EECA

- Energy Efficient Concept Aircraft

A concept study for an environmentally friendly general aviation aircraft.

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ABSTRACT

A concept study for an environmentally friendly aircraft is made with the help of mathematical models. The aircraft has a capacity of 4 passengers and is powered by electrical engines and lithium-air batteries so that no greenhouse gases are emitted. The aim of the study is to find a concept that can compete with today’s petrol powered aircraft, with applications in entertainment flying and transportation. Flight conditions such as steady state flight and steady climbing are modeled to estimate the performance of the aircraft and to find the most efficient way to fly. A static stability analysis is made to determine a geometry for which the aircraft is statically stable in pitch. The study shows that a canard configuration together with pusher contra rotating propellers is a good way to design a very efficient aircraft. It is also found out that the most energy efficient way to climb is to use a high rate of climb and a low velocity and that it can be profitable to increase the weight of the aircraft with more batteries.
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1 Introduction

1.1 Background
It becomes more and more important for everyone to reduce the influence on the greenhouse effect. This leads to a big challenge for the aviation industry when one has to find alternative fuels and new solutions to power aircraft. Today, smaller aircraft (also called general aviation aircraft) is almost exclusively driven by oil based fuels that results in emissions of carbon dioxide in the atmosphere. These kinds of problems are difficult to solve when it comes to aircraft since there are high demands on efficiency in weight and size, reliability and safety. In other words there are very tight frames that one has to work within when solving these problems for the aviation industry.

1.2 Problem
The problem is to find a concept for a more environmentally friendly aircraft. This aircraft should fulfill these following requirements.

- It should be powered in a way that does not contribute to the greenhouse effect by emission of carbon dioxide or other greenhouse gases.
- It should have the capacity to carry 2-4 passengers or a similar load.
- It should also be designed to be statically stable in flight, so that there is no need to have automated electric systems to fly the aircraft.

Since the second requirement left us with a choice of direction for our concept study, we decided to study an aircraft that should have a capacity of 4 passengers. This was based on the thought of an aircraft that should be able to be used in most situations and most importantly, because we thought that the larger scale would be beneficial in the struggle to find a solution. The first requirement could also be solved in many different ways, i.e. electric batteries, combustion of hydrogen etc. We chose to study the possibilities to use electric engines to drive a propeller and batteries as energy source. This was because of that battery technology is evolving rapidly today, and that batteries seem to be the future for the car industry. The electric engine has a big advantage in simplicity which should mean less weight, less space and lower maintenance needs. Electric engines also have a greater grade of efficiency compared to an internal combustion engine. Oil based fuels are getting more and more expensive while electricity still is quite cheap. An electrical system for propulsion should therefore be attractive to costumers because of the lower costs of maintenance and operation compared to similar aircraft today. To enable the aircraft to compete with similar aircraft in performance, we realized that the aircraft had to be able to fly in high cruise speed. This meant a big challenge since the power to propulsion needs to be a lot bigger if the airspeed is higher. This led to the problem that we had to solve which we compiled in a requirement specification (see Appendix A).
2 Method

2.1 Inspiration
To find some kind of starting point, it is very rewarding to do some research about existing aircrafts that are similar in size and power. The following aircrafts are the ones that are found interesting to take inspiration and ideas from.

Cessna 172 Skyhawk
This aircraft is seen as a reference for 4-seated smaller general aviation aircrafts. It is a high winged airplane with a front mounted single propeller and a tail mounted horizontal and vertical stabilizer. The design is over all very simple but Cessna managed to find the exact correct mix in the design. The Cessna 172 Skyhawk is known for its reliability and forgiving flight characteristics which is a success in aircraft design for general aviation. Add to this that it is reputed to be the safest general aviation aircraft ever built. It is equipped with a 180 hp internal combustion engine which gives a cruise speed of about 230 km/h and a maximum range of 1185 km [1].

Figure 2-1. The Cessna 172 Skyhawk in flight.
Velocity XL
This 4-seated kit aircraft is way more unconventional than the Cessna 172 Skyhawk and many other general aviation aircraft. It is completely built with composite materials and has vertical stabilizers mounted on the tips of the main wing, similar to winglets. The main wing is swept backwards and mounted on the rear part of the fuselage, while it has a canard configuration as a horizontal stabilizer. The aircraft is driven by a single pusher propeller mounted in the rear. The propeller is recommended to be powered by an engine with a capacity of 260 to 310 hp. This results in a cruise speed of up to 370 km/h and a range of about 1500 to 1800 km. Thus it has higher power and fuel consumption, but greater speed performance and range than the Cessna 172 Skyhawk [2].

Figure 2-2. The Velocity XL in flight.
Yuneec E430

The Yuneec E430 is the world’s first commercially produced electric powered aircraft. It is a twin seat aircraft that is simple to use and is designed to be easy to fly. It has a conventional design except the V-tail. The design is similar to a sailplane with a large wingspan and a 24:1 glide ratio. It has a single propeller and a 54 hp (40 kW) electric engine that results in a cruise speed of 95 km/h. The batteries are of lithium polymer type and give a flight time of approximately 2 hours. The low speed design and a low power engine makes this aircraft a lot slower, smaller and lighter than the Cessna 172 Skyhawk [3].

Electric Apis

This light-weight one-seated glider is developed by the Slovenian company Pipistrel, and is used by the electric engine manufacturer Enstroj to test their engines in aircraft applications. The aircraft has an empty weight of 280 kg and uses an electric engine, with 40 kW (54 hp) peak power and 30 kW (40 hp) continuous power, for takeoff and climbing. The engine is directly mounted to the propeller and it is possible to retract both propeller and engine during flight, which lowers the resistance when gliding. The aircraft can climb up to altitude 1800 m with a 50% discharge of the batteries, and up to 3000 m with 90% discharge. It has a glide ratio of 41:1 at 95 km/h and a takeoff distance of 50 meters. This aircraft is much smaller and lighter than the concept aircraft in this study, but since it is electrically driven it is interesting to us [4].
2.2 Design aspects

Canard configuration
Since the Cessna 172 Skyhawk apparently is such a success in aircraft design, we decide to aim for a concept that can compete with its simplicity and performance. Thus, the Cessna 172 is chosen as a reference point in the decision of many parameters in the initial design. However we want to find a more unconventional design and a way to make use of new technology that has evolved since the Cessna 172 was designed. Since the main objective of the concept study is to find a concept for an environmentally friendly aircraft it is important to find a design that could result in low air resistance and in turn increased efficiency. When looking on the Velocity XL the canard configuration is interesting, since both the canard and the main wing have to produce positive lift assumed that the center of gravity of the aircraft is placed between the wing and the canard and that the aircraft is statically stable in flight. This is a great opportunity to get rid of the disadvantage that a tail stabilizer has to produce a negative lift to stabilize the moment around the center of gravity in a statically stable conventional design. The stabilizer yields a drag to the aircraft, and it would of course be better to use this drag to both contribute to the overall lifting force instead of allowing it to work against the lift from the main wing.

Pusher propeller
A canard configuration does although lead to another problem. Since the main wing will be placed in the very rear of the aircraft, the center of gravity of the aircraft must also be placed somewhere in the rear. Since the cabin should be placed in the forward part of the fuselage, the engine and other heavy
parts has to be placed in the very rear of the fuselage to get a resulting center of gravity near the resulting lift force. The Velocity XL has solved this with a rear mounted engine and a pusher propeller and this could be our solution too. A general disadvantage with a pusher propeller is that it is harder to cool a rear mounted engine compared to a front mounted engine which can easily be cooled by the surrounding air [5]. However, this does not apply to our aircraft since the electric engine would not require as much cooling as an internal combustion engine. The biggest advantage of a pusher propeller is that it will result in reduced skin friction drag since the aircraft flies in undisturbed air [5]. This is an opportunity to make the design more efficient, both with the canard configuration and the pusher propeller.

**Contra-rotating propellers**

In turn, the pusher propeller leads to yet another problem. During takeoff the aircraft has to rotate to create lift, which creates a problem when the rear mounted propeller may touch the ground. By this reason, this must be considered during the design of the aircraft. The more engine power, the larger is the required propeller diameter and consequently the risk for the propeller to hit the ground. However, the propeller diameter can be reduced by using two separate counter-rotating propellers mounted on the wings or two contra-rotating (concentric rotation in opposite directions) propellers mounted behind each other. Wing mounted engine pods is today used on many aircraft where a single propeller cannot handle the power from the engine, or where the required power for propulsion cannot be generated by a single propeller. The solution with contra-rotating propellers are rarely seen since they require advanced gearboxes which often weighs much and also because the propellers generate higher noise than a single propeller. Nevertheless, the gear box problem does not apply to our aircraft since the use of electric engine enables the use of multiple engines. The engines can then be divided into pairs, where they can direct drive one propeller shaft each, to make the advanced gearbox redundant.

Since the efficiency of the aircraft in this study is so important, the efficiency of the propeller is essential. The efficiency of a contra-rotating propeller is higher than a single propeller, and consequently also higher than two single counter-rotating propellers. This fact comes from that the rear propeller in the contra-rotating configuration, as a consequence from the opposite rotation direction, takes advantage of the residual swirl (also called whirl) from the forward one. In addition, this results in that the roll moment from a single propeller that affects the aircraft is balanced by the rear propeller [6]. Because of this, the contra-rotating configuration is well suited for this study. However, it turns out that it is hard to find a reliable number of by how much the efficiency is increased. An Australian company, which has developed a gearbox that is used for contra-rotating propellers in aircraft applications, claims that the thrust is improved of about 15-20% compared to various single propeller configurations [7]. Together with an overall feeling after researching the internet, this makes it likely to achieve an improvement of the efficiency of the propeller by 12%, which is used in this study.
Winglet vertical stabilizer
In the configuration with canards and pusher propellers it is useful that the vertical stability is secured by two stabilizers on the wing tips as seen on the Velocity XL. This is because of the short fuselage that a canard configuration often brings, and that the pusher propeller benefits from undisturbed air in front of it. A conventionally mounted vertical stabilizer as a fin at the end of the fuselage is by those reasons undesirable since it would be hard to find place for on the short fuselage. In addition, vertical stabilizers on the wing tips will have a longer lever to the center of gravity to the aircraft if the wings are swept.

2.3 Initial sizing
In the initial sizing we are aiming to reach some kind of understanding of the weight and size of the aircraft. This gives information that is useful when deciding important parameters such as wing area and tail size. In the initial estimation of the weight we have to divide the weight of the aircraft in to different parts. These parts are the fuselage including main wing, batteries, engine and passengers. The estimation is made with the maximum takeoff weight in mind, which is with 4 passengers including pilot.
Fuselage
Since the goal of this study is to find a very energy effective concept, we need to work towards a low weight on every part of the aircraft. There is a lot of new technology in lightweight construction, thus it’s hard to find a descent estimation of the weight of the fuselage and the main wing. To find a valid estimation we can take a look at the Skyhawk’s empty weight which is about 780 kg. However, this weight includes the engine, drivetrain and fuel tank etc., basically everything besides passengers, luggage and fuel. Let’s say that a rough estimation of the weight of the Skyhawk’s fuselage, main wing and tail is a weight of about 650 kg. Since it is constructed out of aluminum it weighs more than our aircraft which will be constructed with light weight technology and materials and will also have shorter fuselage. Thus a rough estimation of the weight of our fuselage and main wing will be 400 kg. This also includes interior, instruments, landing gear etc.

