Master of Science Thesis

Running of Neutrino Parameters in Extra Dimensions

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Abstract

In recent years, extra-dimensional models have received new attention as possible extensions to the Standard Model (SM). In this thesis, we study a six-dimensional model with universal extra dimensions (UEDs). The extra dimensions are compactified, and all fields in the SM can probe them. From our four-dimensional point of view, we perceive them as towers of heavy particles, corresponding to the SM fields. In this thesis, we discuss the six-dimensional UED model together with an effective neutrino mass operator. We determine the renormalization group equations (RGEs) relevant to determine the renormalization group running of neutrino parameters. The RGEs are solved numerically and we thus determine the energy-dependence of the lepton mixing angles, the neutrino masses, and the gauge couplings in this model. We find that the mixing angle θ_{12}, the neutrino masses and the gauge couplings exhibit significant running. The running has a power-law dependence, as it is boosted by contributions from the extra dimensions.
The model is of particular interest, since it gives predictions at energies obtained in the Large Hadron Collider (LHC). The experiment have already provided restrictions of the size on the extra dimensions, and will hopefully enlighten us further regarding their existence.

Key words: Universal extra dimensions, neutrino masses, renormalization group running.

Preface

This thesis is the result of my work from September 2011 to March 2012, for the degree of Master of Science in Engineering, in the Theoretical Particle Physics group at the Department of Theoretical Physics, the Royal Institute of Technology (KTH). The work concerns neutrinos, in an extra-dimensional extension to the SM.

Overview of the thesis

The structure of the thesis is as follows. In Ch. 1, we give an introduction to physics in general, and to theoretical particle physics and extra dimensions in particular. In Ch. 2, we introduce quantum field theory (QFT), and we give an introduction to the SM. We also present a six-dimensional model with UEDs, where all the SM fields probe the extra dimensions. In our four-dimensional world, we perceive the extra dimensions as infinite, so-called Kaluza–Klein (KK) towers of heavy particles, corresponding to each SM field. We want to renormalize the SM fields and interactions, and are thus particularly interested in interactions involving at least one SM particle, and two KK modes. In Ch. 2, we have derived the explicit expression used in these interactions of particular interest. Furthermore, we have explicitly derived the Lagrangian terms of specific interest for this thesis. We have introduced the Yukawa interactions and the effective neutrino mass operator in the six-dimensional model and derived the corresponding expressions in the effective four-dimensional Lagrangian. In addition, the Feynman rules corresponding to the effective Lagrangian have been derived, they are presented in appendix A. In Ch. 3, we discuss the concepts of renormalization and renormalization group equations (RGEs). In Ch. 4, the RGEs for the Higgs self-coupling, the Yukawa couplings, and the effective neutrino mass operator are calculated. Together with the ones for the gauge couplings, which have been calculated elsewhere, the RGEs are solved numerically. Thus, the RG evolution of the lepton mixing angle, the neutrino masses, and the gauge couplings is determined.

Notation and conventions

We use the Einstein summation convention throughout the thesis, *i.e.*, the convention that repeated indices are summed over. For the sake of clarity and simplicity, we use natural units, $\hbar = c = 1$. We employ the sign convention for the Minkowski metric tensor given by

$$(\eta_{\mu\nu}) = \mathrm{diag}(1, -1, -1, -1). \tag{1}$$

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Contents

Chapter 1

Introduction

Science is the knowledge that has systematically been acquired through intellectual and practical observations of Nature and the physical world. Even though our time on Earth is scarce, humans strive to obtain as much knowledge as possible about our existence and the world surrounding us. This aspiration drives the human feature of curiosity, which forms the very basis of science.
The scientific branch known as *physics* asks the fundamental questions about Nature, the Universe, matter, and energy; questions about how the Universe began and what matter is made of. Physics can be divided into two distinct branches, experimental and theoretical physics. Experimental physics is the branch concerned with making observations of physical phenomena and quantities in order to gather information about our Universe. Theoretical physics is the branch that uses mathematical models to describe Nature and physical phenomena. In the 16th century, when physics as we know it today took its beginning, was a physicist both an experimentalist and a theoretician. Since then the science has developed, and today is the difference between the two branches striking. Experimental physics has transformed from table-top experiments performed in small laboratories, to huge experiments constructed by enormous collaborations of physicists from all around the globe. Theoretical physics has not experienced such a dramatic development. It is to a large extent still performed with paper and pen, but naturally with aid from computers. As the subject of physics has expanded, however, the theoretical physicist can no longer have an overall knowledge of physics and is thus forced to become more specialized and rely on the knowledge of peers. He or she is therefore often working in small collaborations, instead of locked up alone in a room in accordance with the common picture.
The construction of a physical theory is often driven by beauty and simplicity. In accordance with the principle of Occam's razor, the simplest possible, but still correct, theory ought to be the one held true. However, it is necessary not to emphasize too much on simplicity to the exclusion of accuracy, in the words of the famous physicist Richard Feynman "It doesn't matter how beautiful your theory

is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong."

We need a method to judge whether a physical theory is good or not. It is not a viable option to do as the ancient Greeks, who constructed beautiful theories that would have been rejected, had anyone bothered to make the relevant observations. Of great significance is the concept of *falsifiability*, *i.e.*, the possibility to prove the theory wrong. Contrary to falsifiability, verifiability is not a requirement, since a theory can agree with experiment the first million times it is carried out, but that is no guarantee for agreement the million-and-oneth time. Naturally, the theory also needs to describe all existing experimental data, and furthermore, a good theory ought to make new predictions that distinguish it from other theories.

The road to a new theory might in hindsight seem to be straightforward. Ideally, the development of a new theory is made as for the theory of gravitation. Newton took a giant leap of faith in the 17th century when stating his laws of gravitation. The theory was successful and unchallenged for more than two centuries before Einstein constructed general relativity. Einstein viewed gravitation as an effect of the geometry of spacetime, which was fundamentally different from Newton's view of an absolute space [1]. Although Newton's laws are still valid in the low-energy limit, general relativity gives us an overall description closer to reality, and it has made new predictions which have turned out to be correct. The achievements of both Newton and Einstein are only possible for an extraordinary genius, during the 20th century perhaps only a pocketful of brilliant physicists even came close. Quantum mechanics, together with special relativity, constitute the cornerstones of modern physics and the two different formulations of the theory by Erwin Schrödinger and Werner Heisenberg in the 1920s is another noticable example of remarkable achievements [2, 3]. The ideal development of a physical theory only works in very rare cases, and it gives a slightly misleading picture of how physics actually is performed. In reality, there might be thousands of unsuccessful theories and ideas for each successful one. Every now and then someone has a fruitful idea and then hopefully the joint work of several physicists will render a useful theory.

