dot{m}}{\sqrt{\frac{p_1^2 - p_3^2}{p_1 v_1}}} \quad \text{(Eq.2.26)}$$

The unit for turbine constant is [m$^2$] and $v_1$ implies the specific volume upstream of the stator inlet. This definition is commonly used in SIT steam turbine department.
3 Structure of Computational Methods

In this chapter two widespread computational methods of turbomachinery design (with specific focus on turbines) will be described. Both the approaches are currently used in the preliminary steps of design process where they provide reliable inputs for further detailed calculations such as 3D CFD codes and in the performance analysis segment where they are too vital because of their short run time rather than other evaluation tools and of course experiments.

In the first part, where the one-dimensional process of calculation is followed, the main concentration will be on a global turbine design calculations and useful definition (instead of a pure computational scheme) since a one-dimensional method can be understood as a particular version of a higher order design method where the tangential and radial variations are neglected and an averaged value is considered for different parameters on the mean radius of the annulus flow channel at each computation station.

The second part, is an insight to a common two dimensional calculation method, the streamline curvature. The basic structure of this method is to establish a radial equilibrium equation which can be solved in a space-marching scheme (steady state condition presumed).

3.1 Mean-line Turbine Calculations

The first step in the turbine design process is commonly the one dimensional mean-line calculations. Here, it is assumed that the flow parameters at the mean radius of the flow channel are representative of the flow in the whole flow field. This essentially means that the circumferential and radial flow variations are neglected for the sake of simplicity. However it is worth to say this method may even give a reasonable estimation of the real flow if the tip-to-hub ratio is not too large. In design part, the main duties of the 1D designer are to determine the velocity triangles and to prepare acceptable values for the turbine parameters to be used as the initial value for the further iterative computational methods. A brief explanation about a simple one dimensional mean-line calculation will be given here.

As shown in Figure2-4 at the main core of any initial turbine calculations, there is the velocity triangles diagram which is used to find the absolute and relative velocities and angles through the turbine stages. Also it is important to find a relation between different turbine parameters which were introduced in chapter 2 to find a way to optimize their values.

Starting with a degree of reaction definition by Horlock(1966) it reveals how different stage parameters are related to the velocity triangle:

\[
\Lambda = \frac{h_2 - h_3}{h_{01} - h_{03}} \quad (\text{Eq.3.1})
\]

Where from Eq.2.20 it can be seen that:

\[
h_{01} - h_{03} = \psi u^2
\]

Also the constant total relative enthalpy over the blades, Eq.2.11, gives:

\[
h_2 - h_3 = \frac{1}{2} (w_3^2 - w_2^2) = \frac{c_2^2}{2} (\tan^2 \beta_3 - \tan^2 \beta_2)
\]

and substitution of these terms into Eq.3.1, it gives:

\[
\Lambda = \frac{\phi^2}{2\psi} (\tan^2 \beta_3 - \tan^2 \beta_2) \quad (\text{Eq.3.2})
\]
Also we can use another definition of stage loading based on the tangential velocity change over the blade, Eq.2.22, to write:

\[ \psi = \frac{\Delta c_\theta}{U} = \frac{c_x (\tan \alpha_3 + \tan \alpha_2)}{U} \]

\[ \psi = \phi (\tan \alpha_3 + \tan \alpha_2) \]  

(Eq.3.3)

Also deriving the relative angles is easy as using geometrical relations on velocity triangles. There:

\[ \tan \alpha_2 = \tan \beta_2 + \frac{1}{\phi} \]  

(Eq.3.4)

\[ \tan \alpha_3 = \tan \beta_3 - \frac{1}{\phi} \]

And hence Eq.3.3 can be written as:

\[ \psi = \phi (\tan \beta_3 + \tan \beta_2) \]  

(Eq.3.5)

Now another formula for degree of reaction can be sought by plugging Eq.3.5 into Eq.3.2 which delivers:

\[ \Lambda = \frac{\phi}{2} (\tan \beta_3 - \tan \beta_2) \]  

(Eq.3.6)

This relation shows the dependence of reaction degree on the flow turning relative to the rotor blade. In fact Eq.3.2 and Eq.3.6 illustrate that once the values for reaction degree, stage loading and flow coefficient are decided then all the absolute and relative flow angles can be determined through the velocity triangles diagram. However, it has been noted in the turbine designing literature that setting the values required in these equations involves an iterative procedure and also is mostly relying on the turbine manufacturer previous experiences. The short introduced relations above might give an insight to how the simplified one-dimensional computation setups are going to help and provide the first measures of a turbine outlines.

The number of stages is basically determined by the available enthalpy drop however efficiency considerations also affect this decision. Very roughly and in a specific case of identical stage loading coefficients:

\[ \text{Number of Stages} \geq \frac{\dot{W}}{m\psi \Sigma U^2} \]  

(Eq.3.7)

this shows that a higher stage loading and/or a higher blade rotational speed is favorable to reduce the number of stages. But again the other restrictions such as the mechanical stresses and vibrations should be taken into account. Once an estimated velocity diagram is obtained the other turbine characteristics such as the number of required stages, the number of required blades and preliminary performance estimation can be also determined.

The mean blade speed is mainly decided upon mechanical stress analysis. In addition, in cases where the required shaft rotational frequency is known (e.g. in electrical power generators) the blade speed must satisfy \( \tau_{\text{mean}} = \frac{U}{\omega} \).

Furthermore, it will be described how to derive initial performance estimation for the stage by help of velocity triangle and available enthalpy drop. Starting with the Eq.2.16 where the total- to- static stage efficiency was defined, one can rewrite it as:

\[ \eta_s = \frac{h_{01} - h_{03}}{h_{01} - h_{3ss}} = \frac{h_{01} - h_{03}}{(h_{01} - h_{03}) + (h_{03} - h_3) + (h_3 - h_{3s}) + (h_{3s} - h_{3ss})} \]
\[ \eta_{ts}^{-1} = 1 + \frac{1}{2} \frac{c_2^2 + (h_3 - h_{3s}) + (h_{3s} - h_{3ss})}{h_{01} - h_{03}} \]  
(Eq.3.8)

Employing the constant pressure relation \( \frac{\partial h}{\partial s} \bigg|_p = T \) gives:

\[ (h_{3s} - h_{3ss}) = T_3 (s_{3s} - s_{3ss}) \]  
(Eq.3.9)

\[ (h_2 - h_{2s}) = T_2 (s_2 - s_{2s}) \]  
(Eq.3.10)

and looking to the Mollier diagram it is seen that \( (s_{3s} - s_{3ss}) = (s_2 - s_{2s}) \) then:

\[ (h_{3s} - h_{3ss}) = \frac{T_3}{T_2} (h_2 - h_{2s}) \]  
(Eq.3.11)

If one defines the loss of enthalpy over the blades in term of the exit kinematic energy, for the stator vanes this can be shown as:

\[ (h_2 - h_{2s}) = \frac{1}{2} c_2^2 \zeta_N \]  
(Eq.3.12)

where, \( \zeta_N \) is the enthalpy loss coefficient over the vanes.

In the same way for the blades, it:

\[ (h_3 - h_{3s}) = \frac{1}{2} w_3^2 \zeta_R \]  
(Eq.3.13)

where, \( \zeta_R \) is the enthalpy loss coefficient over the blades. Implementing Eq.3.11, Eq.3.12 and Eq.3.13 into Eq.3.8 it gives:

\[ \eta_{ts}^{-1} = 1 + \frac{\left( \frac{T_3}{T_2} \right) c_2^2 \zeta_N + w_3^2 \zeta_R + c_3^2}{2(h_{01} - h_{03})} \]  
(Eq.3.14)

It seems necessary to have an estimation method for these two enthalpy coefficients introduced in Eq.3.14. In this case, many different approaches have been suggested in the open literature such as Ainley and Mathieson (1951) and Dunham and Came (1970).

Three are also a number of useful relations between the blades geometry and the stage characteristics. Presuming constant axial velocity one can write:

\[ \dot{m} = \rho_1 A_{x1} c_x = \rho_2 A_{x2} c_x = \rho_3 A_{x3} c_x \]  
(Eq.3.15)

and:

\[ A_x = \frac{\dot{m}}{\rho \phi U} \equiv 2\pi r_{\text{mean}} H \]  
(Eq.3.16)

where \( A_x \) is the available area normal to the turbine axis and \( H \) is the blade height. Although it is more reasonable to use a radius which divides the area between the tip and hub into two equal upper and lower sections:

\[ r_{\text{mean}} = \frac{(r_{\text{tip}}^2 + r_{\text{hub}}^2)}{2} \]  
(Eq.3.17)

and then from Eq.3.16:

\[ H = \frac{\dot{m} \omega}{\rho \phi U^2 2\pi} \]  
(Eq.3.18)

In addition, the aspect ratio is usually used to derive the chord length of the vanes and blades at the preliminary design stage. Aspect ratio is defined as the ratio of the vane/blade height to its axial chord:
Aspect ratio = \( \frac{H}{b} \) \hspace{1cm} (Eq.3.19)

where \( b \) is the axial chord length. This ratio is mainly affected by the mechanical and manufacturing observations however the typical range for different applications is almost known.

Here, to close the brief dimensional discussion, the Zweifel criterion for blade loading will be shortly described. This value can be used to determine the pitch to axial chord ratio, \( s/b \), which gives an idea of the dimension of the required pitch of the vane or blade rows.

In fact Zweifel criterion tries to optimize the blades distance in the sake of losses. Considering two adjacent blades in a row it can be inferred that the flow receives more guidance by the blades if these two blades become closer to each other which is of course favorable since it gives a better control of the flow but at the same time it is seen that the friction losses start growing rapidly. On the other hand if the blades space apart each other obviously the friction loss decreases while this jeopardizes the flow attachment to the blade surface (mainly at the suction side) and may cause the flow separation which drastically increases the losses and lessens the efficiency. Zweifel criterion looks for a point between these two extreme conditions. This is shown in Figure 3-1 when the convex and concave curves show the flow velocity non-dimensionalized by the exit velocity on the suction side and pressure side respectively. Figure 3-2 also plots the Mach number variation over a blade in the suction and pressure sides while the typical pressure distribution is shown in Figure 3-3.

Since the driving force on the blade is due to the pressure difference of the two sides of its profile the area between the curves shown in the abovementioned figures can be interpreted as a measure of the actual tangential load of a single blade. Thus, there is no wonder if designers would like to accelerate the flow as soon as possible on the suction side and as far as possible on the pressure side. However it is not necessary to emphasize again that to decide about the blade loading needs indeed a juggling process of different considerations, particularly flow separation control in this approach.

Zweifel (1945) number implies the ratio of the actual to ideal tangential loads acting on the blade. Under many simplifications as done before in this chapter it is defined as:

\[
Z_w = 2 \frac{s}{b} \cos^2 \alpha_2 (\tan \alpha_1 + \tan \alpha_2) \hspace{1cm} (Eq.3.20)
\]

Zweifel reported a value of \( Z_w \approx 0.8 \) as an optimum point from a number of turbine cascade experiments at low Mach number. However, nowadays because of huge improvement in the blade design the state-of-the-art turbines use Zweifel number even larger than one. But anyway, allotting a certain value for \( Z_w \) it is possible to obtain the value of pitch to axial chord ratio or the blade solidity, \( \frac{s}{b} \), and hence the number of required blades.

Figure 3-1. Velocity distribution over suction side and pressure side in three different pitch to axial chord ratios (Dixon, 2010)
3.2 Through-flow Turbine Calculations

After an introduction about a simple one-dimensional calculation, here an overview into the two-dimensional throughflow turbine calculations will be described. In the following text a vast use of meridional plane and the velocity components in this coordinate system make it necessary for the reader to be familiar with this concept. Figure 3-4 and Figure 3-5 illustrate this right handed meridional coordinate system, $(m, \theta, n)$, attached to the streamlines where $c_m, c_\theta, \varphi, \gamma, r_m$ and $l$ are the meridional and circumferential components of velocity, streamline slope, lean angle, curvature radius and the quasi-orthogonal direction respectively. At the same figures the cylindrical coordinate system, $(z, \theta, r)$, is also shown.

The method that will be explained is in fact the most basic throughflow approach, the streamline curvature, which is almost a simple method of solving the radial equilibrium equations in turbomachinery. The radial equilibrium methods have been used since 1940’s and they are still used however many improvements such as implementing different empirical and theoretical loss models progressed their functionality under different designing demands. The works done by Adamczyk(1984), Baralon(2000) and Sturmayer(2004) are good examples of these trials. In the following, it has been tried to clarify the fundamentals of how a radial equilibrium equation can be derived and used as explained by Flydal(n(2010), Schobeiri(2005) and Simon(2007).

To start with this streamline curvature analysis the first step is the inviscid flow assumption which means that the Euler's momentum equations are applicable. Other assumptions are the steady-state and
axisymmetry conditions for the flow. Obviously it means the derivative of all flow parameters in time and tangential direction are equal to zero: \( \frac{\partial}{\partial t} = 0 \) and \( \frac{\partial}{\partial \theta} = 0 \).

**Figure 3-4.** Meridional coordinate plane

**Figure 3-5.** An illustration of the meridional coordinate, velocities and angled definition

Writing Euler’s momentum equation for three different directions in the cylindrical coordinate system and implementing the steady state and axisymmetric assumptions gives:

\[
c_r \frac{\partial c_r}{\partial r} + c_z \frac{\partial c_r}{\partial z} - \frac{c_r^2}{r} = F_r - \frac{1}{\rho} \frac{\partial p}{\partial r} \quad \text{(Eq.3.21)}
\]

\[
c_r \frac{\partial c_\theta}{\partial r} + c_z \frac{\partial c_\theta}{\partial z} - \frac{c}{r} = F_\theta \quad \text{(Eq.3.22)}
\]
\[
    c_r \frac{\partial c_z}{\partial r} + c_z \frac{\partial c_r}{\partial z} = F_z - \frac{1}{\rho} \frac{\partial p}{\partial z} \quad \text{(Eq. 3.23)}
\]

And from Figure 3-5:

\[
    c_r = c_m \sin \varphi \quad \text{(Eq. 3.24)}
\]

\[
    c_z = c_m \cos \varphi \quad \text{(Eq. 3.25)}
\]

Also the first two terms at the left hand side of Eq. 3.21 can be expressed as:

\[
    c_r \frac{\partial c_r}{\partial r} + c_z \frac{\partial c_r}{\partial z} = \frac{1}{dt} \left( \frac{\partial c_r}{\partial r} dr + \frac{\partial c_r}{\partial z} dz \right) = \frac{1}{dt} \frac{\partial c_r}{\partial m} dm = c_m \frac{\partial c_r}{\partial m}
\]

Therefore:

\[
    c_r \frac{\partial}{\partial r} + c_z \frac{\partial}{\partial z} = c_m \frac{\partial}{\partial m} \quad \text{(Eq. 3.27)}
\]

where the derivative corresponding to the meridional direction is:

\[
    \frac{\partial}{\partial m} = \frac{\partial z}{\partial m} \frac{\partial}{\partial z} + \frac{\partial r}{\partial m} \frac{\partial}{\partial r} \quad \text{(Eq. 3.28)}
\]

\[
    \frac{\partial z}{\partial m} = \cos \varphi , \quad \frac{\partial r}{\partial m} = \sin \varphi \quad \text{(Eq. 3.29)}
\]

Replacing radial velocity from Eq. 3.24 into the right-hand-side of Eq. 3.26 and using the chain rule one gets:

\[
    c_m \left( \sin \varphi \frac{\partial c_m}{\partial m} + c_m \cos \varphi \frac{\partial \varphi}{\partial m} \right) = c_m \left( \sin \varphi \frac{\partial c_m}{\partial m} + c_m \cos \varphi \frac{\partial \varphi}{\partial m} \right)
\]

The streamline curvature radius, \( r_m \), as shown in Figure 3-5 can be calculated as:

\[
    \frac{\partial \varphi}{\partial m} = \frac{1}{r_m} \quad \text{(Eq. 3.31)}
\]

Replacing the result of combination of Eq. 3.30 and Eq. 3.31 into radial momentum equation Eq. 3.21 it yields:

\[
    c_m \frac{\partial c_m}{\partial m} \sin \varphi + \frac{c_m^2}{r_m} \cos \varphi - \frac{c_m^2}{r} = F_r - \frac{1}{\rho} \frac{\partial p}{\partial r} \quad \text{(Eq. 3.32)}
\]

Now, following a similar way for the axial momentum equation Eq. 3.23 the left-hand-side terms can be written as:

\[
    c_r \frac{\partial c_z}{\partial r} + c_z \frac{\partial c_z}{\partial z} = c_m \frac{\partial c_z}{\partial m}
\]

based on the same principle as used in Eq. 3.26. Plugging the axial component of velocity from Eq. 3.25 into the derivative term at the RHS of Eq. 3.33 and using the chain rule it gives:

\[
    c_m \left( \cos \varphi \frac{\partial c_m}{\partial m} + c_m \frac{\partial \varphi}{\partial m} \right) = c_m \left( \cos \varphi \frac{\partial c_m}{\partial m} - c_m \sin \varphi \frac{\partial \varphi}{\partial m} \right)
\]

Hence the axial momentum equation may be rewritten as:
\[ c_m \frac{\partial c_m}{\partial m} \cos \varphi - \frac{c_m^2}{r_m} \sin \varphi = F_z - \frac{1}{\rho} \frac{\partial p}{\partial z} \]  

(Eq.3.35)

where \( r_m \) is the streamline curvature as defined in Eq.3.31.

