Real-time wireless communication with per-packet deadlines

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Abstract

The last decades’ tremendous advances in wireless communications have been driven mainly by personal communications. Radio resource allocation mechanisms for optimizing key metrics, such as average throughput and delay, for such traffic are by now rather well-developed. However, with the increased interest in wireless machine-to-machine communication, e.g. for industrial control or monitoring of large-scale infrastructures, new challenges emerge. The performance of an estimator or closed-loop control system that operates over an unreliable wireless network depends on the full latency and loss distributions and not only on their averages. For these applications, more suitable performance metrics are per-packet guarantees on latency and reliability (on-time delivery).

This thesis studies optimal forwarding of deadline-constrained traffic over lossy multi-hop networks. We assume a routing topology in the form of a directed graph with packet loss on links described by finite-state Markov chains, and focus on a single transient packet scenario. Two problems are considered: maximizing the probability that packets are delivered within their specified deadline; and minimizing the expected energy cost with a guaranteed probability of on-time delivery.

The first problem can be formulated as a finite-horizon Markov decision process (MDP), while the second problem is a finite-horizon constrained Markov decision process (CMDP). A general dynamic programming framework that solves a weighted sum of reliability and energy maximization problem is proposed. The maximum deadline-constrained reliability problem is solved by studying a simplification of the general dynamic programming. The minimum energy optimal policy is a random selection between two deterministic and computable forwarding policies, each of which can be found via a dynamic programming framework. Particular instances with Bernoulli and Gilbert-Elliot loss models that admit numerically efficient solutions are discussed. Finally, we show the application of the technique for a co-design of forwarding policies and controller for wireless control. Based on the recent result on the monotonicity of the optimal control loss, the problem of minimum control loss and the problem of minimum energy cost with a guaranteed control performance can be solved optimally.
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Machine-to-machine (M2M) communication paves the way for a broad range of life-changing applications, including smart buildings [1], smart grids [2, 3], wireless health monitoring [4–6], wireless industrial automation and control [7–9]. As one of the underlying technologies, a large research effort has been put in wireless sensor networks in which many low-cost and low-power sensor nodes are distributed over a certain area to monitor the environment and interconnected using wireless communication to be able to convey their information to a destination.

While wireless sensor networks were proposed more than a decade ago, the number of deployments is still limited. One of the main obstacles is the lack of mechanism that ensures timely data transmission over multiple hops. Current studies mainly utilize protocols adapted from the Internet and mobile (cellular) networks, and concentrate on minimizing average delay or maximizing expected network lifetime. Average delivery delay, however, is not a good metric in applications where failure of on-time packet delivery may lead to significant performance loss. The observation is illustrated by the hypothetical example in Figure 1.1. Two schemes provide average delay of 8 and 9 time slots separately, but the scheme with longer average delay has a small delay variation. Suppose that as long as the packet is delivered before the deadline, its contribution to the system performance is the same. Then, if the deadline is 11 time slots, the scheme with the larger average delay is preferable.

While this thesis focuses on real-time communication, there are also other critical issues that need to be addressed to enable wide-spread use of wireless M2M communications. This includes security in smart grids [10], privacy and trustworthiness in wireless health monitoring [4], delay-tolerant packet forwarding in body sensor networks [6], etc. However, we focus completely on real-time wireless communication with per-packet deadlines, and study two important metrics: the probability of on-time delivery and the expected energy cost.
Figure 1.1: A hypothetical example of the importance of real-time communication with per-packet deadlines. Two schemes provide average delay of 8 and 9 time slots, but with different delay variance. The application demands a per-packet delivery delay of 11 time slots. Hence, the scheme with small average delay will perform worse due to the longer tail of its delay distribution.

1.1 Applications

We next discuss two examples, wireless industrial process control and smart grids, where per-packet delivery deadline is the more natural and appropriate metric for real-time wireless communication.

1.1.1 Wireless industrial process control

The integration of wireless networks and control can reduce costs by eliminating the need for installing kilometers of cables, which are not only costly, but also error-prone. Wireless communication also offers greater flexibility, as sensors can easily be added and removed and placed in areas that are not easily reached by cables. This potential of lower deployment cost and high deployment flexibility, has spawned a number of recent standards for industrial wireless, including WirelessHART [11], ISA100 [12], IEEE 802.15.4e [13] and IETF routing protocol for low power and lossy networks (RPL) [14].

In traditional digital control, sensor measurements are sampled periodically and sent over a network to a controller node. Both message latency and data loss degrade performance. It makes sense to require the sampled data to be successfully delivered before the next sample time; otherwise, old packets that are of relatively little value for the controller would compete for networking resources with packets that contain more recent and relevant data. Hence, an appropriate objective is
to maximize the probability that the packet is delivered before the next sampling
time. We will show a concrete example of the co-design of wireless control system
and real-time wireless communication with per-packet deadlines in Chapter 6.

1.1.2 Smart grid

The next-generation electric power system, often called the smart grid, incorporates
a significant amount of renewable energy sources such as wind, hydro, solar, etc.
Many of the power sources will be small, such as a solar panel installed on the
roof of a home. Due to the large number of small energy produces, along with
their variability (e.g. depending on if the source is solar or wind), it is necessary to
monitor the power network in real-time and coordinate producers and consumers.
Many sensors are deployed over the power network to monitor the voltage, power
and frequency. These data should be delivered on time to the control stations
to balance the power supply and demand over time or to detect attacks or other
anomalies. In such a system, long delays and the loss of packets with critical
information can have significant impact on the stability of the system.

1.2 Challenges

Real-time wireless M2M communication is not an easy task due to the following
challenges:

- **Multi-hop connectivity**: Unlike cellular networks, low-power radios usually
  need several hops to cover the distance from source to destination.

- **Lossy and bursty links**: The quality of a wireless link varies over time due
to several factors, including natural channel variations and external interfer-
ence. Modern communication techniques can provide practically error-free
transmission using the appropriate power control, coding and modulation,
but such techniques are hard to implement in low power and low cost wireless
sensor nodes. Although erasure events on links can be considered independent
in some scenarios (e.g. static networks with long inter-packet transmission
times), packet losses are also typically correlated in time [15, 16]. Hence, a
good packet transmission strategy may also need to account for packet cor-
relations, increasing the complexity even further. Several recent papers show
that heuristics that account for channel burstiness become important for ob-
taining strong routing performance [17, 18].

To provide real-time communication with per-packet deadlines is even more
challenging. Most existing schemes focus on the average delay of packet delivery,
and rely on numerous independence assumptions on the packet arrival processes and
packet loss processes. In this way, the average delay can be computed by taking
expectations on each random variable, and many optimal and heuristic protocols
have been proposed. For example, routing protocols for low-delay routing are often
Figure 1.2: A two-hop network. The numbers on the links denote the average success probability. The link expected transmission count (ETX) is simply the inverse of the success probability, and the path ETX is the sum of the link ETX on the path. In this example, the upper path has a smaller ETX value of 10 transmissions than the lower path of 11 transmissions.

Figure 1.3: The cumulative delay distribution of transmitting on two single paths all the time and the deadline-aware optimal policy. The large ETX path has more packets delivered before the deadline than the small ETX path when the deadline is smaller than 10 transmissions. However, the optimal policy is more than a selection between two single paths. It is time-dependent and switches between the two paths. The computation of the optimal policy and its performance is the main focus of this thesis.

based on the expected transmission count (ETX) metric proposed in [19]. The ETX of a link is the expected number of transmissions required to successfully transmit the packet to the next hop node. Since the losses from different links are
1.2. Challenges

assumed to be independent, the ETX of a path is the sum of the ETX of each link in the path, and the ETX-optimal path can be found by solving a classical shortest path problem. However, the shortest ETX path does not provide any guarantees of on-time delivery of deadline-constrained packets. To illustrate this, consider the network shown in Figure 1.2. There are two paths from source to destination: the upper path ($S \rightarrow R_1 \rightarrow D$) has a smaller ETX value, on which the average packet delivery delay is 10 transmissions, while the lower path ($S \rightarrow R_2 \rightarrow D$) is a path with on average 11 transmissions. However, Figure 1.3 shows that the large ETX path is more successful in delivering packets before the deadline than the small ETX path, when the deadline is smaller than 10 transmissions. Therefore, minimum average delay schemes could perform badly in real-time wireless communication with per-packet deadlines. The optimal policy should depend on the deadline of the packet. The computation of this optimal policy and its performance are the main focus of Chapter 4 in this thesis.

Moreover, the optimal policy does not choose only one of the paths based on the deadline of the packet. It also changes the transmission path according to the remaining time to the deadline. Let us consider an example shown in Figure 1.4. The relay path has a reliable connection, while the direct path is lossy. Any good policy with deadline larger than two would choose the relay path. But if a packet has experienced multiple losses and remains at the source when there is only a single slot left until the deadline, the transmission decision will be to try the less reliable direct path. A similar but not so obvious phenomenon can be observed for the network topology in Figure 1.2. The optimal policy switches between the two paths based on the remaining time to deadline, and obtains a better performance especially when the deadline is 9 and 10 shown in Figure 1.3.

![Figure 1.4: The longer path consists of two very reliable links. However, a packet with deadline 1 will be transmitted over the unreliable direct path. Moreover, if the link between source and relay has correlated packet losses, the packet may be transmitted directly on the unreliable path after an unsuccessful transmission from source to relay.](image)

The bursty nature of links further complicates the optimal policy. In the same example of Figure 1.4, suppose the link between source and relay is bursty and correlated in time. It means that the success transmission probability after a failed
transmission is very low. A good strategy is then to forward the packet on the unreliable path after an unsuccessful transmission from source to relay.

Energy efficiency is also critical, since sensor nodes are often battery-powered with life-time targets in the order of years. A natural objective is to minimize the energy cost. However, a lower energy cost usually comes with an increase in the packet delivery delay and loss. The characterization of the tradeoff among these three metrics is interesting and difficult.

1.3 Problem formulation

In this thesis, we consider deadline-constrained packet forwarding over a multi-hop wireless network with lossy and bursty links modeled by Markovian models. Each device is equipped with a half-duplex radio transceiver, hence cannot transmit and receive simultaneously. Communication is slotted, and at each time slot the transmission of a single packet and its associated acknowledgement is allowed.

We focus on the transmission of a single transient packet through the network. In contrast to independent and saturated traffic sources often considered in personal communication, M2M communications are typically lightly loaded and traffic is transient. The majority of control design techniques, for instance, rely on periodic sampling with sampling times longer than the minimal latencies. Moreover, this single transient packet scenario can form the building block for supporting periodic traffic with per-packet deadline smaller than the packet generation period. The single transient packet assumption reduces the complexity of the problem to some extent, as the packet will not encounter queuing delay and interferences from other transmissions in the network.

In this thesis, we first address the deadline-constrained maximum reliability packet forwarding problem

\[
\text{maximize } \pi R^\pi, \quad (1.1)
\]

where \( \pi \) is the packet forwarding policy that determines if a node should forward a received packet or drop it, and to which node it should attempt to transmit, and \( R^\pi \) is the deadline-constrained reliability, \( R \), under policy \( \pi \).

The policy that maximizes the probability that a packet is delivered within its deadline makes full use of all available transmission opportunities as will be shown in Chapter 4. It does not account for the possible energy-inefficiency of always transmitting despite disadvantageous channel states. In order to understand the tradeoff between deadline, reliability and energy cost, we also address the minimum transmission energy cost problem with a requirement of deadline-constrained reliability

\[
\text{minimize } C^\pi \\
\text{subject to } R^\pi \geq R_{\text{req}}, \quad (1.2)
\]
where $C^\pi$ is the deadline-constrained energy cost $C$ under policy $\pi$, and $R_{req}$ is a given deadline-constrained reliability requirement.

1.4 Outline and contributions

The outline of the thesis is as follows. In Chapter 2, we present an overview on real-time wireless communication with per-packet deadlines. Our model and precise Markov decision process formulations for the two problems together with a general dynamic programming framework are detailed in Chapter 3. Chapter 4 and Chapter 5 are devoted to the deadline-constrained maximum reliability forwarding problem and the minimum energy forwarding problem with deadline and reliability constraints, respectively. An application of the real-time scheme with per-packet deadlines in the co-design of a networked control system is given in Chapter 6. Finally, we summarize the thesis and discuss the limitations and future directions.

1.4.1 Contributions

The thesis is built on the following publications. Chapter 3 and Chapter 4 are mainly based on


A preliminary conference version paper appears as


Chapter 5 is covered by


Chapter 6 includes relevant part of the materials from

B. Demirel, Z. Zou, P. Soldati, and M. Johansson, “Modular design of jointly optimal controllers and forwarding policies for wireless control,” Submitted for journal publication (under review).

A preliminary conference version of the paper appears as


Lastly, Section 4.3 is extracted from the following publication. It covers the author’s contribution in this publication.

Chapter 2

Related work

In this chapter, we review recent literature on real-time wireless communication with per-packet deadlines. We progress from the simplest scenario of a single data flow transmitted across a single link to the more complex scenario of multiple data flows over multi-hop networks. We focus on techniques that can solve the problem to optimality under various assumptions and restrictions on traffic arrival model, link model and deadline model. With increased complexity introduced by multiple hops and multiple flows, more restrictions have to be put in place to make the problem tractable.

This chapter is organized as follows. In Section 2.1, we introduce real-time wireless communication with per-packet deadlines and describe the performance metrics and assumptions used in the surveyed literature. In Section 2.2, we focus on the rate adaptation technique to provide minimum energy packet forwarding with a hard real-time per-packet constraint, while in Section 2.3 we consider the problem of maximizing timely-throughput. Finally, in Section 2.4 we discuss the extensions of these two problems in multi-hop networks.

2.1 Introduction and general assumptions

Figure 2.1: Deadline-constrained packet transmission. A delivery of packet after the deadline is regarded as a failure.

In real-time wireless communication with per-packet deadlines, the delivery of
a packet after the deadline is regarded as a failure. On the other hand, we don’t distinguish the reward for the delivery time of packets as long as they are received before the deadline. Hence, the statistics of successfully delivered packets, for instance their average delay, are of no interest.

### 2.1.1 Performance metrics

The first objective is to impose a hard real-time constraint on packet delivery. The delivery of a packet after its deadline is considered as a failure. Our target is to ensure that all the packets are transmitted to the destination before the deadline. For a system for which all the data can be delivered before their deadlines, we also consider minimizing the energy cost while all the packets are still received before their deadlines.

![Figure 2.2: Packet delivery delay distribution.](image)

For saturated or lossy systems, it is not possible that all the packets can be transmitted successfully before their deadlines, see Figure 2.2. In such case, a performance measure, timely-throughput (proposed in [20]), that incorporates both throughput and packet deadline is used. It is defined as the ratio of the packets that are successfully delivered to the destination before their deadlines. Therefore, the natural objective is to maximize the timely-throughput. Note that the timely-throughput has the same meaning as the deadline-constrained packet forwarding reliability.
2.1.2 Coding, modulation and packetization

A reliable transmission is possible by an appropriate channel coding and modulation method, provided that the rate of communication is below the channel capacity. Since the transmission power determines the channel capacity, an arbitrary number of bits can be transmitted reliably with different transmission power in a time interval. A tradeoff between the transmission power and the number of bits can be exploited to provide a minimum energy cost transmission strategy with a guaranteed packet delivery delay bound.

On the contrary, a packet is formed by a fixed number of bits, e.g., in wireless sensor networks and computer communication networks, due to the fact that the physical layer has a fixed coding and modulation, and usually a fixed transmission power. In general, the transmission of a packet is not reliable, but can be improved by retransmissions, at the expense of a longer delivery delay.

2.1.3 Traffic

Figure 2.3: Different packet arrival and deadline patterns. The letter on the arrow denotes the packet. The “Dynamic arrival” shows both the deterministic future packet arrivals and an realization of stochastic packet arrivals. In the “Single relative deadline”, all the packets have the same amount of time for transmission after the packet arrives.

There can be multiple flows to be served in the wireless network with per-packet deadlines. The packet arrival model of each flow can be
• Packets queued in the beginning with no future arrivals,
• Deterministic future packet arrivals,
• Stochastic packet arrivals described by an independent and identically distributed (i.i.d.) random process or a leaky-bucket model.\(^1\)

The deadline model includes the case where all the packets have the same absolute deadline. A slight relaxation is that all the packets may have different deadlines, but the duration between the arrival time and the deadline time is the same. We call this the same “relative deadline”. Lastly, in the most general case, packets have an arbitrary deadline. An illustration of a packet arrival and deadline patterns is shown in Figure 2.3.

### 2.1.4 Topology

The topology can be distinguished into three main types:

• Single link: It models a point-to-point communication, such as long-range telecommunication and the connection between a base station with only a single mobile terminal.

• Star network: Examples include mobile cellular network where the central node is the base station, and wireless LAN where the central node is the access point (AP).

• Multi-hop network: When the physical communication range is too long and/or the device-to-device communication range is small, the packet delivery is extended to a multi-hop network, e.g., wireless sensor network and ad hoc network.

### 2.1.5 MAC and scheduling

An easy way to provide a guaranteed per-packet delivery delay is to have a guaranteed channel access. Most of the work utilize the simple time division multiple access (TDMA) method, since it can eliminate randomization. In this scheme, time is slotted, and the deadline is expressed in number of time slots. Probabilistic medium access control such as carrier sense multiple access (CSMA) is rarely used.

The scheduling policy decides on which link to transmit the packet at each time slot before the packet’s deadline. When there are multiple flows, the scheduling policy should also determine which flow to serve.

\(^1\)In the leaky-bucket traffic model, the total arriving data in any duration \(\tau\) is less than or equal to \(\delta + \tau \rho\), where \(\delta\) is the burst and \(\rho\) is the constant incoming rate.
2.2. Rate adaptation for hard real-time constraints

2.1.6 Channel models
The links are modeled differently. The basic assumption is that the channel state of the link remains static at least for one time slot. In the simplest scenario, the transmission on a certain link is assumed to be fully reliable. However, in most of the work, the link is considered as lossy, and the channel state can be i.i.d. or Markov process modulated across different time slots. The success transmission probability on a link is determined by the channel state, and this probability can be increased with a larger transmission power.

2.1.7 Information on channel states
There are several approaches regarding what is assumed to be known about the channel state. The most prominent in the literature are the following:

(a) the channel state of the current time slot is known before the transmission;

(b) only the channel state at the previous time slot is known before transmission.

Hence, the current channel state is computed by the conditional probability of the Markovian chains of the channel loss model.

In the above two assumptions, we assume that link state estimators can continuously track the link evolution. When link state estimators are not available, the channel state is learned through probing at the cost of time and energy, or through ACK or NAK feedback messages from the actual packet transmissions.

When the communication is extended to a multi-hop network, most work assume only partial knowledge of the channel states in the sense that a node can only access the channel states of its own outgoing links. The reason is that it is unrealistic and costly to maintain up-to-date channel states of all links in the network.

2.2 Rate adaptation for hard real-time constraints

2.2.1 The basic idea
We consider a number of bits of information that needs to be transmitted over a wireless channel before a hard deadline. The Shannon capacity reveals that we can do so by using the appropriate rate specification. For example, the channel capacity of an additive white Gaussian noise (AWGN) channel [21] is

\[ C = W \log_2(1 + \frac{P}{NW}), \]  \hspace{1cm} (2.1)

where \( C \) is the channel capacity (measured in bits per second), \( W \) is the bandwidth of the channel, \( P \) is the transmission power (measured in watts), and \( N/2 \) is the noise spectral density (measured in watts/Hz). The channel capacity is the maximum data rate that can be achieved for an error-free transmission. Suppose
an ideal channel coding is used to approach the channel capacity $C$. Hence, the energy cost per bit for an error-free transmission can be computed through the capacity equation (2.1), and it is a convex and increasing function with respect to data rate $C$,

$$
\mathcal{E}(C) = \frac{P}{C} = NW \cdot \frac{2^{\frac{C}{C}} - 1}{C}.
$$

(2.2)

An example of the relation between energy cost and transmission rate is shown in Figure 2.4.

![Figure 2.4: The bits per second versus energy cost per bit due to Shannon capacity equation. The energy cost is a convex and increasing function of the data rate. The curves show the relation for different channel qualities. Less energy is consumed when the channel is good.](image)

This relation implies that transmitting at a lower rate has less energy cost. For a single packet, the minimum energy cost is found by plugging in Eq. (2.2) with the minimum data rate that guarantees all information can be delivered before the deadline.

However, the problem becomes more interesting when there are more than one packet to be served. We have to determine which packet to be served and the data rate for each packet. This problem was first proposed and solved by Uysal-Biyikoglu et al. [22]. Consider an example with two packets in Figure 2.5. Packet $a$ arrives at time 0, and packet $b$ arrives at time $t$. Both have the same deadline $D$.

![Figure 2.5: Two packets with different arrival time and same deadline.](image)

The problem can first be simplified by considering the necessary conditions for the optimal solution. Since the energy can always be decreased by a longer
transmission duration, the optimal scheduling should be non-idling. Next, due to the convexity of the energy-rate relationship, the transmission rate should be constant. It can be shown by Jensen’s inequality that transmitting at two different rates with the same amount of data results in a higher energy cost than one constant transmission rate.

In this two packet scenario, because of packet b’s arrival at time t, the transmission rate is constant only in the time interval [0, t] and [t, D]. The problem then is to find two constant data rates such that the energy cost is minimized.