The study of *elementary particles*, *i.e.*, particles without any known substructure, and their interactions is one of the central subjects in theoretical physics. As our knowledge of particle physics increases, the length scale of study decreases, which in quantum mechanics is equivalent to an increase in energy. Huge particle colliders, which can accelerate particles to speeds near that of light, are necessary in order to achieve the high energies needed to study the nature of the fundamental particles and their interactions. At these high energies, a large body of new physics appears, and the results from these high-energy experiments need to be interpreted theoretically. This is made in a subfield of theoretical physics called particle physics phenomenology, which acts as a bridge between theory and experiment. Phenomenologists apply existing theories to interactions of elementary particles at collider energies, and interpret the results, ultimately striving to work out the behavior of elementary particle interactions at these energies and predict new interesting phenomena. This inspires a vivid exchange between theory and experiment.

There are four fundamental interactions in Nature; electromagnetism, gravitation, the strong and the weak interaction. An interaction is fundamental if it cannot be described in terms of other interactions, and it describes the way that elementary particles interact. On a daily basis, we are dealing concretely with gravitation and electromagnetism. Gravitation keeps us standing on Earth and is the reason we inevitably hit the ground if we fall out of a tree. Electromagnetism is manifested in the use of electricity to illuminate our homes and empower our computers and mobile phones. Regarding the other two fundamental interactions, the weak and the strong interaction, we do not have the same intuition, though we would not exist without them. The strong interaction is only strong at very short distances, but there, it is so strong that it is able to tie the protons and neutrons in the nucleus together. The weak interaction is responsible for radioactivity, and it exhibits some unusual symmetry-breaking behavior, for instance different strength for a particle and its anti-particle and it makes a distinction between a scattering process and its mirror image. Contrary to what one would expect, gravitation is the weakest interaction by far, the strong interaction is 10^{38} times stronger than gravitation. Gravitation is therefore only important when considering interactions of heavy, macroscopic objects. For particle interactions the gravitational effect is completely negligible.

The ultimate goal is to construct a theory that describes the four fundamental interactions in terms of a unified interaction in one single theory, a theory of everything (ToE). Physicists speculate that there existed a single fundamental interaction immediately after the Big Bang. As the Universe cooled down the fundamental interactions separated from the others, one by one, until we ended up with four separate ones. The first impressive example of a theoretical unification of interactions is that of electricity and magnetism into electromagnetism by James Clerk Maxwell during the 1860s. A century later, Sheldon Glashow, Steven Weinberg, and Abdus Salam showed that electromagnetism and the weak interaction could arise from one unified electroweak theory, nowadays known as the GWS theory [4, 5, 6]. In the unified theory, electromagnetism and the weak interaction start acting on equal terms at high energies, above the electroweak scale. On Earth, such energies can be achieved in enormous particle colliders, which have become the arena for the study of fundamental particle interactions. The Tevatron at Fermilab collided particles up to 1.96 TeV, but it was closed down in the fall of 2011. In 2009, the world's most powerful collider, the Large Hadron Collider (LHC) at CERN, began operating, and is at present colliding particles at an energy of 7 TeV. It will be shut off in the end of 2012 and upgraded to the full operating energy of 14 TeV [7, 8].

The next step towards a ToE should be to include the strong interaction at an even higher energy scale, and thus obtain a Grand Unified Theory. No such theory has yet been constructed, and such high energies have not yet been obtained in existing particle colliders. Pushing the energies even further, beyond the energies that can ever be achieved in earthbound experiments, physicists contemplate the possibility of including gravitation as well in a theory. At the moment, there are some theories that claim to be candidates for a ToE. The theory that has received most the popular interest is string theory, where matter consists of extremely small strings vibrating in extra dimensions. There are other potential theories, such as loop quantum gravitation, but at present none of these candidates are able to make any new predictions, and since the energy scales are immense the theories cannot be tested in the foreseeable future.

The *Standard Model of particle physics* (SM) was finalized in the middle of the 1970s and is at present the best theory we have for describing fundamental particles and their interactions. It includes the GWS theory for electroweak interactions as well as quantum chromodynamics (QCD), which is the theory for strong interactions. Since the advent of the theory, it has succeeded in describing a wealth of physical experiments and predicted several phenomena to very high precision. The existence of the bottom quark, the W and Z bosons, the top quark as well as the tau neutrino was predicted by the SM before their discoveries in 1977, 1983, 1995, and 2000, respectively [9, 10, 11, 12, 13, 14, 15]. As an example of the high precision of the SM, theory and measurement of the anomalous magnetic dipole moment of the electron agrees to a precision better than one part in a trillion [16, 17]. In addition to the twelve leptons and quarks that constitute matter and the gauge bosons that mediate interactions, the SM contains the hypothetical Higgs boson, which is a rest product of the mechanism that generates particle mass. No such particle has been found, although there is a fervent search ongoing at the LHC. In December 2011, data from the ATLAS and CMS collaborations indicated that there might be a signal for a Higgs particle at around 126 GeV, however it is not conclusive, and we need until the end of the year before enough data have been collected [18, 19].

Either way, we will be experiencing an exciting time in the upcoming years, with new experimental results from the LHC. Fortunately, finding the Higgs would not put an end to particle physics, although some slanderers claim so, because even though the SM gives a successful description of a lot of the particle physics that we observe it cannot be the full story. The most obvious shortcoming of the theory is that it does not include gravitation, which has turned out to be very tricky to incorporate at quantum scales. Furthermore, the theory neither explains the existence of dark matter and dark energy, which together constitute more than 95 % of the energy in the Universe, nor does it account for the anti-symmetry between matter and anti-matter in the Universe. In this thesis, we shall focus on an additional problem, namely that of *neutrino* masses.

Wolfgang Pauli suggested the neutrino in 1930 as a solution to the problem of missing energy and momentum in beta decays. About 25 years later, their existence was experimentally confirmed [20, 21]. Neutrinos interact only via the weak interaction, and therefore, they are difficult to detect, we do not notice them even though billions pass through us every second. They come in three flavors; electron, muon and tau neutrino, and at the conception of the SM, these were assumed to be massless. The solar neutrino problem gave a first indication that this was not the truth. Electron neutrinos are produced in enormous numbers in the proton-proton chain reaction in the Sun. This process is assumed to be well understood, as it accounts for the abundance of light elements in the Universe. The measured numbers of electron neutrinos at Earth only corresponds to a third of what could be expected. The most plausible solution to the problem turned out to be *neutrino oscillations*. The neutrinos are then assumed to change flavor during their travel towards Earth. Taking the other neutrino flavors into account, the rate at Earth is that expected from the proton-proton chain. However, neutrino oscillations are only possible if neutrinos have masses, which may however be incredibly small. The oscillations have been measured not only for solar neutrinos, but also in huge neutrino experiments on Earth [22, 23]. The neutrinos have also been on the carpet recently when the OPERA collaboration measured their velocity to be faster than that of light [24]. If this would turn out to be true, physics is on the brink of a revolution. It is, however, necessary for the experiment to be repeated by an independent collaboration before interpreting anything into it.