Furthermore, applying the obtained relation Eq.3.27 to the circumferential momentum equation Eq.3.22 it can be simply manipulated as:

\[ c_r \frac{\partial c_r}{\partial r} + \frac{c_z \frac{\partial c_z}{\partial z}}{r} = \frac{c_m \frac{\partial c_m}{\partial m}}{r} + \frac{c_m \sin \varphi}{r} \frac{\partial c_\theta}{\partial m} = \frac{c_m}{r} \left( \frac{\partial c_\theta}{\partial m} + \frac{c_\theta}{r} \frac{\partial r}{\partial m} \right) = \frac{c_m}{r} \frac{\partial (rc_\theta)}{\partial m} \]  

(Eq.3.36)

\[ \rightarrow \frac{c_m}{r} \frac{\partial (rc_\theta)}{\partial m} = F_\theta \]

Eq.3.32, Eq.3.35 and Eq.3.36 already express the momentum equations of radial, axial and tangential directions in the cylindrical coordinates. These equations can be transferred into meridional coordinate system, \((n, \theta, m)\), if one deploys the following transformations in the derivatives. Figure3-4 helps to illuminate these relations:

\[ \frac{\partial}{\partial n} = \cos \varphi \frac{\partial}{\partial r} - \sin \varphi \frac{\partial}{\partial z} \]  

(Eq.3.37)

\[ \frac{\partial}{\partial m} = \sin \varphi \frac{\partial}{\partial r} + \cos \varphi \frac{\partial}{\partial z} \]  

(Eq.3.38)

\[ e_r = \sin \varphi \; e_m + \cos \varphi \; e_n \]  

(Eq.3.39)

\[ e_z = \cos \varphi \; e_m - \sin \varphi \; e_n \]  

(Eq.3.40)

\[ \frac{\partial}{\partial r} = \sin \varphi \; \frac{\partial}{\partial m} + \cos \varphi \; \frac{\partial}{\partial n} \]  

(Eq.3.41)

\[ \frac{\partial}{\partial z} = \cos \varphi \; \frac{\partial}{\partial m} - \sin \varphi \; \frac{\partial}{\partial n} \]  

(Eq.3.42)

Then the momentum equations in \(n, \theta\) and \(m\) directions respectively will be:

\[ \frac{c_m^2}{r_m} - \frac{c_\theta^2}{r} \cos \varphi = F_n - \frac{1}{\rho} \frac{\partial p}{\partial n} \]  

(Eq.3.43)

\[ \frac{c_m}{r} \frac{\partial (rc_\theta)}{\partial m} = F_\theta \]  

(Eq.3.44)

\[ c_m \frac{\partial c_m}{\partial m} - \frac{c_\theta^2}{r} \sin \varphi = F_m - \frac{1}{\rho} \frac{\partial p}{\partial m} \]  

(Eq.3.45)

Also it is possible to omit the normal direction derivatives, \( \frac{\partial}{\partial n} \), by applying the chain rule to take derivative with respect to the quasi direction, \( l \):

\[ \frac{\partial}{\partial l} = \cos(\varphi - \gamma) \frac{\partial}{\partial n} + \sin(\varphi - \gamma) \frac{\partial}{\partial m} \]  

(Eq.3.46)

and then:

\[ \frac{\partial}{\partial n} = \sec(\varphi - \gamma) \frac{\partial}{\partial l} - \tan(\varphi - \gamma) \frac{\partial}{\partial m} \]  

(Eq.3.47)

The next step is to make the Euler’s momentum equation free of the pressure gradient terms. It can be done by use of thermodynamic relations to close the equations set. The total enthalpy is defined as:
\[ h_0 = h + \frac{c^2}{2} \]  
(Eq. 3.48)

and the first law of thermodynamics yields:

\[ dh = Tds + \frac{dp}{\rho} \]  
(Eq. 3.49)

Knowing that \( c^2 = c_m^2 + c_\theta^2 \) and hence:

\[ cdc = c_m dc_m + c_\theta dc_\theta \]  
(Eq. 3.50)

the combination of Eq. 3.48 and Eq. 3.49 is obtained as:

\[ \frac{dp}{\rho} = dh_0 - Tds - c_m dc_m - c_\theta dc_\theta \]  
(Eq. 3.51)

This equation may also be written in partial form with respect to meridional and normal (which will be transferred to quasi direction via Eq. 3.47) as Eq. 3.52 and Eq. 3.53 expresses:

\[ \frac{1}{\rho} \frac{dp}{dm} = \frac{d h_0}{d m} - T \frac{d s}{d m} - c_m \frac{d c_m}{d m} - c_\theta \frac{d c_\theta}{d m} \]  
(Eq. 3.52)

\[ \frac{1}{\rho} \frac{dp}{dn} = \frac{d h_0}{d n} - T \frac{d s}{d n} - c_m \frac{d c_m}{d n} - c_\theta \frac{d c_\theta}{d n} \]  
(Eq. 3.53)

To insert Eq. 3.52 into Eq. 3.45, the meridional momentum equation and eliminate similar terms on both sides, it yields:

\[ -\frac{c_m^2}{r_m} \sin \varphi = F_m - \frac{d h_0}{d m} + T \frac{d s}{d m} + c_\theta \frac{d c_\theta}{d m} \]  
(Eq. 3.54)

In the same way, Eq. 3.53 into Eq. 3.43 for the normal direction:

\[ \frac{c_\theta^2}{r} \cos \varphi = F_n - \frac{d h_0}{d n} + T \frac{d s}{d n} + c_m \frac{d c_m}{d n} + c_\theta \frac{d c_\theta}{d n} \]  
(Eq. 3.55)

while by implementation of Eq. 3.47 on Eq. 3.55 will transfer it on \( l \) direction, as:

\[ \frac{c_m^2}{r_m} - \frac{c_\theta^2}{r} \cos \varphi = F_m - \frac{d h_0}{d l} + T \frac{d s}{d l} + c_m \frac{d c_m}{d l} + c_\theta \frac{d c_\theta}{d l} \]  
(Eq. 3.56)

To get an expression including the body forces \( F_m \) and \( F_n \) the Eq. 3.54 and Eq. 3.56 will be added to each other. There are a number of algebraic simplifications and terms rearrangements in between and eventually it delivers:

\[ c_m \frac{d c_m}{d l} = c_m \sin(\varphi - \gamma) \frac{d c_m}{d m} + \cos(\varphi - \gamma) \frac{c_m^2}{r_m} - \cos \gamma \frac{c_\theta^2}{r} - \cos(\varphi - \gamma) \frac{d h_0}{d l} + T \frac{d s}{d l} \]  
(Eq. 3.57)

Finally, if the third term at the RHS of the equation be modified using \( \cos \gamma = \frac{\partial r}{\partial l} \), its combination with the fourth term can be simplified like:
\[- \cos \gamma \frac{c^2_\theta}{r} - c_\theta \frac{\partial c_\theta}{\partial l} = - \frac{c_\theta}{r} \left( c_\theta \frac{\partial r}{\partial l} + r \frac{\partial c_\theta}{\partial l} \right) = - \frac{c_\theta}{r} \frac{\partial (rc_\theta)}{\partial l} \quad (Eq.3.58)\]

Thus, Eq.3.57 ends up to its final form as:

\[
c_m \frac{\partial c_m}{\partial l} = c_m \sin(\phi - \gamma) \frac{\partial c_m}{\partial m} + \cos(\phi - \gamma) \frac{c^2_m}{r} - \frac{c_\theta}{r} \frac{\partial r c_\theta}{\partial l} + \frac{\partial h_0}{\partial l} - T \frac{\partial s}{\partial l}
\]

\[- \sin(\phi - \gamma) F_m - \cos(\phi - \gamma) F_n\]

This equation states the radial equilibrium condition and it is important since it gives a correlation associated with the body forces $F_m$ and $F_n$. The third component of body force field $F_\theta$ is already expressed via Eq.3.44. Here, more treatment is required to solve the derived set of equations. First of all the correlation for meridional momentum equation may be written in a way which includes $F_\theta$ also.

Reordering Eq.3.54 gives:

\[
T \frac{\partial s}{\partial m} = \frac{\partial h_0}{\partial m} - c_\theta \frac{\partial c_\theta}{\partial m} - \frac{c^2_\theta}{r} \sin \phi - F_m
\]

\[
\frac{\partial h_0}{\partial m} = \frac{c_\theta}{r} \left( r \frac{\partial c_\theta}{\partial m} - c_\theta \frac{\partial r}{\partial m} \right) - F_m = \frac{\partial h_0}{\partial m} - \frac{c_\theta}{r} \frac{\partial (rc_\theta)}{\partial m} - F_m
\]

Noting that from the velocity triangles the relative and absolute tangential velocities are related to each other as $c_\theta = w_\theta + U$ where $w_\theta$ is the relative tangential velocity and $U$ is the rotational speed, Eq.3.60 and the thermodynamic work definition in turbomachinery combine together which leads to:

\[
\frac{\partial h_0}{\partial m} = U \frac{\partial (rc_\theta)}{\partial m} \quad (Eq.3.61)
\]

Again from the velocity diagram, the angle between the meridional direction and the relative circumferential velocity, $\beta$, will be determined as:

\[
\tan \beta = \frac{w_\theta}{c_m} \quad (Eq.3.62)
\]

Now writing the differential form of Euler’s turbomachinery equation, Eq.3.61 as:

\[
\frac{\partial h_0}{\partial m} = \frac{c_\theta}{r} \frac{\partial (rc_\theta)}{\partial m} - \frac{w_\theta}{r} \frac{\partial (rc_\theta)}{\partial m}
\]

and replacing it into Eq.3.60 it gives:

\[
T \frac{\partial s}{\partial m} = - \frac{w_\theta}{r} \frac{\partial (rc_\theta)}{\partial m} - F_m \rightarrow F_m = - \tan \beta F_\theta - T \frac{\partial s}{\partial m}
\]

To proceed the solution for radial equilibrium equation, consider a body force acting on the flow in $m - \theta$ plane which resists against the fluid flow, $F_s$. In fact this is the force which causes the irreversible increase in the entropy. Thinking of the mid-surface of the blade in addition to the fact that the computational stations are set at the inlet and outlet of blades, one can finds out that this force forms an angle of $\beta$, the relative angle, with the meridional axis. Also to define the mid-surface of blade another vector is needed, the unit vector $\mathbf{B}$. This vector is tangential to the mid-surface of blades and lies in the $l - \theta$ plane and forms an angle of $\epsilon$ to $l$, the quasi direction. Defining these two vectors, the local mid-surface of blade is actually defined. Figure 3-6 shows these vectors positioning in the natural coordinate space.
From the above figure, vector $\mathbf{B}$ and also a unit vector coinciding $\mathbf{F}_s$ can be written as:

$$
\mathbf{B} = \cos \psi \sin (\phi - \gamma) \mathbf{e}_m - \sin \phi \mathbf{e}_\theta + \cos \phi \cos \gamma \mathbf{e}_n \quad \text{(Eq. 3.65)}
$$

$$
\frac{\mathbf{F}_s}{\mathbf{F}_p} = \cos \beta \mathbf{e}_m + \sin \beta \mathbf{e}_\theta \quad \text{(Eq. 3.66)}
$$

The main virtue of composing such vectors is to define the pressure force vector acting on the blade due to the pressure difference of its two sides. Clearly the direction of this vector is perpendicular to the mid-surface plane which can be derived by the product of the vector already obtained in Eq.365 and Eq.3.66. However, since those vector are not orthogonal to each other but build an angle of $\frac{\pi}{2} - (\phi - \gamma)$, the product should be divided by $\cos (\phi - \gamma)$ to get a unit vector for the pressure force, $\mathbf{F}_p$:

$$
\frac{\mathbf{F}_p}{\mathbf{F}_p} = \frac{1}{\cos (\phi - \gamma)} \mathbf{B} \times \frac{\mathbf{F}_s}{\mathbf{F}_s} =
$$

$$
- \sin \beta \cos \psi \mathbf{e}_m + \cos \psi \cos \phi \mathbf{e}_\theta + \left( \frac{\sin \phi \cos \beta}{\cos (\phi - \gamma)} + \sin \phi \cos \gamma \tan (\phi - \gamma) \right) \mathbf{e}_n \quad \text{(Eq. 3.67)}
$$

Figure 3-7 shows this vector.
Now a major simplification can be done on the equilibrium equation if the force vectors of the natural coordinate system \((m, \theta, n)\) be written by use of these opposing and pressure forces. It gives:

\[
F_m = F_s \cos \beta - F_p \sin \beta \cos \epsilon \quad \text{(Eq.3.68)}
\]
\[
F_\theta = F_s \sin \beta + F_p \cos \beta \cos \epsilon \quad \text{(Eq.3.69)}
\]
\[
F_n = F_p \left( \frac{\sin \epsilon \cos \beta}{\cos (\varphi - \gamma)} + \sin \beta \cos \epsilon \tan (\varphi - \gamma) \right) \quad \text{(Eq.3.70)}
\]

and replacing these relations for the forces in meridional and tangential directions into Eq.3.64 it ends up to:

\[
F_s = -\cos \beta \ T \frac{\partial s}{\partial m} \quad \text{(Eq.3.71)}
\]

then imposing this relation to Eq.3.69 will provide an expression for the pressure force based on the tangential force:

\[
F_p = \frac{F_\theta}{\cos \beta \ \cos \epsilon} + \frac{\sin \beta}{\cos \epsilon} T \frac{\partial s}{\partial m} \quad \text{(Eq.3.72)}
\]