Suppose the data rates in these two intervals are $r_a$ and $r_b$, respectively. We first have to ensure that $t \cdot r_a$, the number of bits that served in $[0, t]$, is not larger than the available information bits, i.e., the size of packet a. Such constraint is called the “casuality” constraint. The other constraint is that the total number of transmitted bits, $t \cdot r_a + (D - t) \cdot r_b$, is equal to the size of two packets at the deadline time $D$. Such constraint is called the “deadline” constraint. While, the objective is to minimize the total energy cost,

$$t \cdot r_a E_i(r_a) + (D - t) \cdot r_b E_i(r_b).$$

This is a convex optimization problem with linear constraints, which can be solved efficiently by well-known algorithms [23].

2.2.2 Multiple packets and fading channels

In reality, the channel can be time-varying during the arrival and the deadline of the packet, packets can have different deadlines, and the energy-rate relation can be different and determined by the channel state as seen in Figure 2.4. In order to solve this more complex problem, an epoch, shown in Figure 2.6, is defined. It is a time interval that begins with a packet arrival, a packet deadline or a change in the channel state, and continues until the next arrival, the packet deadline or the state change. The problem is then to find the data rate in each epoch such that the “casuality” and “deadline” constraints are met and the total energy cost is minimized.

A similar convex optimization problem can be formulated. Note that, to solve the convex optimization problem, we need to know the future packet arrival times and the channel state realizations. In general, all the work assume knowledge of future packet arrival times, which is reasonable, for instance, for periodic data flows. Some work assume the channel state is static and known. When the channel is time-varying, some other work assume an infeasible knowledge of future channel realizations.

In the paper by Uysal-Biyikoglu et al. [22], a fixed number of packets arrive with different inter-arrival time, however they have the same absolute deadline and the channel state is static and known. Khojastepour and Sabharwal [24] consider an individual packet deadline scenario, but the optimal scheduler is derived under the condition of no further packet arrival. Chen et al. [25] consider a time-varying channel under the case where packets arrive at different time slots with the same
relative delivery deadline. Zafer and Modiano [26–28] introduce the network calculus approach [29] in modeling the deadline-constrained packet forwarding problem. Several algorithms are derived to solve the convex optimization problem from the perspective of the network calculus model. One advantage of using network calculus is the straightforward illustration of how the rate should be adjusted according to the packet arrivals and their deadlines. In this framework, they study the problem under the assumption that all the packets are queued in the beginning and the packets have different packet deadlines in [26]. The problem with a limit on short-term average power consumption is addressed by a Lagrangian approach in [27]. The scenario with dynamic packet arrivals for both static channels and time-varying channels is studied in [28].

### 2.2.3 Dynamic programming approach for time-varying channels

Several papers consider the time-varying channels with a realistic assumption that future channel states are unknown. However, the channel state can be estimated based on the Markovian property of link models where the state evolves at the boundary of each time slot. This topic has been extensively studied within the dynamic programming (DP) approach. Due to the high complexity of the DP approach, all the work being done assume packets are in the queue at the beginning with the same absolute deadline time. Hence, the time horizon of the DP is the deadline of the packet, and the system state is composed of the channel states, the set of remaining bits/packets and the time to the deadline.

Denote $k$ as the time slot. Let $d_k$ be the number of bits remaining to be sent at time $k$, let $s_k$ be the actual amount of data sent at time $k$, and let the channel...
quality be $q$. Following a DP approach, we obtain the following value function,

$$E_k(d, q) = \min_{0 \leq s \leq d} \left[ \mathcal{E}(s, q) + \sum_{\tilde{q}} \Pr\{\tilde{q}|q\} \cdot E_k(d - s, \tilde{q}) \right],$$

where $E_k(d, q)$ denotes the minimum energy cost when the channel quality is $q$ and there are $d$ number of bits to be sent. The optimal value function for a given number of bits at the beginning of the time horizon $k = 0$ can be computed backwards by DP.

Some specific energy cost versus transmission rate relationship have been exploited to simplify the derivation of the optimal policy, or even to give close form expression. As can be seen in Figure 2.4, in the low transmission power region, the energy cost can be approximated as a linear relation with the rate associated with a maximum transmission power constraint. In the high transmission power region, the energy can be approximated by a high order monomial function of the rate.

Fu et al. [30] study several link models and available information on the links, which include the cases where

1. the channel distribution is known;
2. the channel state follows a Markov process and is observed at the end of the time slot (hence, an estimation of current channel state).

The paper derives a close form expression for a linear energy-bit relationship and a link model where the channel state is restricted to be an integer multiple of some constants. Lee and Jindal [31] study similar problems with an energy-rate relationship governed by the AWGN channel capacity formula with an independent channel fading model. A monomial energy-rate relationship is studied in Lee and Jindal [32]. Srivastava and Koksal [33] study a similar problem to [30], but with fixed data transmission rate. Hence, the decision problem is to decide whether to transmit a packet or not. They extensively consider that the link states are known perfectly with one time slot delay, and that channel state is estimated based on the histories of ACK/NAK messages. All these work reveal the opportunistic strategies in which the optimal policies wait for good channel state when deadline is far away, and transmit aggressively when the deadline is approaching.

The work on minimum energy cost by rate adaptation on a single link topology is summarized in Table 2.1.

2.2.4 Rate adaptation on star networks

The rate adaptation technique can be extended from single link to star network. In the star network, the decision space includes which flow to serve. Moreover the spatial variation can be exploited because the link qualities between the source and different receivers differ.

Some extensions on the two approaches are necessary to solve the star network scenario. In the DP approach, the system state is enlarged to include the remaining
Table 2.1: Summary of papers on minimum energy cost by rate adaptation on a single link topology. Note that all papers assume the future packet arrival times are known.

<table>
<thead>
<tr>
<th>Paper</th>
<th>Packet arrival</th>
<th>Deadline</th>
<th>Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>[22]</td>
<td>dynamic arrival</td>
<td>single absolute</td>
<td>static</td>
</tr>
<tr>
<td>[24]</td>
<td>queued in beginning</td>
<td>multiple</td>
<td>static</td>
</tr>
<tr>
<td>[25]</td>
<td>dynamic arrival</td>
<td>single relative</td>
<td>time varying$^1$</td>
</tr>
<tr>
<td>[26–28]</td>
<td>dynamic arrival</td>
<td>multiple</td>
<td>time varying$^1$</td>
</tr>
<tr>
<td>[30,31]</td>
<td>queued in beginning</td>
<td>single absolute</td>
<td>time-varying</td>
</tr>
</tbody>
</table>

$^1$The future channel state realizations are known.

number of bits and the channel qualities of all receivers. On the other hand, in the convex optimization approach, the problem becomes more difficult because we have to add the constraint that only one receiver can be served at each time epoch. Some work simplify the problem by focusing on a specific service rule, such as first in first out (FIFO) or earliest deadline first (EDF). The problem is then to find an optimal data rate allocation. However, these policies are not always optimal.

El Gamal et al. [34] extend their previous work [22] with multiple receivers. Under the assumption that the channel is static and all the packets have the same single absolute deadline, a FIFO service discipline is optimal. The problem is then to allocate the data rate at each epoch, which is solved by a convex optimization problem. Uysal-Biyikoglu and El Gamal [35] later study a multiple-access channel and broadcast channel in which multiple transmissions can occur. Hence, we can directly apply the convex optimization approach from the single link case. Shuman and Liu [36] consider strict underflow constraints in which each receiver demands a fixed number of bits at a certain frequency. This underflow constraint can be transformed into a set of deadline-constrained packet forwarding problems. Basically, it requires a constantly increasing number of packets to be delivered before different deadlines. The channel is modeled by Markov chains, but the state is known before transmission. The problem can be solved by DP with system state including the receiver buffer queue length and the channel state at each time slot. Tarello et al. [37] study the cases with no dynamic packet arrivals. The first case is the channel state is known before transmission and packets have different deadlines. They specifically show that the optimal service rule is EDF. The problem is then essentially a convex optimization problem. Secondly, they consider a stochastic channel process, and extend Fu et al. [30]’s DP approach by enlarging the system state to include multiple receivers.

Miao and Cassandras [38] extend the work in [34] by considering the case where each packet has different deadlines, and different number of bits. However, they restrict to the non-optimal FIFO service rule, see Figure 2.7. Such non-optimal scheduling policy issues will be revisited in timely-throughput maximization prob-
2.3. Maximize timely-throughput

Due to transmission power constraints and a large amount of data traffic, it is not possible that all the packets can be served before the deadline. Hence, the problem is to maximize the number of packets that can be delivered before the deadline (i.e. timely throughput) and the energy cost is not considered.

Several papers [39–41] consider maximizing the timely throughput on a single-link topology of energy harvesting nodes. The aim is to find an energy allocation at time such that the data transmission rate is maximized and the energy consumption causality constraint is satisfied. Gong *et al.* [42] study the problem of selecting packet size and data rate in a star network such that the reward (related with the packet size) is maximized under deadline and energy constraints. These works utilize rate adaptation technique and are actually the dual problem of minimizing the energy cost surveyed in Section 2.2. Hence, similar approaches are applied.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2_7.png}
\caption{FIFO is not optimal with dynamic packet arrivals and arbitrary packet deadlines. Due to the hard real-time constraint and the FIFO service rule, both packets have to depart the system at time $D_2$. However, it is not optimal, since a better policy is to preempt the transmission of packet 1 after time $t_2$ and resume it after time $D_2$.}
\end{figure}

Table 2.2: Summary of papers on minimum energy cost by rate adaptation on a star network. Note that all papers assume the future packet arrival times are known.

<table>
<thead>
<tr>
<th>Paper</th>
<th>Packet arrival</th>
<th>Deadline</th>
<th>Channel</th>
<th>Service rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>[34]</td>
<td>dynamic arrival</td>
<td>single absolute</td>
<td>static</td>
<td>FIFO</td>
</tr>
<tr>
<td>[35]</td>
<td>dynamic arrival</td>
<td>single absolute</td>
<td>static</td>
<td>unspecified</td>
</tr>
<tr>
<td>[36,37]</td>
<td>queued in beginning</td>
<td>single absolute</td>
<td>time-varying</td>
<td>unspecified</td>
</tr>
<tr>
<td>[37]</td>
<td>queued in beginning</td>
<td>multiple</td>
<td>static</td>
<td>EDF</td>
</tr>
<tr>
<td>[38]</td>
<td>dynamic arrival</td>
<td>multiple</td>
<td>static</td>
<td>FIFO$^1$</td>
</tr>
</tbody>
</table>

$^1$It is not the optimal service rule under [38]'s assumption of the packet arrival and deadline.
Most of the work studying timely-throughput focus on star networks and do not use rate adaptation. We will cover these work in the rest of this section.

2.3.1 Policy adapted from EDF

In EDF service rule, each packet is assigned a deadline, and the scheduler serves packets with the smallest deadline. It has been shown to be optimal in various wireline networks. However, it cannot be directly applied in wireless networks, since it does not consider the lossy and time varying characteristics of wireless links. Li and Zhang [43] study the maximum timely-throughput problem with time-varying channels and the assumption of an error-free transmission depending on the channel quality. They are able to show that EDF service rule is optimal under a special agreeable deadline scenario where the packets that arrive earlier have a smaller deadline.

There have been some other work in the direction of adapting the EDF idea to the wireless star network topology. Shakkottai and Srikant [44] propose a simple EDF based scheduling algorithm to maximize the timely-throughput for different data streams in a wireless start network with time-varying and correlated link losses modeled by the Gilbert-Elliot (GE) model. They assume the GE state is known with good state denoting a successful transmission.

The essential scheduling problem can be described in the example shown in Figure 2.8. There are two flows whose associated wireless transmission links are both at good state. Flow 1 has five packets and longer deadline, but the probability of link being at good state at the next time slot is small. Flow 2, on the other hand, has only one packet and short deadline, but the probability of link being at good state at the next time slot is high. The EDF policy transmits the packet from flow 2, while transmitting the packet from flow 1 is obviously better. Thus, the EDF policy is not optimal.

Due to the correlation introduced by the link loss model, the optimal policy depends on the queue length, the individual packet deadline in the queue, the channel state, and statistics of future packet arrivals. The problem can be solved by DP approach, but it needs a very large state space. Shakkottai and Srikant [44] instead propose a simple feasible EDF policy that schedules the packet which has the earliest time to expire from the set of queues whose channels are marked in good state only (transmission in this state is successful). It is optimal for a class of deterministic arrival processes (e.g. periodic data flows with the same period, but different relative arrival times). It also derives regions of different system parameters (GE loss models, deadlines and number of packets, etc.) such that the proposed policy is indeed an optimal one.

Several follow-up works have appeared. However, they all suffer from the limitation that the proposed policies are heuristic. Kong and Teh [45] propose proactive EDF policy where it adjusts a packet’s deadline in anticipation of an upcoming degradation in its channel quality before transmitting the packet with the smallest adjusted deadline. Khattab and Elsayed [46] propose the channel dependent EDF
2.3. Maximize timely-throughput

Figure 2.8: EDF policy is not optimal for bursty links. In this example, both links are in good state with successful transmission. However, the link for flow 1 has a probability of 0.9 to go to a bad state, and it has a long data queue. EDF policy chooses the packet from flow 2 with shorter deadline, but the optimal policy should be to transmit packet from flow 1. Moreover, if there are future stochastic packets arrivals, the problem becomes more difficult.

policy. The scheduler chooses the queue whose head of line packet has the earliest time to expire and the best channel conditions. A slightly more sophisticated approach appeared in Dua and Bambos [47]. They first consider the scenario with exactly one packet per flow and static channel. These packets have different deadlines. The problem can be solved by DP with system state of the time to expiry of each packet. During the actual packet forwarding, the channel state is estimated before transmission and assumed to be static. The DP solution for the head of line packet for each flow is applied. After the transmission of one packet, the same procedure is repeated.

2.3.2 Hou and Kumar’s framework

In order to make the multiple receiver problem in the star network tractable, Hou and Kumar have proposed a tractable framework [48] with restrictions on the packet arrival pattern and the deadline pattern. The packet arrival and deadlines are deterministic and structured, see Figure 2.9. An interval is composed of a fixed number of time slots. Each client generates one packet at the beginning of each interval and requires the same delay bound equal with one interval. The deadline constraint is then the length of the interval $D$. Moreover, the channel state is assumed to be known and remains static in one interval.

Two sets of problems are studied in this framework and are discussed in turn: (a) feasibility optimal policies for inelastic traffic, and (b) timely-throughput related utility maximization.
Related work

Figure 2.9: Hou and Kumar’s framework proposed in [48]. Packets from different flows arrive at the same time in the beginning of a frame of length $D$, and expire at the end of the frame. Moreover, the transmission link is lossy, but the channel state remains static with a success probability $q$.

Feasibility optimal policies for inelastic traffic

For the inelastic timely-throughput requirements, each client requires a minimum timely-throughput. The authors in [48] first derive the necessary and sufficient conditions for the system that can serve these flows with timely-throughput requirements. These conditions can serve as the admission control for new flows. They then propose the feasibility optimal policies to schedule the flows. The feasibility optimal schedule is defined as the policy that can serve the flows as long as these flows are admissible by the system.

Define the load $w_i$ of a client $i$ as the minimum time it needs in one interval in order to obtain the required timely-throughput. Since the channel state remains static and known, the load is simply the ratio between the timely-throughput requirement $r_i$ and the success transmission probability on the link $q_i$, i.e.,

$$w_i = \frac{r_i}{q_i}.$$ 

A trivial necessary condition is that the sum of the loads of all client should be smaller than the length of the interval. However, it is not a sufficient condition because of the unavoidable idle periods when all the packets have been successfully forwarded in one interval. Hence, the necessary and sufficient condition is that the sum of the loads of the clients should be smaller than the length of the interval minus the idle periods when these clients are served. This condition should be satisfied for all subsets of the nodes.
Feasibility optimal policies, called largest debt first scheduling policies, are proposed. The clients are served in the order of the amount of debts that each AP owns them. The debt is calculated in the beginning of the interval based on the difference between the required service and the actual service the client received. One of the optimal policies calculates the delivery debt as the difference in the required load and the received load,

\[ d_i(k + 1) = k \cdot w_i - \sum_{n=1}^{n=k} u_i(n), \]  

(2.3)

where \( k \) is the index of current time interval, and \( u_i(n) \) is the actual number of time slots that client \( n \) receives at time interval \( n \).

A generalized framework is proposed later in [49] with the following relaxations:

- At the beginning of the interval, the client generates packet probabilistically.
- The deadline constraint can be different, but is smaller than one interval.
- The channel state can be different across different intervals.
- The rate adaption with fixed transmission energy is incorporated to provide error-free transmission with longer transmission time.

A sufficient condition for the feasibility optimal policy for the above generalized framework is derived. The so-called pseudo-debt for each node at each interval is defined as follows. The pseudo-debt increases a constant amount that is related with the timely-throughput requirement. It can be decreased when the client is served. The system is fulfilled when the pseudo-debt goes to zero almost surely. The sufficient condition for a feasibility optimal policy is to maximize a pseudo-debt related payoff function.

It is in general difficult to maximize the payoff function. Nevertheless, Hou and Kumar have proposed two tractable scheduling algorithms:

1. The first case deals with probabilistic packet arrival, rate adaptation and individual packet deadlines. A delivery debt is defined as the difference between timely-throughput requirement and the actual delivered packet for each client similar to Eq. (2.3) for the basic model. The problem is then to find a subset of nodes to be served such that their deadline constraints are satisfied and the sum of the delivery debt is maximized.

2. The second case concerns probabilistic packet arrivals and time-varying channel. The receivers that have packets to be delivered are ordered according to the product of their delivery debt and the success delivery probability. The feasibility optimal policy is to transmit the packet in this order.
Timely-throughput related utility maximization

The second problem deals with elastic timely-throughput requirement. Each client attains a utility based on the acquired timely-throughput, and the aim is to maximize the utility.

The authors study the utility maximization problem under the basic traffic and deadline model in [50]. The feasible set has been characterized by the necessary and sufficient conditions in the feasibility optimal policy problem. A convex optimization problem is formulated, and solved by a bidding procedure found by Lagrange approach. Hou and Kumar also consider a bidding procedure that can prevent clients from faking their utilities [51].

The bidding procedure works as follows. The AP assigns different discounts (correspond to the Lagrange multiplier) to each client at the beginning of the time interval. The client announces a bid to the server knowing that the AP charges for the service. Then, the optimal decision at the AP is to schedule a subset of clients such that the sum of the discount and the bid of these nodes is maximized. Each scheduled client is charged with the minimum bid it should have offered to be scheduled. The aim of the client is to maximize the net reward defined as the utility minus the money paid to the AP. The same procedure is repeated after the AP updates the Lagrange multiplier.

Discussions

Hou and Kumar can solve the problem optimally, but the framework is restrictive on traffic arrival and deadline patterns and it is difficult to study the relaxations:

- They study some scenarios, for example, the data rate can be different, packet arrivals probabilistically, and the deadline can be different but smaller than an interval and only for a rate adaptation technique with fixed transmission power. However, all of these do not break this interval structure.
- Although the framework incorporates the time-varying nature of a wireless channel, there are two main restrictions: first, the channel state remains static over one interval; second, the channel state is known before transmission.

2.3.3 Miscellaneous techniques

Some papers instead study the problem from different perspectives. The probabilistic delay bound, which means the delay violation probability is upper bounded, is considered in [52,53]. Others, for example, study only admission control [54,55].

Probabilistic delay bound

A probabilistic guarantee on the packet delivery bound is very similar to the problems of guaranteed timely-throughput requirement studied by Hou and Kumar. However, the techniques can be very different. One such approach appears in Xu
2.4. Multi-hop network

It focuses on the physical layer techniques that can convert the delay bound violation probability into an easier problem on the conditions of the transmission power, data modulation and time slots allocations in the network. More specifically, they consider the uplink transmission of a CDMA cellular network and utilize existing results of the required bandwidth for a leaky-bucket regulated traffic to guarantee the per-packet delay bound. The statistic bound is guaranteed by ensuring a certain probability that the required bandwidth is allocated. The bandwidth at each single client is determined by the rate and the power allocated for each client in the network.

Admission control

Chaporkar and Sarkar [54] propose an admission control framework that caters towards channel aware EDF policy proposed by Shakkottai and Srikant [44]. They quantify the delay in terms of their arrival processes and the channel characteristics of all the contending flows. The idea is to estimate the maximum number of erroneous slots in a busy period. In an erroneous slot at least one flow has bad channel, and a busy period is the maximum contiguous time interval in which at least one packet is waiting for transmission. With one erroneous slot in the busy period, all flows could experience at most one time slot delay. Hence, the maximum delay is upper bounded by the maximum number of erroneous time slots. The paper then estimates the probability that the maximum number of erroneous slots is larger than a given threshold based on the leaky-bucket traffic model and GE link loss model. The flow is admitted when the required delay is guaranteed with a higher probability than the predefined permissible value.

2.4 Multi-hop network

When packet has to be forwarded on a multi-hop network, more issues arise, such as inter/intra-flow interferences and queueing delays in the routing nodes etc. There are not so many works on multi-hop networks with optimal solutions due to the high complexity and difficulty. However, several interesting papers study the problem by either utilizing a specific protocol to simplify the problem [56–58], or assuming no transmission interference [59,60], e.g., in low duty cycle wireless sensor networks.

2.4.1 WirelessHART network

WirelessHART [61] was recently proposed as an open standard for wireless sensor-actuator networks for process industry. A WirelessHART network consists of field devices, one gateway and a network manager (see, e.g. Figure 2.10). A field device is a sensor or an actuator. The scheduling and routing in WirelessHART network is performed centrally at the network manager. The network manager distributes the schedule among the devices by providing the sender-receiver transmission pair in each time slot to all devices.
The WirelessHART physical layer is compliant with the IEEE 802.15.4-2006 standard, and it supports up to 16 physical channels. However, due to the difficulty in detecting interference between nodes and the variability of interferences, WirelessHART network allows only one transmission in each channel across the entire network, preventing spatial reuse in the same channel. As a direct consequence, the maximum number of concurrent transmissions at any time slot cannot exceed the number of available channels.