The generation of neutrino masses lies beyond the SM and is a problem that needs to be solved by means of new physics. There are several possible ways to extend the SM, the most popular being supersymmetry and extra-dimensional models. In supersymmetry, each SM particle is assumed to have a so-called superpartner with similar properties. None of the hypothetical additional particles have been detected, and thus, they have to be very massive compared to their SM counterparts. In this thesis, we shall study a model with extra dimensions as an extension to the SM. The first extra-dimensional model was proposed in 1921 by Theodor Kaluza, who extended general relativity to five dimensions, four spatial and one temporal, in order to unify electromagnetism and gravitation [25]. Oskar Klein developed this idea further in 1926 when he proposed that the fourth space dimension should be compactified, *i.e.*, small and finite [26]. These ideas led to a group of theories called *Kaluza–Klein (KK) theories*. The extra dimensions must necessarily be very small, since we only perceive three spatial dimensions and have never detected any additional small dimension. Extra-dimensional models have attracted a lot of interest since the birth of string theory in the 1970s. While string theory requires at least seven extra dimensions, there are several other models that include one or two extra dimensions. Perhaps these models will not provide the ultimate answer, but they make predictions at energies achievable at the LHC and might thus be able to enlighten some interesting points and they might give us a hint of whether extra dimensions are a viable way forward in the search for a ToE.

Chapter 2

Universal Extra Dimensions

In this chapter, we shall introduce the concept of quantum field theory (QFT) and review some basic properties of the state-of-the-art QFT, the SM. We shall also discuss its shortcomings, in particular the non-zero neutrino masses, and possible extensions to the SM. Specifically, we shall introduce a model with *Universal Extra Dimensions (UED)*, which is the model studied in this thesis.

2.1 Quantum Field Theory

QFT provides a framework in both particle physics and condensed matter physics for quantum mechanical models of systems parametrized by an infinte number of degrees of freedom, *i.e.*, fields and many-body systems. These quantum fields are operator functions of space and time that obey certain commutation relations. A particle is considered an energy quantum associated with a particular field and QFT provides at present the best starting point for describing fundamental particles and their interactions. The QFT is often formulated in terms of a Lagrangian density, or Lagrangian for short, where the dynamics of the fields in the theory is given by the action

$$\mathcal{S} = \int \mathrm{d}^4x \mathcal{L}(\phi(x), \partial_\mu \phi(x)). \tag{2.1}$$

The Euler-Lagrange equations from classical mechanics can be generalized to the QFT framework as equations of motion

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \right) = \frac{\partial \mathcal{L}}{\partial \phi_i}, \tag{2.2}$$

where ϕ_i is the field of study. The first successful QFT was quantum electrodynamics (QED), the relativistic QFT for electrodynamics which can still be considered a paradigm example of a QFT. QED was developed independently by Shin-Itiro

Tomonaga, Julian Schwinger and Richard Feynman in the 1940s and the theory has since been tested to extreme precision [27, 28, 29, 30]. At the same time, they introduced the concept of *renomalization*, which is used to handle the infinities that appear in QFT. Through renormalization, the bare quantities in the Lagrangian can be related to the physical ones, which we can measure experimentally. We shall discuss this concept in more detail in the subsequent chapter. When Feynman formulated QED, he did it both in terms of *path-integrals* and *Feynman diagrams*. The path-integral formulation is based on the idea that an electron, for example, when passing from one point to another, takes every possible path, and by adding the phases of the different paths together, we obtain the wavefunction, an idea which might seem insane but turns out to give a correct description. Feynman diagrams, on the other hand, offer a simple visualisation of complex particle interactions and can by means of *Feynman rules* be translated into an exact mathematical expression.

In QFT, perturbation theory is a useful tool for expressing approximate solutions to problems which we cannot solve exactly. The corrections at each order in perturbation theory can neatly be expressed in terms of Feynman diagrams, which are added together. However, at a certain order the number of possible Feynman diagrams diverges and the perturbation theory breaks down. Nevertheless, it is often sufficient to make calculations to one or two orders in perturbation theory.

2.2 The Standard Model of Particle Physics

The SM describes the elementary particles that consitute matter and mediate forces, and the interactions between them. Since the advent of the theory in the 1970s, an enormous amount of experimental data has been gathered, supporting the theory. The SM describes the physics of three of the fundamental interactions; the electromagnetic, the weak and the strong interaction. It is a non-Abelian gauge theory with the gauge group $SU(3)_{\rm c} \otimes SU(2)_{\rm L} \otimes U(1)_{\rm Y}$. The gauge group of the GWS model of electroweak interactions is $SU(2)_{\rm L} \otimes U(1)_{\rm Y}$. The subscript L stands for left and implies that only left-handed particles carry the corresponding quantum number. The subscript Y denotes the quantum number of hypercharge, to distinguish it from the gauge group associated with charge, Q. At the electroweak scale, $E_{\rm EW} \approx 246$ GeV, there will be spontaneous symmetry breaking of the electroweak symmetry to QED, with gauge group $U(1)_{\rm Q}$. The last part of the SM gauge group, $SU(3)_{\rm c}$, corresponds to the theory for the strong interaction, QCD. The c denotes the QCD quantum number called color, which comes in three types; red, green and blue.

Group generators

The SM gauge group is a Lie group which is generated by a set of group generators. The group generators constitute a Lie algebra and are a collection of operators T^a, such that the element $A = \exp(\mathrm{i}T^a)$ is an element of the group. We shall state some properties that these generators fulfill, which are used in this thesis. A complete set of their properties are given in Ref. [31]. First, the group generators fulfill the commutation relation

$$\left[T^a, T^b\right] = \mathrm{i}f^{abc}T^c, \tag{2.3}$$

where f^{abc} are the anti-symmetric structure constants of the group. Furthermore, they fulfill

$$T^aT^a = C_2(r) \cdot \mathbf{1}, \tag{2.4}$$

where the quadratic Casimir operator $C_2(r)$ for $SU(N)$, *i.e.*, for $r = N$, is given by

$$C_2(N) = \frac{N^2 - 1}{2N}. \tag{2.5}$$

2.2.1 Particle content

The particles in the SM are categorized by their spins, masses and other quantum numbers. The *fermions*, with spin-1/2, are in turn divided into *leptons* and *quarks* and are the particles that constitute ordinary matter. Interactions in the SM are mediated by the exchange of *gauge bosons*, all of spin-1. In addition, there is the hypothetical Higgs boson, a rest product of the mechanism which generates particle masses. Since it has not been detected, not everyone includes it in the SM, we shall however do so.

