

**Simulation of Human Movements
through Optimization**

by

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Abstract

Optimization has been used to simulate human neural control and resulting movement patterns. The short term aim was to develop the methodology required for solving the movement optimization problem often arising when modelling human movements. A long term aim is the contribution to increased knowledge about various human movements, wherein postures is one specific case. Simulation tools can give valuable information to improve orthopedic treatments and technique for training and performance in sports. In one study a static 3D model with 30 muscle groups was used to analyse postures. The activation levels of these muscles are minimized in order to represent the individual's choice of posture. Subject specific data in terms of anthropometry, strength and orthopedic aids serve as input. The specific aim of this part was to study effects from orthopedic treatment and altered abilities of the subject. Initial validation shows qualitative agreement of posture strategies but further details about passive stiffness and anthropometry are needed, especially to predict pelvis orientation. Four studies dealt with movement optimization. The main methodological advance was to introduce contact constraints to the movement optimization. A free-time multiple phase formulation was derived to be able to analyse movements where different constraints and degrees of freedom are present in subsequent phases of the movements. The athletic long jump, a two foot high jump, a backward somersault and rowing were used as applications with their different need of formulation. Maximum performance as well as least effort cost functions have been explored. Even though it has been a secondary aim in this work the results show reasonable agreement to expected movements in reality. Case specific subject properties and inclusion of muscle dynamics are required to draw conclusions about improvements in the sport activity, respectively.

Descriptors: multibody system, human movements, optimal control, trajectory optimization, long jump, posture, rowing, somersault

Preface

This thesis studies the application of constrained optimization to derive human postures and movements. The first part gives a brief background, description of methods and conclusions. The second part consists of the following papers:

Paper 1. PETERSSON R., BARTONEK Å. AND GUTIERREZ-FAREWIK E.M., 2012
“Posture strategies generated by constrained optimization”,
Journal of Biomechanics 45, 461–468

Paper 2. PETERSSON R. AND ERIKSSON A., 2009
“Movement optimization of multibody system subjected to contact constraint with application to long jump”, Internal report

Paper 3. PETERSSON R., NORDMARK A. AND ERIKSSON A., 2010
“Free-time Optimization of Targeted Movements based on Temporal FE Approximation”, Proceedings of the Tenth International Conference on Computational Structures Technology, Valencia, 2010

Paper 4. PETERSSON R., NORDMARK A. AND ERIKSSON A., 2012
“Optimization of multiple phase human movements”, Submitted to Multibody System Dynamics

Paper 5. PETERSSON R., NORDMARK A. AND ERIKSSON A., 2012
“Simulation of Rowing in an optimization context”, Submitted to Scandinavian Journal of Medicine & Science in Sports

Division of work between authors

The research project was initiated by Prof. Anders Eriksson (AE) who also was the main supervisor and advisor of the work resulting in Papers 2–5. Papers 3–5 were also supervised by Dr. Arne Nordmark (AN) who acted as co-supervisor. Dr. Elena Gutierrez-Farewik (EGF) acted as co-supervisor and was advisor of the work resulting in Paper 1. Robert Pettersson (RP) continuously discussed the progress throughout the work with AE, EGF and AN. Dr. Åsa Bartonek (ÅB) at Department of Women's & Children's Health, Karolinska Institute, was contributing to Paper 1.

Paper 1

The code development and simulations were done by RP with feedback from EGF. The experimental data were supplied from available measurements through ÅB. EGF derived the statistical values for the control study. The paper was written by RP with input from EGF and ÅB.

Paper 2

The code development and simulations were done by RP with feedback from AE. The paper was written by RP with input from AE.

Papers 3–5

The problem definition and simulations were done by RP with feedback from AE and AN. AN coded the wrapper in Matlab for interaction with COMSOL. The paper was written by RP with input from AE and AN.

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Part I

Overview and summary

CHAPTER 1

Introduction

In this thesis simulations of human movements are performed using movement optimization. The intuitive application of movement optimization for human movements are sports in order to improve technique and general performance. Athletes do a high amount of training and have possibilities to learn new techniques to perform a certain task with a very high precision. They are also able to develop physical capacities according to the intended work load which motivates further investigation of optimal movements and development of required methods to do so. Another, but not as obvious, application is also within health care where persons with movement disorders are treated. A common orthopaedic treatment is a prescription of orthosis, which give external support at a joint in the lack of sufficient muscle strength. Other movement disorders can be caused by deformities which require surgical treatment. In both cases simulations can give complementary information in the process of treatment planning.

Optimization of movements is an applicable method for the subject groups above. While the athletes have the best capacities for performance in their activity, respectively, it is still these physical limitations which dictate their performance. The persons with movement disorders are also constrained by their capacity but for daily activities. Therefore the two situations are similar in the sense that each group strives for best performance given a limited capacity. It is only the performance criteria and available capacities which are different.

To express the above described problem using a mechanical approach the physiological properties, technique in the sport or functional task, and performance have to be interpreted to distinguish a movement strategy. Furthermore, how the strategy can be represented mathematically, which method to use, what application specific questions should we be able to answer and what can be said about the results quality. These aspects are dealt with in this thesis with emphasis on method rather than application results.

1.1. Optimality for human movements

To decide what is optimal is one of the major questions when performing movement optimization. The most intuitive choice is probably to use a performance criterion, e.g. to move as fast as possible or jump as high possible, given a specific capacity. Another alternative is to minimize the usage of the capacities while performing a specific task. Examples can be to minimize the energy spent or the muscle activation needed during a movement while a predefined task is accomplished.

In this thesis the movement optimization has been applied to human movements which involve several phenomena which cannot be represented in the optimization. To begin with, sports can contain situations where the athletes are in interaction with each other, for instance in cycling or cross-country skiing where the perpetrators influence the air drag of each other and it suddenly adds a strategic component of planning the intensity according to how the race develops. There is however example of full race optimization where the usage of available energy and constrained power is optimized for best race time (Eriksson 2012). Secondly, psychological aspects are difficult to give a mathematical representation in movement optimisation. Especially when a sport contains phases where athletes are challenging each other and, for instance, one task is to predict the opponent's actions. Lastly, an aspect which is probably present to various extent in all sports is the influence of pain, a very subjective measure which lacks any type of mechanical representation. However, the one who can use his/her capacities in an optimal way in combination with doing optimal strategic performance as well as being able to control the psychological factors advantageously, will determine the total performance which in sports can be the difference between winning or not.

Clinical applications also contains parts which are difficult to find a mechanical representation for. Pain is one aspect as above but also how the situations are experienced, e.g. if balance is experienced during static stance.

1.2. Background

Simulation and modelling of biomechanical systems are used to give increased understanding about functioning from the cell to the musculoskeletal levels. Examples are chemical processes, material properties, blood flow, fractures, motion control, balance, and the areas treated here, human movements and postures.

Something that characterises modelling in biomechanics is the challenge to make a mechanical model, with its well defined degrees of freedom (dof) and properties, to represent a biological system where nonlinear materials, inconsistent centers of rotations, relatively soft tissue motions and subject control based on senses are present. Many times a trade off has to be done concerning which details that have to be represented to keep the most essential features. Modelling of multibody systems (MBS) like the human body are also done in robotics and computer graphics, which fields have developed efficient methods to evaluate the dynamics (Featherstone and Orin 2000).

Optimization is an interesting research area in itself but is also frequently used in engineering sciences, for instance seeking a structural design based on minimal material usage, optimal control of processes and trajectory optimization within space exploration. In biomechanics applied to human musculoskeletal systems the optimization has become useful to solve ambiguous problems and/or to optimize some functional criteria. The load sharing problem in the redundant musculotendinous system (Amankwah et al. 2006; Heintz and Gutierrez-Farewik 2007), the individual's choice of posture (Amankwah et al. 2006) or targeted motions (Kaphle and Eriksson

2008) are examples of ambiguous problems, while performance criteria such as maximum jump height (Andersson and Pandy 1998) or length (Pettersson and Eriksson 2009) are functional tasks. One can distinguish between dynamic and static optimization. Dynamic optimization takes time history into account while static versions only considers one time instant, possibly being one moment of a sequence. Dynamic optimization leads to large numerical costs which limits the number of dof and inclusion of detailed muscle descriptions, but is required when the purpose is to derive novel movements or having performance criteria dependent of time. For analysis of posture and gait, however, static optimization gives comparable results (Andersson and Pandy 1998). With improved computational resources and optimization algorithms the size of the models possible to analyse has increased. Nevertheless, this is still a limiting factor for the simulations, especially in dynamic optimization.

Biomechanical simulations can often be divided into three approaches of analysis. Using inverse dynamic analysis, the forces are sought from known trajectories and in combination with optimization e.g. the load sharing problem can be solved for (Damsgaard et al. 2006). Using forward dynamics analysis the movement, as consequence of control forces, is sought and novel movements can be solved for when put into an optimization context (Ashby and Delp 2006). Using a best fit approach, i.e. the movement and control for a specific model is sought which minimize the deviation from some measured trial, can be used to tune the model's parameter values (Wilson et al. 2006; King et al. 2006). The forward approach is significantly more computationally demanding but is the only way to derive and analyse novel optimal movements. The forward approach is used in this work.

1.3. Sports and medical applications

The application groups of elite athletes and persons with motion disorders appear to be clear contrasts to each other. However, from a mechanical point of view, both groups generally strive for best possible performance, but within their individual limits. The five studies in this work each have their own application. The posture study, Paper 1, is the one where the mechanical model been chosen according to experimental measurements and the results aimed to have clinical relevance. In the sport application studies, Papers 2–5, on the contrary, focus been on development of methodology and only the application essential properties have been used for the athletes representation. This is perhaps most clear in the "two foot high-kick" and "backward somersault", Papers 3 and 4 respectively, where the applications were mainly chosen to test performance of the methodology but still having biomechanical relevance. The simulations of the long jump and of rowing, Papers 2 and 5, respectively, are again more well defined disciplines allowing their results to be compared with the actual sport.

