A 2D Indoor Propagation Model Based on Waveguiding, Mode Matching and Cascade Coupling

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A 2D Indoor Propagation Model Based on Waveguiding, Mode Matching and Cascade Coupling

by

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Abstract

In this thesis a theoretical model for indoor propagation in a straight corridor with adjacent rooms is developed and evaluated. One objective is to assess the effect of different conductivities and permittivities in the walls between rooms have on the power levels, in both the corridor and the rooms. Furthermore, a model of a leaky cable is proposed for which the corresponding propagation characteristics are evaluated and compared to that of a dipole antenna to assess if a leaky cable is a viable alternative for radio coverage in an indoor environment. In order to evaluate the model, a wideband measurement campaign has been conducted at 2.44 GHz with a 40 meter long leaky coaxial cable and two vertically polarized dipole antennas.

The proposed model is based on the waveguide model in 2D, the mode matching method and cascade coupling of scattering matrices. A section of a corridor is modeled as waveguides with different cross section where one waveguide contains a dielectric medium which models the wall between two rooms. Mode matching is used to determine how the waveguide modes are coupled at the boundaries between the waveguides and the result is collected in a scattering matrix. Multiple corridor sections are then connected together, by cascade coupling the corresponding scattering matrices of each section, into a long corridor with adjacent rooms. Point sources are used to excite the waveguides as an approximation of dipole transmitting antennas. Moreover, the radiating slots in the leaky cable are modeled by multiple point sources that are phase and amplitude shifted in order to achieve the same radiation direction and longitudinal loss as the leaky cable. Finally, the inverse discreet Fourier transform is applied to the wideband electromagnetic field distribution in order to determine the propagation characteristics in the time domain.

The results from the model are in good quantitative agreement with the measurement data, and it is shown that a leaky cable give a more even radio coverage in an office corridor compared to a dipole antenna, especially when the internal walls are highly reflective. Moreover, it is shown that the direct path is dominating for transmission between rooms with transparent walls, like plasterboard, while the main propagation path for highly reflective walls is along the corridor.

Index terms: Indoor propagation, waveguide model, mode matching, cascade coupling, leaky cable, indoor environment
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<th>Description</th>
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<tr>
<td>Delta E</td>
<td>Delta Energy</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>FDTD</td>
<td>Finite Difference Time Domain</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
</tr>
<tr>
<td>GO</td>
<td>Geometrical Optics</td>
</tr>
<tr>
<td>HFSS</td>
<td>High Frequency Structural Simulator</td>
</tr>
<tr>
<td>IFFT</td>
<td>Inverse Fast Fourier Transform</td>
</tr>
<tr>
<td>ISM</td>
<td>Industrial Scientific and Medical band</td>
</tr>
<tr>
<td>LCX</td>
<td>Leaky Coaxial Cables</td>
</tr>
<tr>
<td>PEC</td>
<td>Perfectly Electrical Conductor</td>
</tr>
<tr>
<td>PML</td>
<td>Perfectly Matched Layer</td>
</tr>
<tr>
<td>S-Matrix</td>
<td>Scattering Matrix</td>
</tr>
<tr>
<td>S-MRTD</td>
<td>Scaling function based Multi Resolution Time Domain</td>
</tr>
<tr>
<td>TE</td>
<td>Transverse Electric</td>
</tr>
<tr>
<td>TM</td>
<td>Transverse Magnetic</td>
</tr>
<tr>
<td>UTD</td>
<td>Uniform Theory of Diffraction</td>
</tr>
</tbody>
</table>
List of Variable Names

In the variable name, an $i$ represents the waveguide region in which that variable is valid.

- $E_z$: Longitudinal component of the electric field, $E_z = 0$ for TE modes
- $H_z$: Longitudinal component of the magnetic field, $H_z = 0$ for TM modes
- $E^{(i)}$: Transverse electric field for TE modes aligned in the y-direction
- $H^{(i)}$: Transverse magnetic field for TE modes aligned in the x-direction
- $c_0$: Speed of light in vacuum
- $c_{\text{cable}}$: Speed of light in the leaky cable
- $\lambda$: Free-space wavelength
- $\varepsilon_w$: Relative permittivity for the dielectric wall
- $\sigma_w$: Electrical conductivity of the dielectric wall
- $k_0$: Free space wave number
- $k_w$: Complex wave number in the dielectric wall
- $a_0$: Waveguide width in region 3 and 4
- $a_1$: Waveguide width in region 1
- $a_2$: Waveguide width in region 2
- $d_w$: Dielectric wall thickness
- $k_{xm}^{(i)}$: Transverse wave number
- $k_{zm}^{(i)}$: Longitudinal wave number
- $y_m^{(i)TE}$: Mode admittance for TE modes
- $\psi_m^{(i)}(x)$: Mode expansion for the transversal electric and magnetic field components
- $c_m^{(i)+}$: Mode coefficients for a wave propagating in the positive z-direction
- $c_m^{(i)-}$: Mode coefficients for a wave propagating in the negative z-direction
- $c_m^{(i)+}(z)$: Mode coefficients with the z-dependence $e^{\mp ik_{zm}^{(0)}z}$ included
- $h_m^{(i)}(z)$: Mode coefficients, for homogeneous propagation, collected in vectors
- $s_m^{(i)}(z)$: Mode coefficients, excited directly from sources, collected in vectors
- $c_m^{(i+)}(z)$: Total mode coefficients on vector form
- $s^-(z_{h0})$: All source coefficients propagating to the left of one corridor segment
- $s^+(z_h)$: All source coefficients propagating to the right of one corridor segment
- $L^{(w)}$: S-matrix relating homogeneous mode coefficients for the dielectric wall
- $L^{(2)}$: S-matrix relating homogeneous mode coefficients for region 2
- $L$: S-matrix relating homogeneous mode coefficients for one corridor segment
- $S^{(0)}$: S-matrix relating source coefficients in region 3 and 4 to the external boundaries
- $S^{(1)}$: S-matrix relating source coefficients in region 1 to the external boundaries
- $L^{(Vm)}$: Homogeneous S-matrix for $m$ cascade coupled cells from the left of the corridor
- $L^{(Hm)}$: As the matrix above from the right side of the corridor
- $S^{(Vm)}$: S-matrix relating source coefficients for $m$ cascade coupled cells from the left
- $S^{(Hm)}$: As the matrix above from the right side of the corridor
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$L^{(Mm)}$</td>
<td>Homogeneous S-matrix for the internal mode coefficients between two cascaded cells</td>
</tr>
<tr>
<td>$S^{(Mm)}$</td>
<td>As the variable above but relating the source coefficients</td>
</tr>
<tr>
<td>$P_{(i)}$</td>
<td>Diagonal matrix describing homogeneous propagation through one waveguide region</td>
</tr>
<tr>
<td>$D_{(i)}$</td>
<td>Diagonal matrix where the elements are the mode admittance for each mode</td>
</tr>
<tr>
<td>$K_{(1)}$</td>
<td>Matrix describing mode coupling between region 1 and 3 (or 4)</td>
</tr>
<tr>
<td>$K_{(2)}$</td>
<td>Matrix describing mode coupling between region 2 and 3 (or 4)</td>
</tr>
<tr>
<td>$M_0$</td>
<td>Number of waveguide modes included in the numerical treatment in region 3 &amp; 4</td>
</tr>
<tr>
<td>$M_1$</td>
<td>Number of waveguide modes included in the numerical treatment in region 1</td>
</tr>
<tr>
<td>$M_2$</td>
<td>Number of waveguide modes included in the numerical treatment in region 2</td>
</tr>
<tr>
<td>$K$</td>
<td>Number of frequency samples included in the wideband simulations</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of point sources included in the LCX model</td>
</tr>
</tbody>
</table>
1 Introduction

Understanding how electromagnetic waves propagate in indoor environments becomes increasingly important as mobile devices are the typical way for long range communication. Laptops, smart phones and the newer tablet computers are overwhelming the market now more than ever and wireless networks have to work in all environments to satisfy the increasing demand.

Predicting the wave propagation in indoor environments is especially hard due to the wavelength being short compared to the many differently sized and shaped objects in the normal building. Varying room sizes, building materials and furniture placements as well as people moving around all affect the wave propagation in different ways. To predict how the indoor environment affects the wave propagation, different models have been developed. These models are divided into empirical and theoretical models.

1.1 Empirical Models

Empirical models, derived from in situo measurements are straightforward ways to estimate the path loss. The path loss is the difference between the transmitted and the received power. Measurements in a particular indoor environment are used to fix the parameters in a path loss model, which can then be used to predict the path loss in similar buildings. However the empirical models don’t take specific site features into account resulting in lack of accuracy. These models typically give the average power levels in terms of the distance from the transmitting to the receiving antenna and cannot accurately predict the power levels at every point in the environment considered.

Different empirical models have been used for predicting the path loss along corridors in office environments. One of these models [1] is based on the power law and is combined with the recursive model [1] to take into account propagation around corners. In further work [2] other models have been considered to account for the propagation from room to room and to more accurately predict the path loss along a corridor and over corridor intersections. These models have lead to predictions, for one class of office corridors, which are in good agreement with measurements.

1.2 Theoretical models

Some theoretical models are based on solving Maxwell’s equations, which can be done in different ways by making certain assumptions about the wave propagation and the geometry of the environment. Below follows a short description of the models commonly used to predict indoor propagation and some of their advantages and disadvantages.

1.2.1 Ray Tracing

Ray tracing is based on Geometrical Optics (GO) which describes wave propagation in terms of rays, this typically requires that the wavelength is much shorter than the dimensions of the
objects that interact with the wave. Geometrical Optics is normally used for wave propagation in the visible light spectrum, with wavelengths around 500 nanometers. There are two approaches to ray tracing, the imaging approach and the ray launching technique.

In the ray launching technique the propagating wave is modeled as a large number of rays. One of these rays can be seen as a point on the wave front of the outward propagating wave. When a ray comes into contact with a media boundary a new set of rays are excited according to the laws of reflection, refraction and diffraction. A large number of rays are typically required for this approach to give good results.

In the imaging approach an algorithm is used to first calculate, based on the position of the transmitting and receiving antennas, which specific rays will propagate to the receiver point. As a result only the rays that contribute to the field at the receiver needs to be included in the calculations. The name of the imaging approach comes from the simple case of a transmitter located above a ground plane. The resulting reflection is modeled by an image source placed below the ground plane.

Ray tracing is often supplemented with the uniform theory of diffraction (UTD). Diffraction occurs when the rays interact with wedges or smaller objects of similar size as the wavelength. UTD is used to calculate diffraction at sharp corners such as corridor bends [3] and edges of windows.

One problem with the ray launching approach is that it is not certain that all the possible ray paths are taken into account [4], to do that an infinite number of rays have to be sent out from the source. If the indoor environment is geometrically complex a large number of rays are needed for the results to be accurate, which requires substantial computational resources. Detailed information about the building materials and the geometry of the environment is also needed for this approach to give accurate predictions [5].

### 1.2.2 Waveguide Model

It has been shown in simulations [6] and measurements [5], [7] that corridors have a waveguiding effect. Waveguiding has been understood for almost a century and can be implemented quite straightforwardly to model corridors. However, in mobile communication applications the difficulties in modeling a corridor as a waveguide is that the corridor is many times larger than the commonly used wavelengths. A large number of waveguide modes are excited in the corridor, and these modes are scattered differently when interacting with interior objects as well as variations in corridor size.

So far 2D analytical models treating corridors as metallic and later on dielectric waveguides with uniform cross-section have been developed. In the dielectric case the walls have been treated as infinitely thick [5] and later on, more realistically, with finite thickness [7].

### 1.2.3 Numerical Models

Numerical models for indoor propagation are based on solving Maxwell’s equations numerically in the desired domain. Due to the rapid development of computers, numerical full wave solutions of Maxwell’s equations are becoming more feasible. Finite Difference Time Domain (FDTD) is a commonly used numerical technique where Faraday’s law and
Ampere’s law are solved in the time domain. The spatial domain considered is discretized into many cells, and the equations are solved by iteratively stepping forward in time. FDTD is used in [8] to create a 3D indoor propagation model of a multistory building. By computing streamlines of the Poynting vector the dominant propagation mechanisms within the building are determined, including propagation from floor to floor.

Another numerical model, similar to FDTD, is Scaling function based Multi Resolution Time Domain (S-MRTD) [6] in which the field-components are expanded in high-order basis functions in space. This allows for the discretized spatial cells to be larger than in FDTD, resulting in less computational time but with retained accuracy in the solution.

**1.2.4 Hybrid Models**

In hybrid models two or more of the above mentioned theoretical models are combined. For example, one can utilize ray tracing or the waveguide model in simpler regions where these models are valid and can be applied efficiently. A more versatile and detailed numerical model is then used to solve for the field distribution in regions where the previously mentioned models cannot be used for an accurate result. This saves computational time as the detailed numerical model only needs to be implemented in certain regions.

In [7] the waveguide model is used for the guiding effect of corridors and is combined with FDTD to solve for the field distribution in corridor intersections. The intersections are treated as multiports and the corridors act as the different ports. In [9] a 3D hybrid method is used by combining ray tracing and a full wave technique to analyze wave propagation through inhomogeneous periodic walls. In the office environment ray tracing is implemented in the volume between the walls and a commercial full wave solver is used to calculate the propagation through the walls, which can then be modeled as inhomogeneous.

**1.3 Leaky Cables**

Leaky coaxial cables (LCX) have been used for a long time as wireless feeders in subway tunnels and coal mines. A leaky cable is a coaxial cable with periodic slots cut along the outer conductor of the cable. Depending on the placement of the slots and the properties of the cable different radiation patterns can be achieved for different frequencies.

In the last decade leaky cables have been considered for use as antennas for mobile communications in buildings. Different theoretical models have been proposed to examine the radio coverage [10] and the radiation characteristics [11] of leaky cables to see if it is a viable alternative for indoor radio coverage. This thesis simulates propagation from a leaky cable and compares it to corridor measurements to further investigate its usage. Moreover, the results are compared to the simulations of a dipole antenna.

**1.4 Thesis Objectives**

This thesis attempts to simulate an office corridor with adjacent rooms by combining the waveguide model, mode matching and cascade coupling of S-matrices. To evaluate the
predictions of the model, measurements have been conducted with a dipole antenna as well as a leaky cable in a specific office corridor. The measurements were made at the ISM (Industrial Scientific and Medical) band on frequencies 2.40-2.48 GHz. The simulations are done in this frequency range as well. Below follows a list of specific objectives that this thesis handles.

- To determine how propagation from room to room takes place in an office corridor and which effect different conductivities, permittivities and wall thicknesses in the walls separating the rooms have on the power levels, in both the corridors and the rooms.
- To simulate a leaky cable as the transmitter in an indoor propagation scenario and compare to the case with a transmitting dipole antenna. Moreover, to assess if a leaky cable is a viable alternative for a normal indoor cell for wireless connectivity.
- Assessment of the phase difference between leaky cable slots and how it affects the radiation direction and the power levels in the corridor.
- To assess if the waveguide model combined with mode matching and cascade coupling is a viable model for prediction of wave propagation in indoor environments.
- To simulate an office corridor where the power levels in the rooms and in the corridors are compared with measurements. Moreover, to explore propagation characteristics in the time and frequency domain where the following sources and source positions are modeled.
  - Dipole antenna in corridor
  - Dipole antenna in room
  - Leaky cable in corridor

1.5 Thesis Outline

In chapter 2 a background of the theories this thesis is based on is presented. The waveguide model, mode matching and cascade coupling of S-matrices are discussed as well as excitation of waveguides.

The model this thesis proposes is presented in chapter 3. A corridor model is constructed by cascade coupling different corridor sections, referred to as corridor segments. Each corridor segment consists of waveguides with three different cross-sections where one of the waveguides contains a dielectric medium. The electric conductivity and relative permittivity of the medium are changed to simulate different wall materials between the rooms in an office corridor. A full wave numerical solution, made in HFSS [12], is used to verify the analytical calculations made in the corridor model. Furthermore, a leaky cable model is constructed by multiple point sources that are amplitude and phase shifted. Finally, to analyze the propagation characteristics in the time domain the inverse discreet Fourier transform is applied to the time harmonic electromagnetic field distribution.

In chapter 4 a summary of the measurement campaign is given and the corridor model is used to simulate the office corridor with adjacent rooms where the measurements were conducted. The simulations are compared with the measurement data and the results are analyzed.
Finally, in the last chapter conclusions, a summary and suggestions of future work is presented.

2 The Waveguide Model in 2D

Waveguides are normally used to transfer signal power from one point to another with only small radiation losses. This is usually done for wave propagation at higher microwave frequencies, typically at wavelengths below 10 cm [13]. At higher frequencies a waveguide has better electrical and mechanical properties than the normally used transmission line, which results in lower transmission losses. The most common waveguides are the rectangular metallic waveguide and the circular dielectric waveguide, an example of the latter is the optical fiber.

2.1 Rectangular Metallic Waveguides

Maxwell’s equations describe the electric and the magnetic fields, denoted \( E \) and \( H \) respectively, in terms of charge and current densities for all electromagnetic phenomena. The corridor model proposed in this thesis focuses on time harmonic wave propagation, primarily at the frequencies 2.40-2.48 GHz. To theoretically model time harmonic wave propagation in waveguides, equation (2.1) and (2.2) are used to derive Helmholtz equation (2.5) for the electromagnetic fields [13]. Helmholtz equation can be solved analytically for the specific geometry a waveguide constitutes. Maxwell’s equations for a linear, isotropic, homogeneous and non-dispersive media on differential form are

\[
\nabla \times E(r, \omega) = -j \omega \mu H(r, \omega) \tag{2.1}
\]
\[
\nabla \times H(r, \omega) = j \omega \varepsilon E(r, \omega) \tag{2.2}
\]
\[
\nabla \cdot E(r, \omega) = \frac{\rho(r)}{\varepsilon} \tag{2.3}
\]
\[
\nabla \cdot H(r, \omega) = 0 \tag{2.4}
\]

and Helmholtz equation for the electric field in a source free region is

\[
\nabla^2 E(r, \omega) + k^2 E(r, \omega) = 0 \tag{2.5}
\]

where \( k \) is the wave number. In free-space \( k \) is real and is given by (2.6), where \( c_0 \) is the speed of light in vacuum, \( \omega \) is the angular frequency and \( \lambda \) is the free-space wavelength. If the medium in which the wave propagates is dielectric, \( k \) is given by (2.7) where \( \sigma \) is the conductivity and \( \varepsilon_r \) is the relative permittivity of the medium. As no magnetic materials are modeled in this thesis the permeability of free-space is always used.

\[
k_0 = \frac{\omega}{c_0} = \frac{2\pi}{\lambda} \tag{2.6}
\]
\[ k = k_0 \sqrt{\left( \varepsilon_r - j \frac{\sigma}{\omega \varepsilon_0} \right)} \]  \hspace{1cm} (2.7)

Separation of variables is used, in standard waveguide theory, to solve Helmholtz equation for the different spatial coordinates [14]. A fixed direction of propagation is defined along \( z \) and a solution for the \( z \) variable is derived as one wave propagating in the positive \( z \)-direction and one in the negative \( z \)-direction. The \( z \)-dependence for the respective waves is \( e^{-jkzmz} \) and \( e^{jkzmz} \). The variable \( k_{zm} \) is the longitudinal wave number which is given by \( \beta_m - j\alpha_m \) and the following convention for the time dependence is used: \( e^{j\omega t} \).

The transversal field-components are determined from the \( z \)-components of the electromagnetic fields, \( E_z \) and \( H_z \). Therefore, the separated Helmholtz equation only needs to be solved for these two scalar quantities. The differential equations that needs to be solved are

\[ \left( \frac{\partial}{\partial x} + k_{xm}^2 \right) \left\{ H_z \right\} = 0 \]  \hspace{1cm} (2.8)

The solutions are divided into two cases, transverse electric (TE), where \( E_z = 0 \) and transverse magnetic (TM), where \( H_z = 0 \). The boundary conditions at the waveguide walls, \( x=0 \) and \( x=a \) (see Figure 2.1), have to be satisfied to determine the complete expression for the transversal dependencies of the field-components.

For a waveguide with dielectric walls the boundary conditions leads to a transcendental equation for the transverse wave number \( k_{xm} \) which has to be solved numerically. The complexity of the equation depends on if the walls are modeled as infinitely thick or with a finite width, in [7] an analysis of dielectric walls are examined in more detail. For a rectangular metallic waveguide with uniform cross-section, where the walls are treated as perfect electrical conductors (PEC), the transversal boundary conditions are solved analytically [14]. In this thesis the waveguide walls are modeled as perfectly conducting.

Solving the differential equation with PEC walls result in the following solutions
\[ H_z \propto \cos(k_{xm} x), \quad 0 < x < a \]
\[ E_z \propto \sin(k_{xm} x), \quad 0 < x < a \]  
(2.9)

where \( k_{xm} \) is the transversal wave number given by

\[ k_{xm} = \frac{m \pi}{a}, \quad m = 1, 2 \ldots \infty \]  
(2.10)

An infinite number of solutions exist where each solution is referred to as a waveguide mode. From (2.9) the transversal dependencies can be derived as

\[ E_y = \varphi_m(x) = \frac{2}{a} \sin(k_{xm} x), \quad 0 < x < a \]  
(2.11)

\[ H_y = \varphi_m(x) = \frac{2 - \delta_{m,0}}{a} \cos(k_{xm} x) \]

The expansions in (2.11) forms a set of basis functions [13] that are complete and orthogonal over the cross-section \( \Gamma \) in Figure 2.1 and have been normalized according to

\[ (\varphi_m(x)|\varphi_{m'}(x))_R = \int_\Gamma \varphi_m(x)\varphi_{m'}(x)dx = \delta_{mm'}, \quad \delta_{mm'} = \begin{cases} 0 & \text{if } m \neq m' \\ 1 & \text{if } m = m' \end{cases} \]  
(2.12)

This result is utilized to simplify the analytical calculations used in the mode matching method described in the next section.