Fermions

As stated above, the fermions in the SM carry spin-1/2 and can be divided into two categories; leptons and quarks. The fermions are divided into three generations each, which couple identically to the gauge bosons. There is no theoretical reason for there only to be three generations, however, no particle belonging to any other generation has ever been detected. In addition to each fermion in the SM is a corresponding anti-particle.
Leptons are defined to be the particles which do not interact through the strong interaction, and there are six of them. The electron (e^-), the muon (μ^-) and the tau (τ^-) all of charge $Q = -|e|$, each has an associated uncharged neutrino, ν_e, ν_μ and ν_τ. The three lepton generations are

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}.$$

After electroweak symmetry breaking (EWSB), the masses of the generations will increase as one moves to the right.
The strongly interacting fermions are the quarks, and there are six of them; up (u),

down (d), charm (c), strange (s), top (t) and bottom (b). They are also divided into three generations with similar properties

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}.$$

The upper member of each quark doublet carries charge $Q = +\frac{2}{3}|e|$ and the lower members $-\frac{1}{3}|e|$.
Fermions are chiral, either left- or right-handed, and thus not identical to their mirror images. Only the weak interaction makes a distinction between different chiralities, as the gauge bosons corresponding to $SU(2)_\mathrm{L}$ do not interact with right-handed particles. The chirality operators are defined as

$$P_\mathrm{L} = \frac{1-\gamma^5}{2} \quad \text{and} \quad P_\mathrm{R} = \frac{1+\gamma^5}{2}, \tag{2.6}$$

where $\gamma^5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3$ and γ^μ are the Dirac γ-matrices. The left- and right-handed components of a fermionic field, Φ, are thus defined as

$$\Phi_\mathrm{L} = P_\mathrm{L}\Phi \quad \text{and} \quad \Phi_\mathrm{R} = P_\mathrm{R}\Phi. \tag{2.7}$$

In the SM, the left-handed fermions are $SU(2)_\mathrm{L}$-doublets and the right-handed fermions are $SU(2)_\mathrm{L}$-singlets. The content of one fermion generation can then be written as

$$L_\mathrm{L} = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_\mathrm{L}, \quad e^-_\mathrm{R}, \quad Q_\mathrm{L} = \begin{pmatrix} u \\ d \end{pmatrix}_\mathrm{L}, \quad u_\mathrm{R}, \quad d_\mathrm{R}.$$

In particular, it should be noted that there are no right-handed neutrinos in the SM, which implies that neutrinos are massless.

Gauge bosons

The SM particles which mediate forces are the gauge bosons, and there are one or several corresponding to each gauge group in the SM. To the group $U(1)_\mathrm{Y}$ corresponds the B boson. The group generator of this group is the hypercharge, $T^a = Y$. The hypercharges of the SM particles, including the Higgs boson, are given in Table 2.1. We use the definition $Y = Q - I_3$, where Q is the electric charge and I_3 is the weak isospin.

Field	L_L	e_R	Q_L	u_R	d_R	ϕ
Y	$-\frac{1}{2}$	-1	$\frac{1}{2}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$

Table 2.1. The hypercharges for the SM particles.

To $SU(2)_\mathrm{L}$ there is a corresponding non-Abelian spin-1 triplet, $\mathrm{W}^1, \mathrm{W}^2$, and W^3. After EWSB these will combine with the B boson to form the massless photon and

the massive $W^{\pm}$ and Z bosons. The generator of this group is $T^a = i\tau^a/2$, where τ^a are the Pauli matrices.
Finally, corresponding to the $SU(3)_c$ gauge group are eight spin-1 gluons, G^a_μ, $a = 1, 2, ..., 8$, which are an $SU(3)_c$-octet. The generators of this group are $T^a = i\lambda^a/2$, where λ^a, $a = 1, ..., 8$, are the Gell-Mann matrices. A particle transforming non-trivially with respect to the $SU(3)_c$ factor is said to carry color charge.

The Higgs boson

The SM gauge bosons are assumed to acquire mass as a result of EWSB through the so-called Higgs mechanism [32, 33, 34, 35]. As a rest product of such a mechanism there should be a complex scalar $SU(2)$-doublet, which can be written

$$\phi = \begin{pmatrix} ih^+ \\ \frac{1}{\sqrt{2}}(h^1 + ih^2) \end{pmatrix},$$

where the conjugate of h^+ is denoted h^-. Apart from the gauge bosons, the SM fermions are also assumed to acquire mass at EWSB through interactions with the Higgs field, though in a different way compared to the gauge bosons. The SM particles only acquire mass when the symmetry breaks, prior to this only the Higgs field is massive. At the time of writing of this thesis, the search for the Higgs boson is currently underway at the LHC, however no such particle has been detected yet although there have been some positive indications recently [18, 19]. There are several possible mechanisms for generating gauge boson masses, of which the Higgs mechanism is the simplest and the one assumed in this thesis.

2.2.2 Problems with the Standard Model

Even though the SM describes almost all experimental data from particle colliders, it is apparent that it is not the final theory. The most obvious shortcoming is the failure to include gravitational interactions, which has proven very difficult to incorporate in a quantum theory. The SM also contains 19 free parameters which need to be put in by hand; these should rather be given from the theory. Neither does it explain the matter and anti-matter assymmetry in the Universe, nor the existence of dark matter and dark energy.
Furthermore, in the SM neutrinos are assumed to be massless, since there are only left-handed neutrinos present in the theory. However, this is not the case, since the neutrinos have been shown to oscillate, *i.e.*, change flavor. The oscillations have been detected for neutrinos produced in different ways, for solar neutrinos in the SNO experiment, for atmospheric neutrinos in the Super-Kamiokande experiment, and for reactor neutrinos in the KamLAND experiment [36, 37, 22, 23]. In order to describe the neutrino masses, we need to extend the SM.

2.3 Universal Extra Dimensions

One possible way of extending the SM is by adding extra dimensions. The idea was first proposed by Theodor Kaluza in 1921 as a means for unifying gravitation and electromagnetism [25]. In 1926, Oskar Klein proposed that the extra spatial dimension should be circular and finite in size, *i.e.*, compactified [26]. These ideas caused some activity at the time but fell into oblivion for a few decades until they received new attention at the birth of string theory in the late 1960s [38]. In addition to string theory, which is a theory of extended objects, there are several types of extra-dimensional QFTs. Among the most popular such theories are the Arkani-Hamed–Dimopoulos–Dvali model, where the SM fields are confined to a four-dimensional brane but where gravitation is allowed to propagate through extra dimensions [39], and the Randall–Sundrum model, where the extra dimensions are warped [40]. In this thesis, we shall study a third type of extra-dimensional models with Universal Extra Dimensions (UED), *i.e.*, with extra dimensions that all SM fields can propagate through. The first UED model was introduced in 2000 by Appelquist, Dobrescu and Cheng [41]. In our four-dimensional world, we will perceive an effective theory where the extra dimensions are accounted for by towers of heavy KK modes [41]. The simplest model with UEDs is a five-dimensional model with the extra dimension compactified on a circle, corresponding to a S^1/Z_2 orbifold. This model is also the one which has been most extensively studied in the literature; see for example Refs. [41, 42, 43, 44]. In this thesis we shall consider a six-dimensional model with two UEDs [45]. The six-dimensional UED model has several attractive features. As in the five-dimensional case, it contains a dark matter candidate in form of the lightest KK particle and a mechanism for generating EWSB [46, 47, 48, 49, 50]. The model also requires that the number of lepton generations is a multiple of three in order to obtain anomaly cancellations [51], and it ensures proton stability [52].