Saifullah et al. [56] study multiple real-time flow transmissions with per-packet deadlines over WirelessHART network with the restriction on the spatial reuse. Due to this limitation, two transmissions conflict only when they share the same node, which is also called the node-exclusive interference. Moreover, the authors assume a fully reliable link. Hence, as long as the minimum time (considering the node exclusive interference and the availability of the packet) to reach the destination is smaller than the deadline, the schedule is valid. The problem is then to find an optimal algorithm such that all flows can be scheduled whenever a feasible schedule exists. They show that it is NP-complete to decide whether the scheduling problem is feasible or not. A necessary condition for schedulability is derived, which is used to prune the search space with an optimal branch and bound algorithm. The same authors later consider a simple and efficient fixed priority scheduling. Due to the spatial reuse limitation, the scheduling of real-time periodic data flows in a WirelessHART network can be mapped to a real-time multiprocessor scheduling [58]. Combined with the worst-case delay analysis of multiple flows with fixed priorities, an optimal priority assignment is derived [57].

The main restriction of Saifullah et al.’s approach is that it assumes links are fully reliable; hence, it does not guarantee end-to-end delivery reliability. The way to deal with this problem is firstly to use links that are highly reliable only. The other way is to duplicate the data packet at the source, and each duplicated packet is treated as a separate data flow. They derive the schedulability analysis to support as many data flows as possible, hoping that one of them can successfully reach the destination.
2.4.2 Wake-up scheduling in wireless sensor networks

Duty cycle is the main technique for power saving in wireless sensor networks. In such networks, the energy cost can be saved by letting nodes to stay asleep for a longer time, but the packet delivery delay will be inevitably increased. Some papers consider minimizing the energy cost while ensuring that the per-packet delivery delay is bounded. Cohen and Kapchits [59] assume a single route from each sensor to the sink and that the links are fully reliable. Hence, the delay is bounded by the sum of the inverse of the wake up frequency. In Gu et al. [60]’s framework, the links can be lossy, but the time to successfully transmit one packet between two nodes is upper bounded. They solve the problem of finding the minimum wake up time to a working schedule such that the delivery delay is bounded. In both works, the interference due to packet transmission collisions is not modeled due to low collision probability in low duty cycle mode, which simplifies the problem considerably.

2.5 Contributions of this thesis

Most of the work on real-time wireless communications with per-packet deadlines have been devoted to single link and star network topologies under various assumptions on packet arrivals, packet deadlines and channel models. Extensions to some more complex scenarios in star network topologies are already shown to be difficult, e.g. maximizing the timely-throughput under stochastic packet arrivals and time-varying channels [44] and minimizing the energy cost by rate adaptation for dynamic packet arrivals and arbitrary packet deadlines [38]. Optimal and tractable solutions in a star network topology can be developed under additional restrictions, such as the packet arrival and deadline patterns in [48].

In this thesis, we instead restrict the number of flows to one, but extend real-time wireless communications with per-packet deadlines to a multi-hop network topology. This single transient packet scenario roots from the machine to machine communication where the traffic is lightly loaded. Examples include wireless sensor and actuator networks in which the periodic sampling times are usually longer than packet deadlines. In this single transient packet scenario, the complexities due to wireless transmission interferences and queueing delays can be reduced, as mentioned in Section 2.4.

We assume finite-state Markov channel models and that nodes only have access to (possibly delayed) the state of their outgoing links. The physical layer is fixed, and the packet end-to-end delivery reliability can be increased by retransmissions on the link.

We first study the problem of maximizing the probability that the packet is delivered before the deadline. This problem is similar to the problem of maximizing timely-throughput discussed in Section 2.3. It can be efficiently solved by dynamic programming. We also study the minimum energy packet forwarding problem with an arbitrary reliability requirement, while the works mentioned in Section 2.2 consider the minimum energy problem with a hard real-time deadline constraint, i.e.,
the packet has to be delivered before the deadline all the time. In our model, the energy cost can be saved by either waiting or reducing transmission power with lower transmission success probability. This problem is formulated as a constrained Markov decision process and solved by studying a related weighted-sum maximization problem.
In this chapter, we present detailed models and assumptions for multi-hop communication with per-packet deadlines. We formulate the deadline-constrained maximum reliability forwarding problem and show how it can be cast as a Markov decision process (MDP), while the minimum energy problem with a deadline-constrained reliability requirement is formulated as a constrained Markov decision process (CMDP). A general dynamic programming framework to solve these two problems is proposed.

3.1 System model

We consider a scenario where a single packet, generated by an arbitrary node at time $t = 0$, should be transmitted over a multi-hop wireless network to the sink node $N$ within a deadline of $D$ time slots.

The routing topology of the multi-hop wireless network is represented by a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ with nodes $\mathcal{N} = \{1, \ldots, N\}$ and links $\mathcal{L}$. The presence of a directed link $(i, j) \in \mathcal{L}$ means that node $i$ is able to successfully transmit a packet to node $j$. The destination node is labeled $N$. We define $\mathcal{N}_i$ as the set of neighbors of node $i$ in $\mathcal{N}$ and $h_i$ as the minimum hop count from node $i$ to the destination $N$. Examples of such routing graphs include destination-oriented directed acyclic graphs (DODAG) built in the IETF routing protocol for low power and lossy networks (RPL) [14].

Communication links are unreliable and packet loss processes on links are modeled by finite-state Markov chains [62–64]. The Markov chains for different links are independent and each node can only access the state of its own outgoing links. Each Markov chain evolves in discrete time and state changes coincide with the transmission slot boundaries. Let $s \in \mathcal{S}$ denote the Markovian state for a link and $\mathcal{S}$ denote the set of all possible states. Let $r \in \Gamma$ denote the relative power (with respect to full power) used for a single transmission where $\Gamma$ is a finite set with
30 System model and dynamic programming framework

elements between 0 and 1. Similar to the model used in [65], a packet transmission on a link in state $s$ with relative power $r$ is successful with probability $q_r^s$. Let $\omega_i(t) = [\omega_{ij}(t)]_{j \in \mathcal{N}_i}$ represent the state of all links $(i,j)$ outgoing from node $i$ at time $t$, with $\omega_{ij}(t) = s$ if the channel is in state $s$. Furthermore, let $\Omega_i = \mathcal{S}^{||\mathcal{N}_i||}$ be the set of all possible link states for node $i$ such that $\omega_i(t) \in \Omega_i$.

The optimal policy hinges on what knowledge can be assumed about the state of the underlying Markov chain at the time when the forwarding decision is made. Let $I_i(t) \in \mathcal{I}_i$ denote the link state information available at node $i$ and at time $t$, where $\mathcal{I}_i$ is the set of all possible link state information. In this thesis, we consider two information patterns:

(a) $I_i(t) = \omega_i(t)$: each node knows the state of its outgoing links at time $t$ prior to transmitting.

(b) $I_i(t) = \omega_i(t-1)$: each node knows the state of its outgoing links at the previous time $t-1$, which allows to compute the channel state distribution at time $t$.

One could argue that information pattern (a) is not realistic in low-power networks, as it assumes that nodes know the channel states ahead of time. However, it provides an upper bound on the achievable reliability and gives insight into the optimal policies. Also information pattern (b) can be challenged, since it assumes full state knowledge at time slot $t-1$, but it approaches a situation in which link state estimators continuously track the link evolution.

In both information patterns, we neglect the energy and time cost of the link state estimators for acquiring channel state information. Several additional information patterns would be interesting to consider in future work: one could be that the channel state is acquired by probing with an associated cost; another one could be that nodes do not know the state of their links until they are used. Once a link has been used, the conditional probability of successful transmission at any future time can be computed using the Markov chain.

3.1.1 The Bernoulli and Gilbert-Elliot link models

In this section, we present a special two-state Markovian link loss model that covers the Bernoulli and Gilbert-Elliot (GE) link models often used in the remaining part of the thesis.

The two-state Markov model illustrated in Figure 3.1 has a “good” (G) and a “bad” (B) state. We let $\omega_{ij}(t) = G$ if the link is in good state, and $\omega_{ij}(t) = B$ if the link is in bad state. The success probability in good state denoted by $q_G^r$ depends on the transmission power $r$, while the success probability in the bad state is always zero.

If transmissions in good state are always successful (e.g., always transmitting with full transmission power), we recover the basic GE model for correlated link losses [66,67] shown in Figure 3.2, where $\omega_{ij}(t) = G$ means that the transmission
is successful at time $t$ and $\omega_{ij}(t) = B$ means the transmission fails at time $t$. Hence, the conditional success and failure probabilities in good state are $q_G$ and $p_G$, respectively, and the conditional success and failure probabilities in bad state are $q_B$ and $p_B$, respectively. The average unconditional packet loss probability (i.e., the stationary distribution of the bad state of the Markov chain) $\Pi_B$ is computed as

$$\Pi_B = \frac{1 - q_G}{1 - q_G + q_B},$$

and the average unconditional probability of successful transmission is $\Pi_G = 1 - \Pi_B$. Moreover, let $T_B$ be the average error burst length (number of time slots for which the channel stays bad),

$$T_B = \frac{1}{q_B}.$$

The GE model further reduces to the Bernoulli model of independent packet erasures when $p_B = p_G = p$ and $q_B = q_G = q = 1 - p$. 

**Figure 3.1:** Two-state Markov chain model for link packet erasures with transmission power adjustments. The transition probabilities between the “G” and “B” states are denoted by $q_G, q_B, p_G$ and $p_B$. The failure (F) probability in bad state is always one, while the success (S) probability $q'_G$ in good state depends on the transmission power $r$. Specifically, $q'_G = 1$ if $r = 1$, i.e., the transmission in good state with full transmission power is always successful.

**Figure 3.2:** Gilbert-Elliot link loss model.
3.2 MDP and CMDP formulation

The deadline-constrained packet forwarding problem can be formulated as a finite-horizon Markov decision process (MDP) [68] with the horizon equal to the packet deadline $D$.

In what follows, we give a general description of relevant theory for MDPs. In an MDP, at each time $t$, suppose the process is at a state $x(t)$. If the decision maker chooses an action $a(t)$, the process goes to the next state $x(t+1)$ at the time slot $t+1$ with probability $\Pr\{x(t+1)|x(t), a(t)\}$. A reward or cost $\mu_t(a(t), x(t))$ is obtained at each decision epoch $t$ depending on the state and action at current time slot, and there may exist a terminal reward or cost $\mu_D(x(D))$ at the end of the time horizon where no more actions are available.

To give an illustrative MDP example in the context of our deadline-constrained packet forwarding problem, let us study the simplest scenario where the link loss model is Bernoulli and the transmission power adjustment is not used. The MDP state is then the packet location. The process goes to the next state (i.e., the next hop node or the current packet location) depending on the action (i.e., transmitting the packet on a link or holding the packet at the current node). An energy cost is induced if the packet is transmitted. The state transition probability is determined by the link loss probability. An MDP example is illustrated in Figure 3.4 for the network topology in Figure 3.3.

![Figure 3.3: A simple network topology with four nodes. We assume Bernoulli link loss model and no transmission power adjustments. Here, $s$ is the source and $d$ is the destination; $l$ denotes the link and $p$ denotes the average link loss probability.](image)

The MDP becomes more complex when the channel is modeled by a Markovian link loss process. However, the basic structure remains the same. The system state has to be enlarged to include the knowledge of the channel states, and the state transition probabilities are now also influenced by the transitions of the Markov chain of the link models. An example of packet forwarding on a single link with two-state Markov chain link loss model is shown in Figure 3.5. Lastly, the transmission power adjustment can be simply included by adding more actions with different energy cost and success probability tuned by the transmission power.

The core problem of an MDP is to find a sequence of actions such that the sum
Figure 3.4: The Markov decision process for the packet forwarding problem on the network topology in Figure 3.3. This MDP starts with the initial state \( s \) at time 0, which means that the packet is at the source node in the beginning. The state of the MDP is progressed at discrete time according to the action and the link loss probability. Notice that, transmitting the packet to neighbor consumes energy, while withholding the packet does not. A reward is received if the MDP stays at state \( d \) (the destination node) at the deadline time \( t = D \).

of the rewards over time is maximized. We define a decision rule that prescribes the action in each state at a specified time, and a policy composed by a sequence of decision rules at each time. In the most general case, the decision rule should depend on the previous states and actions, and can prescribe different actions randomly. More precisely, let history \( h(t) \) be a sequence of previous states and actions, i.e., \( h(t) \triangleq (x(0), a(0), \ldots, x(t-1), a(t-1), x(t)) \), and \( H(t) \) be the set of all possible histories. The decision rule is a function \( d(t) : H(t) \to \mathcal{P}(A(t)) \) that maps \( H(t) \) into a set of probability distributions on the action space \( A(t) \) of all possible actions. Since the MDP is a process of finite-horizon with length \( D \), the policy is \( \pi \triangleq (d(0), d(1), \ldots, d(D-1)) \), indexed by time. With these expressions, the expected total reward when executing a policy \( \pi \) is written as

\[
\sum_{s} \Pr\{s\} \mathbb{E}^\pi_s \left\{ \sum_{t=0}^{D-1} \mu_t(a(t), x(t)) + \mu_D(x(D)) \right\}, \tag{3.1}
\]

where \( s \) is the initial state of the MDP, and \( \mathbb{E}^\pi_s \) is the expected total reward with initial state \( s \). Notice that the expectation \( \mathbb{E}^\pi_s \) is taken over the probability space induced by policy \( \pi \).
Figure 3.5: The Markov decision process for the packet forwarding on a single link modeled by two-state GE link loss model. We assume that the channel state in the previous time slot is known. The MDP state now includes the channel state good (G) or bad (B). The MDP starts with either good or bad channel state depending on the unconditional good or bad channel state distribution. The MDP state transition probability is determined by the channel states at current time slot and the next time slot. These specific numbers are given by the GE link loss model, see Section 3.1.1 for details.

The deadline-constrained reliability is the total expected reward of the MDP when a reward is only offered at the deadline time $t = D$, which is also the termination time of the MDP with time horizon $D$. Suppose the deadline-constrained reliability under a policy $\pi$ is $R^\pi$. A standard MDP can be formulated for the maximum deadline-constrained reliability forwarding as

$$\max_{\pi} \ R^\pi.$$ 

Furthermore, the minimum energy forwarding with a requirement of deadline-constrained reliability is a constrained MDP,

$$\min_{\pi} \ C^\pi$$

subject to $R^\pi \geq R_{req}$,

where $C^\pi$ is the expected energy cost under a policy $\pi$. 

---

Initial: good channel state at $t=0$ with prob. $\Pi_G$

Initial: bad channel state at $t=0$ with prob. $\Pi_B$

Action | State | Action with energy cost | Action without energy cost
--- | --- | --- | ---
 Transmit on link | | | Hold the packet

A single link topology

s | i | d

s,G | h | s,B

s,G | h | s,B

s,G | h | s,B

s,G | h | s,B
Now, we will detail the state $x(t)$, the action $a(t)$, the state transition probability $\Pr\{x(t+1)|x(t),a(t)\}$ and the reward $\mu_1(a(t),x(t))$ of our deadline-constrained packet forwarding problem in the MDP formulation and the CMDP formulation.

### 3.2.1 MDP formulation

Let $x(t) \triangleq (m(t), I_{m(t)}(t))$ denote the state of the MDP where $m(t)$ is the packet location at time $t$ and $I_{m(t)}(t)$ are the channel state information of node $m(t)$ at time $t$, i.e., $I_{m(t)}(t) = \omega_{m(t)}(t)$ for information pattern (a) and $I_{m(t)}(t) = \omega_{m(t)}(t-1)$ for information pattern (b). The action $a(t) \triangleq (j(t), r(t))$ chooses the next hop node $j(t)$ and the transmission power $r(t)$.

The state transition probability $\Pr\{x(t+1)|x(t),a(t)\}$ is determined by link parameters. For ease of presentation, suppose the MDP state at time $t$ is $x(t) \triangleq (i, I_i(t))$ and the MDP state at time $t+1$ is $x(t+1) \triangleq (k, I_k(t+1))$. Let $\Pr\{I_i(t+1)|I_i(t)\}$ be the channel state information transition probability of node $i$'s outgoing links, and let $\Pr(I_i(t))$ be the stationary channel state information distribution of all links outgoing from node $i$ at time $t$. Notice that $\Pr(I_i(t))$ is the stationary distribution of Markovian chains of the link model in both information pattern (a) and (b).

The state transition probability function $\Pr\{x(t+1)|x(t),a(t)\}$ depends on the action. If the action is to hold the packet (i.e., $j(t) = i$), then the packet still stays at node $i$ in the next time slot (i.e., $k = i$), and the channel states evolve according to the Markov chains of the link models. The probability that the packet is at any other nodes is zero. Hence, the state transition probability when $j(t) = i$ is

$$\Pr\{x(t+1)|x(t),a(t)\} = \begin{cases} 
\Pr\{I_k(t+1)|I_i(t)\} & \text{if } k = i, \\
0 & \text{otherwise}.
\end{cases}$$

On the other hand, if the action is to forward the packet (i.e., $j(t) \neq i$), in the next time slot the packet can be at node $i$ or at neighbor $j(t)$ to which the packet is forwarded. The probability that the packet is at any other nodes is zero. The state transition probability becomes more complex, and it is

$$\Pr\{x(t+1)|x(t),a(t)\} = \begin{cases} 
\sum_{\omega_i(t)} (1 - q_\omega_{ij}(t)) \Pr(\omega_i(t)|I_i(t)) \Pr(I_k(t+1)|\omega_i(t), I_i(t)) & \text{if } k = i, \\
\sum_{\omega_i(t)} q_\omega_{ij}(t) \Pr(\omega_i(t)|I_i(t)) \Pr(I_k(t+1)) & \text{if } k = j(t), \\
0 & \text{otherwise}.
\end{cases}$$

In these expressions, $\omega_i(t)$ is a realization of the channel states at time $t$ depending on node $i$'s information on the channel states at time $t$. The probability that the packet stays at node $i$ is $1 - q_\omega_{ij}(t)$, and the probability that it is at the neighbor $j(t)$ is $q_\omega_{ij}(t)$. Finally, the information of the channel states at the next time slot...
$I_k(t+1)$ is determined by the channel state realization $\omega_i(t)$ and the channel state information $I_i(t)$ if the packet stays at node $i$, and is unconditionally distributed as the channel state information of the next hop node $k$ if the packet is successfully forwarded to the neighbor $j(t)$.

There is no reward for each action, but a terminal reward $\mu_D(x(D))$ is given if the packet arrives at the sink node $N$ at the last time slot $D$,

$$\mu_D(x(D)) = \begin{cases} 1 & \text{if } m(D) = N, \\ 0 & \text{otherwise}. \end{cases}$$

Under a policy $\pi$, the expected reward (deadline-constrained packet reliability) is

$$R^\pi \equiv \sum_{x(0)} \Pr\{x(0)\} \mathbb{E}_x^\pi \{\mu_D(x(D))\},$$

where $x(0)$ is the initial state. It describes that the packet is at the source node and with the unconditional channel states distribution,

$$\Pr\{x(0)\} = \begin{cases} \Pr(I_m(0)) & \text{if } m(0) = \text{src}, \\ 0 & \text{otherwise}. \end{cases}$$

With this notation, the maximum deadline-constrained reliability problem is formulated as

$$\maximize_{\pi} R^\pi. \quad (3.2)$$

### 3.2.2 CMDP formulation and weighted sum maximization

The energy cost $c(x(t), a(t))$ is incurred when the action is to transmit the packet to a neighbor with power $r(t)$,

$$c(x(t), a(t)) = \begin{cases} r(t) & \text{if } j(t) \neq m(t), \\ 0 & \text{otherwise.} \end{cases} \text{ for } t \in \{0, 1, \ldots, D-1\}.$$

Note that a unit of transmission energy cost corresponds to a transmission with full power. The expected energy cost under a policy $\pi$ is

$$C^\pi \equiv \sum_{x(0)} \Pr\{x(0)\} \mathbb{E}_x^\pi \left\{ \sum_{t=0}^{D-1} c(x(t), a(t)) \right\}.$$

In this notation, the minimum energy forwarding with a requirement of deadline-constrained reliability $R_{\text{req}}$ can be formulated as

$$\minimize_{\pi} C^\pi \quad \text{subject to} \quad R^\pi \geq R_{\text{req}}. \quad (3.3)$$
3.3 General dynamic programming framework

This problem falls into the category of constrained MDP (CMDP). The standard solution is to use linear programming [68]. We instead use the Lagrangian approach proposed in [69] for CMDP to convert it to a non-constrained weighted sum problem. The Lagrange dual of the minimum energy problem (3.3) is

$$\max_\lambda \min_\pi (C^\pi + \lambda(R_{\text{req}} - R^\pi))$$

subject to $\lambda \geq 0$

which is equivalent to

$$\min_\delta 1/\delta \cdot \max_\pi \{R^\pi - \delta \cdot C^\pi\} - 1/\delta \cdot R_{\text{req}}$$

subject to $\delta \geq 0$  \hspace{1cm} (3.4)

where $\delta = 1/\lambda$. Our finite-horizon CMDP can be cast into the infinite-horizon case with total cost criterion. The Markov state $x$ is extended to include the time from $t = 0$ to $t = D$. It goes to the next state with time $t + 1$ only if the current state’s time is $t$. We define a termination state to which all the states with time $D + 1$ are directed. This is an absorbing state with no reward and cost. All other parameters including rewards, costs and state transition probabilities remain the same. It can be shown that this is a contracting MDP as defined in [69, Def. 2.4]. Hence, by [69, Thm. 4.8 ii], the duality gap is zero.