Engine
To find an estimated weight of the electric engine we need to know the required power that the engine has to deliver. This information is needed to find one or more engines that are suitable for our aircraft which will give us the weight of the engine. But to find the required power we need to know the total weight of the aircraft together with other parameters such as wing area, climb speed and wing span. The only way to come to a conclusion regarding these parameters is to go through an iterative process where different values are tried out. However, electric engines are quite light weight compared to internal combustion engines so a rough estimation of the weight of the propulsion system is that it weighs about 50 kg.

Passengers
The weight of the passengers including luggage is estimated to be 90 kg per passenger. Thus the maximum weight of 4 passengers is estimated to be 360 kg. Note that this includes personal luggage for the passengers, so that the capacity for luggage depends of the weight of the passengers. Since the use of the aircraft is meant to be personal transportation for shorter distances, there should not be a great need for luggage capacity.

Batteries
As in the case with the fuselage and main wing the battery technology is developing rapidly. Many of the current estimation methods for fuel in aircraft are developed with the use of oil based fuels such as petrol. To be able to estimate the weight of the batteries we need to know the required energy that the batteries have to contain. Compared to the engine the batteries are heavier and will probably be one of the heaviest part of the aircraft. As the specific energy for batteries is much lower than for petrol, we have to compromise the range of the aircraft to get the weight of the batteries sufficiently low. To estimate the required energy we need the weight and geometry of the aircraft together with some kind of desired range or flight time. This will be calculated later on, but for now we need some kind of
estimation to get a rough estimation of the total weight of the aircraft. We can decide some kind of weight that we should aim to reach by looking at similar aircraft and try to match their total take-off weight. The maximum take-off weight for the Cessna Skyhawk is 1157 kg and since we need to get a lighter aircraft we can take 1000 kg as a limit for our maximum take-off weight. Without the batteries we have a weight of 810 kg, which gives us 190 kg for the batteries. However, we need to work for an even lighter aircraft so we are aiming for the batteries to weigh 150 kg which will result in a total weight of 960 kg.

**Maximum take-off weight**

With the above estimations the total maximum take-off weight is 960 kg. Note that this is just a starting point in the iterative process to get number on the parameters.

**Wing geometry**

With an estimation of the weight it is possible to find a wing area that is adequate in relation to the weight of the aircraft. This can be done by using the wing loading, which is the weight divided by the wing area. For a general aviation single engine aircraft this quota should be about 814 N/m² [5]. This gives us that the required wing area is

\[ S = \frac{W}{814} = \frac{960 \cdot 9.82}{814} \approx 11.58 \text{ m}^2. \]  

(2.1)

This is then rounded up to 12 m² which is decided to be the estimated initial wing area. Since this is rounded up it gives us an area of maneuverability when it comes to the total weight of the aircraft.

Another thing that needs to be decided regarding wing geometry is the wing span. Here, the aspect ratio \( AR \) can be used to determine an adequate length of the wing span by the relation

\[ AR = \frac{b^2}{S} \]  

(2.2)

where \( b \) is the wing span and \( S \) is the wing area. A too big aspect ratio results in low critical stall angle, why it is important to take this in consideration when deciding the wing geometry. Typically, the aspect ratio of a general aviation aircraft is 7.6 [5] which can be used to find an initial estimate of the wing span. Inserting numerical values in (2.2) gives

\[ b = \sqrt{7.6 \cdot 12} \approx 9.55 \text{ m}. \]  

(2.3)

The wing chord is then equal to the wing area divided by the wing span, which gives that the wing chord is
\[ c = \frac{S}{b} = \frac{12}{9.55} \approx 1.26 \text{ m}. \] (2.4)

### 2.4 Engine selection

The engine of the aircraft is meant to be one or more electric engines. To find an engine that fits our requirements we can look at similar electric driven aircraft such as the Yuneec E430 and the Electric Apis (see Section 2.1). The Yuneec E430 uses its own in-house engine, which is hard to get any specifications for. Further, it has too low output power because of the lighter aircraft. The engine in the Electric Apis is an EMRAX engine with 30 kW continuous output power and low weight (11.7 kg). It is also possible to mount two or three engines on the same shaft to increase the output power on the shaft without using gearboxes [8].

![Two EMRAX engines mounted on the same shaft.](image)

**Figure 2-6. Two EMRAX engines mounted on the same shaft.**

This enables the use of these engines on the concept aircraft, where we can direct drive the two concentric propeller shafts by mounting one or more of the engines on each shaft. It is also easy to match the available power to the required power by adding or removing engines.

The engines are also dependent on electric controllers in order to work. A tested and verified controller that is air cooled weighs 6.5 kg and can control two engines, so that the number of required controllers is dependent on the number of engines [9].
## Engine specifications

<table>
<thead>
<tr>
<th>Type</th>
<th>Brushless AC motor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum rotation speed</td>
<td>3000 rpm</td>
</tr>
<tr>
<td>Continuous power</td>
<td>30 kW</td>
</tr>
<tr>
<td>Peak power (1 min/ 2 min)</td>
<td>50 kW/40 kW</td>
</tr>
<tr>
<td>Weight</td>
<td>11.7 kg</td>
</tr>
<tr>
<td>External dimensions (diameter / width)</td>
<td>228 / 86 mm</td>
</tr>
<tr>
<td>Motor efficiency, $\eta_{\text{engine}}$</td>
<td>$&gt;92%$ (depends on rpm and torque)</td>
</tr>
</tbody>
</table>

### 2.5 Energy source selection

To find a sufficient storage for energy in the aircraft, to replace today’s fuel tanks, we look at different battery types to store electrical energy. The two top candidates are lithium-ion batteries that are currently used in most hybrid cars and other similar mobile energy storage and lithium-air batteries that are currently being researched. An important factor in the search of an energy source is the specific energy, which states how much energy the battery can hold per mass unit. Simply put, the higher specific energy, the higher is the energy content in the battery. This becomes very important since it decides the total weight of the battery in the aircraft. Also the specific power, which describes how much power the battery can deliver per mass unit, is important to make sure that the battery can deliver a sufficient amount of power to the engine.

#### Lithium-ion

The lithium-ion batteries are currently used in many applications, such as hybrid cars, laptops and cell phones. The specific energy of these batteries varies with the application, since some batteries are optimized for energy storage and other are optimized for high output power. The higher specific energy, the lower is the specific power for a lithium-ion battery. The lithium battery that is used in the upcoming Toyota Prius plug in hybrid car has a specific energy of about 70 Wh/kg but the industry states that it is possible to get a specific energy of up to 200 Wh/kg [10] for energy optimized batteries.

#### Lithium-air

The lithium-air batteries are currently being researched and there are non-rechargeable prototypes that have reached a specific energy of about 700 Wh/kg. These batteries are using the surrounding air together with lithium to create a chemical reaction. The rechargeable batteries are predicted to reach up to 1000 Wh/kg [11]. Since these batteries still is in development, we decide to use a lower specific energy in the comparison to increase the reliability of this study. The estimated specific energy that is used for the battery type in this study is decided to be 750 Wh/kg since this is close to what is achieved in the non-rechargeable prototypes today.
Selecting the battery type

By the above mentioned reason, the lithium-air battery seems to be the better option. On the other hand, the lithium-ion batteries are already used in similar applications which make it more trustworthy that it is actually possible to use these batteries in the concept aircraft in the foreseeable future.

However, since this is a concept study it would be interesting to study how the use of new battery technology could benefit to make the aircraft competitive to today’s petrol-driven aircraft. Because of this, the lithium-air battery with the mentioned estimated specific energy will be used in this study.

Note that an assumption is made, that states that it is possible to get the appropriate voltage and current from the battery. Also, the battery is assumed to have the same density as the lithium-ion batteries (about 1 kg/m$^3$). Since the lithium-air battery technology is still in development, there is no data for the discharge efficiency of the batteries. Thus, this efficiency is assumed to be equal to one in this study so that the extracted energy from the battery is equal to the energy that the engine can use.

2.6 Initial performance

To be able to estimate the amount of energy needed to be stored in the aircraft and the power needed for propulsion we need to use a mathematical model. The model is set up for different flight conditions, such as steady state flight, steady climbing and take-off. A model for landing and gliding flight is not worked out since these flight conditions are less relevant to the objective of this study.

Steady state flight

The following model for steady state flight, where the aircraft is flying at constant velocity and altitude, comes mainly from the force balance equations for the aircraft. Since the velocity and the altitude are constant, the force balance states that the lift $L$ must be equal to the weight of the aircraft $W$ and that the thrust $T$ must be equal to the drag $D$. Expressed with aerodynamic coefficients this gives

$$T = D = qSC_D$$

(2.5)

$$L = W = qSC_L$$

(2.6)

where $C_D$ is the aerodynamic drag coefficient, $C_L$ is the aerodynamic lift coefficient, and $S$ is the reference area of the aircraft (in this case the wing surface area). Further, $q$ is the dynamic pressure defined as

$$q = \frac{1}{2} \rho V^2.$$ 

(2.7)
Here $V$ is the velocity and $\rho$ is the air density. The drag coefficient can then be divided in two parts with one representing the zero-lift drag (also called parasite drag), $C_{D_0}$ and one depending on the lift coefficient. Inserting this in Eq. (2.5) gives

$$T = D = qS(C_{D_0} + KC_L^2)$$  \hspace{1cm} (2.8)

where $K$ will be explained later. From Eq. (2.6) we can get that

$$C_L = \frac{W}{qS}$$  \hspace{1cm} (2.9)

which inserted in Eq. (2.8) gives

$$T = D = C_{D_0}qS + K\frac{W^2}{qS}.$$  \hspace{1cm} (2.10)

This can then be differentiated with respect to $q$ to find the dynamic pressure and the velocity where minimum thrust is required as follows

$$\frac{\partial T}{\partial q} = C_{D_0}S - K\frac{W^2}{q^2S} = 0$$  \hspace{1cm} (2.11)

$$\Rightarrow q_{\text{min}}^2 = \left[\frac{1}{2} \rho q_{\text{min}}^2 \right]^2 = \frac{K}{C_{D_0}}\left(\frac{W}{S}\right)^2,$$  \hspace{1cm} (2.12)

from which the velocity can be solved as

$$V_{\text{min}} = \sqrt{\frac{2W}{\rho S} \frac{K}{C_{D_0}}}.$$  \hspace{1cm} (2.13)

We can now find the velocity for which minimum power is required from the propeller. To find this we can set up the power needed for a certain velocity and differentiate with respect to the velocity. Multiplying Eq. (2.10) with velocity gives

$$P = DV = \frac{1}{2} \rho V^3S(C_{D_0} + K\frac{W^2}{qS})$$  \hspace{1cm} (2.14)

$$\Rightarrow \frac{\partial P}{\partial V} = \frac{3}{2} \rho V^2SC_{D_0} - \frac{2KW^2}{\rho V^2S} = 0$$  \hspace{1cm} (2.15)
Solving this equation for the velocity gives that

\[ V_{\text{power}_{\text{max}}} = \sqrt{\frac{2W}{\rho S}} \sqrt{\frac{K}{3C_{D_b}}} \]  

(2.16)

which is about 0.76 times the speed for minimum thrust [5].

