Real Options in Real Estate Development Investment

WARUT SATTARNUSART

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Warut Sattarnusart

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KTH Industrial Engineering and Management
Industrial Management
SE-100 44 STOCKHOLM
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Abstract

Real estate development investment requires a large capital funding but it has slow payback with many risks and uncertainties in the investment. The current approach by using NPV to evaluate this type of investment is not adequate anymore. This is because NPV does not thoroughly capture the uncertainties in the investment and the method ignores the management flexibility whether to postpone or abandon the project in the future. An alternative approach that addresses these issues is to use real options to evaluate this type of investment.

The thesis uses the real option model that was proposed by McDonald and Siegel (1986) to evaluate real estate development investment. The model captures value and cost uncertainty in the investment and considers that management has the flexibility to defer the investment into the future. The thesis analyzes the model critically by sensitivity analyses and shows that using the model requires the input parameters to be carefully determined, especially the ones that relate to unit rental rate. Furthermore, the paper uses Monte Carlo simulation to determine the optimal ratio between value and cost which suggests that the investment should be deferred or invested now. The result shows that, in general, a real estate project should be invested when the value of the project doubles the cost. Also, the result from the simulation allows investors to adjust the ratio according to their risk behavior. Lastly, the thesis performs another Monte Carlo simulation in order to quantitatively identify the effect of the real option model on the investment decision. The result shows that using only the traditional NPV to evaluate the investment can lead to the wrong investment decision more than 90% of the time. Therefore, using both real options and NPV together can improve investment decisions on the real estate development project.

Key-words

Real estate development investment, Investment evaluation, Real options, Option to defer, Sensitivity analysis, Monte Carlo simulation, Investment decision
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All of the mistakes in this thesis are solely my responsible.

Warut Sattarnusart
Stockholm, Sweden
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1. Introduction

1.1. Background

Nowadays, companies have many financial decisions to take, for example how to finance an investment project, how to control the level of debt financing and equity financing and how to control the risk of companies’ investments. On top of that, one of the most important financial decisions that companies have to take is whether to undertake or leave an investment project such as buying new machines, developing a new product, or building a new operating facility. Undertaking in the right projects that generate positive cash flows higher than the initial investment will bring success and growth to the company. On the contrary, if companies invest in non-profitable projects (both financially and strategically), these can lead to the failure of the firms. With these reasons in mind, it is interesting to conduct a research on the approaches that companies are using to evaluate investments.

According to corporate finance, Brealey, Meyers and Allen (2006, p. 119-139) discuss several ways that companies can evaluate an investment. The commonly used approaches are Net Present Value (NPV), Internal Rate of Return (IRR) and Payback Period (PP). Brealey, Meyers and Allen (2006, p. 119-139) suggested that NPV is the best practice to use when companies want to evaluate investments. NPV is an approach that uses the Discounted Cash Flow (DCF) technique to discount the future cash inflows and outflows back to the present values. The sum of the present values from both cash inflows and cash outflows is called NPV. This method suggests that a firm should invest in a project when NPV is more than zero and abandon the project if it turns out to be negative. This concept is widely used since it provides a clear criteria and quantitative information to managers who are decision makers.

When using NPV to evaluate investments, there are some underlying assumptions that are normally overlooked. One of the most important assumptions is that if the investments are irreversible (once invested in an investment project, the project cannot be undone or the expenditures cannot be recovered after investing), firms have to decide whether to undertake now or never (Dixit and Pindyck, 1994, p. 6). In the real world, most of the investments fit this irreversible characteristic, but companies have a choice to undertake an investment now or postpone the investment. Also, managements have the flexibility to delay, expand, or modify the project in the future. Furthermore, in long-term investments, there are many risks and uncertainties that can occur in the future. The NPV method does not thoroughly capture the management flexibility and the future uncertainties.

With all of the drawbacks of NPV, “Real Options” has been introduced into the corporate finance principle. It was first identified by Myers (1977). The general concept of real options is that investments can be viewed as options that can be exercised now or in the future. In order to properly evaluate an investment, firms should value the investment as an option that accounts for future uncertainties and the management flexibility whether to postpone, expand or abandon of that project in the future. This aspect of real options leads to the interest of this thesis to conduct a research on real options.

From a preliminary literature review, it has been found that real options have been used in many research articles in different industries such as IT, research and development, forestry, and oil indu-
try. Among these industries, the real estate development industry also fits the criteria to use real options. Real estate investments are irreversible. Once a real estate developer decides to construct a building, it cannot be undone because the under-construction building does not have the value before the project is finished and the expenditures invested in the project cannot be recovered. Also, most real estate projects are long-term projects that have many risk factors needed to be considered such as price of construction material, land and selling price per square meters. Furthermore, companies always have options to delay or postpone the projects both before and after investing. With these reasons, it is interesting to conduct a research on how to implement real options in real estate investments. Another reason comes from personal interest of the author because since 2007 there was a big boom of real estate and condominium in Bangkok, Thailand (the author’s home country) due to the development of public transportation, especially the metro system.

1.2. Problem

After searching articles in real option models that are used for evaluating investments under uncertainties, there are many models that have been proposed. For example, McDonald and Siegel (1986) studied the optimal timing investment in an irreversible project that has uncertainties in project value and cost. Also, they proposed a real option model to for valuing such project. Pindyck (1993) examined irreversible investment decisions which are subject to two types of cost uncertainty, technical and input cost uncertainties. Then, he introduced a model for evaluating investments under these types of uncertainty.

Although there are many research papers on real option models, a limited number of articles using real option models to evaluate real estate development investments have been found. This is supported by Lucius (2001). He concluded from his research on the usage of real option in real estate development as highly academic-abstract results with limited practical value and limited number of quantitative studies regarding real estate valuation using real options have been conducted. This reflects the lack of in-depth understanding in the applications of real options and frameworks for using real options in real estate development industry.

On account of this, the value of real options cannot be used effectively for future research and practical implementation in business. This is one of the knowledge gaps that needs to be fulfilled which leads to the research questions of the thesis below.

- “What are the main concerns needed to be considered when using real options to evaluate real estate development projects under uncertainties?”
- “How do real options affect investment decision in real estate development projects under uncertainties?”

1.3. Objective

The objective of this thesis is to analyze the selected real option model in a great detail in order to have a clear understanding on the mechanism and logic behind the model. Then, the thesis determines the applications of the real option model that can be used in an investment evaluation and the effect of the real option in decision making compared with the traditional NPV approach.
1.4. Delimitation

One limitation is that it is impossible to find the real option model that covers all types of uncertainty in a real estate development project. Instead, the selected model accounts for important types of uncertainty only.

1.5. Research Outline

First, the thesis examines the characteristics of real estate development and then identifies uncertainties underlying in a real estate development project. After identifying important risks and uncertainties that affect the project, the appropriate real option model is selected. The detail discussion of these topics is provided in Chapter 2. Chapter 3 discusses and justifies the methodology used in this study in order to answer the research questions and reach the objectives of the thesis. Then, the real option model is critically analyzed in Chapter 4 through a case study. Then, the chosen model is studied in a greater detail to determine the application of the real option model and also the effect on investment decision by using computer simulations. Chapter 5 explains the design of the simulations and the results of the simulation are analyzed in Chapter 6. Chapter 7 concludes the research. Figure 1.1 shows the outline of the research and main outcomes in each step.
Analyze the Real Option Model in Detail through a Case Study
Identify Uncertainties in Real Estate Development
Select a Real Option Model that Captures Identified Uncertainties
Analyze the Real Option Model in Detail through a Case Study
Study the Applications and Effect of the Real Option Model in Investment Decision
Analyze and Summarize the Result

Real Option Model

Answering the first research question:
“What are the main concerns needed to be considered when using real options to evaluate real estate development projects under uncertainties?”

Answering the second research question:
“How do real options affect investment decision in real estate development projects under uncertainties?”

Figure 1.1 Research Outline
2. Theory and Literature Review

This chapter reviews the existing body of knowledge related to the thesis in order to provide the background knowledge for the readers. In addition, the thesis uses the theories presented in this chapter as ground knowledge to conduct the study that answers the research questions.

2.1. Real Estate Development Investment

This section provides the knowledge of real estate development. First, it examines the macro picture of real estate system in order to understand how the system works and how each element in the system connects to the real estate development. Then, the section focuses on the real estate development process in order to identify the criteria used to decide whether to invest in a project or not. The last part discusses risks and uncertainties that affect the decision making criteria and identifies the important uncertainties needed to be considered when evaluating a real estate development project.

2.1.1. Real Estate Development as Part of Real Estate System

In real estate system, there are three major elements that interact together. These elements are space market, asset market and real estate development industry.

To begin with the markets in real estate system, a market is a mechanism which goods and services are exchanged among different owners (Geltner et al., 2006, p. 3). In real estate context, a real estate property is a product that can be traded or provide a service in the market as well. Here are the characteristics of space market and asset market in the real estate system.

Space Market

The space market is the market for the usage or right to use the space of real property, land and built space (Geltner et al., 2006, p. 3). The demand side of this market is individuals, families, or organizations that want to use the space of the real property for living or production purposes. On the other side, the suppliers of the space market are the property owners who rent their property to tenants. These consumers are required to pay the rent for using the space every certain period of time such as monthly or yearly. The rent is normally quoted in price per m².

In general, the rental price of a property is a good indicator in the level of balance in demand and supply of space market and more importantly, the value of the property that is being rented. The rental price depends on various parameters. The research of Marco (2008) on determining the factors that affect the residential rental price in New York City reveals that there are many factors affecting the rental price. These are environmental regulatory and economic factors namely, location of the property, crime rates in the area, rent-regulated or rent-subsidized housing, and household incomes. Another research by Donovan and Butry (2011) focused on the relationship between urban trees and rental price of single-family homes in Portland, Oregon, US. They found a positive correlation between these two parameters which is additional trees in house’s lot and public right of way increased the monthly rental price by $5.62 and $21.00 respectively. When considering commer-
cial real estate rental pricing, Wiley, Benefield and Johnson (2010) presented that the energy-efficient design of the commercial office space earns superior rents and sustains higher occupancy.

The amount of rental payment is very important in real estate system context since it is the main cash flow that involves in this market and has the direct effect on another market in real estate system which is the asset market.

Asset Market

The real estate asset market is the market for the ownership of real estate assets (Geltner et al., 2006, p. 11). The real estate assets include land parcels and property (buildings) that are built on the lands. The supply side of real estate assets is the property owners who want to sell their assets. On the demand side, it is the investors who want to buy in order to increase the holding on their real estate assets. Owning real estate assets means having the claim of future cash flows that the assets can generate. These future cash flows normally come in two forms which are the rental payments of the space market that was mention earlier and the sales of the property (Miles et al., 2007). With this characteristic, real estate assets can be viewed as another type of capital assets, such as stocks or bonds, in a larger capital market. As same as all types of investments, when investors want to invest in any particular capital assets, they need to price those assets first before deciding to invest. There are several ways that real estate assets can be priced.

There are many factors that affect the value of real estate assets. Geltner et al. (2006, p. 14-15) identified that opportunity cost of capital, growth expectations on rental prices, and investor risk perception influence the capitalization rate using for evaluating real estate asset values. Moreover, Hott (2011) explained that mortgage lending behavior of banks affects the real estate prices. This is because real estate prices depend on the level of demand of the property which links together with the level of supply of mortgages for the investors. The research from Wang, Yang and Liu (2011) examined the linkage between urban economic openness, the ratio of trade volume as a percentage of GDP, and urban real estate prices. They empirically found that for every 1% increase in urban economic openness, urban real estate prices will increase significantly by 0.282% and urban economic openness alone accounted for about 15.90% appreciations of Chinese real estate prices. This can be summarized that economic factor affects the real estate asset prices.

To sum up, the main focus on the asset market that investors need to consider is the value of real estate assets. The asset values are also influenced by the space market through the rental price and other factors. The property values play an essential role in the real estate development industry in the next part.

Real Estate Development Industry

Real estate development industry is both supplier and the first owner of real estate assets that can be sold or provide the service in both space market and asset market. The main role of the real estate developers is to convert financial capital into physical capital (Geltner et al., 2006, p. 21) in order to supply the demand from both markets. For this, the key question that real estate developers have to consider is that “is a development worth undertake and profitable or not?” The next section discusses in a greater detail of real estate development in order to determine the key step that answers this question and elements that influence the decision to invest in a project.
Real Estate System

Putting all elements together, the real estate system can be displayed by Figure 2.1. It also shows the connection and logical flow between each component as described earlier in the previous parts. This gives the broad perspective on how real estate development (which is the focus of this research) is affected by different factors in the system.

2.1.2. Real Estate Development Process

Miles et al. (2007, p. 5) define the definition of real estate development as the following.

“A real estate development starts as an idea that comes to fruition when consumers - tenants or owner-occupants - occupy the bricks and mortar (space) put in place by the development team.
Land, labor, capital, management, entrepreneurship, and broadly defined partnerships are needed to transform an idea into reality. Value is created by providing usable space over time with associated services. It is these three things - space, time, and services - in association that are needed so consumers can enjoy the intended benefits of the built space.”

Real estate developers follow a sequence of steps starting from recognizing the need in the market to completing the construction and managing it. All of the important steps are captured in the eight-stage model of real estate development (Miles et al., 2007, p. 7) in Figure 2.2.

Figure 2.2 The Eight-Stage Model of Real Estate Development (Miles et al., 2007, p. 7)
From the model in Figure 2.2, it is obvious that the third stage which is the feasibility analysis is the core step used to answer the question “is a development worth to undertake and profitable or not?” which is the key question for the developers. When considering more carefully, the feasibility analysis is also a very important stage of the whole process since it is the key decision making point that has the effect in the long term. One reason is because of the characteristic of this type of investment that is characterized by large capital funding, long construction period, slow payback and enduring many uncertainties (Rocha et al., 2007; Mao and Wu, 2011; Bulan, Mayer and Somerville 2006). Furthermore, after the evaluating stage of the project, the capital investment and operational costs in the following steps increase significantly compared with the previous steps (for example, these costs are legal and administrative cost, construction cost, and marketing cost). These steps also involve many external parties such as legal, local government, and construction companies. These create much higher complexity in managing and execution.