Now having \(F_s\) and \(F_p\), it is possible to write the normal and meridional forces Eq.3.68 and Eq.3.70 in terms of tangential velocity. This leads to substitution of \(F_m\) and \(F_n\) terms in the available radial equilibrium equation Eq.3.59, with the parameters which are already available or can be calculated in the iterative computational procedure. Noting that the tangential velocity, \(C_\theta\), is also replaced by \(C_m \tan \beta\) it ends up to:

\[
c_m \frac{\partial c_m}{\partial t} = c_m \sin (\varphi - \gamma) \frac{\partial c_m}{\partial m} + \cos (\varphi - \gamma) \frac{c_m^2}{r_m} - c_m \tan \beta \ F_\theta + \frac{\partial h_\theta}{\partial t} - T \frac{\partial s}{\partial l} - \tan \epsilon \ F_\theta + \left[ \sin (\varphi - \gamma) \cos \beta - \tan \epsilon \sin \beta \cos \beta \right] T \frac{\partial s}{\partial m} \quad \text{(Eq.3.73)}
\]

where, from Eq.3.44:

\[
F_\theta = \frac{c_m}{r} \frac{\partial (r \ c_m \ tan \beta)}{\partial m} \quad \text{(Eq.3.74)}
\]

Provided with the simplified radial equilibrium equation Eq.3.73, the flow field in a turbomachine can be divided into two parts: the bladeless part and the bladed one. Firstly considering the bladeless part, all the terms which contain \(\epsilon\) in Eq.3.73 will be removed. But the main important conclusion comes if one investigates the nature of forces on the natural coordinate system of the bladeless gap. It can be justified that the pressure force, \(F_p\), equals to zero in this region where there is no blades. As an immediate outcome, it will be seen from Eq.3.70 that the force acting in the normal direction is also zero while two other orthogonal components of force field can be determined by combination of Eq.3.68 Eq.3.69 and Eq.3.71:

\[
F_m = -\cos^2 \beta \ T \frac{\partial s}{\partial m} \quad \text{(Eq.3.75)}
\]
\[
F_\theta = -\sin \beta \ \cos \beta \ T \frac{\partial s}{\partial m} \quad \text{(Eq.3.76)}
\]

these equations show that in swirling flow the change of entropy in the flow direction creates body forces in meridional and tangential directions.
In the bladed region, the radial equilibrium can be solved under different available data and various iteration procedures. Typically, given the total temperature the iterative procedure deploys Eq.3.73 (or its counterpart in the relative coordinate system) where the variables are $c_m$ and $\beta$. The other variants are either inputs or will be updated through the iteration steps to obtain convergence. In the computational schemes the mass conservation equation and the energy equation will also be discretized and updated as the solution proceeds.

Main common drawbacks of all throughflow solutions are their dependency to the experimental loss modeling correlations and also their inherent incapability to predict the boundary layer development as the flow passes through the blades. Boundary layer formation on the blade surfaces, hub and tip walls cause the flow passage to shrink in a real turbomachine. This phenomenon plus the well-proven effects of the complex flow structures such as hair-pins and/or horse-shoes brings out an essential need of blockage estimation to make the predictions of computational codes closer to the actual flow condition. This blockage factor is from the order of boundary layer thickness at the annulus endwalls and today it is very dependent to the empiricism however the mass flow correct prediction and the stage designs are greatly affected by this factor. A misprediction of this value may lead a number of stages not to work on their design point which reduces efficiency. The necessity for good prediction of flow capacity becomes more significant regarding to its effect on mass flow and turbine upstream pressure where too low flow capacity leads to low mass flow taken into a steam turbine and hence lower power output than desired. The same problem in a gas turbine may cause compressor surge due to higher turbine inlet pressure and then higher compressor pressure ratio. On the other hand, too high flow capacity values of a turbine causes too low isentropic drop at design mass flow rate and hence low power output.

Basically, the throughflow codes can be used in the design stage, when the total temperature, pressure and the angular momentum are set to seek the flow angles either in the analysis mode, where based on the known machine geometry its performance under different working conditions is going to be predicted. The method reliance on the empirical data was explained, hence it can be interpreted that any throughflow code (including the streamline curvature solutions) are calibrated against a number of tested machines. Based on these facts, throughflow models may predict credible design parameters provided that the aimed turbine characteristics are close to the turbine used in the model calibration. Adamczyk (1999) explained that the 3D CFD codes, which work based on inputs from calibrated throughflow codes, result in good designs although he describes that strict reliance on the 2D aerodynamics inputs may decrease the full functionality of CFD programs during the design procedure.
4 Experimental Method

As described in previous chapter, a major resource for calibration, evaluation and validation of any computational method is to compare its predictions against some real flow experiments. Given the fact that many limitations such as the equipment accessibility, probes and sensors working temperature and pressure range, costs, etc. usually restrict researchers to obtain the required data directly from an actual size working turbomachine, the need for conducting different types of test rigs can be understood. However, live monitoring of critical system information is usually possible in the industrial installed equipments (for instance in a power plant) but surely they are not designed to measure the exit flow angles of the blade or to illuminate the boundary layer separation points and so on. In addition, having test facilities makes it possible to gain data under various conditions where a real machine is not allowed to work. Then the test rig flexibility and faster start-up and shut-down procedures rather than the commercial instances can also be added to the abovementioned reason of test equipments necessity.

Focusing on turbines, nowadays two main categories of testing facilities exist worldwide. The first division is the stationary cascade equipments where a number of blades are installed in a flow channel. The cascade tests are usually easy to be controlled. Many different blades configuration and arrangement can be achieved by setting the angles with respect to the inlet flow axis and/or the distance between the blades. Also the possibility of flow visualization and convenience of handling the measurement probes can be added to the counted advantages. However, it seems evident that the results out of the cascade tests are not fully representing the physics of real flow in a rotary turbomachine and this fact has raised the demands for other type of test machines, the rotary cold flow test turbines. The biggest problems in the way are firstly the costs of having such an apparatus and then their low flexibility in case of blading or end-walls geometry. However access to the flow is more difficult in rotary test rigs than the stationary cascades but the outputs eventually can be analyzed more reliable and closer to what happens in an actual turbomachines.

As a part of this thesis a set of experiments were conducted in the Heat and Power Technology department at KTH where a rotary cold flow test rig was used, as will be described more in details in this chapter. The experiments objectives were to measure the performance under different working conditions. Three pressure ratios and two turbulence intensities (only at the medium pressure ratio) were tested and the outputs were recorded for further post-process studies.

To measure the turbulence level, a constant temperature anemometry (CTA) method was used where a hot-wire probe plugged into the flow channel and the inlet velocity profile was sought by radial traverse of the probe from tip to hub.

4.1 Test Turbine Specifications and Instrumentations

The test rig at KTH has been working since 1989. The machine is a cold flow turbine where the air is used as working fluid. A 1MW compressor with the maximum pressure of 4 bars pressurizes the air toward the main turbine inlet. Inlet temperature to the turbine can be set between 30 to 80 ℃.

In an open loop mode, air passes a cooler and condensed water separator, an orifice and a filter before entering the turbine and then it passes through a honeycomb and a converging chamber to achieve a homogenized flow. Right before entering to the stages passage a perforated plate (removable) is installed to create required turbulence intensity. After moving through the vanes and blades in every stage it will leave the building via outlet duct by means of an exhaust fan.

The rig can be used in different configurations either by altering the number of stages up to three or by change of (or removing) the turbulence grid at the upstream of stages inlet. To control the rotation speed a hydraulic water brake system is used. The shaft power is damped into the water brake. The water flow
rate is regulated by a control valve and the torque is measured in a calibrated torsion shaft torque-meter system.

Both the vane and blades profiles have been designed straight in their spanwise direction though while the blades are mounted radially on the rotor disc the vanes have been stacked with a certain lean angle with respect to the stator disc radial direction. According to its highly cambered rotor blade profile, the turbine is of a low reaction degree design which is normally used in steam turbines. As a low speed research turbine (in comparison with the industrial actual size ones), it has been designed to work in the subsonic flow regime and low Mach numbers. Each rotor blade has two sealing strips or shroud platform to control the tip leakage. There is no contact between the shrouds and turbine casing at high speed loading cases.

Figure 4-1 shows the turbine sketch in its 2 stages configuration but the reader has to be aware that the tests were performed on the second stage and the first stage was dismounted (this configuration can be seen in chapter 5, Figure 5-1). Also, a number of geometrical specifications and mid-span characteristics under nominal loading (design point) are detailed in Table 4-1 for this stage. To keep the SIT documentations naming, this configuration will be called 4b which implies that the first stage (4a) has been removed. The stage parameters in this table are in fact calculated based on one-dimensional mean-line calculations during the design phase.

A manual control unit rack in the turbine control room plus controlling software are used to set the desired loading point and to monitor the condition during the experiments. Mainly, load setting is adjusted by varying the shaft speed and the opening percentage of the main compressor bypass valves (that results in a change of mass flow rate into the turbine). In addition a safety annunciator panel on the rack alarms the occurred functional troubles such as low turbine oil pressure level. Figure 4-2 shows the turbine control unit rack.

![Figure 4-1. Test turbine at Department of Energy, KTH. Two stages configuration (4ab).](image)

A datascanner system collects analogue signals from barometers, thermocouples, humidity sensors, etc. and online programs, a Labview code (TURBAQ) and an Excel program TURBAN, are used to save and convert the raw data into inferrable information instantly. As mentioned before the mass flow is measured at the upstream flow where air passes an orifice.

Any major trouble actuates the pneumatic inlet valve and the whole flow will be blocked toward the turbine. Also, the machine vibration level is continuously measured by means of a vibrometer which helps to choose the loading points away from the system igen-frequencies to avoid resonance.
### Stage 2 (4b)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Stator</th>
<th>Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static-to-static pressure ratio</td>
<td>-</td>
<td>1.23</td>
</tr>
<tr>
<td>Mean degree of reaction</td>
<td>-</td>
<td>0.17</td>
</tr>
<tr>
<td>Mean isentropic velocity ratio</td>
<td>-</td>
<td>0.47</td>
</tr>
<tr>
<td>Reynolds number (\gamma \times 10^5)</td>
<td>3.9</td>
<td>1.8</td>
</tr>
<tr>
<td>Shaft speed rpm</td>
<td>-</td>
<td>4450</td>
</tr>
<tr>
<td>Flow turning °</td>
<td>95</td>
<td>134</td>
</tr>
<tr>
<td>TE Relative Mach number</td>
<td>-</td>
<td>0.49</td>
</tr>
<tr>
<td>Number of blades</td>
<td>-</td>
<td>42</td>
</tr>
<tr>
<td>Hub diameter mm</td>
<td>355</td>
<td>355</td>
</tr>
<tr>
<td>Nominal width mm</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Real chord mm</td>
<td>34.4</td>
<td>25.9</td>
</tr>
<tr>
<td>Trailing edge thickness ((t_b/\text{Chord}))</td>
<td>%</td>
<td>0.74</td>
</tr>
<tr>
<td>Setting angle °</td>
<td>46</td>
<td>23</td>
</tr>
<tr>
<td>Tip-to-hub diameter ratio</td>
<td>-</td>
<td>1.149</td>
</tr>
<tr>
<td>Pitch-to-chord ratio</td>
<td>-</td>
<td>0.831</td>
</tr>
<tr>
<td>Aspect ratio</td>
<td>-</td>
<td>0.77</td>
</tr>
<tr>
<td>Trailing edge stacking mm</td>
<td>-12.8</td>
<td>0</td>
</tr>
<tr>
<td>Axial stator-rotor spacing mm</td>
<td></td>
<td>10</td>
</tr>
</tbody>
</table>

Table 4-1. Test turbine geometric parameters and operational characteristics at design loading. One stage configuration (4b).

![Figure 4-2. Test turbine control unit rack located in the control room.](image)
4.2 Performed Tests

Aiming to investigate the effect of pressure ratio on the turbine performance, it was decided to assign three different pressure ratios for the experiments. Therefore, tests were done under the static to static pressure ratios 1.15, 1.23 and 1.35. However, these are nominal values and in each trial the exact value was recorded according to the setting. In addition, since in the computational simulations the total-to-static pressure ratio was supposed to be used, its value was also measured for all the cases to be directly exported to simulations codes later.

In addition, to evaluate the effect of inlet turbulence intensity on the turbine output data two different cases were run and at the medium pressure ratio value, 1.23. In the first configuration a turbulence generator grid was used at the flow inlet to the stage and later this grid was removed and turbine tested under a range of different loadings in both cases. As briefly discussed before the turbulence intensity measurements were done in these conditions by virtue of a CTA method. The results can be found more in detail in Appendix A where turbulence levels were measured as 6% and 1% for the cases with and without turbulence grid respectively.

Results of the two other pressure ratios, 1.15 and 1.35, were acquired where the turbulence grid was installed and it was assumed that their turbulence levels had the same value as what measured in the pressure ratio equals to 1.23. Table4-2 shows the performed tests during the experimental part of this project. It is seen that the measurements were done at 71 different cases.

<table>
<thead>
<tr>
<th>Turbulence Grid</th>
<th>Pressure Ratio (static-static)</th>
<th>Shaft Revolution Speed (RPM)</th>
<th>Velocity Ratio Range</th>
<th>Number of Loading Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perforated plate</td>
<td>1.15</td>
<td>1900…5580</td>
<td>0.25…0.76</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>1.23</td>
<td>2300…6200</td>
<td>0.25…0.67</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>1.35</td>
<td>2750…6050</td>
<td>0.25…0.55</td>
<td>13</td>
</tr>
</tbody>
</table>

Table 4-2. Experimental campaign tested cases.

4.3 Experiments Results and Discussions

Results will mainly be divided into two categories depending on turbulence intensity level. The first set of plots is related to the cases with turbulence generator grid, \( \text{Tu} = 6\% \). After that the results from free-turbulence measurements, \( \text{Tu} = 1\% \), also will be presented and compared to their counterparts in the other turbulence level.

Before going through different plots there is a point that should be explained. Looking to the plots it will be questionable that the number of loading points decreases as the pressure ratio increases. This happened because of the bearing temperature limitations. The turbine safety function is set in such a way that it shuts the equipment down as soon as the bearings temperature rises to 80°C (~ 79°C). Now remembering the Mollier’s diagram it will be seen that in the case of a known exit pressure, the higher pressure ratio indicates a higher isentropic enthalpy drop, \( \Delta h_s \), over the turbine. In addition, the stage loading and static-to-static velocity ratio concepts were introduced in chapter 2 where they were defined as non-dimensionalized ratios of blade speed and enthalpy drop. Thus, there is no wonder that having an identical loading as of a lower pressure ratio in a higher pressure ratio needs a higher blade speed, \( U \). Knowing that
the radius remains the same this means the shaft revolution speed has to increase which leads to a higher bearing temperature. Figure 4-3 enlightens this point. In addition, there is also another hidden fact. Along with the tangential force in any rotating turbomachinery there also exists an axial thrust force which arises as the pressure difference at two sides of the turbine increases. Again considering a known exit pressure, higher pressure ratios indicate higher axial pressure differences and hence higher thrust. This axial force affects the contact forces inside the turbine bearing and causes the temperature to increase.