The total electromagnetic field-expansions for the TE modes are given below (The TM modes can be expressed similarly with \( E_z \neq 0 \)).

\[ E^\pm(r) = \sum_{m=1}^{\infty} E_{tm}^{\pm} e^{\pm ik_{xm}z} \]  
(2.13)

\[ H^\pm(r) = \sum_{m=1}^{\infty} (\pm H_{tm}^{\pm}(x) + 2H_{zm}(x)) e^{\pm ik_{xm}z} \]  
(2.14)

where \( E_{tm}^{\pm}(x) \) and \( H_{tm}^{\pm}(x) \) are the transversal electromagnetic field-components and the transversal electric field is given by

\[ E_{tm}^{\varphi}(x) = \bar{\psi}E_{ym}^{\varphi}(x) = \bar{\psi}\varphi_m(x) = \bar{\psi} \frac{2}{a} \sin(k_{xm} x), \quad m = 1, 2 \ldots \infty \]  
(2.15)

For completeness, the transversal magnetic field expansion for the TM modes is given by

\[ H_{tm}^{\varphi}(x) = \bar{\psi}H_{ym}^{\varphi}(x) = \bar{\psi}\varphi_m(x) = \bar{\psi} \frac{2}{a} \cos(k_{xm} x), \quad m = 0, 1, 2 \ldots \infty \]  
(2.16)

The transversal fields for the TE and TM modes are related as
\( \mathbf{H}_i^{\text{TE}}(x) = Y^{\text{TE}} \mathbf{\hat{z}} \times \mathbf{E}_i^{\text{TE}}(x) \)  
\( \mathbf{E}_i^{\text{TM}}(x) = -Z^{\text{TM}} \mathbf{\hat{z}} \times \mathbf{H}_i^{\text{TM}}(x) \)

where \( Y^{\text{TE}} \) and \( Z^{\text{TM}} \) are the mode admittance and impedance respectively, given by

\[
Y^{\text{TE}} = \frac{1}{Z^{\text{TE}}} = \frac{k_z}{k \eta} 
\]

\[
Y^{\text{TM}} = \frac{1}{Z^{\text{TM}}} = \frac{\eta}{k_z \eta} 
\]

and \( \eta \), the wave impedance is

\[
\eta = \sqrt{\frac{\mu_0}{\varepsilon}} = \frac{\omega \mu_0}{k}.
\]

The model presented here is two dimensional. Hence, the electromagnetic field-components have no \( y \)-dependence. In three dimensions there are two mode indices, \( m \) and \( n \), one for each transverse direction. In two dimensions, the only mode included in the \( y \)-direction is the first, \( n=0 \). Therefore \( k_{xm} \) is actually \( k_{x0} \) but the former expression is used throughout this thesis.

The longitudinal wave number \( k_{zm} \), calculated from \( k \) and \( k_{xm} \), is given by

\[
k_{zm} = \sqrt{k - k_{xm}} = \beta_m - j\alpha_m
\]

For wave propagation in a lossless medium, the longitudinal wave number is either real or imaginary. Therefore, the modes are divided into two groups, propagating modes and non-propagating modes (evanescent modes). For wave propagation in the positive \( z \)-direction (\( e^{-j[k_{zm}z]} \)), a real value of \( k_{zm} \) results in a propagating wave (\( e^{-j\beta_m z} \)) and an imaginary value of \( k_{zm} \) results in a quickly attenuated mode (\( e^{-\alpha_m z} \)).

When the medium is dielectric with a finite conductivity and permittivity, \( k \) is given by (2.7) and \( k_{zm} \) has both a real and an imaginary part for every mode. The imaginary part models the conduction losses in the dielectric medium. In this thesis a dielectric medium is used to model the walls in between rooms, the corridor and rooms are modeled as free-space waveguides.

The wavelength of the propagating wave as well as the cross section of the waveguide determines the number of propagating modes. Waveguides are normally constructed so that only the dominant mode propagates, where the cross section of the waveguide is dimensioned after the intended frequency usage.

When modeling a corridor as a waveguide the cross section is already specified by the environment and the frequencies are specified by the frequency bands used in mobile communication. This results in a large number of propagating modes in a normally sized corridor. For example, in the simulations made herein the cross-section of the widest waveguide is 5.55 meter and the frequency is 2.44 GHz (corresponding to a wavelength of 12.3 cm) which leads to 90 propagating modes. To calculate how modes are coupled at waveguide transitions and intersections of different media the mode matching method, explained in section 2.2, is implemented.
The Poynting vector determines the energy flow in the waveguide [14], and it can be shown that only the transversal field components contribute to the power transport. Furthermore, in the corridor model developed in this thesis the waveguides are excited by a vertical current source (y-direction). In a 2D model such a source excites TE modes which have a transversal electric field component in the y-direction only. Hence, the power distribution in the corridor is completely determined by the y-component of the electric field. However, the expansion for the transversal magnetic field is necessary when the mode coefficients, described below, are to be determined.

The transversal electromagnetic fields for the TE modes, expanded as [15], are given by

\[ E_y(x, z) = \sum_{m=1}^{\infty} \left[ c_m^+ e^{-jkmz} + c_m^- e^{jkmz} \right] \varphi_m(x), \]  

(2.23)

\[ H_x(x, z) = \sum_{m=1}^{\infty} \left[ c_m^+ e^{-jkmz} - c_m^- e^{jkmz} \right] Y_m^{TE} \varphi_m(x), \]  

(2.24)

where \( c_m^\pm \) are the mode coefficients which are determined from the excitation of the waveguide. Because the \( z^- \) and \( x^- \) dependence is known, the mode coefficients only need to be determined for a specific \( z^- \) position. The total electromagnetic field is then given by (2.23) and (2.24).

The fields for the TM modes are expanded similarly; depending on the excitation both TE and TM modes may propagate. Excitation of waveguides is discussed in section 2.4.

### 2.2 The Mode Matching Method

As explained in the section 2.1 a large number of modes are excited in a corridor due to the short wavelength used in wireless communication and the large cross-section of the corridor. At intersections of waveguides with different cross sections or media, each mode is backscattered and transmitted differently. To determine the backscattered and transmitted field for a large number of propagating modes, mode matching is implemented.

The mode matching method [15] is used for analyzing transitions between waveguides with different cross section as well as intersections of waveguides with different media. The electromagnetic fields are expanded in waveguide modes in each region. At the waveguide intersection the electromagnetic boundary conditions are applied to connect the field expansions for the different regions. From the resulting equations an S-matrix is determined. The S-matrix relates the backscattered and transmitted amplitude of each mode coefficient at the output of a region to the amplitudes of the mode coefficient at the input of the region. A general procedure for computing the S-matrix is presented here.

In Figure 2.2 a uniform waveguide is depicted. Region 2,2 is filled with a homogeneous dielectric medium with a finite conductivity and permittivity. This is an example of an intersection between different media where the waveguide modes in region 2,1 and 2,3 are partially backscattered and partially transmitted, both at \( z_1 \) and \( z_2 \).
The mode coefficients for the field expansions in the different regions are collected into vectors as

\[
\mathbf{c}^{+}_{(i)}(z_i) = \begin{bmatrix}
    c^{(i)+}_{2}(z_i) \\
    c^{(i)+}_{2}(z_i) \\
    \vdots \\
    c^{(i)+}_{M}(z_i)
\end{bmatrix}
\]

(2.25)

where \( i \) is the index for the region in which the mode coefficient is valid, \( l \) is the index specifying the \( z \) boundary position and \( M \) is the number of mode coefficients. Note that only the TE mode coefficients have been included in (2.25).

The S-matrix relation for the depicted intersection is

\[
\begin{bmatrix}
    \mathbf{c}_{(2,1)}(z_1) \\
    \mathbf{c}_{(2,3)}(z_2)
\end{bmatrix} = \mathbf{S}^{(w)} \begin{bmatrix}
    \mathbf{c}_{(2,1)}(z_1) \\
    \mathbf{c}_{(2,3)}(z_2)
\end{bmatrix} = \begin{bmatrix}
    \mathbf{S}_{11}^{(w)} & \mathbf{S}_{12}^{(w)} \\
    \mathbf{S}_{21}^{(w)} & \mathbf{S}_{22}^{(w)}
\end{bmatrix} \begin{bmatrix}
    \mathbf{c}_{(2,1)}(z_1) \\
    \mathbf{c}_{(2,3)}(z_2)
\end{bmatrix}
\]

(2.26)

or on equation form

\[
\begin{align*}
    \mathbf{c}_{(2,1)}(z_1) &= \mathbf{S}_{11}^{(w)} \mathbf{c}_{(2,1)}(z_1) + \mathbf{S}_{12}^{(w)} \mathbf{c}_{(2,3)}(z_2) \\
    \mathbf{c}_{(2,3)}(z_2) &= \mathbf{S}_{21}^{(w)} \mathbf{c}_{(2,1)}(z_1) + \mathbf{S}_{22}^{(w)} \mathbf{c}_{(2,3)}(z_2).
\end{align*}
\]

(2.27)

where \( \mathbf{S}^{(w)} \) is the scattering matrix. The part-matrix \( \mathbf{S}_{11}^{(w)} \) gives the mode coefficients of the backscattered field at \( z_1 \) and \( \mathbf{S}_{21}^{(w)} \) gives the mode coefficients of the transmitted field from \( z_1 \) to \( z_2 \), and likewise in the other direction for the other two part-matrices.

The electromagnetic fields are expanded in each region as in (2.23) and (2.24). At the media boundaries \( z_1 \) and \( z_2 \) the electromagnetic boundary conditions, described in detail in [13],
states that the transversal fields have to be continuous, resulting in the following systems of

equations.

\[ E_{t}^{(2,1)}(x, z_1) = E_{t}^{(2,2)}(x, z_1) \] (2.28)

\[ H_{t}^{(2,1)}(x, z_1) = H_{t}^{(2,2)}(x, z_1) \] (2.29)

\[ E_{t}^{(2,2)}(x, z_2) = E_{t}^{(2,3)}(x, z_2) \] (2.30)

\[ H_{t}^{(2,2)}(x, z_2) = H_{t}^{(2,3)}(x, z_2) \] (2.31)

where \((2, i)\) for \(i = 1, 2, 3\) denote the field expansions in the different regions

To reduce this system of equations the orthogonality of the modes is utilized. Equation (2.28)
is taken as an example. The expansion for the electric fields in the two regions is inserted and
both sides of the equation are multiplied with \(\varphi_{m_{i}}(x)\) for a specific mode index \(m_{i}\) resulting in

\[
\sum_{m=1}^{\infty} \left[ c_m^{(2,1)} e^{-j k_{zm}^{(2,1)} z} + c_m^{(2,1)} e^{j k_{zm}^{(2,1)} z} \right] \varphi_{m_{i}}(x) \varphi_{m}(x) = \sum_{m=1}^{\infty} \left[ c_m^{(2,2)} e^{-j k_{zm}^{(2,2)} z} + c_m^{(2,2)} e^{j k_{zm}^{(2,2)} z} \right] \varphi_{m_{i}}(x) \varphi_{m}(x)
\] (2.32)

Both sides of the equation are integrated over the cross section of the waveguide as

\[
\int_{0}^{a} \sum_{m=1}^{\infty} \left[ c_m^{(2,1)} e^{-j k_{zm}^{(2,1)} z} + c_m^{(2,1)} e^{j k_{zm}^{(2,1)} z} \right] \varphi_{m_{i}}(x) \varphi_{m}(x) dx = \int_{0}^{a} \sum_{m=1}^{\infty} \left[ c_m^{(2,2)} e^{-j k_{zm}^{(2,2)} z} + c_m^{(2,2)} e^{j k_{zm}^{(2,2)} z} \right] \varphi_{m_{i}}(x) \varphi_{m}(x) dx
\] (2.33)

The orthogonality relation in (2.12) is applied resulting in

\[
c_{m_{i}}^{(2,1)} e^{-j k_{zm_{i}}^{(2,1)} z} + c_{m_{i}}^{(2,1)} e^{j k_{zm_{i}}^{(2,1)} z} = c_{m_{i}}^{(2,2)} e^{-j k_{zm_{i}}^{(2,2)} z} + c_{m_{i}}^{(2,2)} e^{j k_{zm_{i}}^{(2,2)} z}
\] (2.34)

When using mode matching at the transition between waveguides with different cross section,
the modes in the different regions will couple into each other. How the modes are coupled is
described by integrals in the equations for the boundary conditions. For the waveguide in
Figure 2.2 the integrals are completely solved when the orthogonality condition is applied,
this is not the case when the regions have different cross section.

The mode coupling integrals for TE modes are given by

\[
\int_{0}^{a} \varphi_{m_{i}}^{(1)}(x) \varphi_{m_{j}}^{(3)}(x) dx = \int_{0}^{a} \sin \left( \frac{m_{i} \pi}{a_{i}} x \right) \sin \left( \frac{m_{j} \pi}{a_{j}} x \right) dx
\] (2.35)
where $\varphi_{m_i}^{(1)}$ and $\varphi_{m_i}^{(3)}$ are the basis functions for the two waveguides with different cross section. The parameters $m_i$ and $m_j$ are the number of modes included in each waveguide region. This integral is solved analytically in appendix 6.2.7.

After orthogonalization the remaining equations still consist of an infinite sum that has to be truncated to a limited number of modes for the numerical treatment. It is discussed in section 3.6 how many modes need to be included. After truncation of the modes the remaining equations are solved analytically to determine $S^{(w)}$. The mode coefficients of the wave propagating towards region 2,2 is determined from the excitation of the waveguide. The size of the total S-matrix ($S^{(w)}$) is $2M \times 2M$ and the size of the part-matrices is $M \times M$, where $M$ is the number of modes included in the truncation.

2.3 Cascade-Coupling

When an S-matrix has been determined for a specific region of a waveguide, it describes how the waveguide modes are scattered at the boundaries of that particular region. If a waveguide is divided into more regions with different geometrical properties, a specific S-matrix can be computed independently for each region. These S-matrices can be cascade-coupled [15] together to compute an overall S-matrix for the entire waveguide (see Figure 2.3).

The process of coupling the S-matrices together can be decoupled from the process of determining the S-matrix for each waveguide region. If the parameters of a specific waveguide region need to be changed, then only the S-matrix for that region and the total S-matrix need to be recalculated, saving computational resources. In this thesis, different waveguide regions, or corridor segments, are cascade-coupled together to model a corridor in an office building. For each corridor segment an S-matrix is computed, these are then cascade-coupled into one total S-matrix.

One advantage of this approach is that a real office corridor with rooms of different sizes and corridors of different widths can be modeled more accurately. Even walls between rooms can be modeled with different thickness and material properties along the corridor.
Cascade-coupling of two S-matrices is straightforward and is presented in [15]. If the elements of the S-matrices for the two different waveguide regions are

\[ S^{(1)} = \begin{pmatrix} S_{11}^{(1)} & S_{12}^{(1)} \\ S_{21}^{(1)} & S_{22}^{(1)} \end{pmatrix} \quad \text{and} \quad S^{(2)} = \begin{pmatrix} S_{11}^{(2)} & S_{12}^{(2)} \\ S_{21}^{(2)} & S_{22}^{(2)} \end{pmatrix} \]

these S-matrices are cascade-coupled to a total S-matrix given by

\[ S^{(T)} = \begin{pmatrix} S_{11}^{(T)} & S_{12}^{(T)} \\ S_{21}^{(T)} & S_{22}^{(T)} \end{pmatrix} \]

where the elements of the cascade-coupled matrix are

\[ S_{11}^{(T)} = S_{11}^{(1)} + S_{12}^{(1)} \left( I - S_{11}^{(2)} S_{22}^{(1)} \right)^{-1} S_{11}^{(2)} S_{21}^{(1)} \]

\[ S_{12}^{(T)} = S_{12}^{(1)} \left( I - S_{11}^{(2)} S_{22}^{(1)} \right)^{-1} S_{12}^{(2)} \]

\[ S_{21}^{(T)} = S_{21}^{(2)} \left( I - S_{22}^{(1)} S_{11}^{(1)} \right)^{-1} S_{21}^{(1)} \]

\[ S_{22}^{(T)} = S_{22}^{(2)} + S_{21}^{(2)} \left( I - S_{22}^{(1)} S_{11}^{(1)} \right)^{-1} S_{22}^{(1)} S_{12}^{(2)} \]

and \( I \) is the unit matrix.
2.4 Waveguide Excitation

The field expansions in (2.23) and (2.24) are only valid in a region where there are no sources as they are derived from the source-free Helmholtz equation. Therefore, a waveguide region is divided into different sections to separate sections where sources are present from the sections where the field expansions are valid. To determine which modes are excited in the source-free sections the field expansions need to be related to the sources in some way. A general relation between the mode coefficients outside a section with a localized current source (see Figure 2.4) is derived in [14]. The excited mode coefficients are determined by

\[ s^\pm_m = -\frac{1}{2Y^\text{EX}_m} \int_A \mathbf{J} \cdot \mathbf{E}_m^\mp dA \]  

(2.42)

where \( \mathbf{J} \) is the current distribution, \( dA \) the cross section of the waveguide and \( Y^\text{EX}_m \) the mode admittance for the TE and TM modes respectively. In the region \( z > z_2 \) the current source will only excite modes that propagate in the positive z-direction, represented by the mode coefficients \( s^+_m \). Likewise, in the region \( z < z_1 \) the current source will only excite modes that propagate in the negative z-direction, represented by the mode coefficients \( s^-_m \). It is important to note that the mode coefficients are only valid for the field expansions outside of the area \( A \).

\( Y^\text{EX}_m \) is defined in (2.19) - (2.20) and \( \mathbf{E}_m^\mp \) is given by

\[ \mathbf{E}_m^\mp = (\mathbf{E}_{tm}(x) \mp 2\mathbf{E}_{zm}(x))e^{\mp ik_{zm}z}. \]  

(2.43)

where \( \mathbf{E}_{tm} \) represents the transversal electric field for both TE and TM modes.

In the measurements made in this thesis, (described in section 4.2) a dipole antenna and a leaky cable are used. The dipole measurements are primarily made with vertical polarization.
To model the dipole a point source is used, polarized in the \( y \)-direction, with the following current distribution

\[
J(x, z) = \hat{y} f(x, z) = \hat{y} \delta(x - x_0) \delta(z - z_0).
\] (2.44)

If a three-dimensional view is taken the point source is an infinite line source with uniform current aligned in the \( y \)-direction. In a three-dimensional waveguide model a dipole antenna could be modeled as a line current of finite length in the \( y \)-direction with a non-uniform current distribution, this would lead to the excitation of both TE and TM modes [13].

From equation (2.42) it is found that if the current source is aligned in the \( y \)-direction the \( x \)- and \( z \)-components of the electric field doesn’t contribute to the excited mode coefficients. In a two-dimensional waveguide model the transverse electric field only have a \( y \)-component for the TE modes and only an \( x \)-component for the TM modes [13], therefore only TE modes are excited.

When the current distribution, in Figure 2.4 is modeled as a point source (positioned at \( x = x_0 \) and \( z = z_0 \)) the area \( A \) are reduced to a line positioned at \( z = z_0 \). Because of this the field expansions in the sections outside of \( A \) are valid is the whole waveguide region.

A more compact analytical expression of (2.42) is given below, where the current distribution in (2.43) and the electric field in (2.44) have been utilized as well as \( E_{lm}(x) \) from (2.15). The integrals are solved and the resulting expression is

\[
s_m^+(z) = -\frac{1}{y_m^{TE} \sqrt{2a}} \sin(k_{xm} x_0) e^{-i k_{xm} |x - x_0|} \] (2.45)

where \( s_m^+ \) and \( s_m^- \) are the excited modes for \( z > z_0 \) and \( z < z_0 \) respectively and \( y_m^{TE} \) is given by (2.19).

## 3 Corridor Model with Adjacent Rooms

### 3.1 Background

As mentioned in the introduction, measurements and simulations have shown that corridors have a waveguiding effect. Due to this the waveguide model has been considered as a model for indoor propagation. One indoor propagation model based on waveguiding is proposed in [7] where straight corridor propagation is combined with a method for electromagnetic field coupling at corridor intersections. The corridors are modeled in two dimensions as waveguides with uniform cross sections and the walls are modeled as dielectric with finite thickness. To determine the field coupling at corridor intersections FDTD is used. All simulations are conducted at 900 MHz with a point source as the excitation.

Two cases are primarily considered, the corridor walls modeled as strongly reflecting and weakly reflecting, with a wall conductivity of 0.2 S/m in the former case and 0.01 S/m in the latter. The higher conductivity is set to model highly reflective wall materials such as...
reinforced concrete or brick while the lower conductivity is set to model more transparent wall materials such as plaster board or wood.

Common discontinuities such as doorways and reflecting objects inside adjacent rooms are also included in the model and neither of them have a large impact on the field distribution. It is discovered that the dominant low order modes propagate relatively unaffected by small discontinuities in the environment. It is important to note that the simulations are conducted at 900 MHz, at higher frequencies these discontinuities may have a larger impact on the field distribution.

A full FDTD numerical solution is also implemented to verify the corridor model. The FDTD simulation is made of an entire office floor and for the case with strongly reflective walls the corridors display a strong waveguiding effect. For the case with weakly reflective walls the waveguiding effect is still strong although the wave propagation through walls has increased. In [6] numerical simulations of an office floor using S-MRTD display similar results. In this model two different wall conductivities are also simulated, 0.05 S/m and 0.002 S/m, representing strongly and weakly reflecting walls.

The primary strength of the waveguide model discussed so far is that the corridor walls are modeled as dielectric with a finite thickness. However the corridor is modeled as one waveguide with uniform cross section and the effect the adjacent rooms have on the field distribution is not included. As the rooms are not included in the model the wave propagation from room to room is not analyzed.

### 3.2 Model Overview

In this thesis, mode matching and cascade coupling are implemented, attempting to predict the electromagnetic field distribution in corridors as well as in the adjacent rooms. The corridor and rooms are modeled as rectangular metallic waveguides filled with a homogeneous and isotropic material, because of this the methods presented in section 2.1 can be applied. To model more than one room adjacent to a corridor, waveguides with different cross sections are used.

The walls in between rooms are modeled as waveguides filled with a dielectric medium, these regions can also be extended to model doors or door openings perpendicular to the corridor. By using mode matching at boundaries between waveguide transitions the electromagnetic fields in a small corridor segment is determined. Multiple corridor segments are then connected together by using cascade coupling of S-matrices to model a long corridor with many rooms. An example of two corridor segments which are cascade coupled to model one room is depicted in Figure 3.1.
An office generally has rooms with different sizes and sometimes corridors with different width. In addition the walls between rooms are typically built with the same materials but some variations do occur, for example a whiteboard hanging on the wall in one room will change the effect that room has on the wave propagation. As the corridor model proposed in this thesis is constructed with many different segments, such changes in corridor geometry are possible to account for. The mode matching method combined with cascade coupling allow for every corridor segment to be constructed with different geometric and dielectric parameters to a certain extent. In Figure 3.1 two examples of corridor segments with different geometries and parameters for the dielectric wall is illustrated.

The external building walls are modeled as PEC and so are the walls in between the main corridor and the rooms. The doorway openings in between the main corridor and the rooms are modeled as openings in the PEC wall.