Graaskamp (1972) defines that a real estate development project is feasible when there is a reasonable likelihood of satisfying explicit objectives form the selected plan within the constrains and limited resources. With this definition, it shows that there are several topics that are needed to be covered in a feasibility analysis. Miles et al. (2007) provide a layout that covers essential elements in feasibility analysis as follows.

- Executive summary
- Maps
- Photographs of the site
- Renderings
- Market study
- Electronic valuation model derived from market study
- Documented cost projections
- Development schedule
- Background on key players, including project consultants

Although there are many aspects needed to be covered in feasibility analysis, the bottom line deciding whether the project is feasible or not is still financial aspect of the analysis as Miles et al. (2007, p. 413) mentioned “a project is feasible when the value exceeds all the projected costs of development.”

Regarding the costs of the project, each project has different cost structure which depends on the characteristic of the project such as size, or real estate type. In general, Miles et al. (2007, p. 406) provides the list of costs in a real estate development project as follow.

- Land cost
- Site and infrastructure development costs
- Design fees
  - Architecture
  - Engineering
- Hard costs
  - By category
Labor and materials

- Permitting costs
- Financial costs
  - Permanent loan commitment fees
  - Construction interest
  - Construction loan fees
- Marketing costs
  - Promotion
  - Advertising
  - Leasing commissions
  - Brokers’ fees
- Preopening operating costs
- Legal fees
- Accounting costs
- Field supervision (inspection) costs
- Overhead
- Contingencies
- Development fees

Regarding the value of the project, it can be determined through the market study. The market study is the most crucial item in feasibility analysis and considers as a backbone of the real estate development process (Miles et al. 2007, p. 415). The market study analyses all long term economic trends from global to local perspective and also the demand and supply of real estates in space market. The end result of the market study is the forecast of expected income from rents (price per m²). This is used to derive the cash flows that the project can generate and ultimately, the value of the project (Miles et al. 2007, p. 395-396, 415-417).

Obviously, the costs and value used in the feasibility analysis are the estimated figures. The actual value and total cost of the project can only be known when the project is finished. From the eight stage model, most of the costs consist in the first six stages of the development process and the value of the development is generated in the last two stages. The estimated costs and value can be greatly different from the actual ones because there are many risks in the real estate development. The next section discusses the relationship between the risks and uncertainty of the costs and the value in real estate development.

2.1.3. Risk and Uncertainty in Real Estate Development

Mao and Wu (2011) describes that the typically used method to identify risk is risk list. In their research, they identified that some economical and policy risks such as inflation raising rate, tax rate policy, market and supply demand, and the project management skills are the reasons that cause the uncertainty in income and cost of real estate development project. This finding can be added by the research of Sun et al. (2008) which asked the experts in the real estate industry to identify the risk in real estate development. The result shows that not only economic risk does affect the real estate development, but there are also political risk, social risk, and technical risk. The details of each risk factor are displayed in Figure 2.3.
When considering the level of impact of each risk on real estate development, the result can be found in Linjie (2010) study. He asked industry experts, professors and practitioners, to evaluate the impact of different types of risk on real estate development by categorizing into seven levels as [VL] (very low), [L] (low), [RL] (rather low), [M] (neither high nor low), [RH] (rather high), [H] (high), and [VH] (very high). Table 2.1 shows the results of the study.

<table>
<thead>
<tr>
<th>Order</th>
<th>Evaluation indicator of development risk</th>
<th>Impact</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Environmental risks</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Macroeconomic risk</td>
<td>[H]</td>
</tr>
<tr>
<td>3</td>
<td>Policy risk</td>
<td>[H]</td>
</tr>
<tr>
<td>4</td>
<td>Regional Social risk</td>
<td>[VL]</td>
</tr>
<tr>
<td>5</td>
<td>Natural risks</td>
<td>[RL]</td>
</tr>
<tr>
<td>6</td>
<td>Business risks</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Decision risk</td>
<td>[VH]</td>
</tr>
<tr>
<td>8</td>
<td>Business risks</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Funding risks</td>
<td>[H]</td>
</tr>
<tr>
<td>10</td>
<td>The risk of developing scale and model</td>
<td>[RH]</td>
</tr>
<tr>
<td>11</td>
<td>Project risks</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Technical risk</td>
<td>[RL]</td>
</tr>
<tr>
<td>1</td>
<td>Project risks</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Land price risk</td>
<td>[VH]</td>
</tr>
<tr>
<td>10</td>
<td>Timing risk</td>
<td>[RH]</td>
</tr>
<tr>
<td>11</td>
<td>Market positioning risk</td>
<td>[M]</td>
</tr>
<tr>
<td>12</td>
<td>Construction process of risk</td>
<td>[M]</td>
</tr>
</tbody>
</table>

Table 2.1 Impact of Risks on Real Estate Development (Linjie, 2010)

Now, it can be clearly seen that there are many types of risk that have different levels of impact on real estate development projects. These risks consist in all of the stages of the development process and also are the main cause, directly and indirectly, of the uncertainty in both cost and value of the
project. Putting all of these together, Figure 2.4 can be drawn to show the linkage between the eight stage model of real estate development, risks, and uncertainty in the cost and value.

Figure 2.4 Relationships between Real Estate Development, Risk, and Uncertainties

The uncertainty in the cost and the value of real estate development affect considerably in the feasibility analysis because it can turn a feasible project into a non-profitable project at the end. On account of this, the evaluation method used in the feasibility analysis needs to take these types of uncertainty into the consideration. This leads to the next section of the thesis that focuses on finding the real option models that capture these two types of uncertainty, cost uncertainty and value uncertainty, for the evaluation of real estate development.
2.2. Determining Cost and Value of Project

As mentioned in the section 2.1.2, it discussed roughly on how to determine the costs and the value of real estate investment project. In this part, it examines in greater detail on how real estate developers can calculate the costs and the value of the project more precisely.

Williams (1991) discusses that investors can choose to develop a property with a certain density \((q)\) which is subject by the zoning regulations. The density means the available space that the building will have after finishing the construction. From the basis of this concept, Williams (1991), Quigg (1993), and Sing (2001) said that the cost and the value of the real estate development project can be determined in relationship with the density function.

2.2.1. Determining Cost

According to the list of the costs in section 2.1.2, these costs can be separated into fixed costs and variable costs. The fixed costs are the costs that do not change according to the size or the height of the construction. Here is the example of the fixed costs.

- Land cost
- Design fees
  - Architecture
  - Engineering
- Permitting costs
- Financial costs
  - Permanent loan commitment fees
  - Construction interest
  - Construction loan fees
- Marketing costs
  - Promotion
  - Advertising
  - Leasing commissions
  - Brokers’ fees
- Preopening operating costs
- Legal fees
- Accounting costs
- Field supervision (inspection) costs
- Overhead
- Contingencies

Sing (2001) mentioned that the largest fixed cost in real estate development is the land cost. For the variable costs, this type of costs changes in the relationship with the density of the building. Below is the example of the variable costs in a real estate development project.

- Site and infrastructure development costs
• Hard costs
  – By category
  – Labor and materials
• Overhead
• Development fees

Additionally, Sing (2001) also mentioned that when building reaches a certain size or gets taller, the variable costs, especially the construction cost, increase considerably with every additional unit increased in building height. With all of these, Williams (1991), Quigg (1993), and Sing (2001) determined the total cost in a real estate development project as the following function.

\[ F_0 = q^\gamma K + f \]  

Where \( F_0 \) is the cost of the development project  
\( q \) is the density of the building (m^2)  
\( \gamma \) is the cost elasticity of scale  
\( K \) is the unit variable cost per unit of density, and  
\( f \) is the total fixed cost

The variable \( \gamma \) which is the cost elasticity of scale represents what Sing (2001) mentioned and \( \gamma > 1 \) when the development is at high density (Williams, 1991).

2.2.2. Determining Value

Regarding the value of the project, as mentioned in previous section, the value of the project depends on the expected income in term of rental which is quoted in price per m^2. With this, the value of the development project can be determined as a function of the density and rental price as follow (Williams, 1991).

\[ V_t = qR \]  

From the above equation, Sing (2001) added the price elasticity of scale into the function. So, the final form of the equation is displayed below.

\[ V_t = q^\theta R \]  

Where \( V_t \) is the income of the development project at year \( t \)  
\( q \) is the density of the building (m^2)  
\( \theta \) is the price elasticity of scale, and  
\( R \) is the rental price per m^2

The rental price, \( R \), changes according to the demand and supply in the space market and also economic situation in certain period of time. This value can be determined through the market study or examining the current market price of the area that the investor wants to develop the real estate. Since \( R \) is quoted as a rental price per year, in order to find the total present value that the project
can generate, this stream of cash flows has to be discounted back to the present value. A real estate can be considered as perpetuity project because once finish constructing, the building has an infinite lifetime that can generate cash flow to the owner. On account of this, the present value can be calculated with this formula.

\[ V_0 = \sum_{t=1}^{\infty} \frac{q^t R}{(1+\delta)^t} \]  

This formula can be reduced into the short form as shown in the equation (5) (Brealey, Meyers and Allen, 2006, p. 37).

\[ V_0 = \frac{q^0 R}{\delta} \]  

Where \(\delta\) is the expected rate of return on rent (%)

### 2.3. Real Options in Real Estate Development

This section describes real estate development investment evaluation. At the beginning of the section, general concepts of real options are introduced. After providing the basic concepts of real options, the section focuses on identifying types of real option that consist in real estate development. Then, the existing real option model that captures the important uncertainties in the real estate development project is selected.

#### 2.3.1. Real Options

Financially, options are financial instruments mostly used in stock trading. Options are the “right” but not “obligation” to buy or sell the share at a specific price (exercise or strike price) on or before a specific date. The options that allow investors to buy a share at a specific price in the future are called call options (in short for calls). On the contrary, the options that allow investors to sell a share at a specific price in the future are called put options (in short for puts). Furthermore, if call or put options can be exercised on a particular date only, they are known as European options. If the options can be exercised on or any dates before a particular date, these options are American options. Both calls and puts have a great value for the stock traders since they have the application to hedge the risk and protect the possible losses due to uncertainties in the future of shares.

Myers (1977) is the first one who identified the same application of financial options in capital investment projects. The term for this type of options is real options. He stated that an investment opportunity in the future can be viewed as an American call option. This reflects the management flexibility to modify the projects before and after investing in the project. Managers have the right but not the obligation to whether defer, expand, or abandon a project which depends on the situation in the future. Together with the concept of financial options and real options, the analogies between the elements in these two types of options can be drawn as presented in Table 2.2 (Brach, 2003, p. 43).
For better understanding of real options, the following examples present how real options can be used as an investment evaluation method. In the first example, a digital camera manufacturing company wants to introduce a new product called Mark I to the market because it spots the future boom in the market. The firm evaluates this investment using NPV and it turns out that the NPV of the Mark I project is -$50 million. If the company considers only NPV, it should not invest in this project. However, this new Mark I product can help the company to open another line of new products or an improved sequential product called Mark II. This future investment opportunity can be looked upon as an option to the company and this option is worth $60 million. Together with the option, the Mark I investment changes to be a positive project at $10 million! As a result, the Mark I project seems better to invest in.

Another example is that a company wants to build a duplex house with the cost of $500,000. The house requires one year of construction and it can be sold in the next year. However, the future price of the duplex house can either go up to $1.1 million or go down to $400,000 with the same probability and the price will stay at this value afterwards. When using NPV to evaluate this investment given 10% as the discount rate, the NPV of this project equals $181,818 and the company should build the duplex now since this is a profitable one. However, this is not always true. If the price of the duplex turns out to be $1.1 million, the company will enjoy the benefit of the decision to invest. On the contrary, if the price falls, the investment will turn out to be a lousy one. This is because the NPV implies that the company does not have any flexibility to defer the investment into the future. It has to decide to invest now or never which is not the case in the reality. With the view that the company has flexibility to defer the investment, this duplex project can be viewed as a call option which is worth $311,688. Now, the firm can make the decision based on these two values. If the firm wants to invest now, the project is worth $181,818. However, if the firm decides to wait for the situation to resolve and invests the project if the price goes up, the project is worth $311,688. Therefore, it is better for the company to wait and invest in the project in the future since the value of the project as an option is much higher that the value received from the NPV.

From the given examples, the first example recognizes the future investment as an expand option and uses the future investment to add value to the current project. The second example recognizes the project as a defer option and uses it to compare whether to invest now or wait to invest in the future. The examples show that using real option method to evaluate investments can lead to a dramatic effect on managers’ investment decision.

Also noticed from the examples that using real options does not mean that one can ignore and stop using NPV to evaluate investments. On the other hand, NPV is still the important method in investment evaluations but considering only NPV can lead to wrong decision. Real options help managers to capture management flexibility and future uncertainties and when using the real options together with the NPV, they can lead to a better investment decision. The examples also demonstrate
that different types of real option have different usage in investment evaluation. On account of this, it is essential to determine which types of real option needed to be considered in an investment.

With the benefits of the real option that captures managerial flexibility, it has been found that real options have been used in many different industries. For the oil industries, real options can also be used since the oil production and exploration projects require large capital investment with high fluctuation in the oil price and endure many uncertainties (Cortazar and Schwartz, 1997; Armstrong et al., 2004; Dias, 2004; Fan and Zhu, 2010). Duku-Kaakyire and Nanang (2004) and also Kallio, Kuula and Oinonen (2012) used real options for forestry investment analysis. In Duku-Kaakyire’s and Nanang’s (2004) paper, they discussed that there are four types of real option which are the option to delay, expand, abandon, and multiple options. They concluded that real option method give a better decision since the method values management flexibility, risks, and uncertainties in the project. Furthermore, Cassimon et al. (2011) presented models used for pharmaceutical R&D licensing opportunities. Their study introduced real option model capturing technical risk of new drug development. Then, the study concluded that the NPV can underestimate the value of the project, while the real option evaluation reflects the nature of R&D investment in a better way. So, the real option approach is more reliable than the NPV. Not only can the real option be used solely as investment evaluation tools, it can also be used together with other frameworks such as Mean-Variance (MV), portfolio management analysis, and risk-return tradeoff as presented in the studies of Wu and Ong (2008) and Choungsirakulwit and Sutivong (2007).