1.3.1. *Standing posture*

The medical diagnoses among the subjects which have been studied are cerebral palsy (CP), Myelomenigocele (MMC) and Arthrogryposis Multiplex Congenita (AMC). Biomechanical limitations brought by CP can be different levels of muscle weaknesses, limited ranges of motion and spasticity, i.e., stretch velocity induced tonus

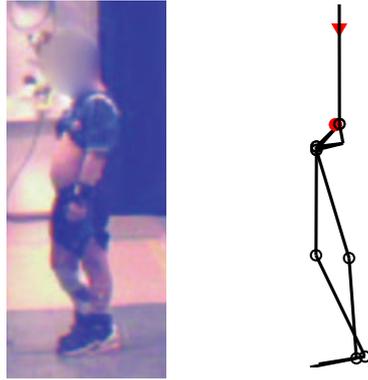


Figure 1.1: Standing posture of a child having movement disorder and wearing orthosis (Case 1 in Paper 1). The orthosis and disabilities causes an asymmetric posture in the frontal plane, and requires to be represented with a 3D model. The stick figure is an example of how the human body can be represented by a reduced mechanical system.

(Gage 2004). Spasticity is therefore impossible to model in a static model unless simplifying it into a limit of ranges of motion or as muscular forces constrained to fixed values. However, studies of the effect of reduced spasticity through treatment can be done. Since also perception and cognition disorders are often present, this group can be difficult to model with only mechanical concepts. Despite the limitations it is still a justified application group because of the varying levels of disability in this large patient group.

MMC is caused by an innate hernia of the spinal cord. The most prominent symptoms of MMC are paresis and reduced sensibility in lower extremities. Dependent on the level of the hernia different muscles are affected. Contractures and skeletal deformities can be present from birth, but also developed during growth because of increasing strength in non-affected muscles (Beckung 2002).

The symptoms of AMC are innate contractures caused by hypoplasia or aplasia, i.e., incomplete developed or absence of muscle groups (Ortopedi 2004). Accordingly, limited ranges of motion and weaknesses are the biomechanical limitations.

A basic condition for quiet standing is to keep the center of mass (COM) of the whole body within the base of support (BoS) formed by the feet. Then the posture is the chosen configuration to fulfil this condition and describes the orientation of the body segments relative to the gravitational vector, i.e., it is an angular measure from the vertical (Winter 1995).

At quiet standing the ground reaction force (GRF) and the gravity force act towards each other and the slightest misalignment will contribute to a moment driving to increase this misalignment, thus the mechanical system is unstable and demands

active control to keep the balance. The human continuously tries to control the musculoskeletal system by activating muscle groups in a coordinated way but cannot control the system without a resulting sway.

To maintain balance, there are two strategies to compensate for a movement of the COM. These are either moving the center of pressure (COP) with respect to the vertical projection of the COM or by counter-rotating segments around the COM (Hof 2007). The first strategy can be used when the COM is within the BoS and has limited velocity. Counter-rotating segments can always be used but is most useful when it is not possible to move the COP any further. The results by Hof (2007) suggest that the counter-rotating strategy is more important with a narrow BoS.

In this work the subject group is able to balance into almost quiet standing so a static model has been assumed to be sufficient to represent the standing posture. However, there are still many factors that can affect the choice of posture. Subject properties in terms of anthropometry and flexibility (Rybski 2004), muscle strength (Kuo and Zajac 1993) and obesity (Gilleard and Smith 2007) together with orthopedic treatments like orthoses (Owen 2004) and heel lifts (Rodriguez and Aruin 2002; Owen 2004; Eslami et al. 2006) influence the posture.

The problem of choosing a posture strategy can thereby be described as a three-dimensional link structure controlled by a redundant control system where load sharing and configuration are sought which minimize the required activation levels.

1.3.2. Long jump



Figure 1.2: Foto sequence of take-off and flight phase of a long jump (http://www.coachr.org/approach_speed_and_performance_in_the_horizontal_jumps.htm, 2012-08-08). The take-off in terms of velocity and configuration determines the length of the flight phase. During flight the forward rotation has to be prevented, in this case done by rotating the limbs. The landing also affects the final jump distance as an active landing, by hitting the ground with the feet, can make the center of mass to reach ground in front of the otherwise free projectile motion during flight.

In sports applications there are many abilities which characterise athletes, such as agility, speed, power, flexibility, strength and coordination. These properties are not independent of each other and some of the properties are more important than the others, dependent on the application. In the athletic long jump (Paper 2) all are

important but also that a balance must exist between speed, strength and technical requirements (Graham-Smith and Lees 2005). Flexibility is also essential but only such that the ranges of motion does not limit the movements. This group of athletes is highly evolvable through technique and specific physical training, which makes it interesting to study possible improvements to their performance.

The athletic long jump and particularly the take off is an action which is totally dependent on high levels of coordination and intensity. The ground contact time is approximately 110ms (Luthanen and Komi 1979) during which the jumper gains the vertical momentum needed for the jump. In simple terms the jumper should gain as much vertical momentum as possible while trying to keep the horizontal speed already gained in the run up. This can however be realized in different ways and important factors have been studied in literature. High correlation between horizontal speed and jumping length has been shown (Hay 1993), optimal take-off angles have been analysed by Linthorne et al. (2005), and the importance of angle of attack and muscle architecture have been studied by Seyfarth et al. (2000). The quick action also emphasises the importance of muscle dynamics. The spring-like effects which can be enhanced by pre-activation and high velocities and forces in the eccentric phase (Kyrolainen et al. 2003) are investigated by Seyfarth et al. (2000) who conclude that the jumping distance is sensitive to this feature. This is a property which justifies the often emphasized importance of footplant velocity. In GRF measurements of a take-off (Seyfarth et al. 2000; Hatze 1981; Muraki et al. 2008), one can observe two separated peaks, where the first is introduced by the initial impact and passive properties and the second by active push-off. These measurements also show the importance of the initial impact, which contributes to approximately 25% of the total change in momentum (Seyfarth et al. 1999).

The mechanical representation of the long jump take off can be summarized as a multibody system subjected to a contact constraint, and for which the coordination is sought to maximize the following flight distance.

1.3.3. *Two-foot high kick*

This application was used to incorporate the multiple phase and free-time concept (Paper 3) into the movement optimization. The two-foot high kick is a discipline in the World Eskimo-Indian Olympics. The event requires the athlete to jump off the ground, kick a suspended object (hanging as high as possible), and land such that balance is maintained again. The three steps should be done with both feet together (http://www.weio.org/the_games.html 2012-07-02). The athlete is therefore required to attain the upward momentum with a two-foot take off with the possibility to push off with whole feet and the forefoot. The movement needs to be controlled such that the feet reach the highest position possible.

The mechanical representation of the two-foot high jump was a multibody system subjected to a two phase contact constraint. The movement time of each phase was free and the coordination was sought to maximize the following two-foot kick. Here the phase after reaching the apex point, e.g. the landing was not included.

1.3.4. *Backward somersault*

The backward somersault was used to illustrate the performance of the further developed free-time multiple phase formulation (Paper 4). The movement includes take-off, flight phase where the rotation takes place and landing. During the take-off the preparing movement should generate the vertical velocity of the centre of mass and the angular momentum about the centre of mass as these factors dictate how much rotation can be achieved (King and Yeadon 2004). The arm movement is important to gain velocity and rotation at the take-off (Ashby and Delp 2006; Hara et al. 2008). The rotation during flight phase is controlled by arranging the body segments such that the moment of inertia is adjusted. Lastly, the landing has to be made with a configuration where balance can be maintained. At the instant of ground contact initiation an impact takes place. The remaining rotation can then be reduced into a static position as the ground reaction force (horizontal and vertical component) act under the foot segment.

In the optimization context of this work the somersault problem is chosen as application to solve for a multiple phase movement with variable time and without boundary conditions. The contact initiations are modelled with impulses.

1.3.5. *Rowing*

Lastly, the developed methodology is applied to rowing which is a cyclic movement with drive and recovery phases and contact constraints which require a free-time multiple phase formulation (Paper 5).

Rowing makes use of the physiological range for several parameters. Strength and endurance as well as technique are important. The length of a race, 2000 m, can be performed in 6 min and 33 s (2009 Mahe Drysdale, <http://www.worldrowing.com/results/2012-08-14>) which require an average speed of 5.1 m/s or over 18 km/h. The rowing event is one of the sports requiring the highest aerobic capacity (Volianitis and Secher 2009), due to the intensity, i.e. power output, required to maintain a high average speed.

The equipment consists of boat and oars. The rower is in contact with this equipment at feet, seat and oar handles. The feet are strapped to the boat, the seat may slide and the contact at the handles are maintained by the rower, which together form the basic constraints of the movement. The cyclic exchange of translational momentum between a rather light boat and a substantially heavier athlete is one of the basic mechanical aspects of rowing. There is also energy dissipation present as the boat and oars are subjected to hydrodynamic — and, to some extent, aerodynamical — forces. For the boat, the relative movement of the rower causes velocity variations throughout the rowing stroke and these variations lead to more energy loss than with constant speed. Approximately these variations lead to an additional time of 5 s for a 2000 m race with 4.7 m/s (Hill and Fahrig 2009). Another source to energy dissipation in the system is the oars contact with water. In Hofmijster et al. (2007) the power loss at the oar blades was estimated to be 19–24% of the total energy loss (10% due to air resistance assumed) and in Hofmijster et al. (2010) they indicate that the power loss can be even substantially higher.

The choices of equipment are important for the performance in rowing as there are numerous alternatives for boats, oars and oar blades. These should be chosen to fit each athlete and their adjustable measures should be tuned for the most effective use (Nolte 2005). For rowers with similar capacities and technique, the rigging would determine their results at competition. Hence, it is very motivating and challenging to do optimization analysis of rowing where parameters as boat, and oar properties and measurements can be varied for various capacities of the rower.

In the optimization analysis of rowing, one stroke during a constant average speed form a multiple phase optimization problem. The movement is made cyclic and the required contacts are modelled. The rigging measurements can also be included in the optimization while solving for the movement and forces of the rower. It is, however, noted that endurance strategy and psychology which are crucial components in the real race situation are neglected.



Figure 1.3: Picture from mid drive phase during a stroke in single scull (<http://www.zimbio.com/photos/Lassi+Karonen/2011+Samsung+World+Rowing+Cup+III+Day+1/IUFSHPc3Plb>, 2012-08-08). The high force produced by the rower causes the oars to deflect.

1.4. Aim and scope

Paper 1, Posture strategies

The main objective was to develop a tool for investigation and prediction of how different aids, such as orthoses and heel lifts, affect standing posture in order to justify and thereby to explain clinical experiences about posture and treatment planning. The

simulations should be subject specific and allow the user to specify the properties found during a clinical assessment. Due to the individual characteristics of the subjects, a 3D model, where asymmetry and relatively detailed properties of the muscle system can be defined, was needed.