To find the required power provided by the propeller at a certain velocity we use that power is thrust times velocity and the balance in Eqs. (2.5) and (2.6) together with Eq. (2.8) which gives that

\[ P = TV = DV = qSV(C_{D_b} + KC_{L}^2). \]  

(2.17)

Substituting the velocity from Eq. (2.16) in (2.6) results in the lift coefficient for minimum power,

\[ C_{L,\text{power}_{\text{max}}} = \sqrt{\frac{K}{3C_{D_b}}}. \]  

(2.18)

which inserted in Eq. (2.17) yields Eq. (2.19). Then Eq. (2.7) is inserted to get Eq. (2.20), in which the velocity from Eq. (2.16) is inserted to get the minimum required power for steady state flight, Eq. (2.21).

\[ P = DV = qSV(C_{D_b} + 3C_{D_b}) \]  

(2.19)

\[ P_{\text{min}} = 2\rho V^{3}_{\text{power}_{\text{max}}} SC_{D_b} \]  

(2.20)

\[ P_{\text{min}} = 4\sqrt{\frac{2W^3}{\rho S}} \left( \frac{KC_{D_b}}{3} \right)^{\frac{1}{2}}. \]  

(2.21)

This is sufficient to give us an estimation of minimum required power from the propeller and at which velocity this minimum occurs. However, we need values on \( K \) and \( C_{D_b} \) before we can get any numerical values. The latter is the zero-lift drag which includes all sorts of drag on the aircraft except the lift induced drag. This zero-lift drag consists mostly of skin friction drag plus a small separation drag, which can be gathered in an “equivalent skin friction coefficient” (\( C_{fe} \)). This can then be used in Eq. (2.22) together with a typical value on \( C_{fe} \) for the type of aircraft in question to get an estimation of the zero-lift drag coefficient.

\[ C_{D_b} = C_{fe} \frac{S_{\text{net}}}{S} \]  

(2.22)
A typical value on $C_{fe}$ for a single engine light aircraft is 0.0055 [5] and the ratio between the wet area of the aircraft and the wing area can be initially estimated to 4 [12] which yields

$$C_{lb} = 0.0055 \cdot 4 = 0.022.$$  (2.23)

$K$ is a factor that is multiplied to the square of the lift coefficient, as seen in Eq. (2.8), to represent the lift induced drag. For a wing with elliptical lift distribution, $K$ should be equal to the inverse of the product of the aspect ratio and $\pi$. However, the actual lift distribution is rarely elliptical. This factor is usually called the drag-due-to-lift factor and can be approximated as [5]

$$K = \frac{1}{\pi AR e}$$  (2.24)

where $e$ is a factor that accounts for the extra drag due to non-elliptical lift distribution and flow separation and $AR$ is the aspect ratio. Typically, $e$ is equal to 0.7 [12] and with the values from Chapter 2.3 where we decided the wing geometry we get

$$K = \frac{1}{0.7 \pi \cdot 9.55^2 / 12} = 0.0598.$$  (2.25)

With these values fixed we can insert them together with the other decided values in Section 2.3 in Eqs. (2.16) and (2.21) to get estimated numerical values on minimum power required and at which velocity this minimum occurs. The density at 1500 m altitude is $\rho = 1.05$ kg/m$^3$ [5] which gives

that

$$V_{power} = \sqrt{\frac{2 \cdot 960 \cdot 9.82}{1.05 \cdot 12} \cdot \frac{0.0598}{3 \cdot 0.022}} = 36.7407 \text{ m/s} \approx 36.7 \text{ m/s}.$$  (2.26)

and

$$P_{min} = 4 \sqrt{\frac{2 \cdot (960 \cdot 9.82)^3}{1.05 \cdot 12} \left(\frac{0.0598 \cdot 0.022}{3}\right)^{\frac{5}{4}}} = 2.9803 \cdot 10^4 \text{ W} \approx 30 \text{ kW}.$$  (2.27)

Eq. (2.17) can also be used to graphically analyze the required power at certain speeds where the lift coefficient can be found in Eq. (2.6). The result from this analysis, based on an altitude of 1500 m together with the estimations of $C_{\alpha h}$ and $K$, is presented in Figure 2-7.
Since we are aiming to compete with the performance of the Cessna 172, the velocity for which minimum power is required seems a bit too low. The cruising speed is supposed to be around 200 km/h, which is about 56 m/s so we need to know what power is needed at that speed and check if this is possible to reach in our aircraft. Inserting this velocity in Eq. (2.17) together with Eqs. (2.6) and (2.7) yields

\[ P_{\text{cruise}} (V_{\text{cruise}} = 56 \text{ m/s}) \approx 40 \text{ kW}, \]  

which can also be seen in the figure above. Note that this is the required propeller power, why we need to divide this with the efficiency for the propeller to get the required output power from the engine so that

\[ P_{\text{engine,cruise}} = \frac{P_{\text{cruise}}}{\eta_{\text{prop}}} \]  

The efficiency \( \eta_{\text{prop}} \) of a single propeller is typically about 0.85 [5], but since we are using contra rotating propellers this is said to be 12% higher. This yields that the efficiency for the propellers is

\[ \eta_{\text{prop}} = 0.85 \cdot 1.12 = 0.952 \]  

(2.30)
which gives that

\[ P_{\text{engine, cruise}} = \frac{40}{0.952} \approx 42 \text{ kW} \]  

However, probably there is higher power required during the climb phase where we are climbing to reach the cruise altitude so we need to estimate the required performance during this phase before we can come to any kind of conclusions regarding required engine power.

Still, we can now estimate the energy consumption during a certain flight time at cruise speed since the energy consumption during climb phase probably is small compared to a two hour flight in cruise speed. This can be done since energy is power multiplied with time \( t \) in hours, but first the efficiency of the engine has to be taken in consideration so that

\[ E = \frac{P_{\text{engine, cruise}} \cdot t}{\eta_{\text{engine}}}. \]  

Here, the discharge efficiency has to be taken in consideration but since the battery in this study is assumed to discharge without any losses this will be equal to one. Also, another loss in drive train is the losses in the gearbox, if one is used. This mechanical efficiency is however very high, why it can be ignored. Thus, the efficiency of the engine and the propeller is the only ones that describe the losses from the battery to the propulsion. The total energy consumption during a 2.5 hour flight in cruise speed is

\[ E_{\text{cruise}} = \frac{42 \cdot 2.5}{0.92} \approx 114.1 \text{ kWh} \]  

**Stall speed**

The lower limit for the speed is important for the performance of the aircraft, since the approach speed is required to be at least 1.3 times the stall speed for which the wing loses its lift [5]. This becomes crucial when deciding in what speed the aircraft can climb, take off and land. The stall speed is determined by the wing loading and the maximum lift coefficient \( C_{L,\text{max}} \) as

\[ \frac{W}{S} = \frac{1}{2} \rho V_{\text{stall}}^2 C_{L,\text{max}} \Rightarrow V_{\text{stall}} = \sqrt{\frac{2W}{S \rho C_{L,\text{max}}}}. \]  

Now, we need the maximum lift coefficient which depends on the availability of flaps and the maximum lift coefficient on the airfoil that is used.
The maximum lift coefficient can now be estimated as [5]

\[ C_{L,\text{max}} = 0.9 \left( c_{l,\text{max,flapped}} \frac{S_{\text{flapped}}}{S_{\text{ref}}} + c_{l,\text{max,unflapped}} \frac{S_{\text{unflapped}}}{S_{\text{ref}}} \right). \] (2.35)

If the wings are swept backwards, then we need to multiply this expression by the cosine of the sweep angle where \( c_{l,\text{max}} \) is for the airfoil section and \( C_{L,\text{max}} \) is for the aircraft. This yields

\[ C_{L,\text{max}} = 0.9 \left( c_{l,\text{max,flapped}} \frac{S_{\text{flapped}}}{S_{\text{ref}}} + c_{l,\text{max,unflapped}} \frac{S_{\text{unflapped}}}{S_{\text{ref}}} \right) \cos \Theta \] [5]. (2.36)

For a typical airfoil used in this kind of applications (NACA 2412), the maximum lift coefficient is about 1.6 [13].

**Full flaps**

If full flaps are used, the stall speed for the aircraft is calculated with the lift contribution \( \Delta c_{l,\text{max}} \) for the flap, which is about 1.3 for a slotted flap [5].

With slotted flaps on two thirds of the wing area and without any wing sweep we get that

\[ C_{L,\text{max}} = 0.9 \left( 2.9 \frac{2}{3} + 1.6 \frac{1}{3} \right) = 2.22. \] (2.37)

This can now be inserted in Eq. (2.34) to give us that

\[ V_{\text{stall,flaps}} = \sqrt{\frac{2 \cdot 960 \cdot 9.82}{12 \cdot 1.05 \cdot 2.22}} \approx 26 \text{ m/s}, \] (2.38)

which tells us that the lower limit for the approach speed of the aircraft is

\[ 1.3V_{\text{stall,flaps}} \approx 33.8 \text{ m/s}. \] (2.39)

**Flaps in takeoff position**

During takeoff, the takeoff speed has to be calculated from the stall speed with the flaps in takeoff position. Usually, small general aviation aircraft takes off without flaps, so that the maximum lift coefficient is given by the maximum lift coefficient for the airfoil. The stall speed in this configuration is calculated based on the air density at sea level \( \rho_s = 1.22 \text{ kg/m}^3 \). With the airfoil NACA 2414, which has a maximum lift coefficient of about 1.6,

\[ C_{L,\text{max}} = 0.9 \cdot 1.6 = 1.44 \] (2.40)
\[ V_{\text{null,lo}} = \sqrt{\frac{2 \cdot 960 \cdot 9.82}{12 \cdot 1.22 \cdot 1.44}} \approx 29.9 \, \text{m/s} \]  

**Steady climbing**

The power required during climbing will probably be the highest power required from the engine since it needs to work both to balance the drag and to change the potential energy of the aircraft. For that reason, this analysis is very important if we want to find the most efficient way to fly. In this section we will use rate of climb \( R/C \) (expressed in meters per second) to describe the vertical velocity of the aircraft. To analyze steady state climbing, we can use a similar approach as in the steady state flight.

The force balance equations now states that

\[ T = D + W \sin \gamma \quad (2.42) \]
\[ L = W \cos \gamma \quad (2.43) \]

where \( \gamma \) is the climbing angle relative the horizontal plane [14]. By extracting the sinus term from Eq. (2.42) and multiplying with the velocity in the direction of the aircraft \( V \) we get

\[ R/C = V \sin \gamma = \frac{(T - D)W}{W} = \frac{P_{\text{prop}} - DV}{W} \quad (2.44) \]

where \( P_{\text{prop}} \) is the power from the propeller. By inserting the relation from Eq. (2.6) in Eq. (2.43) and solving for \( C_L \) we can get that

\[ W \cos \gamma = L = qSC_L \Rightarrow C_L = \frac{W \cos \gamma}{qS}, \quad (2.45) \]

which can now be used together with Eq. (2.8) to yield

\[ D = C_{Dq}qS + K \left( \frac{W \cos \gamma}{qS} \right)^2 qS = C_{Dq}qS + \frac{KW^2 \cos^2 \gamma}{qS}. \quad (2.46) \]

By inserting this in Eq. (2.44) we get

\[ R/C = \frac{P_{\text{prop}}}{W} - \frac{C_{Dq}qS}{W}V - \frac{KW \cos^2 \gamma}{qS}V, \quad (2.47) \]

which with the approximation \( \cos^2 \gamma \approx 1 \) gives the expression
\[
R/C = \frac{P_{prop}}{W} - C_d qS V - \frac{KW}{qS} V. \tag{2.48}
\]

If Eq. (2.48) is solved for the propeller power, we get [5]

\[
P_{prop} = W \frac{R/C}{qS} + C_d qSV + \frac{KW^2}{qS} V. \tag{2.49}
\]

Eq. (2.49) can be used to graphically analyze the required propeller power for different velocities and rates of climb to find the most efficient combination. The rate of climb tells us how long it will take for us to reach cruising altitude, so that the total energy consumption during climbing can be calculated. However, to get to this analysis we need a valid range for the speed and the rate of climb. The lower limit for the speed is \(1.2 V_{stall,lo}\) [5] and the higher limit is picked to be somewhat realistic. If the available propeller power is known, the maximum rate of climb can be found in a graphic analysis where the rate of climb is evaluated for different speeds.