After understanding the general concepts of real options, the effect of the real options in investment evaluation, and the applications of real options in different industries, the next part discusses the types of real option that can be found in real estate development which is the focus of the thesis.

2.3.2. Real Options in Real Estate Development

Trigeorgis (2001, p. 104-105) described that there are seven most common types of real options that can be found in any investments. Also, he specified the types of industries that these options are important in as presented in Table 2.3.

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
<th>Important in</th>
</tr>
</thead>
<tbody>
<tr>
<td>Option to defer</td>
<td>Management hold a lease on (or an option to buy) valuable land or resources. It can wait (x years) to see output price justify constructing a building or plant, or developing a field.</td>
<td>All natural resource extracting industries; real estate development; farming; paper products.</td>
</tr>
<tr>
<td>Time to build option (staged investment)</td>
<td>Staging investment as a series of outlays creates an option to abandon an enterprise in mid-stream if new information is unfavorable. Each stage can be viewed as an option on the value of subsequence stages, and valued as a compound option.</td>
<td>All R&amp;D intensive industries, especially pharmaceutical; long-development capital intensive projects, e.g. large-scale construction or energy generating plants; start-up ventures.</td>
</tr>
<tr>
<td>Option to alter operating scale</td>
<td>If market conditions are more favorable, an option to adjust the size of operation.</td>
<td>Natural resource industries.</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
<th>Important in</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e.g. to expand; to contract; to shut down and restart)</td>
<td>Favorable than expected, the firm can expand the scale of production or accelerate resource utilization. Conversely, if conditions are less favorable than expected, it can reduce the scale of operations. In extreme cases, production may temporarily halt and start up again.</td>
<td>Such as mine operations; facilities planning and construction in cyclical industries; fashion apparels; consumer goods; commercial real estate.</td>
</tr>
<tr>
<td>Option to abandon</td>
<td>If market conditions decline severely, management can abandon current operations permanently and realize the resale value of capital equipment and other assets in secondhand markets.</td>
<td>Capital intensive industries, such as airlines and railroads; financial services; new product introduction in uncertain markets.</td>
</tr>
<tr>
<td>Option to switch (e.g. outputs or inputs)</td>
<td>If prices or demand change, management can change the output mix of the facility (“product” flexibility). Alternatively, the same output can be produced using different types of inputs (“process” flexibility).</td>
<td>Output shift: Any good sought in small batches or subject to volatile demand, e.g., consumer electronics, toys; specialty paper, machine parts; autos. Input shift: All feedstock-dependent industries, e.g., oil; electric power; chemicals; crop switching; sourcing.</td>
</tr>
<tr>
<td>Growth options</td>
<td>An early investment (e.g., R&amp;D, lease on undeveloped land or oil reserves, strategic acquisition, information network/infrastructure) is a prerequisite or link in a chain of interrelated projects, opening up future growth opportunities (e.g., new generation product or process, oil reserves, access to new market, strengthening of core capabilities). Like interproject compound options.</td>
<td>All infrastructure-based or strategic industries, especially, high-tech, R&amp;D, or industries with multiple product generations or applications (e.g., computers, pharmaceuticals); multi-national operations; strategic acquisitions.</td>
</tr>
<tr>
<td>Multiple interacting options</td>
<td>Real-life projects often involve a “collection” of various options, both upward-potential enhancing call and downward-protection put option present in combination. Their com-</td>
<td>Real-life project in most industries discussed above.</td>
</tr>
</tbody>
</table>
Table 2.3 Common Types of Real Options and Industries Consisting These Options (Trigeorgis, 2003, p. 104-105)

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
<th>Important in</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>bined option value may differ from the sum of separate option values, i.e. they interact. They may also interact with financial flexibility options.</td>
<td></td>
</tr>
</tbody>
</table>

From the definition of each type of option given by Trigeorgis (2003, p. 104-105) in Table 2.3, there are two types of options that are mentioned to be important in real estate development, option to defer and option to alter operating scale. But, the thesis focuses only on option to defer with the following reasons.

- **Option to defer** - This type of option is common in real estate development since a company can purchase a land space and wait to build any real estate on that land at any time it wants according to the market situation. The company does not have to build a real estate asset immediately because it does not cost more or the land will not deteriorate through time. The defer option of real estate development is recognized in some research articles as well. Sing (2001) identified that a land parcel entails not only the net value of real estate asset that can be built on the land, but it also includes the premium for the option to wait to develop the land (to defer the investment). This is supported by Cappoza and Sick (1992) and Quigg (1993) that say the current time does not always to be an optimal time to invest in a real estate project. So, the project should be deferred to the future.

- **Option to alter operating scale** - Although the option to alter operating scale is mentioned to be important in commercial real estate according to Table 2.3, there are no research articles that support this type of option properly. Furthermore, a real estate project cannot be altered in term of scale easily (increase or decrease the available space). This is because any construction developments have to follow the construction drawing and the contract that was agreed before. Moreover, there are zoning regulations that developers have to follow. Once the company starts to develop a real estate project in a particular area, the building cannot be easily changed because of the regulations (Williams, 1991). On account of this, the option to alter operating scale is not considered in this thesis.

In the next section, the thesis identifies the mathematical model that is used to find the value of the option to defer. The identified model has to take two types of uncertainty which are value uncertainty and cost uncertainty into the account since these uncertainties are important in real estate development (as mentioned in the section 2.1.3).

### 2.3.3. Option to defer

Option to defer has been examined in several articles. Titman (1985) presented a simple model to evaluate a real estate investment on vacant land when the value of the building and the size are uncertain. Also, Paddock, Siegel, and Smith (1988) developed a model to value a claim on a real asset by focusing on the case of offshore petroleum lease. However, the model proposed by Titman (1985) is quite simple since is not a continuous time model. The model consists only two dates (pre-
sent date and a future date), so the land developer has only two decision points. This is not a realistic assumption. Another model that Paddock, Siegel, and Smith (1988) presented is too industry specific. For this, it is difficult to adapt the model in the real estate development context.

Due to these reasons, the thesis chooses the model from McDonald and Siegel (1986). This model evaluates the value of waiting to invest and it is suitable in this study because the model captures both value and cost uncertainties. Furthermore, the model has a general purpose that can be implemented in real estate development. The following is the description of the model.

At the beginning, the model assumes that at any time \( t \), the firm can invest in a project that costs \( F_t \) and the project has the present value at \( V_t \). Both \( F_t \) and \( V_t \) follow geometric Brownian motion with drift below.

\[
\frac{dV}{V} = \alpha_v dt + \sigma_v dz_v \\
\frac{dF}{F} = \alpha_f dt + \sigma_f dz_f
\]

Where \( dV \) and \( dF \) are the change in the project value and cost
\( \alpha_v \) and \( \alpha_f \) are the drift in the project value and cost (%)
\( \sigma_v \) and \( \sigma_f \) are the degree of uncertainty (volatility) of the project value and cost (%)
\( dt \) is the time interval during the observed period, and
\( dz_v \) and \( dz_f \) are standard Wiener process (Brownian motion).

The assumption that the value and the cost of the project move stochastically as geometric Brownian motion is acceptable. The main driver of the project value is price per \( \text{m}^2 \) while there is no single parameter that represents the cost but most of the costs in real estate development are land price and construction material such as iron and concrete which are similar to commodities. Table 2.4 compares the definition of geometric Brownian motion with drift (Dixit and Pindyck, 1994, p. 63, 71) with the value (price per \( \text{m}^2 \)) and the cost (construction materials) of the project.

<table>
<thead>
<tr>
<th>Geometric Brownian Motion with Drift Characteristic (Dixit and Pindyck, 1994, p. 63, 71)</th>
<th>Price per ( \text{m}^2 ) behavior in a real estate development project</th>
<th>Construction material behavior in a real estate development project</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Brownian motion is a continuous-time stochastic process. The stochastic process is a variable that changes over time in a random way.</td>
<td>Price per ( \text{m}^2 ) in the market changes along the time in a random way with a certain direction (the drift). For example, the price of the area has the overall trend to increase during a period of time (a positive drift) but during this period, it fluctuates in a random way.</td>
<td>Price of construction materials change along the time in a random way with a certain direction (the drift). For example, the price of construction material can have the overall trend to increase (a positive drift) but the price is still fluctuated in random way.</td>
</tr>
<tr>
<td>The probability of distribution of all future values of the property tomorrow depends on today’s</td>
<td>The area price of the property tomorrow depends on today’s</td>
<td>The price of construction materials in tomorrow depends on</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Geometric Brownian Motion with Drift Characteristic (Dixit and Pindyck, 1994, p. 63, 71)</th>
<th>Price per m² behavior in a real estate development project</th>
<th>Construction material behavior in a real estate development project</th>
</tr>
</thead>
<tbody>
<tr>
<td>process depends only on its current value, and it is unaffected by any past values of the process or by any other of the current information. For this, in order to forecast the future value of a variable, it requires only one current value of the variable.</td>
<td>price only.</td>
<td>today’s price only.</td>
</tr>
<tr>
<td>The Brownian motion has independent increments. This means that the probability distribution for the change in the process over any time interval is independent of any other (overlapping) time interval.</td>
<td>The chance that the price per m² will go up or down has the same probability and the degree of change is normally distributed. This means that there is a higher probability that the price will go up or down with in one standard deviation and a lower probability that the price will change within three standard deviations (but it is possible to happen).</td>
<td>The chance that the construction material price will go up or down has the same probability and the degree of change is normally distributed. This means that there is a higher probability that the price will go up or down with in one standard deviation and a lower probability that the price will change within three standard deviations (but it is possible to happen).</td>
</tr>
<tr>
<td>Changes in the process along the time are lognormally distributed.</td>
<td>The changes of price per m² are lognormally distributed, which means that, in the worst case, the price never decreases more than 100 percent (the price equals to zero), but it can increase more than 100 percent.</td>
<td>The price of the construction materials are lognormally distributed, which means, the cost never decreases more than 100 percent and becomes negative value, while there is no limit of the increase in the value.</td>
</tr>
</tbody>
</table>

Table 2.4 Relationship between geometric Brownian motion with drift, price per m², and construction materials

Figure 2.5 shows the sample part of geometric Brownian motion with drift for the better understanding in the movement of these two factors.
In addition, the model assumes that the investment opportunities are infinitely lived. This assumption also matches the real estate development characteristic because in order to build a property on the land, the developer has to acquire the land. After acquiring, the land does not have expiration date in the future, so the investment opportunity to develop a real estate is infinitely lived.

After taking these assumptions in mind, McDonald and Siegel (1986) derived the model that values the opportunity to invest in the future as following.

\[ X = (C^* - 1) F_0 \left( \frac{V_0}{C^*} \right)^\varepsilon \]  

Where

\[ C^* = \frac{\varepsilon}{(\varepsilon - 1)} \]  

\[ \varepsilon = \sqrt{\left( \frac{\delta_f - \delta_v}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{\delta_f}{\sigma^2} + \left( \frac{1}{2} - \frac{\delta_f - \delta_v}{\sigma^2} \right)} \]  

\[ \sigma^2 = \sigma_v^2 + \sigma_f^2 - 2\rho_{vf} \sigma_v \sigma_f \]

Given  
\( X \) is the option value  
\( \delta_v \) is the expected rate of return on asset \( V \) (\%)  
\( \delta_f \) is the expected rate of return on asset \( F \) (\%), and  
\( \rho_{vf} \) is the instantaneous correlation between the drift rate of \( V \) and \( F \).
From the given equation, $X$ is the value of the opportunity to invest in the future, which is the value of deferred option. McDonald and Siegel (1986) also identified the optimal investment rule which tells that one should invest now when the ratio between the present value ($V_0$) and the present cost ($F_0$) exceeds $C^*$ and defer the investment when otherwise. This is because when $V_0/F_0$ equals to $C^*$ the option value (the value of the investment when deferring it into the future) and NPV (the value of the project when investing now) of the project are the same. Figure 2.6 shows the optimal value to invest according to the given rule. The horizontal axis presents the present value of the project and the vertical axis presents the total project value after deducting all the costs.

$C^* = \frac{V_0}{F_0}$

![Figure 2.6 Value of Option to Defer under Value and Cost Uncertainty](image)

Figure 2.6 shows that according to the NPV rule, the investment occurs only the NPV is more than 0 (the dash-line). However, from the figure, it can be seen that even though the project does not have a positive value from the NPV, the real option valuation (the thick line) still has positive value with the lowest possible value equals to zero. So, the option value tells that the investment should be deferred into the future. The reason for this is that NPV does not recognize the possibility that the value of the project can go up in the future. On the other hand, although the NPV is positive in certain range, the investment should still be invested in the future because the value of real option is higher than the NPV. This is because the real option considers that the value of the investment project can go even higher than the current situation or drop to the point that it is not worth to undertake. Then, one should undertake the project only if the value of the project goes up. It is obvious that the real option model provides a very valuable insight that can affect greatly in the decision making whether to invest in the project or not.

Although the real option model already captures the uncertainties when evaluating an investment project, the model requires more input parameters which these parameters are also not certain or can be determined precisely. On the account of this, the project owner needs to perform extra analyses in order to understand how each parameter affects the investment project and know what can
happen in the future. The next section discusses the key methods that are used to perform this kind of the analysis.

2.4. Financial Analysis with Uncertain Variables

In this section, it discusses the methods that are used to analyze investment projects when there is uncertainty in input parameters. The section focuses on the two most recognized approaches in corporate finance namely, sensitivity analysis and Monte Carlo simulation.