### 3.3 One Corridor Segment

In Figure 3.2 an enlarged corridor segment is depicted. Region 1 represent the corridor and region 2 a partial room. Region 2 is divided into 3 sub-regions where region 2,2 models the wall in between two rooms. When two corridor segments are cascade coupled together a complete room is modeled as illustrated in Figure 3.1 in the previous section. Region 3 and 4 are part corridor and part room, their function is to connect different corridor segments together and to model differently sized door openings. Mode matching is used at the dielectric wall and at the transition between waveguides with different cross section, detailed calculations of this is presented in appendix 6.1 and 6.2.
One corridor segment consists of waveguides with three different cross sections. The transversal electromagnetic field dependencies are different for each waveguide cross section and are described by the following basis functions

\[
\varphi_m^{(1)}(x) = \sqrt{\frac{2}{a_1}} \sin\left(\frac{m\pi}{a_1} x\right), \quad 0 < x < a_1
\]

\[
\varphi_m^{(2)}(x) = \sqrt{\frac{2}{a_2}} \sin\left(\frac{m\pi}{a_2} (a_0 - x)\right), \quad a_0 - a_2 < x < a_0
\]

\[
\varphi_m^{(3)}(x) = \sqrt{\frac{2}{a_0}} \sin\left(\frac{m\pi}{a_0} x\right), \quad 0 < x < a_0
\]

where \(\varphi_m^{(1)}(x)\) is the mode expansion used in region 1, \(\varphi_m^{(2)}(x)\) in region 2, \(i = 1, 2, 3\) and \(\varphi_m^{(3)}(x)\) in region 3 and 4. The basis functions have been orthonormalised according to equation (2.12).

Only the TE modes are included in the calculations. No TM modes are excited when a vertically polarized point source is utilized to excite the waveguide modes. The transversal electric field is aligned in the \(y\)-direction and the transversal magnetic field in the \(x\)-direction. TM modes can be implemented analogously. In each region the transversal fields are expanded as (see appendix 6.1 and 6.2 for complete expressions in each region)
\[ E^{(i)}(x, z) = \sum_{m=1}^{\infty} \left[ e_m^{(i)+}(z) + e_m^{(i)-}(z) \right] \varphi_m^{(i)}(x), \quad (3.4) \]

\[ H^{(i)}(x, z) = \sum_{m=1}^{\infty} \left[ e_m^{(i)+}(z) - e_m^{(i)-}(z) \right] \gamma_m^{(i)\text{TE}} \varphi_m^{(i)}(x), \quad (3.5) \]

where the z-dependence have been included in the mode coefficients as

\[ e_m^{(i)\pm}(z) = e_m^{(i)\pm} e^{\pm jk_{zm} z} \quad (3.6) \]

It is only necessary to determine the mode coefficients for a specific z-position in each waveguide region. The propagation through that region is then described by \( e^{\pm jk_{zm} z} \) where the longitudinal wave number for each mode is determined by

\[ k_{zm}^{(i)} = \sqrt{k - \frac{mn}{a_i}}, \quad m = 1, 2, \ldots, \infty \quad (3.7) \]

and the wave number \( k \) is given by \( k = \frac{\omega}{c_0} \) except in the dielectric wall where \( k \) is given by

\[ k_w = k = k_0 \sqrt{\left( \varepsilon_w - \frac{\sigma_w}{\omega \varepsilon_0} \right)} \quad (3.8) \]

where \( \varepsilon_w \) is the relative permittivity and \( \sigma_w \) the conductivity of the dielectric wall.

As an illustration of how the TE modes propagate through the different regions in one corridor segment the magnitude of the electric field (in dB) for the first two modes are shown separately in Figure 3.3. The frequency used is 100 MHz which corresponds to a free space wavelength of 3 m, the geometry of the corridor segment is shown in the figure. The waveguide is excited by a source positioned at \( z=0 \) and \( x=1.28 \).

The amplitude of the modes which are excited in a waveguide depends on where the source is positioned in relation to the maxima or minima of each mode, over the waveguide cross section. Because of the internal scatterers in one corridor segment, more modes than one are excited no matter where the source is positioned. However, how much each mode is excited depends on the source position. The first mode has a maximum in the middle of the waveguide while the second mode is zero at that \( x \)-coordinate and has a maximum at \( x = \frac{a_0}{4} \) and \( x = \frac{3a_0}{4} \) (in region 3 and 4). For the first mode to be the dominant one, the source should be positioned close to the middle of the waveguide. For the second mode to be dominant the source should be positioned close to \( x = \frac{a_0}{4} \) or \( x = \frac{3a_0}{4} \).

In the example two modes propagate in region three and four and based on the position of the source the second TE mode should be the dominating one which is confirmed in the figure. Only the first mode \((m=1)\) propagates in region one and two due to the smaller cross section and it can be seen how the electric field attenuates for the first evanescent mode \((m=2)\) in these regions.
The reason for the discontinuity at $z_1$ and $z_2$ is that only one mode is plotted at a time. The mode matching is done for multiple modes and all of them need to be included for the solution to converge. In Figure 3.4 all the propagating modes have been included and enough evanescent modes for the solution to converge, in total: 22 modes in region three and four, 10 modes in region one and 12 modes in region two. The number of modes that need to be included in the numerical treatment is discussed in section 3.6.

Figure 3.3: Magnitude of the electric field (in dB) for the first two modes, $m=1$ and $m=2$, in one corridor segment. $f=100$ MHz, $\varepsilon_w = 3$, $\sigma_w = 0.1$ S/m and the waveguide is excited by a source positioned at $z=0$ and $x=1.28$.

Figure 3.4: Total electric field for the corridor segment in Figure 3.3. 22 modes (20 evanescent) are included in region 3 and 4, 10 modes (9 evanescent) in region 1 and 12 modes (11 evanescent) in region 2.

3.4 Sources

To model a dipole antenna in rooms as well as in corridors, point sources are inserted in region 1, 3 and 4. To model a leaky cable multiple point sources are positioned after each other in the waveguide regions representing the corridor. It is considered to be sufficient to have sources in these three regions as all the desired antenna positions can be modeled. Although no sources are introduced into region 2, an S-matrix relating the mode coefficients for that region have to be determined. In appendix 6.1 this S-matrix is calculated. In Figure 3.5 the mode coefficients excited by the sources in each region is illustrated. In section 2.4 the theory for waveguide excitation is presented in more detail.
Only the TE modes are excited from a point source aligned in the y-direction, and an analytical expression for the source coefficients, on vector form, is

\[
\mathbf{s}^\pm(z) = -\frac{1}{Y_{\text{TE}}\sqrt{2a}} \sin(k_x x_0) e^{-jk_z z-z_0} \tag{3.9}
\]

where \(s^+(z)\) and \(s^-(z)\) are the excited modes for \(z > z_0\) and \(z < z_0\) in their respective region. The mode admittance \(Y_{\text{TE}}\) is given by (2.19).

In each region the mode coefficients are divided into one homogeneous part and one part representing the contribution from the sources. The medium in the waveguides is linear therefore superposition applies and the coefficients can be determined separately. The total mode coefficients in each region are given by

\[
\mathbf{c}^\pm(z) = \mathbf{h}^\pm(z) + \mathbf{s}^\pm(z) \tag{3.10}
\]

The source coefficients are known in their respective region from (3.9) and the homogeneous mode coefficients are determined from them. This is done on the external boundaries, \(z_v\) and \(z_h\), to determine an S-matrix for the entire corridor segment. For detailed calculations see appendix 6.2 where the source coefficients are included in the boundary conditions for the mode matching analysis. The result is the following expression for the total backscattered and transmitted mode coefficients at the external boundaries.

**Figure 3.5:** One corridor segment with source coefficients and which sub-region they are valid in.
\[
\begin{pmatrix}
 c^- (z_{h0}) \\
 c^+ (z_h)
\end{pmatrix}
= \begin{pmatrix}
 L \\
 S
\end{pmatrix}
\begin{pmatrix}
 h^+ (z_{h0}) \\
 h^- (z_h)
\end{pmatrix}
+ \begin{pmatrix}
 S^{(0)}_{(3)} (z_1) \\
 S^{(0)}_{(4)} (z_2)
\end{pmatrix}
+ \begin{pmatrix}
 S^{+ (1)} (z_2) \\
 S^{- (1)} (z_1)
\end{pmatrix}
+ \begin{pmatrix}
 S^{(3)} (z_{h0}) \\
 S^{(4)} (z_h)
\end{pmatrix}
\] (3.11)

where \( L \) and \( S \) are the notations used for an S-matrix relating homogeneous mode coefficients and source coefficients respectively. Each S-matrix in (3.11) is divided into part-matrices in accordance with standard scattering matrix theory. For example, the part-matrices for \( S^{(0)} \) is given by

\[
S^{(0)} = \begin{pmatrix}
 S_{11}^{(0)} & S_{12}^{(0)} \\
 S_{21}^{(0)} & S_{22}^{(0)}
\end{pmatrix}
\] (3.12)

The source coefficients \( s^{- (3)} (z_{h0}) \) and \( s^{+ (4)} (z_h) \) propagate directly to the external boundaries without internal scattering as illustrated in Figure 3.5. However, the other source coefficients are partially backscattered and transmitted at the internal boundaries, how they are coupled to the external boundaries is described by their respective part-matrix.

For example, \( s^{+ (3)} (z_1) \) are excited by the source in region 3 and propagate in the positive z-direction. Two part-matrices of \( S^{(0)} \) determine how \( s^{+ (3)} (z_1) \) affect the homogeneous mode coefficients at the external boundaries. The part-matrix \( S_{11}^{(0)} \) determines how much of \( s^{+ (3)} (z_1) \) is scattered back in the negative z-direction at \( z_{h0} \) and \( S_{21}^{(0)} \) determines how much of \( s^{+ (3)} (z_1) \) is transmitted in the positive z-direction at \( z_h \). The function of the other part-matrices may be explained similarly.

### 3.5 Multiple Corridor Segments

To determine the effect of each corridor segment on the electromagnetic field distribution, in the other corridor segments, they are cascade coupled together. By cascade coupling, the effect of scattering at waveguide transitions and dielectric walls in every corridor segment is included. Two corridor segments are cascade coupled into one larger cell, this cell is then cascade coupled with the third corridor segment and so on, to build up the entire corridor. The term \( \text{cell} \) is used for one or more cascade coupled corridor segments.

To determine the field distribution in every corridor segment the mode coefficients on the boundaries, \( z_{hn} \) of each corridor segment need to be found (see Figure 3.6). To do so the corridor segments are cascade coupled in each direction, from \( z_{h0} \) and to the right as well as from \( z_{hN} \) and to the left, where \( N \) is the total number of corridor segments. The term internal boundary is used for a transition between two corridor segments while the term external boundary refers to the outer boundaries at the first and last corridor segments.

To model a leaky cable, point sources have to be included in multiple corridor segments. This can be done although it leads to more complicated cascade coupling matrices than if sources are only included in one corridor segment. Therefore, the latter alternative is chosen and the total electromagnetic field distribution, from sources in different corridor segments, is determined by superposition. Sources are placed in one corridor segment at a time and the electromagnetic field distribution is determined for each source position, the solutions are then added together.
A simple illustration of four cascade coupled corridor segments are shown in Figure 3.6 with a source positioned in corridor segment 2. The mode coefficients on the external boundaries are determined by cascade coupling every corridor segment into one cell, and are given by

$$\begin{pmatrix} c^-(z_{h0}) \\ c^+(z_{h4}) \end{pmatrix} = L^{(V4)} \begin{pmatrix} c^+(z_{h0}) \\ c^-(z_{h4}) \end{pmatrix} + S^{(V4)} \begin{pmatrix} s^-(z_{h1}) \\ s^+(z_{h2}) \end{pmatrix}$$

(3.13)

where the contribution from the sources in one corridor segment have been collected into two vectors, described by equation (6.180) in appendix 6.2. The S-matrix $L^{(V4)}$ relates the inward propagating mode coefficients on the external boundaries and $S^{(V4)}$ relates the source coefficients from the corridor segment they are positioned in.

To find the mode coefficients on an internal boundary all the corridor segment to the left are cascade coupled into one cell and likewise for the corridor segments to the right. The mode coefficients on the internal boundaries are given by

$$\begin{pmatrix} c^-(z_{h_{n-1}}) \\ c^+(z_{h_{n+1}}) \end{pmatrix} = L^{(Mn)} \begin{pmatrix} c^+(z_{h_0}) \\ c^-(z_{h_4}) \end{pmatrix} + S^{(Mnm)} \begin{pmatrix} s^-(z_{h_1}) \\ s^+(z_{h_2}) \end{pmatrix}, \quad n = 1, 2, 3$$

(3.14)

Where the index $m$ in $S^{(Mnm)}$ indicate if the sources are to the left ($m=1$) or to the right ($m=2$) of the internal boundary as the matrix elements are different. Expressions of the elements for two cascade coupled cells are given in appendix 6.4.

An example of cascade coupled corridor segments is shown in Figure 3.7 and Figure 3.8. The magnitude of the electric field, excited by a point source at 2.44 GHz, is shown for two different source positions. The model consists of 17 corridor segments with varying geometry to model the differently sized rooms. All visible dielectric walls have the same conductivity and permittivity. Some dielectric walls are modeled with the same parameters as vacuum to create the rooms and walls with different cross sections, these walls are not visible in the figure. The variable $d_w$ defines the thickness of the dielectric walls.
3.6 Mode Truncation Analysis

In a waveguide, the electromagnetic fields are expanded in an infinite series of waveguide modes. These modes may be divided into a finite number of propagating modes and an infinite number of evanescent modes. In the numerical implementation of the model a limited number of modes have to be included. As there is a finite number of propagating modes in each waveguide region, one way would be to only include these modes and discard the rest. All evanescent modes will eventually be zero, at a certain distance from the source, as they attenuate quickly in the waveguide. How quickly they attenuate depend on the frequency, this is analyzed later in this section.

However, if the evanescent modes have not attenuated to insignificant amplitudes, some may still contribute to the scattered fields at a waveguide transition. A limited number of evanescent modes may therefore have to be included in the numerical treatment for the solution to be accurate. A few cases are here examined to illustrate how the evanescent modes affect the wave propagation.

The number of modes included in each waveguide region is denoted by $M_0$, $M_1$ and $M_2$ for the three different waveguide cross sections (see Figure 3.2). The number of modes in region 3 and 4 ($M_0$) are decided manually, how many to include is discussed further down in this section. The number of modes included in the other waveguide regions are then determined in proportion to their cross section as

$$M_1 = \frac{a_1}{a_0} M_0 \quad \text{and} \quad M_2 = \frac{a_2}{a_0} M_0 \quad (3.15)$$

This is done such that the sum of the modes included in region 1 and 2 is equal to the number of modes included in region 3 (and 4).
Boundary discontinuities at waveguide transitions are prominent at lower frequencies if only propagating modes are included. Large variances in the electric field distribution are also observed in some cases. To illustrate this, the magnitude of the electric field is determined in one corridor segment at the frequency 0.4 GHz. The result is shown in Figure 3.9. In (a), only the propagating modes have been included. In (b), all propagating modes as well as six evanescent modes in region three and four and two evanescent modes in region one and two is included.

At higher frequencies the effect is less noticeable and the largest differences are around the source, an example of this is shown in Figure 3.10. This is because the magnitude of the longitudinal wave number is larger at higher frequencies therefore the evanescent modes decay more quickly. At higher frequencies the evanescent modes will affect the electric field distribution to some extent, especially close to the source. However, this will not impact the power levels along the corridor substantially.

To illustrate how the power levels in a longer corridor is affected by including evanescent modes, the corridor model in the previous section is examined with the source positioned closer to a waveguide transition. The source is positioned in cell 1, region 1, on the boundary.
to region 3. Only the propagating modes have been included in Figure 3.11. All propagating modes as well as six evanescent modes in region three and four, three in region one and five in region two is included in Figure 3.12. As the source is on the boundary between two regions, evanescent modes should have to be included for the mode coupling to be accurate. As seen in the figure the evanescent modes have an impact on the field distribution in the close vicinity of the source but elsewhere it is minimal. This can also be seen in Figure 3.13 where the average path loss in the center of the rooms is plotted along the length of the corridor.

<table>
<thead>
<tr>
<th>Figure 3.9 (a)</th>
<th>0.4</th>
<th>4</th>
<th>6</th>
<th>10</th>
<th>0.1</th>
<th>3</th>
<th>10</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 3.9 (b)</td>
<td>0.4</td>
<td>6</td>
<td>8</td>
<td>16</td>
<td>0.1</td>
<td>3</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>Figure 3.10 (a)</td>
<td>1.2</td>
<td>13</td>
<td>18</td>
<td>32</td>
<td>0.1</td>
<td>3</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>Figure 3.10 (b)</td>
<td>1.2</td>
<td>15</td>
<td>20</td>
<td>38</td>
<td>0.1</td>
<td>3</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>Figure 3.11</td>
<td>2.44</td>
<td>27</td>
<td>60</td>
<td>90</td>
<td>0.1</td>
<td>3</td>
<td>10</td>
<td>17</td>
</tr>
<tr>
<td>Figure 3.12</td>
<td>2.44</td>
<td>30</td>
<td>65</td>
<td>96</td>
<td>0.1</td>
<td>3</td>
<td>10</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 3.1: List of parameters for the corridor models illustrated in this section.

Figure 3.11: Number of modes included (only propagating modes): $M_1 = 27, M_2 = 60$ and $M_0 = 90$. $f=2.44$ GHz and the resolution is 950 x 80.

Figure 3.12: Total number of modes included: $M_1 = 30, M_2 = 65$ and $M_0 = 96$. Six evanescent modes in region three and four, three in region one and five in region 2. $f=2.44$ GHz and the resolution is 950 x 80.

The effects of evanescent modes are less significant at higher frequencies, except close to the source. The model of the leaky cable require as good accuracy as possible close to the sources therefore six evanescent modes are included in region 3 and 4. Including these six additional modes doesn’t extend the computational time by much.
3.7 Wideband Power Distribution

Fast fading occur for narrowband signal transmissions in indoor environments due to the many propagation paths. An interference pattern arises from the many reflections and the signal is not very consistent over a small area. This effect is referred to as fast fading and it depends on the bandwidth of the transmitted signal. When sending in wideband the effect is much smaller because the interference pattern is different for each frequency which results in a much smoother power distribution. The fast fading effect is illustrated in Figure 3.14 where the power distribution in the corridor is displayed for 2.44 GHz (upper) and for the frequency interval 2.40-2.48 GHz (lower). In the latter case the average power distribution for 40 frequency samples is displayed.

Figure 3.13: Average power levels in the center of the room for two different numerical truncations.

Figure 3.14: (upper) Corridor power distribution at the frequency 2.44 GHz. (lower) Corridor power distribution at the wideband frequencies: 2.40-2.48 GHz with $K=40$ frequency samples.
The wideband power distribution is determined from the electric field for a discrete number of frequencies in the interval 2.40-2.48 GHz. The average electric field is given by

\[ E(x, z) = \frac{1}{K} \sum_{k=1}^{K} |E_k(f_k, x, z)| \]  

(3.16)

By conducting simulations with the corridor model it is seen that the difference between the average electric field and the average power (which is normally used) is small. Throughout this thesis the average electric field is taken in all simulations. The power distribution in the illustrations above is normalized as

\[ \bar{P}_{\text{wide}}(x, z) = 20 \log_{10} \frac{E(x, z)}{E(x_0, z_0)} \]  

(3.17)

where \( E(x_0, z_0) \) is the average power level close to the source. The discrete frequencies are given by

\[ f_k = 2.40 + k\Delta f \text{ GHz} \quad , k = 1, 2, ..., K \]  

(3.18)

\[ \Delta f = \frac{2.48 - 2.40}{13} \text{ GHz} \]

When the electric field needs to be determined for multiple frequency samples the computational time it takes to reach a solution increases significantly. The corridor model is implemented in matlab which runs on a laptop with an Intel Core i5 2.4 GHz processor, 4 GB of RAM and Windows Vista 32-bit. The total program runtime for the example above is given in Table 3.3. The calculation of S-matrices is done for each corridor segment and every frequency sample while the cascade coupling is done once for every frequency. The mesh generation for the electric field is done for every corridor segment and frequency sample. The latter includes summing up the electric field for every mode at every \( x \)- and \( z \)-position in the corridor segment. It can be seen in Table 3.3 that the E-field mesh generation is the bottleneck in the numerical calculations and the largest influence on program speed is the mesh resolution and the number of modes included.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Time (min)</th>
<th>Iterations</th>
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</thead>
<tbody>
<tr>
<td>Calculation of S-matrices</td>
<td>1</td>
<td>13 \cdot 40</td>
</tr>
<tr>
<td>Cascade coupling process</td>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>E-field mesh generation</td>
<td>9</td>
<td>13 \cdot 40</td>
</tr>
<tr>
<td><strong>Total program runtime</strong></td>
<td><strong>11</strong></td>
<td><strong>-</strong></td>
</tr>
</tbody>
</table>

Table 3.3: Program runtime for the corridor model display in Figure 3.14
3.8 Validation of the Corridor Model

To validate that the analytical calculations are correct a full wave numerical solution is implemented in HFSS [12] for a few cases. To determine a numerical solution for high frequency wave propagation in large geometries, substantial computational resources are required. The frequency 2.44 GHz is too high in relation to the size of a corridor to determine a solution with the available computer. Therefore, a model with a limited number of corridor segments are designed in HFSS and the analysis is conducted at a lower frequency. The HFSS simulations are done on a laptop with an Intel Core i5 2.4 GHz processor, 4 GB of RAM and Windows Vista 32-bit.

HFSS utilizes the finite element method (FEM), which is a numerical technique for solving partial differential equations. The solution domain is discretized into a finite element mesh consisting of a collection of four-sided pyramids called tetrahedra. A solution is generated, referred to as an adaptive pass, and the parameter delta energy (delta E) determines if the solution has converged or not. Delta E is the difference in the relative energy error from one adaptive pass to the next and measure the stability of the computed electromagnetic field. Before analyzing a specific HFSS design, delta E is set manually. During the solution process the mesh is more refined for each adaptive pass until the desired delta E is reached. To demonstrate how many tetrahedra are required for a solution to converge, two consecutive adaptive passes from the HFSS design, are compared to each other. In Figure 3.15 the average power levels along the corridor are shown. It can be seen that the solution has converged well for the first adaptive pass, where Delta E is 0.1141. The average difference between the two adaptive passes is 0.031 %

![Figure 3.15: The average power levels in the corridor, from the HFSS model, for two consecutive adaptive passes.](image)

The HFSS design is constructed with the same geometry as four cascade coupled corridor segments and the subsequent analysis are conducted at 0.9 GHz. The dielectric walls are assigned a material with a variable relative permittivity and conductivity. At both endpoints of the HFSS design a perfectly matched layer (PML) of length 0.25 meter is positioned to absorb outward propagating waves. The PMLs represent matched waveguides so that no power is reflected back into the considered domain.
Designs in HFSS have to be three dimensional therefore a finite height is introduced in the y-direction. For the HFSS model to match the 2D corridor model as closely as possible it is necessary that the electric field remains constant in the y-direction. To achieve this, the height is set to 0.05 meter which is much smaller than the width (5.55 m) and the total length (10.65 m) of the design and the upper and lower planes are modeled as PEC. Figure 3.16 shows the magnitude of the electric field along the corridor for three different y-positions. It can be seen that the electric field is approximately constant in the y-direction. The average difference for the three sampled y-positions in the figure is 0.33 %

![Figure 3.16: The average power levels in the corridor, from the HFSS model, for three different y-positions. The average difference for the three sampled y-positions in the figure is 0.33 %](image)

In HFSS, wave ports or current sources are the two primary ways that are used to excite waveguides. To calculate S-matrices for waveguide regions, wave ports can be implemented. This is useful if scattering from complex geometries have to be determined. However, the maximum number of modes that can be used to excite a port in HFSS is 25. Hence, wave ports cannot be implemented in conjunction with the corridor model except at very low frequencies or for corridors with small cross sections. A current source cannot be used to calculate S-parameters. However, it can be implemented at any frequency and for any waveguide cross section therefore a current source is used as excitation. A uniform current, aligned in the y-direction, is applied to a sheet that is connected to the floor and to the ceiling. The size of the sheet is 0.5 cm in the z-direction and 5 cm in the y-direction. To model a point source as accurate as possible the sheet is small in relation to the waveguide geometry.

