



# Linnæus University

School of Computer Science, Physics and Mathematics

Degree project

## Space Time Coding For Wireless Communication

Master of Science in Electrical Engineering with  
Specialization in Signal Processing & Wave Propagation  
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## Abstract

As the demand of high data rate is increasing, a lot of research is being conducted in the field of wireless communication. A well-known channel coding technique called Space-Time Coding has been implemented in the wireless Communication systems using multiple antennas to ensure the high speed communication as well as reliability by exploiting limited spectrum and maintaining the power. In this thesis, Space-Time Coding is discussed along with other related topics with special focus on Alamouti Space-Time Block Code. The Alamouti Codes show good performance in terms of bit error rate over Rayleigh fading channel. The performance of Alamouti's code and MIMO capacity is evaluated by using MATLAB simulation.

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## Abbreviations

AWGN	Adaptive White Gaussian Noise
ASN	Access Service Network
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
CSI	Channel State Information
ETSI	European Telecommunication Standard Institution
ISI	Inter symbol Interference
ICI	Inter carrier Interference
IEEE	Institute of Electrical and Electronic Engineering
LST	Layered Space Time Code
LOS	Line-of-Sight
MEA	Multi element antenna
MIMO	Multiple-Input Multiple-Output
MRRC	Maximal Ratio Receive Combining
MISO	Multiple-Input Single-Output
NLOS	Non Line-of-Site

R <sub>x</sub>	Receiver
SISO	Single-Input Single-Output
SIMO	Single-Input Multiple-Output
SVD	Singular Value Decomposition
STC	Space Time Coding
STBC	Space Time Block Code
STTC	Space Time Trellis Code
SNR	Signal-to-Noise Ratio
T <sub>x</sub>	Transmitter

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## 1. Introduction

In the field of communication, wireless communication is the fastest growing technology and the growth of the cellular system is exponential over the last decade. Wireless systems have the capacity to cover broad geographical areas, and it excludes the more costly infrastructures to deploy wired links to the individual sites. The main purpose of broadband wireless communication is to provide reliable and high data rate transmission over a larger area. The broadband wireless systems that provide multimedia services such as high-speed internet access, wireless television, online gaming and mobile computing are rapidly growing. Due to the availability of limited radio spectrum, spectrum efficiency must be increased significantly. Considering the reliability of wireless channel, the diversity technique is used so that the receiver will get the independently faded copies of the transmitted signal, and the receiver will be able to receive at least one of these replicas correctly. Channel coding is done to achieve diversity in wireless communication, the channel coding scheme called Space Time Coding (STC) is an efficient scheme used in modern wireless communications employing MIMO system. For the realization of the diversity benefits of multiple transmit antennas, Space Time Coding is used [1][3].

In wireless communications, antenna system is designed for reducing multipath fading, interference and polarization mismatch. The multiple antennas deployed at the transmitting and receiving side called MIMO systems, which provide high data rates in rich scattering environments are being implemented in recent communication systems. MIMO system also improves the data transmission reliability and all these benefits of MIMO system can be achieved without increasing the transmit power and required bandwidth. The spatial diversity obtained from transmit and receive antennas can be combined with the channel coding, and the outcome of the combination is called space-time coding, and the system is referred as coded MIMO system. Coding over MIMO system provides a solution for reliable high-speed wireless communication links. Generally, space time coding can be classified into two types:



Space Time Block Code (STBC) and Space Time Trellis Code (STTC). Both STBC and STTC are used to achieve full spatial diversity in a given number of transmitting and receiving antennas but STTC, based on trellises achieves spatial diversity as well as coding gains. In STBC, data encoding is done using STBC encoder and the encoded data is converted into  $n$  streams. Then  $n$  streams are simultaneously transmitted over  $n$  transmit antennas. At the receiving side of each antenna, the received signal is the superposition of transmitted signal. The Space Time Block Codes are designed in such a way that maximum diversity order for the number of transmit and receive antennas is achieved. STBC can be decoded more efficiently at the receiver by linear processing. In case of two transmit antennas, Alamouti scheme is used to obtain full transmit diversity, and this technique is comparatively simpler to implement. Here, Rayleigh fading channel is considered as it provides good performance for outdoor and long-distance communication [2] [3].

## **2. An Overview of Wireless Communication**

Information can be transferred from one point to another without any physical connections, and this way of communication is termed as wireless communication. The term “communication” includes all form of communications such as telegraphy, radio, television, telephony, computer networking and data communication. [4] Due to the reliability and feasibility, use of wireless communication is rapidly growing in global scenario. The first wireless networks were developed in the age of pre-industrialization. Smoke signals, torch signaling and flash mirrors were used for communication. Stations were built for the observation on hill tops to deliver the message over long distances. Samuel Morse, in 1838 invented telegraph network which replaced the early networks. In 1897, Marconi conducted a demonstration which showed the radio ability to provide continuous connection with ships. Later, in 1934, Mobile communication based on AM was conducted by United States Municipal Police Radio System. Mobiles installed more than five thousands radios for communication but there was the noise problem. In 1935, Edwin Armstrong made a demonstration on FM. In 1946, twenty-five American cities used first public mobile telephone service. A single high-power transmitter was used by each system for the coverage of long distances. The FM push to talk, system was employed with 120 KHz of RF bandwidth in half duplex mode. Federal Communications Commission (FCC) cut that bandwidth into half to double the number of channels. Again in 1960's, bandwidth was reduced to 30 KHz. The

theory of cellular radio telephony was developed during 10-60's by AT and T Bell Lab and other related companies. In a cellular system, a coverage region is broken into smaller cells where each of them can reuse spectrum and hence spectrum efficiency can be increased. First generation (1G) of cellular technology was based on analog transmission system, and this technology was exclusively used for voice. The most popular 1G cellular system are AMPS and ETACS. The second generation (2G) is based on digital radio technology, which can accommodate voice and text messages (SMS). IS-54/136, GSM (Global System for Mobile developed by European Telecommunication Standards Institute (ETSI)), PDC, IS-95 are 2G digital cellular standards. Intermediate technology between 2G and 3G known as 2.5 G is based on cell phone protocols that can transmit limited graphics like picture text messages (EMS), General Radio Packet Service (GPRS) is an example of 2.5G technology. Third generation (3G) is the digital wireless technology that combined the different incompatible network technologies and provided the globally integrated wireless communication system. Technologies such as CDMA 2000, EDGE, HSPA provides 3G services. 3G services include wide area wireless voice telephone, mobile internet access, mobile TV, video calls, etc. As the demand of high data rate increased, fourth generation (4G) has been evolved, which can provide seamless communication and multimedia display can be faster. Wi-max and 3GPP LTE Advanced can provide 4G services and are based on all-IP network. 5G is yet to come and is expected to be more intelligent technology, which will turn this globe to the real wireless world. [19][20]

### **3. Multi antenna system**

In the early days of wireless communication multipath propagation was considered as the undesirable phenomenon of wireless communication. Due to multipath fading caused by hills, buildings, forest and various other obstacles, the waveform of the signal is scattered and takes various paths to reach the destination. Since the wave follows various paths it reaches the destination at different time instants, thus the late coming signals cause various problems such as fading, packet fencing (intermittent reception), cliff effect (cut-out) in case of a single antenna system, Single-Input-Single-Output (SISO). So thus various processes such as diversity array, adaptive array, equalizations were developed to minimize its effect. So basically the traditional wireless communication system consists of a single transmitter and a

single receiver, the total performance of the wireless system depends mostly upon channel behavior and the environment. But today wireless communication has a totally different architecture so the system performance does not only depend upon the environment and the channel behavior as well as on the system architecture.

In today's wireless communication to achieve high spectral efficiency multi element antenna (MEA) arrays are deployed at both the terminal's transmitter, and the receiver ends that best utilizes the multipath scattering in the rich scattering environment. This multiple-input multiple-output communication channel having matrix transfer function of independent Gaussian random variable, the information capacity of such a system grows with no of an element in an antenna array without compromising the total power and the bandwidth. Combining techniques can be implemented in both time and spatial domain, but in multi-antenna system space diversity is mainly concerned. Diversity can be achieved in either of the two sides. The transmitter or the receiver, if the diversity is achieved at the receiver side, i.e. if we have single antennas at the transmitter side and multiple antenna at the receiver it results in a Single-Input-Multiple-Output (SIMO) system, likewise, if the diversity is achieved at the transmitter by having multiple antennas at the transmitter and single antennas at the receiver side, then the resulting system configuration will be Multiple-Input-Single-Output (MISO). If the diversity is achieved at the both ends by implementing multiple antennas at the transmitter and as well as in the receiver, then such system configuration is called Multiple-Input-Multiple-Output (MIMO) systems. [7]

### 3.1 Single-Input-Multiple-Output (SIMO) system

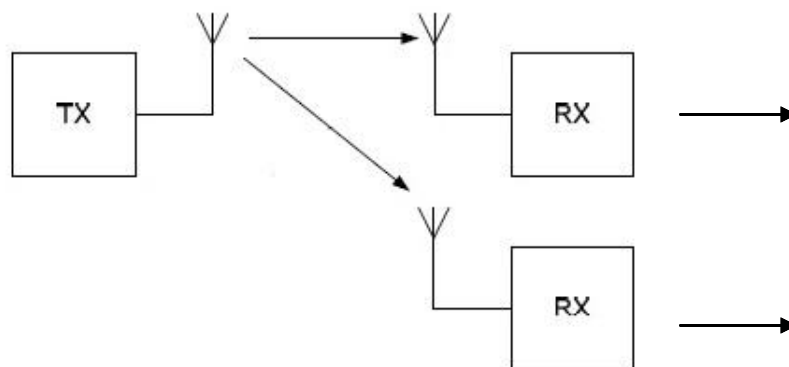


Fig: SIMO System

SIMO (Single-Input-Multiple-Output), is a smart antenna system technology for wireless communication. It utilizes a single antenna at the transmitter and more than one, multiple antennas combined to optimize the data speed and minimize the error at the receiver. [8] Let us consider a SIMO system with one transmitter antenna, and two receive antenna, and suppose that a complex symbol 's' is transmitted in a flat fading environment, then the two received samples can be written as.

$$y_1 = h_1s + n_1 \quad (3.1)$$

$$y_2 = h_2s + n_2 \quad (3.2)$$

Where, the channel gain between the transmitting and the receiving antenna is given by  $h_1$  and  $h_2$  and  $n_1$  and  $n_2$  are the uncorrelated noise.

### 3.2 Multiple-Input-Single-Output (MISO) system

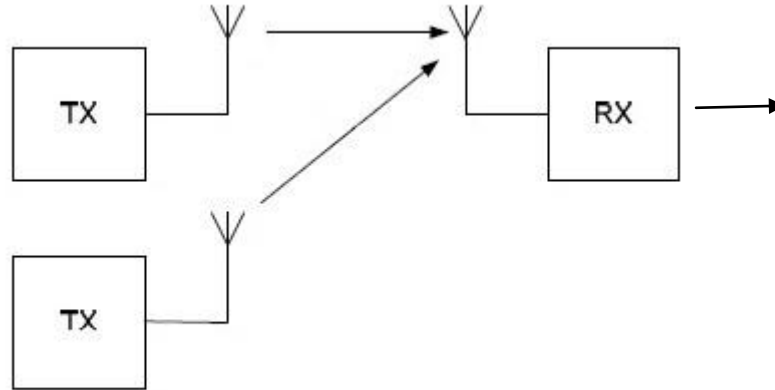


Fig: MISO system

MISO (Multiple-Input-Single-Output): Multiple-Input-Single-Output is an antenna configuration system in wireless communication. This configuration has multiple, more than one antenna combined to optimize the data speed, and minimizes the error at the transmitter section and single antenna at the receiver section. In this configuration system since the transmitters have multiple antennas the data can be distributed between different antennas to

exploit spatial diversity at the transmitter. Different signal processing technique such as Transmit Beamforming when full CSI is available at the transmitter, Antenna Selection when partial CSI is available, Space Time coding when no CSI is available at the transmitter, can be implemented to exploit spatial diversity. [9]

Now let us consider a MISO system with two transmitter antenna and one receiving antenna as shown in the above figure. At any given instant of time , a complex symbol 's' is transmitted that will be pre weighted with  $w_1$  and  $w_2$ , then the received sample will be

$$y=h_1w_1s+h_2w_2s+n, \quad (3.2.1)$$

where  $h_1$  and  $h_2$  are the channel gain between the transmitting antenna and the receiving antenna and  $n$  is the noise sample.[10]

### 3.3 Multiple-Input-Multiple-Output (MIMO) system

When we have an antenna array at ends, transmitter and the receiver then it is called as multiple inputs multiple output MIMO system, so the system that has multiple antennas at the transmitter as well as the multiple antennas at the receiver then such system is termed as MIMO systems.

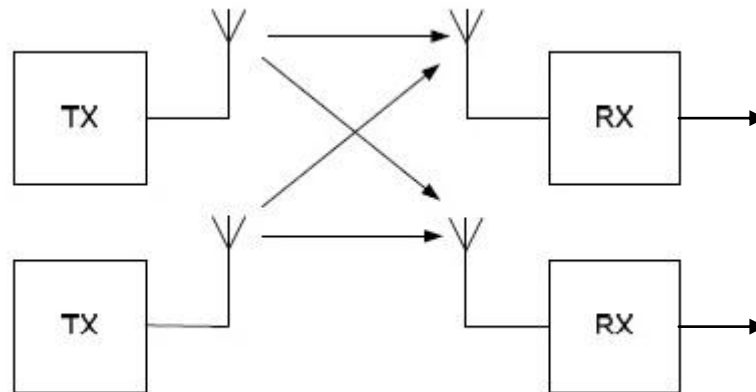


Fig: 2x2 MIMO Systems

The main advantage of MIMO system is that the signals sampled at the spatial domain at both the transmitter, and the receiver sides are combined such that it increases the data rate through multiple parallel data pipes or increase the quality using diversity as BER (Bit Error

probability) decreases thereby improving the quality of communication. With the use of new domain space for communication, MIMO can also be considered as Space-Time wireless antennas as it utilizes both space and time domains. Today MIMO is the most promising terminology in an antenna system because of its multi-benefit features like beam forming and spatial diversity.

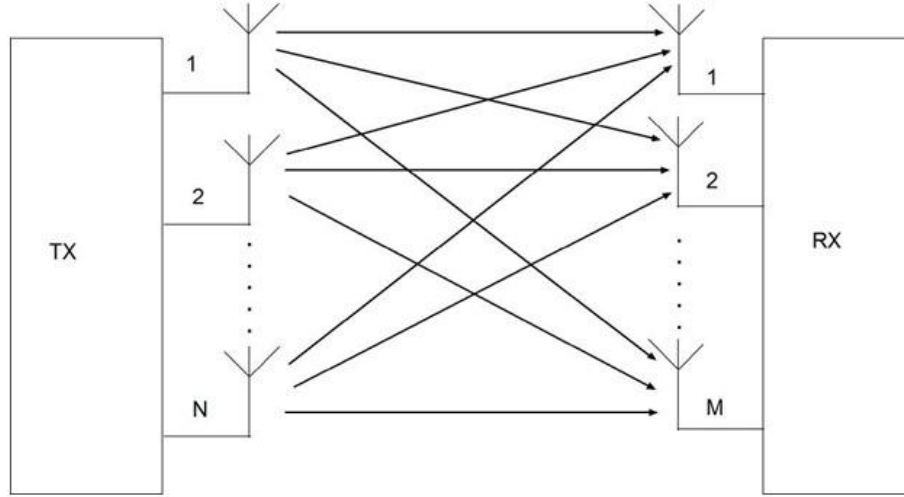


Fig: N×M MIMO System

Spatial diversity utilizes random fading caused by multipath propagation, which in turn increases the SNR (Signal to Noise Ratio) by combining the output of different antenna so that the best one can be gathered from the streams of signals. In another hand beam forming increase the SNR as the energy can be focused in the desired direction. MIMO utilizes the so-called disadvantage of wireless communication such as the random fading, delay spread for the enhancement of transfer rate.