2.4.1. Sensitivity Analysis

“Uncertainty means that more things can happen than will happen.” (Brealey, Meyers, and Allen, 2006, p. 255). This definition is what sensitivity analysis tries to answer. The sensitivity analysis does not focus on answering what will happen to an investment project, but it rather examines what can actually happen to the project in different situations. The sensitivity analysis can be performed by changing some input parameters or one parameter at a time in a certain range while keeping other parameters unchanged. Then, it calculates all of the possible project values according to the changed parameter(s). The process in performing sensitivity analysis forces the project owner to identify all key variables used in the calculation and to determine which input parameters are most likely to deviate from the estimated values.

Sensitivity analysis has many benefits and applications. One application of the sensitivity analysis is to find the pessimistic and optimistic scenario of a project. This can be done by asking relevant departments involving in the project to provide not only the estimated figures, but also to submit the pessimistic and optimistic number for the underlying variables. This helps the analyst to know the possible spread of the project value so that he/she realizes the risk level of the project from miscalculating the variables. Moreover, this application of sensitivity analysis reveals the main source of uncertainty causing the pessimistic scenario. Hence, the company can take a corrective action to resolve the uncertain situation before actually invest in the project (Brealey, Meyers, and Allen, 2006, p. 255-257).

For example, a car manufacturing company wants to introduce new electric scooter to the market. The company calculates the NPV of the project and the result is $15 million. The company performs the sensitivity analysis and found out that the main drivers used in the calculation are new product’s share of the market and the unit production cost of the scooter. So, the finance department asked the marketing and manufacturing department to estimate the pessimistic and optimistic figures of the market share and unit production cost to the department. With these figures, the NPV of the project turns out to be -$5 million and $30 million in pessimistic and optimistic scenario respectively. In addition, the company also found out that the pessimistic scenario in unit production cost is caused by the concern from manufacturing department that the new machine used in the production will not work properly as designed. This leads to an increase in unit production cost of the scooter. To resolve the issue, the company decides to run a pretest on the machine with some cost which reduces the uncertainty in this variable. As a result, the pessimistic scenario turns out to be $1 million and this tells that the company is unlikely to be in a trouble in the project nevertheless there are some uncertainties remaining.
However, the mentioned application of the sensitivity analysis has some limitations. First, the results from the analysis are somewhat ambiguous because pessimistic and optimistic are subjective which leaves the room for different interpretation. One person pessimistic scenario can still be considered as a normal situation or even optimistic to another one. Another drawback of the sensitivity analysis is that the underlying variables are likely to be interrelated such as an increase in market share can lead to an increase in unit sale price. Changing the variables separately causes unreliable results in the sensitivity analysis as well (Brealey, Meyers, and Allen, 2006, p. 257-258).

Break-even analysis is another application of sensitivity analysis. In this case, the objective of the analysis is to find the value of a particular input parameter that makes the project to become feasible, or NPV equals to zero. From the previous example, the company can use break-even analysis to find how many scooters needed to be sold per year so that the project becomes profitable. Also, the company can run the analysis to identify the minimum price per unit of scooter should be sold to the market in order to yield a positive NPV. This type of sensitivity analysis gives different perspective to the project owner on how the company can earn a profit from the investment (Brealey, Meyers, and Allen, 2006, p. 259-261).

Last but not least, sensitivity analysis helps the analyst to understand how each variable affects the project and in what degree. Results from the analysis provide an in-depth understanding of the valuation model that is used to calculate the project value. The degree of the effect on the project value can be considered as a signal that gives a caution to the project analyst on which variable should be carefully estimated and monitored.

### 2.4.2. Monte Carlo Simulation

Sensitivity analysis is a very useful tool used in financial analysis when input variables are uncertain. However, sensitivity analysis only allows the effects of changing one variable at a time to be considered and also the limited number of scenarios can be generated through this method (Brealey, Meyers, and Allen, 2006, p. 263). This is because changing many variables or creating a great number of scenarios brings complexity to the analysis which leads to unreliable result. Furthermore, as mentioned earlier, sensitivity analysis ignores that the variables are likely to be interrelated and interdependent between different periods of time. This is quite a strong assumption that does not reflect the reality.

With these pitfalls, an alternative is to use computer simulation in the financial analysis when there are uncertainties in input variables. Monte Carlo Simulation is a widely recognized tool that was introduced in capital budgeting problems by Hertz (1968). The Monte Carlo simulation can generate all possible combinations of inputs and therefore, it can generate the entire distribution of the possible outcomes (Brealey, Meyers, and Allen, 2006, p. 263). Moreover, the Monte Carlo simulation assists the decision makers to be more rational and consistent in their decisions and the analysts to gain a greater comprehensiveness and understanding in all of the risk factors in the development project (Loizou and French, 2012).

Loizou and French (2012) and Brealey, Meyers, and Allen (2006, p. 263-268) described how to construct a Monte Carlo simulation which has three main steps as the following framework.
Step 1: Modeling the investment project

The first step of the simulation is to give a precise model of the project to a computer. The model in this case is normally a mathematical formula. For example, a company wants to run a Monte Carlo simulation on the electric scooter project to see all of the possible cash flow. The company specifies the model of cash flow as follow.

\[
\text{Cash flow} = (\text{revenue} - \text{costs} - \text{depreciation}) \times (1 - \text{tax rate}) + \text{depreciation}
\]

\[
\text{Revenues} = \text{market size} \times \text{market share} \times \text{unit price}
\]

\[
\text{Costs} = (\text{market size} \times \text{market share} \times \text{unit price}) + \text{fixed cost}
\]

In this step, not only does the company specify the model of the cash flow, but it also has to identify the interrelation between the variables in the model. Using the same example, the company determines that the market size is derived by the expected market size with a forecast error. Also, the expected market size in the following year interrelated with the previous year’s market size. All of these can be described below.

\[
\text{Market size, year } 1 = \text{expected market size, year } 1 \times (1 + \text{forecast error, year } 1)
\]
\[
\text{Market size, year } 2 = \text{expected market size, year } 2 \times (1 + \text{forecast error, year } 2)
\]

Since the expected market size of year 2 is interrelated with the year 1’s market size so the year 2’s market size can be rewritten as follow.

\[
\text{Market size, year } 2 = \text{market size, year } 1 \times (1 + \text{forecast error, year } 2)
\]

The market size after year 3 can be derived in the same way as a function of year 2’s market size and so on. In the market size example, it shows the interdependence in different period. But the model can also have the interdependence between variables. For example, the company identifies that the 10% error in market size will lead to a 3% error in the unit price. So, the function of the unit price is below.

\[
\text{Unit price, year } 1 = \text{expected unit price, year } 1 \times (1 + 0.3 \times \text{forecast error of market size, year } 1)
\]

The unit price in year 2 and after can also be specified in the same way or the company can add the interdependence between periods of the unit price as described in the market size as well. Specifying interdependencies is considered as the hardest and the most important task in Monte Carlo simulation. This process forces the company to understand every aspect of the project which brings others benefits than just the inputs of the simulation.

Step 2: Identifying random variables and probability distributions

After modeling the investment project, the second step is to identify which variables are considered as random inputs in what form of probability distribution. There are many types of probability distribution such as uniform distribution, normal distribution, triangular distribution, or lognormal distribution. This depends on the characteristic of the random variables. A simulation can have more than one random variable according to how many variables that are uncertain or are interesting for the analysis.
In the same example as in step 1, the random variable for market size is forecast error. The company can specify that the forecast error of market size has probability distribution as normal distribution with mean at 0% and 5% of standard deviation. The normal distribution is acceptable because the forecast error can be both positive and negative. Another case for the random variable can be expected unit price. If the company wants to random this input, it has to specify the probability distributions that do not allow that value to go beyond zero, which can be lognormal distribution.

**Step 3: Simulate random variables, calculate, and collect the results**

The last part is to run the simulation by letting the computer samples the random variables as specified probability and then calculate the result according to the model in the first step. The simulation should contain a high enough number of iterations that allows the computer to sample all of the possible random variables with all possible combinations. It can be said that the number of iterations should be at least 10,000 times as a rule of thumb since most of the literatures regarding Monte Carlo simulation use this number (Hoesli, Jani and Bender, 2006; French and Gabrielli, 2004; Gimpelevich, 2011). At the end of each iteration, the result is collected for the analysis.

With all of these, the framework for constructing a Monte Carlo simulation can be summarized in Figure 2.7 below.

![Figure 2.7 Framework for Constructing a Monte Carlo Simulation](image)

Although the Monte Carlo simulation has several benefits, there are some disadvantages in this method. The drawbacks of the Monte Carlo simulation are presented by Loizou and French (2012) which are originally cited by Johnson (1985) and Reutlinger (1970).

- Need to know probability distributions for each outcome of choice
- Historic information not always reliable or appropriate
- Subjective (estimates and guesstimates)
- Continuous distribution of inputs gives rise to an infinite number of outcomes which is unrealistic
- Easier to forecast capital cost item than effective demand, thus more appropriate to questions of cost effectiveness rather than profitability (NPV) of project
- There is correlation between variables (economic, organizational, technical) so that when independent variables are aggregated for risk assessment purposes the effect of the variation of one may be compensated by a variation of another in an opposite direction
- The continuous probability outcome may make data inadequacies and obscure causative relationships
- May distract attention for radical policy and project alternatives
- More staff time for data collection and analysis.
To deal with these drawbacks, the input data, both quantitative and qualitative data, used to construct a Monte Carlo simulation need to be carefully determined. A person building the simulation must have a deep and comprehensive knowledge not only about the investment project itself, but also in an overall perspective related with the project such as, economic outlook, competition in the market and etc. So, the analyst is able to identify all important interdependence between each variable and is able to properly justify the choice of probability distribution for each parameter. Furthermore, before actually performing the simulation, the details regarding the assumptions and how the simulation was built should be discussed with all of the departments related with the project. The reason is to reduce the subjective judgment on building the simulation and to get the opinions from different perspective that might be overlooked.

All of these can reduce the drawbacks of the Monte Carlo simulation and ensure that the result from the simulation is reliable for decision making process. Nevertheless, decision makers still have to remind themselves that the Monte Carlo simulation is only a tool that provides additional information about the possible value of the investment project, not the actual value that the project will be. Making the decision based on the result from the simulation is not suitable.
3. Methodology

This chapter discusses the research methodology used in the thesis and explains how the research is conducted in order to reach the determined objective.

3.1. Research Paradigm

The research paradigm of the thesis can be considered as positivism. According to Collis and Hussey (2009, p. 56), positivism assumes that the reality that is being observed is independent from the researcher and the knowledge is derived by empirical research, which is by observation and experiment. The starting point of positivism is the existing theories and then these theories are used to explain and/or predict the social phenomena by applying logical reasoning. It is important that the researcher is precise, objective and rigorous to the subject that is being studied.

This paradigm aligns with the objective of the thesis because it begins with searching existing real option models in theory first. Then, the selected real option model is analyzed and used in a case study as an experiment in order to reach the objective of the thesis which is to analyze the model, identify the applications, and determine effect of the model when using it in real estate development projects. Hence, the knowledge is derived from the results of the empirical study. Moreover, the models and the case study are not affected by the researcher.

3.2. Research Type

Since this thesis is based on positivistic paradigm, the type of the research that fits to the paradigm is deductive research. A deductive research is carried out by first developing the theoretical structure and then the theory is tested (Collis and Hussey, 2009, p. 7). This thesis is classified as deductive because there is already existing theory in real options which is a general theory for investment evaluation. Then, the real option approach is tested in the real estate development investment context in order to deduce the knowledge that can answer the research questions.

Looking from another point of view, the main themes of the thesis are descriptive and explanatory research (Collis and Hussey, 2009, p. 5-6). To answer the first research question, the model is analyzed critically as it is in order to describe the characteristic of the model. This is viewed as descriptive research since the focus of this part is to explain the model only. Next, the thesis takes another step by determining the applications of the real option model and the effect on the investment decision. This requires more analysis than just considering the model itself. The model has to be used to evaluate a real estate development project and then, the result is used to derive the benefits and the effect of the model on investment decision. Thus, this is considered as explanatory research.

Furthermore, the thesis chose quantitative approach as the main methodology for the research. The reason for choosing this approach is because first, the study in this thesis needs to be precise and objective since it is considered as positivistic paradigm. A quantitative study is suitable because the result and the data used for the analysis are quantifiable. Therefore, a precise definite answer can be derived without subjective interpretation from the researcher. Another reason that quantitative approach was chosen is due to the nature of the research topic itself. To elaborate, investment evalua-
tion is a quantitative study since when companies decide whether to undertake an investment project, they evaluate the investment in terms of quantitative data such as NPV or IRR of the investment as mentioned in the introduction part. The real option approach which is the focus of the study is another quantitative method used to determine the value of the investment as well.

3.3. Research Methodology

According to the objective and the outline of the thesis, there are three main tasks, which are, identification of real option model, analyzing the selected real option model, and determining the applications and effect of the real option model on investment decision. For the identification of real option model, the thesis does not construct real option model by itself, but it identifies the models by choosing from existing models that were derived and published in different journals by the specialists in this field. This method can guarantee the reliability and validity of the real option model used in this research. This part was already covered in the previous section.

In the second task, the experimental study approach is chosen since it is the method used to analyze the relationship between independent variables and a dependent variable (Collis and Hussey 2009, p. 74). The study is carried out by performing the sensitivity analysis. The method fits the objective of the research since it allows the relationship between input variables of the model and the real option value to be studied. Then, the knowledge about the characteristic of the model can be deduced from the study. Moreover, the real option model requires some numerical data as input variables for calculating the result. Therefore, a case study of real estate development is used for this purpose. The case study in this context does not have the same meaning as Collis and Hussey (2009, p. 82-83) described which is the method to explore a single phenomenon in a natural setting in order to derived the theory from the study. On the other hand, the case study is the source of secondary data that have been collected by others (Collis and Hussey 2009, p. 73). This brings the issue of finding a reliable case study that contains all required inputs. To deal with this issue, a case study must be selected from an article that was published in a journal in order to guarantee the quality of the case. After extensive searching, the case study of commercial development at Spitalfields, East London (Sing, 2001) was chosen since it meets all mentioned criteria. The reason that the case study approach was chosen is because the thesis does not have any contacts with companies in the real estate development industry. So, the numerical data used for the model cannot be collected directly. Also, the data from the real case study still gives a realistic picture of the evaluation problem.