To solve problem (3.4), we hence need to solve the weighted sum maximization of reliability and energy, i.e.,

$$\max_\pi \{R^\pi - \delta \cdot C^\pi\}$$

for a given $\delta \geq 0$. Note that the maximum reliability problem (3.2) is a special case of the weighted sum maximization problem (3.5) with $\delta = 0$.

3.3 General dynamic programming framework

Next, we will develop a general dynamic programming framework to solve the weighted sum maximization problem (3.5). Notice that the maximum reliability problem (3.2) can be solved by letting $\delta = 0$ with this dynamic programming framework. By treating the weighted energy cost $\delta \cdot C^\pi$ as a negative reward scaled by $\delta$ in the MDP formulation, a history-independent and deterministic optimal policy can be found by dynamic programming [68].

The MDP state is composed of the packet location and the link state information. Thus at time $t$ and node $i$, let the maximum conditional utility be

$$U_i^* (t|I_i(t)) = R_i^* (t|I_i(t)) - \delta C_i^* (t|I_i(t))$$

that describes the optimal utility for packet delivery within the next $D - t$ time slots. $R_i^* (t|I_i(t))$ and $C_i^* (t|I_i(t))$ are the corresponding optimal reliability and
optimal energy cost, respectively. The unconditional maximum utility is

$$U^*_{i}(t) = \sum_{I_i(t)} \Pr\{I_i(t)\} U^*_{i}(t|I_i(t)),$$

where $\Pr\{I_i(t)\}$ is the probability that the link state information at node $i$ is $I_i(t) \in \mathcal{I}_i$.

To this end, our aim is to develop optimal forwarding policies that maximize the utility at $t = 0$ with deadline of $D$ time slots, $U^*_{i}(0)$, for node $i$. This quantity can be computed recursively by dynamic programming from $t = D - 1$ to $t = 0$, starting from initial condition

$$U^*_{i}(D|I_i(D)) = \begin{cases} 1 & \text{if } i = N, \\ 0 & \text{if } i \neq N; \end{cases} \quad (3.7)$$

$$R^*_{i}(D|I_i(D)) = \begin{cases} 1 & \text{if } i = N, \\ 0 & \text{if } i \neq N; \end{cases} \quad C^*_{i}(D|I_i(D)) = 0. \quad (3.8)$$

At each step with $t < D$, the maximum conditional utility $U^*_{i}(t|I_i(t))$ at node $i$ is characterized by the following optimality equation

$$U^*_{i}(t|I_i(t)) = \max \left\{ \max_{j \in N, r \in \Gamma} U^r_{jr}(t|I_i(t)), U^*_{i}(t|I_i(t)) \right\} \quad (3.9)$$

where $U^r_{jr}(t|I_i(t))$ is the utility of forwarding to a neighbor $j \in N_i$ with power $r$, and $U^*_{i}(t|I_i(t))$ is the utility of withholding the packet at node $i$, respectively. These utilities are computed as

$$U^r_{jr}(t|I_i(t)) = \sum_{\omega_i(t)} \Pr\{\omega_i(t)|I_i(t)\} \left( q_{\omega_{ij}(t)} U^r_{j}(t+1) \right)$$

$$+ (1 - q_{\omega_{ij}(t)}) \sum_{I_{i}(t+1)} \Pr\{I_i(t+1)|\omega_i(t), I_i(t)\} U^*_{i}(t+1|I_i(t+1))$$

$$- \delta \cdot r; \quad \text{Staying at node } i,$$ \quad (3.10)

In these expressions, $\omega_i(t)$ is a specific link state realization for node $i$ at time $t$, $\Pr\{\omega_i(t)|I_i(t)\}$ is the probability that $\omega_i(t)$ occurs at time $t$ conditioned on the
with the power \( \omega \) computed in the previous step. It also needs the maximum link state \( q \) where

\[
q = \arg \max_i \mathbb{P}(I_i(t+1)|I_i(t)) \forall j \in \mathcal{N}, r \in \Gamma.
\]

3.3. General dynamic programming framework

The optimal policy at time \( t \) forwards the packet to the node \( j^*_t(t|\omega(t-1)) \) with the power \( r^*_t(t|\omega(t-1)) \) that maximizes Eq. (3.15). For ease of notation, define \( j^*_t \equiv j^*_t(t|\omega(t-1)) \) and \( r^*_t \equiv r^*_t(t|\omega(t-1)) \). Thus, we have

\[
(j^*_t, r^*_t) = \begin{cases} 
(i, 0) & \text{if } U^*_t(t|I_i(t)) \geq U^*_t(t|I_i(t)) \forall j \in \mathcal{N}, r \in \Gamma; \\
\arg \max_{j \in \mathcal{N}, r \in \Gamma} U^*_t(t|I_i(t)) & \text{otherwise.}
\end{cases}
\]

(3.12)

Note that withholding the packet does not consume energy, and that we break ties arbitrarily. With Eq. (3.12), one can compute \( R^*_t(t|I_i(t)) \) and \( C^*_t(t|I_i(t)) \). If \((j^*_t, r^*_t) \neq (i, 0)\), then

\[
R^*_t(t|I_i(t)) = \sum_{\omega_i(t)} \mathbb{P}(\omega_i(t)|I_i(t)) \left( q^*_i(\omega_i(t)) R^*_t(t+1) \right. \\
+ \left. (1 - q^*_i(\omega_i(t))) \sum_{I_i(t+1)} \mathbb{P}(I_i(t+1)|\omega_i(t), I_i(t)) R^*_t(t+1|I_i(t+1)) \right)
\]

(3.13)

\[
C^*_t(t|I_i(t)) = \sum_{\omega_i(t)} \mathbb{P}(\omega_i(t)|I_i(t)) \left( q^*_i(\omega_i(t)) C^*_t(t+1) \right. \\
+ \left. (1 - q^*_i(\omega_i(t))) \sum_{I_i(t+1)} \mathbb{P}(I_i(t+1)|\omega_i(t), I_i(t)) C^*_t(t+1|I_i(t+1)) \right)
\]

(3.14)

where \( q^*_i(\omega_i(t)) \equiv q^*_{i|\omega_i(t)}(t) \) is the success transmission probability with the policy \((j^*_t, r^*_t)\) under the channel state \( \omega_i(t) \). On the other hand, if \((j^*_t, r^*_t) = (i, 0)\), then

\[
R^*_t(t|I_i(t)) = \sum_{I_i(t+1)} \mathbb{P}(I_i(t+1)|I_i(t)) R^*_t(t+1|I_i(t+1)); \quad (3.15)
\]

\[
C^*_t(t|I_i(t)) = \sum_{I_i(t+1)} \mathbb{P}(I_i(t+1)|I_i(t)) C^*_t(t+1|I_i(t+1)). \quad (3.16)
\]
The unconditional $R^*_i(t)$ and $C^*_i(t)$ can be computed similarly

$$R^*_i(t) = \sum_{I_i(t)} \Pr\{I_i(t)\} \cdot R^*_i(t|I_i(t)); \quad C^*_i(t) = \sum_{I_i(t)} \Pr\{I_i(t)\} \cdot C^*_i(t|I_i(t)).$$

(3.17)

**Remark.** The dynamic programming framework allows nodes to find their optimal forwarding policy based on the statistics of their outgoing links and the “offered deadline-constrained utilities” $U^*_j(t+1)$ of one-hop away parents. This step can be done a priori if the link statistics do not change, or at a slow time-scale using message-passing between nodes. At run-time, nodes can then make optimal forwarding decisions by acquiring the states of their outgoing links and applying the policy.

**Distributed implementation**

This dynamic programming framework can be implemented in a distributed manner. For the destination node $N$, the optimal utility is one, the optimal reliability is one, and the optimal energy is zero at any time $t$. The dynamic programming computes the optimal utility and the optimal forwarding policy for other nodes. It starts with the initial conditions of the dynamic programming at time $t = D$ for all nodes $i \neq N$ in Eq. (3.7) and Eq. (3.8). In step 1, each node $i$ ($i \neq N$) computes $U^*_i(D-1|I_i(D-1))$ with $U^*_i(D|I_i(D))$ and $U^*_i(D)$ for all $I_i(D-1)$ based on Eq. (3.9) - Eq.(3.11). After computing the unconditional utility with Eq. (3.6), it broadcasts $U^*_i(D-1)$ to any node $j$ for which $i \in \mathcal{N}_j$, i.e., $i$ is one of $j$’s next hop nodes. At the beginning of step 2, each node $i$ knows $U^*_j(D-1)$ for $j \in \mathcal{N}_i$ from step 1. Hence, these nodes can compute $U^*_i(D-2|I_i(D-2))$ for all $I_i(D-2)$ using Eq. (3.9) - Eq.(3.11). The unconditional utility is computed, and broadcast to neighbors again. The procedure repeats until $t = 0$. At this point, we have computed $U^*_i(t|I_i(t))$ for all nodes $i \neq N$, for all $t \in [0, D-1]$ and for all $I_i(t)$. A total $(D-1) \cdot (|\mathcal{N}| - 1)$ number of messages have to be passed among the neighboring nodes during this procedure where $|\mathcal{N}|$ is the number of nodes in the network. The optimal forwarding policy depending on the packet location, the time, and the link state information can be derived with Eq. (3.12).

### 3.4 Summary

In this chapter, the models for real-time wireless communication with per-packet deadlines that will be used in this thesis are presented. In particular, our model is composed of a directed graph routing topology and finite-state Markov chains to describe packet loss processes on links. The maximum reliability problem is modeled as a Markov decision process (MDP), while the minimum energy problem is cast as a constrained Markov decision process (CMDP). For the minimum energy problem, we show by Lagrangian approach that it can be solved by studying a set of
weighted sum of reliability and energy maximization problems. Finally, a general dynamic programming framework for the maximum reliability problem and the weighted sum maximization problem was derived.
In this chapter, we consider the deadline-constrained maximum reliability forwarding problem for both the Bernoulli and the Gilbert-Elliot (GE) link loss models. In order to maximize the link reliability, the packet is always transmitted with full power. More specifically, the dynamic programming framework is applied here with \( \delta = 0 \) and the utility function coincides with the reliability function, i.e., \( U_i(\cdot) = R_i(\cdot) \). The sole goal is to find the optimal forwarding node \( j^* \), since the packet is always transmitted with full power. For ease of notation, we let \( R_i^j(t|I_i(t)) \) be the forwarding reliability of node \( i \) to a neighbor \( j \) with full transmission power.

We develop simplified expressions for conditional maximum utility in Eq. (3.9), optimal forwarding policy at each time slot \( t \) in Eq. (3.12) and corresponding maximum utility in Eq. (3.6) for both information pattern (a) and (b) with Bernoulli and GE link loss models. We demonstrate the techniques with numerical examples, and compare with the strategy of dedicated time slot studied in Section 4.3 and the scheme of forwarding on the minimum expected transmission count (ETX) path [19].

### 4.1 Maximum reliability forwarding under Bernoulli loss model

In what follows, we analyze this problem for two different information patterns when erasure events follow a Bernoulli process. Under this loss model, the link state at any time slot does not provide any knowledge on the state at the next time slot and hence, \( \Pr(\omega_i(t)|\omega_i(t-1)) = \Pr(\omega_i(t)) \).
4.1.1 Bernoulli loss model and information pattern (a)

In information pattern (a) with $I_i(t) = \omega_i(t)$ and $\delta = 0$, Eqs. (3.10)-(3.11) become

$$
R_i^j(t|\omega_i(t)) = 1_{\{\omega_{ij}(t)=G\}} R_j^i(t+1) \\
+ 1_{\{\omega_{ij}(t)=B\}} \sum_{\omega_i(t+1)} \Pr\{\omega_i(t+1)|R_j^i(t+1)\} \\
= 1_{\{\omega_{ij}(t)=G\}} R_j^i(t+1) + 1_{\{\omega_{ij}(t)=B\}} R_j^i(t+1) \\
R_i^j(t|\omega_i(t)) = \sum_{\omega_i(t+1)} \Pr\{\omega_i(t+1)|R_j^i(t+1)\} R_j^i(t+1) = R_j^i(t+1),
$$

where $1_{\{\omega_{ij}(t)=s\}}$ for $s = \{G, B\}$ is an indicator function defined as $1_{\{\omega_{ij}(t)=s\}} = 1$ if $\omega_{ij}(t) = s$ and 0 otherwise. We can re-write the update step in Eq. (3.9) as follows

$$
R_j^i(t|\omega_i(t)) = \max \left\{ \max_{j \in N_i} \left\{ 1_{\{\omega_{ij}(t)=G\}} R_j^i(t+1) + 1_{\{\omega_{ij}(t)=B\}} R_j^i(t+1) \right\}, R_j^i(t+1) \right\}
$$

Note $R_j^i(t|\omega_i(t))$ can be either $R_j^i(t+1)$ or $R_j^i(t+1)$, and staying at node $i$ has the highest priority according to Eq. (3.12). Thus, it is sufficient to use $1_{\{\omega_{ij}(t)=G\}} R_j^i(t+1)$ for node $j$.

We present a low complexity algorithm to find the optimal policy in Eq. (3.12) and solve the maximum reliability problem inspired by the technique used in Oger [70]. Let nodes in $N_i$ be sorted in order of decreasing reliability as $(j_{i1}^1, j_{i2}^1, \ldots, j_{i|N_i|}^1)$, i.e.,

$$
R_{j_{i1}^m}^i(t+1) \geq R_{j_{i2}^m}^i(t+1), \quad m = 1, \ldots, |N_i| - 1,
$$

and let $k_i(t) \in \{1, \ldots, |N_i|\}$ denote the largest integer such that

$$
R_{j_{i1}^m}^i(t+1) > R_{j_{i2}^m}^i(t+1), \quad \forall m \leq k_i(t).
$$

The optimal policy forwards the packet on the first link $(i, j_{1}^{k_i(t)})$, $m \leq k_i(t)$ (according to the ordering) that is in good state; if all $k_i(t)$ links are in bad state, node $i$ withholds the packet,

$$
\hat{j}_i^* = \begin{cases} 
  j_{1}^{k_i(t)} & \text{if } \omega_{ij_{1}^{k_i(t)}}(t) = B \quad \forall n < m \text{ and } m \leq k_i(t), \omega_{ij_{1}^{k_i(t)}}(t) = G, \\
  i & \text{if } \omega_{ij_{1}^{k_i(t)}}(t) = B \quad \forall n \leq k_i(t).
\end{cases}
$$

(4.1)
Therefore, one can simplify the maximum conditional reliability $R_i^*(t|\omega_i(t))$ as follows
\[
R_i^*(t|\omega_i(t)) = \begin{cases} 
R_{ij}^m(t + 1) & \text{if } \omega_{ij}^m(t) = B \forall n < m \text{ and } m \leq k_i(t), \ \omega_{ij}^m(t) = G, \\
R_i^*(t + 1) & \text{if } \omega_{ij}^m(t) = B \forall n \leq k_i(t).
\end{cases}
\]

Finally, the computation of $R_i^*(t)$ can be simplified by observing that the set of available information at node $i$ consists of the link state realization $\omega_i(t)$ in which two types of actions can take place: each of the $k_i(t)$ outgoing links is selected; the packet is withheld at node $i$. Let $q_{ij}$ and $p_{ij}$ be the average success and loss probability of transmission from node $i$ to node $j$. Hence,
\[
R_i^*(t) = \sum_{m=1}^{k_i(t)} R_{ij}^m(t + 1) q_{ij}^m \prod_{l=1}^{m-1} p_{ij}^l + R_i^*(t + 1) \prod_{l=1}^{k_i(t)} p_{ij}^l.
\]

The computation involves the highest number of multiplication operations when $R_i^*(t + 1) > R_i^*(t + 1)$ for all $j \in N_i$. Thus, each node performs at most $|N_i|(|N_i| - 1)/2$ multiplication operations.

### 4.1.2 Bernoulli loss model and information pattern (b)

Under information pattern (b), the state of the outgoing links is no longer available prior to transmitting. Since erasure events in the Bernoulli model are independent from one time slot to the next, information about previous link state does not affect the forwarding decision, i.e., $R_i^*(t|I_i(t)) = R_i^*(t)$ and $R_i^*(t|I_i(t)) = R_i^*(t)$. Thus, Eqs. (3.10)-(3.11) become
\[
R_i^*(t) = \sum_{\omega_i(t)} \Pr\{\omega_i(t)\} \left(1_{\{\omega_{ij}(t) = G\}} R_{ij}^*(t + 1) + 1_{\{\omega_{ij}(t) = B\}} R_i^*(t + 1)\right)
= q_{ij} R_{ij}^*(t + 1) + p_{ij} R_i^*(t + 1);
R_i^*(t) = R_i^*(t + 1).
\]

The maximum end-to-end reliability from node $i$ to the destination is
\[
R_i^*(t) = \max \left\{ \max_{j \in N_i} \left\{ q_{ij} R_{ij}^*(t + 1) + p_{ij} R_i^*(t + 1) \right\}, R_i^*(t + 1) \right\}. \tag{4.2}
\]

Given the minimum hop count $h_i$ from node $i$ to the destination, the next theorem simplifies the optimal policy under Bernoulli model and information pattern (b).

**Theorem 4.1.1.** Under information pattern (b) with the Bernoulli loss model, the optimal policy that maximizes the end-to-end reliability at node $i$ and time slot $t$ is
\[
j_i^*(t) = \begin{cases} 
\arg \max_{j \in N_i} \left\{ q_{ij} R_{ij}^*(t + 1) + p_{ij} R_i^*(t + 1) \right\} & \text{for } t + h_i \leq D; \\
& \text{for } t + h_i > D.
\end{cases}
\]
The maximum reliability is

\[
R^*_i(t) = \begin{cases} 
\max_{j \in \mathcal{N}_i} \{q_{ij} R^*_j(t + 1) + p_{ij} R^*_i(t + 1)\} & \text{for } t + h_i \leq D; \\
0 & \text{for } t + h_i > D. 
\end{cases}
\]

Proof. This result is a special case of the optimal forwarding policy under the GE loss model in Theorem 4.2.1. See Section 4.2.2 and the proof in Appendix A.1.

Since the optimal policy is independent from channel states, the computational complexity in terms of the number of multiplication operations at each node is reduced to \(2|\mathcal{N}_i|\).

From the optimal forwarding policy of unconditionally transmitting to the most reliable next-hop for \(t + h_i \leq D\), one might be misled to believe that a shortest-path routing with the appropriate weights would be optimal. However, it is important to note that the optimal path might change when the delay constraint changes. Hence, a series of packets originating from the same source node with the same delay constraint may follow different paths. One such example is given in Figure 4.1, where packets are normally forwarded along the more reliable two-hop path. However, if a packet has experienced multiple losses and remains at the source when there is only a single slot left until the deadline, the transmission decision will be to try the less reliable direct path.

![Figure 4.1](image)

**Figure 4.1:** The longer path consists of two very reliable links. However, a packet with deadline 1 will be transmitted over the unreliable direct path.

### 4.2 Maximum reliability forwarding under Gilbert-Elliot loss model

We next address the problem of maximum reliability routing for the two-state Markov chain model with full transmission power. Links now have memory which correlates information and decisions from one time slot to the next, and in general \(\Pr(\omega(t)|\omega(t-1)) \neq \Pr(\omega(t))\). For information patterns (a) and (b), we find solutions that despite a substantially larger state space, remain tractable for most reasonable network scenarios.
4.2. Maximum reliability forwarding under Gilbert-Elliot loss model

4.2.1 GE loss model and information pattern (a)

Under the GE loss model, the reliability of keeping the packet at node $i$ at a specific time slot will now depend on the state of all the outgoing links of node $i$. With $I_i(t) = \omega_i(t)$ and $\delta = 0$, we have

$$R^*_i(t|\omega_i(t)) = 1_{\omega_{ij}(t)=G} R^*_j(t + 1) + 1_{\omega_{ij}(t)=B} \sum_{\omega_i(t+1)} \Pr\{\omega_i(t+1)|\omega_i(t)\} R^*_i(t + 1|\omega_i(t + 1))$$

and the maximum conditional reliability in Eq. (3.9) becomes

$$R^*_i(t|\omega_i(t)) = \max \left\{ \max_{j \in N_i} \left\{ R^*_j(t|\omega_i(t)) \right\}, R^*_i(t|\omega_i(t)) \right\}$$

The unconditional maximum reliability $R^*_i(t)$ follows directly from Eq. (3.6) with $I_i(t) = \omega_i(t)$ and $\delta = 0$,

$$R^*_i(t) = \sum_{\omega_i(t)} \Pr\{\omega_i(t)\} R^*_i(t|\omega_i(t)).$$

Contrary to the Bernoulli case, it is not easy to analytically characterize the set of optimal forwarding nodes. Therefore, we need to iterate over all possible state vectors to determine the optimal forwarding nodes $j^*_i$ using Eq. (3.12). Every node needs to evaluate $2^{|N_i|}$ possible link state vectors in the next time slot to compute the reliability of staying action for a given channel state, and needs $2^{|N_i|}$ steps to iterate over all possible channel states $\omega_i(t)$ to compute the unconditional $R^*_i(t)$. Thus, the complexity to compute $R^*_i(t)$ in terms of number of multiplication operations at each node in each step is $4^{|N_i|}$. Although the complexity increases exponentially, it remains tractable for most reasonable network scenarios in which each node typically would use a handful (say three to five) of outgoing links [11].