The transmit\receive diversity of the MIMO system will be

$$y_i = \sum_{j=1}^k h_{ij} x_j + n_i \quad (3.3.1)$$

whrere  $i=1,2,\dots,N$ ,  $h_{ij}$  is the fading that corresponds to transmitting antenna 'j' to the receiving antenna 'i', and  $n_i$  is the noise that is received at the corresponding 'i' antenna.[11][12]

#### 4. Information theory for MIMO system

The concept of Information theory relates with the channel capacity. Information Theory is the branch of applied mathematics involving the quantification of information, and it is widely being used in Electrical and communication Engineering. Information is the message that consists of an ordered sequence of symbols or the meaning that can be interpreted from it. Information theory gives an important result for the error-free transmission which states that error-free transmission is theoretically possible even in a noisy channel provided that information rate doesn't exceed the channel capacity.[13][14]

Information theory was introduced by Claude E Shanon during his research on “Fundamental limits in Signal processing” in 1948. His study was a purely statistical study which produced a measurement of Information called as bits. Measurement of information is known as Entropy, which is the average number of bits needed to be transmitted or stored for communicating one symbol of the message. Entropy is the measurement of mean uncertainty when we don't know the outcome of an information source that means it is the measurement of how much of information that we don't have. However, it also means it is the average amount of information we will have when we receive an outcome form an information source.

$$H(y) = \sum_{i=1}^m p_i \log_2 \left( \frac{1}{p_i} \right) \quad (4.1)$$

Information theory defines the data compression and the transmission of data through the channel which we call it as the channel capacity, which is the most important factors in wireless communication. The primary idea of channel capacity lies in the concept of Information content. The qualitative limitations that determines the system capacity is the maximum number information that can be transmitted in bits per second, if the system cannot respond to the instantaneous changes in signal due to presence of energy-storing devices. Time response is limited due to the inherent capacitance and inductance. These factors relate to the useful bandwidth of the system.

In a wireless communication system channel capacity can be broadly classified into two categories, first one, which does not consider the background noise while decoding the transmitted signal at the receiver and the second is the channel capacity derived by Shannon, which considers the additive noise during the transmission. Shannon proposed a model in which he added redundant bits (called as coding) in the information signal at the source before

transmitting, the added bits called as the redundancy bit will make a signal robust to the additive noise that will be added in the channel during the transmission. So, the signal could be properly retrieved at the receiver. We can reduce the signal power with the use of redundant codes. In this case quantized channel capacity is expressed in terms of Background SNR. In case of multiplicative noise, signal interference appears as a convolution; in this scenario the channel capacity discussed in first scenario is more useful. [15]

## 4.1 Entropy and Mutual Information

It is rather hard to find the concrete definition of information as it is a broad area to be defined by a single definition. However, for any probability distribution, we define an abstract quantity called the entropy. Entropy has the property that helps to measure the information, and these properties or notation of entropy could be extended to define mutual information. Mutual Information is the amount of information contained in a random variable about another in a sample. Entropy is the information contained in a random variable, where mutual information is the relative entropy, so we can also state it as the more special case of general quantity entropy. Mutual Information is the measurement differences/distance between two probability distributions. [16]

Now we suppose  $y$  is a random variable with discrete probability mass function  $p_y(y)$ . Then we have the quantity:

$$H(y) = -E_y [\log_2 (p_y(y))] = -\sum_k \log_2 (p_y(y_k)) p_y(y_k), \quad (4.1.1)$$

which is called as the entropy of  $y$ . Entropy is basically the measurement of necessary bits on an average to convey the message contained in  $y$  or it the minimum amount of bits that should be available for decoding the information contained in  $y$ . In this way, a continuous random variable can be defined as:

$$H(y) = -E_y [\log_2 (p_y(y))] = -\int d_y p_y(y) \log_2 (p_y(y)) \quad (4.1.2)$$

In terms of information theory, the continuous probability distribution function  $H(y)$  is called as differential entropy.

In a similar way if  $y$  is a real valued Gaussian random vector having length  $n$ ,  $y \sim N(\phi, \mathbf{P})$

Then the entropy  $H(y)$  which is independent of  $\phi$  is,

$$H(y) = \frac{n}{2} \log_2 (2\pi e) + \frac{1}{2} \log_2 |\mathbf{P}| \quad (4.1.3)$$



If we consider  $y$  to be a circular Gaussian variable of length  $n$ , then the entropy  $H(y)$  is:

$$H(y) = n \log_2(e\pi) + \log_2 |P| \quad (4.1.4)$$

This is also independent of  $\phi$ . In all random vectors  $y$  having zero mean and covariance matrix  $P$ . The entropy  $H(y)$  is maximum if  $y$  is Gaussian. [17]

## 4.2 Capacity of MIMO channel

We can define system capacity as *"the maximum possible rate of transmission where the probability of error is conveniently small. For a given channel the optimum capacity or the Shannon capacity is the maximum mutual information between the received vector  $\{y_n\}$  and the transmitted vector  $\{x_n\}$ . Among all zero mean vectors  $\{x\}$  is Gaussian".* [17]

Then the total transmission power is  $P = \text{Tr} \{P\}$ , so the resulting channel capacity is

$$C(H) = B \log_2 \left| I + \frac{1}{\sigma^2} H P H^H \right| \quad (4.2.1)$$

Here  $H$  is the given channel, and  $B$  is bandwidth of the given channel. The channel capacity  $C(H)$  in the above equation is the highest information rate achievable in bits/sec for a given channel, and this can be achieved only under certain condition. The first and foremost condition is that for the channel to achieve above information rate, the block length of the data block that is considered should be infinitely long. In case of SISO system having transmitted power  $P$ , channel gain is constant and is equal to  $h$ . This system is affected by the AWGN noise, so the channel capacity is given by,

$$C(H) = B \log_2 \left( 1 + \frac{1}{\sigma^2} h^2 \right) \quad (4.2.2)$$

The above equation 3.7 is equal to Shannon capacity.

The channel capacity in case of independent and parallel channels, if two matrices  $P$  and  $H$  are diagonal which results in  $k$  parallel and independent SISO system. Then the total capacity is the cumulative capacity of such  $k$  systems, i.e

$$C(H) = B \log_2 \left| I + \frac{1}{\sigma^2} H P H^H \right| \quad (4.2.3)$$

$$= B \sum_{k=1}^k \log_2 \left( (1 + |H_{k,k}|^2 P_{k,k}) / \sigma^2 \right) \quad (4.2.4)$$

In the similar manner if frequency selective MIMO channel is considered and if  $\{H_t\}$  is the impulse response of the channel which is frequency selective MIMO channel, we can find the channel capacity by integrating the channel capacity with additive Gaussian noise over the frequency band that is available for transmission.

$$C(H(z^{-1})) = \int_{\sigma} dw \log_2 \left| I + \frac{1}{\beta_2} H(W)P(W)H^H(W) \right|. \quad (4.2.5)$$

Here  $H(w)$  is the channel transfer function. [18]

### 4.3 Ergodic Channel capacity

When we consider a dynamic channel like in mobile radio communication, the channel matrix  $H$  is no longer a constant matrix, due to the movement of receiver and or transmitter the channel is the time variant, this is also due to the scattering from the moving objects in the channel. If we consider a time-variant stochastic channel matrix  $H$ , the elements in the matrix  $H$  are random variable, then we can define the ergodic channel capacity as the expected value of the mutual information, that is

$$C_{ST} = R_{cc} > 0, n \{Tr(R_{cc}(H))\} \leq P_T \quad E_H \left[ \log_2 \det \left\{ I_m + \frac{H R_{cc} H^*}{\sigma^2 n} \right\} \right] \quad (4.3.1)$$

over all possible channels, and it is measured in bits per second per hertz. So the ergodic channel capacity is represented in terms of achievable bit rate of the channel that is normalized with the transmission bandwidth averaged over the fading distribution, the bit error rate for this is driven asymptotically to zero. Here we have an average power constraint  $n \{Tr(R_{cc}(H))\} \leq P_T$ , where the maximization is outside the expected value over the channel  $H$ . Here  $R_{cc}$  is dependent on  $H$ . Ergodic channel capacity can be defined as the power adapted over both time and space (Eigen value of channel). The inherent bit rate limit of channel is called the channel capacity, and if perfect channel state information (CSI) is available at the transmitter, channel capacity can be achieved by bit rate relative to the channel quality and transmission power. The channel capacity that we have derived above is with no co-channel interference if we consider co-channel interference the channel matrix  $H$  has to be exchanged with  $H^* = R_{VV}^{-1/2} H$ . Where  $R_{VV}$  is the covariance matrix of noise and interference. [19]

## 4.4 Outage Capacity and Outage Information

When we consider the quasi-static Rayleigh channel model then the fading is non-ergodic, then we do not have codes for different states of channel as it is not possible to code. In this case, there are only limited realizations for the total frame of data, so the ergodic channel capacity or Shannon channel capacity is zero, as we always have a non-zero probability of instantaneous mutual information between channel output and input that is less than a fixed rate unconcern of the frame length. If we consider the instantaneous signal-to-noise ratio, then the channel capacity is a random variable. For quasi-static Rayleigh, flat fading channel capacity is derived as a function of signal-to-noise ratio

$$C(\rho) = \log(1 + \rho). \quad (4.4.1)$$

Here  $\rho$  is an exponential random variable.

So in this channel for any given rate  $R$ , there is always a probability that any of the coding technique is insufficient to support this channel. The outage probability is given by,

$$P_{out} = P(C(\rho) < R). \quad (4.4.2)$$

The outage capacity is associated for every outage probability, so when the system is not outage i.e.  $1 - P_{out}$ , the outcome transmission rate is possible.

From the above equation in (4.4.2) we know that  $P_{out} = P(C(\rho) < R)$  when we have rate of information  $R$ , and a specific channel input, The mutual information can be calculated using specific input constraints computed for the given signal-to-noise ratio.[20]

## 4.5 Mutual Information when CSI available at the Transmitter

This is the condition when the transmitter knows about the channel  $H$ , i.e. the complete channel state information is available to the transmitter. If CSI is available, then we can choose the transmit correlation matrix  $P$  for the maximization of the channel capacity for the given realization of the channel. The maximum channel capacity which is also the informed transmitter (IT) capacity (CIT) is achieved by the technique called as "water-falling". If  $P_i$  is the power of the symbol transmitted in the  $i^{\text{th}}$  channel, then the capacity is given by

$$C = \sum_{i=1}^v \log_2(1 + \rho \lambda_i P_i) \text{ where } \text{Max } P: \sum_{j=1}^v P_j \leq \text{with } P = [P_1, P_2, \dots, P_v]. \quad (4.5.1)$$

We can achieve this capacity if the inputs are independent complex Gaussian.

This equation could be solved to the following general solution.

$$P_{i, out} = (\mu - \frac{1}{\rho \lambda_i})^+. \quad (4.5.2)$$

Where  $(X)^+$  is  $\max[X, 0]$  and  $\mu$  is the result of

$$\sum_{i=1}^v (\mu - \frac{1}{\rho \lambda_i}) (\mu - \frac{1}{\rho \lambda_i})^+ = 1 \quad (4.5.3)$$

Then the capacity is

$$C = \sum_{i=1}^v (\log_2(\mu \rho \lambda_i))^+. \quad (4.5.4)$$

The above equation shows that, depending on the signal to noise ratio the optimal scheme uses only some of the equivalent parallel channels. In case of low signal to noise ratio the channel with the best gain is used for making beam forming optimal. More parallel channel will be used for transmission as signal to noise ratio increases. [21]

## 4.6 The water filling algorithm

When the channel information is known at the transmitter then the channel  $H$  is known in (4.2.1) and the capacity is optimized over  $P$  with the power constraint  $P\rho$ . In this case  $P$  is known and is known as the water filling (WF) solution. This algorithm gives the solution of channel capacity in (4.5.4). The channel capacity can be simulated using (4.5.2)(4.5.3)(4.5.4) . So, the optimum channel capacity can be calculated or simulated for any channel. For a given channel  $H$ , the noise variance is  $\sigma_n^2$  and the total power transmitted is  $P_{total}$  . Then calculate the SVD of  $H$  to get  $H = U \sum V^H$  , Then the rank  $r$  is a singular values .[22]

## 5. Fading

Fading is a phenomenon in which any time variation of phase, polarization and/or level can occur in terms of received signal. In wireless links, there can be random fluctuations in signal level across space, time and frequency, which is termed as fading. Definition of fading depends upon propagation mechanism involved such as reflection, refraction, diffraction, scattering, attenuation or ducting of radio waves. If mechanisms are identified and understood, it will be easier to find the solution for mitigation or can be avoided by applying a suitable method. Certain terrain geometry and meteorological conditions cause the fading

which are not necessarily mutually exclusive. For the frequency range of 0.3 to 300 GHz, all radio transmission systems can suffer fading and the systems can include satellite earth terminals that operate at low elevation angles and/or in heavy precipitation. Multipath fading is most commonly encountered fading type generally on LOS radio links. This is caused by dispersion and basically digital troposcatter and high-rate LOS links suffer by this type of fading. Fading, which a signal experiences while propagating through a mobile radio channel relies on nature of transmitted signal corresponding to the characteristics of channel.

## 5.1 Flat fading

Let us consider that  $B_s$  is the band width of signal,  $B_c$  is the coherence band width of channel,  $T_s$  is symbol period and  $\sigma_T$  is delay spread then,

A signal undergoes flat fading if

$$B_s \ll B_c$$

$$T_s \gg \sigma_T$$

Flat fading channels are also called amplitude varying channels and are considered as narrow band channels because the band width of applied signal is narrow than the band width of channel.

## 5.2 Frequency selective fading

A signal undergoes frequency selective fading if

$$B_s \gg B_c$$

$$T_s \ll \sigma_T$$

Dispersion of transmitted signal causes frequency selective fading which in turn results in inter-symbol interference. It is very difficult to model frequency selective fading channels than to model flat fading channels because each multipath component should be modeled and channel should be regarded as linear filter. As the bandwidth of signal is wider than the bandwidth of channel in public response frequency selective fading are also known as wide band channels.

### 5.3 Fast fading

In fast fading, the coherence time of channel is smaller than symbol period of transmitted signal. Fast fading causes signal distortion and this distortion is directly proportional to the Doppler spread. The impulse response of channel in fast fading changes at a rate faster than transmitted base band signal.