Regarding the method used to identify the applications and the effect of the real option model on investment decision, the repeated-mesures design of experimental study is chosen which is a method that Collis and Hussey (2009, p. 74-76) suggested for a positivistic research. The repeated-mesures design is the experiment that the data is collected many times under different conditions. The issue needed to be considered in this approach is the order effect (Collis and Hussey, 2009, p. 75). This means that interdependence of each data has to be carefully considered and determined correctly to ensure the reliability of the result. Moreover, this method takes considerable amount of time if the data is collected directly. Due to the short period of the research (only five months) and the lack of contact with companies in the real estate development industry, the thesis cannot conduct the experiment in natural setting. To solve these issues, a Monte Carlo simulation is used in this research. The simulation can randomly generate a large number of real estate development projects which allows the thesis to perform repeated-measure experiment within the mentioned constraints. The simulation also requires numerical data as input variables so that the case study of commercial
development at Spitalfields, East London (Sing, 2001) is used again as the data for the input variables. This is for a consistency purpose and the result from the base case in sensitivity analysis can be compared with the result from simulation.

In order to reach the objective of the thesis, there are two simulations needed to be conducted. First one is the simulation related to the $C^*$ parameter of the real option model which is the ratio used as the investment criteria whether to invest now or defer the project. The simulation is used to generate all possible values of $C^*$. The result from the simulation can be used to derive general investment rule that guides the management to decide when to invest in the project. Furthermore, these values will allow managers to adjust the parameter according to their risk behavior which considers as an application of the real option model in investment evaluation. Next, the aim of the second simulation is to quantitatively identify the effect of real option model on investment decision. This can be done by simulating a large number of projects and then, using both the real option and NPV to value these projects. The key point is to identify the difference between the investment decision from the real option and NPV. So, the thesis can quantitatively identify the effect of real option model on investment decision. The details on how to construct both simulations are described in Chapter 5 of the thesis. With all of these, the knowledge on the application and the effect of the real option model on investment decision can be deduced from the simulation results.

3.4. Limitations of the Research

The main limitations of the research concern about the data used in the thesis. Even though the chosen case study has complete numerical data, there is a lack of additional details in term of qualitative data which is also important for constructing the simulation. This leads to some limitations of the research, especially the simulation part, as discuss below.

In order to construct a simulation, it needs to model the investment project that has interdependencies between input variables. This process requires an in-depth knowledge about the investment project that will be simulated. However, the research uses the case study which is a secondary source of data so that there is limited information about the interdependencies between each input variables. As a consequence, the simulation cannot identify all interdependencies between variables that might exist and the results might not fully reflect the characteristics of real estate development investment.

Another limitation is that in Monte Carlo simulation, there is a process to specify a suitable probability distribution for random variables. This step heavily depends on the judgment of the person who designs the simulation. Other readers of the thesis might not agree on the choice of the selected probability distribution. To address this issue, the thesis will elaborately justify the chosen probability distribution used in both simulations, but the issue can still occur.

Furthermore, regarding the second simulation that will generate a large number of investment projects, this requires some information about the characteristics of a real estate development project such as the length of construction period, selling period, and the expected streams of cash flows. This information does not contain in the case study so that some assumptions have to be stated. Although efforts will be taken to determine the necessary assumptions to be as realistic as possible, they still cannot perfectly represent all of the characteristics of the project.
3.5. Reliability and Validity of the Research

After identifying the research paradigm, research type, methodology and limitations, the reliability and validity of the thesis can be discussed in this section. According to Collis and Hussey (2009, p. 64), reliability means when a repeat study is conducted, it should produce the same result as the previous study. For this thesis, it is considered to have a high reliability because the chosen methods for the study, which are sensitivity analysis and Monte Carlo simulation, are objective. The methods do not require interpretation from the researcher when analyzing the results given from the study. The analysis is based on the quantitative results, such as figures and graph that gives a precise and definite answer to the study. Consequently, the research should yield the same result when it is repeated in the future.

On the other hand, the research has a high validity when the findings accurately reflect the phenomena that are studied (Collis and Hussey, 2009, p. 64-65). In this case, the thesis tends to have low validity due to the limitations of the research. As discussed earlier, the thesis lacks the qualitative information from the case study which is important to construct the simulations. So, some assumptions are needed in order to conduct the research. This decreases the validity of the thesis since the assumptions might not accurately reflect the object that is studied, which is the characteristic of real estate development investment. In order to cope with this issue, a great effort will be taken to determine and justify all of the assumptions. Also, before analyzing the results from the simulations, both simulations will be validated in order to confirm that the results can be used.

All of these reflect a nature of the positivistic study which is likely to have high reliability but low validity as Collis and Hussey (2009, p. 64-65) mentioned.
4. Real Option Model Analysis - A Case Study

The main focus of this chapter is to critically analyze the real option model that was purposed by McDonald and Siegel (1986) in order to understand the mechanism and logic behind the model. On account of this, the users of the model know the benefits and drawbacks of the model, including important cautions in using it. The chapter starts with calculating some results according to the data from the case study as a base case. Then, the sensitivity analysis is performed and the results are discussed compared with the base case.

4.1. Base Case

As mentioned in the previous chapter, the case study of commercial development at Spitalfields, East London (Sing, 2001) is used as a numerical input for the study. The following paragraph explains the detail of the case study and then summarizes the key parameters for the calculation.

In this case, the land at Spitalfields in East London has an area of 8,000 m$^2$ was bought at the market price of £44 million, which means that the land has the market value at £5,500 per m$^2$. The company that bought the land plan to develop commercial real estate with the main purpose for office use. The building was designed to have the 70,000 m$^2$ leasable space after finishing the construction. The list of all parameters that relate to this project is given in Table 4.1.

<table>
<thead>
<tr>
<th>Input parameters</th>
<th>Base Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land area (m$^2$)</td>
<td>8,000</td>
</tr>
<tr>
<td>Land cost (£)</td>
<td>$f = 44,000,000</td>
</tr>
<tr>
<td>Maximum density of development (m$^2$)</td>
<td>q = 70,000</td>
</tr>
<tr>
<td>Unit construction cost (£ per m$^2$)</td>
<td>K = 2,000</td>
</tr>
<tr>
<td>Instantaneous drift of unit construction cost (%)</td>
<td>$\alpha_f = 10$</td>
</tr>
<tr>
<td>Volatility of unit construction cost (%)</td>
<td>$\sigma_f = 10$</td>
</tr>
<tr>
<td>Financing cost (%)</td>
<td>$\delta_f = 10$</td>
</tr>
<tr>
<td>Unit rental rate (£ per m$^2$)</td>
<td>R = 250</td>
</tr>
<tr>
<td>Instantaneous rental drift (%)</td>
<td>$\alpha_v = 5$</td>
</tr>
<tr>
<td>Rental volatility (%)</td>
<td>$\sigma_v = 20$</td>
</tr>
<tr>
<td>Rental yield (%)</td>
<td>$\delta_v = 7$</td>
</tr>
<tr>
<td>Correlation between rental and construction cost volatility</td>
<td>$\rho_{vf} = 0.0$</td>
</tr>
<tr>
<td>Cost elasticity of scale</td>
<td>$\gamma = 1.01$</td>
</tr>
<tr>
<td>Price elasticity of scale</td>
<td>$\theta = 0.98$</td>
</tr>
<tr>
<td>Risk free rate (%)</td>
<td>$r = 4.6$</td>
</tr>
</tbody>
</table>

Table 4.1 List of the Parameters of the Case Study (Sing, 2001)

For the NPV calculation, equation (1) and (5) are used to determine the present value ($V_0$) and the present cost ($F_0$) of the project. So, the NPV can be calculated as follow.

$$F_0 = q^r K + f = £200,523,307$$ (11)
\[ V_0 = \frac{q^R}{\delta_v} = £200,003,708 \]  

(12)

\[ NPV = V_0 - F_0 = £-519,599 \]  

(13)

From \( V_0, F_0 \) and the given parameters the value of the project by using option pricing can be calculated as well by using equation (7), (8), (9), and (10). The results are shown below.

\[ \sigma^2 = \sigma_v^2 + \sigma_f^2 - 2\rho_{vf}\sigma_v\sigma_f = 0.05 \]  

(14)

\[ \varepsilon = \sqrt{\left(\frac{\delta_r - \delta_v}{\sigma^2} - \frac{1}{2}\right)^2 + 2\frac{\delta_f}{\sigma^2} + \left(\frac{1}{2} - \frac{\delta_r - \delta_v}{\sigma^2}\right)} = 1.90 \]  

(15)

\[ C^* = \frac{\varepsilon}{(\varepsilon-1)} = 2.11 \]  

(16)

\[ X = (C^* - 1)F_0 \left(\frac{V_0}{C^*}\right) = £53,505,391 \]  

(17)

From the calculation, the NPV suggests that the company should not invest and therefore, leave this project because the net present value of the project is negative at £-519,599. However, the value of the project using real option method which considers the value and cost uncertainty is much higher than the NPV with £53,505,391. The real option model suggests that the company should keep the project alive and defer the investment into the future time. Then, the company should invest when the ratio between \( V_0 \) and \( F_0 \) is higher than 2.11.

This result shows that if the company only considers the NPV and decides whether to undertake the project or not, the NPV can lead to the incorrect decision. This is because the NPV does not take uncertainty in the rental price and the cost into the account. In the future, the rental price has probability to go up, or the variable construction cost can decrease. These uncertainties can turn the unprofitable project to the profitable one. The real option model captures these uncertainties and also considers that the investor has management flexibility to defer the project into the future. The investor can defer the project to wait the situation to be resolve more or have a clear picture of the future. If there is a clear trend that the rental price will decrease or the construction cost will increase, the investor can abandon the project without a great loss.

4.2. Sensitivity Analysis

Next, the base case results can be analyzed more in-depth by using sensitivity analysis. The sensitivity analysis performed in this section does not focus on determining the optimistic or pessimistic scenario of the project. It is rather used to study how each input parameters affects the project value from the real option model. All of the variables in the real option model are critically analyzed and discussed in each section. Then, the results from sensitivity analysis give a better understanding in the characteristic of the real option model which is the main outcome that satisfies the objective of the thesis by answering the first research question.
First, the unit construction cost and unit rental rate can be changed in order to determine the optimal value of these parameters that make the investment become feasible. The investment is feasible to invest now when the NPV of the project equals to zero. Hence, those two parameters can be determined by the backward calculation. The results are presented in Table 4.2. The table also shows the values of unit construction cost and unit rental rate that make the NPV and real option value to be the same. This is called the optimal investment values which in the other word, the values that make the ratio between $V_0$ and $F_0$ equals to 2.11, which is the value of $C^*$. 

<table>
<thead>
<tr>
<th>Parameters</th>
<th>NPV</th>
<th>Real Option</th>
<th>$V_0/F_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit construction cost sensitivity analysis (fix the unit rental rate at $R = 250$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base Case</td>
<td>£519,599</td>
<td>£53,505,391</td>
<td>0.99</td>
</tr>
<tr>
<td>Feasible Case</td>
<td>£0</td>
<td>£53,630,825</td>
<td>1.00</td>
</tr>
<tr>
<td>Optimal Case</td>
<td>£105,133,712</td>
<td>£105,133,712</td>
<td>2.11</td>
</tr>
<tr>
<td>Unit rental rate sensitivity analysis (fix the unit construction cost at $K = 2,000$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base Case</td>
<td>£519,599</td>
<td>£53,505,391</td>
<td>0.99</td>
</tr>
<tr>
<td>Feasible Case</td>
<td>£0</td>
<td>£53,770,155</td>
<td>1.00</td>
</tr>
<tr>
<td>Optimal Case</td>
<td>£222,186,874</td>
<td>£222,186,874</td>
<td>2.11</td>
</tr>
</tbody>
</table>

Table 4.2 Sensitivity Analysis of Unit Construction Cost and Unit Rental Rate

From the analysis, it is obvious that a slight decrease in unit construction cost, from £2,000 per unit to £1,993.36 per unit, and increase in unit rental rate, from £250 per m² to £250.65 per m², turn the project to become feasible. This is because the NPV of the base case has a relatively small negative value. Furthermore, this is a very large scale commercial real estate project with 70,000 m² available space. A minimal change in these parameters results a significant change in the project value.

When considering the optimal value to invest, the unit construction cost has to drop considerably to £649.99, or the unit rental rate has to be as high as £528.38 per m² in order to make this project become optimal to invest now. As discussed above, a slight change in both parameters can turn the project to become favorable or unfavorable one. This means that waiting to invest in the future has the value in itself because waiting can help the owner of the project to resolve the situation and has a better prospect about the future. On account of this, the value of the project now (the NPV) has to be high enough to a certain level to compensate for the risks and uncertainties that might happen in the future. More importantly, it should also be noted that even though £649.99 of unit construction cost and £528.38 per m² of unit rental rate make the project to reach the optimal ratio between $V_0$ and $F_0$ at 2.11, the values of option are not the same. This situation is caused by the degree of uncertainty in the cost and the value which is discussed in more details in the next section of sensitivity analysis.
4.2.2. Unit Value and Option Value

After identifying the feasible and the optimal value of unit construction cost \((K)\) and unit rental rate \((R)\), these two input parameters can be changed as well in order to examine the effect on the option value. This is done by fixing one parameter and changing another one. The rest of the parameters such as level of uncertainty and expected rate of return are also fixed according to the base parameters in the case study. The results of the analysis are displayed in Figure 4.1.