4.2.2 GE loss model and information pattern (b)

With $I_i(t) = \omega_i(t-1)$ and $\delta = 0$ the maximum conditional reliability from Eq. (3.9) can be re-written as

$$R^*_i(t|\omega_i(t-1)) = \max \left\{ \max_{j \in N_i} \left\{ R^*_j(t|\omega_i(t-1)) \right\}, R^*_i(t|\omega_i(t-1)) \right\}, \quad (4.3)$$
where
\[ R^*_i(t|\omega_i(t-1)) = \sum_{\omega_i(t)} \Pr\{\omega_i(t)|\omega_i(t-1)\} \]
\[ \cdot \left(1_{\omega_{ij}(t)=G} R^*_j(t+1) + 1_{\omega_{ij}(t)=B} R^*_i(t+1|\omega_i(t))\right), \]
\[ R^*_i(t|\omega_i(t-1)) = \sum_{\omega_i(t)} \Pr(\omega_i(t)|\omega_i(t-1)) R^*_i(t+1|\omega_i(t)). \]

The following theorem characterizes the optimal forwarding policy.

**Theorem 4.2.1.** Under information pattern (b) with the GE loss model, the optimal policy that maximizes the end-to-end reliability at node \( i \) and time slot \( t \) is
\[ j^*_i = \begin{cases} \arg \max_{j \in N_i} \left\{ R^*_i(t|\omega_i(t-1)) \right\} & \text{for } t + h_i \leq D; \\ i, & \text{for } t + h_i > D. \end{cases} \]
The conditional maximum reliability is
\[ R^*_i(t|\omega_i(t-1)) = \begin{cases} \max_{j \in N_i} \left\{ R^*_i(t|\omega_i(t-1)) \right\} & \text{for } t + h_i \leq D; \\ 0 & \text{for } t + h_i > D. \end{cases} \]

**Proof.** See Appendix A.1.

In essence, the optimal forwarding policy under the GE loss model and information pattern (b) is to always transmit the packet to the node that achieves the maximal \( R^*_i(t|\omega_i(t-1)) \) if \( t + h_i \leq D \), or to hold the transmission if \( t + h_i > D \) in which case the remaining time slots to deadline is smaller than minimum hop count and \( R^*_i(t|\omega_i(t-1)) = 0 \). The unconditional maximum reliability follows from Eq. (3.6) with \( \delta = 0 \) and \( I_i(t) = \omega_i(t-1) \),
\[ R^*_i(t) = \sum_{\omega_i(t-1)} \Pr(\omega_i(t-1)) R^*_i(t|\omega_i(t-1)). \]
The computation of \( R^*_i(t|\omega_i(t-1)) \) for \( t + h_i \leq D \) becomes more involved, compared to information pattern (a) with the GE loss model because only the links states at the previous time slot are available. Specifically, \( R^*_i(t|\omega_i(t-1)) \) can be computed in \( 2^{|N_i|} \) multiplication operations and repeated \( |N_i| \) times of all \( j \) and \( 2^{|N_i|} \) times of all \( \omega_i(t-1) \). Thus to compute \( R^*_i(t) \) each node needs to carry out \( |N_i|4^{|N_i|} \) multiplications per step.

### 4.3 Dynamic scheduling versus dedicated time slot

In this section, we consider a dedicated time slot strategy with which only one link can be allocated the transmission opportunity at a single time slot. Such a strategy
roots from the limitation in the WirelessHART network where only one link can be activated at one channel at each time slot. However, it is not as efficient as the dynamic scheduling scheme where the transmission rights can be allocated to multiple conflict-free links. The optimal dynamic scheduling scheme is computed by the dynamic programming framework proposed in Chapter 3 and detailed in Section 4.1 and Section 4.2.

We study a single line network with independent Bernoulli packet losses and unknown channel state, i.e., the model studied in Section 4.1.2. Let the link loss probability outgoing from node $i$ be $p_i$. Without loss of generality, let the source node be 1 and the destination node be $N$. The hop distance between source and destination is $N - 1$. Let $x_i \in \mathbb{Z}^+$ be the number of transmission opportunities assigned to link $i$ and let $x = [x_i]$. For a given $x$ with $\sum_i x_i = D$, there are up to $D!/(x_1!x_2!\ldots x_{N-1}!)$ possible schedules, depending on how the transmission opportunities are organized. The following lemma characterizes the optimal schedule structure of dedicated time slot scheme.

**Lemma 4.3.1.** Given a slot allocation vector $x$, the dedicated time slot schedule that maximizes the end-to-end reliability assigns $x_i$ consecutive transmission opportunities to each node $i$ in the order $1, 2, \ldots, N$ (i.e. from source to destination).

**Proof.** Let us first consider the transmission opportunities allocation of the source node 1. Clearly, time slot 1 should be allocated to node 1. Suppose we have an allocation, and node $j$ is the first node that is allocated between time slot 1 and any other allocation time of node 1. Firstly, node $j$ should be the next hop node. Otherwise, the transmission opportunity is wasted. Further, we let $t_j$ as the allocation time for node $j$, and $t_1$ as the first allocation time for node 1 after time $t_j$. We can see that swapping allocations at $t_j$ and $t_1$ results in a higher reliability. The reason is that the probability of having a packet transmitted from node $j$ is higher. Repeating this argument for all other allocations from other nodes in between node 1’s allocations, we establish that node 1 should be allocated the first $x_1$ consecutive transmission opportunities. The same arguments can be applied to the remaining nodes and the remaining transmission opportunities.

The success transmission probability from node $i$ to its next hop node using $x_i$ transmission attempts is

$$\sum_{n=0}^{x_i-1} p_i^n (1 - p_i) = 1 - p_i^{x_i}.$$ 

Therefore, given a slot allocation vector $x$, the end-to-end reliability under the optimal scheduling policy of Lemma 4.3.1 is

$$R_{\text{line}}(x) = \prod_{i=1}^{N-1} (1 - p_i^{x_i}).$$  \quad (4.4)
The next result shows that the optimal allocation of $D$ dedicated time slots that maximizes $R_{\text{line}}(\mathbf{x})$ can be efficiently found.

**Theorem 4.3.2.** Algorithm 1 returns the time slot allocation vector $\mathbf{x}^*$ that maximizes $R_{\text{line}}(\mathbf{x})$ under the constraint that $\sum_i x_i = D$.

**Proof.** The time slot allocation $\mathbf{x}^*$ that maximizes (4.4) also maximizes $\log(R_{\text{line}}(\mathbf{x}))$, and can be found by solving

$$\begin{align*}
\text{maximize} & \quad \sum_i \log(1 - p_i^{x_i}) \\
\text{subject to} & \quad \sum_i x_i = D, \\
& \quad x_i \in \{1, \ldots, D\} \quad \forall i.
\end{align*}$$

(4.5)

Since $\log(1 - p_i^{x_i})$ is concave in $x_i$, this is a discrete resource allocation problem with a separable and concave objective function. Such problems can be solved efficiently using discrete convex programming [71]. Specifically, the optimal resource allocation can be characterized by the marginal utility

$$\Delta_i(x_i) = \log(1 - p_i^{x_i+1}) - \log(1 - p_i^{x_i})$$

for resource $i$. The marginal utility describes the increase in utility when one more transmission opportunity is added on node $i$ and satisfies $\log(1 - p_i^{x_i}) = \sum_{m=1}^{x_i-1} \Delta_i(m)$. Moreover, since the function $\log(1 - p_i^{x_i})$ is concave, we have

$$\Delta_i(1) \geq \Delta_i(2) \geq \cdots \geq \Delta_i(D).$$

(4.6)

It follows directly that the optimal allocation consists of the $D$ largest elements in the set $\Delta \triangleq \{\Delta_i(x_i), \forall i, x_i\}$ (see [71, Theorem 4.1.1]). Since $\Delta_i$ is sorted for each $x_i$ by (4.6), we can find the largest elements in a greedy fashion as in Algorithm 1: starting from an initial allocation, the algorithm adds resources one at a time slot to the variable that has the greatest marginal utility. The algorithm stops when the total number of resources are allocated.

**Algorithm 1** Greedy algorithm.

1. Initialize $\mathbf{x} = [1 \ldots 1]$.
2. For $t = N \ldots D$:
   1. Find $\Delta_i(x_i) = \log(1 - p_i^{x_i+1}) - \log(1 - p_i^{x_i}) \forall i$.
   2. Let $i^* = \arg \max_{i=1,\ldots,N-1} \Delta_i(x_i)$.
   3. Set $x_{i^*} = x_{i^*} + 1.$

This optimal dedicated time slot schedule (Algorithm 1) is inefficient, in the sense that after a successful forwarding on a link, the complete network is idle.
4.4 Numerical examples

During the time slots that were reserved for the retransmission attempts. A higher end-to-end reliability can be achieved by allowing nodes to dynamically schedule their transmissions, which can be computed by dynamic programming framework proposed in Chapter 3.

The numerical example in Figure 4.2 traces the set of deadlines and end-to-end reliability for the optimal dedicated time slot schedule and the optimal dynamic scheduling found by the dynamic programming framework along a 10-node line topology. From the case in which all links are equally good, i.e. the same loss probability $p = 0.2$, we observe that the optimal dedicated time slot schedule performs poorly already for good link quality ($p = 0.2$).

![Figure 4.2: Deadline-reliability curves for the optimal policy with the dedicated time slot strategy and the dynamic scheduling in a 10-node line topology. All the links have the same quality with the same loss probability. We consider two loss probabilities: $p = 0.2$ and $p = 0.6$.](image)

4.4 Numerical examples

We are now ready to demonstrate our algorithms on numerical examples. We focus our evaluations on the topology in Figure 4.3 where a source (node 1) sends packets to a sink (node 6), and analyze the end-to-end reliability for both the Bernoulli and GE link loss models. We have chosen this particular network since it is of limited size (hence comprehensible) yet gives results that are representative of a large number of evaluations that we have performed of networks with different sizes, topologies, node densities and link parameters.

We assume homogeneous links with unconditional loss probability $\Pi_B = 0.5$ under full transmission power for both the Bernoulli and GE models. In GE model,
we characterize the bursty link by its average error burst length $T_B = 1/q_B$. See Section 3.1.1 for details of the relations of these parameters.

Figure 4.4 shows the achievable deadline-reliability curves for information pattern (a) under the Bernoulli and GE loss models, respectively. The curves display the natural monotonicity of increasing reliability with increasing deadline. Since the minimum hop-count between source and destination is two, the smallest deadline that admits non-zero reliability is two time slots. The figure also shows that it is harder to maintain high reliability with correlated losses: when a transmission fails, the link is in bad state where it faces a higher conditional loss probability $p_B$ than the average loss rate $\Pi_B$. Furthermore, for fixed $\Pi_B$, $p_B$ increases with error burst length $T_B$, which explains why the achievable reliability in Figure 4.4 decreases with increasing error burst length.

Figure 4.5 shows how the available link-state information impacts the achievable
reliability. Information pattern (a), where nodes know which outgoing links will be successful when they make forwarding decisions, always outperforms information pattern (b), where nodes cannot be certain that transmissions will be successful. The difference between the two decreases as the links get more bursty, since the link state at the previous time slot instant then becomes an increasingly accurate prediction of the link state at the current time slot.

Figure 4.6 shows the average number of times that each link has been used before the packet is dropped or received at the destination under information pattern (b) over $10^6$ realizations of Monte Carlo simulation. In the case of homogeneous links and Bernoulli losses, the single path ($1 \rightarrow 3 \rightarrow 6$) is optimal. However, as links become increasingly bursty, it becomes more and more beneficial to use multi-path routing to avoid links that are in, or likely to be in, bad state. When links are described by the GE model with $T_B = 2.5$, the packets follow three paths $1 \rightarrow 3 \rightarrow 6$, $1 \rightarrow 4 \rightarrow 6$ and $1 \rightarrow 5 \rightarrow 6$. For longer error burst length $T_B = 10$, all possible paths are used.

Figure 4.7 shows the performance benefits of the optimal forwarding scheme compared with the minimum average end-to-end delay routing scheme that chooses the minimum ETX path [19]. Although not true in general, the minimum ETX path always achieves the optimal performance under Bernoulli loss model in this network topology and link parameter setup. On the other hand, the performance of ETX routing worsens as links become more bursty.

Figure 4.5: Deadline-Reliability curves for information patterns (a) and (b) with unconditional loss probability $\Pi_B = 0.5$ and different average error burst lengths $T_B$ for GE loss model.
4.5 Summary

In this chapter, we have studied the deadline-constrained maximum reliability packet forwarding problem in lossy multi-hop networks. We developed simplified optimal forwarding policies when erasure events on links are independent and fol-
low a Bernoulli process and when links are bursty and losses can be described by
the two-state GE model. The comparisons with the dedicated time slot strategy
and the minimum ETX path routing illustrated the power of the dynamic pro-
gramming approach. Numerical examples on a multi-hop network also show some
interesting results. One insight is that as links become more bursty, it becomes
increasingly difficult to achieve high reliability and the optimal solution makes use
of an increasing number of paths.
Chapter 5

Minimum energy forwarding under deadline and reliability constrains

The optimal forwarding policies studied in Chapter 4 on maximum reliability forwarding make full use of the available transmission opportunities with full transmission power to maximize the probability of on-time packet delivery, see Theorem 4.1.1 and Theorem 4.2.1. However, this is energy-inefficient since nodes may persist in forwarding the packet even if all outgoing links are likely to be in bad state. To overcome this, in this chapter, we consider the minimum energy packet forwarding with a guarantee on the deadline-constrained reliability.

We first show that the minimum energy optimal policy is a random selection between two deterministic and computable forwarding policies, each of which can be found via a dynamic programming framework that solves the weighted sum maximization problem. We then study the structure of the optimal forwarding policies and develop simplified expressions for the case where link losses are independent (Bernoulli model) and for the case where packets are routed on a line with Gilbert-Elliot (GE) loss model. A study on the latency-reliability-energy trade-off in numerical examples shows a dramatic energy penalty of aiming for the maximum achievable reliability. We then propose a heuristic policy that strikes a good balance between reliability and energy cost, and develop a dynamic programming-based algorithm to optimize its parameters and estimate its performance.

5.1 Energy optimal forwarding policy

We have shown by Lagrangian approach in Section 3.2.2 that to solve the minimum energy problem in Eq. (3.3), we have to first solve the weighted sum maximization of reliability and energy,

$$\max_\pi \{ R_\pi - \delta \cdot C_\pi \}$$

for a given $\delta \geq 0$. The solution of this weighted sum maximization problem is the dynamic programming framework proposed in Section 3.3. In this section, we show
that the optimal policy for the minimum energy problem is a randomization of two deterministic policies found by the above weighted sum problem with different values of $\delta$. Similar results for infinite-horizon CMDP with total reward and total cost constraint appeared in [72]. We instead clearly state these two policies and specify the probabilities at which they are selected in the optimal randomized policy.

To this end, let $R^\star(\delta)$, $C^\star(\delta)$ and $\pi^\star(\delta)$ be the optimal reliability, energy and policy in the weighted sum problem for a given $\delta$, respectively. Define $\mathcal{R} \triangleq \{R^\star(\delta)\}$, for all $\delta$ and $\Delta_R \triangleq \{\delta : R^\star(\delta) = R\}$ for a given $R \in \mathcal{R}$. We have the following results:

**Lemma 5.1.1.** $\mathcal{R}$ is a finite set. For a given $R \in \mathcal{R}$, $C^\star(\delta)$ is unique for all $\delta \in \Delta_R$.

**Proof.** See Appendix A.2.

**Theorem 5.1.2.** Let $R^{(1)} = \max\{R \in \mathcal{R} : R \leq R_{req}\}$ and $R^{(2)} = \min\{R \in \mathcal{R} : R > R_{req}\}$ with the associated unique energy costs $C^{(1)}$ and $C^{(2)}$. The optimal value of the minimum energy forwarding problem (3.3) is then

$$C^\star = C^{(1)} + \frac{R_{req} - R^{(1)}}{R^{(2)} - R^{(1)}} (C^{(2)} - C^{(1)}).$$

(5.1)

Suppose that the optimal policies that attain $(R^{(1)}, C^{(1)})$ and $(R^{(2)}, C^{(2)})$ are $\pi^{(1)}$ and $\pi^{(2)}$ respectively. An optimal policy $\pi^\star$ for the minimum energy problem is obtained by random selection of policies $\pi^{(1)}$ and $\pi^{(2)}$ with probabilities

$$\theta^{(1)} = \frac{R^{(2)} - R_{req}}{R^{(2)} - R^{(1)}}; \quad \theta^{(2)} = \frac{R_{req} - R^{(1)}}{R^{(2)} - R^{(1)}}.$$

**Proof.** Lemma 5.1.1 shows the existence of $R^{(1)}$ and $R^{(2)}$ and the uniqueness of $C^{(1)}$ and $C^{(2)}$. The rest of the proof is in Appendix A.2.

The theorem states that the optimal forwarding policy is to make a random selection between two history-independent and deterministic policies, each found by the dynamic programming. A naive implementation would be to randomly select one of the deterministic policies when the packet is created, mark the packet accordingly, and let intermediate nodes forward according to the chosen policy. Moreover, the minimum energy for any $R_{req} \in (R^{(1)}, R^{(2)})$ can be computed by Eq. (5.1) and the Pareto frontier of achievable reliability and energy cost can be traced out by linearly interpolating the closest pairs of reliability and energy values obtained from the dynamic programming framework.

### 5.2 Simplified polices for Bernoulli and GE loss models

Due to the similarities of the two information patterns, we focus on information pattern (b) where nodes can access the channel states in the previous time slot with
Bernoulli and GE link loss models. Since the minimum energy policy is a random-
ization of two deterministic policies computed by the general dynamic programming
framework for the weighted sum maximization, we study only the simplification of
the optimal policy that obtains the optimal utility $U^*(\cdot)$ with a given $\delta$. To better
illustrate the maximum utility and the optimal policy in this scenario, let us first
simplify the update step from Eq. (3.9) with channel state $\omega_i(t - 1)$ as

$$U^*_i(t|\omega_i(t-1)) = \max \left\{ \max_{j \in N_i, r \in \Gamma} \left\{ \sum_{\omega_i(t)} \Pr\{\omega_i(t)|\omega_i(t-1)\} \left( q_{rj}(t) U^*_j(t+1) \right. \right. \right.$$

$$\left. \left. + (1 - q_{rj}(t)) U^*_i(t+1|\omega_i(t)) \right) - \delta r \right\} \right., \sum_{\omega_i(t)} \Pr\{\omega_i(t)|\omega_i(t-1)\} U^*_i(t+1|\omega_i(t)) \right\}.$$  

An interesting observation is that the structural results about waiting strategies now
depend on $\delta$. The energy penalty $\delta$ may now refrain a node from transmitting when
links are likely to be in bad state. Intuitively, waiting is optimal if the reliability
gain of forwarding is small compared to $\delta$.

In the following, we consider full transmission power and derive simplified forms
for the optimal policy.

5.2.1 Optimal policy with Bernoulli loss model

We first consider the optimal forwarding policy when events on links are uncorre-
lated in time described by Bernoulli loss model. We let $h_i$ be the minimum hop
count to the sink. Intuitively, the optimal forwarding policy does not forward the
packet at a node if the remaining time to deadline is smaller than the node’s mini-
mum hop count. The next theorem shows that with a transmission energy cost, the
optimal forwarding policy may stop forwarding even when the time to deadline is
higher than the minimum hop count $h_i$. Since the optimal policy and the optimal
utility for Bernoulli model are independent from channel states, we let $U^*_j(t)$ be the
utility of forwarding to node $j$ with full transmission power.

**Theorem 5.2.1.** Under Bernoulli link losses and information pattern (b), the
optimal policy for weighted sum maximization with a given $\delta$ is

$$j_i^*(t) = \begin{cases} \arg \max_{j \in N_i} \{U^*_j(t)\} & \text{for } 0 \leq t \leq D - h_i, \\ i & \text{for } t > D - h_i, \end{cases}$$

where $\overline{h}_i = \arg \min_{h \in [h_i, D-1]} \left\{ \max_{j \in N_i} q_{ij} U^*_j(D - h + 1) > \delta \right\}.$

**Proof.** See Appendix A.3.
The optimal forwarding policy has an interesting structure. There exists a positive number $h_i \geq h_i$ such that if $t > D - h_i$, the optimal policy does not attempt to forward the packet. The value $h_i$ is the "effective minimum hop count" for node $i$ considering transmission energy cost.

### 5.2.2 Optimal policy with line topology and GE loss model

![Figure 5.1: Line topology.](image)

We also study the optimal forwarding policy with GE loss model in the line topology shown in Figure 5.1. Suppose node $j$ is the next hop for node $i$. Let the optimal conditional utility with previous slot in good (G) state and bad (B) state be

$$U^*_i(t|G) = \max\left\{ qG U^*_j(t + 1) + pG U^*_i(t + 1|B) - \delta, qG U^*_i(t + 1|G) + pG U^*_i(t + 1|B) \right\},$$

$$U^*_i(t|B) = \max\left\{ qB U^*_j(t + 1) + pB U^*_i(t + 1|B) - \delta, qB U^*_i(t + 1|G) + pB U^*_i(t + 1|B) \right\},$$

respectively. By comparing the two terms in (5.2) and (5.3), the optimal forwarding policies are

$$j^*_i(t|G) = \begin{cases} i & \text{if } \zeta_t \leq \frac{\delta}{qG}, \\ j & \text{if } \zeta_t > \frac{\delta}{qG} \end{cases},$$

$$j^*_i(t|B) = \begin{cases} i & \text{if } \zeta_t \leq \frac{\delta}{qB}, \\ j & \text{if } \zeta_t > \frac{\delta}{qB} \end{cases},$$

where $\zeta_t = U^*_j(t + 1) - U^*_i(t + 1|G)$.

The optimal forwarding policy with good state observation also stops forwarding even when the time to deadline is larger than the minimum hop count $h_i$. The following theorem states the optimal forwarding policy with good channel state.