Let,  $T_s$  represents symbol period,  $T_c$  represents coherence time, then fast fading occurs if

$$T_s \gg T_c$$

### 5.4 Slow fading

Slow fading also known as shadowing and is caused by buildings, mountains, hills etc. The impulse response of channel in fast fading changes at a rate slower than transmitted base band signal.

Slow fading occurs if

$$T_s \ll T_c$$

### 5.5 Rayleigh fading

If there is no dominant propagation along line of sight between transmitter and receiver then Rayleigh fading is applicable but Rician fading is considered if there is a dominant line of sight between transmitter and transmitter. In Rayleigh fading, it is assumed that the magnitude of signal passing through transmission medium varies randomly or undergoes fading according to the Rayleigh distribution.

Many objects in the environment cause the radio signal to be scattered before reaching to the receiver. According to the central limit theorem, “if there is sufficiently much scatter, the channel impulse response will be well modeled as a Gaussian process irrespective of distribution of individual components” [23]. The probability density function of Rayleigh random process is expressed as:

$$P_R(r) = (2r/\Omega) e^{-r^2/\Omega}, r \geq 0. \quad (5.5.1)$$

Where,  $\Omega = E(R^2)$ ,  $R$  is random variable.

Convenient representation of gain and phase elements of channel distortion is done by a complex number. By independent and identically distributed zero-mean Gaussian process, the real and imaginary parts of responses are modeled in which amplitude of response is an addition of such two processes.

## **6. Classification of channels according to time or frequency response**

### **6.1 Time flat channels**

These types of channels are time invariant channels. The stationary transmitter and receiver with the change in propagation environment is an example

### **6.2 Frequency flat channels**

These are the channels in which the frequency response is approximately flat over a bandwidth which is larger than or equal to bandwidth of transmitted signal.

### **6.3 Time selective channels**

These channels are time varying channels. Any wireless terminal moving in an environment and experiencing Rayleigh fading is an example of such channel.

### **6.4 Frequency selective channels**

In this type of channel, the frequency response is not considered to be flat over the bandwidth of signal. By the significant delay spread relative to the symbol period of transmission, frequency selectivity is achieved. [23]

## **7. Diversity**

In a channel if the signal power drops significantly, the channel is said to be faded and when a channel experiences fading, then this caused high bit error rates (BER). So to combat fading, diversity is used. In wireless communications, different types of diversity techniques such as Spatial, Polarization, time and frequency diversity are used to improve the signal to noise ratio, bit error rate, channel capacity and power saving. Diversity technique is the methods to improve the performance by efficient transmission of the same information multiple times through the independent channels. Diversity can reduce the interference to and from the users and battery life can be increased in peer to peer handheld system. Basically, diversity of

antennas provides two benefits. One is to increase the overall average received signal power. Next one is to improve the reliability in multipath channels in the case where the fading of the received signal is caused by the interference from the reflected signal. [24][25]

## **7.1 Time Diversity**

In this technique, the same signal is transmitted over many times using different time intervals. The separation of a time interval must be larger than the coherence time of channel for the fading channel coefficients to change and to get different channel gains. If we consider idealized channel models, it is possible to achieve time diversity but in case of quasi-static fading channel, slow fading is observed even it takes longer time to transmit replicas.

## **7.2 Frequency Diversity**

Frequency diversity implements the simultaneous use of different frequencies for the transmission of signals. This diversity scheme provides replicas of original signal in a frequency domain. Frequency diversity is used to tackle the channel uncertainties like multipath fading. It means that different replicas of signal are transmitted over different frequency bands. To make the channel independent, frequency bands are separated by more than a coherence bandwidth of the channel [24]

## **7.3 Polarization diversity**

Polarization diversity is a technique in which antenna having different polarizations, horizontal or vertical is used for diversity reception. Using two different polarized antennas horizontal and vertical signals are transmitted and then received by two different polarized antennas. Due to the comparable performance of polarization diversity to that of space diversity and due to the size reduction of antenna array, polarization diversity is being used in these days. In urban, rural and indoor environments polarization diversity has received more attention. Polarization diversity reduces the polarization mismatches due to random handset orientation with significant improvement on SNR. For the base station, polarization diversity is more effective in terms of providing greater space and cost effectiveness. In case



of the worst polarization mismatch, half the best-case received signal power can be provided by the polarization diversity system. [25]

## **7.4 Angle diversity**

Angle diversity is a special case of space diversity in which a number of directional receiving antennas are used to receive message signal simultaneously. In the receiver, the received signals are the scattered waves coming from all directions which are uncorrelated. [26]

## **7.5 Spatial diversity**

In spatial diversity, two or more antennas are separately placed in space for reception or transmission. The purpose of the space diversity is to reduce the multipath fading. The spatial diversity is studied as antenna diversity, which includes transmit diversity and receive diversity. Antenna diversity plays a vital role in the reliability of wireless communication. Due to the different channel condition, if one antenna undergoes deep fade, the signal may not and that adds reliability. Transmit diversity uses multiple antennas at the transmitting side while receive diversity uses multiple antennas at the receiving side. In case of Line of Sight (LOS) condition and Non-Line of Sight (NLOS) condition, each receive antenna has a different fading experience. [24] [27] [28]

### **7.5.1 Transmit diversity**

Transmit diversity implements multiple antennas at the transmitting side. In this diversity scheme, controlled redundancies are introduced at the transmitter and at the receiver side, suitable signal processing technique is implemented. For this technique, channel information at the transmitter is required. But due to the use of Space- Time Coding schemes, such as Alamouti's scheme, it has become possible to achieve transmit diversity without getting channel information. Multiple antennas used either in transmitting and receiving side increases the performance of a communication system in fading environment. In case of a mobile radio communication system, employing multiple antennas at base station is more effective but single or double antennas can be employed in mobile units. In case of transmitting from mobile to base station, the diversity can be achieved through multiple

receive antennas and while transmitting from base station to the mobiles, diversity is achieved through multiple transmit antennas. Transmit diversity is gaining popularity due to its simple implementation and feasible for having multiple antennas at the base station that improves the downlink and is one of the best methods of brushing the detrimental effects in wireless communication. [29]

### **7.5.2 Receive diversity**

Receive diversity implements multiple antennas at the receiving side. In this scheme, maximum ratio is frequently used for the improvement of signal. In cell phones, it is costly and difficult to employ more receivers so that transmit diversity is widely used since it is easier to implement at base station. In receiver diversity, independently faded copies of signals, which are transmitted by the transmitter, are suitably combined to get performance gain. But for transmitter diversity, receiver gets the already combined transmitted signal. Comparatively, transmitter diversity is considered more difficult to exploit than receiver diversity because it is assumed that transmitter knows less about the channel than the receiver, and the transmitter is permitted to generate a different signal at each antenna. [30]

### **7.6 Diversity gain**

Diversity gain measures the improvements in signal level over the strongest branches for certain reliability after the diversity is combined. Diversity gain is obtained by finding the horizontal distance between the combined signals and longest branch where the cumulative distribution value is given. The cumulative distribution function of the signals provides information about the amount of gain that can be achieved through diversity. The diversity gain highly depends on reliability level. [31]

## **8. Space Time Coding**

The capacity limit of today's communication system is predicted by the Shannon's capacity approach. For the effective exploitation of physical radio link, efficient coding and signal processing is required. Capacity limit is caused by bandwidth restriction, and this is a great deal in communication systems. To deploy the new applications for services such as

multimedia, high data rate is required. Since bandwidth is an expensive resource, increasing capacity without increasing the required bandwidth is a major challenge in modern wireless communication systems. Space time coding is the effective way of achieving the capacity of multiple-input multiple-output (MIMO) wireless channel. Multiple antenna system is being used for the increment of the spectral efficiency, and this is achieved by exploiting the resource space. [32]

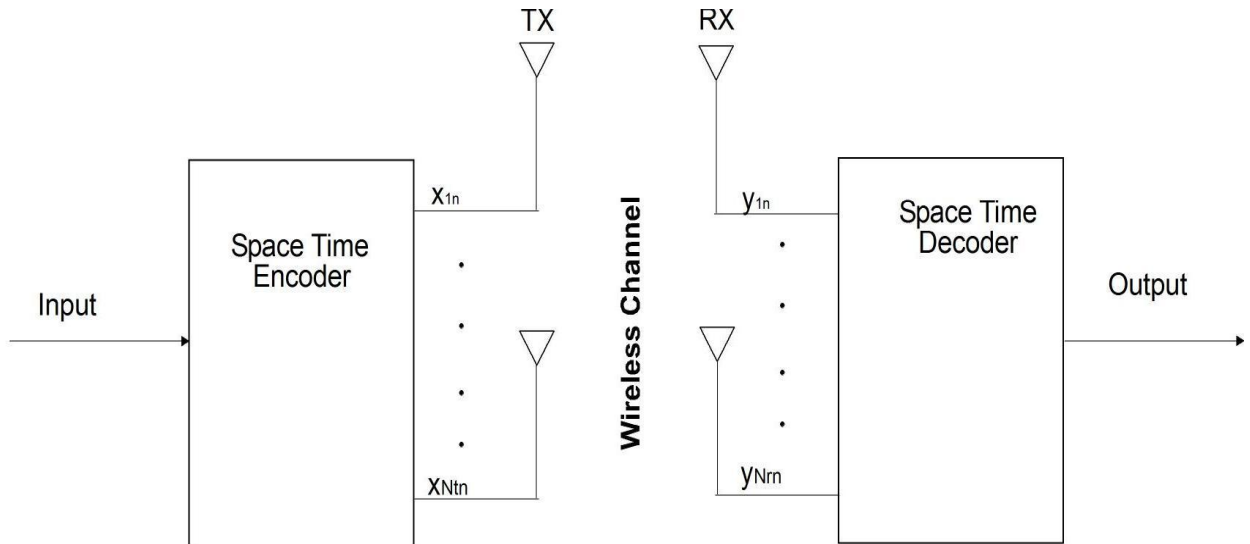


Figure: Generic Space-time system model for  $N_t$  transmit antennas and  $N_r$  receive antennas [33]

Space time coding is the effective way of achieving the capacity of multiple-input multiple-output (MIMO) wireless channel. It is coding technique that has been designed for the multiple transmit antennas. In this scheme, coding is performed in both temporal and spatial domains in order to introduce correlation between the signals from different antennas at different instants of time. This spatial-temporal is used to minimize the transmission error and to accomplish MIMO channel fading at the receiver side. With the use of space time coding, we can achieve transmit diversity as well as the power gain without use of excess bandwidth compared to the spatially uncoded system. There is a wide range of approaches that is followed in space time coding technique, the most popular coding schemes are space-time block code (STBC), Space-time trellis codes(STTC), Space-time turbo trellis codes, and layered space-time (LST) codes are few to name. All these coding schemes are developed for the attainment of multipath effect for performance gain and to get high spectral efficiency.

It is necessary to code both across the time and space to achieve transmits diversity, if the channel state information is not available at the transmitter. We can achieve the transmit diversity by transmitting the same symbol repeatedly over a time interval that equals to the number of transmitting antennas. So, in order to overcome this deficiency, we need a special scheme that can spread symbols over space and time and transmitting these several symbols at the same time. Space-time coding is the method that helps to achieve this scheme.

Due to interference and destructive addition of multi paths in the channel, the wireless channel is affected by attenuation. When the channel statistic is Rayleigh, it is even more difficult for the receivers to decode the transmitted signal, if the least attenuated version of the same signal is not provided to the receiver. Providing the different version of same signal or providing the replica of same signal is diversity and the diversity is more often provided using temporal, polarization, frequency or spatial resources. But when the channel is neither frequency selective nor time varying, in such situation diversity is often achieved by deploying multiple antennas at both receiving and the transmitting side to acquire spatial diversity. [34][35]

Let us consider a  $n \times n$  mimo system that has  $N_t$  transmitter antennas and  $N_r$  receive antennas, as shown in the figure. The input information data symbols  $s(n)$  that belong to a set are coded into blocks  $s(n) = [s(nN_s) \dots s(nN_s + N_s - 1)]^T$  of size  $N_s \times 1$ . The blocks are then encoded by the space time encoder, the encoder then maps the code uniquely to  $N_t \times N_d$  matrix code.

$$C(n) = \begin{bmatrix} c_1(nN_d) & c_1(nN_d + 1) & \dots & c_1(nN_d + N_d - 1) \\ c_2(nN_d) & c_2(nN_d + 1) & \dots & c_2(nN_d + N_d - 1) \\ \vdots & \vdots & \ddots & \vdots \\ c_{N_t}(nN_d) & c_{N_t}(nN_d + 1) & \dots & c_{N_t}(nN_d + N_d - 1) \end{bmatrix}. \quad (8.1)$$

The code symbol is an element of a constellation set  $B$ , the constellation set are different depending upon the scheme used of space time coding. In the matrix  $C(n)$ , the columns  $N_d$  are generated with each  $N_t$  coded symbols at an interval of  $T_c$  at the given column sent through the one of the  $N_t$  transmit antennas simultaneously. The coded symbols  $N_d$  corresponds to  $N_s$  information symbols from each transmit antenna, the overall transmission rate will be given by,

$$R = \frac{N_s}{N_d} \log_2 A \text{ bits/sec/Hz}, \quad (8.2)$$

where absolute value of A is the cardinality of A. If  $E_c$  and  $E_s$  are the average power of  $c_i(n)$  and  $s(n)$  respectively for the total transmitter power to be independent of the Space Time encoding, then  $E_c = \gamma E_s$ , Where  $\gamma = N_s / (N_d N_t)$ . Now, if we assume the channel to be flat faded channel, that is the channels delay spread is small compared to  $T_c$  where as the coherence time is larger than  $M_d T_c$ . Since the channel is flat faded channel, there will be no inter-symbol interference in time domain but the receiver receives the noisy superposition of  $M_t$  signals from the  $M_t$  transmit antenna at any time n. If  $y_j(n)$  is the base band signal received by  $j^{\text{th}}$  receiving antenna then the corresponding data model is:

$$Y_j(n) = \sqrt{\gamma E_s} \sum_{i=1}^{N_t} h_{ij} c_i(n) + v_j(n), j = 1, \dots, N_r. \quad (8.3)$$

The term  $v_j(n)$  is the samples of the zero-mean complex white Gaussian process with spectral density  $N_0/2$  per dimension. The flat fading channel is modeled by the term  $h_{ij}$ , where i is the index for transmit antenna where as j is the index for receive antenna which means that the channel is flat faded from  $i^{\text{th}}$  transmitting antenna to  $j^{\text{th}}$  receiving antenna. [36]

Space time coding has many advantages. First, down link performance can be improved without need of multiple antenna elements at terminals and space time coding provides an efficient method to power control and bandwidth allocation. Second, space time coding can be combined with channel coding such that diversity gain and coding gain can be achieved. Third, CSI is not required at the transmitter that means space time coding operates in open loop mood because of which reverse link is not required. In this coding scheme, the information is encoded spatially and temporally and same bandwidth is used to transmit the encoded sequences over multiple antennas. In case of transmission of the independent uncoded stream of symbols using multiple transmit antenna elements, spatial multiplexing scheme is obtained. Space time coding and beam forming are fundamentally different schemes. In space time coding set-up, the direction of transmission is not certain and specific. The signals that are transmitted are totally different which means the transmitted signals may be from the same encoder and hence may be correlated or these transmitted signals may be totally independent information streams. From the completely different fading channels, received signals are obtained and the main purpose is to obtain diversity so as to increase transmission rate. In case of beam forming, the transmission and reception direction is certain and at the transmitter and receiver, antenna arrays are used to maintain the main beams of antenna pattern in that particular direction considering the suppression of possible

interference. Received signals are the some phase-shifted version of each other while transmitted signals are same but scaled by certain coefficients. This scheme is used to increase the effective signal to noise ratio. [37][38].