The time that it takes to climb a certain altitude \(h\), is then given by

\[
t = \frac{h}{R/C}, \tag{2.50}
\]

from which the total energy consumption during climbing can be calculated as

\[
E_{climb} = \frac{P_{prop}}{\eta_{engine} \cdot \eta_{prop}} \cdot \frac{t}{3600}. \tag{2.51}
\]

To know how much power the propeller can deliver, we need to decide how many engines we need to have a sufficient rate of climb. Since the design needs pairs of engines because of the contra rotating system, we are limited in our options of engine power. Two engines will deliver 60 kW and four will deliver 120 kW and so on. To determine the number of engines required, an analysis of the rate of climb at an altitude of 1500 m for different velocities and powers according to Eq. (2.48) is done and shown in Figure 2-8. 60 kW produces a rate of climb that is too low but 180 kW needs a total of 6 engines which in turn means a lot of wiring and excess weight which seems unnecessary since this maximum power is only required during climbing. We only need 42 kW during level flight thus 120 kW is just enough. From Figure 2-8 we get that this power results in a maximum rate of climb

\[
R/C_{max} \approx 8.9 \text{ m/s} \quad \text{at the velocity } 1.2 V_{stall,lo} = 35.9 \text{ m/s}.
\]
We now need to determine which velocity and rate of climb that results in the lowest energy consumption during climbing. Eqs. (2.49), (2.50) and (2.51) are therefore used in a graphical analysis of the required power and total energy consumption for a climb to an altitude of 1500 m in Figure 2-9 and Figure 2-10. It shows from these figures that the lowest energy consumption occurs when the rate of climb is the highest possible and the velocity is the lowest possible. From this we get that the optimal circumstances during climbing, to minimize energy consumption, is to climb with $\frac{R}{C_{max}}$ and $1.2V_{stall.no}$. The total climb time to 1500 m is approx. 2.8 min and at an engine power of 117 kW the total energy consumption is $E_{climb} \approx 5.3$ kWh. Note that this energy consumption does not take the covered travelled distance during climbing in consideration since the purpose of the aircraft mainly is entertainment flying where the travelled distance is less relevant. By this reasoning the aim of the climbing procedure is to reach cruise altitude with minimum energy consumption.
Figure 2-9. The power required during climb for different rates of climb and velocities at 1500 m.

Figure 2-10. The energy consumption during a climb to 1500 m for different rates of climb and velocities. Note that this is calculated with a constant air density (at 1500 m).
**Total energy required**

The total energy consumption during a climb to 1500 m and a flight time of 2 hours in cruise speed is given by

\[
E_{\text{cruise}} + E_{\text{climb}} = 114.1 + 5.2 = 119.3 \text{ kWh}
\]  
(2.52)

### 2.7 Take-off distance estimation

The take-off procedure can be divided into three parts, where one is the ground roll, the second is the transition and the third is the climb to obstacle clearance altitude. The ground roll includes acceleration up to lift speed, and rotation where the aircraft is rotating to create lift. During the transition phase, the aircraft accelerates to climb speed and the climb to obstacle clearance is the climb until the aircraft has reached a certain obstacle clearance altitude [5].

**Ground roll**

The ground roll distance is calculated by integrating velocity divided by acceleration during the acceleration phase. The integration yields

\[
S_G = \left(\frac{1}{2gK_A}\right) \ln \left(\frac{K_T + K_AV_{\text{takeoff}}^2}{K_T}\right)
\]  
(2.53)

where

\[
K_T = \left(\frac{T}{W}\right) - \mu
\]  
(2.54)

\[
K_A = \frac{\rho}{2(W/S)} \left( \mu C_{t_w} - C_{t_b} - KC_{t_w}^2 \right).
\]  
(2.55)

Here, \( V_{\text{takeoff}} \) is the takeoff velocity, \( T \) is the thrust at 70% of \( V_{\text{takeoff}} \), \( \mu \) is the rolling friction factor and \( C_L \) is the lift coefficient based on wing angle of attack on the ground. The takeoff velocity is 1.1 times the stall speed \( V_{\text{stall, to}} \), where the stall speed should be evaluated for the maximum lift coefficient with the flaps in takeoff position [5]. The thrust is now calculated as

\[
T = \frac{\eta_{\text{prop}, P_{\text{engine}}}}{0.7 \cdot 1.1 \cdot V_{\text{stall, to}}} = \frac{0.952 \cdot 120 \cdot 10^3}{0.7 \cdot 1.1 \cdot 39.9} \cong 4.96 \cdot 10^3 \text{ N}
\]  
(2.56)

which with a rolling friction factor of 0.04 [5] gives

\[
S_G \cong 114.9 \text{ m}
\]  
(2.57)
The time to rotate the aircraft to produce lift is about \( t = 1 \) second for small aircraft [5], which gives that

\[
S_R = V_{\text{takeoff}} \cdot t \tag{2.58}
\]

and together with the acceleration phase it according to [5] yields

\[
S_{GR} = S_G + S_R. \tag{2.59}
\]

This gives that the total ground roll for the concept aircraft is

\[
S_{GR} = 114.9 + 1.1 \cdot 29.9 \equiv 147.8 \text{ m} \tag{2.60}
\]

**Transition**

In this phase, where the aircraft is accelerating from \( 1.1 V_{\text{stall, to}} \) to the climb speed \( 1.2 V_{\text{stall, to}} \), the climb angle \( \varphi \) is given by

\[
\sin \varphi = \frac{T - D}{W}, \tag{2.61}
\]

where the thrust \( T \) and the drag \( D \) must be evaluated at the average speed \( 1.15 V_{\text{stall, to}} \). The thrust can be found by dividing the available propeller power by the velocity, while the drag can be found using Eq. (2.10). The drag is then calculated as in Eq. (2.8). If this is inserted in Eq. (2.61) we get that

\[
\sin \varphi = \frac{\eta_{\text{prop}} P_{\text{engine}}}{1.15 \cdot 1.1 \cdot V_{\text{stall, to}}} - \frac{1}{2} \rho (1.15 \cdot 1.1 \cdot V_{\text{stall, to}})^2 S (C_{\text{a}_{\text{h}}} + K \left[ \frac{\rho (1.15 \cdot 1.1 \cdot V_{\text{stall, to}})^2 W}{2S} \right]^2)
\]

The distance covered during transition can according to [5] be calculated as

\[
S_T = R \sin \varphi, \tag{2.63}
\]

where

\[
R = 0.205 V_{\text{stall, to}}^2. \tag{2.64}
\]

With numerical values we get that

\[
S_T \equiv 45 \text{ m} \tag{2.65}
\]
\textit{Climb to obstacle clearance}

Finally, the distance covered during the climb to obstacle clearance altitude is given by [5] as

\[ S_c = \frac{h_{\text{obstacle}} - h_{TR}}{\tan \varphi}, \tag{2.66} \]

where \( h_{\text{obstacle}} \) is 50 ft. (15.2 meters) and

\[ h_{TR} = R(1 - \cos \varphi). \tag{2.67} \]

With numerical values inserted we get that

\[ S_c \approx 37.8 \text{ m}. \tag{2.68} \]

\textit{Total takeoff distance}

The total distance needed for takeoff is then given by adding these components. This gives that the total takeoff distance to obstacle clearance altitude with the initial values is

\[ S_{TO} = S_{GR} + S_T + S_C \approx 230.7 \text{ m} \tag{2.69} \]

\subsection*{2.8 Static pitch stability analysis}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2-11.png}
\caption{Contributions to the pitching moment for a canard configuration.}
\end{figure}

\textbf{Stability criteria}

In the stability analysis we split the contribution to the total lift \( L \) on the aircraft in to two parts, where one is the lift from the main wing and the other is from the horizontal stabilizer (in this case the canard). We can then set up the pitch moment around the center of gravity of the aircraft, expressed in the lift contributions and its lever to the center of gravity. This moment is defined positive when pitching the nose upwards (clockwise rotation). In this expression, the wing pitching moment \( M_{a.c.} \) is also included for the main wing. This moment comes from that the wing is creating a moment around
its aerodynamic center because of its asymmetrical airfoil. This yields that the pitching moment around the center of gravity is given by

\[ M_{c.g} = M_{a.c.} - l_{c.g} L_w + (l_h - l_{c.g}) L_h = M_{a.c.} - l_{c.g} L + l_h L_h \]  

(2.70)

where \( L_w \) is the lift from the main wing and \( L_h \) is the lift from the stabilizer. \( l_{c.g} \) and \( l_h \) are the distances from the center of gravity to the aerodynamic center of the main wing and stabilizer respectively [15]. Note that the analysis made in Ref. [15] is based on a tail stabilizer, where the sign convention for the distances \( l_{c.g} \) and \( l_h \) is opposite to the convention in this analysis. Here, the sign convention is such that \( l_{c.g} \) is defined as positive when the aerodynamic center of the main wing is behind the center of gravity. Further, \( l_h \) is positive when the aerodynamic center of the canard is ahead of the center of gravity.

If the aircraft should be statically stable, the derivative of the pitching moment with respect to the angle of attack should be negative. Specifically, this means that if the aircraft is disturbed from trim point (where the pitching moment is zero) so that the nose is pitched upwards the angle of attack increases and therefore the pitching moment becomes negative. The negative pitching moment makes the aircraft counteract to the disturbance by pitching the nose down. If the disturbance pitches the nose downwards, the pitching moment becomes positive which raises the nose again. This means that the aircraft is statically stable in pitch and can be expressed in the criterion

\[ \frac{dM_{c.g}}{d\alpha} < 0 \]  

(2.71)

This can also be translated into non-dimensional lift coefficients, similar to what is done earlier in the steady state flight model. The lift contributions in Eq. (2.70) is then expressed in these coefficients, and the above criterion can be translated into

\[ \frac{dC_{m.c.g.}}{dC_L} < 0 \]  

(2.72)

since the lift coefficient is in the same way dependent on the lift coefficient as it is on the angle of attack. The criteria for static pitch stability is then given by [15]

\[ -\frac{dC_{m.c.g.}}{dC_L} = \frac{l_{c.g} - l_{n.p}}{c} > 0 \]  

(2.73)

where
\[ \frac{l_{n.p.}}{c} = \bar{V}_h \frac{a_h}{a} \left(1 - \frac{d\xi_h}{d\alpha} \right) \]  

(2.74)

Here, \( \bar{V}_h \) is the horizontal canard volume coefficient which together with \( a_h \) and \( \frac{d\xi_h}{d\alpha} \) is described below.

\[ \bar{V}_h = \frac{S_{h}f_h}{S_{c}} \]  

(2.75)

\[ a = \frac{AR}{2 + AR} 2\pi \quad \text{and} \quad a_h = \frac{AR_h}{2 + AR_h} 2\pi \]  

(2.76)

\[ \frac{d\xi_h}{d\alpha} = \left(1 + \frac{\sqrt{\xi^2+1}}{\xi} \right) \frac{1}{2 + AR} \]  

(2.77)

In the above equations \( S_{h} \) is the reference area for the horizontal stabilizer, \( AR \) and \( AR_h \) is the aspect ratio of the main wing and the stabilizer respectively. Further, \( \xi = 2(-l_h)/b \) and \( c \) is the aerodynamic chord, which for a tapered wing is given by [15] as

\[ \bar{c} = \frac{1 + \lambda + \lambda^2}{1 + \lambda^3} \frac{2}{3} \quad \text{where} \quad \lambda = \frac{c_{tip}}{c_{root}} \]  

(2.78)

An important parameter in the stability design process is the stability margin, which is given by the ratio \((l_{c.g.} - l_{n.p.})/\bar{c} \). This has to be positive, and the aircraft should be designed so that this margin is between 5-10 % for a general aviation aircraft [15].