The methodological development in this application was primarily related to formulation of an optimization problem such that joint configuration as well as load sharing can be solved for, and to represent the subjects with a mechanical model only requiring data from standard clinical assessment in connection with gait analysis.

Paper 2, Movement optimization with application to long jump

The objective was to formulate a solution methodology for the study of movement optimization of a multibody system subjected to contact constraint, with an application to long jump. Some main problems to solve were to include sufficient numbers of mechanical dof for an accurate representation of the human body, still keeping the number of variables low in the optimization. The interaction of the mechanical model with contact constraints should also be considered in the optimization. Another feature, which has great effect on the characteristics of the problem, is that the simulation should not be based on measured motion, but to create novel movements.

The methodological development in this application was primarily related to explore an alternative efficient formulation of the dynamics as well as include contact constraints in the movement optimization.

Paper 3, Movement optimization applied to two foot high jump

The objective was to introduce a multiple phase and free time formulation into the optimization in order to successfully model contact constraints active only during sub-intervals of a more complex movement. Also to investigate the free time formulation dependence on movement and its role when varying cost functions.

The methodological development in this application was primarily related to the introduction of a time scale parameter as variable in the optimization and the contact model. A restriction deemed necessary was that the sequence of contact conditions is pre-defined.

Paper 4, Movement optimization applied to a backward somersault

The objective was to introduce impact problems into the movement optimization and further investigate the methodology performance when using larger mechanical systems. A basic test problem as well as a complex movement such as a backward somersault were solved for.

The methodological development in this application was primarily related to the introduction of explicit impulses at contact initiation and solving for the accelerations used to apply the contact constraints. The work also led to investigation of compatible interpolations in the different phases.

Paper 5, Movement optimization applied to rowing

The objective was to apply the developed methodology to rowing in order to see its performance but also in order to create an analysis tool for rowing where basic aspects of rowing can be investigated. A more physiological model of the rower should be possible for more realistic simulations of rowing.

The methodological development in this application was primarily related to the inclusion of the cyclic movement and the rower's interaction with boat and oars in the optimization.

1.5. Outline of thesis

The methods used in the papers are described in Chapter 2. Since the methods used are somewhat different in the different projects they are presented with reference to their application, respectively. Chapter 2 also includes a short description of the applications of the methods. In Chapter 3 some main results are presented and discussed. In Chapter 4, the main conclusions from the papers are found. The second part of the thesis consists of the resulting papers.

CHAPTER 2

Methods

2.1. Mechanical representation of the body

2.1.1. *Link structure*

The body segments have been modelled with rigid links in the following applications. This is a well motivated assumption since the skeletal structures are stiff and the compression and deflection are small compared to the relative movements of the body segments. However, when studying the effect of an impulse acting on the body this matter should be considered, since tissue deformation decreases the resulting stress in the system (Gruber et al. 1998; Gittoes et al. 2006). The joints are represented with hinge or ball and socket joints with well defined degrees of freedom (dof). In reality the motion is more complex. At the knee joint, mainly acting like a hinge joint, the motion follows a path containing both flexion/extension, rotation and translation (Hamill and Knutzen 2009). A consequence of using rigid links is also that the inertia properties are constant, which makes modelling less complex.

The above described mechanical model is undamped although in many dynamic applications, including long jump, the inclusion of damping and distributed mass is required (Seyfarth et al. 2000). To represent this, a wobbling mass has been attached to the rigid link structure with spring and damper elements in Paper 2. A drawback is the extra dof which are needed to represent the complete motion.

In Papers 2–5 sagittal models were used, even though an additional out of plane rotation was used in Paper 5, as the main movements in the applications considered are planar movements. However, in addition to the decrease dof by reducing the mechanical system the redundancy in the musculotendonous system is emphasised. Muscles which have different functions in 3D might become parallel in 2D modelling. The complexity of the mechanical model become significantly reduced using this planar representation which is the main advantage for the following optimization.

2.1.2. *Muscle system*

The muscles are the driving component in the musculoskeletal system. The muscle system is complex and needs to be simplified in a numerical model. These simplifications can be done to different extents. Joint actuators can be used to represent the muscle groups acting at each joint (Andersson and Pandy 1998; Ashby and Delp 2006; King et al. 2006). When using joint actuators one can introduce varying maximum strengths along the range of motion to represent the effect of muscle length and moment arm (Paper 2), thereby improving the muscle system model. There are also

various models in literature where passive stiffness, representing the passive resistance within and at the end of range of motion, is included as an additional moment to the muscles (Amankwah et al. 2006). A further step towards a more detailed description of the muscle system is to introduce muscles and muscle groups (Delp et al. 1990; Andersson and Pandy 1998; Spägle et al. 1999). With this formulation biarticular muscles can be included and similarly also muscle lengths and moment arms. In this thesis, Papers 1 and 2, muscle data have been collected from SIMM (Delp et al. 1990) in terms of maximum moment range of motion dependence based on an average male. This implies that the relationship between moment and joint angle was assumed to be the same in all simulations while segment lengths were scaled according to the specific application in each study. Similar assumptions about the relationship were made in Morse et al. (2008) and partly verified in O'Brien et al. (2009). The total strength of each muscle group at a joint was then specified according to the application.

With separate muscle groups, including biarticular muscles, the system becomes redundant such that a specific load can be carried by many different feasible solutions of force distribution in the system. This problem has been dealt with in literature (Andersson and Pandy 1998; Rasmussen et al. 2001; Amankwah et al. 2006; Heintz and Gutierrez-Farewik 2007) and optimization has become a standard method, but together with different optimization criteria. Muscle effort, measured by muscle stress or activation level seen as the ratio between current and maximum available force, is often used.

In a dynamic situation the muscle has variable capacity dependent on contraction velocity, activation history and muscle dynamics (Taylor et al. 1991; Ding et al. 2000; Yeadon et al. 2006; Anderson et al. 2007; Kosterina et al. 2012). In the type of simulations performed here the possibilities to include advanced muscle dynamic description are limited due to computational demands. However, factors compensating for contraction velocity can be included without complications (Ashby and Delp 2006). In Paper 2 the total moment about a joint was chosen to be

$$M = \alpha \theta(q) \phi(\dot{q}) f_{\max} \quad (2.1)$$

where α is the activation, f_{\max} the maximum isometric force, θ a factor for moment arm and muscle length, and ϕ a factor compensating for concentric or eccentric movements. A simple constraint on activation/deactivation rate was introduced in Paper 2 to explore its dependence.

One component in the muscle dynamics which is mentioned by Luthanen and Komi (1979); Seyfarth et al. (2000) to be important in the long jump take-off, is the spring-like effect in a muscle when working interactively with its tendon, which is the stretch-shortening cycle (SSC), (Komi 2000; Stålbom 2008). However this feature is not modelled in this work.

In the method development in Papers 3, 4 and 5, the forces are dependent neither on joint configuration nor velocity and are assumed to be infinitely quickly controlled as this more clearly show the characteristics of the solutions, e.g. discontinuities which would be difficult to detect otherwise. However, in Paper 5 a constraint on power is introduced which is a simplified velocity dependence compared to Eq. (2.1)

Another aspect in the application to rowing in Paper 5 is the duration of the exercise, which causes fatigue in the muscles. The cause of fatigue to the muscles is complex but a simplified model could likely be included into the present model without causing major difficulties in the optimization, as this aspect is not really considered in this study. The only consideration is in the composite cost function used, where the joint moments relative their maximum values are minimized during recovery as a control strategy during this motion. This relative level of load is one of the components causing fatigue (Ding et al. 2000). The phenomenon of fatigue will likely dictate what would be an optimal variation of the intensity considering a full race.

2.2. Equations of motion

The following formulations of the dynamic equilibrium equations make use of generalised coordinates:

$$\mathbf{q}(t) = [q_1(t), q_2(t), \dots, q_{N_d}(t)]^T, \quad (2.2)$$

and forces:

$$\mathbf{c}(t) = [c_1(t), c_2(t), \dots, c_{N_c}(t)]^T \quad (2.3)$$

where N_d and N_c are the numbers of dof and control forces, respectively.

2.2.1. Euler-Lagrange equations

There exist several alternative procedures to derive the equations of motion for a multi-body system (MBS). Dependent on the size and topology of the systems representing the human body three approaches have been used. The natural approach to derive the equations of motion for a system of moderate size is to use the Euler-Lagrange equations. These are derived from energy expressions using any symbolic mathematical and analytical software, here Maple (Maplesoft, Waterloo Maple Inc, Canada), resulting in N_d equations and N_d unknowns.

$$\frac{\partial \mathcal{L}}{\partial \mathbf{q}} - \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\mathbf{q}}} \right) = \mathbf{0} \quad (2.4)$$

where a dot denotes a time derivative, and \mathcal{L} the Lagrangian

$$\mathcal{L}(t) = \mathcal{T}(\mathbf{q}(t), \dot{\mathbf{q}}(t)) - \mathcal{V}(\mathbf{q}(t), \mathbf{c}(t)) \quad (2.5)$$

where \mathcal{V} and \mathcal{T} are the potential and kinetic energies, respectively. This is a convenient approach but the computational cost increases drastically with increasing size of the MBS in terms of dof. The derivation and size of the analytical expressions becomes unmanageable with the software and computer resources used. Furthermore, the computational cost to solve the $N_d \times N_d$ system of equations increases by $O(N_d^3)$. This method was used in the application to posture strategies, Paper 1, where the expressions became manageable by describing the system from its most center positioned segment (pelvis in this application) in order to have as short kinematic chains as possible. The system was also reduced by cancelling the terms including $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ for the static situation.

2.2.2. Articulated body algorithm

In order to enable modelling of future increasing sizes of systems an alternative method was explored in Paper 2. In computer graphics and robotics the need for efficient algorithms to model realtime situations is present. There are several methods developed for this purpose (Ju and Mansour 1989; Balafoutis 1994; Featherstone and Orin 2000), but the Articulated Body Algorithm (ABA) (Featherstone 1987) is one of the fastest linear complexity algorithms for linked bodies without closed loops (Kokkevis and Metaxas 1998).