Figure 4-3. Shaft rotational speed (rpm) vs. velocity ratio. In the same velocity ratios required rpm increases as pressure ratio increases

### 4.3.1 Mass Flow, Turbine constant and Flow Coefficient

#### 4.3.1.1 Mass Flow

As Figure 4-4 shows mass flow rates were measured under different loading points in each pressure ratio. It can be seen that the overall range of mass flow rate increases as pressure ratio raises. This can be figured out by thinking of a higher flow velocity due to driving force in the continuity equation. Also in a certain pressure ratio, mass flow rate increases as the velocity ratio decreases. It can be explained by the effect of loading variation on the velocity triangle and pressure differences. In fact flow acceleration in rotor reduces due to a lower blades speed (and hence lower velocity ratio) and thus pressure drop over rotor should decrease which leads to a lower value of $p_2$ on the Mollier diagram, Figure 2-4, since $p_3$ is already held constant. Considering the difference between the total inlet pressure and the static outlet pressure of each row as the driving force, then it increases over the vanes as $p_{01} - p_2$ rises and the mass flow rate through the stator also increases as a result. The same mass flow must passes through the rotor blades as well where the relative inlet velocity to the rotor, $w_2$, and the relative total inlet pressure, $p_{02,rel}$, are increasing. Higher values of $p_{02,rel}$ explains the higher mass flow rate over the blades.

Take a look into the mass flow graph at the medium pressure ratio (1.23) clarifies why using raw mass rate is not so appropriate. It is seen that the values are more scattered around the plotted trend-line and this happened since the test trials of this particular pressure ratio were done during different days and minor variations in the humidity, temperature and pressure caused measured values of different working conditions not to follow a smooth curve in case of mass flow. As described in chapter 2, to avoid such
problem it is wiser to use the turbine constant or flow capacity (swallowing capacity) curves to present the variation of mass flow.

4.3.1.2 Flow Capacity

Figure 4-5 shows the flow capacity, Eq.2.25, curves measured at three pressure ratios. In comparison to Figure 4-4 it can be seen that the acquired values for different loadings on a specific pressure ratio follow the mean thread since they have somehow normalized and the effects of pressure and temperature are also included in the representative output. Notable that here the horizontal axis is the stage loading since this is the usual way of mass flow presentation in gas turbine literature. However in comparison to Eq.2.20 here the total-to-static enthalpy drop has been used and hence the exit kinetic energy from the blade, $(\frac{c_3^2}{2})$, is also included. It is observed that the turbine swallowing capacity increases by increase of the pressure ratio and stage loading which can be described by the same reasons already stated for the mass flow.
4.3.1.3 Turbine Constant

Turbine constant, Eq.2.26, is plotted in Figure 4-6 below. On the horizontal axis the static-to-static velocity ratio has been used to keep the track of previous studies on the test turbine. This is seen that the turbine constant level decreases as the pressure ratio increases and also in a pressure ratio it again goes down as the velocity ratio increases. The fact that higher pressure ratios have lower turbine constant can be explained if one notices to its definition where the pressure ratio is taken into account inherently as:

\[ C_T = \frac{\dot{m}}{\sqrt{\frac{p_1^2 - p_2^2}{p_1 v_1}}} = \frac{\dot{m}}{p_3} \sqrt{\frac{(\frac{p_1}{p_2})^2 - 1}{p_1\rho T_1}} \]

\( \frac{\dot{m}\sqrt{T_1}}{p_1} \cdot \frac{\sqrt{R}}{p_3} = \frac{\dot{m}\sqrt{T_1}}{p_1} \cdot \frac{\sqrt{R}}{1 - 1/(\frac{p_1}{p_3})^2} \) (Eq.5.1)

actually the first fraction reminds a similar concept as the flow capacity. From the second fraction it can be inferred that in comparison of two cases while assuming the first term i.e. \( \frac{\dot{m}\sqrt{T_1}}{p_1} \) is the same for more clarification, the one with a higher pressure ratio has smaller turbine constant.

![Figure 4-6. Turbine constant (Eq.2.26) vs. static-t-static velocity ratio. Tu=6%.

4.3.2 Stage Efficiency

Figure 4-7 shows the stage total-to-static efficiency, Eq.2.16, values for three tested pressure ratios normalized by their maximum quantity. To understand the overall shape of efficiency curves, one should notice the outstanding role of incidence losses in efficiency drop. It has known that the flow over the blades is more susceptible to the positive incidence (Moustapha, 2003). From the velocity triangle, Figure 2-4, it is seen that while the outlet absolute velocity from the stator (inlet to blade), \( c_2 \), is only slightly affected by the rotational speed the relative velocity, \( w_2 \), tends to the tangential direction (positive incidence) as the blade speed decreases. On the other hand, the incidence becomes negative as the rotational speed increases. Combining all the facts together it is realized that moving from the peak
efficiency point where the flow angles have their optimum quantities toward lower velocity ratios (higher loading) the efficiency drops faster in comparison with the opposite direction.

It can be seen that the curves completely coincide with each other which was expected before for such a low Mach number turbine which is working under relatively low pressure ratio. For more clarification if one rewrites Eq.2.26 as:

$$\eta_{ts} = \frac{h_{01} - h_{03}}{h_{01} - h_{3ss}} = \frac{h_{01} - h_{03}}{(h_{01} - h_{03}) + (h_{03} - h_{03ss}) + (h_{03ss} - h_{3ss})}$$

and also an acceptable assumption from Mollier’s diagram is \( h_{03ss} - h_{3ss} = \frac{c_3^2}{2} \), then the stage total-to-static efficiency can be written as:

$$\eta_{ts} = \frac{h_{01} - h_{03}}{(h_{01} - h_{03}) + (h_{03} - h_{03ss}) + \frac{c_3^2}{2}}$$  \hspace{1cm} (Eq.5.2)

Eq.5.2 indicates that the effect of pressure ratio in the stage total enthalpy drop is included in both nominator and the first term of denominator. For the second term in denominator, it can be assumed to remain fairly constant in different pressure ratios (though the curve of constant pressure \( p_{03} \) can shift up or down). Variations of the exit kinetic energy term in denominator with respect to change of pressure ration cannot be too drastic when the turbine is working in a subsonic flow regime and low Reynolds numbers. Hence, it seems a slight change of pressure ratio from 1.15 to 1.35 does not affect the turbine efficiency.

In pressure ratio 1.35, there is a mere point close to the curve peak which does not follow perfectly the same trend as the other tested points. This maybe happened as a result of not fully stabilized measurement. At higher velocity ratios, the rise of bearing temperature impeded time to elapse for a good

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Figure 4-7. Stage total-to-static efficiency (nondimensionalized by the its maximum value) vs. velocity ratio, \( Tu=6\% \).
while before data acquisition. However, not surprisingly it is realized large variation of pressure ratio and higher Mach numbers definitely affects the stage efficiency. Figure 4-8 also confirms that to make a detectable change in efficiency a larger quantity of pressure ratio should be applied.

Figure 4-8. Stage efficiency vs. pressure ratio. Characteristics of P&WC High Work Research Turbine. (Moustapha, 2003)

4.3.3 Degree of Reaction

In chapter 2 it was described that there are different definitions for the stage reaction degree in literature and even between industrial turbine manufacturers. Here the correlation based on isentropic enthalpy drop (Eq.2.19) is used and its value can be directly read from TURBAN. Figure 4-9 shows reaction degree curves in three pressure ratios against the velocity ratio and the plot of reaction degree versus stage loading can also be seen in Figure 4-10.

Figure 4-9. Stage degree of reaction (based on isentropic enthalpy drop, Eq.2.19) vs. velocity ratio, Tu=6%.
From the above figures it can be seen that the degree of reaction decreases as stage loading values become larger. In fact higher stage loading means a higher change in tangential velocity through the blades and hence higher flow turning (less flow acceleration) in rotor which causes a lower pressure difference over the blade. Therefore, recalling the definition of reaction degree based on pressure drops it can be explained that how higher loadings lead to smaller reaction degree values.

4.4 Turbulence Intensity Effect

In section 4.3 the experimental measurements when a perforated plate was mounted upstream of the stator vane was presented (Tu=6%). Here those data will be compared to the measurement when this plate was removed (Tu=1%).

To investigate the effect of free stream turbulence level on flow in an axial rotary turbomachine more in depth, a number of variables should be taken into account amongst them Reynolds number and turbulence micro and macro length scales are of high importance. Butler et al. (2001) showed that the effect of free stream turbulence level is amplified at higher Reynolds number values while this is well-proven by many other researchers. In fact elevated turbulence intensity moves the onset of laminar to turbulent transition region toward the blade leading edge and reduces the probability of a laminar separation. Generally in low turbulence intensities the flow is more susceptible to the adverse pressure gradients. The turbulence intensity effects will be more studied later where the computational results will be discussed. In the following the acquired data during the experimental campaign are shown.

Figures 4-11 to Figure 4-14 show the results of these trials in comparison to their counterparts in Tu=6%. They reveal that for all the studied parameters, surprisingly there is no detectable deviation between outputs from two cases of turbulence level though a difference in efficiency levels was expected at least.
This may imply existence of a certain level for turbulence intensity to be influential on the turbine performance.

![Figure 4-11](image1)

**Figure 4-11.** Measured flow coefficient vs. stage loading for two different levels of turbulence intensity. Static to static pressure ratio is 1.23 for both cases.

![Figure 4-12](image2)

**Figure 4-12.** Measured turbine constant vs. velocity ratio for two different levels of turbulence intensity. Static to static pressure ratio is 1.23 for both cases.
Figure 4-13. Measured total to static efficiency vs. velocity ratio for two different levels of turbulence intensity. Static to static pressure ratio is 1.23 for both cases.

Figure 4-14. Measured degree of reaction (based on isentropic enthalpy drop) vs. velocity ratio for two different levels of turbulence intensity. Static to static pressure ratio is 1.23 for both cases.
Although, using a set of experiments on a cold flow turbine which works under low Reynolds numbers, rather low Mach numbers and pressure ratios it is not so wise to draw a general and definite conclusion but it is worth to be investigated more. For example knowing that the profile loss contributes as a major source of efficiency losses, Figure4-15 shows outputs from a number of tests in Moscow Energy Institute where profile and tip leakage losses are plotted versus different turbulence levels for reaction and impulse turbine types. It is seen that at turbulence intensities lower than 5–6% the variations in the abovementioned losses are less detectable in comparison to the cases with turbulence intensity larger than 6% (Dejc, 1973).

![Figure 4-15](image1.png)

**Figure 4-15.** Profile loss coefficient and tip-leakage loss coefficients vs. inlet turbulence intensity for impulse and reaction turbine. Experimental data from Moscow Energy Institute. (Dejc, 1973).

In other experimental trials, Choi et al. (2004) sought the onset of boundary layer transition region. They measured the distance where an immediate ascent of Nusselt number occurs under different turbulence intensity and Reynolds number values. Figure4-16 shows their acquired data for two different Reynolds numbers and five turbulence levels. Looking on the suction side, it is seen that transition point is almost the same for the turbulence levels up to 6 percent (at $\frac{x}{C} \approx 1$) but a change to 15 percent in turbulence intensity causes a visible shift of transition point toward the leading edge ($\frac{x}{C} \approx 0.5$).

![Figure 4-16](image2.png)

**Figure 4-16.** Nusselt number vs. distance to the leading edge (nondimensionalized by chord length). Data acquired from a cascade test done by Choi et al. (2003) to determine the boundary layer transition point. (a) Re=105000 (b) Re=52000. The blue vertical line indicates transition point of cases with turbulence levels up to ~6% and the red one shows that point for $Tu=15%$. 

-35-
It has to be mentioned that different studies dedicated to turbulence level influence has been done recently and they include a broad range of conclusions which are sometimes even opposing each other. For example the results of experiment from Schulz (1986) in Figure 4-17 shows a different behavior of Nusselt number in different turbulence levels in comparison to what was seen in Figure 4-16. Then, the importance of Reynolds number is also highlighted.

Mayle et al. (1997) introduced idea of an effective turbulence where they have tried to combine the turbulence level with its length scale parameters. As they concluded, the effect of turbulence can be understood just when its intensity and length scales spectrum is known. For the gas turbine designers, they have suggested to deploy turbulence macroscale (e.g. the same order as the gap to the upstream row) to calculate a turbulence Reynolds number and use it along with the effective turbulence level to predict the aerodynamic performance.

To wrap up, the experimental outputs of this thesis state that changing turbulence intensity from 1% to 6% does not affect the turbine investigated parameters. Albeit, as explained before it is too difficult to count it as a general and definite behavior of turbines under different turbulence levels, Reynolds number and turbulence micro and macro length scales.
5 Simulation Method

This chapter is dedicated to the computational segment of the thesis. Through this part the procedure and results of mean-line (1D) and throughflow (2D) solutions by use of SIT computational codes, MAC1 and BETA2 respectively, are discussed.

5.1 Geometry Simulation

As the first step, the codes must be provided with nicely defined geometrical parameters of both the blades and computation region. Specifications of the test turbine were described more in details through the experimental method chapter. Figure 5-1 shows the sketch of that turbine. Given that the experiments were just executed on the second stage and the first stage was removed, inner wall of the inlet cone was directly moved toward the stator row and its outer surface was replaced by another one to match the new location.

![Figure 5-1](image)

Figure 5-1. Test turbine at KTH, one stage (4b) configuration. The first stage has been removed, the inner inlet cone was shifted toward the stator and the outer inlet cone has been replaced with an appropriate one.

5.1.1 Computational Region

According to the test turbine drawing, coordinates of the outer and inner walls are extracted. In case of the curve for stator blade tip wall, the extracted coordinates from the drawing was compared to the document (Paulsson, 1998) which explicitly states how these points coordinates are defined and they showed a very good coincidence to each other. Thus, it was decided to keep the drawing coordinates as a reference to generate the wall geometry since it gives smoother transition when it connects to the next part of the wall that was generated just based on drawing coordinate extraction. Figure 5-2 shows that they can be hardly distinguished from each other.
Seeking 1D and 2D solutions then this region can be assumed as a channel. It also has to be considered that MAC1 and BETA2 solve the problem using the values at the inlet, throat and outlet of each row of blades. Thus, exact geometry of the channel and its extensions - ahead of the stator blade inlet and after the rotor blade outlet - has been just produced for future 3D tasks. Figure 5-3 shows the generated channel. In addition, coordinates for both walls are attached in Appendix B where the stator inlet was decided as x=0. In addition, the leakage gaps and rotor blade shrouding have been sealed geometrically but their effects can be modeled in computations.

5.1.2 Vanes and Blades Geometry

To generate the blade sections an in-house SIT software, CATO (vr.1.9.7.0), has been used. Given the provided coordinate points CATO basically deploys Bezier function to generate trailing edge(TE), suction side(SS), leading edge(LE) and pressure side(PS) curves. These Bezier functions are controlled manually to obtain a blade section(profile section) with an acceptable deviation from a section which is based on the coordinate points(coordinate section). Since the blades of both stator and rotor are stretched linearly over their spans it is not necessary to create more than two sections for each one to form a blade. Even it does not seem reasonable to have a third section in this case since any new section adds up its own deviation(error) to the supposedly straight blade. But it must be known that for non-linear blades, the
A larger number of sections is of course required to produce the exact span-curve of the blade as much as possible.

Knowing that MAC1 and BETA2 ignore any geometric entity outside of their defined computational field, here the channel, two sections have been generated at radiiueses R=170mm and R=230mm outside of the channel inner and outer walls. The lean of the stator blades has been set by a shift in y-direction of the section local coordinates system in such a way that there is distance of 12.84mm from the trailing edge to the local x-axis. Figure 5-4 to Figure 5-10 show the sections which are created to generate the stator and rotor blades, their deviation from the coordinate section, the generated blades and the final geometry that includes both the channel and blades.