![Figure 4.1 Sensitivity Analysis of Unit Value and Option Value](image)

Figure 4.1 shows that the option value falls when the unit construction cost increases. Conversely, the option value rises when the unit rental rate increases. This aligns with an intuition in any investment problems. Anyway, it still should be noted that the effect from increasing the rental value on the option value is substantially higher than the effect from the increase in unit construction cost.

The reason for this is because the unit rental rate is the perpetuity income of the project that the owner of the building will receive forever after finishing the construction. On the other hand, the unit construction cost is the single time cost that the owner has to pay during the construction phase and then once the project is finished, there is no more cost in the project. This is why a slight increase in unit rental rate considerably affects the option value.
4.2.3. Level of Uncertainty and Option Value

Next, the analysis examines the sensitivity of the option value according to the levels of unit construction cost and unit rental rate uncertainty ($\sigma_f$ and $\sigma_v$). As previous, this analysis can be done by changing the level of uncertainty in one type ($\sigma_f$ or $\sigma_v$) of the time while fixing all of the parameters as in the base case. The results of this sensitivity analysis are shown in Figure 4.2 below.

![Figure 4.2 Sensitivity Analysis of Level of Uncertainty and Option Value](image)

From the results, it can be clearly seen that the level of uncertainty (volatility) affects greatly on the option price and the higher the volatility, the higher the option value is. This is because the volatility is the indicator measuring the possible spread of the unit construction cost and unit rental rate. The higher volatility means that there is a wider range of the possible values. This can be described by Figure 4.3 which shows the spread of possible unit rental rate in different level of uncertainty.
From the figure, it can also be seen that with 20% volatility, the unit rental rate can reach £400 per m², while the value can reach only £350 per m² in case of 10% volatility. Since the real option considers the investment project as the right but not an obligation to invest, the owner is not obliged to invest now if the unit rental rate falls beyond the feasible value at £250.65 per m². On the other hand, the real option model values the project base on possible positive upside of the project. As a consequence, the option must have a higher value when there is a probability that the unit rental rate can go as high as £400 per m² than only £350 per m².

This characteristic of the real option model answers the note at the end of section 4.2.1 that there is a difference in the value of the option at the optimal point to invest when calculating by the unit construction cost and the unit rental rate. When the unit construction cost reaches the optimal point at £649.99, the value of the project as an option is £105,133,712. However, when considering the unit rental rate at the same optimal point of £528.38 per m², the value of the option is £222,186,874. This difference is caused by the different levels of uncertainty in each parameter. The unit rental rate volatility is higher than the volatility of the unit construction cost at 20% and 10% respectively. As a result, the option value calculated based on the unit rental rate has to be higher than another one calculated by the unit construction cost.

### 4.2.4. Expected Rate of Return and Option Value

The last sensitivity analysis examines the relationship between the expected rate of return, both on the unit construction cost and unit rental rate, and the option value. From the case study, the expected rate of return on cost is the financing cost (δ_f) and the expected rate of return on value is the rental yield (δ_r). The sensitivity analysis can be done in the same approach as previous by fixing one parameter and changing another one in order to examine the effect of expected rate of return on the option value. The results are presented in Figure 4.4.
From the figure, the analysis starts at the 4.6% expected rate of return from both financing cost and rental yield because the lowest expected rate of return has to be the same as risk free rate, which is 4.6% in this case. As the results, it is apparent that the option value has a positive correlation with financing cost, but exponentially negative correlation with rental yield. The relationship can be explained by the concept of discounted cash flow (DCF). The DCF concept has two basic rules which are the today’s money has more value than the future’s money, and the safer money has more value than the riskier money. With these concepts, it means that the future cash flows from a particular asset have to be discounted back into the present values by the expected rate of return. The higher the expected rate of return means the asset is riskier, thus the future cash flows are discounted more and have less value today.

Considering the unit construction cost which is the cash outflow of the project, the real option model considers that the project owner can develop the commercial real estate in the future at £2,000 per unit. For this, the £2,000 per unit cash outflow in the future has less value today. Increasing the financing cost leads to the even lower today’s unit construction cost because the cost is discounted more. On the account of this, the construction project costs less and the value of the project calculated by the real option increases. The same line of reasoning can be applied to the unit rental rate. When the rental yield rises, the future cash inflows from rent are discounted more and have the lower value at present. This results in the decrease in the income and then, the project value. However, increasing the rental yield severely affects the project value because the option value also depends on the expected project value ($V_0$). The expected project value is calculated according to the equation (12) as a perpetuity income. So, not only does an increase in rental yield affect directly on the real option model, but it also significantly affects the expected value of the project. As a consequence, the option value falls exponentially when the rental yield rises.
4.2.5. Summary of Sensitivity Analysis

From all of the analysis, the effects from changing different parameters on the option value can be summarized according to Table 4.3 below.

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Unit Construction Cost ((K, f))</th>
<th>Unit Rental Rate ((R, v))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feasible Value</td>
<td>£1,993.36</td>
<td>£250.65</td>
</tr>
<tr>
<td>Optimal Value</td>
<td>£649.99</td>
<td>£528.38</td>
</tr>
<tr>
<td>Unit Value*</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Level of Uncertainty ((\sigma))*</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Expected Rate of Return ((\delta))*</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

*Changes in the option value when the input parameters increase

Table 4.3 Summary of the Sensitivity Analysis

As shown in Table 4.3, it is very interesting to see that these characteristics of real option correspond with the financial call option in stock as described by Brealey, Meyers, and Allen, F. (2006, p. 581). To elaborate, an increase in unit rental rate (stock value) and level of uncertainty of unit rental rate (volatility of the stock) have a positive effect in the option price. Inversely, an increase in the unit construction cost (exercise price) negatively affects the option price. In addition, this analysis adds the knowledge of the effect on the option value from the volatility of the unit construction cost (exercise price) which normally does not have in financial call option in stock. Also, the analysis presents the effect on the option value from expected rate of return of unit construction cost (exercise price) and unit rental rate (stock value) which are not covered in financial call option as well. The sensitivity analysis yields an interesting result as discussed in details in the previous parts.

In conclusion, the real option model has many variables as the inputs and the results from sensitivity analyses reveal that the input parameters that relate to the unit rental rate have much greater effect on the option value than the parameters related to the unit construction cost. This suggests that the owner of the project must carefully determine all of the input parameters, especially ones that related to the unit rental rate, so that result in project value from the real option model is reliable. In addition, the company should keep a good track on the fluctuation in unit rental rate since it is the key variable that can lead to a dramatic change in the project value as discussed earlier. These comments show that the real option model has higher complexity in term of usage of the model compared with the traditional NPV approach.

All of these characteristics help financial analysts to have a better understanding of the real option model that captures value and cost uncertainties and know how to use the model correctly with some cautions for any real estate development projects.
5. Monte Carlo Simulation Design

After calculating the base result from the case study and analyzing the real option model in a great detail by using sensitivity analysis in chapter 4, the Monte Carlo simulation is introduced as a study to determine the application of the real option model and the effect on the management decision. There are two simulations needed to be performed as mentioned in the methodology chapter which are $C^*$ simulation, and NPV and real option value simulation. This chapter focuses on stating the purpose of each simulation and explaining how to construct them in order to insure reliability and validity of the thesis. The information used to build the simulations also comes from the case study of commercial development at Spitalfields, East London (Sing, 2001) as in the previous chapter.

5.1. $C^*$ Simulation

According to the real option model, $C^*$ is the ratio between the total present value of the project ($V_0$) and the total present value of the cost ($F_0$). As the investment rule that was described in section 2.3.3, the company should invest in the project now when $V_0/F_0$ equals $C^*$. With this, $C^*$ is a very important parameter since it tells when the company should invest in the project. Therefore, determining the expected value of $C^*$ in general case is very useful in decision making process. In order to find the expected value of $C^*$, a Monte Carlo simulation is used to generate all possible values of $C^*$ by changing some input variables randomly. The results from the simulation can be used to derive the expected value of $C^*$ in general case and also allow managers to adjust this parameter according to their risk behavior. This considers as an application of the real option model. Hence, the objective of the simulation can be defined as follow.

**Simulation Objective:** to find the expected value and all possible values of $C^*$

After defining the objective of the simulation, the design of the simulation follows the framework that was discussed in section 2.4.2.

**Step 1: Modeling the investment project**

In this case, the model for the investment project means the model used for calculating the $C^*$ which is equation (8), (9), and (10) as presented again below.

$$C^* = \frac{\varepsilon}{(\varepsilon - 1)}$$

(8)

Where

$$\varepsilon = \sqrt{\left(\frac{\delta_f - \delta_v}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\delta_f}{\sigma^2} + \frac{1}{2} - \frac{\delta_f - \delta_v}{\sigma^2}}$$

(9)

$$\sigma^2 = \sigma^2_v + \sigma^2_f - 2\rho_{vf}\sigma_v\sigma_f$$

(10)

Regarding the interdependence between variables, in this simulation, there is neither time nor variable interdependence between level of uncertainty in unit construction cost ($\delta_f$) and unit rental rate ($\delta_v$), or the expected rate of returns ($\delta_f$ and $\delta_v$). This is because these variables do not change ac-
cording to time and also this simulation consider at one point of the time which is at the present time \( t_0 \). For the interdependence between variable, there is no interdependence as well since from the case, the correlation between rental and construction cost volatility \( \rho_{vr} \) equals to zero and there is no correlation between expected rate of return.

**Step 2: Identifying random variables and probability distributions**

The next step is to identify the random variables to be sampled by the simulation. \( C^* \) is calculated by using equation (8), (9), and (10) and from these equations, it can be seen that the value of \( C^* \) depends on the levels of uncertainty of unit construction cost \( (\sigma_f) \), uncertainty of unit rental rate \( (\sigma_v) \), and the expected rate of returns \( (\delta_f \text{ and } \delta_v) \). This simulation focuses on changing the level of uncertainties on \( C^* \) only because these two variables are not certain since they are determined from the market study and forecast. While, the expected rate of returns are much more certain input because the expected rate of return on the rent \( (\delta_v) \) and the financing cost \( (\delta_f) \) are determined by the company and the financial institution themselves.

Regarding the probability distributions for these random variables, the key characteristics of the level of uncertainties is that first, the level of uncertainties has the minimum value at 0, while the maximum value cannot exceed 1. The second is that the possible values of these variables are the real number. On account of these, the selected probability distribution has to be a continuous function but it cannot be uniform or normally distributed. This is because the uniform distribution means that there is an equal probability for the random variables to be any values from 0 to 1. For example, there is the equal probability that the level of uncertainty in unit construction cost is 0.01 or 1. This is not correct in this case since the case already estimated the values for both uncertainties which are 0.1 and 0.2 for volatility of unit construction cost and rental volatility respectively. These estimated value can be consider as the averages of the level of uncertainties. On the other hand, the probability distribution of the random variables cannot be the normal distribution because the boundary for the changes of these values do not equal. To elaborate, for the level of uncertainty in unit construction cost which has the estimated (mean) value at 0.1, this means that the value can decrease only for 0.1 while it can increase for 0.9 more. This is the same for the level of uncertainty in unit rental rate. This characteristic causes a right skew to the distribution and hence, the normal distribution does not fit to the random variables.

With all of these reasons, the probability distribution which is a good representative for the random variables is lognormal distribution. The lognormal distribution is a continuous probability distribution of a random variable whose logarithm is normally distributed. This distribution is commonly used in computational finance for non-negative variables (Jäckel, 2002, p. 14-15). It has two control parameters which are the mean of \( \ln(\sigma) \) and the standard deviation of \( \ln(\sigma) \). Due to the lag of information from the case study, these two parameters are derived from the following assumptions.

- Assume that mean of \( \ln(\sigma) \) equals to \( \ln(\text{mean of } \sigma) \), given that
  - The mean of the level of uncertainty in unit construction cost equals to 0.1
  - The mean of level of uncertainty in unit rental rate equals to 0.2
- The company is certain that the lowest possible of level of uncertainties is 0. This leads to
  - The values of \( \pm 3 \) standard deviation of the level of uncertainty in unit construction cost are 0 and 0.2.
The values of ±3 standard deviation of the level of uncertainty in unit rental rate are 0 and 0.4.

With the assumptions, Table 5.1 shows the calculation of the control parameters for lognormal distribution.

<table>
<thead>
<tr>
<th>Random Variables</th>
<th>Mean</th>
<th>Value of +3 SD</th>
<th>ln(mean)</th>
<th>ln(value of +3 SD)</th>
<th>+3 SD of ln</th>
<th>+1 SD of ln</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level of uncertainty in unit construction cost (σ_f)</td>
<td>0.1</td>
<td>0.2</td>
<td>-2.30</td>
<td>-1.61</td>
<td>0.69</td>
<td>0.23</td>
</tr>
<tr>
<td>Level of uncertainty in unit rental rate (σ_v)</td>
<td>0.2</td>
<td>0.4</td>
<td>-1.61</td>
<td>-0.92</td>
<td>0.69</td>
<td>0.23</td>
</tr>
</tbody>
</table>

*SD means standard deviation and ln means natural logarithm

Table 5.1 The Calculation of Control Parameters for Lognormal Distribution for C* Simulation

From the given parameter, the lognormal probability distributions of both random variables can be drawn as in Figure 5.1.

Figure 5.1 The Lognormal Probability Distribution of the Random Variables in C* Simulation
Step 3: Simulate random variables, calculate, and collecting results

The procedure of running the simulation can be described by the flow chart in Figure 5.2 with 50,000 iterations. The remaining variables used in the simulation are the same as in the case study in chapter 4. Table 5.2 shows the values of these variables. The results from the simulation are presented and discussed in the chapter 6 of the thesis.