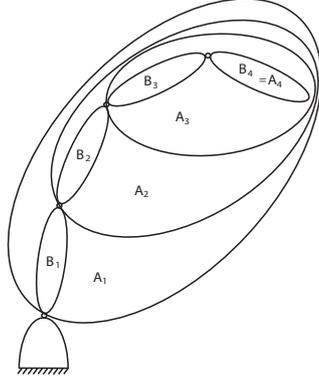


Figure 2.1: Kinematic chain with defined articulated bodies. Re-drawn from Featherstone (2008).

The main idea is to determine properties of groups of bodies, articulated bodies (Fig. 2.1), allowing them to be treated as single rigid bodies but with articulated body properties. The equation of motion for a body in a linked structure can be written as

$$\mathbf{f}_i = \mathbf{I}_i^A \mathbf{a}_i + \mathbf{p}_i^A \quad (2.6)$$

using spatial notation, i.e. vectors consisting of 3 translational and 3 rotational dof (Featherstone 2010). Here, \mathbf{f}_i is the force in joint i , \mathbf{I}_i^A is the articulated-body inertia, \mathbf{a}_i the spatial acceleration of body i and \mathbf{p}_i^A the bias force for body B_i in the articulated-body A_i . The articulated-body inertia depends on the rigid-body inertia of the members, and of their instantaneous kinematics, and the bias force depends on velocity effects and forces acting on the articulated body. Now, using that the equations of motion of two linked bodies are rather simple (Eq. 2.6), the problem is turned into evaluation of the properties of the articulated bodies (Fig. 2.1). Recurrence relations, where \mathbf{I}_i^A and \mathbf{p}_i^A can be expressed in terms of \mathbf{I}_{i+1}^A and \mathbf{p}_{i+1}^A , allow the equations of motions be evaluated linearly in time.

The accelerations are then derived in three loops: one outwards to calculate velocities and forces arising from velocities, one inwards loop for the articulated-body inertias and bias forces, and a second one outwards to calculate the accelerations.

The original ABA (Featherstone 1983) is a forward dynamics algorithm for a kinematic chain with fixed base. However, with the modifications presented in Featherstone (2008) a floating base (free to move in plane) was introduced as well as the possibility to solve hybrid dynamics problem (forward and inverse kinematics at different joints, i.e., rheonomic constraints). The dof in the contact were here chosen to be evaluated as forward dynamics such that the displacements in these dof do not have to be included as variables in the following optimization problem. This avoided the convergence difficulties with the compliant contact model. The final function call uses the position and velocity in all dof, the acceleration in the inverse dof and the forces in the forward dof as input:

$$[\tau_{\text{Inv}}, \ddot{\mathbf{q}}_{\text{Fwd}}] = \text{DynamicsFunction}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}_{\text{Inv}}, \tau_{\text{Fwd}}) \quad (2.7)$$

where the subsets of indices ‘Inv’ corresponds to joint dof and ‘Fwd’ to the contact and wobbling mass dof.

2.2.3. Weak form

In Papers 3–5 a weak form of the equilibrium equations was used which is derived by a general variation (Reddy 1984) of the Lagrangian, Eq. (2.5):

$$\int_0^T \left(\frac{\partial \mathcal{L}}{\partial \mathbf{q}}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{c}) \delta \mathbf{q} + \frac{\partial \mathcal{L}}{\partial \dot{\mathbf{q}}}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{c}) \delta \dot{\mathbf{q}} \right) dt \quad (2.8)$$

$$+ \sum_{t \in T_b} \frac{\partial \mathcal{L}}{\partial \dot{\mathbf{q}}}(\mathbf{q}, \mathbf{u}, \mathbf{c}) n \delta \mathbf{q} = \mathbf{0}$$

where the set T_b contains the times at the boundary, \mathbf{u} the velocities at the boundary and n is the outward normal vector at the boundaries: $n(0) = -1$, $n(T) = 1$.

One advantage when using the weak form is that it makes use of only up to first derivatives in the displacements. Low order of interpolation can therefore be used. In addition, the equations can be entered with general syntax and in very compact and convenient form. This also makes the problem definition more lucid and increases the chance of detecting coding errors.

Equation (2.8) requires that the Euler-Lagrange equations Eq. (2.4) are fulfilled for $t \in [0, T]$. It also gives a boundary condition of the velocities \mathbf{u} at $t \in \{0, T\}$ through the momenta:

$$\frac{\partial \mathcal{L}}{\partial \dot{\mathbf{q}}}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{c}) = \frac{\partial \mathcal{L}}{\partial \dot{\mathbf{q}}}(\mathbf{q}, \mathbf{u}, \mathbf{c}). \quad (2.9)$$

which can be seen as the required impulse to bring the system from rest into the actual modelled situation and back to rest. The velocities in \mathbf{u} were introduced as independent variables in the optimization to allow constraints controlling the initial and final velocities.

Solving for the accelerations

In the following contact formulation, see 2.3.2, the accelerations are used, as contact constraints applied directly on the coordinate were not sufficient to limit the velocities

in the optimization (Paper 4). There are three alternative ways of determining the accelerations. The first is to use a higher order of interpolation, but then some of the benefits with the weak form would be lost.

A second method is to solve for the accelerations from a strong form of the equilibrium equations. With the chosen type of mechanical system the kinetic energy can be written as:

$$\mathcal{T} = \frac{\dot{\mathbf{q}}^T \mathbf{M}(\mathbf{q}, t) \dot{\mathbf{q}}}{2} \quad (2.10)$$

where \mathbf{M} is the mass matrix. Accordingly, the generalised momenta become:

$$\frac{\partial \mathcal{L}}{\partial \dot{\mathbf{q}}} = \dot{\mathbf{q}}^T \mathbf{M} \quad (2.11)$$

Equation (2.4) gives the differential equations of motion and can be written as:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{q}} - \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\mathbf{q}}} \right) = \frac{\partial \mathcal{L}}{\partial \mathbf{q}} - \dot{\mathbf{q}}^T \dot{\mathbf{M}} - \dot{\mathbf{q}}^T \mathbf{M} = \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{c}, t) - \dot{\mathbf{q}}^T \mathbf{M} = \mathbf{0} \quad (2.12)$$

where $\mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{c}, t)$ is introduced to represent the non-acceleration terms in the equilibrium equations. Thus, the accelerations can be solved for from Eq. (2.12) as:

$$\mathbf{a}^T \mathbf{M}(\mathbf{q}) - \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{c}) = \mathbf{0} \quad (2.13)$$

This method was used in Paper 3.

A third alternative was used in order to reduce the computational cost of inverting the mass matrix symbolically as it grows rapidly with the number of dof. This method introduces the accelerations as independent variables similar to the coordinates and forces as:

$$\mathbf{a}(t) = [a_1(t), a_2(t), \dots, a_{N_d}(t)]^T, \quad (2.14)$$

These variables are then solved for from:

$$\int_0^T (\mathbf{a}^T \mathbf{M}(\mathbf{q}) - \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{c})) \delta \mathbf{a} dt = \mathbf{0} \quad (2.15)$$

This way of determining the accelerations was applied in Paper 4.

2.3. Contact

Modelling movements where the body is in contact with the ground is a well known challenge in human movement simulations. There are mainly two ways to model the contact (Featherstone 2008): rigid body contact (Hatze 1981; Ashby and Delp 2006, e.g.) in the form of kinematic constraints, and compliant contact (Andersson and Pandy 1998; Wilson et al. 2006; King et al. 2006; Gittos et al. 2006, e.g.) .

2.3.1. *Compliant contact*

In Paper 2 a compliant contact was used since it can represent foot deformation at contact in the present application to long jump. This was expressed as:

$$\tau = kq + c\dot{q}q \quad (2.16)$$

where k and c are the stiffness and damping coefficients, respectively. The multiplication by q in the damping term is used to make the constraint continuous which was a requirement in the optimization. Advantages are that initiating contact and contact loss can be solely determined from position and velocity; no linear complementary problem has to be solved as when using kinematic constraints. In addition to the constants k and c not being measurable, the main drawback is that this formulation introduces a tendency to high frequency vibrations in the system, requiring smaller steps in the integration or increasing numbers of time stations when discretizing the trajectories. Compliant contact has been used by Andersson and Pandy (1998); Wilson et al. (2006). None of these mention the convergence problem with optimization encountered in this work. However, with the formulation where the contact was integrated forwards, and the dof representing the contact were not included as optimization variables, the convergence problems were avoided.

2.3.2. *Non-compliant contact*

When modelling non-compliant contact one can choose to work with either maximal or reduced coordinate formulations. Using the first, with explicit constraints for modelling joints, would require more dof and the needed constraint equations become complex. Also drift in the constraints can occur which needs to be damped out, e.g. by using the Baumgarte stabilization method (Baumgarte 1972). When reducing the system according to the joint type connecting the bodies, the contact becomes fulfilled implicitly.

However, contact between the structure and the environment in biomechanical applications is often alternating during subintervals of a movement. It is still possible to reduce the system for each contact initiation or release but the generality of the model would decrease. Therefore, in this work a constraint in the contact dof was applied even when using reduced coordinate formulations. For this an efficient formulation of constraints has been developed to be used together with ABA (Kokkevis and Metaxas 1998) which should be considered if more complex contact simulations are modelled for a MBS.

In Papers 3 and 4, a similar constraint is used but implemented on the weak form. This contact is represented by zero accelerations. The complete constraint defining the contact is then ensured by a constraint in the coordinate when initiating contact, zero acceleration during the contact phase and a constraint in the coordinate at the final boundary of the contact phase. When, as an example, the contact takes place in the coordinate q_1 during a time subinterval set $T_c(1) = \{[0, t_2], [t_4, t_6]\}$, the contact constraint can be written as:

$$\int_{T_c(1)} \delta c_1 a_1 dt = 0 \quad (2.17)$$

$$q_1(0) = q_1(t_2) \quad , \quad q_1(t_4) = q_1(t_6) \quad (2.18)$$

where c_1 and a_1 are the contact force and acceleration corresponding to the displacement coordinate q_1 . The displacement condition at the phase boundaries, Eq. (2.18), can thus be predetermined or solved for in the optimization. As the problem is solved using a discretised representation, a resulting numerical error is present. The violation of the contact constraints is one of the factors which determine the need for fineness of the discretization.