Among other diversity techniques, Space time coding is transmit diversity technique which can be applied to both MIMO and MISO system. Considering cost and space limitations, STC is an effective signaling scheme if the receiver has one antenna. The temporal and spatial correlation can be introduced between the signals which are transmitted from various antennas to provide diversity at the receiver and hence reception reliability of transmitted signals is increased. Space time coding is applicable in cellular communication and wireless local-area networks. [33]

## 8.1 Space Time Block Code

For the wireless transmission over Rayleigh fading channels implementing multiple transmit antennas, Space time block codes provide a paradigm in the field of wireless communication. Space time block coding follows the theory of orthogonal design and is used to obtain full diversity with low decoding complexity. Encoding of data is done by using space time block codes and these encoded data is converted into  $n$  streams such that these streams are transmitted simultaneously through  $n$  transmit antennas. Each receiving antenna gets the signals which are the linear superposition of the  $n$  transmitted signals along with the noise. Decoupling of the signals from different transmit antennas is done to achieve maximum likelihood decoding instead of using joint detection. Orthogonal structure of space time block code is used to obtain maximum likelihood decoding algorithm and the processing at the receiver is completely linear processing. Space time block coding with multiple transmit antennas can provide remarkable performance without the requirement of extra processing. Most remarkably, Space time block coding method implements simple encoding and decoding technique. [38]

Use of STTC, a part of space time coding, combines the diversity advantage and coding advantage but due to the use of Viterbi detector additional complexity in detection increases exponentially, when the  $M_d$ - array modulation is used then number of states increases as  $r_d^{nt}$ . To overcome this drawback orthogonal space time block codes are used which are the family of linear codes. CSI at the receiver is required for space time codes and using training

sequence which has to be estimated in FDD system. As an alternative, differentially detected space time block codes can be used which doesn't require CSI at the receiver.

For the given sequence of symbols, ML detector remains linear then decoupled for orthogonal space time block code. In the receiver, linear detector implements a simple algorithm so STBC is attractive. STBC code words are semi-unitary so that channel estimation process which uses pilot symbols goes to be trivial. Use of training symbols for the ML estimate of channel involves the inverse of the matrix and due to its diagonal form this is trivial. [39]

### 8.1.1 Space Time Block Code Encoder

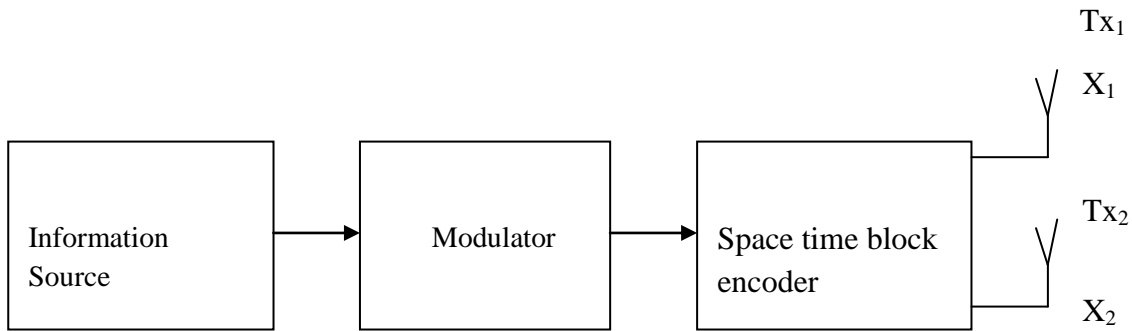


Fig: Space Time Block Encoder

The Alamouti algorithm became the most popular algorithm because of its properties to utilize full diversity without the channel state information (CSI) available at the transmitter and due to simple decoding system (maximum likelihood decoding). Full diversity gain  $N_r$  at the receiver can be achieved with maximum likelihood decoder. Therefore this system guarantees the total diversity of  $2N_r$ , without the CSI available at the transmitter. This is because of the orthogonality between the time sequence of the signals generated by the two transmitting antenna. With the extension of this property, the algorithm was generalized to arbitrary number of transmit antennas by applying the theory of orthogonal design. The generalized scheme that is capable of full transmit diversity of  $N_t N_r$  are referred to as space-time block codes (STBCs). These codes can be easily recovered using simple likelihood decoding algorithm which is based on the linear processing of the received signals.[40][41]

If  $N_t$  and  $p$  represent the number of transmit antennas and number of time periods for transmission of one block of coded symbols respectively then  $N_t \times p$  defines transmission matrix  $X$ . Let us consider that the signal constellation consists of  $2^m$  points. A block of  $k$  information bits are mapped into the signal constellation at each encoding process for the selection of  $k$  modulated signals  $x_1, x_2, \dots, x_k$ . Here, a constellation signal is selected by

each group of  $m$  bits. Space time encoder encodes  $k$  modulated for the generation of  $N_t$  parallel signal sequences of length  $p$  in accordance with transmission matrix  $X$ . Simultaneous transmission of these signal sequences takes place through  $N_t$  transmit antennas in time periods  $p$ .

The encoder takes  $k$  as its input in each coding operation where  $k$  is the number of symbols. For each block of  $k$  input symbols, each multiple transmit antenna transmits  $p$  space- time symbols. We can define the rate of space-time block code as the ratio of the number of symbols the encoder takes as its input to the number of space time coded symbols transmitted from each antenna and mathematically can be expressed as:

$$R = k/p \quad (8.1.1)$$

Also, the spectral efficiency of space-time block code can be written as:

$$n = r_b/B = r_s m R / r_s = km/p \text{ bits/sec/Hz} \quad (8.1.2)$$

Where,

$r_b$ ,  $r_s$ ,  $B$  are bit rate, symbol rate and bandwidth respectively.

The linear combination of  $k$  modulated symbols  $x_1, x_2, \dots, x_k$  and their conjugates  $x_1^*, x_2^*, \dots, x_k^*$  are the entries of transmission matrix  $X$ . Based on the orthogonal design, the transmission matrix  $X$  is constructed so that full diversity of  $N_t$  is achieved.

$$X \cdot X^H = C (|x_1|^2 + |x_2|^2 + \dots + |x_k|^2) I_{N_t} \quad (8.1.3)$$

Where  $X^H$  is Hermitian of  $X$ ,  $C$  is constant and  $I_{N_t}$  is the  $N_t \times N_t$  identity matrix. The symbols transmitted consecutively during transmission periods  $p$  from the  $i^{\text{th}}$  transmit antenna is represented by  $i^{\text{th}}$  row of  $X$ . On the other hand, simultaneously transmitted symbols through  $N_t$  transmit antennas at time  $j$  is represented by  $j^{\text{th}}$  column of  $X$ . The  $j^{\text{th}}$  column of  $X$  is considered as space-time symbols transmitted at time  $j$ . The  $i^{\text{th}}$  row and  $j^{\text{th}}$  column of  $X$ ,  $x_{i,j}$ ,  $i = 1, 2, \dots, N_t$ ,  $j = 1, 2, \dots, p$ , represents the signal transmitted from the antenna  $i$  at time  $j$ .

It is considered that  $R \leq 1$ , that means the rate of space-time block code with full transmit diversity is equal to one or less than one. For the code with full rate  $R=1$ , no bandwidth expansion is required but for the code with  $R<1$ , bandwidth expansion of  $1/R$  is required. If space-time block codes have  $N_t$  transmit antennas, the transmission matrix is represented by  $X_{N_t}$  and code is known as space-time block code with size  $N_t$ .



For the construction of space-time block codes, orthogonal designs are applied. If  $X_{N_t}$  is transmission matrix, then the rows of this matrix are orthogonal to each other. Signal sequences are orthogonal from any two transmit antennas in each block. Let us consider that  $i^{\text{th}}$  antenna transmits the sequence  $x_i = (x_{i,1}, x_{i,2}, \dots, x_{i,p})$  where  $i = 1, 2, \dots, N_t$ , then we can write the following expression:

$$x_i \cdot x_j = \sum_{t=1}^p x_{i,t} \cdot x_{j,t}^* = 0, \quad i \neq j, \quad i, j \in \{1, 2, \dots, N_t\} \quad (8.1.4)$$

The inner product of  $x_i$  and  $x_j$  is represented by  $x_i \cdot x_j$ . For given number of transmit antennas, the orthogonality helps to attain the full transmit diversity. It also enables the receiver for decoupling the transmitted signals from different transmit antennas. Along with this it is possible to employ a simple maximum likelihood decoding which depends only upon linear processing of received signals. [37]

### 8.1.2 Alamouti Space-Time Code

By using multiple antennas at receiver, it is easier to achieve spatial diversity. The uplink where transmission takes place from mobile to base station of cellular telephone system is an example. It is possible to implement multiple antennas at the base station with easy required antenna separation and the signal transmitted by the mobile terminals can be received by multiple receive antennas. The received signal can be combined by using appropriate combining technique such as maximal-ratio combining, selection combining, equal gain combining, etc. But in case of down link transmission where transmission takes place from base-station to the mobile terminals, it is complicated to achieve a diversity gain because the mobile units have limited size and employing multiple antennas with suitable separation is difficult for the reception of multiple copies of the transmitted signals. So, necessity was felt to have such a scheme that takes the benefits of spatial diversity through transmit diversity. In 1998, Alamouti developed that scheme to obtain transmit diversity for multiple transmit antennas. This scheme is a systematic method for the construction of full-rate space time block codes with a full diversity order.[33]

Alamouti coding scheme is a simple space time block code which provides full transmit diversity for the system using two transmit antennas. Delay diversity scheme can also be used but it introduces interference between symbols and receiver.

### 8.1.2.1 Alamouti space time encoding

Let us consider M-ary modulation process is implemented. For modulation, each group of  $m$  information bits is used first where  $m = \log_2 M$ . A block of two modulated symbols  $x_1$  and  $x_2$  is taken in each operation of encoding by the encoder which is mapped to the transmit antennas in accordance with a code matrix expressed as:

$$X = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \quad (8.1.5)$$

From two transmit antennas, the encoder output is transmitted in two consecutive transmission periods. Simultaneous transmission of two signals  $x_1$  and  $x_2$  is performed during the first transmission period from antenna one and antenna two respectively. In second transmission period,  $x_2^*$  and  $x_1^*$  is transmitted from antenna one and two respectively where  $x_2^*$  and  $x_1^*$  are the complex conjugate of  $x_2$  and  $x_1$  respectively. The overall transmission rate is one symbol per channel use because two symbols are transmitted in two time slots. Here encoding is done in both space and time domains. If  $x^1$  and  $x^2$  represent transmit sequence from antennas one and two respectively then can be expressed as:

$$x^1 = [x_2^*, x_1^*]. \quad (8.1.6)$$

$$x^2 = [x_2, x_1]. \quad (8.1.7)$$

$$x^1 \cdot x^2 = x_1 x_2^* = x_2 x_1^* = 0. \quad (8.1.8)$$

The transmit sequences from two transmit antennas are orthogonal because,

$$x^1 \cdot x^2 = 0. \quad (8.1.9)$$

The property of code matrix is written as:

$$X X^H = \begin{bmatrix} |x_1|^2 + |x_2|^2 & 0 \\ 0 & |x_1|^2 + |x_2|^2 \end{bmatrix}. \quad (8.1.10)$$

$$= (|x_1|^2 + |x_2|^2) I_2 \quad (8.1.11)$$

$I_2$  represents 2×2 identity matrix.[37]

### 8.1.2.2 Optimal Receiver for the Alamouti Scheme

#### 8.1.2.2.1 Single Receive Antenna System

Assume that one antenna is used at the receiver. Let  $y_1(1)$  and  $y_1(2)$  be the received signals in the first time slot and second time slot respectively.

$$y_1(1) = \sqrt{\rho}(h_{1,1}x_1 + h_{2,1}x_2) + n_1(1). \quad (8.1.12)$$

$$y_1(2) = \sqrt{\rho}(-h_{1,1}x_2^* + h_{2,1}x_1^*) + n_1(2). \quad (8.1.13)$$

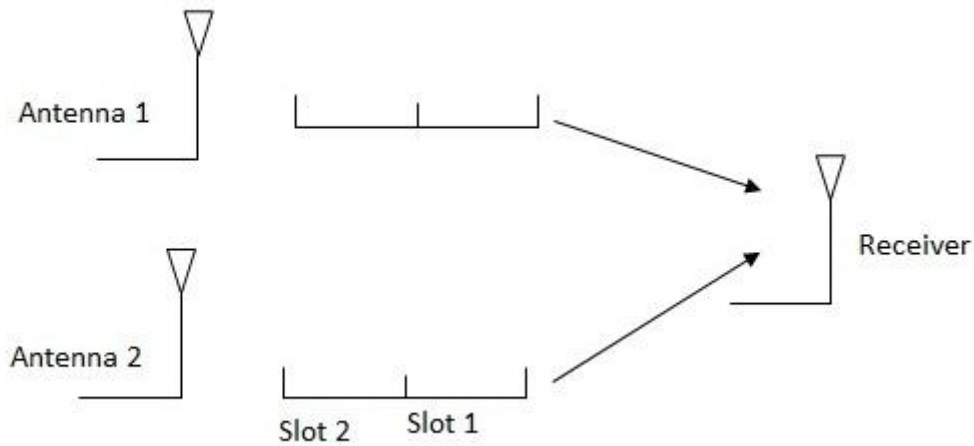


Fig: Alamouti Scheme

The channel is assumed to be Rayleigh fading which means  $h_{1,1}$  and  $h_{2,1}$  are zero mean complex Gaussian random variables having unit variance (1/2 per dimension). And  $h_{1,1}$  and  $h_{2,1}$  remain constant for two consecutive time intervals. Here  $n_1(1)$  and  $n_1(2)$  are additive noises and also are complex AWGN having variance 1/2 per dimension.  $\rho$  is signal to noise ratio.

The vector of the received signal can be written as:

$$\mathbf{y} = \begin{bmatrix} y_1(1) \\ y_1^*(2) \end{bmatrix}. \quad (8.1.14)$$

Or

$$\mathbf{y} = \sqrt{\rho} \begin{bmatrix} h_{1,1} & h_{2,1} \\ h_{2,1}^* & -h_{1,1}^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1'(1) \\ n_1^*(2) \end{bmatrix}. \quad (8.1.15)$$

It is assumed that receiver has perfect knowledge of channel state information then the optimal receiver minimizes the probability of error and selects  $x_1$  and  $x_2$  as given below:

$$(\widehat{x}_1, \widehat{x}_2) = \underset{(x_1, x_2)}{\arg \max} P(x_1, x_2 | y, h_{1,1}, h_{2,1}) \quad (8.1.16)$$

Or

$$(\widehat{x}_1, \widehat{x}_2) = \underset{(x_1, x_2)}{\arg \max} p(x_1, x_2 | H^H \mathbf{y}, h_{1,1}, h_{2,1}). \quad (8.1.17)$$

Where

$$H = \begin{bmatrix} h_{1,1} & h_{2,1} \\ h_{2,1}^* & -h_{1,1}^* \end{bmatrix}. \quad (8.1.18)$$

$H^H \mathbf{y}$  is one-to-one transformation.