Also, there is a second criterion for the aircraft to be statically stable in pitch that states that when the lift coefficient is zero, the pitch moment coefficient has to be positive [15]. For a canard configuration, this can be described as

\[ C_{mb} = C_{m,w,a.c.} + a_h i_h \bar{V}_h > 0 \]  

(2.79)

Typically, \( C_{m,w,a.c.} < 0 \) which gives that the tail incidence \( i_h \) (the difference in angle between the angle of attack for the stabilizer and the main wing) has to be positive to satisfy this second criterion.

With the initial numerical values, different geometries of the aircraft can then be tried out to find adequate values on the distances \( l_{c.g.} \) and \( l_h \) and the geometry of the canard. In this moment, the stall performance has to be taken in consideration. In a canard configuration it is crucial that the canard
stalls before the main wing which if designed properly means that the aircraft is very stall proof. When the canard stalls, the lifting moment in front of the aircraft is lost, why the aircraft pitches down to automatically recover from the stall. If the aircraft is badly designed, the main wing stalls first making the aircraft tip over on its back because of the remaining lift on the canard. This property of the aircraft can be obtained by using an airfoil on the canard that stalls in a lower angle of attack than the main wing airfoil, or by designing the canard so that its aspect ratio is higher than the main wing. In this study, the airfoil used for the canard is the GU25-5(11)8 [16]. This airfoil stalls at a lower angle of attack than the NACA 2412 airfoil that is used on the main wing. With a canard aspect ratio of 8 and an area of $1.6 \text{ m}^2$ the canard wing span becomes 3.58 m. Hence, the aspect ratio is higher than for the main wing ($AR = 7.6$). Further, $l_{c.g.} = 0.5 \text{ m}$ and $l_h = 2.6 \text{ m}$ results in a stability margin of approx. 8.7%. This geometry requires a canard incidence $i_h > 0.1082^\circ$

**Positive lift for the canard**

For a canard aircraft, the lift on the canard should be positive for all speeds. To investigate this we start by solving Eq. (2.70) for the lift on the canard $L_h$ with $M_{c.g.} = 0$ in the trim state.

$$L_h = \frac{l_{c.g.} L - M_{a.c.}}{l_h} \quad (2.80)$$

This yields that the criterion for positive lift on the canard is that

$$l_{c.g.} L - M_{a.c.} > 0 \quad (2.81)$$

The pitch moment can be expressed in with a non-dimensional coefficient for the pitching moment and the lift has to be equal the weight of the aircraft in the trim state. This gives that

$$l_{c.g.} W - C_{m.a.c.} \frac{1}{2} \rho V^2 \overline{c} > 0 \quad (2.82)$$

Since $C_{m.a.c.}$ usually is negative and $l_{c.g.}$ and $W$ is strictly positive we get that

$$l_{c.g.} W + \left( -C_{m.a.c.} \frac{1}{2} \rho V^2 \overline{c} \right) > 0 \quad (2.83)$$

which implies that the left hand side is positive for all velocities. In practice, this means that the canard is producing positive lift for all velocities. In the case with a tail stabilizer there is a velocity for which the load on the tail changes signs from negative to positive [15].
2.9 Positioning the main wing

An important part of the stability analysis is the weight distribution of the aircraft. The goal of this analysis is to find out where the aerodynamic center of the main wing should be positioned, to meet the requirements for static pitch stability. When the stability criterion is fulfilled the weight distribution is calculated. The only thing that is carried over from the previous installment is the distance between the main wing and the center of gravity of the aircraft $l_{c.g.}$. The main problem here is that when we start to move the main wing the center of gravity shifts. It becomes a linear matrix problem. We have two criteria. The first one is that

$$
(M_1 + M_w) \cdot x_m = x_i \cdot M_1 + M_w \left[ x_v + 0.17 \cdot c \cdot \cos(\alpha_m) \right].
$$

(2.84)

Here, $M_1$ is the mass of the aircraft without the main wing and $M_w$ is the mass of the main wing. The last term contains the contributions from the main wing, where the cosine comes from that the aerodynamic center of the wing is moved when the wing is mounted on the fuselage in the angle of attack required at cruise speed $\alpha_m$ so that the aircraft fuselage is in level flight. This angle is found by calculating the required lift coefficient from Eq. (2.9). In the data for the airfoil that is used in this study (NACA 2412), the required angle of attack can be found. Further, $0.17c$ is the distance between the aerodynamic center of the main wing and its center of gravity.

$\overline{x}_i$ is calculated with a simple moment equation as

$$
\overline{x}_i = \frac{M_{batt} \cdot l_{batt} + M_{eng} \cdot l_{eng} + 2 \cdot M_{pass} \cdot l_{pass1} + 2 \cdot M_{pass} \cdot l_{pass2} + M_{body} \cdot l_{body} + M_{stab} \cdot l_{stab}}{M_{batt} + M_{eng} + 4 \cdot M_{pass} + M_{body} + M_{stab}}.
$$

(2.85)
One thing that has to be noted is that the position of the stabilizer is based on the position of the wing. But, since it is such a small mass we decide to simplify the calculations by neglecting it in the following calculations. Our second criteria is that

\[ x_v - x_m = \left| l_{cg} \right| \].

Eq. (2.84) is reorganized to

\[ \frac{x_v \cdot M_v}{M_i + M_v} - x_m = \frac{-\left( x_l \cdot M_i + 0.17 \cdot c \cdot M_v \right)}{(M_i + M_v)} \] (2.87)

and then combined with Eq. (2.85) to form the matrix problem

\[
\begin{bmatrix}
1 \\
\frac{M_v}{M_i + M_v} \\
-1 \\
-1
\end{bmatrix}
\begin{bmatrix}
x_v \\
x_m
\end{bmatrix}
=
\begin{bmatrix}
l_{cg} \\
-\left( x_l \cdot M_i + 0.17 \cdot c \cdot M_v \right) \\
(M_i + M_v)
\end{bmatrix},
\]

This is then solved to find the position of the aerodynamic center of the wing \( x_v \) and the center of gravity \( x_m \). The position of the center of gravity happens to be one the same spot as the backseat passengers which was not planned from the beginning. This however a good thing, since it means that the pilot does not need to change the trim of the aircraft depending on the amount of passengers.

With the initial values and the geometry from the stability analysis, we get the following values for the distances from the reference point.

\[ x_v = 3.16 \text{ m} \quad \text{and} \quad x_m = 2.26 \text{ m} \]
It shows that it is hard to get satisfactory distance between the center of gravity and the aerodynamic center of the main wing since the heavy batteries are placed together with the engines in the very rear of the aircraft. This is solved by sweeping the wing backwards 10 degrees, which moves the aerodynamic center so that the distance is correct. This sweep affects the stall speed of the aircraft, which has to be recalculated. The clearance between the propeller and the wing is also considered during the positioning of the wing.

2.10 Component build-up method for zero-lift drag
At our initial calculation of the aircraft's performance we used a very simple way to calculate the zero-lift drag \( C_{D_0} \). That method is based on an assumption that all small aircraft in principal look the same. But since our aircraft is more of the unconventional type a more refined method is needed. The component build up method is a way to take the, now known, geometry of the aircraft in consideration when calculating the zero-lift drag [5]. This method works by adding a specific \( C_{D_0} \) for every separate component of the aircraft's exterior to get this coefficient more fine-tuned. The total \( C_{D_0} \) of the aircraft is according to [5] calculated as

\[
(C_{D_0})_{subsonic} = \sum \left( \frac{C_{f_c}FFQ_{swt}}{S_{ref}} \right) + C_{D_{aux}} + C_{D_{t.r.p}} \tag{2.89}
\]

Here, \( C_{f_c} \) is the skin friction coefficient which is calculated based on the Reynolds number of the component as

\[
C_{f_{c,ref}} (R) = 1.328 / \sqrt{R} \tag{2.90}
\]

if its laminar flow and with

\[
C_{f_{c,sub}} (R) = \frac{0.455}{(\log_{10} R)^{2.58} \left(1 + 0.144M^2 \right)^{0.65}} \tag{2.91}
\]

if it is turbulent flow, where \( M \) is the Mach number. Reynolds number for each component is calculated as

\[
R_c = \frac{l_c \rho V}{\mu} \tag{2.92}
\]

where \( \mu \) is the viscosity in the air (16.7 \( \cdot \) 10^{-6} Pa \cdot s at 0°C), \( \rho \) is the air density at 1500 m and \( V = V_{cruise} \). Further, \( l_c \) is the characteristic length of the component. For the wings and canard this is the airfoil mean chord length. For the fuselage it is the length of the entire fuselage [5].
To find where the flow goes from laminar to turbulent on each component, we are calculating how far in on every component the flow reaches a Reynolds number of \( R_{\text{trans}} = 5 \cdot 10^5 \), for which the flow becomes turbulent [17]. This distance is calculated from the Reynolds number as
\[
I_{\text{lam}} = R_{\text{trans}} \mu / \rho V
\]  
(2.93)

Then, the total \( C_{f_c} \) for each component is calculated as
\[
C_{f_c} = \frac{C_{f_{\text{lam}}} I_{\text{lam}} - C_{f_{\text{sub}}} (R_{\text{trans}} - I_{\text{sub}}) + C_{f_{\text{sub}}} (R_c - I_c)}{I_c}
\]  
(2.94)

Since the winglets are placed on the wing tip which is one of the most turbulent parts of the aircraft, \( C_{f_{\text{wing}}} = C_{f_{\text{sub}}} (R_c) \).