Figure 5-4. (Left) Stator Blade profile (P21), generated in CATO at R=170mm, (Right) Deviation from coordinate profile

Figure 5-5. (Left) Stator Blade profile (P21), generated in CATO at R=230mm, (Right) Deviation from coordinate profile
The x-axis scale in the deviation plots includes both suction side and pressure side curves. It starts from zero at the trailing edge on the suction side and ends to one at the trailing edge on the pressure side. The vertical dashed line indicates the throat position where the smallest possible deviation of coordinates and produced profile has been tried to be obtained because of the importance of throat position in the computations.
Figure 5-8. (Left) Stator vane, (Right) Rotor blade. Linear Bezier points distribution over the blades span has been shown.

Figure 5-9. 3D visualization of the geometry simulation.

Figure 5-10. 2D visualization of the geometry simulation. Radius on vertical axis and x-position on horizontal axis.
5.2 Calculations

5.2.1 Setup

Defining geometry of the computational field, now it is possible to write the input files for the solver codes. The basic core of turbine iterative calculation schemes was described before and here the main concentration will be on the outputs from the codes (1D and 2D) and to compare them with the already available experimental data. As described in the methodology of this work, previous experimental data was used to write the main structure of the input code. However the results of those computations are not going to be discussed here very detailed but it is worth to state that the channel blockage estimation was decided during that stage. Later after gathering new experimental data - see chapter 4 – the new boundary conditions were implemented into the codes. Therefore, the same as experimental part there are totally four different boundary conditions. Three different pressure ratios where the perforated plate is used as the turbulence generator in addition to one case of calculations where there is no turbulence grid, at the medium pressure ratio case.

For the sake of work pace it is wiser to first write the 2D input file for BETA2 and solve the case. As one of its outputs BETA2 generates the input file for the one dimensional code, MAC1. The geometry file (path file) which is used for both cases is the same and it has to be defined in a MERCOO file attribution. As mentioned before there are many correlations inserted in the codes to model different losses, also a certain value can be allotted to a factor to adjust their influence level. During the present study it was decided to leave all of them at their default values in the code (i.e. to set the coefficients equal to 1). Among different losses the codes provide two approaches for the friction loss modeling. The first approach is based on the loss correlations by Mamaev – see Dahlquist(2008)- and had already been included in the codes and the other which has been recently added to the solvers works on the basis of an estimation of laminar to turbulent boundary layer transition point. In the rest of this text, they will be called Old approach and Boundary Layer (BL) approach respectively. The calculations then were executed by use of both approaches to provide a set of data that might help to make their difference more clear.

To have a rough estimation of laminar to turbulence boundary layer transition point on the vane and blade (which are required as input data of 1D and 2D codes in BL approach) a MISES calculation was performed in two different turbulence levels, 6% and 1%, just at the design loading. The obtained transition points were applied in their related input files but also the free-turbulence case was once again solved with the transition points of the other case (i.e. transition point was assumed the same). It was done since in previous chapter there was a claim that in such range of turbulence intensity variations the transition point may not change too drastically. The comparison of results against the measured data can help to judge how MISES transition point prediction is reliable to be used in BL approach.

In addition, it was decided to use a blockage of 2.5% based on the codes calibration process against previous available data at the design point (pressure ratio of 1.23). As the project went on, because of a detectable deviation of predicted mass flow parameters from experimental measured data at pressure ratio 1.36 it was decided to run a set of calculations in this pressure ratio while the blockage was set 1.5%. This change just applied once (under BL approach and when the turbulence grid was installed) to check if it corrects the deviation. However the main idea was to keep the blockage percentage constant to judge its effect on the deviations.

All computations have been done without any cooling air flow and also no diffuser was included. In addition the flow inlet angle was considered completely axial and the fillet radius for both the vanes and blades was set to zero.

The inlet temperature and pressure profiles were assumed to have constant values over the spanwise direction. This can be considered as an acceptable assumption except a small region near the hub and tip walls. Theoretically the Prandtl’s boundary layer theory states that there should be no pressure gradient in
the wall normal direction inside the boundary layer, \( \frac{\partial p}{\partial y} = 0 \). Albeit, distortion of inlet pressure profile due to formation of an impinging jet from the perforated plate holes toward the inlet cone walls may cast doubt on this assumption.

Considering the rather large number of loading points for each case and also the demanding job of replacing new boundary condition parameters (such as pressures, rotation speed, gas composition, etc.) one by one in the codes that could also increase the user’s error possibility, a more systematic procedure was designed. In fact a MATLAB code was employed to create the 2D input script. The required data was automatically picked from the TURBAN Excel sheets of experimental campaign and inserted into the input file that its main structure was provided. In addition, this code categorizes various cases based on their different pressure ratios and turbulence levels into separate folders.

The solving and results post-processing stages were also done under control of a number of MATLAB programmes. The input files were initially sent to BETA2 to be solved and the produced 1D input files were directly sent to MAC1 for one-dimensional calculation. Output parameters were also read from one and two dimensional output files to be used in post computations directly or after some manipulation to achieve desired plots and study the trends.

Excluding the vast number of preliminary computational runs, a total number of 173 computations were done in BETA2 as well as MAC1. Table 5-1 shows them in detail.

<table>
<thead>
<tr>
<th>Turbulence Grid</th>
<th>Friction Loss Model</th>
<th>Pressure Ratio Static/Static</th>
<th>Transition Point (x/C) Vane-Blade</th>
<th>Blockage %</th>
<th>Number of Loading Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perforated Plate</td>
<td>OLD</td>
<td>1.15</td>
<td>Not Applicable</td>
<td>2.5</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.23</td>
<td></td>
<td></td>
<td>18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.35</td>
<td></td>
<td></td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>BL</td>
<td>1.15</td>
<td>0.5-0.2</td>
<td></td>
<td>22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.23</td>
<td></td>
<td></td>
<td>18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.35</td>
<td></td>
<td></td>
<td>13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.35</td>
<td>1.5-0.2</td>
<td></td>
<td>13</td>
</tr>
<tr>
<td>No Grid</td>
<td>OLD</td>
<td>1.23</td>
<td>Not Applicable</td>
<td>2.5</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>BL</td>
<td>1.23</td>
<td>0.5-0.2</td>
<td></td>
<td>18</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.8-0.8</td>
<td></td>
<td>18</td>
</tr>
</tbody>
</table>

Table 5-1. Calculation scope of the thesis. Categorized by different boundary conditions and solver parameters.

The calculations were done by the latest available versions of MAC1 (version 1.03a) and BETA2 (version 2.01a). It has to be mentioned that since these codes are not supposed to be used in such boundary condition, typical of test turbines, the codes updated specifically to meet the needs of this thesis. An important check point for the users of these codes is the mass flow and axial velocity convergence status in its related text output file since the graphical convergence interface just indicates the pressure ratio convergence as the iterations forward.

5.2.2 Blockage - A Suggestion

For the present thesis the blockage value was decided upon preliminary computations where the experimental data available from a set of tests reported by Hedlund(2004) was used. To determine this value, turbine constant (Eq.2.26) was used as a criterion. Figure 5-11 shows how turbine constant shifts toward its measured value while the blockage increases to 2.5% (or the Throat Coefficient factor in the codes
goes down to 0.975). In all cases the turbulence intensity was assumed 7% and the Old correlations were used for profile loss modeling and the results from BETA2 were compared with experiments.

Figure 5-11. Turbine constant vs. static to static velocity ratio. Outputs from BETA2 and measurements performed on the test turbine reported by Hedlund(2004).

Here another estimation procedure will be suggested. However this method is not also able to calculate an exact value (in fact no method can) but it seems useful to be used when there are a number of available experimental data. The main advantage is that before writing any code and doing some try and error runs, the designer may come up with an idea of blockage. At its core, it senses how imperfect the theoretically calculated mass flow might be versus the measured data and that value can be used as the blockage percentage. Notable, the blockage should not be considered just as a result of boundary layer growth but also many other complex phenomena (which are not possible for a calculative scheme to capture) are included.

This approach will be described by use of the current available data (measured in this project). Firstly, a very well-known correlation between the turbine pressure ratio and flow capacity, the Stodola’ Ellipse, states:

\[
Flow \ Capaciry = \frac{\dot{m} \sqrt{T_{01}}}{p_{01}} = k \sqrt{1 - \left(\frac{p_{03}}{p_{01}}\right)^2}
\]

(Eq.5.1)

where \(k\) is the Sotola’s constant of the turbine. Although this is the most basic correlation for a single stage turbine and there are modified versions for multi-stage calculations, it has been kept here for the simplicity. Figure5-12 shows how this formula results in an ellipsoid shape.
There, it can be seen that the ellipse radius on the y-axis represents the Stodola’s constant. Thus, fitting a second degree curve to the measured data (all in grid-mounted condition) at design loading gives this constant for the test rig at KTH as Figure 5-13 shows, $k = 41.8679$.

![Figure 5-13. Stodola’s Ellipse based on the measured data for the test rig at KTH. Stodola's constant $k = 41.8679$](image)

Now the Ellipse formula, Eq.5.1 can be treated as follows:

$$
\dot{m} = k \frac{p_{01}}{\sqrt{T_{01}}} \sqrt{1 - \left( \frac{p_{03}}{p_{01}} \right)^2}
$$

(Eq.5.2)

and by use of compressible flow relations total temperature and pressure can be written in terms of the static quantities. Since the outlet static pressure, $p_3$, is known it is more convenient to work with static pressures. Also in this particular case the value of static to static pressure was the quantity that kept constant during the experiment campaign. Thus:

$$
\frac{T_{01}}{T_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2 \right) \rightarrow T_{01} = T_1 \left(1 + \frac{\gamma - 1}{2} M_1^2 \right)
$$

(Eq.5.3)

$$
\frac{p_{01}}{p_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2 \right)^{\gamma - 1} \rightarrow p_{01} = p_1 \left(1 + \frac{\gamma - 1}{2} M_1^2 \right)^{\gamma - 1}
$$

(Eq.5.4)

$$
\frac{p_{03}}{p_3} = \left(1 + \frac{\gamma - 1}{2} M_3^2 \right)^{\gamma - 1}
$$

(Eq.5.5)

also inlet static pressure can be rewritten as:

$$
p_1 = p_3 \left( \frac{p_1}{p_3} \right) = p_3 \cdot \pi_{ss}
$$

(Eq.5.6)

where $\pi_{ss}$ is the static to static pressure ratio. Now, if one writes the total to total pressure ratio term in the right-hand-side of Eq.5.2 as:

$$
\frac{p_{03}}{p_{01}} = \frac{p_{03}}{p_3} \cdot \frac{p_3}{p_1} \cdot \frac{p_1}{p_{01}}
$$

(Eq.5.7)

and replaces corresponding acquired relations into Eq.5.7 it gives:

$$
\left( \frac{p_{03}}{p_{01}} \right)^2 = \left(1 + \frac{\gamma - 1}{2} M_3^2 \right)^{2\gamma} \cdot \left( \frac{p_3}{p_1} \right)^2 \cdot \left(1 + \frac{\gamma - 1}{2} M_1^2 \right)^{-2\gamma}
$$

(Eq.5.8)
Finally the mass flow rate, Eq.5.2 can be re-written by use of the already derived correlations. But before that, a coefficient will be introduced to represent the theory deviation from measurements, \( f_{\text{mass}} \). In fact the aim of this suggested method is to determine this coefficient. Regardless of every complex scheme used in computational codes, this proposed quantity may represent an appropriate order of deviation of mass flow prediction from what has been measured. Of course the experimental data uncertainty can be included also. Eventually Eq.5.2 delivers:

\[
\dot{m} = f_{\text{mass}} \cdot K \frac{p_3 \pi s s}{T_1} \left( 1 + \frac{\gamma - 1}{2} M_1^2 \right)^{\frac{2 \gamma - 1}{(\gamma - 1)}} \sqrt{1 - (1 + \frac{\gamma - 1}{2} M_3^2)^{\frac{2 \gamma}{\gamma - 1}}} \left( \frac{1}{\pi s s} \right)^{\frac{2 \gamma}{\gamma - 1}} \left( 1 + \frac{\gamma - 1}{2} M_1^2 \right)^{\frac{2 \gamma}{\gamma - 1}} \]  

(Eq.5.9)

Figure 5-14 shows the plotted curves of above equation for different values of inlet and outlet Mach numbers at static to static pressure ratio 1.23 for a mass correction factor of 0.975. The other parameters are already known. In fact, by knowing Mach numbers and a measured mass flow rate at a certain pressure ratio it is possible to change the correction factor to obtain a correct set of \( \dot{m}, M_3 \) and \( M_1 \). In the presented sample (the specific point on the curves, Figure.5.14) it was surprisingly seen that the acquired value nicely matches to the throat coefficient that was previously decided by a try and error approximation. Also it must be said that the mass flow variation range is rather small and even falls into the measurement device uncertainty but one can still see the differences in mass flow if the blockage value varies.

\[\text{Figure 5-14. Mass flow rate calculated by Eq.5.9 in different Inlet/Exit Mach numbers. Pressure ratio=1.23 , Correction factor=0.975 and design point loading.}\]

### 5.2.3 Shrouding

Based on the turbine geometric specifications and the available shrouding types in BETA2 and MAC1, labyrinth type 3 in Figure 5-15 was selected to model the rotor blades shrouding parameters. The number of labyrinth combs is 4 (rotor and casing together). Table 5-2 shows the set shrouding parameters.

<table>
<thead>
<tr>
<th>Number of Labyrinth combs</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial clearance in the seal</td>
<td>0.2 mm</td>
</tr>
<tr>
<td>Average labyrinth radius</td>
<td>208 mm</td>
</tr>
<tr>
<td>Labyrinth pitch</td>
<td>8 mm</td>
</tr>
<tr>
<td>Comb edge thickness</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Comb Height</td>
<td>5 mm</td>
</tr>
<tr>
<td>Inclination angle of the forward comb wall</td>
<td>90</td>
</tr>
<tr>
<td>Inclination angle of the rear comb wall</td>
<td>90</td>
</tr>
<tr>
<td>Comb width on the stator</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 5-2. Test turbine shrouding parameters.
Figure 5-15. Shrouding types in BETA2 and MAC1.

5.3 Simulation Results

In this section different outputs from the simulations will be presented and they will be discussed under different boundary condition. Also the acquired results will be compared with the measured data. It has to be mentioned that the solution was obtained using 20 streamlines in the 2D part, see Figure 5-16. In Figure 5-17 the computational stations used by the codes are illustrated where \( n \) implies the throat condition.

For BETA2, an example of predicted e.g. radial distribution of pressure, temperature, angles, etc. in different computational stations can be found in Appendix D. But here the main focus will be again on the stage performance predictions via investigation of the mass flow parameters, total to static efficiency and degree of reaction. The stage velocity ratio which will be mentioned in many plots, is picked equal to the predicted Parson number:

\[
\nu_{Pss} = \frac{\sqrt{\sum U^2}}{\sqrt{2\Delta h_s}}
\]  

(Eq.5.10)
where $\nu_{ps}$ is the Parson number. Fridh (2012) has shown this value is equal to the velocity ratio in a full admission single stage turbine. Parson number can be directly picked from the computation outputs.

The same as previous chapter, firstly the obtained results from grid-mounted ($Tu=6\%$) case will be presented and later the influence of turbulence intensity variation will be also studied.

### 5.3.1 Flow Capacity

Figure 5-18 shows the flow capacity predicted by the codes in different pressure ratios while the BL approach has been used to model the profile loss. The transition point is assumed to be the same for all pressure ratios as 0.5 and 0.2 in terms of $x/C$ on the vane and blade respectively. The blockage coefficient is 2.5% for all cases mentioned above. It is seen that at the off-design (high loading) region the discrepancies between predictions and measured data are higher than the lower loading cases. It is more critical for the one dimensional code, MAC1. The reasons will be explained later in this chapter more in detail. In 1D results, the maximum relative deviation decreases from 4.7% at pressure ratio 1.15 to 2% at pressure ratio 1.35.