In four dimensions, the fermions have a definite chirality, they are either left-handed or right-handed. This chirality needs to be preserved in the four-dimensional effective theory that is obtained after integration over the two extra dimensions, and thus the compactification of the two extra dimensions must be made accordingly. The simplest compactification that fulfills this criterion is the *chiral square*. The compactification is made on the square in Fig. 2.1 with sides of length L such that, $0 \leq x_4, x_5 \leq L$, and the adjacent sides are identified. This compactification is equivalent to the T^2/Z_4 orbifold, where T^2 is a torus and Z_4 is a rotation by 90 degrees. The topology is that of a sphere with three singular points $(0,0)$, (L,L) and $(L,0) \sim (0,L)$, where there can be localized kinetic terms. We shall also define the compactification radius $R = L/\pi$. The points on the chiral square are identified according to

$$(x^\mu, 0, y) = (x^\mu, y, 0), \tag{2.8}$$

$$(x^\mu, L, y) = (x^\mu, y, L), \tag{2.9}$$

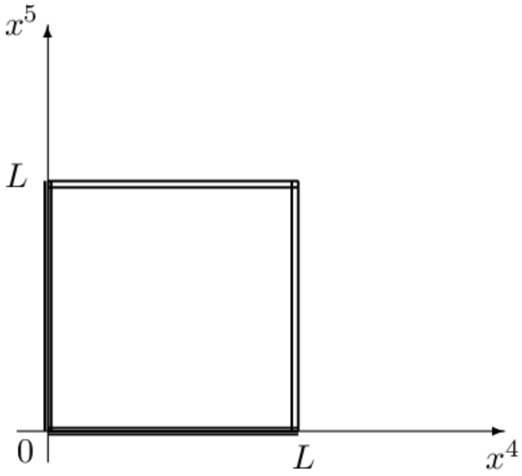


Figure 2.1. The chiral square on which the extra dimensions are compactified. The thick sides are identified as well as the sides with double lines. The picture is taken from Ref. [53].

where $\mu = 0, 1, 2, 3$. We require the physics at these points to be identical, which can be done by requiring that the Lagrangian takes the same value at these identified points

$$\mathcal{L}(x^\mu, 0, y) = \mathcal{L}(x^\mu, y, 0), \tag{2.10}$$
$$\mathcal{L}(x^\mu, 0, L) = \mathcal{L}(x^\mu, y, L). \tag{2.11}$$

The six-dimensional lepton and quark fields are chiral, with four components each, compared to the four-dimensional fermions, which have only two components. This six-dimensional chirality is labeled $+/-$, compared to the left- and right-handedness of the SM particles. The zero mode components of the six-dimensional fields, *i.e.* the fields with zero momentum along the extra dimensions, are identified with the SM particles. The definite chiralitites of the four-dimensional particles allow us to put a restriction on the six-dimensional fields which have a zero mode component. For anomaly cancellation and fermion mass generation we require the six-dimensional chiralities to be opposite to the four-dimensional case. Thus, the quark content for one generation will be $Q_+ = (u_+, d_+)$, u_-, d_- and the lepton content $L_+ = (\nu_+, l_+^-)$, l_- [54].

2.4 Kaluza-Klein Decomposition

In the two extra dimensions the fields are only allowed to propagate on the chiral square of side L and since these extra dimensions are compact, we can make an expansion of the fields without loss of generality. As an example we shall study the six-dimensional Abelian gauge field $A^\alpha(x^\beta)$, which has six components and is allowed to propagate through all six dimensions. Hence, the Lorentz indices take on the values $\alpha, \beta = 0, 1, ..., 5$ and the field can be written as

$$A^\alpha(x^\beta) = (A^\mu(x^\nu, x^4, x^5), A^4(x^\nu, x^4, x^5), A^5(x^\nu, x^4, x^5)), \tag{2.12}$$

where x^μ, x^ν are the Minskowski spacetime coordinates and thus $\mu, \nu = 0, 1, 2, 3$. We can make an expansion of the field, a *KK decomposition*, in a complete set of KK functions, f_n, given by [53]

$$f_n^{(j,k)}(x^4, x^5) = \frac{1}{1 + \delta_{j,0}\delta_{k,0}} \left[e^{-\mathrm{i}n\pi/2} \cos\pi \left(\frac{jx^4 + kx^5}{L} + \frac{n}{2} \right) + \cos\pi \left(\frac{kx^4 - jx^5}{L} + \frac{n}{2} \right) \right], \tag{2.13}$$

where j and k are integers. These functions satisfy the two-dimensional Klein-Gordon equation

$$\left(\partial_4^2 + \partial_5^2 + M_{j,k}^2 \right) f_n = 0, \tag{2.14}$$

where the KK mass is given by

$$M_{j,k}^2 = \frac{j^2 + k^2}{R^2}. \tag{2.15}$$

The functions are normalized so that

$$\frac{1}{L^2} \int_0^L \mathrm{d}x^4 \int_0^L \mathrm{d}x^5 [f_n^{(j,k)}(x^4, x^5)]^* f_n^{(j',k')}(x^4, x^5) = \delta_{j,j'}\delta_{k,k'}. \tag{2.16}$$