The component form factors \( FF_c \) takes the shape and size of the component in consideration. Different types of components have different equations to calculate \( FF_c \). For wings and canards it is calculated as [5]
\[
FF_c = \left[ 1 + \frac{0.6}{(x/c)_m} \left( \frac{t}{c} \right) + 100 \left( \frac{t}{c} \right)^2 \right] \left[ 1.34M^{0.18} \cos(\Theta) \right]
\]  
(2.95)

Here, \((x/c)_m\) is the position along the chord where the airfoil is the thickest and \( t \) is the maximum thickness of the airfoil. For the fuselage \( FF_f \) is given as [5]
\[
FF_f = \left[ 1 + \frac{60}{f^2} + \frac{f}{400} \right]
\]  
(2.96)

where
\[
f = \frac{l}{\sqrt{(4/\pi)A_{\text{max}}}}
\]
(2.97)

Here, \( A_{\text{max}} \) is the maximum cross section area of the fuselage. In Eq. (2.89) \( Q_c \) is the interference factor. It takes in to account the zero-lift drag that occurs between components. For a mid-wing aircraft like ours the interference is minimal for the wing. Hence, \( Q_{\text{wing}} = 1 \) [5]. The same goes for the canards but since the winglet vertical stabilizers are placed in the most troubled place on the wing (the tip) they will have a \( Q_{\text{winglet}} = 1.2 \) [5]. In the calculator for the canard, the airfoil GU25-5(11)8 is used
The fuselage has negligible interference which gives $Q_{fuselage} = 1$. Further, $S_{wet}$ is the wet area of the component.

This procedure is done for the fuselage, the wing, the canard, and the winglets. They are then summed up and divided by the reference area $S_{ref}$ which is the wing area. The landing gears and wing flaps contribution to $C_{Dh}$ come in the form of $C_{D_{misc}}$. The landing-gear $C_{Dh}$ is estimated by comparison to test data for a similar gear arrangement [5]. A factor for the design of landing-gear is multiplied with the frontal area and the divided with the reference wing area. The factor for streamlined wheel and tire is 0.01672 and the factor for round strut is 0.02787 [5]. The estimated total area of the wheels is 0.1 m$^2$ and the struts have a total area of 0.027 m$^2$ (estimated from the landing gear on the Velocity XL, see Section 2.1). Thus the calculation for the landing gear is done as

$$C_{D_{landing}} = \frac{0.01672 \cdot 0.1 + 0.02787 \cdot 0.027}{S_{ref}}.$$  \hfill (2.98)

For the wing-flaps the estimation is made by this equation

$$C_{D_{0,flap}} = 0.00023 \frac{\text{flap span}}{\text{wing span}} \delta_{flap},$$  \hfill (2.99)

where $\delta_{flap}$ is the flaps angle from the wing. The only time we have flaps down is during landing why $\delta_{flap}$ is set to zero at all other times.

There is one more contribution to the total zero-lift drag, which comes from the fuselage upsweep at the end of the aircraft. The upsweep angle of the concept aircraft is 12° ($u=0.2$ rad), so that this contribution is calculated as [5]

$$C_{D_{upsweep}} = \frac{3.83u^2 A_{max}}{S_{ref}}.$$  \hfill (2.100)

Since all of these zero-lift drag factors are dependent on the velocity, the above calculation has been implemented in the below described mathematical model so that $C_{Dh}$ is calculated for different flight conditions (see Appendix B). For example, $C_{Dh}$ in steady state flight at cruise speed is calculated to 0.018 compared to the initial estimate at 0.022.

2.11 Absolute ceiling estimation

From Eq. (2.48) can be solved for the air density to find the maximum altitude for the aircraft at a given speed. This altitude occurs when the rate of climb is zero, so that
\[
\frac{P_{prop}}{W} - \frac{C_{D_p} \rho S V_{cruise}^3}{2W} - \frac{2KW}{\rho S V_{cruise}} = 0.
\] (2.101)

With the altitude density variation included in the calculation for the zero-lift drag, a graphical analysis in Figure 2-14 shows that the rate of climb becomes zero when the density is about 0.16 kg/m\(^3\). This density occurs at an altitude of about 30000 meters [5]. However, it is impossible to fly at that altitude without a pressurized cabin and in addition the concept aircraft probably does not have enough batteries to climb to this altitude.

\[\text{Figure 2-14. Rate of climb for different air densities.}\]
2.12 Computerized mathematical model
When the number of engines has been determined based on the calculations of the required power for climbing, the required energy capacity for the battery in the aircraft is known. This can then be used as a starting point in an iterating process to optimize the performance to given requirements. A computerized mathematical model is now created to enable this iterating process, where different design parameters such as the number of engines, number of passengers, canard position and area and battery capacity can be tried out to study how they affect the overall performance of the aircraft (see Appendix B).
3 Result
The iterating process results in a higher amount of batteries in the concept aircraft, since it is found to increase the maximum range and flight time of the aircraft even if a higher weight means that the maximum rate of climb is lower. Since the weight is increased, both wing area and wing span are also increased to preserve the wing loading and aspect ratio. Further, the distances $l_{c,g}$ and $l_h$ has to be adjusted to retain a stability margin of 8%. The overall geometry of the aircraft is described in Figure 3-1.

\[ \text{Figure 3-1. Overall geometry of the aircraft.} \]

\[ \text{Figure 3-2. Side view of the aircraft.} \]
<table>
<thead>
<tr>
<th>Cruise speed</th>
<th>201.6 km/h</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum speed</td>
<td>331.9 km/h</td>
</tr>
<tr>
<td>Economy speed</td>
<td>145.1 km/h</td>
</tr>
<tr>
<td>Maximum Range¹ at cruise speed</td>
<td>796.7 km</td>
</tr>
<tr>
<td>Maximum flight time² at economy speed</td>
<td>5h 23min</td>
</tr>
<tr>
<td>Takeoff distance</td>
<td>277 m</td>
</tr>
<tr>
<td>Ground roll</td>
<td>181 m</td>
</tr>
<tr>
<td>Maximum rate of climb</td>
<td></td>
</tr>
<tr>
<td>at 1500 m</td>
<td>7.2 m/s</td>
</tr>
<tr>
<td>at sea level</td>
<td>7.5 m/s</td>
</tr>
<tr>
<td>Stall speed with full flaps</td>
<td>24.7 m/s</td>
</tr>
<tr>
<td>$C_{D_0}$</td>
<td>0.018</td>
</tr>
<tr>
<td>$K$</td>
<td>0.0598</td>
</tr>
<tr>
<td>Power plant</td>
<td></td>
</tr>
<tr>
<td>Type</td>
<td>Brushless AC electric engine</td>
</tr>
<tr>
<td>Manufacturer</td>
<td>Enstroj</td>
</tr>
<tr>
<td>Model</td>
<td>EMRAX</td>
</tr>
<tr>
<td>Power output</td>
<td>30 kW continuous / 50 kW peak</td>
</tr>
<tr>
<td>Maximum takeoff weight</td>
<td>1119.8 kg</td>
</tr>
<tr>
<td>Battery type</td>
<td>Lithium-air</td>
</tr>
<tr>
<td>Battery capacity</td>
<td>225 kWh</td>
</tr>
<tr>
<td>Battery weight</td>
<td>300 kg</td>
</tr>
<tr>
<td>Baggage capacity</td>
<td>Dependent on passenger weight¹</td>
</tr>
<tr>
<td>Length</td>
<td>5.7 m</td>
</tr>
<tr>
<td>Wing span</td>
<td>10.13 m</td>
</tr>
<tr>
<td>Wing area</td>
<td>13.51 m²</td>
</tr>
<tr>
<td>Canard area</td>
<td>1.8 m²</td>
</tr>
<tr>
<td>Canard span</td>
<td>3.8 m</td>
</tr>
<tr>
<td>Seating capacity</td>
<td>4</td>
</tr>
</tbody>
</table>

¹with 45 min reserve flight time
²after climb to 1500 m
³the maximum weight for passengers and baggage is 360 kg
4 Discussion

4.1 Conclusion and discussion

The most efficient flight in terms of loiter endurance is obtained when flying at the velocity for which minimum power is required, while it in terms of maximum range is obtained when flying at the velocity for minimum thrust. In the steady state flight analysis it shows that these velocities are dependent on the square root of the wing loading. Hence, if a larger wing loading is accepted the velocity for minimum power or thrust is higher. The minimum power is also dependent on the square root of the wing loading, which implies that if the wing loading is raised the required power is raised by the square root so that it pays off to have a higher wing loading. In turn, this shows that it pays off to have a higher weight in the aircraft in relation to required power if the higher wing loading can be handled. Unfortunately, this cannot be used in this study since we are not sure that the higher wing loading can be handled.

Another interesting result is that the steady climbing analysis shows that the lowest energy consumption during a climb to a certain altitude is obtained for the highest rate of climb and the lowest speed possible for the aircraft. However, the power required during climbing is also dependent on the square of the weight so that the required power is higher for higher weights. This contradicts what is valid for steady state flight. Hence, a larger weight and therefore a larger wing loading pays off during steady state flight but means that the power during climbing needs to be much higher with the same rate of climb and velocity. Further, the energy consumption during climbing can be lowered by allowing lower velocities or higher rates of climb. The velocity is limited by the stall speed, so the use of high lift devices during climbing would allow a lower velocity and in turn result in lower energy consumption. The rate of climb is mainly limited by the available engine power, which has to be increased. A higher engine power does however lead to higher weight, which in turn requires even higher power.

A higher weight of the aircraft appears to pay off if the added weight consists of batteries so that the energy capacity in the aircraft is increased. This was found to increase the available flight time and range of the aircraft, despite the fact that a higher weight requires a larger wing area, which in turn leads to higher drag. This explains why the iterating process led to a higher battery weight than the initial values.

The stability for a static stable aircraft is also important when discussing efficient flight conditions. For a canard aircraft it is shown that the load on the canard is positive for all speeds, while it for a conventional tail stabilizer is shown that the load on the stabilizer is negative for low speeds. This is an important difference when it comes to finding the most efficient way to fly. At lower speeds, when a tail stabilizer is producing negative lift, it is still producing drag. In addition, the main wing has to compromise for the negative lift force at the tail by producing even more lift, which produces even
more drag. For this reason, the canard configuration can give a more efficient flight. This becomes important during climbing, since the most efficient climb is at low speeds. This leads to the conclusion that a canard aircraft is able to climb more efficiently than a conventional tail stabilized aircraft due to the lower drag. Further, an aircraft that is neutral stable in pitch can result in lower drag, since the stabilizer in this case is more of a control surface which does not have to produce any lift and therefore no lift induced drag. A neutrally stable aircraft can be controlled in pitch by using computerized systems and control surfaces similar to a tail stabilizer or canard. However, this cannot be applied in this study since one of the initial requirements is that the concept aircraft should be statically stable in pitch.

The component build up method showed that the zero-lift drag of the aircraft became lower than the estimation. This implies that the zero-lift drag on the concept aircraft is low compared to similar aircraft of this type. This is a consequence of the shorter fuselage that comes with the canard configuration, which results in a smaller wetted area. In the initial estimation, the wetted area was estimated to be 4 times the wing area but it turns out that this factor is lowered to 3.3 on the concept aircraft. It is also possible that the shorter fuselage can lead to lower weight of the aircraft, which in turn gives lower wing area for a similar capacity aircraft. In that case, this would also contribute to a lower zero-lift drag. However, this has not been accounted for in this study.

To be able to find a concept that is environmentally friendly the energy source in the aircraft is important. Today, most of the aircraft uses oil based fuels with very high specific energies. This makes it hard to find an electrical concept that can compete with the performance of current day aircraft, mainly in range and flight time. Another drawback with an electric powered aircraft is that the battery weight is constant even though the battery drains out during flight, in comparison to petrol powered aircraft where the fuel weight decreases when the fuel is burned. This enables petrol powered aircraft to reach a longer range. However, the study shows that the use of lithium-air batteries makes it possible to at least compete with performance of similar petrol powered aircraft, except when it comes to range. For example, the Cessna 172 Skyhawk has a maximum cruise speed at 230 km/h and a maximum range of 1185 km with 45 min fuel reserve left. The concept aircraft has a top speed that is a lot higher, and a cruise speed that is similar to what is useful in most situations. The range of the concept aircraft should be high enough to make it applicable for entertainment flying or in other situations where similar aircraft is used today.