The electric field data in HFSS is mapped to a grid of equal size as in the corridor model and is exported from HFSS into matlab for comparison with the data from the corridor model. Comparisons between the models are made for a different set of source positions, conductivities and permittivities. One example, with the source positioned in corridor segment one, is illustrated in Figure 3.17 and Figure 3.18. The power levels for the corridor model and the HFSS model are shown respectively. The average power levels, in the room and in the corridor, with respect to the length of the corridor are shown in Figure 3.19 and Figure 3.20.
<table>
<thead>
<tr>
<th></th>
<th>f (GHz)</th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$M_0$</th>
<th>$\sigma_w$ (S/m)</th>
<th>$\varepsilon_w$</th>
<th>$d_w$(cm)</th>
<th>Delta S</th>
<th>Tetrahedra</th>
<th>Resolution $(z \times x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HFSS model</td>
<td>0.9</td>
<td>-</td>
<td>-</td>
<td>0.2</td>
<td>3</td>
<td>10</td>
<td>0.0053</td>
<td>143065</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Corridor model</td>
<td>0.9</td>
<td>12</td>
<td>27</td>
<td>39</td>
<td>0.2</td>
<td>3</td>
<td>10</td>
<td>-</td>
<td>-</td>
<td>$560 \times 220$</td>
</tr>
</tbody>
</table>

Table 3.4: Parameter data for the HFSS model and the corridor model in the example illustrated below.

Figure 3.17: Corridor model: Power levels in four cascade coupled cells. $f=0.9$ GHz.

Figure 3.18: HFSS model: Average power levels (over the height, $y$) from a sheet with a uniform current. $f=0.9$ GHz.

The field distributions for the two models are in good agreement for most parts of the corridor. At most, a difference of 5 dB is observed which is seen at $z = 5.5$ m in Figure 3.19. In the rooms the average difference in the power levels is 3.3 % and in the corridor 8.4 %. Similar differences are found when the source is positioned in a room or in an internal...
However, the general agreement of the two models remains for different source positions. Furthermore, the power distribution pattern is more detailed in the HFSS design than in the corridor model. If comparisons to measurements are to be made, the higher resolution of the HFSS design is not necessary as the indoor measurements are not made with such a high resolution.

Figure 3.19: Average (over the total room cross section, $a_2$) power levels in the rooms.

Figure 3.20: Average (over the total corridor cross section, $a_3$) power levels in the corridor.
3.9 Inverse Discrete Fourier Transform

In section 3.7 it is discussed how the electric field distribution for a discrete number of frequency samples is used to determine the wideband power distribution in the corridor. In this section a discrete number of frequency samples are used to inverse Fourier transform the time harmonic description of the electric field into the time domain. This is done to illustrate how the wave propagates forward in time from the source.

The electric field, \( E_k(f_k, x, z) \), is determined for a discrete number of frequency samples between 2.40 and 2.48 GHz which is the same frequency interval that was used in the measurement campaign (see section 4.1). 40 frequency samples are included which give a good resolution in the time domain and a reasonable short computational time. The electric field is the frequency response for a small sampled frequency interval and by applying the inverse discrete Fourier transform (DFT) the corresponding impulse response can be determined. The impulse response describes how the system reacts to a brief impulse sent to the antenna.

The inverse fast Fourier transform (iFFT) in matlab is utilized for the calculation. A hanning filter is applied to the electric field before performing the iFFT. The hanning filter is a sine shaped window function used in spectral analysis to filter out unwanted side lobes. Side lobes appear because the inverse transform is done for a small number of samples in a limited frequency interval. Side lobes are still present, and the field distribution for each step in the time domain will have a spatial extension which is proportional to the bandwidth in the frequency domain. A large bandwidth results in a higher resolution in the time domain. If the bandwidth where infinite the first step in the time domain would be a Dirac pulse with no spatial extension.

Matlab defines the inverse dft [16] as

\[
E_n(t_n, x, z) = \frac{1}{K} \sum_{k=1}^{K} E_k(f_k, x, z) e^{\frac{j2\pi(n-1)(k-1)}{K}}
\]  

(3.19)

where \( K \) is the number of frequency samples. The bandwidth is determined by: \( bw = K(f_2 - f_1) \) and the sample interval in the time domain by: \( dt = \frac{1}{bw} \).

An example is shown on the next page where the impulse response is determined for a small corridor with the source positioned in corridor segment one. The wavefront is shown for six consecutive time intervals. The bandwidth for the frequency interval 2.40-2.48 GHz is 82.1 MHz which results in a time resolution of 12.2 ns. The thin vertical blue strips, seen best in the first two figures between 7.5 and 17.5 meters, are side lobes which are an effect of the inverse transform.

<table>
<thead>
<tr>
<th>( f ) (GHz)</th>
<th>( K )</th>
<th>( M_0 )</th>
<th>( d_w ) (cm)</th>
<th>( \sigma_w ) (S/m)</th>
<th>( \varepsilon_w )</th>
<th>segments</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.40-2.48</td>
<td>40</td>
<td>94.97</td>
<td>10</td>
<td>0.1</td>
<td>3</td>
<td>13</td>
</tr>
</tbody>
</table>

Table 3.5: Parameters for the impulse response shown on the next page.
3.10  Leaky Cable Model

In this section a model of the leaky coaxial cable (LCX model) used in the measurement campaign (the measurement campaign is described in section 4.1) is presented. The LCX is made by RFS and is constructed for operation up to 2.66 GHz [17]. A leaky cable can radiate in two different modes, either coupled mode or radiating mode. The mode and the radiation angle are determined by the frequency of operation and the slot positions along the extension of the cable. At 2.44 GHz the RFS cable operates in the radiating mode with a 60 degree radiation angle [2]. The slots are spaced out in groups along the cable and they are aligned perpendicularly to the extension of the cable as illustrated in Figure 3.21 (upper).

![Figure 3.21: (upper) Illustration of the slot positioning and slot spacing of the RFS LCX. (lower) LCX model with evenly spaced point sources.](image)

The proposed model of the LCX is constructed by point sources, positioned along the extension of the corridor. The electrical excitation of the point sources is directed out of the horizontal plane, in the $y$-direction. The reason for this orientation and corresponding limitations are discussed at the end of this section. To achieve the same radiation angle in the model as for the LCX the distance between the slots should be the same as in the cable. However, in the measurement campaign a 40 meter long LCX was used, which corresponds to approximately 1700 point sources in the model. The computational time for that many sources is too long and therefore a model with evenly spaced slots is used instead. With a slot spacing of 7.6 cm a radiation direction of 60 degrees can be achieved and only 527 point sources are needed. In Figure 3.22 the radiation angle with respect to frequency is displayed for the RFS LCX and the LCX model. It can be seen that the radiation angle is 120 degrees at 2.44 GHz for the LCX model. To change this to 60 degrees in the right direction the point sources are phase shifted from the right to the left instead of the other way around. How the phase shift is implemented in the LCX model is explained further down in this section.
There are two kinds of losses in a leaky cable, longitudinal loss and coupling loss. The longitudinal loss of the cable depends on the frequency, as shown in Figure 3.23. It describes the signal loss along the length of the cable as power radiates out in each slot. The coupling loss is defined in [10] as “the ratio of the power carried in the cable to the median power received by a half-wave dipole placed at a standard distance from the cable”.

The longitudinal loss for the RFS cable is at 2.44 GHz approximately 24 dB/100m, (see Figure 3.23). It is used to calculate how the amplitude of the electric field for each point source changes along the corridor. With the specified longitudinal loss the amplitude change per point source $n$ becomes

$$A_n = \sqrt{10^{-9(n-1)ds/1000}}$$

(3.20)

where $ds$ is the distance between each point source.

The phase difference between each slot in the leaky cable depends on the distance between the slots and the speed of light in the cable. The explicit phase difference between two point sources in the model is determined by $e^{i\omega dt}$, where $dt$ is the time it takes for the wave to propagate from one slot to the next. The phase difference for each slot is
\[
\psi_{nk}(f_k) = e^{j2\pi f_k \frac{n ds}{c_{\text{cable}}}}
\]  
(3.21)

where \(c_{\text{cable}} = \frac{20}{23}c_0\) is the speed of light in the cable and \(f_k\) is the \(k\):th discrete frequency sample.

The electric field distribution for the point sources is expressed as: \(E_{pnk}(f_k, x_s, z_{sn}, x, z)\) where \(x_s\) and \(z_{sn}\) are the source positions. As the phase shift depends on the frequency and on the position of the source, the field distribution has to be phase shifted differently for each source position and frequency and are therefore collected into a vector as

\[
E_{\text{leakyn}}(x, z) = \begin{bmatrix}
A_n\psi_{n1}(f_1)E_{pn1}(f_1, x_s, z_{sn}, x, z) \\
A_n\psi_{n2}(f_2)E_{pn2}(f_2, x_s, z_{sn}, x, z) \\
\vdots \\
A_n\psi_{nk}(f_k)E_{pnk}(f_k, x_s, z_{sn}, x, z)
\end{bmatrix}
\]  
(3.22)

To find the complete field distribution from all point sources superposition is used

\[
E_{\text{leaky}}(x, z) = \sum_{n=1}^{N} E_{\text{leakyn}}(x, z)
\]  
(3.23)

Where \(N\) is the number of point sources used to build up the cable and \(E_{\text{leaky}}(x, z)\) consists of a vector with \(K\) frequency samples. The average electric field for \(K\) frequency samples is

\[
E_{\text{leaky}}(x, z) = \frac{1}{K} \sum_{k=1}^{K} |E_{\text{leaky}k}|
\]  
(3.24)

A limitation of the LCX model is that the current for the point sources follows the \(y\)-direction i.e. out of the horizontal plane. The slots in the RFS LCX are positioned perpendicularly to the extension of the cable and a more accurate model would be a current distribution in the direction of the corridor (\(z\)-direction). In the former case only TE modes are excited and in the latter only TM modes. For TM modes the boundary conditions at the waveguide transitions are different than for TE modes which result in different expressions for the S-matrices. The S-matrices for TM modes have not been calculated in this thesis due to time considerations and is suggested as a possible future work. The S-matrix calculations for the TE modes, described in appendix 6.1 and 6.2, can be used as a starting point to more easily determine the S-matrices for the TM modes.

The boundary conditions for the TM modes lead to a different power distribution pattern in the corridor. However, local differences in the power distribution patterns may not affect the average power levels along the corridor and the rooms much. In [6] the power distribution in an office floor for TE and TM modes are compared to each other. A point source is used as excitation and S-MRTD is used to determine the electromagnetic field distribution. The results show that there are local differences between the two cases but the overall power distribution is similar. In order to evaluate the LCX model it is compared to the LCX measurements in section 4.8.
4 Results and Analysis

In the first part of this section a short summary of the measurement campaign is presented as well as a geometrical description of the corridor in which the measurements were conducted. Furthermore, the dielectric parameters and the limitations of the corridor model are discussed. After that, the dipole antenna simulations are compared to the measurements for three different dielectric wall conductivities. Moreover, the propagation pathways are examined for two cases with different conductivities. The effect that the dielectric wall thickness and permittivity have on the power levels in the corridor is also analyzed. In the last part the LCX model is compared with the leaky cable measurements and with the dipole antenna model.

4.1 Corridor Measurements

A measurement campaign has been performed in order to evaluate the corridor model. Corresponding results for empirical modeling are reported in [2]. The objective therein was to conduct measurements in order to improve and evaluate an empirical model for path loss in indoor environments as well as to investigate the usage of leaky cables.

The measurements are compared with the corridor model developed in this thesis in order to evaluate its accuracy. In this section the relevant measurement scenarios are summarized shortly, for a more detailed exposition see [2]. The corridor in the office building where the measurements were conducted is depicted in Figure 4.1 (for a complete floor plan see Figure 15 in [2]). In the campaign, two kinds of antennas were used, a half-wave vertically polarized dipole antenna and a leaky coaxial cable. All measurements are conducted in the wideband frequency range 2.40-2.48 GHz.

The measurements are divided into five cases. In the first case, the receiving dipole is stationary in the corridor (Rx2 in Figure 4.1) and the transmitting dipole is moved along the 74 meter long corridor. This is referred to as the corridor to corridor case. In the second case the receiving dipole is positioned in a room (Rx1 in Figure 4.1) and the transmitting dipole is moved along the corridor, this is referred to as the room to corridor case. In the third case the receiving dipole is positioned as in case two and the transmitting dipole is moved along the rooms marked with stars in Figure 4.1. This is referred to as the room to room case.

In the final two cases a 40 meter long leaky cable is used as the receiving antenna (LCX in Figure 4.1) and a dipole as the transmitting antenna. In the fourth case the dipole is moved along the corridor and in the fifth case the dipole is moved along the rooms marked with stars in Figure 4.1. The measurement results, together with the corridor model simulations, are presented in section 4.4 and 4.8.
4.2 Geometry of the Corridor

A part of the corridor where the measurements were conducted is simulated with the corridor model. The dashed black rectangle in Figure 4.1 indicates the part of the corridor being modeled and the geometrical parameters of that section are displayed in Table 4.1. The width of the central corridor is 1.70 meter \(a_2\) and the width of the rooms is 3.75 meter \(a_2\). At \(z=47.5\) meter a door opening in the corridor is modeled as part of a dielectric wall sticking out into the central corridor. The rooms are modeled with three different lengths and the doors with three different doorway widths. The thickness of the walls separating the rooms is 0.10 meter.

<table>
<thead>
<tr>
<th>Corridor Model geometry (m)</th>
<th>(a_0)</th>
<th>corridor width (a_1)</th>
<th>room width (a_2)</th>
<th>Room length</th>
<th>Doorway width</th>
<th>Corridor length</th>
<th>(d_w) (m)</th>
<th>Corridor obstacle</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_w = 0.10) m</td>
<td>5.55</td>
<td>1.70</td>
<td>3.75</td>
<td>2.2/3.0/5</td>
<td>1.1/1.5/1.5</td>
<td>76.6</td>
<td>0.10</td>
<td>(z=47.5)</td>
</tr>
</tbody>
</table>

Table 4.1: Geometrical parameters of the corridor model used to simulate a part of the left corridor in Figure 4.1.

The width of the walls separating the rooms from the corridor is 0.10 meter thick and consists of plaster board. Apart from the walls, large glass windows lined with metallic frames separates the rooms from the corridor as well as wooden doors, 0.8 meter wide. Recall from section 3.2, that the walls separating the corridor from the rooms are modeled as metal with an opening that represents the doorways. The open doorways are made larger than the normal doors to represent wave propagation through the glass windows as well as the open wooden doors.

4.3 Corridor Model Parameters and Limitations

One objective of this thesis is to determine how propagation from room to room takes place in an office corridor and which effect different conductivities and permittivities in the dielectric walls have on the power levels in the corridor. The walls are considered homogeneous and are modeled with an average conductivity and permittivity. The conductivity depends on the wall materials and if there are support studs in the walls. Because the walls are built with more
than one material it is difficult to determine an accurate value for the average conductivity. Therefore, three different conductivities are implemented in the simulations. A relative permittivity of 3 is used in all cases except in section 4.7 where the effect of a higher wall permittivity is analyzed. A conductivity of 0.2 S/m is used to model highly reflective walls, like reinforced concrete, and a conductivity of 0.01 S/m is used to model more transparent walls, like plaster board with metallic support studs. An additional conductivity of 0.05 S/m is added to include the effect of whiteboards. In the corridor where the measurements were conducted a small or a large whiteboard hangs on the wall in most of the rooms.

In order to compare the results from the corridor model to the measurements the limitations of the model need to be discussed. The largest limitation of the model is that the external walls are modeled as PEC. Because of this no power is absorbed in the external walls and no power is transmitted to the outside through the walls and windows. Another limitation is that the model is two-dimensional. Hence, the power leakage through the floor and ceiling is not included.

It is desired to include the effect of the third dimension in the corridor model, when comparing the model to the measurement data. To do this an approximate method is used. The vertically polarized half wave dipole antenna, used in the measurements, is modeled as an infinite line source in the y-direction. In [18], an approximate method was used to transform the electric field from a line source to the corresponding field from a point source. The final electric field distribution is multiplied with \( \frac{1}{\sqrt{r}} \) where \( r \) is the distance from the transmitting antenna to the receiving antenna. In [18] this approximate method is used for line of sight wave propagation in free space. For comparison to the measurements this approximate method is applied to the corridor model in order to adjust the final electric field from that of a line source to that of a point source in three dimensions. This is done by multiplying the power distribution with \( 1/r \). Henceforth, this change is denoted by the \( 1/r \) adjustment.

### 4.4 Comparison with Dipole Antenna Measurements

The power distribution, extracted from the corridor model, is shown along the main corridor and in the rooms separately. The data are collected along the measurement paths described in Figure 4.1 (along the dashed red line and along the star marked rooms). The electric field data extracted from the corridor model are normalized to the measurement data in the corridor to corridor case. The normalization distance is chosen to be 1.0 meter from the transmitting half wave dipole antenna where the average received power is \(-41 \, \text{dB}\). In the different simulation cases described below, the index \( c \) (c1, c2 etc) denote that the antenna is positioned in the corridor. An \( r \) (r1, r2 etc) denote that the antenna is positioned in the room furthest to the left in the corridor.

#### 4.4.1 Corridor to Corridor

The dipole antenna is positioned in the beginning of the corridor and three different dielectric wall conductivities are analyzed, the specific parameters are displayed in Table 4.2.
Table 4.2: Parameters for the simulation cases used in this section. The source is positioned in the corridor.

<table>
<thead>
<tr>
<th>Case</th>
<th>f (GHz)</th>
<th>$K$</th>
<th>$M_0$</th>
<th>$\sigma_w$ (S/m)</th>
<th>$\varepsilon_w$</th>
<th>$d_w$ (m)</th>
<th>segments</th>
<th>$(z_s, x_s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>2.40-2.48</td>
<td>40</td>
<td>94-97</td>
<td>0.2</td>
<td>3</td>
<td>0.10</td>
<td>33</td>
<td>(0.5, 0.85)</td>
</tr>
<tr>
<td>$c_2$</td>
<td>2.40-2.48</td>
<td>40</td>
<td>94-97</td>
<td>0.01</td>
<td>3</td>
<td>0.10</td>
<td>33</td>
<td>(0.5, 0.85)</td>
</tr>
<tr>
<td>$c_3$</td>
<td>2.40-2.48</td>
<td>40</td>
<td>94-97</td>
<td>0.05</td>
<td>3</td>
<td>0.10</td>
<td>33</td>
<td>(0.5, 0.85)</td>
</tr>
</tbody>
</table>

The power levels in the corridor are compared to the measurement data in Figure 4.2. Both the raw data from the corridor model and the data with the $1/r$ adjustment are shown. The power levels in the corridor for the two higher conductivities ($c_1$ and $c_3$) are approximately equal. For the case with the lowest conductivity the dielectric walls are more transparent and less power is absorbed which results in slightly higher power levels overall. At $z=47.5$ the effect of the open doorway in the corridor can be observed in the data from the model. The reason that the dielectric wall conductivity doesn’t affect the power distribution much in the main corridor is that the source is positioned in the corridor, which has a waveguiding effect. Because of the PEC walls, the effect is amplified.

It is seen that with the $1/r$ adjustment the results from both of the two higher conductivities are consistent with the measurement results. For both the case of the adjusted and the unadjusted power levels it can be concluded that the conductivity of the walls in between the rooms doesn’t have a large impact on the power distribution in the main corridor.

In Figure 4.3 the unadjusted power distribution in the corridor and in the rooms is shown for the three different cases. It can be seen that the power distribution in the rooms is much more even for the conductivity 0.01 S/m. This is because of the highly transparent walls through which the wave propagates without much attenuation. In the case for the two higher conductivities the main path of propagation is through the open doorways and shadowing occurs in parts of the rooms due to the highly reflective walls.
Figure 4.3: The unadjusted power distribution in the corridor for three different wall conductivities with the source positioned in the corridor. The power levels at the end of the main corridor, marked with a red dot, for the three different cases are shown as well.

- $c_1$: $\sigma_w=0.2$ S/m, $\varepsilon_w=3$, $d_w=10$ cm
- $c_2$: $\sigma_w=0.01$ S/m, $\varepsilon_w=3$, $d_w=10$ cm
- $c_3$: $\sigma_w=0.05$ S/m, $\varepsilon_w=3$, $d_w=10$ cm

Power levels at the end of the corridor:
- $c_1$: -53.0 dB
- $c_2$: -52.6 dB
- $c_3$: -52.1 dB
4.4.2 Room to Corridor and Room to Room

The dipole antenna is positioned in the room furthest to the left and three different dielectric wall conductivities are analyzed as in the previous section, the specific model parameters are displayed in Table 4.3.

<table>
<thead>
<tr>
<th>Case</th>
<th>$f$ (GHz)</th>
<th>$K$</th>
<th>$M_0$</th>
<th>$\sigma_w$ (S/m)</th>
<th>$\varepsilon_w$</th>
<th>$d_w$ (m)</th>
<th>segments</th>
<th>$(z_s, x_s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>r1</td>
<td>2.40-2.48</td>
<td>40</td>
<td>94-97</td>
<td>0.2</td>
<td>3</td>
<td>0.10</td>
<td>33</td>
<td>(2.23, 2.74)</td>
</tr>
<tr>
<td>r2</td>
<td>2.40-2.48</td>
<td>40</td>
<td>94-97</td>
<td>0.01</td>
<td>3</td>
<td>0.10</td>
<td>33</td>
<td>(2.23, 2.74)</td>
</tr>
<tr>
<td>r3</td>
<td>2.40-2.48</td>
<td>40</td>
<td>94-97</td>
<td>0.05</td>
<td>3</td>
<td>0.10</td>
<td>33</td>
<td>(2.23, 2.74)</td>
</tr>
</tbody>
</table>

Table 4.3: Parameters for the two corridor models illustrated in this section. The source is positioned in a room.