In addition, at the highest pressure ratio, 1.35, it seems there is a parallel shift between BETA2 result and measurement. This can be interpreted as an excessive blockage in the flow channel. The current blockage percentage is set based on the design loading condition at pressure ratio, 1.23. At least it is known that one of the effective parameters on the blockage, boundary layer thickness, tends to decrease as the velocity and hence Reynolds number are increasing. This can be found from a boundary layer thickness relation in its simplest form on a flat plate, $\delta \propto \frac{x}{\sqrt{Re}}$, where $\delta$ is the boundary layer thickness. Therefore it was decided to reduce blockage factor (equivalently increase throat coefficient in the codes input script) to 1.5% at this pressure ratio. Figure 5-19 shows the resultant flow capacity plot where it can be seen that BETA2 prediction has become closer to the experimental data. Although this value can be fine tuned more to get a better agreement but the objective is set to present and justify the effects of solver and boundary condition variables not necessarily to imitate the experimental data by calculations. Making this modification, deviation between MAC1 prediction and measurement however increases since MAC1 curve will also shift up as the shrinkage decreases. Figure 5-20 depicts these two cases versus each other.

Also the absolute flow exit angle from rotor blade, predicted by MAC1, for these two different blockage percentages can be seen in Figure 5-21. It reveals that the maximum deviations observed in the presented cases have in fact occurred where the blade speed is rather small and hence the exit flow is more tangential. In contrast to what Hedlund (2004) reported as prediction of another 1D code, AXIAL, here the trend of exit angle is smoothly decreasing and therefore 1D flow capacity prediction keep the correct curvature shape regardless of their accuracy. In his report, he explains the different shape of measured and predicted flow capacity as a result of predicted exit angle from the rotor.

Figure 5-22 shows the predictions in comparison to the measurements in different pressure ratios while the aforementioned Old approach has been used. It reveals that however both codes show an acceptable agreement in lower loading points but again MAC1 predicts a larger deviation in high loading cases and the relative error decreases as the pressure ratio increases. At the highest pressure ratio, 1.35, the maximum relative error between MAC1 output and experiments is about 1.4% (the blockage is 2.5%). The curvature of BETA2 curves looks larger than the experimental curve and the relative error is smaller at the high and low loading points (in pressure ratio 1.35 it is predictable that correction of blockage from 2.5% to 1.5% will support this fact). It seems that BETA2 somehow predicts a separation via its embedded correlations as the loading increases. For more clarification Figure 5-23 shows a comparison of BL and Old approaches at pressure ratio 1.23 as an example while they have same blockage.
Figure 5-18. Predicted (MAC1 and BETA2) and measured flow capacity vs. stage loading. Pressure ratio=1.15(top), 1.23(middle) and 1.35(bottom). Turbulence intensity=6%, BL approach (Trans. Point=0.5 at vane and 0.2 at blade), Blockage=2.5%.
Figure 5-19. Predicted (MAC1 and BETA2) and measured flow capacity vs. stage loading. Pressure ratio=1.35, Turbulence intensity=6%, BL approach (Trans. Point=0.5 at vane and 0.2 at blade), Blockage=1.5%.

Figure 5-20. Flow capacity prediction. Effect of two different blockage value on the results is illustrated. Solid lines: Blockage=1.5% and dashed lines:Blockage=2.5%.

Figure 5-21. MAC1 predicted flow exit angle from the rotor blade. Two different blockage. BL approach. Tu=6%.
Figure 5-22. Predicted (MAC1 and BETA2) and measured flow capacity vs. stage loading. Pressure ratio=1.15(top), 1.23(middle) and 1.35(bottom). Turbulence intensity=6%, OLD approach. Blockage=2.5%.
Figure 5.23. An example to compare predicted flow capacity using BL and OLD approaches. Pressure ratio=1.23, Tu=6%, Blockage=2.5%. Transition point for BL approach: 0.5 (x/c) for the vane and 0.2 (x/C) for the blade.

5.3.2 Turbine Constant

Figure 5-24 shows the turbine constant predicted by the codes and the measured data in different pressure ratios when the boundary layer approach has been used and the blockage is 2.5%. Since originally the throat coefficient was decided by use of turbine constant, it can be seen that the agreement between prediction and measurement looks better in comparison to what displayed in flow capacity plots. Notice that here the x-axis is static to static velocity ratio. For pressure ratios 1.15 and 1.23 the 2D turbine constant predictions almost match the ones of experiment for a broad range of velocity ratios. Also 1D predictions are quite acceptable where \( v_{se} > \sim 0.4 \). The reasons for larger deviation at smaller velocity ratios will be discussed later when off-design performance is studied. The maximum relative error of MAC1 results are 4%, 3.7% and 1.9% for different pressure ratios in ascending order.

Looking at the turbine constant plot in highest pressure ratio it confirms the necessity for a parallel shift of the 2D curve. Figure 5-25 shows the graphs after modifying blockage form 2.5% to 1.5%. The same as what was observed in flow capacity, the turbine constant predicted curve shifts to higher ranges. The maximum relative error of MAC1 prediction jumps to 2.6% from its previous value of 1.9% however the reduction trend of relative error by increasing velocity ratio conserves.

Also as an example, the case that was shown in Figure 5-24(middle) has been plotted again in Figure 5-26 but the stage loading has been used instead of velocity ratio on the x-axis.

In addition, Figure 5-27 shows the same as what Figure 5-24 displays but using the Old loss correlation. An important feature that can be detected is different trends of 2D results and measured data at higher loadings. There can be seen that the predicted one tends to lean down as the velocity ratio decreases to low levels (similar to observations in high loading prediction of flow capacity when Mamaev’s loss correlation was deployed, Figure 5-22). The 1D predictions however show more consistent trend in comparison to the experiment but their relative error is still large at lower velocity ratios (4%, 3.3% and 1.3% in different pressure ratios. Note that all calculated under blockage 2.5%). Figure 5-28 shows the turbine constant predicted by BL and Old approaches at pressure ratio 1.23.
Figure 5.24. Predicted (MAC1 and BETA2) and measured turbine constant vs. velocity ratio. Pressure ratio=1.15(top), 1.23(middle) and 1.35(bottom). Turbulence intensity=6%, BL approach (Trans. Point=0.5 (x/C) at vane and 0.2 (x/C) at blade), Blockage=2.5%.
Figure 5-25. Predicted (MAC1 and BETA2) and measured turbine constant vs. velocity ratio. Pressure ratio=1.35, Turbulence intensity=6%, BL approach (Trans. Point=0.5 at vane and 0.2 at blade), Blockage=1.5%.

Figure 5-26. Turbine constant versus stage loading. Pressure ratio=1.23, Tu=6%, BL approach, Blockage=2.5%.
Figure 5-27. Predicted (MAC1 and BETA2) and measured turbine constant vs. velocity ratio. Pressure ratio=1.15(top), 1.23(middle) and 1.35(bottom). Turbulence intensity=6%, OLD approach. Blockage=2.5\%.
5.3.3 Degree of Reaction

Before presenting the results for degree of reaction it has to be mentioned that this parameter is largely affected by the measurement of static pressure between the stator and rotor during the tests. As the vortex theory states the pressure at the tip is usually slightly larger than the hub when there is a whirl component of velocity (Saravanamuttoo, 2009). In the test turbine measurements, the mean value is established by use of pressure taps at the endwalls. It has to be known that these measured data are affected by the stator tangential position due to the specific distribution of pressure taps on the hub and casing. However, clocking was done before the test campaign but the effect of probable errors in clocking should be kept in mind.

Degree of reaction (based on the isentropic enthalpy drop) is explicitly provided as an output of the computational codes but its definition - see 2nd part of the CTC program documents by Mamaev and Ryabov (2003) - is different from the reaction degree calculated by the test data. Therefore reaction degree was recalculated based on isentropic relations and the assumption of specific heat in constant pressure, $c_p$.

This assumption is fairly reasonable since the temperature range and its variation in the investigated cases are rather small. Knowing that $\Delta h = c_p \Delta T$, then Eq.2.19 can be written as:

$$\Lambda_{ent \, h.} = \frac{\Delta T_{s,R}}{\Delta T_{s,S} + \Delta T_{s,R}} = \frac{T_2 - T_{3s}}{(T_1 - T_{2s}) + (T_2 - T_{3s})} \quad (Eq.5.11)$$

the isentropic temperatures $T_{2s}$ and $T_{3s}$ will be calculated by isentropic relation while $p_{2s} = p_2$ and $p_{3s} = p_3$. It gives:

$$T_{2s} = T_1 \left(\frac{p_1}{p_2}\right)^{\frac{1-\gamma}{\gamma}} \quad (Eq.5.12)$$

$$T_{3s} = T_2 \left(\frac{p_2}{p_3}\right)^{\frac{1-\gamma}{\gamma}}$$
and the values of different required parameters can be read from the output files. For the 2D case, these values are picked on the mean streamline. Also, later the reaction degree calculated by use of Eq.3.6., where relative angles into and out from the blades are employed will be presented and the results will be compared.

Figure5-29 shows the degree of reaction, \( \Lambda \), predicted in three pressure ratios while BL approach is used. Despite a good agreement between predictions and test data in high velocity ratios it is seen that in all cases there is large discrepancy in high loaded points. Both codes predict lower values of reaction degree than measurements in a way that MAC1 outputs are smaller than those of BETA2. The trends of curves are almost similar to each other. The range of velocity ratios where the agreement is poor becomes wider as the pressure ratio increases. Reaction degree may be has to be known as the most difficult parameter to be predicted correctly since many flow parameters are influential on its value e.g. loading, flow coefficient, flow turning, etc.

Figure5-30 shows the same issue as Figure5.29 but there the Old approach has been used. It does not show any qualitative change in MAC1 outputs in comparison to BL model however the predicted quantities are quite larger. This can be seen in Figure5-31 where the results from these approaches are compared to each other at pressure ratio 1.23. Also, looking at the shape of BETA2 prediction in Old case it can be seen that the three lowest velocity ratios have almost the same reaction degree and the curve does not follow the trend of measurement data.

In addition to see the effect of flow channel blockage, the predictions have plotted in two different blockage values as Figure5-32 shows. There it is seen that such variation does not affect the reaction degree detectably. The most important effect occurred when the formula of reaction degree calculation changed from the abovementioned isentropic relation, Eq.5.11, to Eq.3.6. Figure5-33 and Figure5-34 shows the effect for MAC1 and BETA2 respectively. Also the default calculated reaction degree by the codes themselves, (can be found under \( R_{hn,mid} \) in the outputs), is displayed on the graphs and it can be seen that it is almost the same as use of isentropic temperature to calculate reaction. A drastic change is observed in the agreement level of the predictions and measurement. In velocity ratios larger than 0.4, the codes show good functionality however in high loading case they are still off. The trends are similar to those AXIAL predicted in previous works done on the test turbine where the formulation of reaction degree which has the straightforward isentropic drop over the stage (as a whole) at denominator (e.g. from stage 1 or 01 to state 3 or 03 and so on) has been used. Obviously, to compare \( \Lambda \) by two different definitions is not wise but it may lead to a revise in AXIAL formulation.

Figure5-35 illustrates how relative flow turning in the rotor affects the reaction degree. As described before a sharper turn in the blade causes less acceleration and hence lower pressure change through the rotor. In such way that angle \( \Theta \) is defined in that figure it can be seen that smaller values of \( \Theta \) means the larger flow turnings. The observed discrepancies in high loadings may correspond to mis-prediction of this angle and also the flow coefficient (which relates to the axial velocity and then mass flow rate predictions). Probable reasons for off design discrepancies will be discussed later.

Also it must be known that the reaction degree for 2D prediction has been presented by use of values on the mid-streamline whereas looking to the radial distribution of stage reaction degree computed by BETA2 (while its default formula is used) shows a noticeable variation of degree of reaction over the channel height. Figure5-36 shows this fact in three sample velocity ratios at pressure ratio 1.23. A broad range of \( \Lambda \) radial distribution, particularly in high loadings, affirms that it is more difficult to obtain accurate value by use of just one streamline i.e. MAC1 calculation method.

On the other hand, the test data are acquired at the Euler’s radius which is larger than the mid radius where the reaction degree computations were done. There, the static pressure at the trailing edge is estimated by a cubic interpolation where the hub and tip pressures are known. This profile changes a lot in the gap between rotor and stator (especially at low reaction degrees) so it affects the enthalpy drop calculation over the rotor blade.
Figure 5-29. Predicted (MAC1 and BETA2) and measured reaction degree vs. velocity ratio. Pressure ratio=1.15(top), 1.23(middle) and 1.35(bottom). Turbulence intensity=6%, BL approach. Blockage=2.5%.
Figure 5-30. Predicted (MAC1 and BETA2) and measured reaction degree vs. velocity ratio. Pressure ratio=1.15(top), 1.23(middle) and 1.35(bottom). Turbulence intensity=6%, OLD approach. Blockage=2.5%.
Figure 5-31. Degree of reaction predicted by BL and OLD approaches. Pressure ratio=1.23, Tu=6%, Blockage=2.5%. Transition point for BL approach: 0.5 (x/c) for the vane and 0.2 (x/C) for the blade.

Figure 5-32. Degree of reaction predicted by MAC1 and BETA2. Effect of two different blockage value on the results is illustrated. Solid lines: Blockage=1.5% and dashed lines: Blockage=2.5%.
Figure 5-33. Degree of reaction predicted by MAC1. Two different correlations to calculate \( \Lambda \) compared (the one which uses isentropic relations and constant \( c_p \) versus the \( \Lambda \) equation based on flow coefficient and blades relative angles). The test data and default MAC1 output (\( R_{hn,mid} \)) are also displayed. BL approach.

Figure 5-34. Degree of reaction predicted by BETA2. Two different correlations to calculate \( \Lambda \) compared (the one which uses isentropic relations and constant \( c_p \) versus the \( \Lambda \) equation based on flow coefficient and blade relative angles). The test data and default MAC1 output (\( R_{hn,mid} \)) are also displayed. BL approach.
Figure 5-35. Relative flow turning predicted by MAC1 and BETA2 at pressure ratio 1.23. Smaller $\theta$ means sharper turns. Here $\theta = 180 - \beta_2 - \beta_3$.

Figure 5-36. Radial distribution of stage reaction degree for three different loadings (or velocity ratios; radially averaged) predicted by BETA2 at pressure ratio=1.23.
5.3.4 Total-to-static Efficiency

One of the most significant outputs from any stage simulation is the efficiency prediction. When the result is compared against measurement it can show the quality of loss models which are employed in the program. It is also known that over a range of different loadings different losses take the leading role in efficiency reduction then it is of high importance to see if the codes predict the correct order of losses in a low stage loading point for instance. Figure5-37 shows the total to static efficiency predicted by the codes in different pressure ratios when the boundary layer approach has been used to model the friction losses. Figure5-38 also shows the same as Figure5-37 but there the Old friction modeling correlations have been used. In all cases it is seen that the calculated efficiencies are higher than the measurement. MAC1 has predicted larger efficiencies than BETA2. It might be questionable that unlike the other parameters studied before the predicted values seem to have better agreement with test data in high loadings. It can be explained by the order of dominant losses in that region. It was told that while the blade speed decreases the inlet flow to the blades becomes more tangential, in other words the incidence angle tends to be more positive. Effect of incidence angle on the profile loss has been studied widely. Knowing that inlet metal angle, \( \beta_{1m} \), for the blades is calculated by CATO as 31.7\(^\circ\) then Figure5-39 may help to investigate the highest loading tested point. In this figure the effect of incidence angle on the profile loss has been illustrated by use of correlations from different authors when both the metal inlet angle and the flow relative exit angle are 30\(^\circ\). Looking at positive incidence region on the graphs, apart from the diverse prediction of different formulas it reveals that for such particular angles setting a small variation in incidence may lead to a quite large change in the predicted loss. Also it can be said that a better agreement between measurement and computations at low velocity ratios happened due to this large contribution of incidence angle on losses.