Further, it is interesting to check how the use of the lithium-ion batteries would affect the performance. Lithium-ion batteries can have a specific energy of 200 Wh/kg compared to the specific energy at 750 Wh/kg for the lithium-air batteries used in this study. If the same amount (300 kg) of lithium-ion batteries is used, the maximum range drops to 81 km which limits the application of the aircraft. Additionally, the flight time is reduced to 24 min + 45 min reserve flight time. This shows the
importance of the specific energy in the energy source, if the aircraft should be useful and able to compete with petrol driven aircraft.

4.2 Reliability of the study
To enable the reader to analyze the reliability of this study it is important to note that this study is based on future battery technology, which is not available on the market today. The lithium-air batteries used in this study is currently being researched, with the goal to make them useful in the vehicle industry as a fuel that has the capacity to replace the oil based fuels currently used [18]. As of today, a full scale prototype is expected to be ready in 2013 and the batteries are expected to be commercialized by around 2020 [19]. The reason for using this battery technology instead of current day battery technology (i.e. lithium-ion) in this study was that we thought it would be interesting to study how future technology would be able to replace the available fuels of today in this application. It is difficult for current day battery technologies to compete with standard oil based fuels, because of the low energy density. Also, the assumption that states that the battery discharges without any losses affects the performance of the concept aircraft which is why this has to be accounted for when analyzing the reliability of this study.

Another aspect that affects the reliability of this study is the assumption of the lower structural weight as a consequence of the use of light-weight materials. The lower weight is essential for the performance of the aircraft, why it is important to note the influence of this aspect to the overall result. However, the light-weight technology that is used in this study is meant to be based on technology and materials that are available today.

4.3 Suggestions for further research
For further research it would be interesting to analyze how different parameters are connected to the final performance of the aircraft in a more analytical way. For example the research could deal with how the final performance depends on the amount of batteries. Further, the consequences of a larger scale of the aircraft could be examined to find out if it is possible to find a concept for an aircraft that can carry more passengers or cargo. It would also be interesting to investigate if a higher wing loading can be handled on an aircraft of this type, and how it would be done, since this study shows that the wing loading is important to find the most efficient way to fly.

Since this study is limited to lower speeds, the use of electric engines in higher speeds could be investigated. It would also be interesting to find out how laminar flow on a larger part of the wing can be obtained to reduce the drag, as well as how the presence of a stabilizer affects the drag compared to a neutrally stable aircraft where the stability can be obtained from computer controlled control surfaces.
5 Division of labour
A large part of the work has been done in cooperation between us. However, we had a few focus areas where one of us accounted for a slightly larger part of the labour. For Linus Flodin, these areas have been the static stability analysis together with the editing of this paper. For Oscar Hag, these areas have been the geometry and weight distribution of the aircraft together with the component build up method. The remainder of the labour has been distributed equally between us.
References


Appendix A

Specification of requirements
Concept study
Version 1.1, created 2012-02-06.
Code name: Princess of Elair

The requirement specification refers to the concept study of an environmentally friendly aircraft. The requirements are the goals that the study is aiming to reach. The desired features explained below are aspects that have to be taken in consideration.

Requirements

- Capacity of 4 passengers.
- Cruise speed of at least 200 km/h.
- Flight time of 2 hours and 15 minutes plus 45 minutes reserve time in case of any coincidence where you have to fly to another airfield or interrupt and redo the landing procedure.
- Should be environmentally friendly. This means that the aircraft should be powered by a renewable energy source.
- Should not emit any greenhouse gases such as carbon dioxide.
- Stall speed below 50 knots (approx. 90 km/h or 26 m/s).

Desired features

- Pusher propeller
- Canard configuration for horizontal stability.
- High number of propeller blades since this reduces the risk for the propeller to hit the ground during landing or takeoff.
- Esthetically appealing and unconventional design for marketing reasons.
- The transition between the fuselage and main wing and stabilizing surfaces should be blended design. This means that the transition should be smooth and without sharp edges.
- Retractable landing gear to minimize air resistance during flight.
- Low cost in operation.
- Low weight.
- Good maneuverability performance, i.e. low stall speed, forgiving flight characteristics.
Project description
Concept study
Version 1.1, created 2012-02-06).
Code name: Princess of Elair

1. Project title
Code name: Princess of Elair

Official name not decided in the time of writing.

2. Members of the project
Linus Flodin, linusflo@kth.se
Oscar Hag, oscarhag@kth.se

3. Supervisor
Arne Karlsson, akn@kth.se

4. Purpose of the project etc.
4.1 Background
It becomes more and more important to reduce the influence on the greenhouse effect and this is a big challenge for the aviation industry. There is a need to find alternative fuels and new solutions to power the aircraft. This is also valid for smaller general aviation aircraft which have capacity for a smaller (usually 2-4) passengers.

4.2 Purpose and goals
To solve the problem described above, a concept study of an environmentally friendly aircraft will be done. The aircraft will have capacity for 4 passengers including the pilot. The fact that it should be environmentally friendly means that the influence on the environment should be minimized, with focus on the emission of greenhouse gases such as carbon dioxide. The study will aim to find a concept for an aircraft that is able to fly shorter distances that is suitable for hobby usage. The purpose is to find out how far you can go regarding the performance and if this aircraft would be able to compete with today’s petrol powered aircraft in similar size.

5. Method
Information will be gathered mainly from lectures by our supervisor and literature. It is also possible that information will be asked for from different companies which are working with production or reselling of components that might be used in the aircraft. Current aircraft in similar size and performance will also be studied, which hopefully will result in ideas and inspiration to our project. The process will include many hard decisions, where it is important to have sufficient information to make the right decision to reach the purpose of the project. Mathematical models for the performance
of the aircraft will be generated and used in an iterative process to maintain optimized performance and reach the requirements. The project will result in a first draft in the process to reach a complete concept, from which you should be able to determine what needs to be improved and what problems that has to be solved before you can start the work for a prototype. The result will include calculated expected performance of the aircraft. During the process a sketch will be produced that will show the overall shape and appearance of the aircraft.
Appendix B

MATLAB® code, component build-up method

```matlab
function [Cd0]=component_Cd0(V,S,b,Sh,bh,ra,flap)
my=16.7*10^-6;

c_wing=S/b;
c_canard=Sh/bh;

l=5; %Length of fuselage
Amax=2*536885/10^6; %Maximum crosssection area
k=5.08*10^-7;
Rey=5*10^5;
x=Rey/(ra*V)*my;

Rv=ra*V*c_wing/my;
Rc=ra*V*c_canard/my;
Rbody=ra*V*l/my;

M=V/340.27;

f=l/sqrt(4*pi*Amax);
body=((100000+793252+290000+337721+390000)*2+(136162+611032+253901+
...305653+332877)*4)/10^6; % Body wet area

% Wing

Cwing1=1.328/sqrt(Rey)*2*S*1*(1+0.6/0.4*0.12+100*0.12^4)*(1.34*M^0.18*cosd(10)^0.28);
Cwing2=0.455/((log10(Rey))^2.58*(1+0.144*M^2)*0.65)*2*...S*1*(1+0.6/0.4*0.12+100*0.12^4)*(1.34*M^0.18*cosd(10)^0.28);
Cwing3=0.455/((log10(Rv))^2.58*(1+0.144*M^2)*0.65)*2*...S*1*(1+0.6/0.4*0.12+100*0.12^4)*(1.34*M^0.18*cosd(10)^0.28);
Cvinge=(x*Cwing1-x*Cwing2+Cwing3*(c_wing-x))/c_wing;

% Canard

Ccanard1=1.328/sqrt(Rey)*2*Sh*1*(1+0.6/0.4*0.20+100*0.20^4)*(1.34*M^0.18*cosd(0)^0.28);
Ccanard2=0.455/((log10(Rey))^2.58*(1+0.144*M^2)*0.65)*2*...Sh*1*(1+0.6/0.4*0.20+100*0.20^4)*(1.34*M^0.18*cosd(0)^0.28);
Ccanard3=0.455/((log10(Rc))^2.58*(1+0.144*M^2)*0.65)*2*...Sh*1*(1+0.6/0.4*0.20+100*0.20^4)*(1.34*M^0.18*cosd(0)^0.28);
Ccanard=(x*Ccanard1-x*Ccanard2+Ccanard3*(c_canard-x))/c_canard;

% Body

Cbody1=1.328/sqrt(Rey)*(1+60/f^3+f/400)*body;
Cbody2=0.455/((log10(Rey))^2.58*(1+0.144*M^2)*0.65)*...body*1*(1+60/f^3+f/400);
Cbody3=0.455/((log10(Rbody))^2.58*(1+0.144*M^2)*0.65)*...body*1*(1+60/f^3+f/400);
Cbody=(x*Cbody1-x*Cbody2+Cbody3*(l))/l;

% Control winglets

Clets=1.1*0.455/((log10(Rc))^2.58*(1+0.144*M^2)*0.65)*2*...Sh*1*(1+0.6/0.4*0.10+100*0.10^4)*(1.34*M^0.18*cosd(30)^0.28);
Cland=(0.3*0.03*0.05*3+0.18*0.1)*0.09290304/S; % 0.09290304 is m^2 till Ft^2

Cd0=Cvinge/S+Cbody/S+Ccanard/S+3.83*0.20^2.5*Amax/S+Cland+...
Clets/S+0.00023*2/3*45*flap;
```
MATLAB® code, computerized model

```matlab
%% Mathematical model for aircraft performance
% Concept study of an environmentally friendly aircraft
% Linus Flodin & Oscar Hag
% Bachelor Thesis in flight mechanics
clc
clear
close all

% Specific input data
Vssf=[10:0.1:100];  % velocities [m/s]
g=9.82;               % gravity acceleration
motor=4;               % # of engines
Wpass=360*g;          % weight of passengers, [N]
Wbatt=300*g;          % weight of batteries, [N]
Wmotor=(11.7+6.5/2)*motor*g;  % weight of engines + controllers [N]
Wtom=400*g;           % empty weight of aircraft [N]
W=(Wpass+Wbatt+Wmotor+Wtom);  % total weight [N]
tid=2.5;              % Desired flight time after climb [h]
S=W/814;              % Wing area [m^2]
ra=1.05;              % air density at 1500m [kg/m^3]
b=sqrt(7.6*S);        % wing span [m]
Pa=motor*30e+3;       % max. power from engine [W]
eta_pr=0.85*1.12;     % rate of efficiency, propeller
eta_m=0.92;           % rate of efficiency, engine
h=1500;              % desired cruise altitude [m]
Sh=1.8;              % area of stabilizer [m^2]
my=0.04;             % rolling friction constant, runway
se_batt=750;         % spec.energy batteries, [Wh/kg]
ra_g=1.22;           % air density sealevel [kg/m^3]
bh=sqrt(8*Sh);       % stabilizer span [m]
WS=W/S;              % wing loading [N/m^2]

%% Steady state flight
Cfe=0.0055;
Cd0=Cfe*4;

for n=1:length(Vssf)
    Cd0n=component_Cd0(Vssf(n),S,b,Sh,bh,ra,0);
    q=1/2*Vssf(n)^2*ra;  % dyn pressure
    Cl=W/(q*S);
    K=1/(pi*0.7*b^2/S);
    Cd=Cd0n+K*Cl^2;   % drag and req. power
    D=Cd*q*S;
    Pr=D^*Vssf(n);  % power
    Pr(0+n)=W./(Cl.^((3/2))./Cd)*sqrt(2*W/(ra*S));
end

%available power vector
Pa_vek=ones(1,length(Vssf))*Pa*eta_pr;
Pr=Pr/eta_pr;  % req. power
figure(1)
plot(Vssf,Pr,Vssf,Pa_vek,'r-'),title('Required power'),xlabel('Velocity[m/s]'),ylabel('P_r[W]

top speed
diff=abs(Pr-Pa_vek);
```
\[ [Pr_{\text{topp}}, B] = \min(\text{diff}); \]
\[ V_{\text{topp}} = V_{\text{ssf}}(B); \]