**Theorem 5.2.2.** When packet losses on links are described by the two-state GE model with information pattern (b), the optimal policy for weighted sum maximization for a given $\delta$ with good channel state observation on a line topology is

$$j^*_i(t|G) = \begin{cases} j & \text{for } 0 \leq t \leq D - \hat{h}_i, \\ i & \text{for } t > D - \hat{h}_i \end{cases},$$

where $\hat{h}_i = \arg\min_{h \in [h_i, D-1]} \zeta_{D-h_i} > \frac{\delta}{qG}$. 


Proof. See Appendix A.4.

On the other hand, the optimal forwarding policy conditioned on a previously bad channel state is different and it allows the packet to wait. Intuitively, it waits for good state when there are plenty of transmission opportunities. The policy eventually transmits the packet when the remaining time is small. However, there does not exist a single threshold type optimal policy with bad state observation. In order to illustrate the waiting strategy, let us consider the optimal forwarding policy with a single link where node $j$ is the sink and $U_j^*(t) = 1$ for all $t$. The optimal forwarding policy is

$$j^*(t|B) = \begin{cases} i & \text{if } q_B - \delta \leq q_B U_j^*(t+1|G), \\ j & \text{if } q_B - \delta > q_B U_j^*(t+1|G). \end{cases}$$

Since $U_j^*(t|G)$ decreases as $t$ becomes larger (See Appendix A.4), the optimal forwarding policy at node $i$ conditioned on a previously bad channel state may change with the number of remaining time slots. For instance, a large $U_j^*(t+1|G)$ may induce node $i$ to withhold the transmission allowing the channel to turn good. As the deadline approaches, however, $U_j^*(t+1|G)$ becomes smaller and smaller to the point that it is optimal to transmit despite a bad channel observation. The parameter $\delta$ trades between energy and reliability by affecting the moment when the optimal policy with bad channel state switches from waiting to transmitting.

### 5.3 Numerical examples

In this section, we illustrate our technique with numerical examples under information pattern (b) where the channel states in the previous time slot are known. It allows us to trace out the trade-off between reliability and transmission energy cost for different deadline constraints, see the example with GE loss model in Figure 5.2.

We first study a small illustrative example on a link to show the potential of large energy saving with bursty links. We then use the larger network shown in Figure 4.3 where a source (node 1) sends packets to a sink (node 6) to analyze the end-to-end reliability and transmission energy cost for different deadline constraints. Both the Bernoulli link loss model and the GE link loss model are considered. For the Bernoulli link loss model, the loss probability $\Pi_B$ is randomly generated in $[0.2, 0.9]$. For the GE link loss model, the links are homogeneous with unconditional loss probability $\Pi_B = 0.5$, and the burstiness parameter $p_B$ of each link is uniformly chosen in the range $[0.75, 0.95]$.

#### 5.3.1 A small illustrative example

To illustrate the tradeoff between reliability and transmission energy cost, consider the two-node topology in Figure 5.3, with deadline $D = 2$ and link parameters $\Pi_B =$
0.5, \( p_G = 0.9 \) and \( p_B = 0.1 \). The deadline-constrained reliability is maximized by transmitting unconditionally with full transmission power, yielding

\[
R_{\text{max}} = \Pi_G + \Pi_B q_B = 0.55 \quad \text{and} \quad C_{\text{max}} = 1 + \Pi_B = 1.5.
\]

When accounting for the transmission cost, e.g. by letting \( \delta = 0.02 \) and disabling transmission power adjustment, on the other hand, the optimal policy holds the packet at node 1 at \( t = 0 \) if the channel is in bad state and transmits otherwise, resulting in a deadline-constrained reliability of \( R \) and energy cost \( C \) as follows

\[
R = \Pi_G(q_G + p_G p_B) + \Pi_B(q_B q_G + p_B q_B) = 0.545;
C = \Pi_G(1 + p_G \cdot 1) + \Pi_B \cdot 1 = 1.055.
\]

The transmission energy cost is decreased by around 30% with a very small reliability loss. In general, \( \delta \) trades between transmission energy cost and reliability by affecting the point when the optimal policy in bad link state switches from waiting to transmitting. A large \( \delta \) yields more waiting decisions, thus larger energy savings, but also lower reliability.
5.3.2 Reliability and energy tradeoff without power adjustments

Next, we analyze the end-to-end reliability and transmission energy cost for different deadline constraints without power adjustments.

A comparison of performance between GE loss model and Bernoulli loss model without power adjustments (PA) is shown in Figure 5.4. The maximum reliability is obtained for $\delta = 0$, i.e., when no considerations for the transmission cost are made. In this case, nodes will always try to transmit, even if all its links are likely to be in a bad state, provided that the remaining time to deadline is not less than the minimum hop count to the sink.

We note that when link losses are correlated in time (GE model), the energy penalty of aiming for the maximum reliability is substantial. For instance, for $D = 12$, the final 3% of reliability demands approximately double the energy. We further observe that higher energy gains typically occur with larger deadlines since the energy-optimal forwarding policy then can wait the appropriate time when links are in bad state. For the Bernoulli loss model, on the other hand, the expected transmission energy increases linearly with the reliability, and there is no threshold value after which the energy cost for additional reliability increases dramatically.

Furthermore, a clearly better performance of the optimal scheme over the minimum expected transmission count (ETX) path scheme is seen in Fig 5.5. In the minimum ETX path scheme, the packets can only be forwarded on the single minimum ETX path. The same techniques are used to evaluate the reliability-energy tradeoff on this path.
5.3.3 Reliability and energy tradeoff with power adjustments

We also evaluate the energy-reliability tradeoff with power adjustments. The success probability in good state now depends on the transmission power $r$, and is denoted by $q^r_G$. The success probability in bad state is always zero. We use values of success probability in GE good state $q^r_G$ in Table 5.1 that mimic what can be expected on an IEEE 802.15.4 platform and show a waterfall type relation between reliability and transmission power (see, e.g., [73, Figure 2.1]). The link reliability does not decrease linearly with the reduced transmission power in the beginning. A drop of 10% transmission power, for example, results in only a 2% decrease in the reliability. Nevertheless, the reliability falls sharply when the transmission power is lower than 80%.

Table 5.1: An example of power versus reliability in the good state of GE model.

<table>
<thead>
<tr>
<th>$r$</th>
<th>100%</th>
<th>90%</th>
<th>85%</th>
<th>83%</th>
<th>80%</th>
<th>78%</th>
<th>76%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q^r_G$</td>
<td>100%</td>
<td>98%</td>
<td>94%</td>
<td>90%</td>
<td>80%</td>
<td>70%</td>
<td>60%</td>
<td>50%</td>
</tr>
</tbody>
</table>

Figure 5.6 and Figure 5.7 show that power adjustments always allow to reduce energy cost for a given target reliability for both Bernoulli and GE model. According to Theorem 5.1.2, the optimal policy without power adjustments forwards the packet on a link with probability, e.g., $\alpha$, to achieve $\alpha$ portion of reliability and energy. However, with power adjustments, the same energy saving can be achieved by transmitting with relative power $\alpha$, but with a smaller decrease in reliability.
5.3. Numerical examples

Figure 5.6: Reliability-Energy curves with and w/o power adjustments (PA) under GE model.

Figure 5.7: Reliability-Energy curves with and w/o power adjustments (PA) under Bernoulli model.

To illustrate the benefits of power adjustments, Figure 5.8 shows the achievable reliability-energy pairs and the Pareto frontier with Bernoulli model under all deterministic policies from the dynamic programming solutions for $D = 2$. There are more pairs of achievable reliability and energy due to power adjustments. The
Pareto frontier is the convex hull of these points, and a better Pareto frontier is obtained by linearly interpolating the appropriately chosen pairs of reliability and energy.

5.4 A heuristic policy for correlated link losses

The energy cost under correlated losses increases dramatically when reliability requirement exceeds a certain value, see Figure 5.4. The main reason for this behavior is that the final reliability gains are obtained by transmitting, even when the channel is likely to be in a bad state. In this section, we develop a heuristic policy that attempts to operate at the “knee” of the energy-reliability trade-off curve by avoiding, to the extent possible, to transmit on channels that were in bad state during the last period. This policy is computed in two steps. First, we compute the maximum deadline-constrained reliability that can be achieved by policies that only transmit if the channel state in the previous time slot was good. We call this value $R_{knee}$. Then, we apply the technique in Section 5.1 to compute the energy-optimal policy that achieves this specific reliability value.

A variation of the dynamic programming framework in Section 3.3 can be applied to compute $R_{knee}$. We let $\delta = 0$, and $R^*(\cdot)$ coincides with $U^*(\cdot)$ in the dynamic programming framework. At each time $t$, a negative penalty $-1$ is induced if the packet is forwarded on the link in bad state. The dynamic programming procedure remains the same otherwise. It chooses the optimal policies that maximize the reliability $R^*_i(t)$ at each step $t$, and returns the reliability value $R^*_i(0)$, i.e., the estimated reliability $R_{knee}$ of the knee location. More specifically, at each time $t$, the
maximum conditional reliability \( R_i^*(t|\omega_i(t-1)) \) is computed similar to Eq. (3.9), and the computation of the reliability of staying \( R_i^t(t|\omega_i(t-1)) \) is the same as the Eq. (3.11). However, the reliability of forwarding \( R_j^i(t|\omega_i(t-1)) \) have two cases conditioned on the channel state. If \( \omega_{ij}(t-1) = G \) (good state), then

\[
R_j^i(t|\omega_i(t-1)) = \sum_{\omega_j(t)} \Pr(\omega_j(t)|\omega_i(t-1)) \cdot \left( q_{\omega_{ij}(t)}^R R_j^i(t+1) + (1 - q_{\omega_{ij}(t)}^R) R_i^j(t+1|\omega_i(t)) \right).
\]

If \( \omega_{ij}(t-1) = B \) (bad state), then \( R_j^i(t|\omega_i(t-1)) = -1 \).

\[\text{Figure 5.9: Reliability and energy values for the heuristic policy under GE loss model.}\]

We evaluate the heuristic policy with GE link loss model and no transmission power adjustment. Figure 5.9 shows that such policies strike a nice balance between energy and reliability.

5.5 Summary

In this chapter, we studied the minimum-energy packet forwarding policies with a guarantee on the deadline-constrained reliability. We proved that the optimal policy is a random selection between two deterministic policies found by the general dynamic programming framework for the weighted sum maximization problem. A simple threshold-type optimal policy was derived for Bernoulli link loss model, and for GE link loss model with good channel state observation in a line topology. Numerical examples of the energy-reliability tradeoff show that the energy cost
of achieving reliabilities close to the maximum is dramatic when links are bursty. Furthermore, the energy cost can be reduced by transmission power adjustments. Finally, a heuristic policy that strikes a good balance between energy and reliability was proposed.
Applications in networked control systems

Networked control has been an active area of research for more than a decade (e.g., [74] and the references therein) and the literature is by now rather extensive. However, the current research has mainly focused on control design methods for a high-level abstraction of the communication network in terms of average packet delay and/or loss, and efforts on the networking side to provide relevant services and primitives for networked control are still scarce. In this chapter, we will develop a co-design framework for wireless control systems, and show how the availability of a deadline-constrained packet forwarding primitive allows to establish a modular co-design framework that is provably optimal.

We consider a networked control system, where sensor measurements are sent over an unreliable multi-hop wireless network to the controller node. By parameterizing the system design in terms of the sampling time of the digital control loop, the co-design problem for single-loop wireless control systems can be separated into two well-defined subproblems: to schedule the multi-hop network with per-packet
deadline constraint, and to design a controller with optimum performance under (independent) packet losses, see Figure 6.1. The first subproblem has been considered in Chapter 4 and Chapter 5. The solution of the second subproblem is derived in [75] that includes the computation of the optimal controller and the estimation of the associated closed-loop performance. With this co-design framework, we address two networked control design problems: the optimal controller design with minimum performance loss, and the minimum energy packet forwarding with a guaranteed control performance.

6.1 Model and problem formulation

The network model is essentially the same as the one in Chapter 3. Communication is time slotted and each time slot is $t_s$ milliseconds long. The packet is forwarded on a multi-hop network with independent Bernoulli packet losses. Nodes do not have access to channel state, only their statistics, i.e., packet loss probabilities on their outgoing links. The design problem for the network is to develop a forwarding policy $\pi$ that determines if a node should forward a received packet or drop it, and to which node it should attempt to transmit.

In the following, we present the model of the process and controller and the control performance followed by the formulation of two networked control design problems.

6.1.1 Process and controller

We consider the control of a stochastic linear system

$$dx = Ax dt + Bu dt + dv_c$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control signal, $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ are the system matrices, and $v_c$ is a Wiener process with incremental covariance
6.1. Model and problem formulation

We assume that a noisy measurement of the system output

\[ y(kh) = \hat{C}x(kh) + w(kh) \]

is taken every sample period \( h \). Here \( w(kh) \) is a discrete-time zero-mean white noise Gaussian process, independent of the disturbance \( v_c \), and with covariance \( R_w \).

The controller and actuator nodes are assumed to be synchronized to the global clock and operate with a fixed lag \( \tau \leq h \) relative to the sampling times of the sensor. The controller uses the information available at times \( kh + \tau \) to compute the control action, and the actuator uses zero-order-hold and maintains the same control action between the controller updates \( [kh + \tau, (k + 1)h + \tau) \) for \( k = 1, 2, \ldots \) as in Figure 6.2.

6.1.2 Control performance

We consider a linear quadratic loss

\[
J = \mathbb{E} \left\{ \int_0^T \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}^T \begin{bmatrix} Q_{xx}^c & Q_{xu}^c \\ Q_{ux}^c & Q_{uu}^c \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} dt + x(T)^T Q_0^c x(T) \right\},
\]

where the matrices \( Q_{xx}^c \) and \( Q_0^c \) are symmetric and positive semi-definite while \( Q_{uu}^c \) is symmetric and positive definite. The expectation is taken over the disturbance, the measurement errors, and the random packet losses in the network.

The loss \( J \) is a function over the sampling interval \( h \), the forwarding policy \( \pi \), and the control law \( u \). Let \( D(\pi, h) \) denote the packet delay distribution for the forwarding policy \( \pi \) when sensor packets are injected with the rate \( 1/h \), and let \( J^*(D(\pi, h), h) \) denote the optimal control loss for the given networking parameters. Although it is possible to compute \( J^*(D(\pi, h), h) \) (see e.g. [76]), little is known about how the delay distribution depends on the forwarding policy and how the optimal control performance depends on these networking parameters. Hence, we restrict our attention to the time-triggered control architecture described in Section 6.1.1.

For this reason, only the packets that are received within \( \tau \) seconds from when they were sampled are used by the controller. Let \( \rho(\pi, h, \tau) \) be the probability that packets injected at the rate \( 1/h \) and forwarded using policy \( \pi \) arrive at the controller within \( \tau \) seconds. Hence, the controller performance can be written as \( J(\tau, \rho(\pi, h, \tau), h, u) \).

6.1.3 Objectives

We study the minimum control performance loss problem

\[
\min_{h, \pi, u} J(\tau, \rho(\pi, h, \tau), h, u),
\]

(6.2)
and the problem of minimum energy packet forwarding for a guaranteed control performance

\[
\begin{align*}
\text{minimize} & \quad E^\pi / h \\
\text{subject to} & \quad J(\tau, \rho(\pi, h, \tau), h, u) \leq J_{\text{req}}, \quad (6.3)
\end{align*}
\]

where \( E^\pi \) is the energy cost per injected packet under policy \( \pi \).

### 6.2 A modular co-design framework

In this section, we first summarize the main results on the optimal control design from [75] with a given deadline-constrained reliability \( \rho \) under sampling period \( h \) and time lag \( \tau \). Please refer to [75] for details if necessary. An optimal modular co-design framework is then proposed by using the monotonicity property of the optimal control loss function.

#### 6.2.1 Optimal linear quadratic Gaussian control

The optimal control is derived with a given deadline-constrained reliability \( \rho \) under sampling period \( h \) and time lag \( \tau \).

The time-triggered controller structure imposes that the network delivers sensor packets with a fixed delay of \( \tau = D t_s \) seconds and losses samples independently with probability \( 1 - \rho \), where \( \rho \) is the deadline-constrained reliability for the source node with deadline of \( \tau / t_s \) time slots. This time-triggered controller works on the information available at times \( kh + \tau \), holds the control signal constant over intervals \( [kh + \tau, (k+1)h + \tau) \), and uses control actions that are optimal in terms of the linear-quadratic loss function (6.1).

For notational convenience, we assume that \( T = Nh \) for some integer \( N > 0 \). Let \( u_k = u(kh) \) be the control signal computed at time \( kh + \tau \) and applied to the process during the time interval \( [kh + \tau, (k+1)h + \tau) \) and let \( x_k = x(kh) \). Then, the continuous-time loss function (6.1) can be transformed into an equivalent discrete-time loss

\[
J = \mathbb{E} \left\{ \sum_{k=0}^{N-1} \begin{bmatrix} x_k \\ u_{k-1} \\ u_k \end{bmatrix}^T \begin{bmatrix} Q_{xx}(\tau) + \Phi^T(\tau)Q_{xx}(\tau)\Phi(\tau) & Q_{xu}(\tau) + \Phi^T(\tau)Q_{xu}(h-\tau)\Gamma(\tau) \\ * & * \\ Q_{uu}(\tau) + \Phi^T(\tau)Q_{uu}(h-\tau)\Gamma(\tau) \end{bmatrix} \begin{bmatrix} x_k \\ u_{k-1} \\ u_k \end{bmatrix} \right\}
\]

\[
= \mathbb{E}_0 \mathbb{E}_u \mathbb{E}_u = \mathbb{E}_0 \mathbb{E}_u \mathbb{E}_u
\]

where \( \mathbb{E}_0 \) and \( \mathbb{E}_u \) are defined in (6.1).
where $\Phi(t) = e^{At}$, $\Gamma(t) = \int_0^t \Phi(s)Bds$, $Q_{xx}(t) = \int_0^t \Phi^T(s)Q_{xx}\Phi(s)ds$, $Q_{uu}(t) = \int_0^t \Phi^T(s)Q_{uu}\Phi(s)ds$ and $Q_{uv}(t) = \int_0^t \Phi^T(s)(Q_{xx}\Gamma(s)+Q_{xu})ds$.

The optimal control problem is then to compute the control sequence $\{u_k\}$ that minimizes the discrete-time loss function. The evolution of the system between sampling instants can be described in terms of the extended state vector $\xi_k \triangleq \text{col}\{x_k, u_{k-1}\}$ as

$$
\xi_{k+1} = \begin{bmatrix}
    e^{Ah} & \int_0^h e^{As}Bds
    \\
    0_{m \times m} & I_{m \times m}
\end{bmatrix} \xi_k + \begin{bmatrix}
    \int_0^h e^{As}Bds
    \\
    I_{m \times m}
\end{bmatrix} u_k + \begin{bmatrix}
    v_k
    \\
    0_{m \times n}
\end{bmatrix},
$$

where $v_k$ and $w_k$ are zero mean discrete-time Gaussian white noise with

$$
\mathbb{E}\left[\begin{bmatrix}
    v_k
    \\
    w_k
\end{bmatrix}\begin{bmatrix}
    v_k^T
    \\
    w_k^T
\end{bmatrix}\right] = \begin{bmatrix}
    R_v & 0
    \\
    0 & R_w
\end{bmatrix},
$$

where $R_v = \int_0^h e^{As}R_c e^{A^T s}ds$.

Note that $y_k$ is not computed until time $t = kh + \tau$, at which time $y_k$ is available to the controller unless it has been dropped by the network. Hence, the controller has access to the following information set when computing $u_k$:

$$
\mathcal{I}_k \triangleq \{y_k, U_{k-1}, R_k\},
$$

Here, $y_k = (y_k, \ldots, y_1)$, and $U_{k-1} = (u_{k-1}, \ldots, u_1)$, while $R_k = (\rho_k, \ldots, \rho_1)$ is the realization of the Bernoulli random variable $\rho_k$ that models successful packet transmissions.