The power levels in the corridor for the different conductivities are compared to the measurements in Figure 4.4. Both the unadjusted data from the corridor model and the $1/r$ adjustment is shown. To avoid overlapping the unadjusted power distribution has been displaced with plus 10 dB. It can be seen that the power levels, for case r1 and r3, are close throughout the corridor while the power levels for case r2 are about eight dB higher than both of the other two cases. When adjusting the results from the corridor model with $1/r$, case r3 is the best fit to the measurement data. However, case r1 is also close which indicates that it is hard to actually determine which wall conductivity is a good representation of the walls.

Figure 4.4: The power levels in the center of the main corridor, for three different conductivities. The source is positioned in the room furthest to the left (the $z$-position is marked with Rx1 in the figure). The unadjusted cases (r1-r3) are displaced 10 dB to better illustrate the different cases.

For the room to room case the unadjusted and the adjusted data are compared to the measurement data separately. The results are shown in Figure 4.5 and Figure 4.6 respectively. The positions of the dielectric walls can be seen by the recurring dips in the power levels, this is most notable for the highest wall conductivity. As for the room to corridor case the wall conductivity 0.05 S/m is the best fit to the measurement data after the $1/r$ adjustment.
The reason that the power levels for case r1 is very close to case r3, both in the corridor and in
the rooms, is probably because of the dielectric wall furthest to the left. Because of the high
wall conductivity in case r1, this wall reflects more power back into the corridor and the
rooms than in case r3. In Figure 4.7 the unadjusted power distribution in the simulated
corridor is shown for the three different cases.

Figure 4.5: The power levels along the rooms, for the measurement data and three different conductivities. The source
is positioned in the room furthest to the left (the z-position is marked with Rx1 in the figure).

Figure 4.6: The power levels along the rooms, for three different conductivities with the 1/r adjustment as well as the
measurement data. The source is positioned in the room furthest to the left (the z-position is marked with Rx1 in the
figure).
Figure 4.7: Power distribution in the corridor for three different conductivities with the source positioned in the room. Note that the scale for the corridor power levels is different from Figure 4.3.
4.5 Propagation Path for High and Low Wall Conductivity

In section 0 it is described how the fast Fourier transform is applied to determine the electric field distribution in the time domain. In this section, the time dependent field distribution is shown for the first three time steps \((t_1 - t_3)\) to illustrate the different propagation paths for high and low dielectric wall conductivity. First the source is positioned in a room and then in the corridor. The same corridor geometry as in section 4.4 is used although only a part of the corridor is shown to better illustrate the details in the propagation path.

4.5.1 Point Source in Room

The source is positioned in the room to the left of the corridor and the case with a conductivity of 0.2 S/m and 0.01 S/m are illustrated next to each other. In Figure 4.8 the power distribution at the first time step is shown, followed by the next two discrete time steps in Figure 4.9 and Figure 4.10. For the high conductivity case, it can be seen that the wave propagates primarily through the door opening, while for the low conductivity case the propagation path through the walls is dominant.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>(f) (GHz)</th>
<th>(K)</th>
<th>(M_0)</th>
<th>(\sigma_w) (S/m)</th>
<th>(\varepsilon_w)</th>
<th>(d_w) (m)</th>
<th>segments</th>
<th>((x_1, x_2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>r1</td>
<td>2.40-2.48</td>
<td>40</td>
<td>94-97</td>
<td>0.2</td>
<td>3</td>
<td>0.10</td>
<td>6</td>
<td>(0.5, 2.74)</td>
</tr>
<tr>
<td>r2</td>
<td>2.40-2.48</td>
<td>40</td>
<td>94-97</td>
<td>0.01</td>
<td>3</td>
<td>0.10</td>
<td>6</td>
<td>(0.5, 2.74)</td>
</tr>
</tbody>
</table>

Table 4.4: Parameters for the two corridor models illustrated in this section.

Figure 4.8: \(t_1\): Power levels in the corridor for \(\sigma_w = 0.2\) S/m (upper) and \(\sigma_w = 0.01\) S/m (lower).
Figure 4.9: Power levels in the corridor for $\sigma_w = 0.2 \text{ S/m (upper)}$ and $\sigma_w = 0.01 \text{ S/m (lower)}$.

Figure 4.10: Power levels in the corridor for $\sigma_w = 0.2 \text{ S/m (upper)}$ and $\sigma_w = 0.01 \text{ S/m (lower)}$. 
4.5.2 Point Source in Corridor

The source is positioned furthest to the left in the main corridor and the dielectric wall conductivities 0.2 S/m and 0.01 S/m are shown next to each other. In Figure 4.11 the power distribution at the first time step is shown, followed by the next two discrete time steps in Figure 4.12 and Figure 4.13. It can be seen that the propagation paths for the two cases are approximately equal along the main corridor and through the doorway openings. The difference is the propagation through the dielectric walls, where more power is scattered and absorbed in the case with the high wall conductivity.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>f (GHz)</th>
<th>K</th>
<th>M₀</th>
<th>σₗ (S/m)</th>
<th>εₗ</th>
<th>dₗ (m)</th>
<th>segments</th>
<th>(zₛ, xₛ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>c₁</td>
<td>2.40-2.48</td>
<td>40</td>
<td>94-97</td>
<td>0.2</td>
<td>3</td>
<td>0.10</td>
<td>6</td>
<td>(0.5, 0.85)</td>
</tr>
<tr>
<td>c₂</td>
<td>2.40-2.48</td>
<td>40</td>
<td>94-97</td>
<td>0.01</td>
<td>3</td>
<td>0.10</td>
<td>6</td>
<td>(0.5, 0.85)</td>
</tr>
</tbody>
</table>

Table 4.5: Parameters for the two corridor models illustrated in this section.

Figure 4.11: t₁: Power levels in the corridor for \( \sigmaₗ = 0.2 \text{ S/m} \) (upper) and \( \sigmaₗ = 0.01 \text{ S/m} \) (lower).
Figure 4.12: t₂: Power levels in the corridor for $\sigma_w = 0.2 \text{ S/m (upper)}$ and $\sigma_w = 0.01 \text{ S/m (lower)}$.

Figure 4.13: t₃: Power levels in the corridor for $\sigma_w = 0.2 \text{ S/m (upper)}$ and $\sigma_w = 0.01 \text{ S/m (lower)}$. 
4.6 Influence of Dielectric Wall Thickness

The dielectric wall thickness is changed in order to determine the effect it has on the power distribution in the corridor. A simulation is done where the dielectric wall thickness is increased by 50 % from 0.10 meter, which have been used so far. The changes in corridor geometry when the dielectric wall thickness is increased are displayed in Table 4.6. The source is positioned in a room because variations of the dielectric wall parameters have a larger impact on the power distribution than if the source is positioned in the main corridor. The parameters for the two illustrated cases are shown in Table 4.7. In Figure 4.14 and in Figure 4.15 the power levels along the corridor and along the rooms are shown respectively.

Due to the waveguiding effect in the main corridor the power levels are only slightly lower for the case with the thicker dielectric walls. The largest observed difference between the two cases is two dB. In the rooms the dielectric wall thickness has a larger effect on the power levels and a difference of two to four dB is observed throughout the corridor. As expected, the largest difference is observed furthest down the corridor as the wave have propagated through more walls there and hence more power have been absorbed.

<table>
<thead>
<tr>
<th>Corridor Model geometry (m)</th>
<th>$a_0$</th>
<th>corridor width ($a_1$)</th>
<th>room width ($a_2$)</th>
<th>Room length</th>
<th>Doorway width</th>
<th>Corridor length</th>
<th>$d_w$ (m)</th>
<th>Corridor obstacle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_w = 0.10$ m</td>
<td>5.55</td>
<td>1.70</td>
<td>3.75</td>
<td>2.2/3.0/5</td>
<td>1.1/1.5/1.5</td>
<td>76.6</td>
<td>0.10</td>
<td>$z=47.5$</td>
</tr>
<tr>
<td>$d_w = 0.15$ m</td>
<td>5.55</td>
<td>1.70</td>
<td>3.75</td>
<td>2.15/2.95/5</td>
<td>1.1/1.5/1.5</td>
<td>76.6</td>
<td>0.15</td>
<td>$z=47.5$</td>
</tr>
</tbody>
</table>

Table 4.6: Geometrical parameters of the corridor for two different dielectric wall thicknesses.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$f$ (GHz)</th>
<th>$K$</th>
<th>$M_0$</th>
<th>$\sigma_w$ (S/m)</th>
<th>$\varepsilon_w$</th>
<th>$d_w$ (m)</th>
<th>segments</th>
<th>$(x_g, y_g)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>r2</td>
<td>2.40-2.48</td>
<td>40</td>
<td>94-97</td>
<td>0.01</td>
<td>3</td>
<td>0.10</td>
<td>33</td>
<td>(0.5, 2.74)</td>
</tr>
<tr>
<td>r4</td>
<td>2.40-2.48</td>
<td>40</td>
<td>94-97</td>
<td>0.01</td>
<td>3</td>
<td>0.15</td>
<td>33</td>
<td>(0.5, 2.74)</td>
</tr>
</tbody>
</table>

Table 4.7: Parameters for the two corridor models illustrated in this section. The source is positioned in a room.

Figure 4.14: The average power along the main corridor for two different dielectric wall thicknesses.
In this section the dielectric wall permittivity is changed in order to determine the effect it has on the power distribution in the corridor, two cases with different wall permittivities are illustrated. The source is positioned in the corridor and the conductivity is set to 0.01 S/m. In Figure 4.16 and Figure 4.17 the power levels in the main corridor and in the rooms are shown. In section 4.4.1, also with the source positioned in the corridor, it was concluded that the wall conductivity didn’t affect the main corridor power levels much. In Figure 4.16 it can be seen that the corridor power levels do not change much for the different wall permittivities either. The power levels in the rooms are slightly higher for the increased wall permittivity. This is attributed to the fact that the walls are more reflective when the permittivity increases therefore less power is absorbed.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$f$ (GHz)</th>
<th>$K$</th>
<th>$M_0$</th>
<th>$\sigma_w$ (S/m)</th>
<th>$\varepsilon_w$</th>
<th>$d_w$ (m)</th>
<th>segments</th>
<th>$(z_s, x_s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>c2</td>
<td>2.40-2.48</td>
<td>40</td>
<td>94-97</td>
<td>0.01</td>
<td>3</td>
<td>0.10</td>
<td>33</td>
<td>(0.5, 0.85)</td>
</tr>
<tr>
<td>c4</td>
<td>2.40-2.48</td>
<td>40</td>
<td>94-97</td>
<td>0.01</td>
<td>5</td>
<td>0.10</td>
<td>33</td>
<td>(0.5, 0.85)</td>
</tr>
</tbody>
</table>

Table 4.8: Parameters for the two corridor models illustrated in this section. The source is positioned in the corridor.
Figure 4.16: The average power along the main corridor for two different dielectric wall permittivities.

Figure 4.17: The average power along the rooms for two different dielectric wall permittivities.
4.8 Leaky Cable Model Compared to Measurements

In this section the LCX model, described in section 3.10, is compared to the leaky cable measurements (see section 4.1) in order to evaluate the accuracy of the model. Moreover, a simulation with a random phase difference between the leaky cable slots is made to model a leaky cable that is not completely straight. Furthermore, the LCX model is compared to the dipole antenna model in order to determine if a leaky cable is a viable alternative to a traditional point antenna deployment.

4.8.1 The Simulated Corridor

In Figure 4.18 the corridor where the measurements were conducted is shown and in Figure 4.19 the simulated corridor is shown. The position of the leaky cable is indicated by the green line in both figures. In Figure 4.18 the measurement paths are indicated by a red dashed line and red stars for the corridor and room measurements respectively. The power levels from the simulation are extracted along the dashed red lines in Figure 4.19. The radial distance from the leaky cable to the two measurement paths is 1.5 and 5.0 meters. Because of the two dimensional model the electric field cannot be sampled at the same radial distances from the leaky cable. Therefore, in the simulation, the corresponding distances are chosen as 1.0 and 4.0 meters. The geometry of the corridor is the same as described in section 4.2 except that the length of the simulated corridor is 71.6 instead of 76 meter. The 1/r adjustment is included in the model as in section 4.4.

The power level from the LCX model is normalized to the measurement data. The normalization distance is 1.0 meter to the right of the leaky cable (at \( z = 1.0 \) m), where the average received power is \(-59\) dB. The parameters for the leaky cable simulation are given in Table 4.9 as well as the parameters for two of the dipole antenna models used in section 4.4.
Table 4.9: Parameters for the LCX model and the dipole antenna model with the conductivity 0.05 S/m. \(N\) is the number of point sources included in the simulation.

<table>
<thead>
<tr>
<th></th>
<th>(f) (GHz)</th>
<th>(K)</th>
<th>(N)</th>
<th>(M_0)</th>
<th>(\sigma_w) (S/m)</th>
<th>(\varepsilon_w)</th>
<th>(d_w) (m)</th>
<th>segments</th>
<th>((z_{sn}, x_{sn})) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCX3</td>
<td>2.40-2.48</td>
<td>20</td>
<td>527</td>
<td>94-97</td>
<td>0.05</td>
<td>3</td>
<td>0.10</td>
<td>31</td>
<td>((1 + 0.076\pi, 1.27))</td>
</tr>
<tr>
<td>c3</td>
<td>2.40-2.48</td>
<td>40</td>
<td>1</td>
<td>94-97</td>
<td>0.05</td>
<td>3</td>
<td>0.10</td>
<td>31</td>
<td>((0.5, 0.85))</td>
</tr>
<tr>
<td>r3</td>
<td>2.40-2.48</td>
<td>40</td>
<td>1</td>
<td>94-97</td>
<td>0.05</td>
<td>3</td>
<td>0.10</td>
<td>31</td>
<td>((0.5, 2.74))</td>
</tr>
</tbody>
</table>

4.8.2 Comparison with Measurements

Recall that the model with the dielectric wall conductivity 0.05 S/m was the best fit when the dipole antenna model was compared to the dipole measurement data. Therefore, the same conductivity is used in the leaky cable simulations. Furthermore, the LCX model is built by 527 point sources and the computational time for that many sources is around 70 hours which is not efficient if multiple simulations needs to be done. The matlab code is not optimized which allows significant improvement of the computational effort in the future.

In Figure 4.20 the power levels in the main corridor are shown for the LCX measurement data and the LCX model with the correct phase as well as the LCX model with a random phase. How the phase shift is implemented is described in section 3.10. It can be seen that after the end of the leaky cable the power levels for the LCX model with correct phase start to diverge from the measurement data. This is attributed to the fact that the LCX in the corridor model is completely straight and therefore a precise radiation angle of 60 degrees is achieved, reducing the radiation in the forward direction. The leaky cable used in the measurement campaign was suspended from the ceiling with elastic rubber bands positioned every three meters. Because of this the cable was slightly curved at some points, most noteworthy at the end of the cable (see the appendix of [2] for pictures). The interference pattern from the curved cable is different from a completely straight one which can explain why the power levels after the end
of the leaky cable are high. The simulated case with a random phase is included to simulate errors in the position of the leaky cable, the phase delay between each point source in the LCX model is picked at random. As seen in Figure 4.20 the power levels for the LCX model with the random phase matches the measurement data, after the end of the leaky cable, better.

In Figure 4.21 the power levels in the rooms are shown. At the first measurement position ($z=1$ m) the power level of the LCX model with correct phase is seven dB lower than the measurement data. This is because of the 60 degree radiation angle and the PEC wall in between the main corridor and the first room. The LCX model with random phase is a better match at the start position of the leaky cable because it radiates almost equally in all directions. This effect can be seen in Figure 4.22 (LCX3 and LCX3 with random phase). The power levels from the LCX model are slightly higher than the measured power levels along the extension of the leaky cable. This may be attributed to the fact that the radial distance from the cable to the room measurement path is one meter longer than in the model. After the end of the leaky cable the model matches the measurements better.

![Room power levels](image)

**Figure 4.21:** Power levels in the rooms for the room measurement data, the LCX model with correct phase and with random phase, both with the $1/r$ adjustment.

In Figure 4.22 the power distribution in the corridor is shown for four cases. Furthest to the left is the LCX3 case with the $1/r$ adjustment and the correct phase, the radiation angle of 60 degrees can be seen clearly and is indicated by a blue line. Second from the left is LCX3 with random phase where no specific radiation direction can be detected. As mentioned before, it can be seen that the power levels in the main corridor, after the extension of the leaky cable, is slightly higher than in the previous case. To the right case c3 and r3 are shown, a dipole antenna in the corridor and in the first room, with the same dielectric wall conductivity as the LCX model. As expected, a more even power distribution is achieved in the rooms when using the leaky cable as a transmitting antenna.
Figure 4.22: (from the left) Power distribution in the corridor (in dB) for the LCX model with correct phase and with random phase, both with the $1/r$ adjustment. The black line indicates the position of the leaky cable and the blue lines mark the 60 degree radiation angle. To the right is the dipole antenna model for the source positioned in the corridor and positioned furthest to the left in a partial room for the same dielectric wall conductivity as the leaky cable.
5 Conclusions

5.1 Summary

In this thesis a corridor model with adjacent rooms is developed. The model is based on waveguiding, mode matching and cascade coupling. It is shown that the results from the model are in good quantitative agreement with the measurement data, when the $1/r$ adjustment is included in the model. The best fit to the measurement data is the simulations with the wall conductivity 0.05 S/m. Moreover, the simulations with the wall conductivity 0.2 S/m show only slightly lower power levels. These conductivities are not a good model for the walls in the building where the measurement campaign was conducted. These walls are made of plasterboard with metallic support studs, for which the conductivity 0.01 S/m is a better fit. The simulations with this wall conductivity shows higher power levels than the measurement data, especially with the source positioned in a room. The reason for this is probably that the external walls are modeled as perfectly conducting. If they were modeled as dielectric, the additional power losses would result in better agreement with the measurement data for the lowest wall conductivity.

When the walls in between rooms are modeled as plaster board with metallic support studs they don’t absorb or reflect much power and the direct path is dominating for transmissions between rooms. When the walls are modeled as reinforced concrete they are highly reflective and for transmissions with a dipole antenna positioned in a room the main propagation path into subsequent rooms is from the corridor, through the open doorways. The results from the LCX model simulations show that a leaky cable gives a more even power distribution in a corridor compared to a dipole antenna, especially for buildings with highly reflective walls between the rooms. The results also indicate that the cable should be mounted as straight as possible to avoid radiation along the corridor outside the extension of the cable.

5.2 Suggested Future Work

In this section some suggestions are given for what can be done to further develop the corridor model presented in this thesis. To include the effect of power leakage to the outside of the building the external walls should be modeled with a dielectric medium. A suggested starting point to do that is [7] where the external walls are modeled as dielectric with a finite thickness. The model should also be taken into the third dimension where a point source excites TM modes as well as TE modes. A more accurate model of a leaky cable can be developed if S-matrices are determined for the propagation of TM modes. Furthermore, a method to handle wave propagation around corners is required to model a larger office environment with multiple intersecting corridors, this is done in [7] by using FDTD.
6 Appendices

6.1 S-Matrix Calculation for the Dielectric Wall

A detailed calculation of the S-matrix relating the homogeneous mode coefficients at the boundaries of region two is determined in this section. Only the TE modes are included in the calculations, but with a few changes the resulting S-matrix is valid for the TM modes also. The boundary conditions at the dielectric wall are set up and mode matching is used to analytically solve the resulting system of equations, finally \( L^{(2)} \) is determined. The theory behind the process used is described in detail in section 2 and 3.