2.4. Free time-formulation

When simulating movements where contact takes place in subintervals, it is desirable that the optimization solves for the duration of these intervals. In the case when using compliant contact this will be the case without any extra variables (Paper 2), while when using a multiple phase formulation (Papers 3–5) the time durations have to be solved for as separate variables. It is worth noting that the whole movement is considered when the optimal movement is determined by using this approach, as an alternative to a recent study, where the phases of the movement were optimized sequentially but separately such that each phase have no bearing on subsequent phases (Allen et al. 2010).

The free-time property is represented as a separate time scale variable. When having N_t phases, the time scale variable changes with each phase, hence a time scale parameter per phase, τ_i , is introduced as a variable in the optimization, such that the real time increment:

$$dt = \tau_i d\hat{t} \quad (2.19)$$

where \hat{t} is a non-dimensional time variable, which varies by unity over each phase. The interior time boundaries and the whole time interval considered can be written as $t_i = \sum_{k=1}^i \tau_k$ and $T = \sum_{k=1}^{N_t} \tau_k$, respectively, Fig. 2.2. Thereby, the scaling of time affects the derivatives, $\frac{d}{dt}$, and the time differential, dt , in the time integration of Eq. (2.8). With this formulation, the sequence of the phases has to be predefined in the sense that the mechanical properties of each phase are completely defined.

By the introduction of separate time scale variables, the optimization problem gets some new characteristics. For instance, equilibrium equations describing linear mechanical properties, i.e. only a linear dependence of the velocities in the weak form, become nonlinear through the scaling term on the time derivatives which follows from Equation (2.19).

Another aspect is the non-unique properties that can arise when there exist neutral positions and movements, giving no contribution to the cost function. An example would be when an optimal performance can be achieved from a specific configuration. Then, any feasible motion prior to reaching this configuration does not contribute, and thereby is neglected. This is shown in the numerical examples in Paper 3. In practise, this non-uniqueness deteriorates the convergence in the optimization.

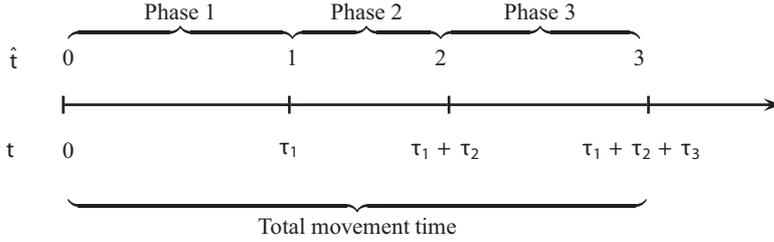


Figure 2.2: Example of simulation time divided into phases with unit lengths in the non-dimensional time \hat{t}

A phenomenon to be aware of is that the time scales also affect the accuracy of the discretisation. Obviously, the time scales change the time each element represents, which directly influences the interpolation error. In most cases this means that one needs to use some upper limit on the time interval lengths, τ_i . Otherwise the optimization solver will, if it happens to gain the objective cost, increase the simulation time so that numerical accuracy is lost. Accordingly, when the time scale constraint is active, one needs to increase the number of elements as one increases the timescale limit.

Thus, the number of elements finally used to represent the resulting movement, depends both on the duration of the movement, characteristic velocities and accelerations, and the constraint fulfilment, i.e., can differ between phases. In this work successive mesh refinement was performed until constraints were satisfactory fulfilled and the optimum converged to a specific value. Though there are more systematic mesh refinement procedures (Betts and Huffman 1998).

2.5. Optimization

The basic problem formulation in all studies in this work is based on optimization. The general constrained optimization problem can be stated as

$$\begin{aligned} \min_{\mathbf{x}} f(\mathbf{x}) \quad & \text{subject to} \\ \mathbf{g}(\mathbf{x}) & \leq \mathbf{0} \\ \mathbf{h}(\mathbf{x}) & = \mathbf{0} \\ \mathbf{x}_l & \leq \mathbf{x} \leq \mathbf{x}_u \end{aligned} \quad (2.20)$$

where \mathbf{x} is a vector containing all variables, f is the scalar cost function to be minimized, \mathbf{g} and \mathbf{h} are inequality and equality constraints formulated from the variables in \mathbf{x} , respectively, and \mathbf{x}_l and \mathbf{x}_u are the lower and upper bounds for the variables. To solve problems of this type, the functions `fmincon` in Matlab (MathWorks Inc., Natick, USA) optimization toolbox and SNOPT (Gill et al. 2006) were used even if also the MMA (method of moving asymptotes) algorithm has been used in similar contexts (Eriksson and Svanberg 2011). The functions are both based on sequential quadratic programming and requires continuous cost function and constraints. The

convergence is significantly improved by supplying derivatives for constraints as well as for the cost function.

2.5.1. *The static optimization problem*

The static optimization formulation used in Paper 1 is relatively straightforward and similar formulation can be found in literature (Amankwah et al. 2006; Heintz and Gutierrez-Farewik 2007) to solve the load sharing problem. However, the present optimization also solves for the configuration of the multibody system. So the activation level for each muscle group together with a coordinate and a moment in each dof are the variables in the optimization. The cost function which represents the individual's choice of posture is here representing the needed effort for a specific posture. The basic cost function is $f = \sum \alpha_i^2$, but in one application an additional term is added: a penalty simulating discomfort for resting at the end of range of motion, $f = \sum \alpha_i^2 + \frac{\text{passive moment}}{\text{nominal moment value}}$.

In the inequality constraints we find the requirement for static stability that the projection of center of mass should be within the base of support, the parallelogram formed by the feet. The inequality constraints also ensure that the friction forces are less than available with the specific friction coefficient used.

The equality constraints consist of the equilibrium equations, the orthosis functions and the predefined intersegment motion, if applied.

2.5.2. *The dynamic optimization problem*

Several methods exist for solving trajectory optimization problems in literature, e.g. indirect and direct shooting methods, and global and local collocation methods (Betts 1998; Huntington and Rao 2007). Dependent on the available type of nonlinear optimization solver the problem should be formulated to fit the general form of optimization (Eq. 2.20). One common strategy in movement optimization in biomechanics is to introduce time discretization (Pandy et al. 1992; Fang and Pollard 2003; Eriksson 2007) in order to have discrete variables representing the motion over time.

Hybrid dynamics formulation

A local collocation was used in Paper 2. The dof which represent physical joints in the application, and which are controlled by the subject, were represented by fifth-order time polynomials. In order to have variables representing physical measurements, i.e., position, velocity and acceleration, shape functions were used between the nodes. Hence the trajectory in each dof can be evaluated between the time nodes. It was also experienced that the use of the naive form of polynomial $P = a + bt + ct^2 + dt^3 + et^4 + ft^5$ gives poor behaviour in the optimization, due to lacking orthogonality of the polynomial terms. The coefficients, in the optimization, got values in different orders of magnitude and solutions which were close in the physical solutions but separated in the numerical ones.

In the dof which represented non-physically controlled dof (contact and wobbling mass) forward integration was used. The use of forward integration rather than having design variables representing the contact and wobbling mass avoided convergence

problems with the used solver (`fmincon`). With this formulation, the gradients of constraints and object function are not available, reducing the convergence speed.

In Paper 2, two fifth-order polynomials were used in each dof to keep the number of variables in the optimization as low as possible. This was possible, as the expected motion did not contain any oscillations. A variable transition time was used to let the structure have an ability to rapidly adjust for the ground contact. The number of time steps used was $n_t = 44$, so the total number of design variables in the optimization became $270 = (9 + (n_t + 1)) \times 5$

The cost function was the negative of the horizontal length of the flight trajectory which only is dependent on the positions and velocities in the last time node. Since the contact and wobbling mass were forward integrated, the dynamics function had to be integrated over the whole simulation time in order to evaluate the cost function.

The equality constraints ensured that the required generalized forces were equal to the generated moments of the muscles (activation times the factors considering moment arm, contraction velocity and maximum isometric strength at the specific joint) for any feasible solution of the optimization problem:

$$h_i^j = \tau_i^j - \alpha_i^j \theta(q_i^j) \phi(\dot{q}_i^j) f_{i, \max} \quad (i = \{\text{number of dof}\}; j = \{\text{number of time step}\}) \quad (2.21)$$

In Papers 3–5 where the free-time multiple phase formulation was used a more general discretization was applied.

Multiple phase formulation

A more sophisticated method for discretization of the problem is a temporal finite element formulation (Eriksson 2007). This method was used in Papers 3–5. The displacements and forces were discretised according to a chosen interpolation, and as the weak form of the equilibrium equations only makes use of first order derivatives, continuous and piecewise differentiable interpolation is allowed. It was concluded in Eriksson and Nordmark (2010) that the complexity of the expressions makes linear interpolation preferable for robustness, with no benefits from a higher order interpolation. The resulting variables existing over time intervals in the discretised problem are the values of the forces, displacements and accelerations at the time nodes. These time nodes are set by the chosen mesh. The discrete values of the displacements, the forces and accelerations (if used) are collected in the vectors, \mathbf{Q} , \mathbf{C} and \mathbf{A} , respectively. It is then possible to independently choose interpolation order for the variables. In Paper 4, the continuous values were interpolated as:

$$\mathbf{q}(t) = \mathbb{N}_Q(t) \mathbf{Q} \quad , \quad \mathbf{c}(t) = \begin{bmatrix} \mathbb{N}_{CC}(t) \\ \mathbb{N}_{CJ}(t) \end{bmatrix} \mathbf{C} \quad \text{and} \quad \mathbf{a}(t) = \mathbb{N}_A(t) \mathbf{A} \quad (2.22)$$

where the $\mathbb{N}(t)$ contains the chosen sets of interpolation functions. These were of linear Lagrangian type for the coordinates and the forces, \mathbb{N}_Q and \mathbb{N}_{CJ} respectively, while the accelerations and contact forces were interpolated with discontinuous piecewise

linear elements, \mathbb{N}_A and \mathbb{N}_{CC} , respectively. From Eq. (2.22), the variations become:

$$\delta q_i = \mathbf{N}_{Q,i} \delta \mathbf{Q} \quad \rightsquigarrow \quad \delta \dot{q}_i = \dot{\mathbf{N}}_{Q,i} \delta \mathbf{Q} \quad (2.23)$$

with $\mathbf{N}_{Q,i}$ one row from $\mathbb{N}_Q(t)$, and similarly holds for the forces and accelerations. The discretized displacements, forces and accelerations were then used together with the weak equilibrium equations, Eq. (2.8), and in the equation used to determine the accelerations, Eq. (2.15). A resulting set of discrete constraints defining the dynamics is thereby achieved, and is used as equality constraints in the optimization problem, Eq. (2.20).