In addition to the effect of incidence angle on the profile losses, almost the same phenomenon occurs for the secondary losses, i.e. in the rather small positive incidences the effect of incidence on the secondary losses will be intensified as Figure5-40 shows. Table5-3 clarifies how friction, incidence and secondary losses contribute in three sample loadings: high, design and low. The data are what BETA2 has predicted by use of BL approach at the blade throat and exit computational stations (distinguished by subscripts \( n \) and 2 respectively). Basically what happens is that the curves of MAC1 and BETA2 in Figure5-37 and Figure5-38 are shifted down at high loading points due to the considerable effect of incidence angle on profile and secondary losses and hence they look more in agreement with the test data curve.

Figure5-41 displays the efficiency predicted by Old and BL correlations at the pressure ratio 1:23. It was observed that in lower loadings the former calculates higher efficiency although at high loadings they are almost the same. It can be seen for both 1D and 2D results. In addition the effect of blockage percentage on efficiency was checked by a change from 2.5\% to 1.5 \% as before. Figure5-42 reveals that the efficiency prediction was not affected by this implemented change.

A general point that should be described is that the loss correlations deployed in the codes are mainly based on a number of experiments on different profiles. For instance in Figure5-40 it can be seen that data from three different profiles have been used to fit a curve. Remembering that the blade profile of the test turbine is of a thick highly cambered type which is not typically used in gas turbine designs, one can relate a major source of deviation of predictions from measurement to this intrinsic bias. However, in all cases investigated above it was seen that the codes have correctly calculated the loading point for the maximum efficiency which occurs at velocity ratio around 0.55.

Again it has to be emphasized that the rather large observed discrepancies should not be interpreted as incapability of the codes to predict more exact values. It was said that the input files attributions were deliberately set to their most basic form (then the outputs may be understood as the roughest possible results) to just study the effect various parameters. Albeit allotting proper values to different modifiers is also a matter of experience from previous designs and predictions.
Figure 5-37. Predicted (MAC1 and BETA2) and measured total to static efficiency (nondimensionalized by experimental \( \eta_{\text{max}} \)) vs. velocity ratio. Pressure ratio = 1.15 (top), 1.23 (middle) and 1.35 (bottom). Turbulence intensity = 6%, BL approach. Blockage = 2.5%.
Figure 5-38. Predicted (MAC1 and BETA2) and measured total to static efficiency (nondimensionalized by experimental $\eta_{max}$) vs. velocity ratio. Pressure ratio=1.15(top), 1.23(middle) and 1.35(bottom). Turbulence intensity=6%, OLD approach. Blockage=2.5%.
Figure 5-39. Effect of incidence angle on profile losses for a profile with inlet metal angle of $30^\circ$. Different correlations are compared together.

Figure 5-40. Effect of incidence angle on secondary loss. Acquired data from tests on three profiles (upper left box) are accumulated and the incidence angle is represented by a nondimensional correlation on x-axis. Y-axis shows the ratio of secondary loss at a certain incidence over the zero incidence case.

<table>
<thead>
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<th>Loss Coeff.</th>
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<th>$\nu = 0.47$</th>
<th>$\nu = 0.67$</th>
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</thead>
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<tr>
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<td>Blade_{n} 1.87 Blade_{2} 4.32</td>
<td>Blade_{n} 1.86 Blade_{2} 4.3</td>
<td>Blade_{n} 1.87 Blade_{2} 4.32</td>
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<tr>
<td>Angle of attack</td>
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<td>Blade_{n} 3.46 Blade_{2} 3.46</td>
<td>Blade_{n} 0.52 Blade_{2} 0.52</td>
</tr>
<tr>
<td>(Incidence)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Secondary</td>
<td>Blade_{n} 9.32 Blade_{2} 11.5</td>
<td>Blade_{n} 2.74 Blade_{2} 4.78</td>
<td>Blade_{n} 1.44 Blade_{2} 4.05</td>
</tr>
</tbody>
</table>

Table 5-3. Comparison of predicted(BETA2) Friction, Incidence and Secondary losses over the rotor blades in 3 different loadings. Subscriptions n and 2 correspond to the throat and outlet computational stations respectively.
Figure 5-41. OLD approach versus BL approach (in friction loss modeling). Predicted (MAC1 and BETA2) and measured total to static efficiency (nondimensionalized by experimental $\eta_{max}$). Pressure ratio=1.23, Turbulence intensity=6%. Blockage=2.5%.

Figure 5-42. Effect of channel blockage on the total-to-static efficiency (nondimensionalized by experimental $\eta_{max}$). Blockage= 2.5% and 1.5%, Pressure ratio=1.35, Tu=6%.
5.4 Turbulence Intensity Effect

To observe the effect of turbulence intensity on the computational outputs, all calculations at pressure ratio 1.23 were repeated by a turbulence intensity of 1% which is equal to the measured level at the experimental phase when the turbulence grid was removed. Thus the measured boundary conditions were directly picked from measurements. However it was possible to change the turbulence intensity merely in the input codes to check more turbulence level cases, but it was avoided since there was no test data for those results to be compared with. In addition, to combine a set of measured boundary condition with a turbulence intensity that does not correspond to them may question the results validity.

As discussed in section 4.4, there seems that the influence of turbulence intensity on the results starts being detectable after a certain level (dependency on the Reynolds and Mach number should also be studied more in detail later). Combining this assumption with the fact that BL approach needs an estimation of boundary layer transition point as input, it was decided to run two sets of computations while BL correlation is going to be used in Tu=1% calculations. Once, based on what MISES estimated the transition points were allowed to be up to maximum 0.8 (x/C) for both vane and blade. In the other case, the transition points were kept the same as what MISES calculated for Tu=6% i.e. 0.5 (x/C) for the vane and 0.2 (x/C) for the blade as the maximum allowance. Notable that BETA2 calculates transition points about 0.5(x/C for vane)-0.5(x/C for blade) when the friction loss is calculated by the default mode: boundary layer equations with the maximum transition point on suction side at the peak velocity (for the code users: leave the friction loss field blank).

Figure 5-43 and Figure 5-44 show the turbine constant and flow capacity when the transition points are the same for both turbulence levels as 0.5(vane) and 0.2(blade). Apart from the output accuracy which was discussed before in this chapter, it can be seen that the difference between predictions in Tu=1% and Tu=6% is rather negligible (even predicted flow capacities by MAC1 in very high loadings are not too different, Figure 5-44). This is in good agreement with the experimental data. Under the same transition point setting, effect of applied turbulence level change on the efficiency and reaction degree can be also seen in Figure 5-45 and 5-46 respectively. Again no drastic effect was detected as well as what the measurements show.

These results strengthen the idea that at least in such Reynolds numbers (less than and about $5 \times 10^5$) a change of turbulence level up to 6% does not affect the transition point (larger levels have to be studied later).

![Figure 5-43](image.png)

Figure 5-43. Effect of turbulence intensity on predicted turbine constant by MAC1 and BETA2. BL approach is used while the transition points are set the same for both levels as 0.5 (x/C for vane) and 0.2 (x/C for blade).
Figure 5-44. Effect of turbulence intensity on predicted flow capacity by MAC1 and BETA2. BL approach is used while the transition points are set the same for both levels as 0.5 (x/C for vane) and 0.2 (x/C for blade).

Figure 5-45. Effect of turbulence intensity on predicted total to static efficiency by MAC1 and BETA2. BL approach is used while the transition points are set the same for both levels as 0.5 (x/C for vane) and 0.2 (x/C for blade).

Figure 5-46. Effect of turbulence intensity on predicted degree of reaction by MAC1 and BETA2. BL approach is used while the transition points are set the same for both levels as 0.5 (x/C for vane) and 0.2 (x/C for blade).
As stated before, to reaffirm this acclaimed idea the computations once again were done based on MISES suggestion: maximum allowance of transition points were set to 0.5(vane)-0.2(blade) for the case with Tu=6% and 0.8(vane)-0.8(blade) for the case with Tu=1%. Figure5-47 to Figure5-50 show the outcomes. It is seen that applying different transition points caused the predictions to deviate from each other for these two turbulence levels inconsistent with the measurements. In particular, looking at the efficiency plot, Figure5-49, it is seen the predicted behavior indeed reminds the rough pre-judgment that higher turbulence level and thus higher losses will lead to lower efficiency. It turned out not always true though, as experiment indicates. Table5-4 portrays the very abstract of the performed job.

<table>
<thead>
<tr>
<th>CASE</th>
<th>Different Turbulence intensity</th>
<th>Different transition point</th>
<th>Outputs agreement with Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>YES (1% &amp; 6%)</td>
<td>NO (Tu=6% trans. Points for Tu=1%)</td>
<td>YES</td>
</tr>
<tr>
<td>2</td>
<td>YES (1% &amp; 6%)</td>
<td>YES</td>
<td>NO</td>
</tr>
</tbody>
</table>

Table 5-4. Turbulance intensity effect investigation. Procedure and result.

Figure 5-47. Effect of turbulence intensity on predicted turbine constant by MAC1 & BETA2. BL approach. Transition points are set 0.5 (x/C, vane) and 0.2 (x/C, blade) for Tu=6% and 0.8 (x/C, vane) and 0.8 (x/C, blade) for Tu=1%.

Figure 5-48. Effect of turbulence intensity on predicted flow capacity by MAC1 & BETA2. BL approach. Transition points are set 0.5 (x/C, vane) and 0.2 (x/C, blade) for Tu=6% and 0.8 (x/C, vane) and 0.8 (x/C, blade) for Tu=1%.
In addition, all these calculations were again repeated while the Old approach was used to model the frictional losses. In this method it is not possible to determine boundary layer transition point (or its maximum allowance). As a sample, Figure 5-51 shows the effect of turbulence level on efficiency predicted by Old correlation (the other studied parameters also experience almost the same effect). The predicted influence may be referred as acceptable.

To wrap up the discussion, the present thesis claims that change of turbulence level up to 6% (with no idea about the higher levels) in the Reynolds number about $5 \times 10^5$ does not cause the transition point to move (regardless of where it exactly is). It is also of interest if the turbulence length scales (mainly macro) will be treated as a variable in the computational codes inputs later.
**5.5 Off-Design Calculations**

As can be perceived (and briefly described) through this chapter, obtaining accurate results in off-design high loadings seems to be more difficult. Particularly the discrepancies become larger when the 1D code, MAC1, is used and this fact may give a clue to explain the reasons. Actually, it was found that the drastic deviation of high-loading points in numerical simulations is a common and even expected incident. Bohn and Kim (1998) stated that as the flow conditions deviate from its nominal design point, the gradient of flow parameters will grow which may cause numerical instability. Stronger spanwise variation of the flow field can result in a much distorted incidence distribution and specially causes large losses near the endwalls. Based on this presumption it is evident that a mean-line code which picks the values of the mid-streamline can be so susceptible in off-design regions. 2D codes also will require a larger number of streamlines to capture the flow physics more accurate though it brings its own difficulties like need of exact inlet pressure profile. In addition a more distorted flow field essentially means a flow with stronger 3D features i.e. tangential derivatives are not easily negligible (like what happened in radial equilibrium method derivation) any longer and 1D and 2D codes may give poor outputs in off-design loads (depends on their incorporated correlations). The same authors explicitly concluded that it is generally difficult to obtain accurate solution in much deviated loads and the flow radial gradient can be known as reliable indicator for performance prediction quality. In Figure5-52 they have compared efficiency prediction of four different computational codes with their measured data. It is notable that two codes even failed to converge in the off design point.

![Efficiency prediction survey](image)

Figure 5-52. An efficiency prediction survey. Four computational codes versus experimental data (Bohn and Kim, 1998)
Primarily it was known that deploying computational codes, which are mainly equipped with correlations based on a number of tests on typical gas turbine blade profiles, to predict the test turbine performance is not the ideal solution (due to its rather thick and sharply curved blade profile) though may be the best possible one at the current state. In addition, the low Mach number, temperature and pressure ranges in comparison to real turbines (where the codes are designed to work) have to be taken into account.

- Predictions for mass flow parameters including flow capacity and turbine constant were come out to be highly dependent on the throat area calculation. Applying a correction factor (the so-called blockage) is necessary to gain a decent flow rate prediction. The required blockage however decreased as the pressure ratio increased i.e. 2.5% and 1.5% at pressure ratios 1.23 and 1.35 respectively. MAC1 does not show qualitative difference in mass flow prediction when the friction loss approach switches from Old to BL whereas BETA2 predictions does not follow the measurement data trend in high-loadings when the Old approach is used. The overall range of discrepancies between prediction and test is larger in the meanline code and it deteriorates in high-loading region. For flow capacity the most reliable solution was seen to be BETA2 in its BL mode provided that a good estimation of transition point position is set.

- Measurement shows that total-to-static efficiency does not change by the applied pressure ratios in the present work. Both codes also predict this fact however their predicted efficiency is higher than measurement. Blockage percentage does not affect the results. It was also seen that the Old approach predicted higher efficiency than BL mode. It can be due to better calculation of laminar to turbulent boundary layer transition point. In the off-design region the predicted values of secondary and profile loss coefficients increases and causes the efficiency to rapidly decrease. This large loss values are mainly due to the effect of incidence angle in the used loss modeling correlations where the small positive incidences ends up to high loss coefficients. However, the correct trend of efficiency curves was observed in all computed cases. The codes showed their best performance in prediction of loading point to gain maximum efficiency.

- Good prediction of degree of reaction turned out to be difficult for the codes since it indeed requires a fine calculation of turning angle flow coefficient. The very important fact is that the isentropic relations with constant $c_p$ were used to calculate $\Lambda$ and the required temperatures where picked on the mid-streamline but the broad spanwise distribution of reaction degree indicates the imperfection of this choice. The discrepancies are larger for MAC1 in high-loadings. The overall shape of predicted curves does not change for MAC1 when Old or BL models are used but BETA2 shows a more similar trend to the experimental data in the BL approach.

- Outcomes from both experimental and simulation phases imply that change of turbulence intensity up to at least 6% does not affect the studied parameters. To study the effect of Reynolds and Mach numbers to generalize such conclusion seems to be so vital. Applying different transition points for different turbulence levels in BL mode ended up with results in contrast with measurements and according to that it is claimed that transition point will not move due to such turbulence level variation in the test turbine at the tested conditions.

- The discrepancies at high-loadings seems to be mainly due to large spanwise gradients of flow characteristic and much distorted incidence distribution which results in 3D features to become stronger. Hence, picking values from a single mid-streamline or limited number of streamlines causes larger discrepancies.
7 Future Work

Performing a set of measurements either at new pressure ratios or new turbine configuration (i.e. two stages) may extend the available data to be applied in simulation phase. The modeled geometry has to be upgraded then however it is not too time consuming since the profiles are already generated in the present task.

It should also be interesting to see the effect of very exact defined inlet pressure profile on the results later. Also, it is worth to perform a set of more extensive and elaborate MISES calculations to study the effect of boundary layer transition point estimation on the BL approach deeply.

In addition, inserting a new turbulence grid in the test turbine to generate higher values of turbulence intensity seems to be too critical to gain a more general view of free-stream turbulence effects. In codes development side, implementation of turbulence length scales is suggested to be studied though it will need experimental proofs alongside.