All the input symbol pairs are assumed to be equally likely. According to Baye's rule, optimal decoded symbols can be written as:

$$(\widehat{x}_1, \widehat{x}_2) = \underset{(x_1, x_2)}{\arg \max} P(H^H \mathbf{y}, |x_1, x_2, h_{1,1}, h_{2,1}). \quad (8.1.19)$$

$$H^H \mathbf{y} = \sqrt{\mathbf{P}} \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & 0 \\ 0 & |h_{1,1}|^2 + |h_{2,1}|^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n'_1(1) \\ n'_1(2) \end{bmatrix}. \quad (8.1.20)$$

And the noise is:

$$\begin{bmatrix} n'_1(1) \\ n'_1(2) \end{bmatrix} = \begin{bmatrix} h_{1,1}^* & h_{2,1} \\ h_{2,1}^* & -h_{1,1}^* \end{bmatrix} \begin{bmatrix} n_1(1) \\ n_1^*(2) \end{bmatrix}. \quad (8.1.21)$$

They are jointly Gaussian and are independent because they are uncorrelated.

Each  $n'_1(1)$  and  $n'_1(2)$  has zero mean and variance of  $\frac{1}{2}(|h_{1,1}|^2 + |h_{1,1}|^2)$  per dimension.

The optimal decisions  $x_1$  and  $x_2$  undergoes decoupling that helps in the minimization of the euclidean distance between the transmitted symbols and vector components of  $H^H \mathbf{y}$ . Therefore,

$$\widehat{x}_1 = \arg \min_{x_1} \left| h_{1,1}^* y_1(1) + h_{2,1} y_1^*(2) - \sqrt{\rho}(|h_{1,1}|^2 + |h_{2,1}|^2) x_1 \right|. \quad (8.1.22)$$

$$\widehat{x}_2 = \arg \min_{x_2} \left| h_{2,1}^* y_1(1) - h_{1,1} y_1^*(2) - \sqrt{\rho}(|h_{1,1}|^2 + |h_{2,1}|^2) x_2 \right|. \quad (8.1.23)$$

This decoding rule used in Alamouti scheme or orthogonal space time coding has made this scheme very useful. The search space for optimal selection of the transmitted symbols is reduced by decoupling of the optimal decisions so that considerable simplification is possible in the receiver structure.

If we consider the constant energy constellation such as BPSK, QPSK, 8-PSK, it is possible to write optimal decision rule as the usual correlation maximization.

If we take a special case of BPSK modulation, we can write  $\widehat{x}_1$  and  $\widehat{x}_2$  as

$$\widehat{x}_1 = \begin{cases} 1, & \text{if } \operatorname{Re} \{ h_{1,1}^* y_1(1) + h_{2,1} y_1^*(2) \} > 0 \\ 0, & \text{otherwise} \end{cases} \quad (8.1.24)$$

$$\widehat{x}_2 = \begin{cases} 1, & \text{if } \operatorname{Re} \{ h_{2,1}^* y_1(1) - h_{1,1} y_1^*(2) \} > 0 \\ 0, & \text{otherwise} \end{cases} \quad (8.1.25)$$

Without using multiple antennas at the receiver, it is possible to obtain spatial diversity and hence this Alamouti scheme is very applicable for this purpose.

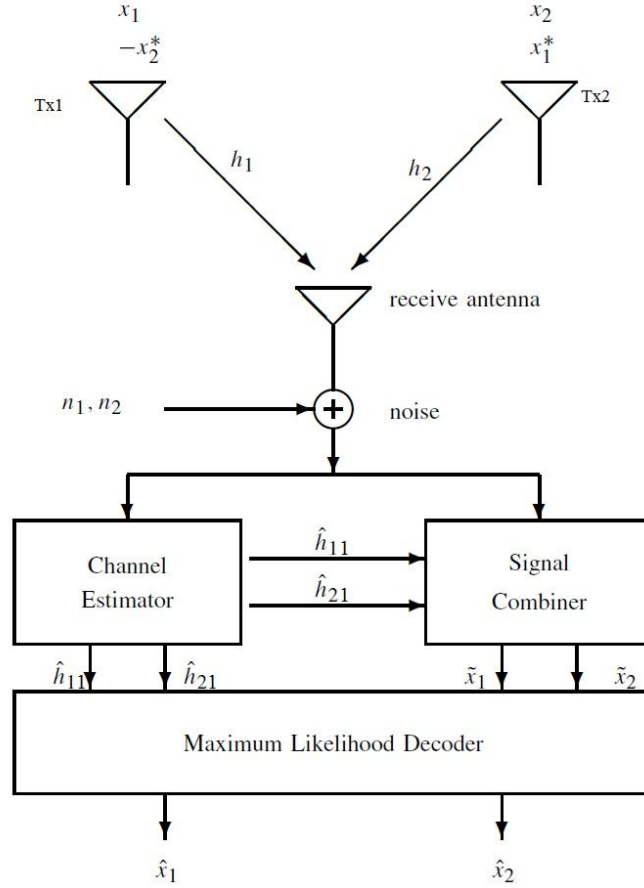


Fig: 5. Receiver for Alamouti Scheme [4]

#### 8.1.2.2.1 Maximum Likelihood (ML) Decoder

The optimum decoder based on likelihood function is known as ML decoder. The ML decoder is not necessarily optimum in the case where the input or code sequences are not equally likely, however it is considered as the best feasible decoding rule. In ML receiver, input symbols are code word spanning space and time where number of code words is finite. In this detection technique, testing is done for all possible code words and then one which best fits the received signal based on the ML criterion is selected for the estimation of code word that was really transmitted. [50] [51]

#### 8.1.2.2.2 Multiple receiving antennas at receiver

Alamouti scheme can also be used for multiple antennas at receiving side to achieve receive diversity. Twice of the number of receive antennas is the available diversity order and by using simple linear receiver it can be achieved.

Let  $y_j(k)$  be received signal at  $j$ th receive antenna during  $k^{\text{th}}$  time slot where  $k = 1, 2, \dots, N_r$ .

$$y_j(1) = \sqrt{\rho}(h_{1,j}, x_1 + h_{2,j}, x_2) + n_j(1). \quad (8.1.26)$$

$$y_j(2) = (-h_{1,j}, x_2^* + h_{2,j}, x_1^*) + n_j(2). \quad (8.1.27)$$

where  $h_{i,j}$  denotes channel coefficients from  $i^{\text{th}}$  transmit antenna to the  $j^{\text{th}}$  receive antenna.

$N_j(k)$  represents AWGN term during time slot  $k$  and receive antenna  $j$ .

If we combine the received signals from all the receive antennas linearly and scaled by the factor

$\frac{1}{\sqrt{|h_{1,j}|^2 + |h_{2,j}|^2}}$ , then the statistics for optimal receiver is written as:

$$y_j(1) = \sqrt{\rho} \sqrt{|h_{1,j}|^2 + |h_{2,j}|^2} x_1 + n_j''(1). \quad (8.1.28)$$

$$y_j(2) = \sqrt{\rho} \sqrt{|h_{1,j}|^2 + |h_{2,j}|^2} x_2 + n_j''(2). \quad (8.1.29)$$

We know,  $j = 1, 2, \dots, N_r$

Each corresponding to one receiver antenna, there are  $N_r$  possible parallel channels which are corrupted by independent Gaussian noise terms having equal variances. As combining technique, maximum ratio combining is used and the decision variables of the two transmitted symbols can be written as below:

$$y(k) = \sum_{j=1}^{N_r} \sqrt{|h_{1,j}|^2 + |h_{2,j}|^2} y_j(k) \quad k = 1, 2. \quad (8.1.30)$$

Then comparison of these decision variables is made with possible “clean” signals which are transmitted. As the optimal one, the one closest to  $y(k)$  is selected in the squared Euclidean sense. The decision rule is can be expressed as:

$$\hat{x}_1 = \arg \min_{x_1} \left| \sum_{j=1}^{N_r} h_{1,j}^* y_j(1) + h_{2,j} y_j^*(2) - \sqrt{\rho}(|h_{1,j}|^2 + |h_{2,j}|^2) x_1 \right|^2. \quad (8.1.31)$$

$$\hat{x}_2 = \arg \min_{x_2} \left| \sum_{j=1}^{N_r} h_{2,j}^* y_j(1) - h_{1,j} y_j^*(2) - \sqrt{\rho}(|h_{1,j}|^2 + |h_{2,j}|^2) x_2 \right|^2. \quad (8.1.32)$$

For the two symbols transmitted, the decisions are decoupled and minimization is done separately over the probable constellation symbols. If we use a constant energy constellation like phase shift keying, the further simplification is possible in optimal decision rule.

If we employ BPSK modulation then it can be written as:

$$\hat{x}_1 = \begin{cases} 1, & \text{if } \text{Re} \left\{ \sum_{j=1}^{N_r} h_{1,j}^* y_j(1) + h_{2,j} y_j^*(2) \right\} > 0 \\ 0, & \text{otherwise} \end{cases} \quad (8.1.33)$$

$$\hat{x}_2 = \begin{cases} 1, & \text{if } \text{Re} \left\{ \sum_{j=1}^{N_r} h_{2,j}^* y_j(1) - h_{1,j} y_j^*(2) \right\} > 0 \\ 0, & \text{otherwise} \end{cases} \quad (8.1.34)$$

Here, we should note that after this stage the decision rule is same as for the case of one receive antenna i.e.  $N_r = 1$ . [42]

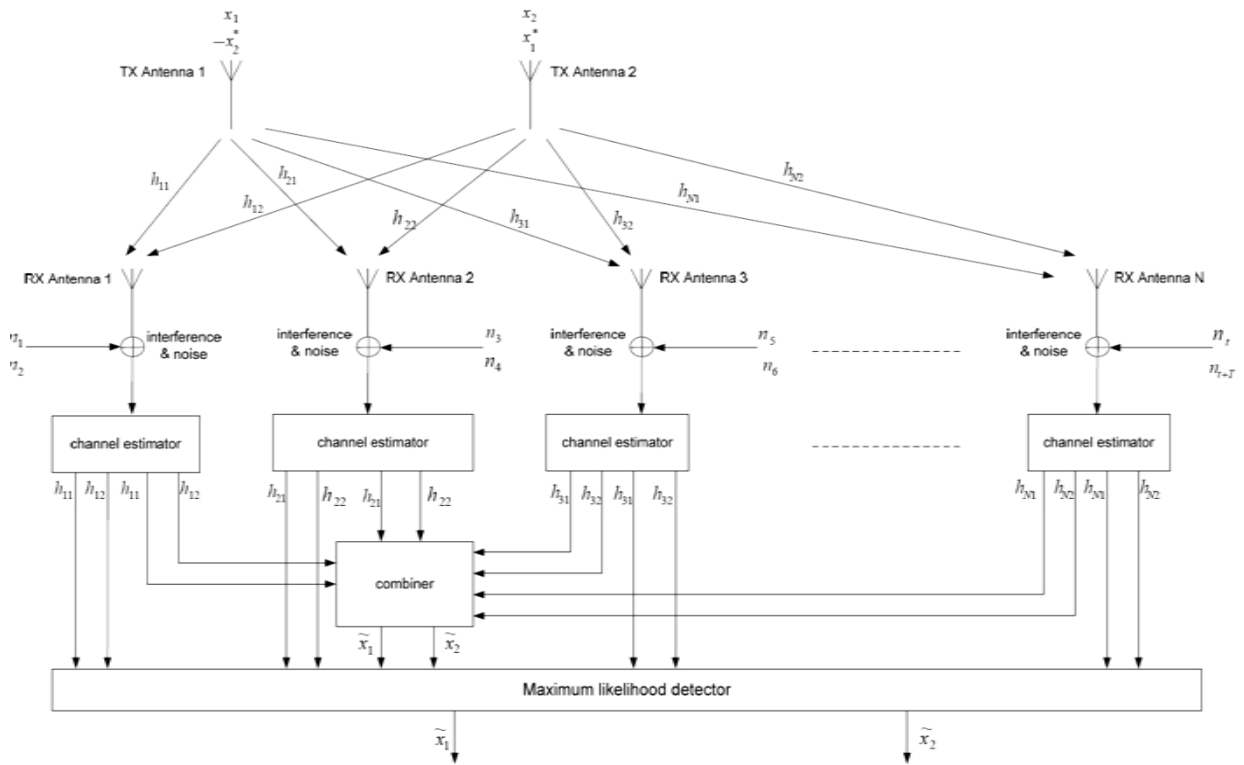


Fig: 6. Receiver for Alamouti scheme for multiple antennas

### 8.1.3 Orthogonal space time block codes

The design of Alamouti scheme is based on two transmit antennas but it is possible to consider arbitrary number of transmit antennas. Orthogonal space time block codes are the general space time block codes which are based on the theory of orthogonal design.



### 8.1.3.1 STBC for real signal Constellation

There are two type of signal constellation based on the type of signal constellation, space-time block code with real signals and space-time block code with complex Signals.

Now if  $N_T \times p$  is a real transmission matrix  $X_{N_T}$  with  $x_1, x_2, \dots, x_k$  are variables that satisfies

$$X.X^H = c(|x_1|^2 + |x_2|^2 + \dots + |x_k|^2)I_{N_T} . \quad (8.1.35)$$

This space-time block code can provides the full transmission diversity of  $N_T$  with code rate  $k/p$ .