The cost functions used together with the multiple phase formulation were of the type least effort or max performance. In Paper 3 the performance criteria of minimizing time for targeted movements, maximizing final velocity and maximizing a vertical position, were used. In papers 4–5 cost functions of least effort was used; minimizing the integrated sum of control forces squared, minimized the integrated sum of the forces squared but normalized by their maximum value, and minimization of the integrated power loss. The cyclic movement required the cost function to be normalized with the total time T such that the cost function become neutral to the frequency, i.e. what is called stroke rate in rowing, and allows to be solved for by the optimization.

2.6. Problem characteristics and practical experiences

The static optimization problem in Paper 1 forms a well defined optimization problem with a limited number of variables and is thereby solved with limited computational effort. The optima are usually well defined and convergence to non-global optima are usually not a major problem.

The dynamic optimization problems derived in these biomechanical applications of movement optimization generally get some specific characteristics. The cost function is often a very simple function which is continuous and differentiable. What makes the optimization problem complex is instead the many non-linear constraints and variables which define and describe the dynamics of the mechanical system when discretized in time.

As it is only possible to guarantee local optimality of the optimum found, an important part of the analysis is to increase the likeliness that the result is an estimate of the global optimum. Local optima in this thesis can be qualitatively different movements or postures, or they can be small deviations from an optimum. These small deviations are possible since their contribution to the cost function is very limited and alternative solutions cannot be distinguished by the tolerances in the optimization. Solutions which are neutral to the cost function for part of the movements and therefore are by the problem definition undetermined increase the complexity significantly as then infinitely many solutions exist.

In some applications, the optimization can be sensitive to the initial guess. It is then difficult to find a feasible solution. Also when a feasible solution is found, the optimization can have difficulties to leave this solution in the space of trajectories. The solution is then dependent of the initial guess. The experience from this work is that both these matters are dependent on the discretization of the problem. It is more

likely to find a feasible solution with a limited number of elements and the resulting solution is also less dependent on the initial guess. When traces of the initial guess can be suspected to affect the result, a random perturbation of the solution can sometimes make it possible to leave the local optimum.

The convergence of the optimum dependent on mesh also require some caution. Qualitatively different solutions can be obtained when the mesh is refined. It is neither possible to predict how the optimum will change with a refined mesh even if staying in the same optimum in the space of trajectories.

The discretization also affects the solution by its ability to represent the theoretically optimal solution. In the situation of having a discontinuity in the optimal solution, continuous interpolation would make its best to represent this. This has in performed experiments been observed to result in overshoots and oscillations in the neighbouring time elements, which are completely artificial. Matching discretization, i.e., a discontinuous interpolation when the optimal solution is expected to be discontinuous, has in several cases in the present work improved the convergence properties. However, when being aware of the phenomenon this will not cause any major issues, especially in these biomechanical applications, as refinement of the mechanical model's representation of muscle dynamics would introduce longer timescales.

When having difficulties to find a feasible solution, sometimes relaxation of constraints, i.e., variable bounds and inequality constraints, can improve convergence. In some experiments performed, additional constraints on selected time instances during a complex movement can guide the solver to find a feasible solution. In the backward somersault in Paper 4 an initial constraint for the rotation of the structure was applied in the middle of the flight phase. Once a feasible movement was found, the constraint was removed and a new optimization performed.

Another practical experience is that superfluous constraints can cause convergence problems. This has for instance been seen when using higher order constraint evaluation then used discretization causing dependent constraints.

2.7. Implementation

In Papers 1 and 2 the problem was mainly implemented in Matlab (MathWorks Inc., Natick, USA). For the optimization the NLP solver `fmincon` (sequential quadratic programming) was used. The equations of motion were derived using the symbolic mathematical software Maple (Maplesoft, Waterloo Maple Inc, Canada) with a computer algebra system Sophia (Lesser 1995). In order to derive parameters for the maximal torque-joint angle relationship the software SIMM (Delp et al. 1990) for modelling musculoskeletal systems was used.

In Papers 3–5, Comsol Multiphysics (version 3.5a, Comsol AB, Stockholm, Sweden) was used to specify the problem, to assemble the matrices and to derive the required derivatives for the Jacobian. For the optimization, the NLP solver SNOPT (Gill et al. 2006) was used. This uses sparse sequential quadratic programming, and is developed to handle the large-scale optimization at hand.

2.8. Applications

2.8.1. 3D model for standing posture

In order to allow studies of a subject with asymmetries, an 8 segment 3D model was created in Paper 1 (Fig. 2.3). The static optimization formulation was used. A muscle system including 30 muscles or muscle groups (Table 2.1) was used. With the varying moment arms and maximum strengths along the range of motion, a intersegment relation was introduced. The posture and load sharing for a given problem definition is then solved by minimizing the activation level in the muscle system. Here, the user can study posture strategy, joint moments, muscle activation level and orthosis effect for an individual.

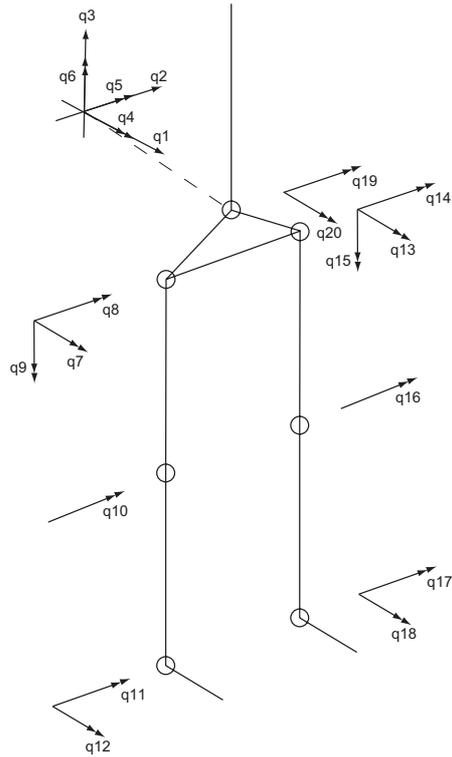


Figure 2.3: The mechanical model for standing posture (Paper 1)

Table 2.1: Muscle groups and their functions.

Function	Muscles	No.
dorsiflexion	tib_ant, per_t, ext_d, ext_h	1
plantarflexion	med_gas, lat_g	2
	soleus, tib_p, flex_h, flex_d, per_b, per_l	3
knee flexion	med_gas, lat_g	2
	semimem, semit	4
	bifemsh	14
knee extension	rect_fem	6
	vas_med, vas_i, vas_l, pat_l	7
hip flexion	rect_fem	6
	add_long, add_b, pect, grac	8
	glut_med1, glut_min1, sar, tfl	9
	psoas, iliacus	15
hip extension	semimem, semit, bifem	4
	glut_max2, glut_max3	5
	add_mag1, add_mag2, add_mag3	10
	glut_med3, glut_max1, glut_min3	11
hip abduction	glut_med1, glut_min1, sar, tfl	9
	glut_med3, glut_max1, glut_min3	11
	glut_med2, glut_min2, peri	12
hip adduction	semimem, semit, bifem	4
	add_long, add_b, pect, grac	8
	add_mag1, add_mag2, add_mag3	10
	psoas, iliacus	15
hip internal rotation	glut_med1, glut_min1, sar, tfl	9
hip external rotation	quad_fem, gem	13
	psoas, iliacus	15

2.8.2. Long jump

A six link model with a wobbling mass was used in Paper 2 to represent the human body in the long jump take-off (Fig. 2.4). The foot contact was modelled with spring and damper elements. A hybrid dynamics formulation of the ABA was used to evaluate the dynamic functions. The dynamic optimization formulation was used, where the model finds optimal movement in terms of a following jump distance from an initial configuration. Activation history, ground contact time and movement are all determined by the optimization.

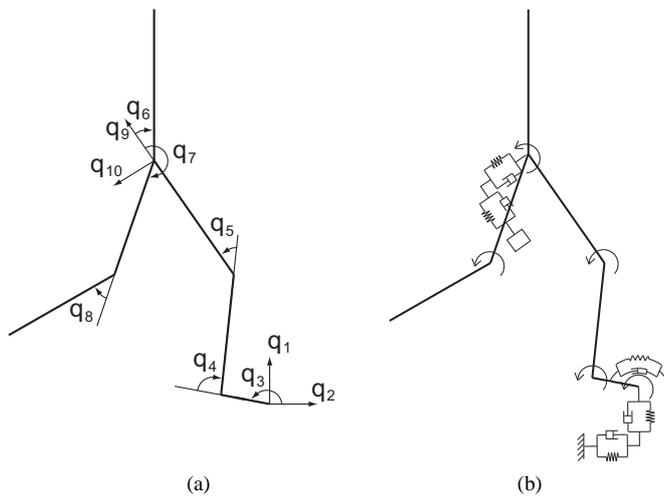


Figure 2.4: The mechanical model for long jump take-off (Paper 2): a) degrees of freedom, b) properties for the forward integration.

2.8.3. *Two foot high kick*

A sagittal four link model was used in Paper 3 to represent the human body neglecting transversial movements and arm-movements, Fig. 2.5. Six dof were required to describe in-plane motion. Non-compliant contact was used and the contact was ensured by three control forces in the contact dof. The movement was divided into three phases: whole foot contact, forefoot contact, and flight phase. The movement was simulated until the foot reached its highest position. The landing was excluded in this study.

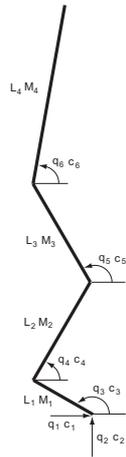


Figure 2.5: The mechanical system for the two foot high kick (Paper 3) with its degrees of freedom representing the human body. This very reduced system was used in this first study applying the free-time multiple phase formulation. The control forces 1–3 are acting in their corresponding degree of freedom while $c_{4...6}$ act between the links.

2.8.4. *Backward somersault*

A sagittal six link structure was used in Paper 4 when simulating the somersault. Five control moments represented the muscles about each joint, and three control forces represented the contact, Fig. 2.6. Six successive phases were used to represent whole foot contact, forefoot contact, two non-contact phases, forefoot contact and whole foot contact, respectively. Explicit impulses were introduced in the contact dof to handle the contact initiations. These were also determined by the optimization.