Performing 3D calculations (with both commercial and SIT in-house codes) based on the created geometry and comparing the results with those of 1D and 2D simulations may be defined as a future task.
Bibliography


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Appendixes
Appendix A – Turbulence Measurement

To measure the turbulence level at the entrance of the flow passage upstream of the vanes, hot-wire measurements were conducted. A Dantec single-sensor probe with straight prongs as shown in Figure A-1 was calibrated and used during the tests.

![Figure A-1. DANTEC Miniature single probe, 55P11 straight](image)

### A.1 Calibration

The probe calibration task was done using a TSI Model-1128 hotwire calibrator as shown in Figure A-2. According to the calibrator instruction manual (2000) and the required experiments air inlet velocity range (expected not larger than 50 m/sec), nozzle #2 was installed in the calibrator. In such setup it would be also possible to capture boundary layer velocity profile. It has to be known that using nozzle set #2 the velocity should be recalculated given the formula stated in the calibrator document:

\[
\text{Calibration velocity} = k + aV + bV^2 + cV^3 \quad \text{(Eq.A.1)}
\]

where \( k = 1.8022 \times 10^{-2} \), \( a = 3.9066 \times 10^{-1} \), \( b = -9.8070 \times 10^{-5} \) and \( c = 5.4915 \times 10^{-7} \).

![Figure A-2. TSI air velocity calibrator model 1128](image)

During the calibration data was gathered, monitored and analyzed by virtue of an IFA-300 computer unit equipped with Thermalpro software. (Eq.A.1) calculates a velocity of 129.3 m/sec as the calibration...
velocity representing the upper bound actual flow velocity of 50m/sec. Figure A-3 shows a velocity-voltage calibration curve example where a mean square error of 2.77% was obtained. As can be seen acquisition of decent data is more difficult in lower velocities than higher ones which makes the calibration process quite time consuming. Also coefficients for a 4th-order polynomial fit curve are shown in the same figure.

![Figure A-3. Calibration curve sample, Mean Square Error=0.02772](image)

It has to be noted that the sensor working temperature was set to $T_{\text{sensor}} = 250 \, ^\circ\text{C}$ where the probe resistance was calculated using the given equation on the probe protective bulb:

$$Probe \, resistance = R_{\text{tot}} + R_{20} \cdot \alpha_{20} \cdot (T_{\text{sensor}} - T_{\text{ambient}}) \quad (\text{Eq.A.2})$$

where the technical specifications $R_{\text{tot}}$, $R_{20}$ and $\alpha_{20}$ are provided by the manufacturer for each probe. This gave a resistance value of 6.6758 ohms for the probe which was employed in the current experiments. Also the offset and gain values as well as the cable resistance were calculated by use of the inherent auto-features of IFA 300 incorporated with Thermalpro.

### A.2 Measurements

After calibrating the probe then it can be put in the flow channel. The hot-wire sensor attached to its support stem was exposed to the stage inlet flow by use of a servo traverse system as shown in Figure A-4 which made it possible to control the radial position in addition to set the appropriate yaw angle.

![Figure A-4. Probe radial traverse equipment mounted on the test turbine.](image)
A.2.1 Cosine Law

An important issue that must be considered is the probe direction to the flow. Obviously the maximum velocity will be obtained when the wire is set perpendicular to the flow. The so-called Cosine-law articulates this fact.

\[ u_{eff} = u \cdot \cos \alpha \]  

(Eq.A.3)

where \( \alpha \) is the complementary angle of the angle between flow direction and the wire, \( u \) is the flow velocity and \( u_{eff} \) is the velocity that the hot-wire senses. Figure A-5 shows the experimental result where the probe was turned and reached the correct yaw angle after a number of trials. The acceptable agreement with the Cosine-law can be seen.

![Cosine Law - Effect of Yaw Angle on the measured velocity](image)

Figure A-5. Cosine Law - Studying the effect of probe Yaw Angle.

A.2.2 Turbulence Intensity Formulation

While the IFA computer unit recorded the data a simple formulation was coded to calculate the turbulence intensity as suggested in IFA-300 Operation Manual (2010). Here it should be noted that all the tests were done as the sample rate was set to 416.6 kHz and sample size to 256 kPts/ch (each kPts includes 1024 samples) and hence sample time calculated as 0.6291 sec from the equation:

\[ \text{Sample time} = \frac{\text{Sample size} \times 1024}{\text{Sample rate (Hz)}} \]  

(Eq.A.4)

The applied turbulence intensity calculations will be introduced here. From the document mentioned above, for a set of acquired data \((x_i's)\) of length \(n\) the \(k^{th}\) moment around the origin is:

\[ M_k(x_i, 0) = \frac{1}{n} \sum_{i=1}^{n} x^k \]  

(Eq.A.5)

and this definition can be used to define the mean (here mean velocity) as:

\[ \bar{x} = M_1 \]  

(Eq.A.6)

also the normal stress(variance or the second moment about the mean) can be written as:

\[ \mu_2 = M_2 - M_1^2 = \bar{x}^2 - \bar{x}^2 \]  

(Eq.A.7)

and then the standard deviation:
Finally the turbulence intensity in percent is formulated as:

$$Tu = \frac{\tau}{M_1} \times 100$$  \hspace{1cm} (Eq.A.9)

A.2.3 Test Results

As frequently mentioned in the main text the turbulence intensity measurements were done in two cases where once a turbulence generator grid was installed upstream of the stator vanes and again after removing this grid. In both cases yaw angle was adjusted as described in section A.2.1 and the probe radially traversed the flow channel from tip to hub. It was tried to get close to the endwalls as much as possible to depict the velocity profile inside the boundary layer. However effect of the cavity region, where the probe was sent in, must be noticed. In addition the effect of heat transfer between the probe (here working in 250 °C) and the metal endwalls is a well-known phenomenon when the probe reaches too close to the walls. A good illustration of this can be found in Zanoun(2009) and Satyaprakash(1991). Figure A-6 and Figure A-7 show the velocity and turbulence intensity radial distribution profiles respectively in the case that the perforated plate was used as the turbulence grid (PPG). In addition, the probe was kept constnet at the channel middle height and a number of measurements were conducted subsequently. Figure A-8 shows the turbulent intensity calculated through these trials. Notable that the presented results were measured at pressure ratio 1.23.

The boundary layer region may be recognized in the plots and eventually a value of 6% was picked to be an acceptable approximation of the flow turbulence intensity in this case.

Figure A-6. Velocity profile. Measured at the upstream of vane1 leading edge. Perforated plate installed.

Figure A-7. Turbulence intensity radial distribution. Upstream of vane1 leading edge. Perforated plate installed.
Figure A-8. Subsequent sampling of Turbulence intensity measurement at the channel mid-height. Perforated plate installed.

Figure A-9 to Figure A-11 show the same as above but for the case without the turbulence grid. Here as can be seen, there is an increase in turbulence intensity at the upper half of the channel which might be due to some leakage from the probe entrance hole. Unfortunately this leakage was detected after the test had been done. In fact, data analysis led to a leakage probability which was then proven when liquid soap was sprayed on the surface. It was decided to accept the current curves due to the time schedule restrictions. Also, since two probes were broken after touching the walls it was risky to jeopardize the only remaining probe before the whole required measurements were done. According to the acquired turbulence intensity values (and giving higher importance to the lower half of the channel), a value of 1% was picked to be representative of turbulence level.

Figure A-9. Velocity profile. Measured at the upstream of vane1 leading edge. Without turbulence grid.

Figure 0A-10. Turbulence intensity radial distribution. Upstream of vane1 leading edge. Without Turbulence gird.
The absence of a boundary layer region at the hub in Figure A-9 might be majorly due to mis-positioning of the probe inside the flow channel (i.e. the probe is not exactly at the position where it was supposed to be).

Figure A-12 shows an example of the log-log energy spectral density. In fact this diagram indicates the energy distribution between different frequencies. As one can plot the measured velocity data versus measurement time, a Fast Fourier Transform (FFT) can also be applied to move to the frequency domain. It actually shows how much energy is available in different frequency levels. This is usually called the energy cascade in turbulence theory literature. It can be seen that the energy level decreases as the frequency increases (which means the turbulence length scale reduces). At the end of cascade the viscous effects overcome the dwindled turbulent structures.

Figure A-12. An Example of Energy spectral density. (x-axis: Energy content of each frequency, y-axis:frequency)
Appendix B – Endwalls Coordinates

Table B-1 and Table B-2 show the coordinate points used to simulate the tip and hub walls respectively. The coordinates were extracted from a CAD drawing file and the coordinate system origin was set at the turbine rotation axis and aligned with the stator vane leading edge. Figure5-3 and Figure5-10 can be used for more illustration. It must be noted that the available gaps in the geometry were capped with straight lines.

### Table B-1

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Table B-1. Coordinates used to simulate Tip. Extracted from CAD drawing.
# Coordinates used to simulate Hub. Extracted from CAD drawing.

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Table B-2. Coordinates used to simulate Hub. Extracted from CAD drawing.
Appendix C – MISES Calculations

C.1 Overview

The MISES system is a collection of programs for cascade analysis and design. Its basic features are grid generation, flow initializing and interpretation of results (Drela, 2008). In the current study a version of MISES code incorporated in CATO was used mainly to determine a reliable approximation of boundary layer transition point. As stated before, to use the BL approach in meanline and throughflow calculations the onset of transition region of laminar to turbulent boundary layer should be provided for the codes in their input files.

Although an extensive MISES calculation could not be performed through this thesis but it was decided to pick at least two cases to represent two main categories of tests: with and without perforated plate upstream of the stage, $Tu=6\%$ and $Tu=1\%$ respectively (see Appendix A). The simulation cases were chosen at the nominal loading (velocity ratio=0.47) and pressure ratio of 1.23. MISES required in-data picked from the available BETA2 outputs when a blockage of 2.5% was set and the OLD profile loss correlation was deployed.

It has to be re-emphasized that a broader set of MISES computations is suggested to provide more specific output for each individual case. Indeed the calculated transition points in this report can be inferred as some data which is mainly used to show how MAC1 and BETA2 results might be affected by this setup parameter when BL correlation is going to be used. Furthermore it was surprisingly shown in chapter 5 that using different transition points for these two resulted in outputs in contrast with measurements data and hence the transition points were set the same for the both cases.

C.2 Calculations

C.2.1 Case 1: With the Turbulence Grid

Figure C-1 shows the static pressure contour nondimensionalized by the total pressure value at the inlet at the stator hub. The shape factor (shape parameter) defined as the ratio of boundary layer displacement thickness to the momentum thickness is also shown in Figure C-2 in two different curves of suction and pressure sides.

![Figure C-1. Pressure distribution at the stator hub. Pressure ratio=1.23. Velocity ratio=0.47. Grid installed.](image1)

![Figure C-2. Shape parameter curves on the suction side(red) and pressure side(blue). Stator hub section. Pressure ratio=1.23. Velocity ratio=0.47. Grid Installed.](image2)

Figure C-3 and Figure C-4 also show the same concepts at the stator tip position. Figure C-5 is shown here as an example of Mach number distribution over the blade profile calculated by MISES.
Figure C-3. Pressure distribution at the stator tip. Pressure ratio=1.23. Velocity ratio=0.47. Grid installed.

Figure C-4. Shape parameter curves on the suction side (red) and pressure side (blue). Stator tip section. Pressure ratio=1.23. Velocity ratio=0.47. Grid Installed.

Figure C-5. Mach number distribution over Vanes at tip section. Red: Suction side, Blue: Pressure side. Grid installed.

As can be seen in the shape factor plots the transition point is predicted almost near the half of the vane axial chord on the suction side, hence it was decided to use the value of 0.5 (x/C) in the computational codes input file as transition point for the stator vanes in the cases with Tu=6%. Figure C-6 and Figure C-7 show the pressure distribution and shape factor curves for the rotor blades tip section in the same condition as above. Mach number is also plotted over the rotor suction and pressure sides at the rotor tip section in Figure C-8.

Figure C-6. Pressure distribution at the Rotor tip. Pressure ratio=1.23. Velocity ratio=0.47. Grid installed.

Figure C-7. Shape parameter curves on the suction side (red) and pressure side (blue). Rotor tip section. Pressure ratio=1.23. Velocity ratio=0.47. Grid Installed.
In Figure C-7 it is seen that the transition point is predicted to be about 0.2 (x/C) on the rotor blades suction side. Notable that the same value obtained using the hub profile similar to what happened for the vanes and was already shown in Figure C-2 and Figure C-4.

The reader can recognize this case through the main script and graphs of the report wherever the conjugate (0.5-0.2) has been mentioned.

C.2.2 Case 2: Without the Turbulence Grid

Another set of computations were also done when the turbulence intensity was set to 1%. The same approach as previous section was followed. Here the results including pressure distribution, shape factor and Mach number curve at the tip sections of both the vanes and the blades are presented.

Figure C-9. Pressure distribution at the stator tip. Pressure ratio=1.23. Velocity ratio=0.47. Without Turbulence Grid.

Figure C-10. Shape parameter curves on the suction side(red) and pressure side(blue). Stator tip section. Pressure ratio=1.23. Velocity ratio=0.47. Without Grid.

Figure C-11. Mach number distribution over Vane at tip section. Red: Suction side, Blue: Pressure side. Without Grid.
Figure C-12. Pressure distribution at Blade tip. Pressure ratio=1.23. Velocity ratio=0.47. Without Turbulence Grid.

Figure C-13. Shape parameter curves on the suction side(red) and pressure side(blue). Blade tip section. Pressure ratio=1.23. Velocity ratio=0.47. Without Grid.

Figure C-14. Mach number distribution over Blade at tip section. Red: Suction side, Blue: Pressure side. Without Grid.

According to these shape factor curves it can be seen that the point at about 0.8 (x/C) has been predicted as the boundary layer transition point for both vanes and blades. In the main report, reader can recognize the results corresponding to this prediction wherever the conjugate (0.8-0.8) has been referred to.
Appendix D – Radial Distribution of Parameters (A sample case)

As a throughflow 2D computational code, BETA2 calculates different stage parameters at different radial positions for every computation station e.g. vane inlet, blade outlet, etc. In this section a sample case at pressure ratio 1.23 and static to static velocity ratio 0.47 with the turbulence intensity of 6% and blockage value of 2.5% has been chosen. BETA2 deployed BL model for the profile loss calculation and the transition points were defined at 0.5-0.2 (x/C) on the vanes and blades respectively (see Appendix C).

In the following, radial distribution of different parameters out from the mentioned simulation will be presented. Notable that a number of these curves are in fact plotted based on user defined values implemented in the input file, such as the inlet temperature and pressure profile, metal angles and cooling air. To find more details about the applied correlations and the specific nomenclature see Mamaev (2003).

Figure D-1. Parameters radial distribution TURBINE INLET. (Left to right: Radius, Total temperature, Total Pressure, Total Enthalpy, Flow absolute angle.)

Figure D-2. Parameters radial distribution VANE INLET. (i: Enthalpy, G:Mass flow, Lam: Laval Number, M:Mach number, ro:Density, inclANG: Lean angle, alfIn: Inlet metal angle.)
Figure D-3. Parameters radial distribution VANE OUTLET. Note: Subscript “n” implies the throat station. (Cm: Curvature in meridional coordinate, alf1: effective exit angle)

Figure D-4. Parameters radial distribution BLADE Inlet.
Figure D-5. Parameters radial distribution BLADE Outlet. Note: Subscript “n” implies the throat station.

Figure D-6. Parameters radial distribution TURBINE EXIT.