Only fields expanded in terms of the function f_0 can have a zero mode. The KK decomposition of the Abelian gauge field in terms of these functions is then given by

$$A_\mu(x^\nu, x^4, x^5) = \frac{1}{L} \left[A_\mu^{(0,0)}(x^\nu) + \sum_{j\geq 1} \sum_{k\geq 0} f_0^{(j,k)}(x^4, x^5) A_\mu^{(j,k))}(x^\nu) \right], \tag{2.17}$$

$$A_+(x^\nu, x^4, x^5) = -\frac{1}{L} \sum_{j\geq 1} \sum_{k\geq 0} f_3^{(j,k)}(x^4, x^5) A_+^{(j,k)}(x^\nu), \tag{2.18}$$

$$A_-(x^\nu, x^4, x^5) = \frac{1}{L} \sum_{j\geq 1} \sum_{k\geq 0} f_1^{(j,k)}(x^4, x^5) A_-^{(j,k)}(x^\nu), \tag{2.19}$$

where $A_\pm = A_4 \pm \mathrm{i}A_5$ and $A_\mu(x^\nu)$ and $A_\pm(x^\nu)$ are canonically normalized KK modes. The case when $j = 0, k = 0$ corresponds to the SM particle, and otherwise, *i.e.*, $k \geq 0$, $j \geq 1$, it corresponds to KK modes. The conservation of momentum along the extra dimensions will translate into conservation of KK numbers at vertices at tree level. For the gauge field, we have a four-dimensional tower of heavy spin-1 KK modes and two spin-0 towers. At each KK level we can define two real scalars $A_{\rm H}$ and $A_{\rm G}$ such that

$$A_{\pm}^{(j,k)} = r_{j,\pm k}\left(A_{\rm H}^{(j,k)} \mp {\rm i}A_{\rm G}^{(j,k)}\right), \tag{2.20}$$

where the complex phases are defined by $r_{j,\pm k} = (j \pm {\rm i}k)/\sqrt{j^2+k^2}$.
In addition a KK-parity can be imposed for scenarios with a stable lightest KK particle as a dark matter candidate. This is given by the Z_2 symmetry

$$\Upsilon^{(j,k)}(x^\mu) \to (-1)^{j+k}\Upsilon^{(j,k)}(x^\mu), \tag{2.21}$$

where Υ is a fermion or a scalar field and the quantity $(-1)^{j+k}$ is the KK parity [53].

2.4.1 Fermionic fields

The KK expansion of a fermionic field is given in [45]. We are only interested in fields corresponding to a SM particle. A six-dimensional field with + chirality must then have a left-handed zero mode component and its expansion is given by [53].

$$\Psi_{+\rm L} = \frac{1}{L}\left[\Psi_{+\rm L}^{(0,0)}(x^\nu) + \sum_{j\geq 1}\sum_{k\geq 0} f_0^{(j,k)}(x^4,x^5)\Psi_{+\rm L}^{(j,k)}(x^\nu)\right] \otimes \begin{pmatrix}1\\0\end{pmatrix} \tag{2.22}$$

$$\Psi_{+\rm R} = -\frac{\rm i}{L}\left[\sum_{j\geq 1}\sum_{k\geq 0} r_{j,k} f_3^{(j,k)}(x^4,x^5)\Psi_{+\rm R}^{(j,k)}(x^\nu)\right] \otimes \begin{pmatrix}0\\1\end{pmatrix} \tag{2.23}$$

This field corresponds to a SM doublet [54]. The singlet with a right-handed zero-mode component is obtained from a six-dimensional field of − chirality. The expansion for this field is obtained by interchanging + and − as well as L and R in Eqs. (2.22) and (2.23) [53].

2.4.2 Scalar field

The KK decomposition of a scalar field is given by [53]

$$\Phi(x^\mu, x^4, x^5) = \frac{1}{L}\sum_{(j,k)} \Phi^{(j,k)}(x^\mu) f_n^{(j,k)}(x^4,x^5), \tag{2.24}$$

where $n = 0,1,2,3$. For the field to have a zero component corresponding to the SM particle we need to make the expansion in terms of the function f_0. The scalar KK modes will have masses given by $M_\phi^{(j,k)} = \sqrt{M_\Phi^2 + M_{j,k}^2}$, where M_Φ is the mass of the SM particle and $M_{j,k}$ is the KK mass.

2.5 The UED Lagrangian Density

We shall proceed to study the interactions in the six-dimensional UED model. All corresponding Feynman rules are given in appendix A. In the present formulation of the Lagrangian, we shall define the covariant derivative as

$$D_\alpha = \partial_\alpha - \mathrm{i} g_6 A_\alpha, \tag{2.25}$$

where $\alpha = 0, 1, \ldots, 5$, A_α are the gauge fields, and g_6 is the six-dimensional gauge couplings with a mass dimension of -1, fulfilling $g_4 = g_6/L$.

2.5.1 Delta functions

To formulate the Lagrangian in an effective way, we shall introduce a generalized form of delta functions in six dimensions defined by [45]

$$\delta^{(j_1,k_1),\ldots,(j_r.k_r)}_{n_1,..,n_r} = \frac{1}{L^2}\int_0^L \mathrm{d}x^4 \int_0^L \mathrm{d}x^5 f^{(j_1,k_1)}_{n_1} \cdots f^{(j_r,k_r)}_{n_r}. \tag{2.26}$$

Since we are only interested in interactions with at least one SM particle, we shall be particularly interested in the case with one zero mode. In Ref. [53], the delta functions are computed for $r = 3$ to be

$$\begin{aligned} \delta^{(j_1,k_1)(j_2,k_2)(j_3,k_3)}_{(n,\bar{n},0)} &= \frac{1}{2(1+\delta_{j_1,0})(1+\delta_{j_2,0})(1+\delta_{j_3,0})} \\ &\times [7\delta_{j_1,0}\delta_{j_2,0}\delta_{j_3,0} + \delta_{j_1,j_3-j_2}\delta_{k_1,k_3-k_2} \\ &+\mathrm{i}^n(\delta_{j_1,j_3+k_2}\delta_{k_1,k_3-j_2} + \delta_{j_1,k_2-k_3}\delta_{k_1,j_3-j_2}) \\ &+(-\mathrm{i})^n(\delta_{j_1,j_3-k_2}\delta_{k_1,k_3+j_2} + \delta_{j_1,k_3-k_2}\delta_{k_1,j_2-j_3}) \\ &+(-1)^n(\delta_{j_1,j_2+j_3}\delta_{k_1,k_2+k_3} + \delta_{j_1,j_2-j_3}\delta_{k_1,k_2-k_3} \\ &+\delta_{j_1,j_2-k_3}\delta_{k_1,j_3-k_2} + \delta_{j_1,j_2+k_3}\delta_{k_1,k_2-j_3})]. \end{aligned} \tag{2.27}$$

Taking into account that j and k are non-negative numbers and only considering the cases when at least one field fulfills $j_n = 0, k_n = 0$, we obtain the relations

$$\delta^{(j_1,k_2)(j_2,k_2)(0,0)}_{0,0,0} = \delta_{j_1,j_2}\delta_{k_1,k_2}, \tag{2.28}$$

$$\delta^{(j_1,k_2)(j_2,k_2)(0,0)}_{n,\bar{n},0} = -\delta_{j_1,j_2}\delta_{k_1,k_2}, \quad n \neq 0, \tag{2.29}$$

for $(j_3, k_3) = (0, 0)$ and equivalently for $(j_1, k_1) = (0, 0)$ and $(j_2, k_2) = (0, 0)$. Hence we can conclude that the two KK numbers are conserved separately under these conditions. Furthermore, note that the generalized delta functions for $r = 4$ fulfill [45]

$$\delta^{(j_1,k_1)(j_2,k_2)(j_3,k_3)(0,0)}_{n,\bar{n},0,0} = \delta^{(j_1,k_1)(j_2,k_2)(j_3,k_3)}_{n,\bar{n},0}. \tag{2.30}$$

2.5.2 Gauge interactions

For a non-Abelian gauge field, the KK decomposition and identification of physical states are completely analogous to that of the Abelian fields. In this section, we shall discuss the self-interactions of the non-Abelian gauge fields, as well as the gauge interactions of the fermionic and scalar fields.