**Estimator design**

The Kalman filter is the optimal estimator. The minimum mean square error (MMSE) estimate $\hat{\xi}_{k|k}$ of $\xi_k$ given by $\hat{\xi}_{k|k} = \mathbb{E}\{\xi_k | \mathcal{I}_k\}$ can be computed recursively starting from the initial conditions $\hat{\xi}_{0|-1} = \text{col}\{0_{n \times 1}, 0_{m \times 1}\}$ and $P_{0|-1} = P_0$. The innovation step is

$$
\hat{\xi}_{k+1|k} \triangleq \mathbb{E}\{\xi_{k+1} | \mathcal{I}_k\} = \Phi \hat{\xi}_{k|k} + \Gamma u_k \quad (6.4)
$$

$$
\hat{e}_{k+1|k} \triangleq \xi_{k+1} - \hat{\xi}_{k+1|k} = \Phi \hat{e}_{k|k} + G v_k \quad (6.5)
$$

$$
P_{k+1|k} \triangleq \mathbb{E}\{\hat{e}_{k+1|k}^T \hat{e}_{k+1|k} | \mathcal{I}_k\} = \Phi P_{k|k} \Phi^T + \tilde{R}_v \quad (6.6)
$$
Proposition 6.2.2. The optimal control law for the finite horizon LQG control is

\[ u_k = - (\Gamma^T S_{k+1} \Gamma + \Xi_{uu})^{-1} (\Gamma^T S_{k+1} \Phi + \Xi_{\xi u}) \hat{\xi}_{k|k} \]

which is a linear function of the estimated state. The matrix \( S_k \) evolves as the backward Riccati recursion

\[ S_k = \Phi^T S_{k+1} \Phi + \Xi_{\xi \xi} - (\Phi^T S_{k+1} \Gamma + \Xi_{\xi u})(\Gamma^T S_{k+1} \Phi + \Xi_{\xi u})^{-1} (\Gamma^T S_{k+1} \Phi + \Xi_{\xi u}) \]

where \( \hat{\xi}_{k|k} \) is the MMSE estimate of the state \( \xi_k \) based on the information set \( \mathcal{I}_k \) computed with the Kalman filter (6.4)–(6.10). As \( k \to \infty \), the Riccati recursion converges to a unique stationary solution \( S_\infty \) satisfying

\[ S_\infty = \Phi^T S_\infty \Phi + \Xi_{\xi \xi} - (\Phi^T S_\infty \Gamma + \Xi_{\xi u})(\Gamma^T S_\infty \Phi + \Xi_{\xi u})^{-1} (\Gamma^T S_\infty \Phi + \Xi_{\xi u}) \]

for which the associated stationary controller gain is

\[ L_\infty \triangleq \lim_{k \to \infty} L_k = - (\Gamma^T S_\infty \Gamma + \Xi_{uu})^{-1} (\Gamma^T S_\infty \Phi + \Xi_{\xi u}) \].
Optimal control loss

The loss function of the finite horizon LQG for the networked control system is

\[ J_N^\star(\rho) = \xi_0^T S_0 \xi_0 + Tr(S_0 P_0) \]
\[ + \sum_{k=0}^{N-1} Tr(S_{k+1} \hat{R}_v) + \sum_{k=0}^{N-1} Tr((\Phi^T S_{k+1} \Phi + \Xi \xi_\xi - S_k) E_\rho(P_{k|k})) , \]

where the expectation is taken over a Bernoulli sequence \( \{\rho_k\} \) with \( E\{\rho_k\} = \rho \). Since there does not exist any known efficient way of computing the expectation, one can use the upper and lower bounds on \( E_\rho(P_{k|k}) \) given in Proposition 6.2.1 to compute associated upper and lower bounds on the finite-horizon control loss \( J_N^{\text{min}}(\rho) \leq J_N^\star(\rho) \leq J_N^{\text{max}}(\rho) \). For the infinite horizon case, the bounds become

\[ J_\infty^{\text{min}} \triangleq \lim_{N \to \infty} \frac{1}{N} J_N^{\text{min}} \]
\[ = \text{Tr}(S_\infty \hat{R}_v) + (1 - \rho) Tr((\Phi^T S_\infty \Phi + \Xi \xi_\xi - S_\infty) P_\infty) , \]
\[ J_\infty^{\text{max}} \triangleq \lim_{N \to \infty} \frac{1}{N} J_N^{\text{max}} \]
\[ = \text{Tr}(S_\infty \hat{R}_v) + \text{Tr}((\Phi^T S_\infty \Phi + \Xi \xi_\xi - S_\infty)(P_\infty - \rho P_\infty C^T (C P_\infty C^T + R) C P_\infty^{-1})) , \]

where the matrices \( P_\infty \), \( P_\infty \) and \( S_\infty \), are given in Proposition 6.2.1 and Proposition 6.2.2, respectively.

6.2.2 Monotonicity of the optimal loss

The optimality of the co-design framework hinges on the observation that the achievable loss \( J \) for a fixed sampling interval \( h \) is monotone decreasing in the end-to-end reliability \( \rho \).

**Theorem 6.2.3.** For given sampling interval \( h \), the optimal control loss \( J_N^\star(\rho) \) is monotone decreasing in end-to-end reliability \( \rho \).

The proof relies on a coupling argument [77] on the underlying end-to-end loss processes; please see [75] for details. One issue is that there is no efficient way to compute the expectation of the covariance matrix with respect to the loss process apart from e.g. Monte Carlo simulation [78]. To overcome this problem, one could replace the true performance expression by the upper bound for the infinite-horizon case, \( J_\infty^{\text{max}}(\rho) \). The next result states that \( J_\infty^{\text{max}} \) has the same monotonicity properties as \( J_N^\star \).

**Corollary 6.2.4.** The upper bound on the control loss \( J_\infty^{\text{max}}(\rho) \) is monotone decreasing in end-to-end reliability \( \rho \).
6.2.3 A modular co-design framework

With the monotonicity property of the optimal control performance shown in Theorem 6.2.3, we develop an optimal modular co-design framework that separates the problem into well-defined and meaningful networking and control tasks.

Minimum control performance loss

The minimum control performance loss problem is

$$\min_{h, \pi, u} J(\tau, \rho(\pi, h, \tau), h, u).$$

After solving the optimal control problem, we can re-write the optimization problem as

$$\min_{h, \pi} J^*(\tau, \rho(\pi, h, \tau), h).$$

To limit the number of free parameters, we set the transmission delay $\tau$ equal to the sampling interval $h$, which results in the following form

$$\min_{h, \pi} J^*(\rho(\pi, h), h).$$

Note that for a fixed sampling interval $h$, the optimal control loss $J^*$ is monotone decreasing in the end-to-end reliability $\rho(\pi, h)$. It is, hence, optimal to schedule the network so as to maximize the deadline-constrained reliability, i.e. the probability that a packet will arrive at the controller node within a fixed delay $h$. This maximum deadline-constrained reliability problem has been solved in Chapter 4.

This implies that for a given sampling interval $h$ and under the restriction of one-sample delayed time-triggered control architectures, the optimal co-designed system is obtained by (a) scheduling the network to maximize deadline-constrained reliability, and (b) computing the control action using the optimal linear-quadratic controller under packet loss. The system configuration that minimizes the linear quadratic loss can then be obtained by a one-dimensional search over the sampling interval.

Minimum energy forwarding for a guaranteed control performance

In the minimum energy forwarding for a guaranteed control performance problem, we also let the fixed transmission lag $\tau$ equal to the sampling interval $h$ to limit the number of free parameters. The problem then becomes

$$\min_{h, \pi} E^\pi/h$$

subject to $J^*(\rho(\pi, h), h) \leq J_{req}$. 
6.3. Numerical examples

Using the monotonicity property of the control loss function $J^*(\rho(\pi, h), h)$, we can replace the performance constraint $J^*(\rho(\pi, h), h) \leq J_{\text{req}}$ with the reliability constraint $\rho(\pi, h) \geq \rho_{\text{min}}(h)$, where $\rho_{\text{min}}(h)$ is the unique end-to-end reliability for which $J^*(\rho_{\text{min}}(h), h) = J_{\text{req}}$. This value can be readily found by bisection due to the monotonicity property of $J^*$.

After replacing the control performance constraint with the associated end-to-end reliability requirement, the minimum energy problem is

$$\begin{align*}
&\text{minimize } \frac{E^\pi}{h} \\
&\text{subject to } \rho(\pi, h) \geq \rho_{\text{min}}(h).
\end{align*}$$

For a fixed $h$, this problem amounts to finding the policy of minimum energy that guarantees that the probability that packets arrive at their destination within the deadline $h$ is greater than $\rho_{\text{min}}$. An optimal solution to this problem is given in Chapter 5. Let $E_{\text{min}}(h)$ be the minimum energy of the optimal policy for a given $h$. The original problem can be reduced to

$$\begin{align*}
&\text{minimize } \frac{E_{\text{min}}(h)}{h}.
\end{align*}$$

The optimal solution is simply found by sweeping over all admissible sampling periods $h$.

6.3 Numerical examples

We are now ready to demonstrate our co-design procedure on numerical examples. We consider the following second-order linear system

$$dx = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & -2\alpha\zeta\omega_0 \end{bmatrix} xdt + \begin{bmatrix} 0 \\ \omega_0^2 \end{bmatrix} udt + dv_c,$$

$$y(\text{kh}) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(\text{kh}) + w(\text{kh}),$$

(6.11)

where $v_c$ has incremental covariance $R_c^v = \text{diag}\{0.5, 0.5\}$, and $w(\text{kh})$ has covariance $R_w = 10^{-4}$. Moreover, we let $Q_{xx}^c = \text{diag}\{2, 1\}$, $Q_{xz}^c = 0_{2 \times 1}$, $Q_{zu}^c = 1$, and $Q_0^c = 0_{2 \times 2}$ in Eq. (6.1) for computing linear quadratic loss.

Minimum control loss - maximum reliability application

We first consider the unstable system (6.11) with $\alpha = -1$, $\zeta = 1$, and $\omega_0 = 1$. Periodic samples of the output $y(\text{kh})$ are transmitted over the multi-hop wireless network. The network topology is shown in Figure 6.3. We consider sampling times $h \in [0, 45]t_s$ with $t_s = 10\text{ms}$ and three scenarios in which the network becomes increasingly unreliable.
Figure 6.3: A network topology with a sensor at 6-hop from the gateway.

Figure 6.4: The optimal control performance (upper bound $J_{\infty}^{\max}$) for a stable second-order linear system under a range of sampling periods. We show the performances on three network scenarios with different deadline-reliability curves. Note the optimal control loss is marked with a square.

Figure 6.4 shows the optimal closed-loop control losses for varying sampling intervals under different network scenarios. We note that there is no universal sampling interval or target end-to-end reliability. The optimal sampling interval ranges from 90ms for the most reliable network scenario to 250ms for the least reliable case, corresponding to a required end-to-end reliability of 65% and 82%, respectively. As the network becomes less reliable, more retransmissions are required to guarantee a sufficiently high reliability which causes the associated control loss to increase. Specifically, the optimal control loss (marked with a square in Figure 6.4) increases about a factor of ten from the most to the least reliable network scenarios.

We also consider the case when the system (6.11) is stable, and $\zeta \geq 0$, $\alpha = 1$, and $\omega_0 = \pi / \sqrt{1 - \zeta^2}$. This parameterization is chosen to yield roughly the same
optimal sampling interval independent of $\zeta$. Figure 6.5 shows how the trade-off curves change when we vary the damping ratio $\zeta$. Similar to the unstable open-loop systems, distinct co-design optima exist for the open-loop stable systems, but they become less relevant when the system poles are better damped.

**Guaranteed control performance - minimum energy application**

We demonstrate the minimum energy packet forwarding problem with a guaranteed control performance on an unstable second-order linear system in Eq. (6.11) with the parameters of $\alpha = -1$, $\zeta = 1$, and $\omega_0 = 1$. We use the network topology shown in Figure 4.3 where node 1 is the sensor and node 6 is the gateway. The loss probability on each link is randomly chosen between 0.1 and 0.8. Each time slot is 0.02 seconds long. We assume the transmission energy cost is the same and fixed for all nodes, and normalize this value to 1.

Figure 6.6 shows the minimum achievable energy cost for a range of control performance requirements. In this numerical example, the optimal control loss $J^*(\rho, h)$ is measured by the upper bound for the infinite horizon case. The minimum energy cost naturally increases when the control performance requirement becomes increasingly stringent. Moreover, the optimal sampling periods are shown in gray scale with shorter periods for smaller control loss. The minimum sampling period required to obtain a finite control loss is 40ms which corresponds to the minimum hop count of two in network topology of Figure 4.3. The associated optimal control
cost is four. An interesting observation is that it is very costly to obtain the minimal control loss, and that significant energy savings can be obtained by accepting a relatively small deterioration in the control performance.

Figure 6.6: The minimum energy cost for different control loss requirements and the corresponding optimal sampling periods (shown in gray scale).

Figure 6.7 shows the minimum energy cost for different sampling periods. The energy cost increases as the performance requirements become more stringent. However, for a given control performance requirement, the relation between sampling period and energy cost is neither monotone decreasing nor monotone increasing except for very lax control performance requirements. This necessitates the procedure of sweeping all sampling periods to obtain the minimum energy.

6.4 Summary

In this chapter, we considered the applications of real-time wireless communication with per-packet deadlines to a networked control system that uses multi-hop wireless communication for transmitting sensor data from process to controller. By utilizing the recent result that the minimal linear-quadratic control loss is monotonically decreasing in the end-to-end deadline-constrained reliability for a given sampling period, we proposed a modular co-design framework that can separate the problem into two well-defined networking and control design subproblems.

In the minimum control performance loss problem, the network should be operated to maximize the delay-constrained reliability, and the control design should optimize the closed-loop performance under packet loss and latency. In the minimum-energy packet forwarding problem for a guaranteed closed-loop control performance
problem, the control subproblem is to find the minimum end-to-end reliability that achieves a guaranteed control performance solved by bisection algorithm, and the network subproblem is to minimize the energy cost subject to a deadline-constrained reliability. With the optimal solutions of the two subproblems, the jointly optimal design is found by a one-dimensional search over sampling periods. This chapter ended with numerical examples of these two applications.
7.1 Conclusions

In this thesis, we have developed optimal policies for forwarding a single transient packet over a multi-hop network with lossy and bursty links, so as to maximize the probability that packets arrive within a given deadline. The routing topology is represented by a direct graph, and the links are modeled by a finite-state Markov chains with success probabilities jointly determined by the channel state and the transmission power. We first considered the problem of maximizing the probability that the packet is delivered within the deadline. We also addressed the problem of minimizing the energy cost while a deadline-constrained reliability is guaranteed. With these solutions, we are able to obtain the achievable deadline, reliability and energy cost triplets for a given network.

The maximum deadline-constrained reliability problem can be formulated as a Markov decision process, and solved by dynamic programming. The minimum energy problem is cast as a constrained Markov decision process. By Lagrangian approach, we showed that it is related to weighted sum of reliability and energy maximization problems, which can also be solved by our general dynamic programming framework. Further, we proved that the optimal policy is a random selection between two deterministic policies found by the dynamic programming.

We derived explicitly the solutions and the optimal policies of these two problems for the independent Bernoulli loss model and the two-state Gilbert-Elliot loss model under two information patterns. By several numerical examples, we tried to convey some insight and intuition that our solutions reveal: as links become more bursty, it becomes increasingly difficult to achieve high reliability and the optimal solution exploits an increasing number of paths. Moreover, the energy cost of achieving reliabilities close to the maximum is dramatic when links are bursty.

Finally, we applied the techniques to a networked control system that uses multi-hop wireless communication for transmitting sensor data from process to controller. The controller has a time-triggered architecture with a fixed time lag equal to the sampling period. We studied two problems: first, minimizing the control perfor-
Conclusions and future work

Based on the result that for a fixed sampling period the minimal control performance loss is monotonically decreasing in the deadline-constrained reliability, the networking design subproblem is decoupled from the control design and can be solved optimally with the techniques developed in this thesis.

7.2 Future work

We are currently investigating several extensions.

Firstly, we want to relax the assumption that a node can only access the states of their own outgoing links in the network. One future work is to use channel state information from immediate downstream nodes to improve forwarding performance. Another one is to develop solutions for other information patterns where link state estimators can only track part of the outgoing links or nodes do not know the states of their outgoing links until they are used for data transmission. Since the channel states of relevant links are not fully observable, the problems fall into the category of partially observable Markov decision processes. It can be solved by including the node’s belief on unobservable channel states in the dynamic programming [79, 80]. However, the direct application of these techniques faces the notorious “curse of dimensionality” problem.

Secondly, we are extending our work to include multiple heterogeneous real-time flows using the single-packet results. However, several issues arise for multiple flow case. The first issue is to define an appropriate objective. It could be to maximize the minimum on-time delivery ratio, to maximize a weighted sum of the deadline-constrained reliabilities, or to maximize the probability that all or some of the flows meet their deadline. The second issue is that related versions of multi-flow scheduling problems are known to be NP-hard [56], so tractable solutions are unlikely. Hence, we plan to develop heuristic techniques to deal with multiple flows.

Thirdly, the energy and the time cost for acquiring channel state information can be included. The fact that time cost for probing is usually smaller than one time slot breaks the Markov property of the decision problem. The extensions to learn the link parameters using multi-arm bandit formulation are also interesting.

Finally, we plan to implement the techniques on a real wireless sensor network. The main issue here is the bursty link model. Recent works on bursty link model propose either a higher-order and more complex Markovian link loss model [81] or a single parameter bursty link metric [16]. The tradeoff between the complexity of the bursty link model and the performance has to be studied.
Chapter A
Proofs

A.1 Proof of Theorem 4.2.1

Intuitively, Eq. (4.3) suggests that if there exists a $j \in \mathcal{N}_i$ such that $R_j^*(t + 1) > R_i^*(t + 1 | \omega_i(t))$ for all $\omega_i(t)$, then it is optimal to forward the packet to a neighbor, i.e., withholding the packet is not optimal. The next lemmas are necessary for the proof of Theorem 4.2.1.

Lemma A.1.1. For any node $i$ and time $t$, $R_i^*(t) \geq R_i^*(t + 1)$.

Proof. The optimal end-to-end reliability for any node $i$ and time $t$ under the GE loss model and the information pattern (b) with $\delta = 0$ and $I_i(t) = \omega_i(t - 1)$ follows from Eq. (3.6)

$$R_i^*(t) = \sum_{\omega_i(t-1)} \Pr\{\omega_i(t-1)\} R_i^*(t | \omega_i(t-1)).$$

Moreover, from Eq. (4.3), we can see that

$$R_i^*(t | \omega_i(t-1)) \geq R_i^*(t | \omega_i(t-1)) = \sum_{\omega_i(t)} \Pr\{\omega_i(t) | \omega_i(t-1)\} R_i^*(t + 1 | \omega_i(t)).$$

Thus

$$R_i^*(t) = \sum_{\omega_i(t-1)} \Pr\{\omega_i(t-1)\} R_i^*(t | \omega_i(t-1))$$

$$\geq \sum_{\omega_i(t-1)} \Pr\{\omega_i(t-1)\} \cdot \sum_{\omega_i(t)} \Pr\{\omega_i(t) | \omega_i(t-1)\} R_i^*(t + 1 | \omega_i(t))$$

$$= \sum_{\omega_i(t)} \Pr\{\omega_i(t)\} R_i^*(t + 1 | \omega_i(t)) = R_i^*(t + 1).$$
Lemma A.1.2. For a given $t$, if there exists a node $j \in \mathcal{N}_i$ such that $R_j^*(t+1) > R_j^*(t+1|\omega_i(t)) \forall \omega_i(t)$, we can also find a node $j \in \mathcal{N}_i$ such that $R_j^*(t) > R_j^*(t|\omega_i(t-1)) \forall \omega_i(t-1)$.

Proof. Assume that at time $t$, $R_j^*(t+1) > R_j^*(t+1|\omega_i(t))$ for all $\omega_i(t)$ at some nodes $j \in \mathcal{N}_i$. Then, the maximum conditional reliability from Eq. (4.3) is

$$R_j^*(t|\omega_i(t-1)) = \max_{j \in \mathcal{N}_i} R_j^*(t|\omega_i(t-1))$$

$$= \sum_{\omega_i(t)} \Pr\{\omega_i(t)|\omega_i(t-1)\}$$

$$\left(1_{\{\omega_j\ast(t)=B\}} R_j^*(t+1) + 1_{\{\omega_j\ast(t)=R\}} R_j^*(t+1|\omega_i(t))\right),$$

where $j^*$ is the forwarding node that obtains $R_j^*(t|\omega_i(t-1))$ for which, by assumption, we have $R_j^*,(t+1) > R_j^*(t+1|\omega_i(t))$ for all $\omega_i(t)$. Hence, we have

$$R_{j^*}^*(t|\omega_i(t-1)) < \sum_{\omega_i(t)} \Pr\{\omega_i(t)|\omega_i(t-1)\} R_{j^*}^*(t+1) = R_{j^*}^*(t+1).$$

By Lemma A.1.1, $R_j^*(t) \geq R_j^*,(t+1)$. Then, it follows that $R_j^*,(t) > R_j^*(t|\omega_i(t-1)) \forall \omega_i(t-1)$, i.e., we have found the node $j^* \in \mathcal{N}_i$. \hfill $\square$

Lemma A.1.3. For any node $i$ with minimum hop-count $h_i$, the optimal end-to-end reliability $R_i^*(t)$ at time $t$ under the GE loss model and information pattern (b) satisfies

$$R_i^*(t|\omega_i(t-1)) = R_i^*(t) = 0 \text{ if } t + h_i > D, \text{ and } R_i^*(t) > 0 \text{ if } t + h_i \leq D.$$ 

Proof. Transmitting a packet on each hop takes at least one time slot. Thus, at least $h_i$ time slots are required to deliver a packet to the destination. Since the deadline time is $D$, $R_i^*(t) = R_i^*(t|\omega_i(t-1)) = 0$ for all $t + h_i > D$ and $\forall \omega_i(t-1)$. Moreover, since any link with zero reliability in both good and bad states would be removed from the routing topology, the reliability at time $t + h_i \leq D$ is strictly positive, i.e., $R_i^*(t) > 0$. \hfill $\square$

We are now ready to prove Theorem 4.2.1.

Proof of Theorem 4.2.1:

Proof. Consider node $i$ with hop count $h_i$ holding a packet at time $t$. It follows from Lemma A.1.3 that for any time $t + h_i > D$, $R_i^*(t) = R_i^*(t|\omega_i(t-1)) = 0$. Thus if $t + h_i > D$, it is optimal to withhold the transmission. Now let us consider $t = D - h_i$.

We can find a node $j^*$ such that the minimum hop count of $j^*$ is $h_j^* = h_i - 1$. We have $R_j^*(D-h_i+1) = R_j^*(D-h_i+1|\omega_i(D-h_i)) = 0$ and $R_j^*(D-h_i) = R_j^*(D-h_i+1) > 0$ from Lemma A.1.3. Hence, $R_j^*(D-h_i+1) > R_i^*(D-h_i+1|\omega_i(D-h_i))$. 