For this let us consider a square transmission matrix  $X_{N_T}$ , and these square transmission matrix exist only if the number of transmission antennas are 2,4 or 8. i.e  $N_T = 2,4$  or 8. These codes are of full rate and full transmit diversity  $N_T$  as the transmission matrix is square, the transmission matrix are given by

For  $N_T = 2$ , i.e 2 transmit antennas

$$X_2 = \begin{bmatrix} x_1 & -x_2 \\ x_2 & x_1 \end{bmatrix} . \quad (8.1.36)$$

For  $N_T = 4$ , i.e 4 transmit antennas

$$X_4 = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 \\ x_2 & x_1 & x_4 & -x_3 \\ x_3 & -x_4 & x_1 & x_2 \\ x_4 & x_3 & -x_2 & x_1 \end{bmatrix} . \quad (8.1.37)$$

For  $N_T = 8$ , i.e 8 transmit antennas

$$X_8 = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & -x_5 & -x_6 & -x_7 & -x_8 \\ x_2 & x_1 & -x_4 & x_3 & -x_6 & x_5 & x_8 & -x_7 \\ x_3 & x_4 & x_1 & -x_2 & -x_7 & -x_8 & x_5 & x_6 \\ x_4 & -x_3 & x_2 & x_1 & -x_8 & x_7 & -x_6 & x_5 \\ x_5 & x_6 & x_7 & x_8 & x_1 & -x_2 & -x_3 & -x_4 \\ x_6 & -x_5 & x_8 & -x_7 & x_2 & x_1 & x_4 & -x_3 \\ x_7 & -x_8 & -x_5 & x_6 & x_3 & -x_4 & x_1 & x_2 \\ x_8 & x_7 & -x_6 & -x_5 & x_4 & x_3 & -x_2 & x_1 \end{bmatrix}. \quad (8.1.38)$$

The square transmission matrix consist of orthogonal rows elements  $\pm x_1, \pm x_2, \dots, \pm x_k$ . from the matrix we can observe that all the above matrices have independent and orthogonal rows, so their dot product will be zero for any real constellation such as M –ASK, which satisfies the equation  $X.X^H = c(|x_1|^2 + |x_2|^2 + \dots + |x_k|^2)I_{N_T}$ , to prove that the code rate of all the matrices is unity let us take an example, let us consider a MIMO system with 4 transmission antennas then the matrix with four transmit antennas is  $4 \times 4$  matrix, since we have four rows in the corresponding matrix, the size of space-time block code is 4, and since there are four columns in the transmission matrix therefore the transmission time p is 4 as it corresponds to the column of the transmission matrix, and the symbols will also be  $k = 4$ , i.e  $x_1, x_2, x_3$  and  $x_4$ . Now during the first transmission sequence  $x_1, x_2, x_3$  and  $x_4$  are transmitted where  $x_1$  is transmitted from first antenna  $x_2$  from second antenna and so on. where as in next transmission sequence  $-x_2, x_1, -x_4$  and  $x_3$  are transmitted, here  $-x_2$  is transmitted from first antenna  $x_1$  from second antenna and so on. These results in

$$R = \frac{k}{p} = \frac{4}{4} = 1. \quad (8.1.39)$$

Therefore the code rate is unity.

Now to construct the transmission scheme with any number of antennas having code rate = 1 as we always desire of full code rate as these are bandwidth efficient, we have to consider another system consisting of both square and non square matrices. In this system with  $N_T$  transmit antennas, to achieve full rate the minimum value of transmission period p is given by

$$p = \min(2^{4c+d}). \quad (8.1.40)$$

where the minimization is taken over the set

$$c, d \mid 0 \leq c, 0 \leq d \leq 4, \text{ and } 8c + 2d \geq N_T$$

For  $N_T \leq 8$ , the minimum value of p is given by

$$N_T = 2, \quad p = 2$$

$$N_T = 3, \quad p = 4$$

$$N_T = 4, \quad p = 4$$

$$N_T = 5, \quad p = 8$$

$$N_T = 6, \quad p = 8$$

$$N_T = 7, \quad p = 8$$

$$N_T = 8, \quad p = 8$$

These values provide guidelines to construct full rate space-time block codes. Now based on the above equations we can construct we can make a non square matrix of size 3, 5, 6 and 7, for real numbers this gives full diversity and full rate.

The matrices will be as follows,

$$X_3 = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 \\ x_2 & x_1 & x_4 & -x_3 \\ x_3 & -x_4 & x_1 & x_2 \end{bmatrix}. \quad (8.1.41)$$

$$X_5 = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & -x_5 & -x_6 & -x_7 & -x_8 \\ x_2 & x_1 & -x_4 & x_3 & -x_6 & x_5 & x_8 & -x_7 \\ x_3 & x_4 & x_1 & -x_2 & -x_7 & -x_8 & x_5 & x_6 \\ x_4 & -x_3 & x_2 & x_1 & -x_8 & x_7 & -x_6 & x_5 \\ x_5 & x_6 & x_7 & x_8 & x_1 & -x_2 & -x_3 & -x_4 \end{bmatrix}. \quad (8.1.42)$$

$$X_6 = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & -x_5 & -x_6 & -x_7 & -x_8 \\ x_2 & x_1 & -x_4 & x_3 & -x_6 & x_5 & x_8 & -x_7 \\ x_3 & x_4 & x_1 & -x_2 & -x_7 & -x_8 & x_5 & x_6 \\ x_4 & -x_3 & x_2 & x_1 & -x_8 & x_7 & -x_6 & x_5 \\ x_5 & x_6 & x_7 & x_8 & x_1 & -x_2 & -x_3 & -x_4 \\ x_6 & -x_5 & x_8 & -x_7 & x_2 & x_1 & x_4 & -x_3 \end{bmatrix}. \quad (8.1.43)$$

$$X_7 = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & -x_5 & -x_6 & -x_7 & -x_8 \\ x_2 & x_1 & -x_4 & x_3 & -x_6 & x_5 & x_8 & -x_7 \\ x_3 & x_4 & x_1 & -x_2 & -x_7 & -x_8 & x_5 & x_6 \\ x_4 & -x_3 & x_2 & x_1 & -x_8 & x_7 & -x_6 & x_5 \\ x_5 & x_6 & x_7 & x_8 & x_1 & -x_2 & -x_3 & -x_4 \\ x_6 & -x_5 & x_8 & -x_7 & x_2 & x_1 & x_4 & -x_3 \\ x_7 & -x_8 & -x_5 & x_6 & x_3 & -x_4 & x_1 & x_2 \end{bmatrix}. \quad (8.1.44)$$

Now in this case we can take a MIMO system with 7 transmission antenna so that we can consider transmission matrix  $X_7$ , here we have  $k = 8$  since there are 8 symbols  $x_1, x_2, \dots, x_8$ , with 8 transmission period we have  $p = 8$ . Here these eight symbols are transmitted over eight transmission period from seven antenna as in the case of previous system, this results in the full transmit diversity of  $N_T = 8$  and code rate will be

$$R = \frac{k}{p} = \frac{8}{8} = 1 \text{ which is also unity.}$$

### 8.1.3.2 STBC for Complex Signal Constellation

In case of complex signals constellation the matrices are also defined as the matrices of size  $N_T \times p$ , where the elements of the matrix will be complex entries of  $X_1, X_2, \dots, X_k$ , and the conjugates such that it satisfies  $X.X^H = c(|x_1|^2 + |x_2|^2 + \dots + |x_k|^2)I_{N_T}$ , such matrices has a full transmit diversity of  $N_T$  and code rate  $k/p$ .

The well known Alamouti principle is space- time block code with complex signal constellation with transmit antennas, where the transmission matrix is represented by

$$X_2 = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}. \quad (8.1.45)$$

Using this scheme we can have a full diversity of 2 and also full code rate 1. It is the only matrix with code rate unity so it is regarded as unique in case of complex entries, so this has wide applications for higher order modulation other than BPSK binary phase shift keying.

The design rules for transmission matrix is nearly same as of real entries, ie here also we design for full diversity satisfying  $X.X^H = c(|x_1|^2 + |x_2|^2 + \dots + |x_k|^2)I_{N_T}$  and in order to minimize the decoding delay we minimize the value of  $p$ .

Now let us consider an example of complex transmission matrix with three and four transmit antennas have orthogonal design for space-time block codes. Such codes have  $1/2$  code rate.

$$X_3 = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & x_1^* & -x_2^* & -x_3^* & -x_4^* \\ x_2 & x_1 & x_4 & -x_3 & x_2^* & x_2^* & x_4^* & -x_3^* \\ x_3 & -x_4 & x_1 & x_2 & x_3^* & x_3^* & x_1^* & x_2^* \end{bmatrix}. \quad (8.1.46)$$

$$X_4 = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & x_1^* & -x_2^* & -x_3^* & -x_4^* \\ x_2 & x_1 & x_4 & -x_3 & x_2^* & x_2^* & x_4^* & -x_3^* \\ x_3 & -x_4 & x_1 & x_2 & x_3^* & x_3^* & x_1^* & x_2^* \\ x_4 & x_3 & -x_2 & x_1 & x_4^* & x_3^* & -x_2^* & x_1^* \end{bmatrix}. \quad (8.1.47)$$

Here we can easily verify that the inner product of any of the two rows of these matrices is zero .which suggest the result that the matrix is orthogonal and is of full rank with diversity of  $N_T = 3$  and  $N_T=4$  resp.

Here in case of first transmission matrices with three transmission antennas there are four symbols  $x_1, x_2, x_3$  and  $x_4$  and their respective conjugates, with transmission period  $p=8$  and  $k=4$ . Then the code rate will be  $R = \frac{k}{p} = \frac{4}{8} = \frac{1}{2}$ . Similarly the transmission matrix with four transmitting antennas code rate is  $\frac{1}{2}$  but the diversity is  $N_T = 4$ .

For higher code rate more complex linear processing is required, here we have the example with three and four transmitting antenna and code rate  $\frac{3}{4}$ .

$$X_3 = \begin{bmatrix} x_1 & -x_2^* & \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} \\ x_2 & x_1^* & \frac{x_3^*}{\sqrt{2}} & \frac{-x_3^*}{\sqrt{2}} \\ \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} & \frac{(-x_1 - x_1^* + x_2 - x_2^*)}{2} & \frac{(x_2 + x_2^* + x_1 - x_1^*)}{2} \end{bmatrix}. \quad (8.1.48)$$

$$X_4 = \begin{bmatrix} x_1 & -x_2 & \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} \\ x_2 & x_1 & \frac{x_3^*}{\sqrt{2}} & \frac{-x_3^*}{\sqrt{2}} \\ \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} & \frac{(-x_1 - x_1^* + x_2 - x_2^*)}{2} & \frac{(x_2 + x_2^* + x_1 - x_1^*)}{2} \\ \frac{x_3}{\sqrt{2}} & \frac{-x_3}{\sqrt{2}} & \frac{(-x_2 - x_2^* + x_1 - x_1^*)}{2} & \frac{-(x_1 + x_1^* + x_2 - x_2^*)}{2} \end{bmatrix}. \quad (8.1.49)$$

[43][44][45]

## 8.2 Space Time Trellis Codes

Space time block codes provide maximum diversity advantage by employing simple decoding techniques but these codes don't provide coding gain. So, another coding scheme known as Space time trellis coding is considered as an effective signaling scheme. STTC was first introduced by Tarokh, Seshadri and Calderbank and it became very popular due to its capability of offering coding gain with spectral efficiency as well as full diversity over fading channels. The inherent nature of STTC itself offer coding gain to be achieved which is different from the coding gain offered by temporal block codes and convolution codes. [46]

Space time trellis coding is a basic method of coding for MIMO systems. In this method, basic trellis structure determines the symbols that have to be transmitted from different antennas. Let us look about the notation and preliminaries for the discussion of the general STTC codes. Space time trellis codes are class of nonlinear Space time codes which provide maximum diversity and high coding gain for the certain bandwidth efficiency. STTCs are represented by a trellis structure and cab be decoded by using Viterbi's algorithm. Encoding and decoding is complex than that of Space time blocks coding.

Using space time trellis codes input scalar symbol stream is encoded into an output vector symbol stream. Consider  $N_t=2$  transmit antennas and code is described by specifying what is to be transmitted by using two transmit antennas considering a frame of data. Suppose four-state trellis used for space-time coding as shown below in figure:

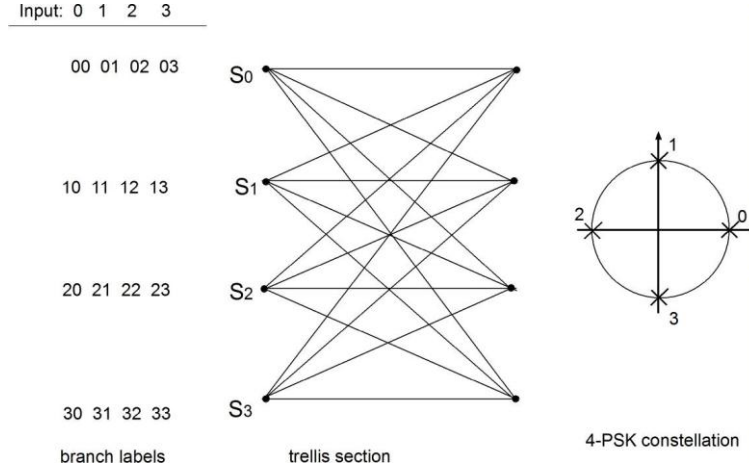


Fig: Example of four-state space time trellis code

Let  $x_i(k)$  be the signal transmitted from antenna  $i$  at time  $k$  then the  $j^{\text{th}}$  antenna corresponding to this time interval receives the following signal:

$$y_j(k) = \sqrt{\rho} \sum_{i=1}^{N_t} h_{ij} x_i(k) + n_j(k). \quad (8.2.1)$$

Here,  $i=1, 2, 3, \dots, N_t$ ,  $j=1, 2, 3, \dots, N_r$ ,  $t=1, 2, 3, \dots, N$  and  $N$  is the frame length.

From this, the channel is Rayleigh or Rician fading and over an entire frame length  $N$ , channel gains are constant. Spatially and temporally white noise samples are considered and normalization is done for both the channel gains and additive noise samples such that their variance per dimension is  $1/2$ , normalization for constellation energy is  $1/N_t$ , for each receive antenna,  $\rho$  denotes the signal to noise ratio.

The input-output relation is expressed as:

$$Y = \sqrt{\rho} XH + N \quad (8.2.2)$$

Here  $X$  is  $N \times N_t$  transmitted code word,  $x_i(k)$  is  $(k, i)^{\text{th}}$  element and  $H$  represents  $N_t \times N_r$  matrix of channel coefficient,  $h_{ij}$  is  $(i, j)^{\text{th}}$  element,  $Y$  is  $N \times N_r$  received matrix,  $y_j(k)$  is its entries,  $N$  denotes  $N \times N_r$  noise matrix.

Since Quasi-static fading occurs in real scenario, it is introduced here. Space Time Trellis Codes along with MIMO communication system are focused for such channels and achieving spatial diversity is crucial if there is no possibility of time diversity.[47]

### 8.2.1 Space Time Trellis Code decoding

If the channel coefficients can be accessed by the receiver then optimal decision rule in terms of minimizing the probability of error can be expressed as:

$$\hat{X} = \arg \min_X p(X|Y, H). \quad (8.2.3)$$

This is called maximum posteriori (MAP) decoding rule and is equivalent to ML decoding rule provided that if the symbols are equally likely and written as:

$$\hat{X} = \arg \min_X p(X|Y, H). \quad (8.2.4)$$

For the given set of channel coefficients and transmitted signal matrix, the received matrix elements are combinely Gaussian because additive noise is Gaussian and spatially, temporally white. Above likelihood function can again be expressed as below:

$$\hat{X} = \arg \min_X \|Y - \sqrt{p}XH\|^2 \quad (8.2.5)$$

The sum of the norm squares of the elements of the matrix is represented by  $\|\cdot\|^2$

And this likelihood function seems to be proportional to negative of the squared Euclidean distance and the distance is the distance between transmitted and received matrix.

The decision rule as result can be written as:

$$\hat{X} = \arg \min \sum_{k=1}^N \cdot \sum_{j=1}^{Nr} |y_j(k) - \sqrt{p} \sum h_{i,j} x_i(k)|^2. \quad (8.2.6)$$

While talking about the way of construction, paths are provided through the code trellis by the space time trellis code words. Here above, minimization process is applied to find a path through space-time code trellis such that Euclidean distance is minimum.