Self-interactions of gauge fields

We start by examining the part of the Lagrangian that treats the self-interactions of the gauge fields. The gauge interactions, together with the gauge-fixing term, are given by [55]

$$\mathcal{L}_{\mathrm{G}} = \int_0^L \mathrm{d}x^4 \int_0^L \mathrm{d}x^5 \sum_{i=1}^{3} \left\{ -\frac{1}{4} F^i_{\alpha\beta} F^{i\alpha\beta} - \frac{1}{2\xi} \left[\partial_\mu A^{i\mu} - \xi(\partial_4 A^i_4 + \partial_5 A^i_5) \right]^2 \right\}, \tag{2.31}$$

where ξ is the gauge fixing parameter. The field strengths are

$$F^i_{\alpha\beta} = \partial_\alpha A^i_\beta - \partial_\alpha A^i_\beta - \mathrm{i} g^i_6 [A^i_\alpha, A^i_\beta], \tag{2.32}$$

where $\alpha, \beta = 1, 2, \ldots, 6$ for the gauge groups in the SM; $U(1)_{\mathrm{Y}}$ ($i = 1$), $SU(2)_{\mathrm{L}}$ ($i = 2$) and $SU(3)_{\mathrm{c}}$ ($i = 3$). The interaction terms in the Lagrangian that are of particular phenomenological interest are those involving two KK modes and at least one SM field. These interactions are given by

$$\begin{aligned}
\mathcal{L}_{\mathrm{gauge}} = & - g_4 f^{abc} \left(A^{(j,k)a}_\mu A^{(j,k)b}_\nu \partial^\mu A^{(0,0)c\nu} + A^{(j,k)a}_\mu A^{(0,0)b}_\nu \partial^\mu A^{(j,k)c} \right. \\
& + \left. A^{(0,0)a}_\mu A^{(j,k)b}_\nu \partial^\mu A^{(j,k)c\nu} \right) - \left(\frac{g_4}{2} f^{abc} \left\{ A^{(j,k)a}_{\mathrm{H}} \left(\partial^\mu A^{(j,k)b}_{\mathrm{H}} \right) \right. \right. \\
& + \mathrm{i} \left[A^{(j,k)a}_{\mathrm{H}} \left(\partial^\mu A^{(j,k)b}_{\mathrm{G}} \right) - A^{(j,k)a}_{\mathrm{G}} \left(\partial^\mu A^{(j,k)b}_{\mathrm{H}} \right) \right] \\
& + \left. \left. A^{(j,k)a}_{\mathrm{G}} \left(\partial^\mu A^{(j,k)b}_{\mathrm{G}} \right) \right\} A^{(0,0)c}_\mu + \mathrm{h.c.} \right) \\
& - \frac{g_4^2}{4} f^{abc} f^{ade} \left[\delta^{(j_1,k_1)\ldots(j_4,k_4)}_{0,0,0,0} A^{(j_1,k_1)b}_\mu A^{(j_2,k_2)c}_\nu A^{(j_3,k_3)d\mu} A^{(j_4,k_4)e\nu} \right. \\
& - 2 \left(A^{(j,k)c}_{\mathrm{H}} A^{(j,k)e}_{\mathrm{H}} + \mathrm{i} A^{(j,k)c}_{\mathrm{G}} A^{(j,k)e}_{\mathrm{H}} \right. \\
& - \left. \left. \mathrm{i} A^{(j,k)c}_{\mathrm{H}} A^{(j,k)e}_{\mathrm{G}} + A^{(j,k)c}_{\mathrm{G}} A^{(j,k)e}_{\mathrm{G}} \right) A^{(0,0)b}_\mu A^{(0,0)d\mu} \right]
\end{aligned} \tag{2.33}$$

Gauge interactions of fermions

The four-dimensional Lagrangian for the gauge interactions of the fermionic sector is given by [55]

$$\begin{aligned}
\mathcal{L}_{\mathrm{fermion}} = & \int_0^L \mathrm{d}x^4 \int_0^L \mathrm{d}x^5 \left(\overline{L_+} \Gamma^\alpha D_\alpha L_+ + \overline{Q_+} \Gamma^\alpha D_\alpha Q_+ \right. \\
& \left. + \overline{e_-} \Gamma^\alpha D_\alpha e_- + \overline{u_-} \Gamma^\alpha D_\alpha u_- + \overline{d_-} \Gamma^\alpha D_\alpha d_- \right),
\end{aligned} \tag{2.34}$$

where the Dirac adjoint is defined as $\overline{\Psi} = \Psi^\dagger\gamma^0$. The representation of the Clifford algebra used here is

$$\Gamma^\mu = \gamma^\mu \otimes \mathbf{1}, \qquad \Gamma^{4,5} = \mathrm{i}\gamma_5 \otimes \sigma^{1,2}, \tag{2.35}$$

where $\mathbf{1}$ is the 2×2 unit matrix and σ^i the Pauli matrices [45]. The relevant terms in the four-dimensional Lagrangian can be split into two parts corresponding to each chirality

$$\begin{aligned}\mathcal{L}_{\text{fermion},+} &= g_4\delta_{0,0,0}^{(j_1,k_1)(j_2,k_2)(j_3,k_3)}\overline{\Psi_{+\mathrm{L}}^{(j_1,k_1)}}A_\mu^{(j_2,k_2)}\gamma^\mu\Psi_{+\mathrm{L}}^{(j_3,k_3)} \\ &+ g_4\overline{\Psi_{+\mathrm{R}}^{(j,k)}}A_\mu^{(0,0)}\gamma^\mu\Psi_{+\mathrm{R}}^{(j,k)} + \left[\overline{\Psi_{+\mathrm{L}}^{(0,0)}}\left(A_\mathrm{H}^{(j,k)} + \mathrm{i}A_\mathrm{G}\right)\Psi_{+\mathrm{R}}^{(j,k)} + \text{h.c.}\right]\end{aligned} \tag{2.36}$$

and

$$\begin{aligned}\mathcal{L}_{\text{fermion},-} &= g_4\delta_{0,0,0}^{(j_1,k_1)(j_2,k_2)(j_3,k_3)}\overline{\Psi_{-\mathrm{R}}^{(j_1,k_1)}}A_\mu^{(j_2,k_2)}\gamma^\mu\Psi_{-\mathrm{R}}^{(j_3,k_3)} \\ &+ g_4\overline{\Psi_{-\mathrm{L}}^{(j,k)}}A_\mu^{(0,0)}\gamma^\mu\Psi_{-\mathrm{L}}^{(j,k)} + \left[\overline{\Psi_{-\mathrm{R}}^{(0,0)}}\left(A_\mathrm{H}^{(j,k)} + \mathrm{i}A_\mathrm{G}\right)\Psi_{-\mathrm{L}}^{(j,k)} + \text{h.c.}\right].\end{aligned} \tag{2.37}$$