<table>
<thead>
<tr>
<th>Input parameters</th>
<th>Base Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financing cost (%)</td>
<td>$\delta_f = 10$</td>
</tr>
<tr>
<td>Rental yield (%)</td>
<td>$\delta_v = 7$</td>
</tr>
<tr>
<td>Correlation between rental and construction cost volatility</td>
<td>$\rho_{vf} = 0.0$</td>
</tr>
</tbody>
</table>

Table 5.2 List of the Input Parameters for $C^*$ Simulation
Figure 5.2 Flow Chart of $C^*$ Simulation

- Start the simulation
- Samples a pair of unit construction cost and unit rental rate uncertainty ($\sigma_f$ and $\sigma_v$)
  - Calculate $\sigma^2$ according to the equation (10)
  - Calculate $\varepsilon$ according to the equation (9)
  - Calculate $C^*$ according to the equation (8)
  - Collect the value of $C^*$ that was calculated
- Does the simulation reach 50,000 iterations?
  - No
  - Yes: Stop the simulation
5.2. NPV and Real Option Value Simulation

From the section 2.3.3, the theory describes that although the project has negative NPV value, the project still has the value when considering it as an option that can be deferred to the future. This is because the valuation of the project using the real option model considers that the company has management flexibility. Also, it takes the uncertainty in the unit construction cost and unit rental rate into the account. Then, the real option model calculates the project value based on possible positive outcome of the project and suggests that the project should be deferred as long as the option value of the project is higher than the NPV value. These applications of the real option model can considerably affect the management decision compared with making the decision solely on the NPV. On account of this, it is interesting to quantitatively identify the effect of the real option model on the NPV. Hence, the objective of the simulation can be defined as follow.

Simulation Objective: to quantitatively identify the effect of real option model on the investment decision by measuring four indicators.

- The value of real option compared with NPV
- The percentage that the project is rejected by NPV and accepted by the real option model.
- The percentage that both NPV and the real option accept the project but the real option suggests that these projects should be deferred.
- The percentage that both NPV and real option accept the project and both methods suggest investing now.

After identifying the objective of the simulation, it can be designed by following the framework that was discussed in the section 2.4.2.

Step 1: Modeling the investment project

The key variables that are used to calculate the present cost \( F_0 \) and the present value \( V_0 \) of the project are the unit construction cost \( K \) and the unit rental rate \( R \). These two variables have the interdependence between periods which follows the geometric Brownian motion with drift as the main assumption of the real option model. On account of this, this simulation has to state some assumptions regarding the time period of the project in the case study due to the lag of this information as follow.

- Assume that the construction period of the project takes 3 years since the construction of a multi-storey commercial complex generally takes more than 2 years (Sing, 2001).
- Assume that the company buys the land \( f \) at the present time \( t = 0 \).
- Assume that the unit construction cost \( K \) is observed every year and it fluctuates according to the geometric Brownian motion with drift.
- The progress in the construction is assumed to be a linear function, which means that for the project with density development of \( q \) m\(^2\), the project is constructed at \( q/3 \) m\(^2\) per year.
• Assume that the project is fully rented after finishing the development.
• Assume that the unit rental price \( R \) is observed every year since the beginning of the development and it fluctuates according to the geometric Brownian motion with drift.
• Assume that after 5 years of renting the property, the unit rental rate is a constant afterwards with the same value at 8\(^{th}\) year.

These assumptions can be summarized in term of the streams of cash flows according to Figure 5.3 below.

![Figure 5.3 Streams of Cash Flows for NPV and Real Option Value Simulation](image)

Figure 5.3 Streams of Cash Flows for NPV and Real Option Value Simulation

With these assumptions, first the models for the unit construction cost and the unit rental rate that have interdependence between period can be built into the equations below (noted that, the details of deriving these models are explained in the appendix).

\[
K_t = (1 + \alpha_f)K_{t-1} + \sigma_f K_{t-1} dZ_f
\]  
(18)

\[
R_t = (1 + \alpha_v)R_{t-1} + \sigma_v R_{t-1} dZ_v
\]  
(19)

Then, the unit construction cost \( K_t \) and the unit rental rate \( R_t \) are used to calculate the present cost \( F_0 \) and the present value \( V_0 \) of the project by modifying the equation (1), (4), and (5). This is because the given equations do not properly consider the time value of money which is the key aspect of the simulation. Here are the modified models for calculating the present value and the present cost of the project.

\[
F_0 = f + \sum_{t=1}^{3} \frac{(q/3)^yK_t}{(1 + \delta_f)^t}
\]  
(20)

\[
V_0 = \sum_{t=3}^{8} \frac{q^\theta R_t}{(1 + \delta_v)^t} + V_{terminal}
\]  
(21)

\[
V_{terminal} = \frac{q^\theta R_8}{\delta_v(1 + \delta_v)^8}
\]  
(22)
After calculating the project’s present cost \( F_0 \) and present value \( V_0 \), the NPV and the real option value \( X \) of the project can be found by these equations.

\[
NPV = V_0 - F_0
\]  
(23)

\[
X = (C^* - 1) F_0 \left( \frac{V_0}{F_0} \right)^{C^*}
\]  
(7)

Where

\[
C^* = \frac{\varepsilon}{(\varepsilon-1)}
\]  
(8)

\[
\varepsilon = \sqrt{\left(\frac{\delta_f - \delta_v}{\sigma^2} - \frac{1}{2}\right)^2 + 2 \frac{\delta_f}{\sigma^2} \left(1 - \frac{\delta_f - \delta_v}{\sigma^2}\right)}
\]  
(9)

\[
\sigma^2 = \sigma_v^2 + \sigma_f^2 - 2 \rho_{vf} \sigma_v \sigma_f
\]  
(10)

**Step 2: Identifying random variables and probability distributions**

Although there are several models in this simulation, there are only two random variables needed to be sampled by the computer. The random variables are \( dz_f \) and \( dz_v \) which are standard Wiener process (Brownian motion). The probability distributions of \( dz_f \) and \( dz_v \) are normal distribution with zero mean and unit standard deviation (Dixit and Pindyck, 1994, p. 65-66). Figure 5.4 displays the probability distributions of \( dz_f \) and \( dz_v \).

![Probability Density](image)

Figure 5.4 The Normal Probability Distribution of the Random Variables in NPV and Real Option Value Simulation
Step 3: Simulate random variables, calculate, and collecting results

The input parameters for the models used in this simulation are shown in Table 5.3. In addition, the procedure in running the simulation is explained in Figure 5.5. The simulation runs for 50,000 iterations.

<table>
<thead>
<tr>
<th>Input parameters</th>
<th>Base Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land cost (£)</td>
<td>$f = 44,000,000$</td>
</tr>
<tr>
<td>Maximum density of development (m$^2$)</td>
<td>$q = 70,000$</td>
</tr>
<tr>
<td>Unit construction cost at year 0 (£ per m$^2$)</td>
<td>$K_0 = 2,000$</td>
</tr>
<tr>
<td>Instantaneous drift of unit construction cost (%)</td>
<td>$\alpha_f = 10$</td>
</tr>
<tr>
<td>Volatility of unit construction cost (%)</td>
<td>$\sigma_f = 10$</td>
</tr>
<tr>
<td>Financing cost (%)</td>
<td>$\delta_f = 10$</td>
</tr>
<tr>
<td>Unit rental rate at year 0 (£ per m$^2$)</td>
<td>$R_0 = 250$</td>
</tr>
<tr>
<td>Instantaneous rental drift (%)</td>
<td>$\alpha_v = 5$</td>
</tr>
<tr>
<td>Rental volatility (%)</td>
<td>$\sigma_v = 20$</td>
</tr>
<tr>
<td>Rental yield (%)</td>
<td>$\delta_v = 7$</td>
</tr>
<tr>
<td>Correlation between rental and construction cost volatility</td>
<td>$\rho_{vf} = 0.0$</td>
</tr>
<tr>
<td>Cost elasticity of scale</td>
<td>$\gamma = 1.01$</td>
</tr>
<tr>
<td>Price elasticity of scale</td>
<td>$\theta = 0.98$</td>
</tr>
</tbody>
</table>

Table 5.3 List of the Input Parameters for NPV and Real Option Value Simulation
Figure 5.5 Flow Chart of NPV and Real Option Value Simulation
6. Simulation Result and Analysis

This chapter presents and discusses the results from $C^*$ simulation and NPV and real opiton value simulation. Before discussing and analyzing the results, each simulation is validated first in order to make sure that the simulations are valid so that the results generated by the simulations can be used for the discussion. Then, the chapter discusses and notes the important points according to the objective of each simulation.

6.1. $C^*$ Simulation

6.1.1. $C^*$ Simulation Validation

After running the simulation, first, the distribution of random variables should be examined in order to confirm that the simulation sampled these variables according to the assigned probability distribution. In the $C^*$ simulation, there are two random variables namely, the level of uncertainty in unit construction cost ($\sigma_f$) and level of uncertainty in unit rental rate ($\sigma_v$), which are assigned to have lognormal distribution with the distribution parameters according to Table 5.1. Figure 6.1 shows the distribution of both variables, and also the average and standard deviation of natural logarithm of the sampled variables are presented in Table 6.1.

![Figure 6.1 Distributions of Random Unit Construction Cost Uncertainty and Unit Rental Rate Uncertainty from $C^*$ Simulation](image)
<table>
<thead>
<tr>
<th>Random Variables</th>
<th>Mean of $\ln$</th>
<th>SD of $\ln$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level of uncertainty in unit construction cost ($\sigma_f$)</td>
<td>-2.30</td>
<td>0.23</td>
</tr>
<tr>
<td>Level of uncertainty in unit rental rate ($\sigma_y$)</td>
<td>-1.61</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Table 6.1 Average and Standard Deviation of Random Unit Construction Cost and Unit Rental Rate Uncertainty from $C^*$ Simulation

Figure 6.1 confirms that the random variables that were sampled by the simulation follow the lognormal distribution with the given distribution parameters because the result in Figure 6.1 shows that the distributions are very similar to the Figure 5.1. In addition, the average and standard deviation of natural logarithm of both random variables have the same values as defined in Table 5.1. This confirms the reliability and validity of the simulation.

6.1.2. $C^*$ Simulation Result and Analysis

Next, possible values of $C^*$ are plotted to examine the distribution of the results and then, the key statistical measurements are calculated for the analysis of the results. Figure 6.2 illustrates the distribution of all possible values of $C^*$ and Table 6.1 presents the statistical results.

![Figure 6.2 Distribution of Possible Values of $C^*$](image)

<table>
<thead>
<tr>
<th>Statistical Measurement</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>1.63</td>
</tr>
<tr>
<td>Maximum</td>
<td>4.25</td>
</tr>
<tr>
<td>Mean</td>
<td>2.16</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.22</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.14</td>
</tr>
</tbody>
</table>

Table 6.2 Statistical Measurements of $C^*$ Simulation
The histogram and skewness measurement show that the results of $C^*$ from the simulation are not normally distributed. The distribution is right skew as it can be seen from the histogram and the positive skewness measurement. The result is reliable because the random variables, the level of uncertainty in unit construction cost ($\sigma_f$) and level of uncertainty in unit rental rate ($\sigma_v$), which are the inputs of the simulation are assigned to be lognormal distribution with a right skew.

As mentioned in the theory, $C^*$ is the ratio between the present value ($V_0$) and present cost ($F_0$) and from the investment rule of the real option model, the company should invest in the project now when the value of $V_0/F_0$ is equal or higher than $C^*$. If the ratio is less than the value of $C^*$, the company should defer the investment. From the statistical measurements, the results suggest that the company should invest in the project now only when the present value ($V_0$) is 2.16 times higher than the present cost ($F_0$) which is the average value of $C^*$ received from the simulation. This value is slightly higher than the value of $C^*$ at 2.11 which was calculated in the base case (section 4.1). The reason for the difference is that the $C^*$ from the base case considered the given level of uncertainties ($\sigma_f$ and $\sigma_v$) to be precisely determined. This is not the case in reality; no company can determine these values without uncertainty. In conclusion, the result of the $C^*$ at 2.16 can be used as a general decision criteria which suggests that a firm should invest in a real estate project when the value of the project doubles the development cost. This can significantly change how companies decide to invest in a project. With this fact in mind, investing in a project when the value is higher than the cost according to NPV rule is not applicable anymore. The value of the project must be much higher than the cost, or 2.16 times higher to be precise. This is because the real option method considers that the value of the project can either increase or decrease in the future due to uncertainties, and deferring the investment to receive more information is valuable. So, the present value of the project has to be high enough to compensate for the risk that the value of the project can decrease. This high present value also needs to cover for the company’s right to wait in order to receive more information in the future.

Considering further, the value of $C^*$ at 2.16 is the expected optimal ratio to invest now for a normal investor. However, the company can characterize itself to be more risk taker or risk adverse investor. If the company characterize itself to be a risk taker investor, the company can decide to invest in the project now when the ratio between $V_0$ and $F_0$ is lower than 2.16. Conversely, the risk adverse company will invest in the project when the ratio between $V_0$ and $F_0$ is higher than 2.16. The standard deviation of $C^*$ can be used for adjusting the expected value of $C^*$ according to the risk characteristic of the company. For example, if the company has a positive aspect of the unit rental rate, the company can adjust the $C^*$ to be one standard deviation lower than the expected value. So, the adjusted $C^*$ will equal to 1.94. Furthermore, the simulation also tells the company that the $C^*$ can go as high as 4.25 or can be as low as 1.63 which is the extreme case of the project. These results provides the insight of the most optimistic and pessimistic case to the company without any subjective decision as the sensitivity analysis. Moreover, these two values also specify the appropriate range that $C^*$ can be adjusted according to the risk characteristic of the company.

To sum up, the thesis found that in general case, a company should invest in a real estate project when the value doubles the cost of development. Otherwise, it is better to defer the investment. Furthermore, the results from the simulation allow the company to properly adjust the value of $C^*$ according to their risk characteristic which is an application of the real option model.
6.2. NPV and Real Option Value Simulation

6.2.1. NPV and Real Option Value Simulation Validation

Again, it is needed to check for the distribution of the random variables for validating the simulation. In NPV and real option value simulation, the random variables are the standard Wiener process of unit construction cost ($dz_f$) and unit rental rate ($dz_v$). The standard normal distribution with zero mean and a unit standard deviation was assigned to both variables. After performing the simulation, the histograms of the random variables are plotted as shown in Figure 6.3. Also, Table 6.3 presents the mean and the standard deviation that were calculated from the random $dz_f$ and $dz_v$.