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Now, applying Lemma A.1.2 recursively from \( t = D - h_i \) to \( t = 1 \), one can always find a node \( j \in N_i \) such that \( R^*_j(t) > R^*_i(t) \) for \( 1 \leq t \leq D - h_i + 1 \). By checking Eq. (4.3), it means that \( R^*_i(t|\omega_i(t)) > R^*_i(t|\omega_i(t - 1)) \) for \( t + h_i \leq D \). Hence the optimal forwarding policy at node \( i \) at time \( t + h_i \leq D \) is to forward the packet.

\[ \square \]

## A.2 Proof of Theorem 5.1.2

### Lemma A.2.1

\( R^*(\delta) \) and \( C^*(\delta) \) are non-increasing functions for \( \delta \geq 0 \).

**Proof.** Since \( \delta \geq 0 \), increasing \( \delta \) results in a smaller utility of forwarding to neighbors in Eq. (3.10), and the optimal policy will not use more transmissions or higher transmit power, hence the optimal reliability and the optimal energy cost cannot become larger.

\[ \square \]

**Proof of Lemma 5.1.1:**

At each step of the dynamic programming, the number of choices is limited by the number of neighboring nodes and channel states. We have a finite number of policies that lead to a finite number of optimal reliabilities. Thus, \( \mathcal{R} \) is a finite set.

For any given \( \delta_1, \delta_2 \in \Delta_R \), we have \( R^*(\delta_1) = R^*(\delta_2) \) with optimal policies \( \pi^*(\delta_1) \) and \( \pi^*(\delta_2) \), respectively. Suppose that \( C^*(\delta_1) \neq C^*(\delta_2) \). Without loss of generality, we let \( \delta_1 < \delta_2 \). According to Lemma A.2.1, we have \( C^*(\delta_1) \geq C^*(\delta_2) \) and since, by assumption, \( C^*(\delta_1) \neq C^*(\delta_2) \), we have \( C^*(\delta_1) > C^*(\delta_2) \). The optimal utility with \( \delta_1 \) is then \( R^*(\delta_2) - \delta_1 \cdot C^*(\delta_2) \). However if we apply policy \( \pi^*(\delta_2) \), the utility with \( \delta_1 \) is \( R^*(\delta_2) - \delta_1 \cdot C^*(\delta_2) > R^*(\delta_1) - \delta_1 \cdot C^*(\delta_1) \), which contradicts the optimality of \( \pi^*(\delta_1) \). Hence, for all \( R \in \mathcal{R} \), \( C^*(\delta) \) is unique for all \( \delta \in \Delta_R \).

\[ \square \]

**Proof of Theorem 5.1.2:**

Let \( g(\delta) \triangleq 1/\delta \cdot \max_\tau \{ R^\tau - \delta \cdot C^\tau \} - 1/\delta \cdot R_{\text{req}} \) and \( h(\delta) \triangleq \max_\tau \{ R^\tau - \delta \cdot C^\tau \} \). The proposed dynamic programming framework computes \( h(\delta) \) for a given value of \( \delta \) and returns an optimal history-independent and deterministic policy. According to Markov decision process theory [68], \( h(\delta) \) can be formulated as a linear program whose objective function coefficients depend on \( \delta \), and it can be shown that \( h(\delta) \) is a continuous function over \( \delta \). Hence, \( g(\delta) \) is also a continuous function over \( \delta \) because \( g(\delta) = 1/\delta \cdot h(\delta) - 1/\delta \cdot R_{\text{req}} \).

For a given \( R^{(m)} \in \mathcal{R} \), by Lemma A.2.1 and the fact \( \mathcal{R} \) is a finite set from Lemma 5.1.1, we have that \( \Delta_{\mathcal{R}^{(m)}} \) is an interval. Moreover, \( h(\delta) = R^*(\delta) - \delta C^*(\delta) \) is a continuous function over \( \delta \) and \( C^*(\delta) \) is unique for \( \delta \in \Delta_{\mathcal{R}^{(m)}} \). Hence, this interval is closed; let us denote it \( \Delta_{\mathcal{R}^{(m)}} = [\delta_{m-}, \delta_{m+}] \). Then, the function \( g(\delta) \) is

\[
g(\delta) = 1/\delta (R^{(m)} - R_{\text{req}}) - C^{(m)}, \quad \delta \in [\delta_{m-}, \delta_{m+}]. \tag{A.1}
\]
Now let $R^{(1)} = \max\{ R \in \mathcal{R} : R \leq R_{\text{req}} \}$ and $R^{(2)} = \min\{ R \in \mathcal{R} : R > R_{\text{req}} \}$ with associated energy costs $C^{(1)}$ and $C^{(2)}$. Note that $R^{(1)} \leq R_{\text{req}} < R^{(2)}$. Their associated $\delta$ range is $\Delta_{R^{(1)}} = [\delta_{1-}, \delta_{1+}]$ and $\Delta_{R^{(2)}} = [\delta_{2-}, \delta_{2+}]$. Furthermore, we have $\delta^* \triangleq \delta_{2+} = \delta_{1-}$ because $h(\delta)$ is a continuous function. Since $R^*(\delta)$ is a non-increasing function over $\delta$ from Lemma A.2.1, we have $R^*(\delta) \leq R^{(1)} \leq R_{\text{req}}$ for $\delta \geq \delta_{1+}$ and $R^*(\delta) \geq R^{(2)} > R_{\text{req}}$ for $\delta \leq \delta_{2-}$. Thus, we have $R^*(\delta) \leq R_{\text{req}}$ for $\delta \geq \delta_{1-}$ and $R^*(\delta) > R_{\text{req}}$ for $\delta \leq \delta_{2+}$. Furthermore, by Eq. (A.1), $g(\delta)$ is a decreasing function for $\delta \leq \delta_{2+}$ and a non-decreasing function for $\delta \geq \delta_{1-}$, so the minimum value of $g(\delta)$ is obtained for $\delta = \delta^* = \delta_{2+} = \delta_{1-}$. The optimal $\delta^*$ can be found from

$$1/\delta^*(R^{(2)} - R_{\text{req}}) - C^{(2)} = 1/\delta^*(R^{(1)} - R_{\text{req}}) - C^{(1)},$$

and the minimal energy cost is

$$C^* = C^{(1)} + \frac{R_{\text{req}} - R^{(1)}}{R^{(2)} - R^{(1)}} (C^{(2)} - C^{(1)}).$$

Suppose the optimal policies to obtain $(R^{(1)}, C^{(1)})$ and $(R^{(2)}, C^{(2)})$ are $\pi^{(1)}$ and $\pi^{(2)}$ respectively. The policy $\pi^*$ that randomizes between $\pi^{(1)}$ and $\pi^{(2)}$ with probabilities

$$\theta^{(1)} = \frac{R^{(2)} - R_{\text{req}}}{R^{(2)} - R^{(1)}} \quad \text{and} \quad \theta^{(2)} = \frac{R_{\text{req}} - R^{(1)}}{R^{(2)} - R^{(1)}},$$

achieves this minimum energy. Thus, it is an optimal policy, which concludes the proof.

### A.3 Proof of Theorem 5.2.1

Let $p_{ij}$ and $q_{ij}$ denote the packet loss and success probability of link $(i, j)$, respectively. Since packet losses are independent between two consecutive time slots, the optimal policy and the maximum utility are independent from channel states. Thus, the initial condition becomes

$$U^*_i(D) = \begin{cases} 1 & \text{if } i = N, \\ 0 & \text{if } i \neq N, \end{cases}$$

and the update at each step from Eq. (3.9) can be re-written as

$$U^*_i(t) = \max_{j \in \mathcal{N}_i} \{ \max\{U^*_j(t), U^*_i(t)\} \}
= \max_{j \in \mathcal{N}_i} \{ \max\{q_{ij}U^*_j(t+1) + p_{ij}U^*_i(t+1) - \delta, U^*_i(t+1)\} \}. \quad (A.2)$$

Note that $U^*_j(t)$ is utility of forwarding to the neighbor $j$ with full transmission power.
The next lemma shows that the optimal policy for a node \( i \) with minimum hop count \( h_i \) is to withhold the packet for any time \( t > D - h_i \).

**Lemma A.3.1.** Given a node \( i \) with minimum hop count \( h_i \),

\[
j^*_i(t) = i \quad \text{and} \quad U^*_i(t) = 0, \quad \forall t > D - h_i.
\]

**Proof.** The minimum time for the packet to be forwarded to the destination is \( h_i \). If \( t > D - h_i \), there are not enough time slots. Thus, \( R^*_i(t) \) is zero for any policies. Any forwarding decisions incur an energy cost leading to the utility less than zero. However, the policy of waiting all the time has zero utility. Thus, the waiting policy is optimal. This proves the lemma. \( \square \)

The next lemma shows the induction step for the proof. Note that \( U^*_i(t) = U^*_i(t + 1) \) with Bernoulli loss model.

**Lemma A.3.2.** For a given \( t \), if \( \max_{j \in N_i} U^*_j(t) > U^*_i(t + 1) \), then \( \max_{j \in N_i} U^*_j(t - 1) > U^*_i(t) \).

**Proof.** Since \( \max_{j \in N_i} U^*_j(t) > U^*_i(t + 1) \), we have \( U^*_i(t) = \max_{j \in N_i} U^*_j(t) > U^*_i(t + 1) \) according to Eq. (A.2). Suppose \( j^* \in N_i \) is the optimal forwarding node at time \( t \). Hence, we have \( U^*_j(t) = q_{ij} \cdot U^*_i(t + 1) + p_{ij} \cdot U^*_i(t + 1) - \delta \). According to Eq. (A.2), \( U^*_j(t) \geq U^*_i(t + 1) \) for all \( i \). Thus it holds also for node \( j^* \), i.e., \( U^*_j(t) \geq U^*_i(t + 1) \). Finally,

\[
\max_{j \in N_i} U^*_j(t - 1) \geq q_{ij} \cdot U^*_j(t) + p_{ij} \cdot U^*_i(t) - \delta \\
> q_{ij} \cdot U^*_j(t + 1) + p_{ij} \cdot U^*_i(t + 1) - \delta \\
= U^*_i(t)
\]

This proves the lemma. \( \square \)

**Proof of Theorem 5.2.1:**

Let us consider node \( i \) with minimum hop count \( h_i \) holding a packet at time \( t \). It follows from Lemma A.3.1 that the optimal policy for time \( t > D - h_i \) is to withhold the packet, yielding \( U^*_i(t) = 0 \). Starting from \( t = D - h_i \), the optimal forwarding policy becomes

\[
j^*_i(D - h_i) = \arg \max \left\{ \max_{j \in N_i} \{ U^*_j(D - h_i) \}, U^*_i(D - h_i + 1) \right\}
\]

\[= \arg \max \left\{ \max_{j \in N_i} \{ q_{ij} U^*_j(D - h_i + 1) - \delta \}, 0 \right\}, \tag{A.3}
\]

since \( U^*_i(D - h_i + 1) = 0 \). Note that if \( j^*_i(D - h_i) = i \), then \( U^*_i(D - h_i) = 0 \). Therefore, by applying Eq. (A.3) recursively from \( t = D - h_i \) to \( t = D - h_i + 1 \), we have

\[
j^*_i(t) = i \quad \text{for} \quad t > D - h_i
\]
with \( \overline{t}_i = \arg \min_{h \in [h_i, D-1]} \left\{ \max_{j \in N_i} q_{ij} u_j^*(D-h+1) > \delta \right\} \). Thus,

\[
U_i^*(t) = 0 \quad \text{for} \quad t > D - \overline{t}_i
\]

Now let us consider \( t = D - \overline{t}_i \). From the definition of \( \overline{t}_i \), \( \max_{j \in N_i} \left\{ q_{ij} u_j^*(D - \overline{t}_i + 1) \right\} - \delta > u_i^*(D - \overline{t}_i + 1) = 0 \). We also have \( \max_{j \in N_i} \left\{ q_{ij} u_j^*(D - \overline{t}_i + 1) \right\} - \delta \). Thus we can apply Lemma A.3.2 from \( t = D - \overline{t}_i \) and obtain

\[
\max_{j \in N_i} \left\{ u_i^* \right\} \quad \text{for} \quad 0 \leq t \leq D - \overline{t}_i.
\]

\[
\square
\]

### A.4 Proof of Theorem 5.2.2

**Lemma A.4.1.** At any node \( i \) and time \( t \), we have

\[
U_i^*(t|G) \leq U_i^*(t-1|G), U_i^*(t|B) \leq U_i^*(t-1|B) \quad \text{and} \quad U_i^*(t) \leq U_i^*(t-1).
\]

**Proof.** We prove this lemma by induction. Suppose at node \( i \) and a given \( t \), we have \( U_i^*(t+1|G) \leq U_i^*(t|G), U_i^*(t+1|B) \leq U_i^*(t|B) \) and \( U_i^*(t+1) \leq U_i^*(t) \) where \( j \) is its next hop node. Hence,

\[
\begin{align*}
U_i^*(t-1|G) & = \max \left\{ q_G u_j^*(t) + p_G u_i^*(t|B) - \delta, q_G u_j^*(t|G) + p_G u_i^*(t|B) \right\} \\
& \geq \max \left\{ q_G u_j^*(t+1) + p_G u_i^*(t+1|B) - \delta, q_G u_j^*(t+1|G) + p_G u_i^*(t+1|B) \right\} \\
& = U_i^*(t|G)
\end{align*}
\]

and

\[
\begin{align*}
U_i^*(t-1|B) & = \max \left\{ q_B u_j^*(t) + p_B u_i^*(t|B) - \delta, q_B u_j^*(t|G) + p_B u_i^*(t|B) \right\} \\
& \geq \max \left\{ q_B u_j^*(t+1) + p_B u_i^*(t+1|B) - \delta, q_B u_j^*(t+1|G) + p_B u_i^*(t+1|B) \right\} \\
& = U_i^*(t|B)
\end{align*}
\]

Since \( U_i^*(t) = \Pi_B U_i^*(t|B) + (1 - \Pi_B) u_i^*(t|G) \), we have \( U_i^*(t-1) \geq u_i^*(t) \).

At the one-hop away node and at time \( t = D - 1 \), the condition for inductive step holds since \( U_i^*(t) = 0 \) for all \( t \) and \( U_i^*(D|G) = U_i^*(D|B) = 0 \). Thus, we apply this inductive step at one-hop away node from \( t = D - 1 \) to \( t = 0 \). As a result, we get \( U_i^*(t) \leq U_i^*(t-1) \) for all \( t \) at one-hop away node. Therefore, the same procedure can be applied for two-hop away node. We can keep this procedure till the source node.

\[
\square
\]
Lemma A.4.2. At any node $i$ and time $t$, we have

$$U_i^*(t|G) \leq U_j^*(t+1), \quad U_i^*(t|B) \leq U_j^*(t+1).$$

Proof. We prove this lemma by induction. Suppose at node $i$ and time $t$, we have $U_i^*(t+1|G) \leq U_j^*(t+2)$ and $U_i^*(t+1|B) \leq U_j^*(t+2)$. Then

$$U_i^*(t|G) = \max\{q_G U_j^*(t+1) + p_G U_i^*(t+1|B) - \delta \cdot q_G U_i^*(t+1|G) + p_G U_i^*(t+1|B)\}$$

$$\leq \max\{q_G U_j^*(t+1) + p_G U_j^*(t+2) - \delta \cdot q_G U_j^*(t+2) + p_G U_j^*(t+2)\}$$

$$\leq \max\{q_G U_j^*(t+1) + p_G U_j^*(t+1) - \delta, U_j^*(t+1)\} \leq U_j^*(t+1).$$

Note that the condition $U_j^*(t+2) \leq U_j^*(t+1)$ from Lemma A.4.1 is used. Similarly,

$$U_i^*(t|B) = \max\{q_B U_j^*(t+1) + p_B U_i^*(t+1|B) - \delta \cdot q_B U_i^*(t+1|G) + p_B U_i^*(t+1|B)\}$$

$$\leq \max\{q_B U_j^*(t+1) + p_B U_j^*(t+2) - \delta \cdot q_B U_j^*(t+2) + p_B U_j^*(t+2)\}$$

$$\leq \max\{q_B U_j^*(t+1) + p_B U_j^*(t+1) - \delta, U_j^*(t+1)\} \leq U_j^*(t+1).$$

The same induction procedure as the proof in Lemma A.4.1 can be applied here to prove this Lemma.

Lemma A.4.3. At any node $i$ and time $t$, we have

$$U_i^*(t|B) \leq U_i^*(t|G).$$

Proof. We prove this lemma by induction from $t = D-1$ to $t = 0$ at node $i$. Suppose $U_i^*(t+1|B) \leq U_i^*(t+1|G)$ for a given time $t$. Moreover, from Lemma A.4.1 and Lemma A.4.2, we have $U_i^*(t+1|B) \leq U_j^*(t+1)$. Moreover, we assume $q_G \geq q_B$. Hence, by comparing Eq. (5.2) and Eq. (5.3), we can easily see that $U_i^*(t|G) \geq U_i^*(t|B)$. Since $U_i^*(D|G) \geq U_j^*(D|B)$, the lemma can be proved by induction.

Lemma A.4.4. At any time $t$, if $\zeta_t > \frac{\delta}{q_G}$, then $\zeta_t-1 > \frac{\delta}{q_G}$.

Proof. Since $\zeta_t = U_j^*(t+1) - U_i^*(t+1|G) > \frac{\delta}{q_G}$, the optimal policy is to forward to node $j$, yielding optimal utility

$$U_j^*(t|G) = q_G U_j^*(t+1) + p_G U_i^*(t+1|B) - \delta.$$
Hence,
\begin{align*}
\zeta_{t-1} &= U^*_i(t) - U^*_i(t|G) \\
&= U^*_i(t) - q_G U^*_j(t + 1) - p_G U^*_i(t + 1|B) + \delta \\
&\geq p_G [U^*_j(t + 1) - U^*_i(t + 1|B)] + \delta \\
&\geq p_G [U^*_j(t + 1) - U^*_i(t + 1|G)] + \delta \\
&= p_G \zeta_t + \delta > p_G \frac{\delta}{q_G} + \delta = \frac{\delta}{q_G}.
\end{align*}

Note that we use $U^*_i(t) \geq U^*_j(t + 1)$ from Lemma A.4.1, and $U^*_i(t + 1|B) \leq U^*_i(t + 1|G)$ from Lemma A.4.3.

\textbf{Proof of Theorem 5.2.2:}

\textit{Proof.} For any time $t > D - h_i$, the number of time slots to forward the packet is smaller than the hop count. Thus, the optimal policy is to stay at the current node $i$.

For any time $t \in [D - \tilde{h}_i + 1, D - h_i]$, we have $\zeta_t \leq \frac{\delta}{q_G}$ based on the definition of $\tilde{h}_i$. Hence, the optimal forwarding policy is to wait at node $i$.

At time $t = D - \tilde{h}_i$, we have $\zeta_t > \frac{\delta}{q_G}$. According to Lemma A.4.4, we have $\zeta_t > \frac{\delta}{q_G}$ for all $t \leq D - \tilde{h}_i$. Hence, the optimal forwarding policy is to transmit to node $j$ for all time $t \in [0, D - \tilde{h}_i]$. This concludes the proof.  \hfill \□

\textbf{Proof of Theorem 5.2.2:}

\textit{Proof.} For any time $t > D - h_i$, the number of time slots to forward the packet is smaller than the hop count. Thus, the optimal policy is to stay at the current node $i$.

For any time $t \in [D - \tilde{h}_i + 1, D - h_i]$, we have $\zeta_t \leq \frac{\delta}{q_G}$ based on the definition of $\tilde{h}_i$. Hence, the optimal forwarding policy is to wait at node $i$.

At time $t = D - \tilde{h}_i$, we have $\zeta_t > \frac{\delta}{q_G}$. According to Lemma A.4.4, we have $\zeta_t > \frac{\delta}{q_G}$ for all $t \leq D - \tilde{h}_i$. Hence, the optimal forwarding policy is to transmit to node $j$ for all time $t \in [0, D - \tilde{h}_i]$. This concludes the proof.  \hfill \□
### List of abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACK</td>
<td>Acknowledgement</td>
</tr>
<tr>
<td>AP</td>
<td>Access point</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive white Gaussian noise</td>
</tr>
<tr>
<td>CMDP</td>
<td>Constrained Markov decision process</td>
</tr>
<tr>
<td>CSMA</td>
<td>Carrier sense multiple access</td>
</tr>
<tr>
<td>DP</td>
<td>Dynamic programming</td>
</tr>
<tr>
<td>EDF</td>
<td>Earliest deadline first</td>
</tr>
<tr>
<td>ETX</td>
<td>Expected transmission count</td>
</tr>
<tr>
<td>FIFO</td>
<td>First in first out</td>
</tr>
<tr>
<td>GE</td>
<td>Gilbert-Elliot</td>
</tr>
<tr>
<td>i.i.d.</td>
<td>Independent and identically distributed</td>
</tr>
<tr>
<td>LQG</td>
<td>Linear-quadratic Gaussian</td>
</tr>
<tr>
<td>M2M</td>
<td>Machine to machine communication</td>
</tr>
<tr>
<td>MDP</td>
<td>Markov decision process</td>
</tr>
<tr>
<td>MMSE</td>
<td>Minimum mean square error</td>
</tr>
<tr>
<td>NAK</td>
<td>Negative-acknowledge</td>
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<tr>
<td>PA</td>
<td>Power adjustments</td>
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<tr>
<td>TDMA</td>
<td>Time division multiple access</td>
</tr>
<tr>
<td>w/o</td>
<td>Without</td>
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<tr>
<td>WSN</td>
<td>Wireless sensor network</td>
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</table>


[13] *IEEE 802.15.4e*, IEEE 802.15 WPAN Task Group 4e Std.


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