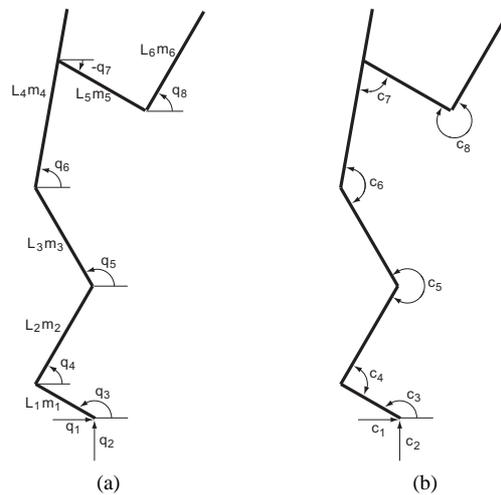


Figure 2.6: The mechanical system for the backward somersault (Paper 4) with its degrees of freedom. The control forces $c_{1...3}$ are acting in their corresponding degree of freedom while $c_{4...8}$ act between the links

2.8.5. Rowing

A multibody system was used in Paper 5 to represent rower, boat and oars, Fig. 2.7. The boat was only allowed to move in one dof, translation in fore and aft direction, while the oars had two rotational dof. The rower was in contact with the boat at foot-strecher and seat. The feet were assumed to be joined with hinge joints to the foot-strecher while the seat contact was represented with a separate contact force. Hydrodynamic loads were assumed to act at boat and oars. Four phases were used representing the cyclic movement: oars partially in the water, oars completely in the water, oars partially in the water and oars completely above the water.

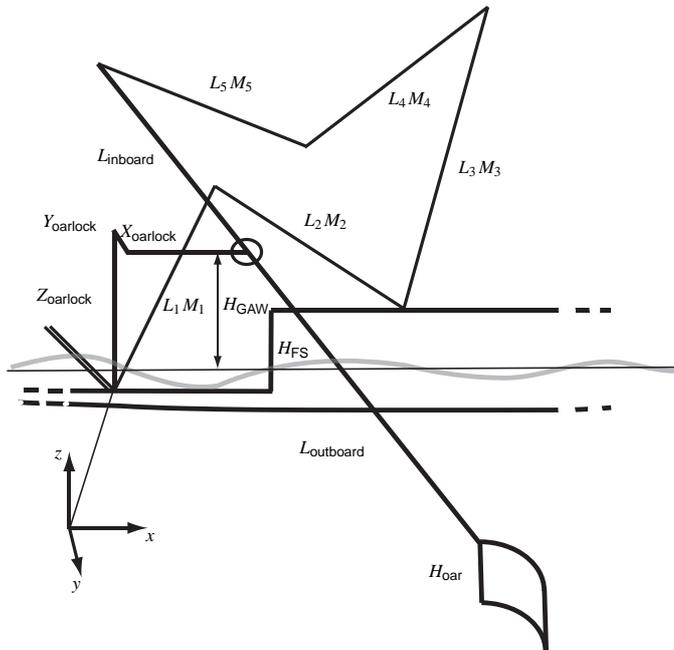


Figure 2.7: The side view of the mechanical model for rowing (Paper 5) with its measurements. As full left-right symmetry is assumed the two oars were modelled as one unit.

Results and discussion

3.1. Posture strategies

A model for studying postural strategies has been developed. The main results are the ability to predict postural changes due to heel elevation of the able-bodied control group, and the good qualitative agreement with captured data of for three cases with persons with movement disorders. The model considers asymmetries, orthoses, weaknesses and limited ranges of motion while solving for the posture resulting in minimum muscle activation.

The differences observed between the simulated and observed angles are mainly related to the pelvic-HAT segment connection, evident in the large differences in anterior pelvic tilt and hip flexion (a relative angle), and decreasing differences in the more distal joints. The present model assumes that the HAT's weight is transferred only through the joint between the HAT and pelvis, and its location is therefore important for the determination of the neutral angle of the pelvis. In a recent study by Murphy et al. (2011) a fairly accurate estimate by a regression expression based on palpable anatomical landmarks is determined. Inclusion of such models could improve the presented mechanical representation. Furthermore, the model does not consider stretch resistance within the range of motion at any joint, even though pelvic orientation is dependent on the trunk and femur orientation (Chaffin and Andersson 1991) due to acting muscles about the hip joint. Therefore, it is often optimal to strive for more vertical orientation of the pelvis segment (resulting in large anterior tilt). One possible improvement to the model would be to include passive stretch resistance with a subject-specific neutral position.

The posture strategies are based on the cost function of muscle activations. As this is an assumption, alternative cost functions would be interesting to investigate. An expected improvement can probably be achieved only by introducing some weights of each activations to represent that some muscles are designed to serve as antigravity muscles, i.e., perform static contraction during longer periods.

Compared to the study of Amankwah et al. (2006), probably the most comparable study to ours, the present model can be tailored to suit a wide variety of biomechanical constraints, and allows comparison to real subjects; our simulations can be performed without the artificial posture constraints such as fixed stance width or enforced symmetry applied in the referred study. Also, the present model applied minimization of muscle activations, instead of joint moments followed by a separate optimization

for load-sharing, making the actual posture strategy based on the subject's individual muscle groups' capabilities.

In a clinical study (Bartonek et al. 2011), where the author of this work also been involved, investigate heel heights influence on the standing posture. In this study it is concluded that some heel heights can contribute to improved biomechanical alignment, i.e. unloading the muscles acting at the joints, while it is necessary to apply heel heights adequate to each individual's orthopaedic and neurologic condition.

3.2. Long jump

The main accomplishment in this study was the possibility to solve for the contact constrained movement in optimization context. The use of the articulated body algorithm was explored and found to be worth considering as an alternative to the classical Lagrangian approach, especially for larger mechanical systems as the computational time increases only linearly with the number of dof.

The use of `fmincon` as NLP solver with this problem setup gave relatively poor convergence properties compared to the size of the optimization problem. With the presented formulation, the gradients were not available and therefore these had to be numerically approximated by the solver as well and this slowed down the simulation time. Even with a good initial guess simulation time of more than 24h was common using an ordinary workstation of 2GHz CPU.

One of the limitation in this study was the representation of the contact. It was not possible to increase the stiffness further in order to result in realistic deflection at the ground contact. In the lack of experimental data the input of the model was based on available published data but not for a single individual. The specified strength was hence not adjusted to initial velocities used. This, together with the limited dof to adjust for the contact due to the choice of interpolation, made it impossible for the simulated athlete to withstand the footplant. As a consequence we had to use softer contact than expected to be valid for the application.

The wobbling mass influence on the movement was not investigated here. Tuning of these properties would likely, in combination with the ground contact properties, determine the characteristics of the ground reaction force as experienced in Seyfarth et al. (2000). Now the initial force peak found in experimental data is not present.

3.3. Free-time multiple phase optimization

The main accomplishment in this study was the introduction of multiple phases where different contact constraints were active. Using the free-time formulation in optimization added some aspects to the simulations in terms of numerical accuracy and undetermined movements according to the cost function.

The multiple phases of contact constraint motivates having a free-time formulation to make the contact interaction as adaptive as possible in order to represent the application. What limits the contact modelling using this formulation is that the sequence of phases has to be predefined. Anyway the multiple phases increase the formulation's applicability to biomechanic applications.

In this study we solve for optimal movements using a performance criterion instead of minimizing the required effort, e.g. the control forces. From these simulations it was clear that some new phenomena can occur dependent of the problem definition, like undetermined movements, which should be avoided to improve the optimization.

The introduction of a contact constraint through requiring the accelerations to be zero was a successful choice and gave good numerical properties where no forces at the time boundary could affect the coordinate derivatives at the boundary.

SNOPT compared to `fmincon` was, for the here presented problem setup, far better in terms of convergence rate and was used in the following studies. The usage of COMSOL Multiphysics involved many advantages compared to working with complete coding in Matlab during the development as it simplified the specification of the optimization problem. It was also used to evaluate the required derivatives.

3.4. Multiple phase optimization including impacts

The main contribution of this study was the inclusion of contact and impacts in the movement optimization. Also, compared to the previous studies a larger system and a more complex numerical example was used.

Even though its possible to show that the resulting optimal control when minimizing the sum of the squared forces is continuous, a discontinuity can be introduced by constraints. This is present in the numerical test problem in Paper 4. Some overshoot takes place, but is decreased by a refined mesh. To completely remove the overshoot, a discontinuous interpolation had to be used. In this study, piecewise linear elements were used. The use of discontinuous interpolation in such a case improved the convergence in the optimization even if this implied more optimization variables.

The modelling of impacts by explicit impulses was successful for the contact initiation in the movement optimization, although neglecting the actual deformation at impacts in a physiological system. This is, however, a successful method if the impact response is not of importance for the analysis.

In order to be able to use accelerations in the problem definition in terms of constraints for larger mechanical systems, these are solved for from the strong form of the equilibrium equations. The extra variables and constraints needed for the determination of the accelerations did not cause any significant lengthening of the iterations.

The numeric result in the application to the backward somersault showed realistic movement pattern compared to an actual gymnastic performed somersault, Fig. 3.1 and Fig. 3.2. The push-off, tuck and landing are present. However, it should be noted that this exercise can be performed with various movement patterns and in gymnastics predefined stylistic requirements exist which do not correspond to minimizing the forces. The optimal movement is derived with a variation of the constraints for the ranges of motion and strengths (20° decreased range of motion and 50% decreased strength) as the original problem setup were not case specific but served as a baseline estimate. The altered capacities significantly affected the optimal movement. Hence, this example emphasizes the importance of individual capacities to determine what is optimal. This has been observed in all applications in this thesis but became very obvious in this application to the backward somersault.