![Figure 6.3 Distributions of the Random Standard Wiener Process of Unit Construction Cost and Unit Rental Rate from NPV and Real Option Value Simulation](image)

<table>
<thead>
<tr>
<th>Random Variables</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Wiener process of unit construction cost ($dz_f$)</td>
<td>-0.00165</td>
<td>1.00068</td>
</tr>
<tr>
<td>Standard Wiener process of unit rental rate ($dz_v$)</td>
<td>-0.00010</td>
<td>0.99918</td>
</tr>
</tbody>
</table>

Table 6.3 Average and Standard Deviation of the Random Standard Wiener Process of Unit Construction Cost and Unit Rental Rate from NPV and Real Option Value Simulation

Figure 6.3 and the results in Table 6.3 shows that the random generator from the simulation worked properly because the distribution of the random variables is normally distributed. Also, the actual mean and the standard deviation of the random variables are very close zero and one respectively. Noted that, the reason the distribution of $dz_v$ has higher frequency than the distribution of $dz_f$ is because the simulation needs to sample eight $dz_v$ s per iteration, while it sampled only three $dz_f$ s in an iteration. This due to the assumption that the project takes three years for construction and eight years before the unit rental rate becomes stable. With all of these, the simulation is considered to be reliable and valid.
Next, Figure 6.4 and Figure 6.5 illustrate the stochastic process of the unit construction cost ($K$) and the unit rental rate ($R$) that were assigned to follow geometric Brownian motion with drift. This is also to prove that the simulation correctly simulates both variables according to the assumption of the real option model.

Figure 6.4 Sample Path of Geometric Brownian Motion with Drift of Unit Construction Cost from NPV and Real Option Value Simulation

Figure 6.5 Sample Path of Geometric Brownian Motion with Drift of Unit Rental Rate from NPV and Real Option Value Simulation
6.2.2. NPV and Real Option Value Simulation Result and Analysis

After validating the simulation, the results from the simulation which are values of NPV and real option from different projects are plotted in Figure 6.6 (noted that, the distributions in Figure 6.6 do not contain all of the results from the simulation because the figure wants to focus on the peak areas and the shape of the distribution. Plotting all of the results will deteriorate this focus). Also, Table 6.4 presents the key statistical measurements of NPV and real option value.

![Figure 6.6 Distributions of Possible NPV and Real Option Values](image)

<table>
<thead>
<tr>
<th>Statistical Measurement</th>
<th>NPV</th>
<th>Real Option</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>£223,240,916</td>
<td>£483,245</td>
</tr>
<tr>
<td>Maximum</td>
<td>£1,244,792,054</td>
<td>£1,244,792,054</td>
</tr>
<tr>
<td>Mean</td>
<td>£35,385,190</td>
<td>£88,552,314</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>£125,699,163</td>
<td>£92,121,958</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.42</td>
<td>2.61</td>
</tr>
</tbody>
</table>

Table 6.4 Statistical Measurements of NPV and Real Option Simulation

From the illustrations and the values of skewness, it is obvious that the possible NPV and real option value are not normally distributed. Both distributions are right skew since the skewness values are positive. This characteristic of the distributions is caused by the assumption in the fluctuation of the unit construction cost \((K)\) and the unit rental rate \((R)\). The unit construction cost and the unit rental rate are assumed to follow geometric Brownian motion with drift which means that the lowest possible value is limited to zero, while there is no limit of the highest value. As the result, the NPV and real option value which were calculated based on these inputs have lognormal distributions in the same way as the unit construction cost and the unit rental rate.
Another interesting point needed to be mentioned is that according to the theory, the lowest value of the option is zero which will occurs only if the present value of the project is zero. The simulation result aligns with this theory as the minimum value of the project from the real option model is still a positive number at £483,245. This means that even though the NPV of the project is very bad, the project still has a positive value to defer the investment. This is not practical in reality because, sometimes, the company has to pay a fee in order to keep the project alive. Holding on the project that has a negative value at present (negative NPV) and a small chance to be positive in the future (low option value) is not suitable. Hence, the company should set an acceptable value of the option value as well and then keep the project alive and defer the investment only if the option value is higher than the acceptable value.

Next, the results from the simulation are used to quantitatively measure the effect of the real option on the investment decision. This was done by first determining the investment decision based on NPV and real option value in each project generated by the simulation. Then, the number of the projects that each investment decision occurs was collected from all of the simulated projects. The investment decision of a project can be one of the three alternatives as follow.

- NPV rejects the project \( (NPV < 0) \) but real option accepts and suggests to defer the investment \( (\text{option value} > 0 \text{ and option value} > \text{NPV}) \)
- NPV accepts the project \( (NPV > 0) \) but real option suggests to defer the investment \( (\text{option value} > \text{NPV}) \)
- NPV and real option accept the project and suggest to invest now \( (\text{NPV} > 0 \text{ and } V_0/F_0 \geq C^*) \)

After collecting and analyzing the NPV and real option value of all projects, the number and percentage of each investment decision are presented in Figure 6.7 and Table 6.5 below.
<table>
<thead>
<tr>
<th>Investment Decision</th>
<th>Number of Projects</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV rejects, but real option accepts and suggests to defer the projects</td>
<td>23,095</td>
<td>46%</td>
</tr>
<tr>
<td>NPV accepts the projects but real option suggests to defer the projects</td>
<td>22,599</td>
<td>45%</td>
</tr>
<tr>
<td>NPV and real option accept the projects and suggest to invest now</td>
<td>4,306</td>
<td>9%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>50,000</strong></td>
<td><strong>100%</strong></td>
</tr>
</tbody>
</table>

Table 6.5 Number of Each Investment Decision from NPV and Real Option Simulation

From the figure, the NPV rejects almost half of the results from the simulation, while the real option suggests that the projects should be deferred. Furthermore, although a large portion of the projects, at 45%, is accepted by NPV, these projects are again should be deferred according to the option value since the option value is still higher than the NPV. Lastly, only 9% or 4,306 projects have the same investment decision from NPV and real option which is to invest in the projects now.

In conclusion, these results quantitatively show that the real option model greatly affects the investment decision by more than 90% at a time when there are cost and value uncertainty in the investment project. This means that making the decision based only on the NPV can lead to a huge error or a wrong timing to invest. Therefore, the traditional way of using only NPV as a decision making tool is not applicable anymore. It is essential for companies to use the real option valuation together with the NPV to evaluate their investment projects. So, they can correctly invest in the projects at the right moment and do not abandon the projects that still have positive aspect in the future.
7. Conclusion

This chapter summarizes all of the findings in relation with the posted research questions. Also, the chapter discusses the knowledge gap that the thesis does not cover and then provides suggestions for the future research.

7.1. Summary of the Findings

According to the introduction part, the thesis posted two research questions which are “What are the main concerns needed to be considered when using real options to evaluate real estate development projects under uncertainties?” and “How do real options affect investment decision in real estate development projects under uncertainties”.

Regarding the first question, the research found out from the study in chapter 4 that the real option model has higher complexity in term of usage compared with traditional NPV approach. When using the real option model, the input parameters should be carefully determined, especially the ones related to the unit rental rate of the project, which are level of uncertainty ($\sigma_{u}$) and expected rate of return ($\delta_{u}$). This is because the study found that a slight change in the variables related to the unit rental rate causes a significant change in the real option value. On the contrary, a change in parameters related to the unit construction cost has much less effect on the real option value. The thesis also presents the relationship between each input variable and the option value in Table 4.3. The finding provides a better understanding of the mechanism and the logic behind the mathematical formula of the real option model. This helps analysts who want to use the real option to be able to use it more effectively in real estate development evaluation problem.

Next, the thesis answers the second question by the finding from Monte Carlo simulations in chapter 6. First, the study found that the value of $C^*$ which is one of the results from the real option model plays an important role in the investment decision. This is because $C^*$ indicates that a real estate development project should be invest now only when the ratio between present value ($V_0$) and present cost ($F_0$) is higher or equal than the value of $C^*$. Chapter 6.1 found that in general case, the value of the $C^*$ is expected to be around 2.16 which means that the value of the project should double the cost before investing. This totally contradicts with the NPV which suggests that the project should be invested when its value is higher than its cost. On account of this, the real option method greatly affects the traditional decision that is based on the NPV only. Furthermore, the thesis identifies the application of the real option model by showing that a company can adjust the value of $C^*$ according to its risk behavior. The study found that the value of $C^*$ ranges from 1.63 to 4.25 so that the firm can adjust this value to be lower if it can accept a higher risk, while increases the value of $C^*$ if the company has a risk adverse behavior.

Regarding another finding, the thesis also quantitatively identifies the effect of the real option on the investment decision in Chapter 6.2. The study found that more than 90% of the time, the real option model changes that investment decision based on the NPV. This is, by 46% of the time, the real option changes the decision from not investing in the project to defer the investment into the future, and, by 45% of the time, the real option changes the decision from invest in the project now to defer the project. This finding can be concluded that using only NPV to evaluate the investment and
making the decision based on that leads to a huge error. Implementing the real option valuation method can improve the decision making process for companies in real estate development industry. This finding can be supported by the empirical studies from Quigg (1993), and Sing and Patel (2001). The studies statistically present that using real option in real estate development projects is better than the NPV in real estate development investment.

7.2. Academic Significance

First, the thesis contributes to the knowledge in the area of corporate finance by showing how to use the real option model developed by McDonald and Siegel (1986) in real estate development investment. Furthermore, the result from the sensitivity analysis does not restrict only in the real estate development context. When using in other types of investment, the relationship between each input variable and the option value is still the same, but the degree of the effect from each input variable might change only. All of these can be used as a guideline to implement the real option model to evaluate other types of investment in other industries.

Another academic significance of this thesis is that it clearly explains the method to construct a Monte Carlo simulation in investment evaluation problem. Other research papers can follow the framework explained in the thesis and use the explanation in Chapter 5 as the examples for a better understanding when constructing a Monte Carlo simulation. This has academic benefits since most of the textbooks regarding Monte Carlo simulation in finance are highly theoretical. The thesis already simplified these concepts so that they are easier to understand and to be used practically.

Finally, the thesis enriches the knowledge on the real options approach and adds the quantitative findings that show how the real option model affects management decision. In this endeavor, it can raise awareness in the existing alternative approach for investment evaluation instead of the well-known NPV method.

7.3. Suggestions for the Future Research

This study mentioned that there are many variables used in the real option model and it is important to carefully determine these parameters since they have a significant effect on the reliability of the result. However, the thesis does not study on how to collect or determine these data which is one knowledge gap that can be filled by other researches.

Furthermore, due to the complexity of the real option model, the future research can extend the current study by conducting a qualitative research in order to see the opinions from the real estate development industry. This can gain an insight on the practical aspect of the real option method and determine other important issues on using the real option for evaluating the investment.

This thesis commented on completeness of the data from the case study which is one of the weaknesses of the research. The case study still lacks the qualitative information that is necessary for constructing the Monte Carlo simulations. One suggestion for the future research is to repeat the study by using the same option model and the structure of this study with better rich data. So, the results of the new study will have higher credibility than the current one.
As highlighted in Chapter 6.2, one important thing of the real option model is that the value of the option will still have some small positive value although an investment project has a very bad negative NPV. The thesis pointed out that sometimes it is not practical to defer the project with this valuation result but the thesis does not address this issue since it is out of the scope of the study. On account of this, the future study can be performed to solve this problem by conducting a study to identify what is the appropriate minimum value of real option that it is still applicable to defer the investment. Alternatively, the future research can conduct a study that modifies the real option model used in this thesis in order to address the issue.

Lastly, the thesis focuses only one type of real option which is the option to defer. However, there might have other types of real option incorporated in real estate development investment such as the option to expand or abandon the project in the future. Another suggestion for the future research is to include other types of real option when evaluating a real estate project. On the other hand, the future research can use the concept of deferred option and the model in other contexts outside real estate development investment such as investments in new product development and manufacturing.
References


Trigeorgis, L. (2011) Real Options and Investment under Uncertainty: Classical Reading and Recent Contributions, Massachusetts: MIT Press.


According to the assumption that the unit construction cost \( (K) \) and unit rental rate \( (R) \) are observed every year and they fluctuate according to the geometric Brownian motion with drift. So, the value of unit construction cost and unit rental rate in each year can be model as follow.

\[
K_t = K_{t-1} + dK \\
R_t = R_{t-1} + dR
\]  

\( \Delta K \) and \( \Delta R \) follow geometric Brownian motion with drift. So, both variables can be calculated by using equation (6a) and (6b).

\[
\frac{dK}{K_{t-1}} = \alpha_f dt + \sigma_f dz_f
\]
\[
\frac{dR}{R_{t-1}} = \alpha_v dt + \sigma_v dz_v
\]

Since both values are observed every year, \( dt \) equals to 1. Substituting this value and rearrange the equations as follow.

\[
dK = \alpha_f K_{t-1} + \sigma_f K_{t-1} dz_f
\]
\[
dR = \alpha_v R_{t-1} + \sigma_v R_{t-1} dz_v
\]

Replacing equations (3a) and (3b) in (1a) and (1b).

\[
K_t = K_{t-1} + \alpha_f K_{t-1} + \sigma_f K_{t-1} dz_f
\]
\[
R_t = R_{t-1} + \alpha_v R_{t-1} + \sigma_v R_{t-1} dz_v
\]

Then, rearranging the both equations yields the models used in the simulation.

\[
K_t = (1 + \alpha_f)K_{t-1} + \sigma_f K_{t-1} dz_f
\]
\[
R_t = (1 + \alpha_v)R_{t-1} + \sigma_v R_{t-1} dz_v
\]