**Velocity for min power**
\[ [Pr_{\text{min}}, B] = \min(Pr); \]
\[ V_{\text{prmin}} = V_{\text{ssf}}(B); \]

\[ V_{\text{prmin}}^2 = \sqrt{\frac{2W}{raS} \times \sqrt{\frac{K}{3Cd0}}}; \]

**Cl and Cd for Vprmin**
\[ Cl_{\text{Vprmin}} = \frac{2W}{(ra \times V_{\text{prmin}})^2 S}; \]
\[ Cd_{\text{Vprmin}} = Cd0 + K \times Cl_{\text{Vprmin}}^2; \]

**Power and energy required at Vprmin**
\[ Pr_{\text{Vprmin}} = \frac{W}{Cl_{\text{Vprmin}}^{3/2} / Cd_{\text{Vprmin}}}; \]
\[ Pf_{\text{Vprmin}} = Pr_{\text{Vprmin}} / \eta_{\text{pr}}; \]
\[ E_{\text{Vprmin}} = Pf_{\text{Vprmin}} / \eta_{\text{m}} \times \text{tid}; \]

**Steady-state flight economy speed**
\[ V_{\text{prmin}} = \frac{56 m/s (200 m/s)}{q = \frac{1}{2} V_m^2 ra}; \]
\[ Cl_m = \frac{2W}{(ra \times V_m^2 S)}; \]
\[ \theta_m = 1; \]

**Component build up method for Cd0**
\[ D_m = Cd_m q S; \]
\[ L_m = Cl_m q S; \]
\[ Pr_m = W / (Cl_m^{3/2} / Cd_m) \times \sqrt{2W/(raS)}; \]
\[ Pf_m = Pr_m / \eta_{\text{pr}}; \]
\[ E_m = Pf_m / \eta_{\text{m}} \times \text{tid}; \]

**Stall speed**
\[ \text{Clmax} = 2.9 \times \frac{2}{3} + 1.6 \times \frac{1}{3} \times 0.9 \times \cosd(10); \]
\[ V_{\text{stall}} = \sqrt{\frac{WS \times 2}{ra \times \text{Clmax}}}; \]
\[ \text{Vmin} = 1.3 \times V_{\text{stall}}; \]

**Stall speed w/o flaps**
\[ \text{Clmax}_{uf} = 1.6 \times 0.9 \times \cosd(10); \]
\begin{verbatim}
Vstall_uf = sqrt(WS^2/ra_g/Cmax_uf);
disp('');
disp(['Stallspeed w/o flaps: ',num2str(Vstall_uf),' m/s']);

%% Steady climbing
Vstig=[Vstall*1.1:0.5:Vtopp]; %velocity
for n=1:length(Vstig)
    %rate of climb calculation with component build up Cd0
    qstig=1/2*Vstig(n)^2*ra; %dyn tryck sfa hastighet
    Clstig=W/(qstig*S);
    if Clstig > 1.6
        flap=(Clstig-1.6)/1.3*3/2;
    else
        flap=0;
    end

    RC(0+n)=Pa*eta_pr/W-
    component_Cd0(Vstig(n),S,b,Sh,bh,ra,flap)/2*ra*Vstig(n).^3*...
    S/W-K*2/ra*W/S./Vstig(n);
end
figure(2)
plot(Vstig,RC)
[RCmax,B]=max(RC);
Vstig_RCmax=Vstig(B);
qstig2=1/2*Vstig_RCmax^2*ra; %dyn pressure
Clstig_behovs=W/(qstig2*S);

%required propeller power for different RC and speeds
RCvek=[1:0.1:RCmax];
Ps=zeros(length(RCvek),length(Vstig));
Es=Ps;
theta=zeros(1,length(RCvek));
t=theta;
n=0;
thetaatill=[];
Vstigtill=[];
Pstill=[];
Estill=[];
RCtill=[];
ttill=[];
for i=1:length(RCvek)
    for j=1:length(Vstig)
        theta(i)=asind(RCvek(i)./Vstig(j)); %climb angle
        Ps(i,j)=(RCvek(i)+Cd0./2*ra*Vstig(j)^3*(S/W)+K*2/ra*W/S*...
                cosd(theta(i))).^2./Vstig(j))*W;
        t(i)=h/RCvek(i)/3600; %climb time
        Es(i,j)=Ps(i,j)*t(i);

        %saving combinations where propeller power is lower than
        %available power
        if Ps(i,j)<Pa*eta_pr
            n=n+1;
            thetaatill(n)=theta(i);
            Vstigtill(n)=Vstig(j);
            Pstill(n)=Ps(i,j);
            Estill(n)=Es(i,j);
            RCtill(n)=RCvek(i);
            ttill(n)=t(i);
        end
    end
end

\end{verbatim}
end

figure(3)
surf(Vstig,RCvek,Es), title('Energy consumption')
xlabel('Vstig'), ylabel('R/C')
figure(4)
surf(Vstig,RCvek,Ps), title('Power')
xlabel('Vstig'), ylabel('R/C')

%values for lowest energy consumption
[Esmin,B]=min(Estill);
Vstig_minEs=Vstigtill(B);
theta_minEs=thetatill(B);
RC_minEs=RCtill(B);
Ps_minEs=Pstill(B);
t_minEs=ttill(B);

disp( ' ');
disp('---Climbing--- ');
disp(['R/C: ',num2str(RC_minEs),', m/s']);
disp(['Climb time to ',num2str(h),', m: ',num2str(t_minEs*60),', min']);
disp(['Engine power: ',num2str(Ps_minEs/eta_pr/1000),', kW']);
disp(['Rate of climb: ',num2str(Vstig_minEs),', m/s']);
disp(['Climb angle: ',num2str(theta_minEs),', grader']);
disp(['Energy consumption: ',num2str(Esmin/1000),', kWh']);

%% Total energy consumption
Etot=Esmin+E_m;
disp( ' ');
disp(['Total energy consumption(cruise 2h + climb) ',num2str(Etot/1000),', kWh']);

%% Batteries
%batteries required to fly 2h cruise + climb to 1500m
M_batt=Etot/se_batt;
cruise flight time at 56 m/s with Wbatt/g kg batteries
Etot_200=se_batt*Wbatt/g;
Emarsch=Etot_200-Esmin;
tid_200=Emarsch/(Pf_m/eta_m);
disp( ' ')
disp(['With ',num2str(Wbatt/g),', kg batteries: '])
disp(['Flight duration after climb(1500m & 56 m/s cruise): ',num2str(tid_200),', h'])

range_reserv=(tid_200-0.75)*V_m*3.6

Emarsch_Prmin=Etot_200-Esmin;
tid_2=Emarsch_Prmin/(Pf_Vprmin/eta_m);

%% Takeoff distance
Clmaxstart=0.1; % lift coefficient at runway
Vto=1.1*Vstall_uf;
Vf=Vto;
Ta=eta_pr*Pa/(0.7*Vto);
Kt=(Ta/W)-my;
Ka=ra/(2*WS)*(my*Clmaxstart-Cd0-K*Clmaxstart^2);
Sg=(1/(2*g*Ka))*log((Kt+Ka*Vf^2)/Kt);

%rotation
Srot=Vto;
Sgr=Sg+Srot;

%transition
Ta_trans=eta_pr*Pa/(1.15*Vto);
q_trans=1/2*(1.15*Vto)^2*ra_g; %dyn press
CL_trans=W/(q*S);
Cd_trans=Cd0+K*CL_trans.^2;
D_trans=Cd_trans*q*S;
s_trans=(Ta_trans-D_trans)/W;
R=0.205*Vstall_uf^2;
St=R*s_trans;
fi=asin(s_trans);
ht=St*tan(fi);

%climb to obstacle clearance
hobst=15.2;
hti=R*(1-cos(fi));
Sc=(hobst-hti)/tan(fi);
Sto_tot=Sgr+St+Sc

%% Static stability analysis

lambda=1; %tipchord/rootchord
l_cg=0.5; %distance between a.c. for main wing and c.o.g.
lh=2.6; %distance between stab. a.c. and main wing a.c.
c=(1+lambda+lambda^2)/(1+lambda)*2/3*S/b; % aerodynamic mean chord
Vh=Sh*lh/(S*c);
ARh=bh^2/Sh; %aspect ratio stabilizer
AR=b^2/S; %aspect ratio main wing

a=AR/(2+AR)*2*pi;
ah=ARh/(2+ARh)*2*pi;

xi=2*(-lh)/b;
de=(1+(sqrt(xi^2+1)/xi)))*1/(2+AR);

l_np=c*Vh*ah/a*(1-de);

dC_mcg=-(l_cg-l_np)/c

st_marg=-dC_mcg; %stability margin, >0 if stable

C_mwac=-0.15; %pitch moment coeff. for mainwing referred to an axis %through a.c. From NACA airfoil 2412, table C_m_c/4

%angle between zero-lift line of main wing and horizontal tail
ih = \frac{-C_{\text{mwc}}}{(ah\cdot Vh)}

%% Positioning the main wing
mbat = Wbatt / g;
mmot = \text{motor} \times (11.7 + 1/2 \times 6.5);
Mv = 100;
mkropp = 300;
mstab = 5;
mlast = 0;
Mu = mbat + mmot + 4 \times mper + mkropp + mstab + mlast;
temp = (3.3 - 0.2) \times mbat + 1.3 \times 2 \times mper + 2.3 \times 2 \times mper + 1.8 \times mkropp + (3.3 + 0.2) \times mmot;
xu = temp / (Mu);
A = [1 -1; Mv/(Mv+Mu) -1];
B = [\text{abs(l_cg)}; -(xu \times Mu + 0.17 \times c \times Mv) / (Mv + Mu)];
X = A \times B

%% Absolute ceiling
ra_ceil = [1.25: -0.01: 0.15]; % air densities
for i = 1:length(ra_ceil)
    Cd0_ceil(i) = component_Cd0(V_m, S, b, Sh, bh, ra_ceil(i), 0);
    RC_ceil(i) = \text{Pa/\eta_pr/W-Cd0_ceil(i)\times ra_ceil(i)\times S/(2\times W)} \times V_m^3 - 2 \times K \times ...
                 W / (ra_ceil(i) \times S \times V_m);
end
figure
plot(ra_ceil, RC_ceil)
Cd0 = component_Cd0(56, S, b, Sh, bh, ra, 0)