The relation between the unknown mode coefficients and the mode coefficients of the inward propagating wave is

\[
\begin{pmatrix}
    h_{(2,1)}^{-}(z_1) \\
    h_{(2,3)}^{-}(z_2)
\end{pmatrix} = L^{(2)} \begin{pmatrix}
    h_{(2,1)}^{+}(z_1) \\
    h_{(2,3)}^{+}(z_2)
\end{pmatrix}
\]

(6.1)

The following relation between the coefficients at the boundary of the dielectric wall is first determined. \( L^{(w)} \) is then extended with propagation matrices over region 2,1 and 2,3 to determine \( L^{(2)} \).

\[
\begin{pmatrix}
    h_{(2,1)}^{-}(z_{w1}) \\
    h_{(2,3)}^{-}(z_{w2})
\end{pmatrix} = L^{(w)} \begin{pmatrix}
    h_{(2,1)}^{+}(z_{w1}) \\
    h_{(2,3)}^{+}(z_{w2})
\end{pmatrix}
\]

(6.2)
6.1.1 Boundary Conditions

As no sources are being modeled in region 2, the mode coefficients only have a homogeneous part, given by

\[ c_{m}^{(2,i)}(z) = h_{m}^{(2,i)}(z), \quad i = 1, 2, 3 \]  

(6.3)

The electric and magnetic fields are expanded as in (3.4) and (3.5) for the different regions depicted in Figure 6.1.

\[ E^{(2,i)}(x, z) = \sum_{m=1}^{\infty} \left[ h_{m}^{(2,i)}(z) + h_{m}^{(2,i)}(z) \right] \varphi_{m}^{(2)}(x), \quad i = 1, 2, 3 \]  

(6.4)

\[ H^{(2,i)}(x, z) = \sum_{m=1}^{\infty} \left[ h_{m}^{(2,i)}(z) - h_{m}^{(2,i)}(z) \right] \gamma_{m}^{(2,i)} \varphi_{m}^{(2)}(x), \quad i = 1, 2, 3 \]  

(6.5)

The mode admittance and the longitudinal wave number for each region is given by

\[ y_{m}^{(2,i)} = \frac{k_{zm}^{(2,i)}}{k \eta} \]  

(6.6)

\[ k_{zm}^{(2,i)} = \sqrt{k - k_{xm}^{(2,i)}} \quad i = 1, 2, 3 \]  

(6.7)

where the wave number is \( k_{0} \) in region 2,1 and 2,3. In region 2,2 \( k \) is given by

\[ k_{w} = k_{0} \sqrt{\left( \varepsilon_{w} - \frac{\sigma_{w}}{\omega \varepsilon_{0}} \right)} \]  

(6.8)

and the transversal wave number is

\[ k_{xm}^{(2)} = \frac{m \pi}{a_{z}} \]  

(6.9)

which is the same for each region as the cross section does not change. The free space wave impedance is given by

\[ \eta_{0} = \frac{\omega \mu_{0}}{k_{0}} \]  

(6.10)

and the wave impedance in the dielectric wall region is given by

\[ \eta_{w} = \frac{\omega \mu_{0}}{k_{w}} \]  

(6.11)

At \( z_{w1} \) and \( z_{w2} \) the transversal electric and magnetic field have to be continuous, this lead to the following boundary conditions

\[ E^{(2,1)}(x, z_{w1}) = E^{(2,2)}(x, z_{w1}) \]  

(6.12)

\[ H^{(2,1)}(x, z_{w1}) = H^{(2,2)}(x, z_{w1}) \]  

(6.13)

\[ E^{(2,2)}(x, z_{w2}) = E^{(2,3)}(x, z_{w2}) \]  

(6.14)
\[ H^{(2,2)}(x, z_{w2}) = H^{(2,3)}(x, z_{w2}) \] (6.15)

which after insertion of the field expansions become

\[
\sum_{m=1}^{\infty} \left[ h_{m}^{(2,1)+}(z_{w1}) + h_{m}^{(2,1)-}(z_{w1}) \right] \varphi_{m}^{(2)}(x) = \sum_{m=1}^{\infty} \left[ h_{m}^{(2,2)+}(z_{w1}) + h_{m}^{(2,2)-}(z_{w1}) \right] \varphi_{m}^{(2)}(x) \]

(6.16)

\[
\sum_{m=1}^{\infty} \left[ h_{m}^{(2,1)+}(z_{w1}) - h_{m}^{(2,1)-}(z_{w1}) \right] Y_{m}^{(2,1)}(x) = \sum_{m=1}^{\infty} \left[ h_{m}^{(2,2)+}(z_{w1}) - h_{m}^{(2,2)-}(z_{w1}) \right] Y_{m}^{(2,2)}(x) \]

(6.17)

\[
\sum_{m=1}^{\infty} \left[ h_{m}^{(2,2)+}(z_{w2}) + h_{m}^{(2,2)-}(z_{w2}) \right] \varphi_{m}^{(2)}(x) = \sum_{m=1}^{\infty} \left[ h_{m}^{(2,3)+}(z_{w2}) + h_{m}^{(2,3)-}(z_{w2}) \right] \varphi_{m}^{(2)}(x) \]

(6.18)

\[
\sum_{m=1}^{\infty} \left[ h_{m}^{(2,2)+}(z_{w2}) - h_{m}^{(2,2)-}(z_{w2}) \right] Y_{m}^{(2,2)}(x) = \sum_{m=1}^{\infty} \left[ h_{m}^{(2,3)+}(z_{w2}) - h_{m}^{(2,3)-}(z_{w2}) \right] Y_{m}^{(2,3)}(x) \]

(6.19)

The equations are multiplied with \( \varphi_{m}^{(2)} \), for a certain mode number \( m_{2} \) and integrated over the waveguide cross section. The inner product (2.12) is then utilized, resulting in the following systems of equations.

\[
h_{m_{2}}^{(2,1)+}(z_{w1}) + h_{m_{2}}^{(2,1)-}(z_{w1}) = h_{m_{2}}^{(2,2)+}(z_{w1}) + h_{m_{2}}^{(2,2)-}(z_{w1}) \]

(6.20)

\[
\left[ h_{m_{2}}^{(2,1)+}(z_{w1}) - h_{m_{2}}^{(2,1)-}(z_{w1}) \right] Y_{m_{2}}^{(2,1)}(x) = \left[ h_{m_{2}}^{(2,2)+}(z_{w1}) - h_{m_{2}}^{(2,2)-}(z_{w1}) \right] Y_{m_{2}}^{(2,2)}(x) \]

(6.21)

\[
h_{m_{2}}^{(2,2)+}(z_{w2}) + h_{m_{2}}^{(2,2)-}(z_{w2}) = h_{m_{2}}^{(2,3)+}(z_{w2}) + h_{m_{2}}^{(2,3)-}(z_{w2}) \]

(6.22)

\[
\left[ h_{m_{2}}^{(2,2)+}(z_{w2}) - h_{m_{2}}^{(2,2)-}(z_{w2}) \right] Y_{m_{2}}^{(2,2)}(x) = \left[ h_{m_{2}}^{(2,3)+}(z_{w2}) - h_{m_{2}}^{(2,3)-}(z_{w2}) \right] Y_{m_{2}}^{(2,3)}(x) \]

(6.23)

### 6.1.2 Truncated Numerical Analysis

The electromagnetic field expansions are truncated into a finite number of modes, \( M_{2} \) where the homogeneous mode coefficients (2.25) are collected into vectors as

\[
h_{l}(z_{w1}) = \begin{bmatrix} h_{1}^{(2,l)+}(z_{w1}) \\ h_{2}^{(2,l)+}(z_{w1}) \\ \vdots \\ h_{M_{2}}^{(2,l)+}(z_{w1}) \end{bmatrix}, \quad l = 1, 2 \]

(6.24)

A mode admittance matrix for each region is defined as
\[
D_{(2,i)} = \text{diag}\left\{\{y^{(2,l)}_m\}_m\right\}^{M_z}, \quad i = 1, 2, 3
\]  
(6.25)

\(D_{(2,i)}\) is a diagonal matrix where the diagonal elements consist of the mode admittance for each mode.

(6.20)-(6.23) are rewritten as a truncated system of equations as

\[
h_{(2,1)}^+(z_{w1}) + h_{(2,1)}^-(z_{w1}) = h_{(2,1)}^+(z_{w1}) + h_{(2,2)}^-(z_{w1})
\]  
(6.26)

\[
D_{(2,1)}\left[h_{(2,1)}^+(z_{w1}) - h_{(2,1)}^-(z_{w1})\right] = D_{(2,2)}\left[h_{(2,2)}^+(z_{w1}) - h_{(2,2)}^-(z_{w1})\right]
\]  
(6.27)

\[
h_{(2,1)}^-(z_{w2}) + h_{(2,2)}^+(z_{w2}) = h_{(2,3)}^+(z_{w2}) + h_{(2,3)}^-(z_{w2})
\]  
(6.28)

\[
D_{(2,2)}\left[h_{(2,2)}^+(z_{w2}) - h_{(2,2)}^-(z_{w2})\right] = D_{(2,3)}\left[h_{(2,3)}^+(z_{w2}) - h_{(2,3)}^-(z_{w2})\right]
\]  
(6.29)

### 6.1.3 Resulting S-matrix

The field dependency in the \(z\)-direction is known and described by \(e^{\pm jkzm^z}\). Because of this the homogeneous mode coefficients only need to be determined for one \(z\)-position in every waveguide region. The propagation through that region can then be described by a propagation matrix which is defined by

\[
P^{\pm}(\Delta z) = \text{diag}\left\{\{e^{\pm jkzm^z}\}_m\right\}^{M_z}
\]  
(6.30)

\(P^{\pm}(\Delta z)\) is a matrix with only diagonal elements which describe the propagation for each mode included in the truncation for that waveguide region.

A specific propagation matrix for the dielectric wall is introduced from (6.30) and is defined as

\[
P_{(2,2)} = P^\pm(z_{w2} - z_{w1}) = P^-(z_{w1} - z_{w2})
\]  
(6.31)

This leads to the following relation between the homogeneous mode coefficients at the boundaries of the dielectric wall.

\[
h_{(2,2)}^+(z_{w2}) = P_{(2,2)}h_{(2,2)}^+(z_{w1})
\]  
(6.32)

\[
h_{(2,2)}^-(z_{w1}) = P_{(2,2)}h_{(2,2)}^-(z_{w2})
\]  
(6.33)

These relations are inserted into equation (6.26) - (6.29)

\[
h_{(2,1)}^+(z_{w1}) + h_{(2,1)}^-(z_{w1}) = h_{(2,2)}^+(z_{w1}) + P_{(2,2)}h_{(2,2)}^-(z_{w2})
\]  
(6.34)

\[
D_{(2,1)}\left[h_{(2,1)}^+(z_{w1}) - h_{(2,1)}^-(z_{w1})\right] = D_{(2,2)}\left[h_{(2,2)}^+(z_{w1}) - P_{(2,2)}h_{(2,2)}^-(z_{w2})\right]
\]  
(6.35)

\[
h_{(2,3)}^+(z_{w2}) + h_{(2,3)}^-(z_{w2}) = P_{(2,2)}h_{(2,2)}^+(z_{w1}) + h_{(2,2)}^-(z_{w2})
\]  
(6.36)

\[
D_{(2,2)}\left[h_{(2,3)}^+(z_{w2}) - h_{(2,3)}^-(z_{w2})\right] = D_{(2,2)}\left[h_{(2,2)}^+(z_{w1}) - h_{(2,2)}^-(z_{w2})\right]
\]  
(6.37)

Equation (6.34) and (6.36) are rewritten and inserted into each other, which after some algebraic manipulations results in
\[ h^+_1(z_{w1}) = A^{-1}[h^+_2(z_{w1}) + h^-_2(z_{w1})] - A^{-1}P_{12} [h^+_2(z_{w2}) + h^-_2(z_{w2})] \quad (6.38) \]
\[ h^-_1(z_{w2}) = -A^{-1}P_{12} [h^+_2(z_{w1}) + h^-_2(z_{w1})] + A^{-1}[c^+_2(z_{w2}) + h^-_2(z_{w2})] \quad (6.39) \]

where

\[ A = (I - P^2_{12}) \quad (6.40) \]

and \( I \) is the \( N \times N \) sized unit matrix.

Equation (6.38) and (6.39) are inserted into (6.35) and (6.37), which after some algebraic manipulations results in

\[ D_{12} [h^+_2(z_{w1}) - h^-_2(z_{w1})] = M [h^+_2(z_{w1}) + h^-_2(z_{w1})] - G [h^+_2(z_{w2}) + h^-_2(z_{w2})] \quad (6.41) \]
\[ D_{23} [h^+_2(z_{w2}) - h^-_2(z_{w2})] = G [h^+_2(z_{w1}) + h^-_2(z_{w1})] - M [h^+_2(z_{w2}) + h^-_2(z_{w2})] \quad (6.42) \]

where

\[ M = D_{12}A^{-1} + D_{12}P_{12}A^{-1}P_{12} \quad (6.43) \]
\[ G = D_{12}A^{-1}P_{12} + D_{12}P_{12}A^{-1} \quad (6.44) \]

(6.41) and (6.42) are rearranged and written on matrix form as

\[
\begin{pmatrix}
M + D_{12} & -G \\
G & -(M + D_{23})
\end{pmatrix}
\begin{pmatrix}
h^+_2(z_{w1}) \\
h^-_2(z_{w2})
\end{pmatrix}
= \begin{pmatrix}
D_{12} - M & G \\
-G & M - D_{23}
\end{pmatrix}
\begin{pmatrix}
h^+_2(z_{w1}) \\
h^-_2(z_{w2})
\end{pmatrix}
\quad (6.45)
\]

Which lead to the desired relation for the homogeneous mode coefficients at the dielectric wall boundaries

\[
\begin{pmatrix}
h^+_2(z_{w1}) \\
h^-_2(z_{w2})
\end{pmatrix}
= L^{(w)} \begin{pmatrix}
h^+_2(z_{w1}) \\
h^-_2(z_{w2})
\end{pmatrix}
\quad (6.46)
\]

where \( L^{(w)} \) is given by

\[
L^{(w)} = \begin{pmatrix}
L^{(w)} & L^{(w)} \\
L^{(w)} & L^{(w)}
\end{pmatrix}
= \begin{pmatrix}
M + D_{12} & -G \\
G & -(M + D_{23})
\end{pmatrix}^{-1}
\begin{pmatrix}
D_{12} - M & G \\
-G & M - D_{23}
\end{pmatrix}
\quad (6.47)
\]

### 6.1.4 Propagation Shift

The geometrical properties of region 2,1 and 2,3 are the same therefore the admittance matrices for these regions are the same

\[ D_{12} = D_{12} = D_{23} \quad (6.48) \]

To determine \( L^{(2)} \) the following propagation matrices are introduced
\[ P_{(2,1)} = P_{(2,1)}^+ (z_{w1} - z_1) = P_{(2,1)}^- (z_1 - z_{w1}) \]  
\[ P_{(2,3)} = P_{(2,3)}^+ (z_2 - z_{w2}) = P_{(2,3)}^- (z_{w2} - z_2) \]  
(6.49)  
(6.50)

This leads to the following relation for the homogeneous mode coefficients in region 2,1 and 2,3

\[ h_{(2,1)}^+ (z_{w1}) = P_{(2,1)} h_{(2,1)}^- (z_1) \]  
(6.51)

\[ h_{(2,1)}^- (z_1) = P_{(2,1)} h_{(2,1)}^- (z_{w1}) \]  
(6.52)

\[ h_{(2,3)}^+ (z_2) = P_{(2,3)} h_{(2,3)}^+ (z_{w2}) \]  
(6.53)

\[ h_{(2,3)}^- (z_{w2}) = P_{(2,3)} h_{(2,3)}^- (z_2) \]  
(6.54)

These relations are inserted into (6.46), which after some rearranging lead to

\[
\begin{bmatrix}
  h_{(2,1)}^- (z_1) \\
  h_{(2,3)}^+ (z_2)
\end{bmatrix} =
\begin{bmatrix}
P_{(2,1)} L_{11}^{(w)} P_{(2,1)} & P_{(2,1)} L_{12}^{(w)} P_{(2,3)} \\
P_{(2,3)} L_{21}^{(w)} P_{(2,1)} & P_{(2,3)} L_{22}^{(w)} P_{(2,3)}
\end{bmatrix}
\begin{bmatrix}
  h_{(2,1)}^+ (z_1) \\
  h_{(2,3)}^- (z_2)
\end{bmatrix}
\]  
(6.55)

or equivalently

\[
\begin{bmatrix}
  h_{(2,1)}^- (z_1) \\
  h_{(2,3)}^+ (z_2)
\end{bmatrix} = L^{(2)}
\begin{bmatrix}
  h_{(2,1)}^+ (z_1) \\
  h_{(2,3)}^- (z_2)
\end{bmatrix}
\]  
(6.56)

where the S-matrix for region 2 is given by

\[
L^{(2)} =
\begin{bmatrix}
P_{(2,1)} L_{11}^{(w)} P_{(2,1)} & P_{(2,1)} L_{12}^{(w)} P_{(2,3)} \\
P_{(2,3)} L_{21}^{(w)} P_{(2,1)} & P_{(2,3)} L_{22}^{(w)} P_{(2,3)}
\end{bmatrix}
\]  
(6.57)

### 6.2 S-Matrix Calculation for One Corridor Segment

A detailed calculation of the S-matrix relating the mode coefficients at the boundaries of a complete corridor segment (see Figure 6.2) is determined in this section. S-matrices relating the mode coefficients excited by the sources to the mode coefficients at the external boundaries of the corridor segment are also determined. Only the TE modes are included in the calculations, with a few changes the resulting S-matrices are also valid for the TM modes. The process is the same as in section 6.1 but transitions between wave guides with different cross sections complicates the calculations somewhat.

Boundary conditions at \( z_1 \) and \( z_2 \) are set up and mode matching is used to analytically solve the resulting systems of equations. Propagations matrices are then defined for region 3 and 4 to determine the S-matrices in (6.58) for the entire corridor segment. The theory behind the process used is described in detail in section 2.3.3 and 3.4.

\[
\begin{bmatrix}
  c^- (z_{h0}) \\
  c^+ (z_h)
\end{bmatrix} = L
\begin{bmatrix}
  h^+ (z_{h0}) \\
  h^- (z_h)
\end{bmatrix} + S^{(0)}
\begin{bmatrix}
  s_{(3)}^+ (z_1) \\
  s_{(4)}^- (z_2)
\end{bmatrix} + S^{(1)}
\begin{bmatrix}
  s_{(1)}^+ (z_2) \\
  s_{(4)}^- (z_1)
\end{bmatrix} + (s_{(3)}^- (z_{h0}))
\]  
(6.58)
6.2.1 Boundary Conditions

The field expansions, mode admittance and wave numbers for the sub-regions to region 2 have already been defined in appendix 6.1 and are not repeated here.

The fields in region 1, 3 and 4 are expanded as in (3.4) and (3.5)

\[
E^{(i)}(x, z) = \sum_{m=1}^{\infty} \left[ c_{m}^{(i)+}(z) + c_{m}^{(i)-}(z) \right] \phi_{m}^{(i)}(x),
\]

\[
H^{(i)}(x, z) = \sum_{m=1}^{\infty} \left[ c_{m}^{(i)+}(z) - c_{m}^{(i)-}(z) \right] Y_{m}^{(i)} \phi_{m}^{(i)}(x),
\]

Where \( \phi_{m}^{(i)}(x) \) are defined in (3.1) - (3.3) and \( \phi_{m}^{(4)}(x) = \phi_{m}^{(3)}(x) \) as these regions have the same cross section. The mode admittance and the longitudinal wave number for region 1, 3 and 4 is given by

\[
Y_{m}^{(i)} = \frac{k_{zm}^{(i)}}{k_{0} \eta_{0}},
\]

\[
k_{zm}^{(i)} = \sqrt{k_{0} - k_{xm}^{(i)}} \quad i = 1, 3, 4
\]
where
\[
k_{\text{xm}}^{(4)} = \frac{m\pi}{a_1} \\
k_{\text{xm}}^{(3)} = \frac{m\pi}{a_0}
\]

The boundary conditions for the electric and magnetic field expansions results in the following equations (see section 2.2)

at \(z=z_1\)
\[
\sum_{m=1}^{\infty} \left[ c_m^{(3)+}(z_1) + c_m^{(3)-}(z_1) \right] \varphi_m^{(3)}(x)
= \begin{cases} 
\sum_{m=1}^{\infty} \left[ c_m^{(1)+}(z_1) + c_m^{(1)-}(z_1) \right] \varphi_m^{(1)}(x), & 0 < x < a_1 \\
0, & a_1 < x < a_2 \\
\sum_{m=1}^{\infty} \left[ c_m^{(2,1)+}(z_1) + c_m^{(2,1)-}(z_1) \right] \varphi_m^{(2)}(x), & a_0 - a_2 < x < a_0
\end{cases}
\]

at \(z=z_2\)
\[
\sum_{m=1}^{\infty} \left[ c_m^{(4)+}(z_2) + c_m^{(4)-}(z_2) \right] \varphi_m^{(3)}(x)
= \begin{cases} 
\sum_{m=1}^{\infty} \left[ c_m^{(1)+}(z_2) + c_m^{(1)-}(z_2) \right] \varphi_m^{(1)}(x), & 0 < x < a_1 \\
0, & a_1 < x < a_2 \\
\sum_{m=1}^{\infty} \left[ c_m^{(2,3)+}(z_2) + c_m^{(2,3)-}(z_2) \right] \varphi_m^{(2)}(x), & a_0 - a_2 < x < a_0
\end{cases}
\]
The electromagnetic field expansions are truncated into a finite number of modes,

\[
\sum_{m=1}^{\infty} \left[ c_m^{(4)}(z_2) - c_m^{(4)}(z_1) \right] Y_m^{(3)}(x) = \sum_{m=1}^{\infty} \left[ c_m^{(2,3)}(z_2) - c_m^{(2,3)}(z_1) \right] Y_m^{(2)}(x), \quad a_0 - a_2 < x < a_0
\]  

(6.70)

\[
\text{Equation (6.65) and (6.68) are multiplied with } Y_{m_0}^{(3)}(x) \text{ for a certain mode number } m_0 \text{ and integrated over the waveguide cross section } (0 \to a_0). \text{ The inner product (2.12) is then utilized, resulting in}
\]

\[
Y_{m_0}^{(3)} \left[ c_{m_0}^{(3)}(z_1) + c_{m_0}^{(3)}(z_2) \right] = \sum_{m=1}^{\infty} \left[ c_m^{(1)}(z_1) + c_m^{(1)}(z_2) \right] Y_m^{(3)} \left( \varphi_m^{(1)}(x) \right) \varphi_m^{(3)}(x) \right] \right] \Gamma_1
\]  

(6.71)

\[
Y_{m_0}^{(3)} \left[ c_{m_0}^{(4)}(z_1) + c_{m_0}^{(4)}(z_2) \right] = \sum_{m=1}^{\infty} \left[ c_m^{(2,1)}(z_1) + c_m^{(2,1)}(z_2) \right] Y_m^{(3)} \left( \varphi_m^{(2)}(x) \right) \varphi_m^{(3)}(x) \right] \right] \Gamma_2
\]  

(6.72)

\[
\text{Equation (6.66) and (6.69) is multiplied with } \varphi_m^{(1)}(x) \text{ for a certain mode number } m_1 \text{ and integrated over the waveguide cross section } ((a_0 - a_2) \to a_0). \text{ The inner product (2.12) is then utilized, resulting in}
\]

\[
Y_{m_1}^{(1)} \left[ c_{m_1}^{(1)}(z_1) - c_{m_1}^{(1)}(z_2) \right] = \sum_{m=1}^{\infty} \left[ c_m^{(3)}(z_1) - c_m^{(3)}(z_2) \right] Y_m^{(3)} \left( \varphi_m^{(1)}(x) \right) \varphi_m^{(3)}(x) \right] \right] \Gamma_1
\]  

(6.73)

\[
Y_{m_1}^{(1)} \left[ c_{m_1}^{(1)}(z_2) - c_{m_1}^{(1)}(z_2) \right] = \sum_{m=1}^{\infty} \left[ c_m^{(4)}(z_2) - c_m^{(4)}(z_2) \right] Y_m^{(3)} \left( \varphi_m^{(1)}(x) \right) \varphi_m^{(3)}(x) \right] \right] \Gamma_1
\]  

(6.74)

\[
\text{Finally equation (6.67) and (6.70) is multiplied with } \varphi_m^{(2)}(x) \text{ for a certain mode number } m_2 \text{ and integrated over the waveguide cross section } (0 \to a_1). \text{ The inner product (2.12) is then utilized, resulting in}
\]

\[
Y_{m_2}^{(2)} \left[ c_{m_2}^{(2,1)}(z_1) - c_{m_2}^{(2,1)}(z_1) \right] = \sum_{m=1}^{\infty} \left[ c_m^{(3)}(z_1) - c_m^{(3)}(z_2) \right] Y_m^{(3)} \left( \varphi_m^{(2)}(x) \right) \varphi_m^{(3)}(x) \right] \right] \Gamma_2
\]  

(6.75)

\[
Y_{m_2}^{(2)} \left[ c_{m_2}^{(2,3)}(z_2) - c_{m_2}^{(2,3)}(z_2) \right] = \sum_{m=1}^{\infty} \left[ c_m^{(4)}(z_2) - c_m^{(4)}(z_2) \right] Y_m^{(3)} \left( \varphi_m^{(2)}(x) \right) \varphi_m^{(3)}(x) \right] \right] \Gamma_2
\]  

(6.76)

\[
\textbf{6.2.2 Truncated Numerical Analysis}
\]

The electromagnetic field expansions are truncated into a finite number of modes, \( M \) for region 1 and 2. In region 3 and 4 the same number of modes is included, denoted by \( M_0 \), as these regions have the same cross section.