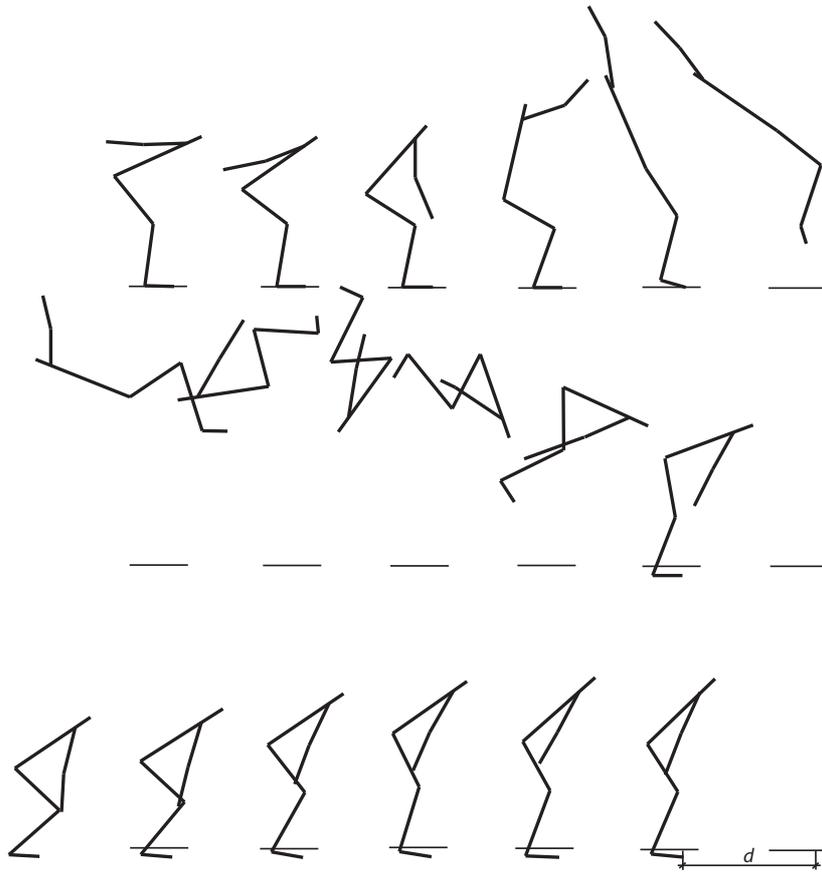


Figure 3.1: Gymnastic performed backward somersault. Extracted from existing video from http://www.ehow.com/video_4993919_backflip.html (2012-07-30), where a frame sequence, equally separated in time by 0.1 s, was chosen and a stick figure was overlaid, approximately based on the right side anatomical landmarks represented in the mechanical model. Observe that the structure has been plotted relative to the initial contact position, thus a backward translation (d) has taken place. The lengths of the segments in this figure appears to change due to out of plane movements. Also some out of plane movement caused the landing not to be aligned with the reference starting ground position

3.5. Rowing

The main methodological advance in this study was the inclusion of the cyclic rowing motion into a movement optimization. This has, to the knowledge of the author, not been dealt with in an optimization context before.

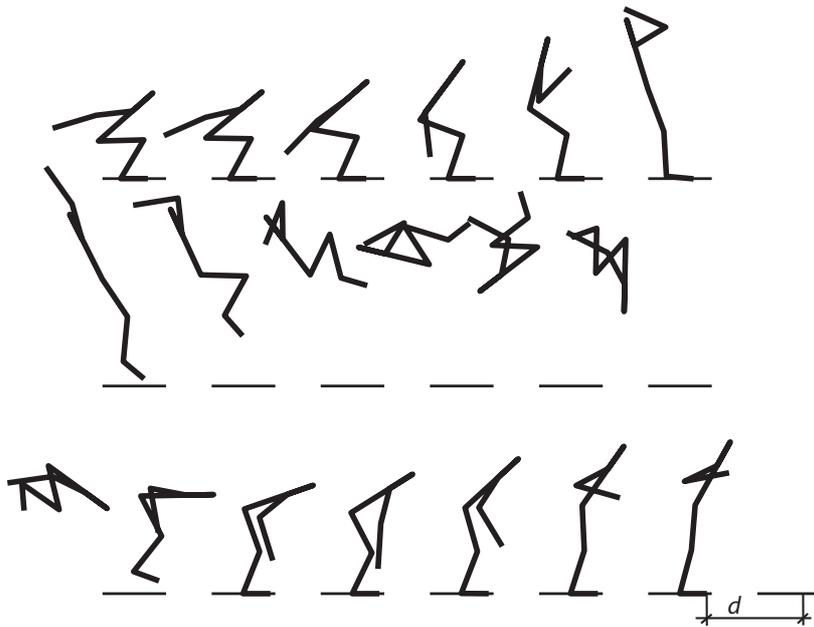


Figure 3.2: The resulting optimal movement with decreased capacities in terms of 20° reduced ranges of motion and 50% less strength. The time increment between each picture is as above 0.0833 s. Observe that the structure been plotted relative to the initial contact position and the horizontal translation d has taken place

The structure's interaction with the equipment and water through boat and oars is optimized. Without predefined input to the movement, the optimal movement of the rower according to the chosen cost function is found, determining stroke rate as well as oar path.

Cost functions consisting of the forces or energy loss were applied. However, both resulted in deviating movements compared to the expected in terms of stroke rate and oar path. It was obvious that something else had to be used as cost function even for these very simplified simulations of rowing. A composite cost function, which was minimizing mechanical energy loss during the driving phase and the control forces during recovery, was explored. The composite cost function resulted in reasonable movements but a cost function based on energy consumption would be preferred since there are movements which consumes metabolic energy but not mechanical, e.g. for eccentric contraction where a resisting moment acts to decelerate a movement or for static contraction. The fatigue aspect should also be considered as it is an important factor due to the duration of a complete rowing race.

The rigging setup was not tuned to the rower's properties in the simulation where the baseline parameters were used. A simulation where the rigging parameters were included as variables in the optimization resulted in a significant effect on the optimal movement. The movement pattern became more realistic and the power loss decreased. In addition, torque-velocity characteristics about the joints are suggested to be important for the rigging setup (Baudouin and Hawkins 2002). This suggests that when tuning the rigging setup one should consider both power loss and torque-velocity characteristics. Including muscle dynamics would hence make this type of analysis even more interesting when aiming for individual specific results.

CHAPTER 4

Conclusions

In this project a central part of the work has been to represent human movement and posture strategies using optimization.

The use of optimization to model standing posture strategies seems promising. With such a formulation many parameters such as weaknesses, contractures and orthoses, can be considered which would be difficult without a systematic procedure. The results can also be analysed on different physiological levels, ranging from specific muscle activations, to joint moments and to overall effort.

The use of muscle activation and coordinates as design variables allows the optimization to solve the load sharing problem, but also to represent the choice of posture, which has been proved to be numerically infeasible in an application of dynamic optimization (Andersson and Pandy 1998). Orthopedic aids, like orthoses, can be modelled. This is a valuable possibility, since creation of the aids can be based on more information and thorough understanding of the effects of the design and properties in treatment planning. Further validation to larger groups of patients would make it possible to study the capabilities and shortcomings of the model for standing postures, as well as to explore alternative cost functions and mechanical representations. The presented model can thereby be valuable in the early stage of prescription or investigation of possible orthopedic treatments to improve static posture strategy. The promising results with subject-specific simulations, even for patients rather severely affected by their disabilities, also demonstrates the model's applicability, especially the model's ability to predict postural changes due to common orthopedic interventions.

In four studies various formulations for movement optimization have been developed (Papers 2–5). All the studies consider the finding of optimal movement of a MBS subjected to contact constraints in various situations.

With presented methods it is possible to solve movement optimization problems for various situations such as targeted movements, unknown boundary states, least effort or maximizing performance, contact constraints and impacts, and cyclic movements. The formulation does not require any predefined information on the motion except from the sequence of phases, which make it possible to solve for novel optimal movements.

In the presented studies rather simplified mechanical representations have been used. Even though these obviously can give qualitative results about the specific application, subject specific simulations would be desirable as the subject's capabilities are important for what is considered as an optimal movement. For this, a substantial

effort would be needed to increase the biofidelity in terms of, e.g., muscle dynamics and muscle architecture. Given such representation the movement optimization methodology developed here could be a powerful tool to analyse movement patterns in sports as well as in medical applications.

Summary of Papers

Paper 1

The objectives of this study were to create a musculoskeletal model to run subject-specific simulations allowing the user to specify lower extremity properties found at routine clinical assessment in order to have a feasible tool in a clinical context, and to explore the model's design and potential in predicting the standing posture in three subjects with motion disorders as well as in a group of able-bodied controls. The simulations are based on optimization as the posture were chosen such the muscle activations are minimized. The simulation showed reasonable to good agreement and an ability to predict the effect of motion disorders and of external support. An example of application in parameter studies was also presented wherein ankle orthosis angles were varied.

Paper 2

The aim was to formulate a solution methodology for movement optimization of multibody systems subjected to contact constraints. In this work the application of the long jump take off was studied. A sagittal model with six rigid links and one wobbling mass was used. The contact was modelled with a soft contact, with its pros and cons. The movement in each dof representing the physical joints was described by two fifth-order polynomials, while the motion in the dof of the contact and wobbling mass was integrated forwards, as a consequence. Using a functional performance criterion in an optimization, novel movements were derived from an initial configuration. The solution maximizes the ballistic flight distance following take-off. The optimization determines contact time, end configuration, muscle activation and interaction with the ground.

Paper 3

In this study the aim was to introduce a free-time formulation as well as multiple phases while using a temporal finite element approach for movement optimization. The time scale parameter in relation to the objective function is discussed and verified by numerical examples. For movements with partial contact multiple phases with different mechanical properties are included. The free-time formulation allows these phases to be determined by the optimization. A three-phase two-foot high jump is

simulated where the movement optimization solves for the whole movement including the prior motion preparing for the subsequent phases with different mechanical properties.

Paper 4

The aim was to further improve the multiple phase formulation and increase its application possibilities in terms of handling larger models as well as more general contact situations where impacts are present. Impulses were introduced to handle contact initiation and an alternative way to solve for the accelerations was introduced to handle larger structures. A two link test problem is used to investigate some basic properties while a six link structure is used to simulate a backward somersault consisting of six phases. It is possible to solve for the complete movement without any predefined input about the movement to the model.

Paper 5

The aim was to develop an optimization model for rowing which can be used to analyse rowing technique and rigging setup, and to gain general insight into the system. A sequence of driving and recovery phase during a stroke is simulated being a cyclic movement when rowing with constant speed. Both the driving and recovery phase are considered when the optimal movement is determined. Objective functions based on control forces and energy loss were investigated. The measurements in the rigging setup were also included as variables in the optimization. The movement strategies of the rower can thereby be analysed and tuning of the equipment be done with the developed method. Even though no direct comparison with experimental data was made, the characteristics of the resulting movements seem reasonable considering the simplified mechanical representation.

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