### 8.2.2 Viterbi decoding

The Viterbi decoding technique is optimal decoding technique because it observes the maximum likelihood criterion. Practically, it is very useful technique and is also available in VLSI form. Viterbi decoding of convolution code is very fascinating for satellite channels. The frame by frame operation takes place in Viterbi algorithm over a finite number of frames and the trellis path used by coder is found. For any input frame the decoder doesn't know about the node that the coder reached due to which all possible nodes are labeled with metrics. In the next frame, these metrics are used by the decoder for the deduction of the most likely path. [51]

Considering Viterbi algorithm, let us suppose the encoder is in state  $s_0$  such that  $k = 0$ . The paths emanating from this state is extended and the value of the path metric is computed using the following:

$$\sum_{j=1}^{Nr} |y_j(l) - \sqrt{p} \sum h_{i,j} x_i(l)|^2. \quad (8.2.7)$$

There will be one path through the trellis at time  $k$  for each state of the encoder in addition with the corresponding value of the accumulated path metric. We can extend each of these paths by using time at  $k+1$  for each state and the computation of possible path metrics are performed by adding the following

$$\sum_{j=1}^{Nr} |y_j(k+1) - \sqrt{p} \sum h_{i,j} x_i(k+1)|^2. \quad (8.2.8)$$

to the current path metrics. With the time index increment, all of these extensions are discarded but the one with the minimum accumulated path metric is not discarded. Termination of the trellis is done in the final steps back to state  $s_0$  [48]

## 8.3 Basic code design principles for space-time codes over quasi-static Rayleigh fading channels

### 8.3.1 Rank Criterion

If  $X_1$  and  $X_2$  are two distinct code words, then the maximum diversity is achieved if the matrix

$A = (X_1 - X_2)^H (X_1 - X_2)$  is full rank for all the pairs of  $X_1$  and  $X_2$ . If  $N_t$  is greater than the minimum rank,  $r$  of  $A$  among all the codeword pairs, a diversity of order  $rN_r$  is obtained.

### 8.3.2 Determinant Criterion

If full rank is obtained from the rank criterion then the determinant Criterion becomes the more interesting case. For the possible maximum coding advantage, the product of the eigen values of A over all pairs of distinct codewords (the minimum determinant of possible A matrices) which is minimum should be maximized.

For the full diversity advantage, rank criterion is more significant code design principle. Until the rank criterion is satisfied it is not necessary to try for the optimization of determinant criterion. The rank criterion is applicable for space time block codes to show that they will provide full rank. As a simple example, suppose the Alamouti scheme having the pairs of distinct codewords in the following form:

$$X_1 = \begin{bmatrix} x_{1,1} & x_{1,2} \\ -x_{1,2}^* & x_{1,1}^* \end{bmatrix}. \quad (8.3.1)$$

$$X_2 = \begin{bmatrix} x_{2,1} & x_{2,2} \\ -x_{2,2}^* & x_{2,1}^* \end{bmatrix}. \quad (8.3.2)$$

The corresponding matrix A is given by

$$A = \begin{bmatrix} |x_{1,1}|^2 + |x_{2,1}|^2 & 0 \\ 0 & |x_{1,1}|^2 + |x_{2,1}|^2 \end{bmatrix}. \quad (8.3.3)$$

Here, the code words are distinct so that the rank of above matrix is two and hence full diversity is achieved.

For the Alamouti scheme, the fourth power of the minimum Euclidean distance of the underlying signal constellation gives the minimum of the determinants among all codeword pairs and that can be very small. So, the benefits of determinant criterion can't be taken by this code and it can't provide coding gain.

Let us consider, 4x4 orthogonal block code, the pair of code words have the following form:

$$X_1 = \begin{bmatrix} x_{1,1} & x_{1,2} & x_{1,3} & x_{1,4} \\ -x_{1,2} & x_{1,1} & -x_{1,4} & x_{1,3} \\ -x_{1,3} & x_{1,4} & x_{1,1} & -x_{1,2} \\ -x_{1,4} & -x_{1,3} & x_{1,2} & x_{1,1} \end{bmatrix}. \quad (8.3.4)$$

$$X_2 = \begin{bmatrix} x_{2,1} & x_{2,2} & x_{2,3} & x_{2,4} \\ -x_{2,2} & x_{2,1} & -x_{2,4} & x_{2,3} \\ -x_{2,3} & x_{2,4} & x_{2,1} & -x_{2,2} \\ -x_{2,4} & -x_{2,3} & x_{2,2} & x_{2,1} \end{bmatrix}. \quad (8.3.5)$$

The corresponding matrix A is written as:

$$\left( (x_{1,1} - x_{2,1})^2 + (x_{1,2} - x_{2,2})^2 + (x_{1,3} - x_{2,3})^2 + (x_{1,4} - x_{2,4})^2 \right) I_4$$

From this, it is obvious that the transmitted symbols are different (at least one of them), the rank of the matrix is four and hence full diversity is achieved. But the minimum determinant can be very small that means there is no coding gain as in Alamouti scheme. [49]

## 9. Simulation and Results

Simulation is performed using MATLAB that provides good platform for simulating the technical problems. Under the consideration of BPSK modulation, the performance of Alamouti scheme is evaluated along with the theoretical values and MRRC technique for the comparison. Also, MIMO capacity, capacity for Rayleigh fading channel and error probability is plotted with respect to the different values of SNR.

### MIMO Capacity

Channel capacity is the ultimate data transmission rate. MIMO system offers potential spatial diversity that can be exploited by Space-time codes. The channel capacity of a communication link can be increased significantly by utilizing spatial diversity. The MIMO system helps to improve the spectral efficiency and link reliability. Shannon's capacity provides the information about the maximum possible rate of information transmission through a given channel in the presence of noise. The rate of information transmission depends upon the band width, the signal level and the noise level. The figure below shows the MIMO capacity along with the Shannon's capacity. The capacity curves are shifted upwards as the number of antennas increases.

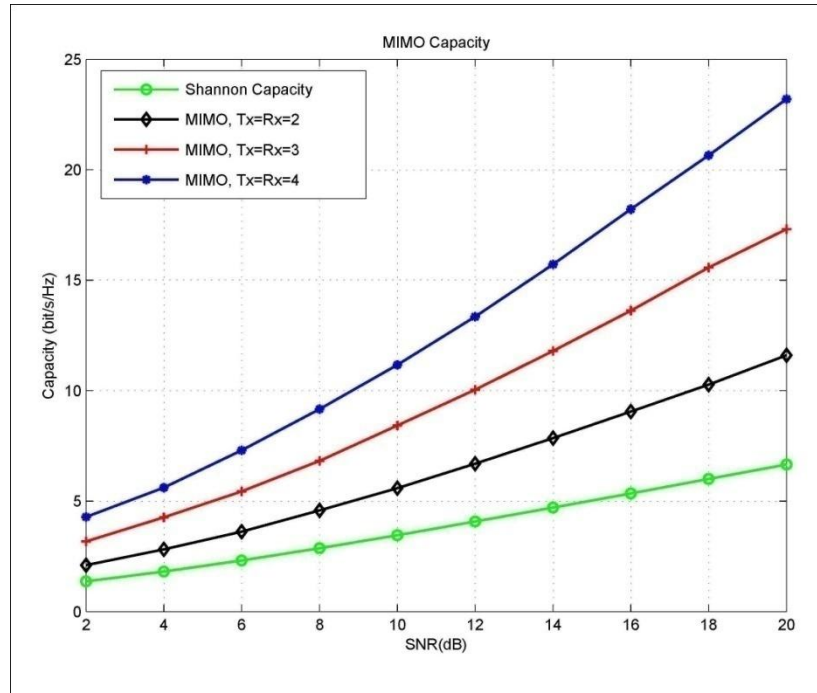


Figure: Plot for MIMO and Shannon's Capacity

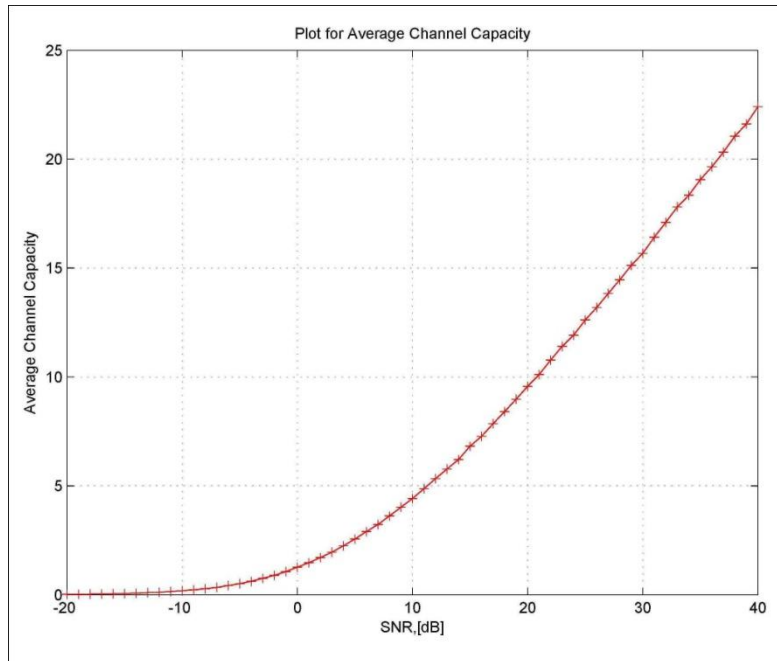


Fig: Average Channel Capacity over Rayleigh Fading Channel.

In Rayleigh fading, it is assumed that the magnitude of signal passing through transmission medium varies randomly or undergoes fading according to the Rayleigh distribution. The Rayleigh fading channel shows better performance for long distance communication.

It is very difficult to obtain small value of error probability in wireless channel due to multipath fading. Receive or transmit diversity is used for the small value of error probability.

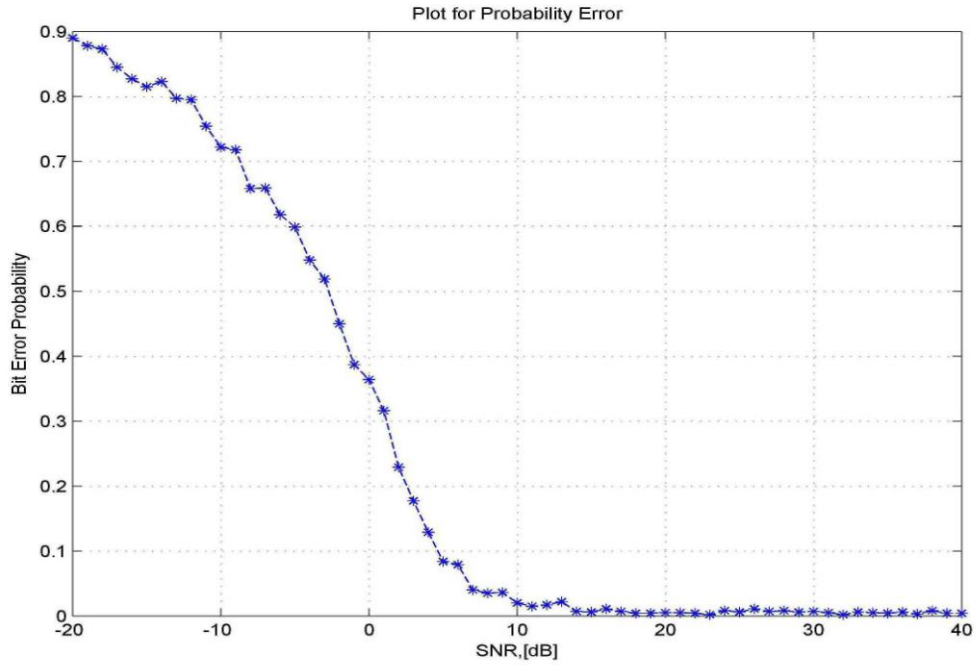


Fig: Error probability of Alamouti 2x2 over Rayleigh fading channel

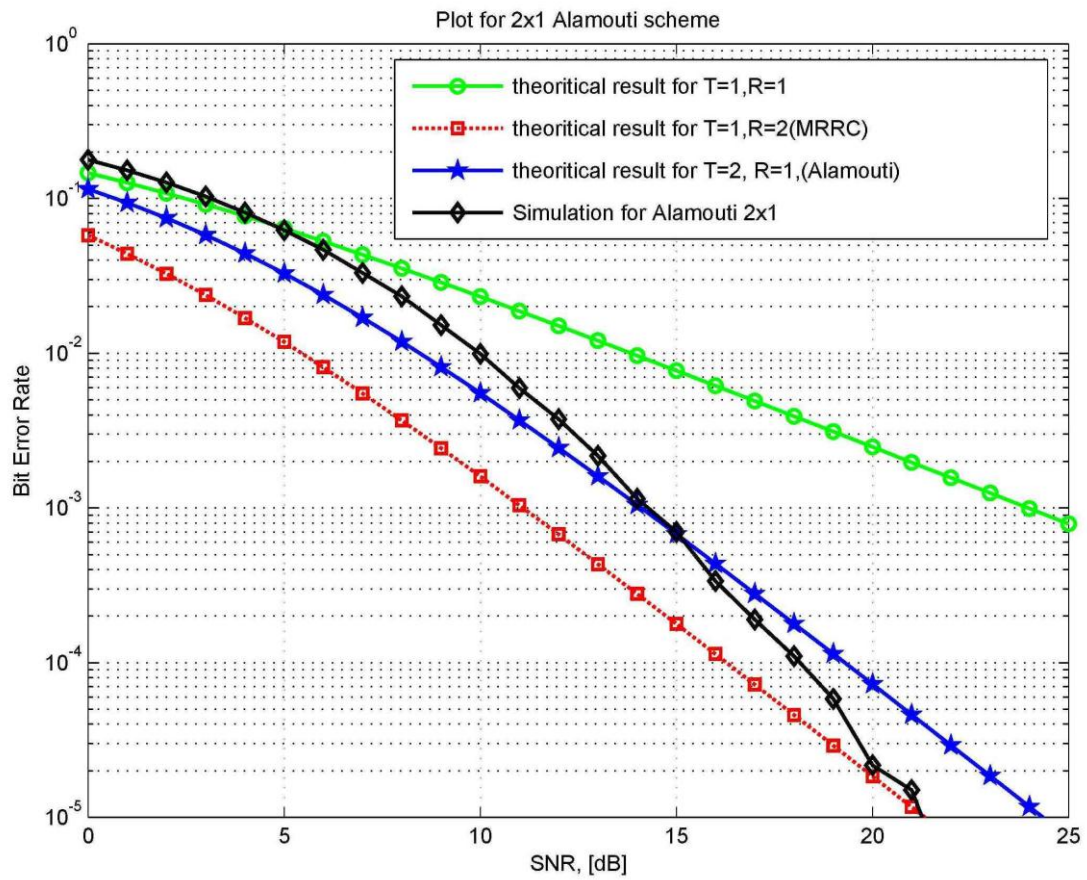


Fig: Comparisons of Alamouti 2x1 (T=2, R=1), SISO (1x1) and MRRC (1x2)

Alamouti 2×1 performance seems to be identical to the theoretical value but the performance of 1×2 (MRC) is better due to the receive diversity.