The mode coefficients for each region are collected into vectors as in (2.25)
A mode admittance matrix for each region is defined as

\[
D_i = \text{diag} \left\{ \{Y_m\}_{m=1}^{M_i} \right\}, \quad i = 1, 3, 4
\]

\(D_i\) is a diagonal matrix where the diagonal elements consist of the mode admittance for each mode.

In the systems of equations presented above the following two types of integrals appear for mode coupling between region 3 (or 4) and 1 and between region 3 (or 4) and 2

\[
\begin{align*}
\left[ \varphi_m^{(k)}(x) \right] \varphi_m^{(3)}(x) & = \int_{\Gamma_k} \varphi_m^{(k)}(x)\varphi_m^{(3)}(x)dx \\
k & = 1, 2; \quad m_i = 1, 2 \ldots M_k; \quad m_j = 1, 2 \ldots M_0
\end{align*}
\]

These mode coupling integrals are collected into two matrices, \(K_{(1)}\) and \(K_{(2)}\) with the following matrix elements

\[
K_{(k),m,m,j} = Y_m^{(3)} \left[ \varphi_m^{(k)}(x) \right] \varphi_m^{(3)}(x) \Gamma_k \\
k = 1, 2; \quad m_i = 1, 2 \ldots M_k; \quad m_j = 1, 2 \ldots M_0
\]

With the definitions above equation (6.71)-(6.76) can be rewritten as

\[
\begin{align*}
D_{(3)}(c_3^+(z_1) + c_3^-(z_1)) & = K_{(1)}^T(c_3^+(z_1) + c_1^-(z_1)) + K_{(2)}^T(c_{(2,1)}^+(z_1) + c_{(2,1)}^-(z_1)) \\
D_{(4)}(c_4^+(z_2) + c_4^-(z_2)) & = K_{(1)}^T(c_4^+(z_2) + c_1^-(z_2)) + K_{(2)}^T(c_{(2,3)}^+(z_2) + c_{(2,3)}^-(z_2)) \\
D_{(1)}(c_1^+(z_1) - c_1^-(z_1)) & = K_{(1)}(c_3^+(z_1) - c_3^-(z_1)) \\
D_{(2)}(c_2^+(z_2) - c_2^-(z_2)) & = K_{(1)}(c_4^+(z_2) - c_4^-(z_2)) \\
D_{(2)}(c_{(2,1)}^+(z_1) - c_{(2,1)}^-(z_1)) & = K_{(2)}(c_{(2,3)}^+(z_1) - c_{(2,3)}^-(z_1)) \\
D_{(2)}(c_{(2,3)}^+(z_2) - c_{(2,3)}^-(z_2)) & = K_{(2)}(c_{(4)}^+(z_2) - c_{(4)}^-(z_2))
\end{align*}
\]

where \(K_{(k)}^T\) is the transpose of \(K_{(k)}\).

The following notations for the mode coefficients in region 3 and 4 are temporarily used to simplify the calculations

\[
\begin{align*}
c_3^{(\pm)} & = c_3^{(\pm)}(z_1) + c_3^{(\pm)}(z_1) \\
c_3^{(-)} & = c_3^{(-)}(z_1) - c_3^{(-)}(z_1) \\
c_3^{(+)} & = c_3^{(+)}(z_2) + c_3^{(+)}(z_2)
\end{align*}
\]
6.2.3 Mode Coefficients in Region 1

The homogeneous propagation through a region are described by a propagation matrix defined as

$$P^\pm(\Delta z) = \text{diag}\left\{ (e^{ik_m\Delta z})^m \right\}$$  \hspace{1cm} (6.92)

$P^\pm(\Delta z)$ is a matrix with only diagonal elements which describe the propagation for each mode included in the truncation for that waveguide region.

A specific propagation matrix for region 1 is introduced as

$$P_{(1)} = P_{(1)}^+(z_2 - z_1) = P_{(1)}^-(z_1 - z_2)$$  \hspace{1cm} (6.93)

The mode coefficients on the boundaries of region one is split into two terms representing the homogeneous solutions and the contribution from the sources in region 1

$$c_{(1)}^+(z_1) = h_{(1)}^+(z_1)$$  \hspace{1cm} (6.94)

$$c_{(1)}^-(z_1) = h_{(1)}^-(z_1) + s_{(1)}^-(z_1)$$  \hspace{1cm} (6.95)

$$c_{(1)}^+(z_2) = h_{(1)}^+(z_2) + s_{(1)}^+(z_2)$$  \hspace{1cm} (6.96)

$$c_{(1)}^-(z_2) = h_{(1)}^-(z_2)$$  \hspace{1cm} (6.97)

The homogeneous mode coefficients on the boundaries of region 1 are related in the following way

$$h_{(1)}^+(z_2) = P_{(1)} h_{(1)}^+(z_1)$$  \hspace{1cm} (6.98)

$$h_{(1)}^-(z_1) = P_{(1)} h_{(1)}^-(z_2)$$  \hspace{1cm} (6.99)

(6.94) - (6.99) are inserted into (6.84) and (6.85) to solve for $h_{(1)}^+(z_1)$ and $h_{(1)}^-(z_2)$ in terms of $c_{(3)}^-$, $c_{(4)}^-$, $s_{(1)}^-$ and $s_{(1)}^+$

$$D_{(1)} \left( h_{(1)}^+(z_1) - P_{(1)} h_{(1)}^-(z_2) - s_{(1)}^-(z_1) \right) = K_{(1)} c_{(3)}^{(-)}$$  \hspace{1cm} (6.100)

$$D_{(1)} \left( P_{(1)} h_{(1)}^+(z_1) + s_{(1)}^+(z_2) - h_{(1)}^-(z_2) \right) = K_{(1)} c_{(4)}^{(-)}$$  \hspace{1cm} (6.101)

After some algebraic manipulations these two equations are rewritten as

$$h_{(1)}^+(z_1) = T_1 c_{(3)}^{(-)} - T_2 c_{(4)}^{(-)} + (I - P_{(1)}^2)^{-1} \left( P_{(1)} s_{(1)}^+(z_2) + s_{(1)}^-(z_1) \right)$$  \hspace{1cm} (6.102)

$$h_{(1)}^-(z_2) = T_2 c_{(3)}^{(-)} - T_1 c_{(4)}^{(-)} + (I - P_{(1)}^2)^{-1} \left( s_{(1)}^+(z_2) + P_{(1)} s_{(1)}^-(z_1) \right)$$  \hspace{1cm} (6.103)

where

\[ c_4^{(-)} = c_{(4)}^+(z_2) - c_{(4)}^-(z_2) \]  \hspace{1cm} (6.91)
6.2.4 Mode Coefficients in Region 2

An expression giving the homogeneous mode coefficients at the boundaries of region two, \( h_{(2,1)}^+(z_1) \) and \( h_{(2,3)}^-(z_2) \) in terms of the total mode coefficients in region 3 and 4, \( c_3^-(\cdot) \) and \( c_4^-(\cdot) \) is derived. The S-matrix relating the homogeneous mode coefficients in region 2, restated below, is utilized.

\[
\begin{align*}
T_1 &= (I - P_{(1)}^2)^{-1} D_{(1)}^{-1} K_{(1)} \quad (6.104) \\
T_2 &= (I - P_{(1)}^2)^{-1} P_{(3)} D_{(1)}^{-1} K_{(1)} \quad (6.105)
\end{align*}
\]

\[
\begin{align*}
T_1 &= (I - P_{(1)}^2)^{-1} D_{(1)}^{-1} K_{(1)} \\
T_2 &= (I - P_{(1)}^2)^{-1} P_{(3)} D_{(1)}^{-1} K_{(1)}
\end{align*}
\]

which after some algebraic manipulations become

\[
\begin{align*}
h_{(2,3)}^-(z_2) &= T_3 h_{(2,1)}^+(z_1) - T_4 c_3^-(\cdot) \quad (6.110) \\
h_{(2,1)}^+(z_1) &= T_5 h_{(2,3)}^-(z_2) + T_6 c_4^-(\cdot) \quad (6.111)
\end{align*}
\]

where

\[
\begin{align*}
T_3 &= (L_{12}^{(2)})^{-1} (I - L_{11}^{(2)}) \quad (6.112) \\
T_4 &= (L_{12}^{(2)})^{-1} D_{(1)}^{-1} K_{(2)} \quad (6.113) \\
T_5 &= (L_{21}^{(2)})^{-1} (I - L_{22}^{(2)}) \quad (6.114) \\
T_6 &= (L_{21}^{(2)})^{-1} D_{(2)}^{-1} K_{(2)} \quad (6.115)
\end{align*}
\]

To determine the desired relation (6.110) are inserted into (6.111) and vice versa resulting in

\[
\begin{align*}
h_{(2,3)}^-(z_2) &= T_3 \left( T_5 h_{(2,3)}^-(z_2) + T_6 c_4^-(\cdot) \right) - T_4 c_3^-(\cdot) \quad (6.116)
\end{align*}
\]
\[ h_{(2,1)}^+(z_1) = T_5 \left( T_3 h_{(2,1)}^+(z_1) - T_4 c_3^- \right) + T_6 c_4^- \]  

which finally become

\[ h_{(2,1)}^+(z_1) = -T_9 c_3^- + T_{10} c_4^-(6.118) \]
\[ h_{(2,3)}^- (z_2) = -T_7 c_3^- + T_8 c_4^- \]  

where

\[ T_7 = (I - T_3 T_5)^{-1} T_4 \]  
\[ T_9 = (I - T_3 T_5)^{-1} T_3 T_6 \]  
\[ T_{10} = (I - T_5 T_3)^{-1} T_5 T_4 \]  
\[ T_{11} = (I - T_5 T_3)^{-1} T_6 \]  

### 6.2.5 Resulting S-matrix

Equation (6.82) and (6.83) are used to get the backscattered and transmitted homogeneous mode coefficients \( h_{(3)}^+(z_1) \) and \( h_{(4)}^+(z_2) \) in terms of the homogeneous mode coefficients \( h_{(3)}^+(z_1) \) and \( h_{(4)}^+(z_2) \) as well as the mode coefficients excited by the sources in region 1, 3 and 4.

The homogeneous mode coefficients on the boundaries of region 1 and 2 (restated below) are inserted into (6.82) and (6.83)

\[ c_{(1)}^+(z_1) = h_{(1)}^+(z_1) \]  
\[ c_{(1)}^-(z_2) = h_{(1)}^-(z_2) \]  
\[ c_{(2,1)}^+(z_1) = h_{(2,1)}^+(z_1) \]  
\[ c_{(2,3)}^+(z_2) = h_{(2,3)}^+(z_2) \]  

resulting in

\[ D_{(3)} c_{(3)}^+ = K_{(1)}^T (h_{(1)}^+(z_1) + c_{(1)}^+(z_1)) + K_{(2)}^T (h_{(2,1)}^+(z_1) + h_{(2,1)}^-(z_1)) \]  
\[ D_{(4)} c_{(4)}^+ = K_{(1)}^T (c_{(1)}^+(z_2) + h_{(1)}^-(z_2)) + K_{(2)}^T (h_{(2,3)}^+(z_2) + h_{(2,3)}^-(z_2)) \]  

The expressions for the mode coefficients on the right hand side of the above two equations have been determined in the previous sections as well as in appendix 6.1 and are given by

\[ h_{(1)}^+(z_1) = T_1 c_3^- - T_2 c_4^- + (I - P_{(1)}^2)^{-1} \left( P_{(1)} s_{(1)}^+(z_2) + s_{(1)}^-(z_1) \right) \]  
\[ h_{(1)}^-(z_2) = T_2 c_3^- - T_1 c_4^- + (I - P_{(1)}^2)^{-1} \left( s_{(1)}^+(z_2) + P_{(1)} s_{(1)}^-(z_1) \right) \]
\[ c^{(1)}(z_1) = P_{(1)} h^{(1)}_{(1)}(z_2) + s^{(1)}_{(1)}(z_1) \quad (6.132) \]
\[ c^{(1)}_t(z_2) = P_{(1)} h^{+}_{(1)}(z_1) + s^{+}_{(1)}(z_2) \quad (6.133) \]

and

\[ h^{(1)}_{(2,1)}(z_1) = L^{(2)}_{11} h^{+}_{(2,1)}(z_1) + L^{(2)}_{12} h^{(2)}_{(2,3)}(z_2) \quad (6.134) \]
\[ h^{(1)}_{(2,3)}(z_2) = L^{(2)}_{21} h^{+}_{(2,1)}(z_1) + L^{(2)}_{22} h^{(2)}_{(2,3)}(z_2) \quad (6.135) \]
\[ h^{+}_{(2,1)}(z_1) = -T_9 c^{(-)}_3 + T_{10} c^{(-)}_4 \quad (6.136) \]
\[ h^{+}_{(2,3)}(z_2) = -T_7 c^{(-)}_3 + T_8 c^{(-)}_4 \quad (6.137) \]

These are inserted into (6.128) and (6.129), resulting in two extensive algebraic expressions, but by inserting the equations two at a time it is manageable, only the end result is given here:

\[ D_{(0)} c^{+}_{(3)} = U_1 c^{(-)}_3 + U_2 c^{(-)}_4 + U_5 s^{(1)}_1(z_2) + U_6 s^{(1)}_1(z_1) \quad (6.138) \]
\[ D_{(0)} c^{+}_{(4)} = U_3 c^{(-)}_3 + U_4 c^{(-)}_4 + U_5 s^{(1)}_1(z_2) + U_6 s^{(1)}_1(z_1) \quad (6.139) \]

where

\[ D_{(0)} = D_{(3)} = D_{(4)} \quad (6.140) \]

have been used as the geometrical properties of region 3 and 4 are the same hence the admittance matrices for these regions are the same,

The part matrices are given by

\[ U_1 = K_{(1)}^T \left[ T_1 + P_{(1)} T_2 - K_{(2)}^T \left[ L^{(2)}_{12} T_7 + \left( I + L^{(2)}_{11} \right) T_9 \right] \right] \quad (6.141) \]
\[ U_2 = -K_{(1)}^T \left[ T_2 + P_{(1)} T_1 \right] + K_{(2)}^T \left[ L^{(2)}_{12} T_8 + \left( I + L^{(2)}_{11} \right) T_{10} \right] \quad (6.142) \]
\[ U_3 = K_{(1)}^T \left[ T_2 + P_{(1)} T_1 \right] - K_{(2)}^T \left[ L^{(2)}_{21} T_9 + \left( I + L^{(2)}_{22} \right) T_7 \right] \quad (6.143) \]
\[ U_4 = -K_{(1)}^T \left[ T_1 + P_{(1)} T_2 \right] + K_{(2)}^T \left[ L^{(2)}_{21} T_{10} + \left( I + L^{(2)}_{22} \right) T_8 \right] \quad (6.144) \]
\[ U_5 = K_{(1)}^T \left[ \left( I - P^2_{(1)} \right)^{-1} P_{(1)} + P_{(1)} (I - P^2_{(1)})^{-1} \right] \quad (6.145) \]
\[ U_6 = K_{(1)}^T \left[ P_{(1)} \left( I - P^2_{(1)} \right)^{-1} P_{(1)} + \left( I - P^2_{(1)} \right)^{-1} + I \right] \quad (6.146) \]

where \( T_1 \) - \( T_{10} \) are given by equation (6.104) - (6.105), (6.112) - (6.115) and (6.120) - (6.123) and \( L^{(2)} \) is given by (6.57)

On the boundaries \( z_1 \) and \( z_2 \) the mode coefficients from region 3 and 4 are split into two terms representing the homogeneous solutions and the contribution from the sources in those regions

\[ c^{+}_{(3)}(z_1) = h^{+}_{(3)}(z_1) + s^{+}_{(3)}(z_1) \quad (6.147) \]
\[ c^{-}_{(3)}(z_1) = h^{-}_{(3)}(z_1) \quad (6.148) \]
\[ c_{(4)}^-(z_2) = h_{(4)}^-(z_2) + s_{(4)}^-(z_2) \] (6.149)

\[ c_{(4)}^+(z_2) = h_{(4)}^+(z_2) \] (6.150)

These relations are inserted in the temporary notations defined in (6.88) - (6.91) resulting in

\[ c_3^{(-)} = h_3^+(z_1) + s_3^+(z_1) - h_{(3)}^-(z_1) \] (6.151)

\[ c_4^{(-)} = h_{(4)}^+(z_2) - h_{(4)}^-(z_2) - s_{(4)}^-(z_2) \] (6.152)

\[ c_3^{(+)} = h_3^+(z_1) + s_3^+(z_1) + h_{(3)}^-(z_1) \] (6.153)

\[ c_4^{(+)} = h_{(4)}^+(z_2) + h_{(4)}^-(z_2) + s_{(4)}^-(z_2) \] (6.154)

These are inserted into the equations (6.138) and (6.139) which after some algebraic manipulations become

\[
\begin{align*}
[U_1 + D(0)]h_3^-(z_1) - U_2 h_{(4)}^+(z_2) \\
= [U_1 - D(0)]h_3^+(z_1) - U_2 h_{(4)}^-(z_2) + [U_1 - D(0)]s_{(3)}^+(z_1) \\
- U_2 s_3^-(z_2) + U_5 s_{(1)}^+(z_2) + U_6 s_{(1)}^-(z_1)
\end{align*}
\] (6.155)

\[
U_3 h_{(3)}^+(z_1) + [D(0) - U_4]h_{(4)}^+(z_2) \\
= U_3 h_{(3)}^+(z_1) - [U_4 + D(0)]h_{(4)}^+(z_2) + U_3 s_{(3)}^+(z_1) \\
- [U_4 + D(0)]s_{(4)}^-(z_2) + U_5 s_{(1)}^+(z_2) + U_6 s_{(1)}^-(z_1)
\] (6.156)

and finally on matrix form

\[
R_1 \begin{pmatrix} h_3^-(z_1) \\ h_{(4)}^+(z_2) \end{pmatrix} = R_2 \begin{pmatrix} h_3^+(z_1) \\ h_{(4)}^-(z_2) \end{pmatrix} + R_2 \begin{pmatrix} s_{(3)}^+(z_1) \\ s_{(4)}^-(z_2) \end{pmatrix} + R_3 \begin{pmatrix} s_{(1)}^+(z_1) \\ s_{(1)}^-(z_1) \end{pmatrix}
\] (6.157)

where

\[
R_1 = \begin{pmatrix} U_1 + D(0) & -U_2 \\ U_3 & D(0) - U_4 \end{pmatrix}
\] (6.158)

\[
R_2 = \begin{pmatrix} [U_1 - D(0)] & -U_2 \\ U_3 & -D(0) - U_4 \end{pmatrix}
\] (6.159)

\[
R_3 = \begin{pmatrix} U_5 \\ U_6 \\ U_5 \end{pmatrix}
\] (6.160)

and the S-matrices for the region in between \( z_1 \) and \( z_2 \) is given by

\[
\begin{pmatrix} h_{(3)}^-(z_1) \\ h_{(4)}^+(z_2) \end{pmatrix} = R_1^{-1} R_2 \begin{pmatrix} h_{(3)}^+(z_1) \\ h_{(4)}^-(z_2) \end{pmatrix} + R_1^{-1} R_2 \begin{pmatrix} s_{(3)}^+(z_1) \\ s_{(4)}^-(z_2) \end{pmatrix} + R_1^{-1} R_3 \begin{pmatrix} s_{(1)}^+(z_1) \\ s_{(1)}^-(z_1) \end{pmatrix}
\] (6.161)

The following S-matrices are temporarily used to calculate the propagation shift from \( z_1 \) and \( z_2 \) to \( z_{h0} \) and \( z_h \)

\[
\begin{pmatrix} h_{(3)}^-(z_1) \\ h_{(4)}^+(z_2) \end{pmatrix} = L^{(i)} \begin{pmatrix} h_{(3)}^+(z_1) \\ h_{(4)}^-(z_2) \end{pmatrix} + L^{(i)} \begin{pmatrix} s_{(3)}^+(z_1) \\ s_{(4)}^-(z_2) \end{pmatrix} + s^{(i)} \begin{pmatrix} s_{(1)}^+(z_1) \\ s_{(1)}^-(z_1) \end{pmatrix}
\] (6.162)

where

\[
L^{(i)} = R_1^{-1} R_2
\] (6.163)
6.2.6 Propagation Shift

The following propagation matrices for region 3 and 4 are introduced

\[
P_{(h_0)} = P^+_{(3)}(z_1 - z_{h_0}) = P^+_{(3)}(z_{h_0} - z_1) \quad (6.165)
\]

\[
P_{(h)} = P^+_{(4)}(z_2 - z_h) = P^+_{(4)}(z_h - z_2) \quad (6.166)
\]

This leads to the following relation for the homogeneous mode coefficients in region 3 and 4

\[
h^+_3(z_1) = p_{(h_0)} h^+(z_{h_0}) \quad (6.167)
\]

\[
h^-_{h_0}(z_h) = p_{(h_0)} h^-(z_h) \quad (6.168)
\]

\[
h^+(z_h) = p_{(h)} h^+(z_2) \quad (6.169)
\]

\[
h^-_{(4)}(z_2) = p_{(h)} h^-_{(4)}(z_h) \quad (6.170)
\]

These expressions are manipulated and inserted into (6.162), which after some algebraic manipulations result in

\[
\begin{pmatrix} h^-_{(3)}(z_0) \\ h^+(z_0) \end{pmatrix} = L \begin{pmatrix} h^+(z_{h_0}) \\ h^-_{(h)} \end{pmatrix} + S^{(0)} \begin{pmatrix} s^+_{(3)}(z_1) \\ s^+_{(4)}(z_2) \end{pmatrix} + S^{(1)} \begin{pmatrix} s^+_{(3)}(z_1) \\ s^+_{(4)}(z_2) \end{pmatrix} \quad (6.171)
\]

where the following notation for the homogeneous mode coefficients at the boundaries of the entire corridor segment have been introduced. This is done to clearly separate the calculations for a corridor segment from the calculations for the cascade coupling of different corridor segments.

\[
h^\pm_{(h_0)}(z_{h_0}) = h^\pm_{(3)}(z_{h_0}) \quad (6.172)
\]

\[
h^\pm_{(h)}(z_h) = h^\pm_{(4)}(z_h) \quad (6.173)
\]

The S-matrices are given by

\[
L = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{21} \end{pmatrix} = \begin{pmatrix} P_{(h_0)} L^{(1)}_{11} P_{(h_0)} & P_{(h_0)} L^{(1)}_{12} P_{(h)} \\ P_{(h)} L^{(1)}_{21} P_{(h_0)} & P_{(h)} L^{(1)}_{22} P_{(h)} \end{pmatrix} \quad (6.174)
\]

\[
S^{(0)} = \begin{pmatrix} S^{(0)}_{11} & S^{(0)}_{12} \\ S^{(0)}_{21} & S^{(0)}_{22} \end{pmatrix} = \begin{pmatrix} P_{(h_0)} L^{(1)}_{11} P_{(h_0)} & P_{(h_0)} L^{(1)}_{12} \\ P_{(h)} L^{(1)}_{21} P_{(h_0)} & P_{(h)} L^{(1)}_{22} \end{pmatrix} \quad (6.175)
\]

\[
S^{(1)} = \begin{pmatrix} S^{(1)}_{11} & S^{(1)}_{12} \\ S^{(1)}_{21} & S^{(1)}_{22} \end{pmatrix} = \begin{pmatrix} P_{(h_0)} S^{(1)}_{11} P_{(h_0)} & P_{(h_0)} S^{(1)}_{12} \\ P_{(h)} S^{(1)}_{21} P_{(h_0)} & P_{(h)} S^{(1)}_{22} \end{pmatrix} \quad (6.176)
\]

The total mode coefficients on the external boundaries with the inclusion of the coefficients excited by the sources in region 3 and 4 is given by
\[ c^-(z_{h0}) = h^-(z_{h0}) + s_{(3)}^-(z_{h0}) \]  \hspace{1cm} (6.177)
\[ c^+(z_h) = h^+(z_h) + s_{(4)}^+(z_h) \]  \hspace{1cm} (6.178)

These are inserted into (6.171) leading to
\[
\begin{pmatrix}
  c^-(z_{h0}) \\
  c^+(z_h)
\end{pmatrix}
= L
\begin{pmatrix}
  h^+(z_{h0}) \\
  h^-(z_h)
\end{pmatrix}
+ S(0)
\begin{pmatrix}
  s_{(3)}^+(z_1) \\
  s_{(4)}^+(z_2)
\end{pmatrix}
+ S(1)
\begin{pmatrix}
  s_{(1)}^+(z_2) \\
  s_{(1)}^+(z_1)
\end{pmatrix}
+ (s_{(3)}^-)(z_{h0})
+ (s_{(4)}^-)(z_h)
\]  \hspace{1cm} (6.179)

which is the desired S-matrix relation for the entire corridor segment.

In order to simplify the cascade coupling calculations all sources in one corridor segment are collected into two vectors as
\[
\begin{pmatrix}
  c^-(z_{h0}) \\
  c^+(z_h)
\end{pmatrix}
= L
\begin{pmatrix}
  h^+(z_{h0}) \\
  h^-(z_h)
\end{pmatrix}
+ s^-(z_{h0})
+ s^+(z_h)
\]  \hspace{1cm} (6.180)

where \( s^-(z_{h0}) \) and \( s^+(z_h) \) is the source contribution to the modes propagating in the negative and positive \( z \)-direction respectively. The resulting relation for the total mode coefficients of one corridor segment then becomes
\[
\begin{pmatrix}
  c^-(z_{h0}) \\
  c^+(z_h)
\end{pmatrix}
= L
\begin{pmatrix}
  h^+(z_{h0}) \\
  h^-(z_h)
\end{pmatrix}
+ s^-(z_{h0})
+ s^+(z_h)
\]  \hspace{1cm} (6.181)

6.2.7 Determining the Internal Mode Coefficients

To determine the electromagnetic field in each waveguide region the homogeneous mode coefficients need to be determined. An expression for the mode coefficients on the external boundaries are given in the previous section and these are valid in region 3 and 4. In the process of determining the S-matrices, expressions for the internal homogeneous mode coefficients in region 1 and 2 have been found, these are restated below.

The homogeneous mode coefficients on the external boundaries are propagated in to the \( z_1 \) and \( z_2 \) with the propagation matrices, \( P_{(h0)} \) and \( P_{(h)} \).

Equation (6.102) and (6.103) give two of the homogeneous mode coefficients in region 1 as
\[
h_{(1)}^+(z_1) = T_1 c_3^+ - T_2 c_4^+ + (I - P_{(1)}^2)^{-1} \left( P_{(1)} s_{(1)}^+(z_2) + s_{(1)}^-(z_1) \right) \]  \hspace{1cm} (6.182)
\[
h_{(1)}^-(z_2) = T_2 c_3^- - T_1 c_4^- + (I - P_{(1)}^2)^{-1} \left( s_{(1)}^+(z_2) + P_{(1)} s_{(1)}^-(z_1) \right) \]  \hspace{1cm} (6.183)

and the other two are determined by \( P_{(1)} \).

Equation (6.118) and (6.119) give two of the homogeneous mode coefficients in region 2,1 and 2,3 as
\[
h_{(2,1)}^+(z_1) = -T_0 c_3^- + T_{10} c_4^- \]  \hspace{1cm} (6.184)
\[ h_{(2,3)}^{(2)}(z_2) = -T_c c_i^{(-)} + T_b c_i^{(-)} \]  
\hspace{1cm} (6.185)

and the other two are given by the S-matrix for region 2 (6.56) as

\[
\begin{pmatrix}
    h_{(2,1)}^{(2)}(z_1) \\
    h_{(2,3)}^{(2)}(z_2)
\end{pmatrix} = L^{(2)} \begin{pmatrix}
    h_{(2,1)}^{(1)}(z_1) \\
    h_{(2,3)}^{(1)}(z_2)
\end{pmatrix}
\hspace{1cm} (6.186)

The mode coefficients in region 2,2 are given by equation (6.38) and (6.39):

\[
h_{(2,2)}^+(z_{w1}) = A^{-1}[h_{(2,1)}^+(z_{w1}) + h_{(2,1)}^-(z_{w1})] - A^{-1}P_{(2,2)}[h_{(2,3)}^+(z_{w2}) + h_{(2,3)}^-(z_{w2})] \hspace{1cm} (6.187)
\]

\[
h_{(2,2)}^-(z_{w2}) = -A^{-1}P_{(2,2)}[h_{(2,1)}^+(z_{w1}) + h_{(2,1)}^-(z_{w1})] + A^{-1}P_{(2,2)}[h_{(2,3)}^+(z_{w2}) + h_{(2,3)}^-(z_{w2})] \hspace{1cm} (6.188)
\]

where the homogeneous mode coefficients in region 2,1 and 2,3 have been propagated in to \( z_{w1} \) and \( z_{w2} \).

All the part-matrices in the above expressions have been calculated in appendix 6.1 and 6.2 and complete expressions for them are given in each section.

**6.3 Mode-Coupling Integral Solution**

The mode coupling integral from equation (2.35) is solved for the transversal mode expansions used for excitation of TE modes. An analogous solution with the cosine function needs to be done to handle mode coupling of TM modes.

The mode coupling integral to be solved is

\[
\int_{x_1}^{x_2} \varphi_m^{(k)}(x)\varphi_m^{(3)}(x)dx, \hspace{0.5cm} k = 1,2 \hspace{1cm} (6.189)
\]

where \( \varphi_m^{(k)} \) and \( \varphi_m^{(3)} \) are given by

\[
\varphi_m^{(1)}(x) = \frac{2}{a_1} \sin \left( \frac{mn}{a_0} x \right) \hspace{1cm} 0 < x < a_1 \hspace{1cm} (6.190)
\]

\[
\varphi_m^{(2)}(x) = \frac{2}{a_2} \sin \left( \frac{mn}{a_2} (a_0 - x) \right) \hspace{1cm} a_0 - a_2 < x < a_0 \hspace{1cm} (6.191)
\]

\[
\varphi_m^{(3)}(x) = \frac{2}{a_0} \sin \left( \frac{mn}{a_0} x \right) \hspace{1cm} 0 < x < a_0 \hspace{1cm} (6.192)
\]

A general form for the mode-coupling integral is

\[
\int_{x_1}^{x_2} \sin(Ax + q_1) \sin(Bx) \, dx. \hspace{1cm} (6.193)
\]
To solve the integral, the following trigonometric identity is utilized:

$$\sin(Ax + q_1) \sin(Bx) = \frac{1}{2} \left[ \cos(Ax + q_1 - Bx) - \cos(Ax + q_1 + Bx) \right].$$

(6.194)

(6.194) is inserted into (6.193)

$$\frac{1}{2} \int_{x_1}^{x_2} \left[ \cos((A - B)x + q_1) - \cos((A + B)x + q_1) \right] dx$$

(6.195)

This integral is solved analytically resulting in

$$\frac{1}{2} \int_{x_1}^{x_2} \left[ \frac{\sin((A - B)x + q_1)}{(A - B)} - \frac{\sin((A + B)x + q_1)}{(A + B)} \right] dx$$

(6.196)

The limits are inserted and the resulting expression is

$$\frac{1}{2} \left\{ \frac{\sin((A - B)x_2 + q_1)}{(A - B)} - \frac{\sin((A - B)x_1 + q_1)}{(A - B)} \right\}$$

$$+ \frac{\sin((A + B)x_1 + q_1)}{(A + B)} - \frac{\sin((A + B)x_2 + q_1)}{(A + B)} \right\},$$

(6.197)

(6.197) is the solution to the mode coupling integral stated in (6.189) where \(x_1\) and \(x_2\) are the coordinates for the cross section the integral is taken over. \(A\) and \(B\) represent the transversal wave number, \(k^{(i)}_{x_\text{m}}\), these are all identified from the expansions in (6.190) - (6.192). \(q_1\) are identified as the constant term from the sine-function in (6.191).

### 6.4 Cascade Coupling of Corridor Segments

Two corridor segments are cascade coupled to determine the elements of the resulting S-matrices, the calculations the based on the theory discussed in section 2.3. In the corridor model proposed in this thesis sources in different corridor segments are modeled therefore additional S-matrices need to be introduced. As illustrated in Figure 6.3 expressions for the internal as well as the external total mode coefficients are determined.

![Figure 6.3: Internal and external mode coefficient for two corridor segments.](image-url)
In Figure 6.4 two cascade coupled cells are depicted as well as the matrices included in the total cascade coupling. The S-matrix relating the homogeneous mode coefficients and the source coefficients are defined as $L^{(V2)}$ and $S^{V2m}$ respectively. The latter determines how the sources in one corridor segment are coupled along the corridor to the other corridor segments. $m$ specifies in which of the two cascade coupled cells sources are present hence $m$ only take the value 1 or 2.

The mode coefficients on the internal boundary between the two corridor segments have to be equal for the electromagnetic field to be continuous. The mode coefficients on the external boundaries of a corridor segment have been redefined from the coefficients in region 3 and 4 of their respective corridor segments, see (6.172) and (6.173). For example, one of the internal mode coefficients in corridor segment 1 is

$$c^-(z_{h1}) = h_{(4)}^{-}(z_{h1})$$

(6.198)

And in corridor segment 2

$$c^-(z_{h1}) = h_{(3)}^{-}(z_{h1}) + s_{(3)}^{-}(z_{h1})$$

(6.199)

When cascade coupling corridor segments, the total mode coefficients, $c^\pm(z_{hn})$ are used. They are separated when the mode coefficients for the regions in each corridor segment are determined.

Two corridor segments have the following relation for the mode coefficients (see appendix 6.2)

$$
\begin{pmatrix}
  c^-(z_{h0}) \\
  c^+(z_{h1})
\end{pmatrix}
= L^{(1)}
\begin{pmatrix}
  c^+(z_{h0}) \\
  c^-(z_{h1})
\end{pmatrix}
+ S^{(V11)}
\begin{pmatrix}
  s^-(z_{h0}) \\
  s^+(z_{h1})
\end{pmatrix}
$$

(6.200)

$$
\begin{pmatrix}
  c^-(z_{h1}) \\
  c^+(z_{h2})
\end{pmatrix}
= L^{(2)}
\begin{pmatrix}
  c^+(z_{h1}) \\
  c^-(z_{h2})
\end{pmatrix}
+ S^{(V12)}
\begin{pmatrix}
  s^-(z_{h1}) \\
  s^+(z_{h2})
\end{pmatrix}
$$

(6.201)

$S^{(V11)}$ and $S^{(V12)}$ is defined as
The reason for including $S^{V1m}$ is so that the total cascade coupled matrices can be expressed in terms of an already known matrix. By doing this an iterative process can be created where every corridor segment are cascade coupled together easily.

(6.200) and (6.201) are written on equation form as

$$
\begin{align*}
\begin{bmatrix}
\mathcal{S}^{(V11)}_{11} & \mathcal{S}^{(V11)}_{12} \\
\mathcal{S}^{(V11)}_{21} & \mathcal{S}^{(V11)}_{22}
\end{bmatrix} &= \begin{bmatrix}
\mathcal{S}^{(V12)}_{11} & \mathcal{S}^{(V12)}_{12} \\
\mathcal{S}^{(V12)}_{21} & \mathcal{S}^{(V12)}_{22}
\end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}
\end{align*}
$$

(6.202)

6.4.1 Internal mode coefficients

An expression for the internal boundary coefficients between the two cascade coupled cells is determined by inserting equation (6.204) into (6.205) and vice versa. The resulting expressions are rearranged which is a straightforward process therefore only the end result is stated here:

$$
\begin{align*}
\begin{bmatrix}
\mathcal{C}^{-}(z_{h1}) \\
\mathcal{C}^{+}(z_{h1})
\end{bmatrix} &= \mathcal{L}^{(M1)} \begin{bmatrix}
\mathcal{C}^{+}(z_{h0}) \\
\mathcal{C}^{-}(z_{h2})
\end{bmatrix} + \mathcal{S}^{(M11)} \begin{bmatrix}
\mathcal{S}^{-}(z_{h0}) \\
\mathcal{S}^{+}(z_{h1})
\end{bmatrix} + \mathcal{S}^{(M12)} \begin{bmatrix}
\mathcal{S}^{-}(z_{h1}) \\
\mathcal{S}^{+}(z_{h2})
\end{bmatrix}
\end{align*}
$$

(6.207)

As sources are only present in one cell (6.207) can also be expressed as

$$
\begin{align*}
\begin{bmatrix}
\mathcal{C}^{-}(z_{h1}) \\
\mathcal{C}^{+}(z_{h1})
\end{bmatrix} &= \mathcal{L}^{(M1)} \begin{bmatrix}
\mathcal{C}^{+}(z_{h0}) \\
\mathcal{C}^{-}(z_{h2})
\end{bmatrix} + \mathcal{S}^{(M1m)} \begin{bmatrix}
\mathcal{S}^{-}(z_{hm-1}) \\
\mathcal{S}^{+}(z_{hm})
\end{bmatrix}
\end{align*}
$$

(6.208)

where $m=1$ if the sources are in cell one and $m=2$ if the sources are in cell 2.

The elements of $\mathcal{L}^{(M1)}$ are given by

$$
\begin{align*}
\mathcal{L}^{(M1)}_{11} &= \left( I - \mathcal{L}^{(2)}_{11} \mathcal{L}^{(1)}_{22} \right)^{-1} \mathcal{L}^{(2)}_{11} \mathcal{L}^{(1)}_{21} \\
\mathcal{L}^{(M1)}_{12} &= \left( I - \mathcal{L}^{(2)}_{11} \mathcal{L}^{(1)}_{22} \right)^{-1} \mathcal{L}^{(2)}_{12} \\
\mathcal{L}^{(M1)}_{21} &= \left( I - \mathcal{L}^{(2)}_{22} \mathcal{L}^{(1)}_{11} \right)^{-1} \mathcal{L}^{(2)}_{21} \\
\mathcal{L}^{(M1)}_{22} &= \left( I - \mathcal{L}^{(2)}_{22} \mathcal{L}^{(1)}_{11} \right)^{-1} \mathcal{L}^{(2)}_{22} \mathcal{L}^{(1)}_{12}
\end{align*}
$$

(6.209-6.212)

And the elements of $\mathcal{S}^{(M11)}$ and $\mathcal{S}^{(M12)}$ (where the definition in (6.202) have been inserted)

$$
\mathcal{S}^{(M11)}_{11} = 0
$$

(6.213)
\[
S_{12}^{(M11)} = (I - L_{11}^{(2)} L_{22}^{(1)})^{-1} L_{11}^{(2)} S_{22}^{(V11)}
\]

\[
S_{21}^{(M11)} = 0
\]

\[
S_{22}^{(M11)} = (I - L_{22}^{(1)} L_{11}^{(2)})^{-1} S_{22}^{(V11)}
\]

\[
S_{11}^{(M12)} = (I - L_{11}^{(2)} L_{22}^{(1)})^{-1} S_{11}^{(V12)}
\]

\[
S_{12}^{(M12)} = 0
\]

\[
S_{21}^{(M12)} = (I - L_{22}^{(1)} L_{11}^{(2)})^{-1} L_{22}^{(1)} S_{11}^{(V12)}
\]

\[
S_{22}^{(M12)} = 0
\]

### 6.4.2 External mode coefficients

The total matrices for the two cascade coupled corridor segments are derived by inserting the relations found in the previous section (6.207) into equation (6.203) and (6.206) which result in

\[
\begin{pmatrix}
  c^-(z_{h0}) \\
  c^+(z_{h2})
\end{pmatrix} = L^{(V2)} \begin{pmatrix}
  c^+(z_{h0}) \\
  c^-(z_{h2})
\end{pmatrix} + S^{(V21)} \begin{pmatrix}
  s^-(z_{h0}) \\
  s^+(z_{h1})
\end{pmatrix} + S^{(V22)} \begin{pmatrix}
  s^-(z_{h1}) \\
  s^+(z_{h2})
\end{pmatrix}
\]

As sources are only present in one cell (6.221) can also be expressed as

\[
\begin{pmatrix}
  c^-(z_{h0}) \\
  c^+(z_{h2})
\end{pmatrix} = L^{(V2)} \begin{pmatrix}
  c^+(z_{h0}) \\
  c^-(z_{h2})
\end{pmatrix} + S^{(V2m)} \begin{pmatrix}
  s^-(z_{hm-1}) \\
  s^+(z_{hm})
\end{pmatrix}
\]

where \(m=1\) if the sources are in cell one and \(m=2\) if the sources are in cell 2.

The elements of \(L^{(V2)}\) are given by

\[
L_{11}^{(V2)} = L_{11}^{(1)} + L_{12}^{(1)} \left( I - L_{11}^{(2)} L_{22}^{(1)} \right)^{-1} L_{11}^{(2)} L_{21}^{(1)}
\]

\[
L_{12}^{(V2)} = L_{12}^{(1)} \left( I - L_{11}^{(2)} L_{22}^{(1)} \right)^{-1} L_{12}^{(2)}
\]

\[
L_{21}^{(V2)} = L_{21}^{(2)} \left( I - L_{22}^{(1)} L_{11}^{(2)} \right)^{-1} L_{21}^{(1)}
\]

\[
L_{22}^{(V2)} = L_{22}^{(2)} + L_{21}^{(2)} \left( I - L_{22}^{(1)} L_{11}^{(2)} \right)^{-1} L_{22}^{(1)} L_{12}^{(2)}
\]

And the elements of \(S^{(V21)}\) and \(S^{(V22)}\) (where the definition in (6.202) have been inserted)

\[
S_{11}^{(V21)} = I
\]
\[ S_{12}^{(V21)} = S_{12}^{(V11)} + L_{12}^{(1)} \left( I - L_{11}^{(2)} L_{22}^{(1)} \right)^{-1} L_{11}^{(2)} S_{22}^{(V11)} \]  
(6.228)

\[ S_{21}^{(V21)} = 0 \]  
(6.229)

\[ S_{22}^{(V21)} = L_{21}^{(2)} \left( I - L_{22}^{(1)} L_{11}^{(2)} \right)^{-1} S_{22}^{(V11)} \]  
(6.230)

\[ S_{11}^{(V22)} = L_{12}^{(1)} \left( I - L_{11}^{(2)} L_{22}^{(1)} \right)^{-1} S_{11}^{(V12)} \]  
(6.231)

\[ S_{12}^{(V22)} = 0 \]  
(6.232)

\[ S_{21}^{(V22)} = S_{21}^{(V12)} + L_{21}^{(2)} \left( I - L_{22}^{(1)} L_{11}^{(2)} \right)^{-1} L_{22}^{(1)} S_{11}^{(V12)} \]  
(6.233)

\[ S_{22}^{(V22)} = I \]  
(6.234)

### 6.4.3 Multiple Corridor Segments

Both the matrices for the internal coefficients \((L^{(M1)}, S^{(M1m)})\) and the external \((L^{(V2)}, S^{(V2m)})\) are expressed in terms of the homogeneous S-matrices for the two cascade coupled cells and \(S^{(V)}\). This is utilized to create an iterative process that cascade couple two corridor segments together into one cell and then couples that new cell with the third corridor segment and so forth to construct a matrix for the entire corridor.

![Figure 6.5: Four cascade coupled corridor segments with a source in cell 2, region 4, where \(L^{(V)} = L^{(1)}\) and \(L^{(H)} = L^{(4)}\)](image-url)

An example of this is illustrated in Figure 6.5 with four corridor segments. The corridor segments are cascade coupled from both the left and the right of the corridor. The direction is indicated in the matrices by the superscript V and H respectively. The matrix relating the
source terms ($S$) are only calculated when the corridor segment containing sources are one of the two cells being cascade coupled. The arrows in the figure indicate the internal boundaries, and the matrices stated to the left and to the right are needed to determine the mode coefficients at each boundary.

The relation in (6.222) determines the mode coefficients on the external boundaries of the entire corridor. The expression for the mode coefficients of the corridor in the example is

$$
\begin{pmatrix}
  c^{-}(z_{h0}) \\
  c^{+}(z_{h4})
\end{pmatrix} = L^{(V4)} \begin{pmatrix}
  c^{+}(z_{h0}) \\
  c^{-}(z_{h4})
\end{pmatrix} + S^{(V4)} \begin{pmatrix}
  s^{-}(z_{h1}) \\
  s^{+}(z_{h2})
\end{pmatrix} \tag{6.235}
$$

The mode coefficients on the internal boundaries are determined by (6.208), in the example they are

$$
\begin{pmatrix}
  c^{-}(z_{h1}) \\
  c^{+}(z_{h4})
\end{pmatrix} = L^{(Mn)} \begin{pmatrix}
  c^{+}(z_{h0}) \\
  c^{-}(z_{h4})
\end{pmatrix} + S^{(Mnm)} \begin{pmatrix}
  s^{-}(z_{h1}) \\
  s^{+}(z_{h2})
\end{pmatrix}, \quad n = 1, 2, 3 \tag{6.236}
$$

Where $S^{(Mnm)}$ depends on $S^{(V)}$ and $S^{(H)}$, which of them is used depends on if the sources are to the left ($m=1$) or to the right ($m=2$) of the internal boundary.
7 References