Here, it is better to make some comparison between the MRRC and Alamouti scheme. In terms of power, if two transmit antennas are used to transmit data, more power is radiated from the transmitter than the receiver deploying single antenna. Since their slopes are identical, the diversity order that is achieved from Alamouti 2×1 is same to that of MRRC 1×2. But it is seen that MRRC leads by 3dB. The total power remains the same for both Alamouti and MRRC technique if the power radiated from the two transmit antennas is half of total power. If the power limitation is not considered then same error performance is observed between MRRC and Alamouti scheme.

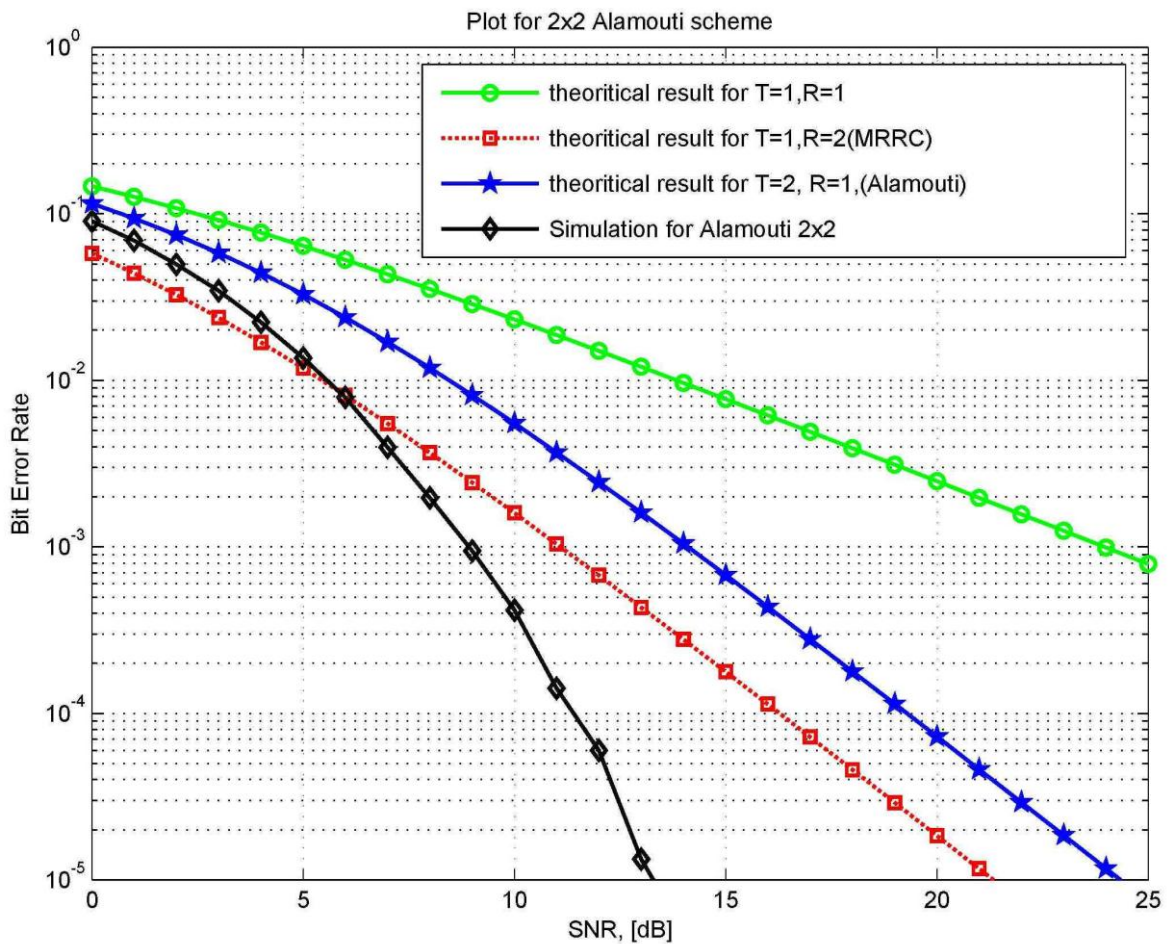


Fig: Comparisons of Alamouti 2×2 (T=2, R=2), SISO(1×1) and 1×2(MRRC)

Here, Alamouti 2×2 Scheme shows better BER performance than 1×1(SISO), 1×2 (MRC) and Alamouti 2×1. BER gets reduced as the number of antenna increases in the receiving side.



## 10. Conclusion and Future Works

### 10.1 Conclusion

Throughout this thesis work, we have tried to express and explain the basic concepts with the knowledge required for the understanding of space-time coding. For this, we have also focused our study on the theoretical part as much as possible in the topics related to space-time coding.

In the initial part of simulation, the capacity of MIMO system with Shannon's capacity is simulated. It is shown that the MIMO capacity increases as the number of antenna increases. From this we can say that diversity can increase the channel capacity. In the next part of simulation, performance of Alamouti code is evaluated. The performance of Alamouti  $2 \times 1$  seems to be identical to the theoretical value while the performance of  $1 \times 2$  (MRC) is better due to the receive diversity. In comparison with  $1 \times 1$  (SISO),  $1 \times 2$  (MRC), Alamouti  $2 \times 1$  and Alamouti  $2 \times 2$ , Alamouti  $2 \times 2$  has shown better bit error performance. If more antennas are employed in receiving side, Bit error rate also gets reduced. From this we conclude that receive diversity contributes for the reduction of bit error rate (BER) and hence Alamouti scheme is good coding technique for the data transmission by minimizing the error.

### 10.2 Future Works

We noticed that employing more antennas in both transmitting side and receiving side can increase the system performance in term of bit error rate and capacity. Further works can be carried out with the increased diversity for better system performance with the proper consideration of system complexity and power control. Also, MIMO-OFDM system may be the area of study for high spectral efficiency and increased throughput.

## References

- [1] T.M. Duman, A.Ghrayeb, "Space-Time Block Codes," in Coding for MIMO Communication Systems, John Wiley and Sons, 2007, PP.71-72
- [2] T.M. Duman, A.Ghrayeb, "Need for MIMO Systems," in Coding for MIMO Communication Systems, John Wiley and Sons, 2007, PP.1-2
- [3] Z. Liu, G.B.Giannakis, B. Muquet, S.Zhou "Space-Time Coding for Broadband Wireless Communication," USA, 2000.
- [4] M. S. Ullah, M. J. Uddin, " Performance Analysis of Wireless MIMO System by Using Alamouti's Scheme and Maximum Ratio Combining Technique" , Vol No. 8, 2011.
- [5] P.M. Chidambara Nathan, "History of wireless communications" in Wireless Communications, PHI Press, PP.1-2
- [6] "History and Evolution of the mobile and Wireless Communications Business Models" in Globalization of mobile and Wireless technology, Springer, PP.48-49.
- [7] Sarkar, Tapan K. Salazar-Palma, Magdalena Mokole, Eric L, "Multiple-Input-Multiple-Output (MIMO) Antenna Systems," in Physics of Multi antenna Systems and Broadband Processing, Hoboken, NJ, USA ,2008, pp 172-173.
- [8] Tech Target, (November, 2004). SIMO single input multiple output [online], Available: <http://searchmobilecomputing.techtarget.com/definition/SIMO>.
- [9] Y. -W. Peter Hong, Wan-Jen Huang, C. -C. Jay Kuo, "review of wireless communication and MIMO technique," in Cooperative Communications and Networking, springer, 2009, pp-26-35.
- [10] Erik G larsson, petre stoica, "MIMO information Theory," in Space-Time Block Coding for Wireless Communication, Cambridge UK, Cambridge University Press, 2008. pp 4-7
- [11] AvClaude Oestges, Bruno Clerckx, "Brief history of Array processing," in MIMO wireless communications: from real-world propagation to space-time code design, Academic press, 2007, pp 1-3
- [12] Ilan Hen, "MIMO Architecture for Wireless Communication," Intel technology Journal, Volume 10 Issue 02 , May 15, 2006.
- [13] A.Neubauer, J.Freudenberger, V.Kunn, "Information Theory" in Coding theory, Algorithms, Architectures and Applications, John Wiley and Sons, 2007, PP.17
- [14] Berta Delango, "Performance Evaluation of Simple Space-Time Block Coding on MIMO Communication System," School of Computer Science Physics and Mathematics, Linnaeus University, Vaxjo, Feb. 2010



- [15] Sarkar, Tapan K. Salazar-Palma, Magdalena Mokole, Eric L., "Channel Capacity From a Maxwellian Viewpoint," in *Physics of Multiantenna Systems and Broadband Processing*, Hoboken, NJ, USA: Wiley, 2008. pp 114-116
- [16] Thomas M. Cover, Joy A. Thomas, "Entropy Relative Entropy and Mutual Information," in *Elements of Information Theory*, New York, John Wiley & Sons, Inc., 1991. pp12-23.
- [17] Erik G larsson, petre stoica, "MIMO information Theory," in *Space-Time Block Coding for Wireless Communication*, Cambridge UK, Cambridge University Press, 2008. pp 22-25.
- [18] Jankiraman, Mohinder, "Space-Time Block Coding," in *Space-Time Codes and MIMO Systems*, Norwood, MA, USA , Artech House , July 2004. pp 103-132.
- [19] On MIMO systems and Adaptive array for wireless communication, Mattias Wennstrom, Uppsala University, 2002 .pp 35 - 36.
- [20] Ghrayeb, Ali Duman, Tolga M, " Capacity and Information Rates of MIMO Channels," in *Coding for MIMO Communication Systems*. Hoboken, NJ, USA, Wiley, 2008 pp 45-50.
- [21] A.Neubauer, J.Freudenberger, V.Kunn, "Information Theory" in *Coding theory, Algorithms, Architectures and Applications*, John Wiley and Sons, 2007, PP.17
- [22] David Gesbert, Mansoor Shafi, Da-shan Shiu, Peter J. Smith, Ayman Naguib, "From Theory to Practice: An Overview of MIMO Space–Time Coded Wireless Systems", *Selected Areas In Communications*, Vol. 21, NO. 3, APRIL 2003.
- [23] R.L.Freeman, "Radio propagation And Fading Effects" in *Radio System Design for Telecommunications*, John Wiley and Sons, 2007, PP.19-22
- [24] T.M. Duman, A.Ghrayeb, "Fading Channels and Diversity Techniques" in *Coding for MIMO Communication Systems*, John Wiley and Sons, 2007, PP.20-22
- [25] C.B. Dietrich, K. Dietze, J. R. Nealy, W. L. Stutzman, "Spatial, polarization and Pattern Diversity for Wireless Handeld Terminals" 2001
- [26] Vikrant Vij, "Diversity" in *Wireless Communication*, University Science Press, New Delhi, 2010, PP.24
- [27] D.Popovic, Z. Popovic, "Multibeam Antennas with Polarization and Angle Diversity," Vol NO, 8, 2002
- [28] M. Kar, P. Wahid, "Two-Branch Space and Polarization Diversity Schemes for Dipoles"
- [29] A. G. Dabak, S. Hosur, T.Schmidl, C.Sengupta "A comparison of the open loop transmit diversity schemes for third generation wireless Systems," Vol NO. 1, 2000
- [30] M.Jankiraman, "MIMO Wireless Channel" in *Space-Time Trellis Codes and MIMO Systems*, Artech House, 2004, PP.15-17

- [31] P.M. C. Nathan, "Interpretation of diversity measurements, and extraction of statistics" in *Wireless Communications*, PHI Press, PP.111-113
- [32] A. Neubauer, J. Freudenberger, V. Kunn, *Space-Time Codes in "Coding theory, Algorithms, Architectures and Applications"*, John Wiley and Sons, 2007, PP.229
- [33] G.B. Giannakis, Z. Liu, Xiaoli, MA, S. Zhou "Fundamentals of ST wireless Communications" in *Space Time Coding For Broadband Wireless Communications*, PP.24
- [34] Vahid Tarokh, Hamid Jafarkhani, A. Robert Calderbank, "Space-Time Block Coding for Wireless Communications: Performance Results," *Selected Areas In Communications*, Vol. 17, NO. 3, March 1999.
- [35] Branka Vucetic, Jinhong Yuan, "Space-Time Block Code," in *Space-Time Coding*, University of Sydney and University of New South Wales, Australia, John Wiley & Sons, England, 2003. pp 91-113
- [36] Z. Liu, G. B. Giannakis, B. Muquet, S. Zhou, "Space-Time Coding for Broadband Wireless Communications," *Wireless Communication and Mobile Computing*, Volume 1, pp 35-53, January/March 2001 [DOI: 10.1002/1530-8677(200101/03)1:1<35]
- [37] T.M. Duman, A. Ghayeb, "Space-Time Coding -Basic ideas" in *Coding for MIMO Communication Systems*, John Wiley and Sons, 2007, PP.37-38
- [38] N. AL-Dhahir, A.R. Calderbank, S.N. Diggavi, "Space-Time Coding Principles" in *Space Time Coding for Wireless Communications: Principles and Applications*, Kluwer Academic publishers, PP.20
- [39] ] M. Wennstrom, "Space time coding" in *On MIMO Systems and Adaptive Array for Wireless Communication*, Uppsala University, 2002, PP.43
- [40] Jankiraman, Mohinder, "Space-Time Block Coding," in *Space-Time Codes and MIMO Systems*, Norwood, MA, USA, Artech House, July 2004. pp 80-82.
- [41] Space time coding, Branka Vucetic University of Sydney, Australia Jinhong Yuan University of New South Wales, Australia, John Wiley & Sons Ltd, 2003 pp 91-104 ]]
- [42] T.M. Duman, A. Ghayeb, "Space-Time Block Codes" in *Coding for MIMO Communication Systems*, John Wiley and Sons, 2007, PP.71-75
- [43] B. Vucetic, J. Yuan, "Space-Time Block Codes", in *Space-Time Coding*, John Wiley and Sons, 2003, PP.91-104
- [44] L. Hanzo, O. Alamri, M. El-Hajjar, N. Wu, "Space-Time Block Code Design using Sphere packing," in *Near-Capacity Multi-Functional MIMO Systems: Sphere-Packing*, John Wiley & Sons, 2009, pp.69-72.

- [45] M.Jankiraman, "Space-Time Trellis Codes" in Space-Time Codes and MIMO System, Artech House, PP.80-87.
- [46] M.Jankiraman, "Space-Time Trellis Codes" in Space-Time Trellis Codes and MIMO Systems, Artech House, 2004, PP.103
- [47] T.M. Duman, A.Ghrayeb, "General Space-Time Trellis Codes" in Coding for MIMO Communication Systems, John Wiley and Sons, 2007, PP.94-95
- [48] T.M. Duman, A.Ghrayeb, "Space-Time Trellis Codes" in Coding for MIMO Communication Systems, John Wiley and Sons, 2007, PP.93-97
- [49] T.M. Duman, A.Ghrayeb, "Basic Space-Time Code Design Principles" in Coding for MIMO Communication Systems, John Wiley and Sons, 2007, PP.100-101
- [50] T.Brown, E.D. Carvalho, P.Kyritsi, "Maximum Likelihood Receiver" in Practical Guide to the MIMO Radio Channel, John Wiley and Sons, 2012, PP.78-79
- [51] G.wade, "Maximum Likelihood Decoding" in Signal Coding and Processing, Cambridge University Press, 1994, PP.184-185



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