Gauge interactions of scalars

Next, we shall consider the gauge interaction of a scalar field. The four-dimensional Lagrangian is given by

$$\mathcal{L}_\text{scalar} = \int_0^L \mathrm{d}x^4 \int_0^L \mathrm{d}x^5 \left[(D_\alpha\Phi)^\dagger(D^\alpha\Phi) \; - M_\Phi^2\Phi^\dagger\Phi \; - \frac{\lambda_6}{2}(\Phi^\dagger\Phi \;)^2\right], \tag{2.38}$$

where λ_6 is a parameter of mass dimension -2 that fulfill $\lambda_4 = \lambda_6/L^2$. The terms of phenomenological interest to us are

$$\begin{aligned}\mathcal{L}_\text{scalar} &= \left[\mathrm{i}g_4\delta_{0,0,0}^{(j_1,k_1)(j_2,k_2)(j_3,k_3)}\Phi^{(j_1,k_1)\dagger}A_\mu^{(j_2,k_2)}\partial^\mu\Phi^{(j_3,k_3)} + \text{h.c.}\right] \\ &+ g_4^2\delta_{0,0,0,0}^{(j_1,k_1)\ldots(j_4,k_4)}\Phi^{(j_1,k_1)\dagger}A_\mu^{(j_2,k_2)}A^{(j_3,k_3)\mu}\Phi^{(j_4,k_4)} \\ &+ \left[\mathrm{i}g_4M_{j,k}\Phi^{(0,0)\dagger}A_\mathrm{G}^{(j,k)}\Phi^{(j,k)} + \text{h.c.}\right] \\ &- g_4^2\Phi^{(0,0\dagger}\left(A_\mathrm{H}^{(j,k)} - \mathrm{i}A_\mathrm{G}\right)\left(A_\mathrm{H}^{(j,k)} + \mathrm{i}A_\mathrm{G}\right)\Phi^{(0,0)} \\ &- \frac{\lambda}{2}\delta_{0,0,0,0}^{(j_1,k_1)\ldots(j_4,k_4)}\Phi^{(j_1,k_1)\dagger}\Phi^{(j_2,k_2)}\;\Phi^{(j_3,k_3)\dagger}\Phi^{(j_4,k_4)}\end{aligned} \tag{2.39}$$

If explicitly not stated otherwise, we imply for the gauge fields $A_\mu = T^aA_\mu^a$. Furthermore, we are only interested in the interactions where only two of the (j_n, k_n) are non-zero in the terms where $\delta_{0,0,0}^{(j_1,k_1)(j_2,k_2)(j_3,k_3)}$ and $\delta_{0,0,0,0}^{(j_1,k_1)\ldots(j_4,k_4)}$ have been kept.

2.5.3 Yukawa interactions

The Yukawa interactions in the SM are used to describe the coupling between the Higgs field, leptons, and quarks. They are given by

$$\mathcal{L}_{\mathrm{Y,SM}} = -(Y_e)_{fg}\overline{L^f_{\mathrm{L}}}\phi e^g_R - (Y_d)_{fg}\overline{Q^f_{\mathrm{L}}}\phi d^g_{\mathrm{R}} - (Y_u)_{fg}\overline{Q^f_{\mathrm{L}}}\phi^C u^g_{\mathrm{R}} + \mathrm{h.c.}, \tag{2.40}$$

where the charge conjugate of the Higgs field is defined as

$$\phi^C = \mathrm{i}\sigma_2\phi^*. \tag{2.41}$$

The Yukawa interactions in the six-dimensional UED model are given in analogy with the SM case [55]

$$\mathcal{L}_{Y,\mathrm{6D}} = -(\hat{Y}_e)_{fg}\overline{L^f_+}\phi e^g_- - (\hat{Y}_d)_{fg}\overline{Q^f_+}\phi d^g_- - (\hat{Y}_u)_{fg}\overline{Q^f_+}\phi^C u^g_- + \mathrm{h.c.} \tag{2.42}$$

By integrating out the two extra dimensions, we can obtain the four-dimensional Lagrangian from the six-dimensional Lagrangian

$$\mathcal{L}_Y = \int_0^L \mathrm{d}x^4 \int_0^L \mathrm{d}x^5 \mathcal{L}_{Y,6D}. \tag{2.43}$$

As, for all the gauge couplings, the six-dimensional Yukawa coupling constants are dimensionful and the relation between the six-dimensional and four-dimensional coupling constants are given by $Y = \hat{Y}/L$. We insert the expansions of the scalar and fermion fields given in Eqs. (2.22) and (2.24) and integrate over the two extra dimensions. Furthermore, we can use the fact that $(\mathrm{i}\sigma_2)_{ij} = \varepsilon_{ij}$, where ε_{ij} is the two-dimensional permutation tensor that fulfills $\varepsilon_{ij} = -\varepsilon_{ji}$. Thus $\phi^C_i = \varepsilon_{ij}\phi^*_j$ and the Yukawa interactions are then given by

$$\begin{aligned}
\mathcal{L}_{Y_e} = &- (Y_e)_{fg}\Big(\delta^{(j_1,k_1)(j_2,k_2)(j_3,k_3)}_{0,0,0}\delta_{ba}\overline{L^{(j_1,k_1)f}_{+\mathrm{L},b}}\phi^{(j_2,k_2)}_a e^{(j_3,k_3)g}_{-\mathrm{R}} \\
&+ \delta_{ba}\overline{L^{(j,k)f}_{+\mathrm{R},b}}\phi^{(0,0)}_a e^{(j,k)g}_{-\mathrm{L}} + \mathrm{h.c.}\Big), \qquad (2.44)\\
\mathcal{L}_{Y_d} = &- (Y_d)_{fg}\Big(\delta^{(j_1,k_1)(j_2,k_2)(j_3,k_3)}_{0,0,0}\delta_{ba}\overline{Q^{(j_1,k_1)f}_{+\mathrm{L},b}}\phi^{(j_2,k_2)}_a d^{(j_3,k_3)g}_{-\mathrm{R}} \\
&+ \overline{Q^{(j,k)f}_{+\mathrm{R},b}}\phi^{(0,0)}_a d^{(j,k)g}_{-\mathrm{L}} + \mathrm{h.c.}\Big), \qquad (2.45)\\
\mathcal{L}_{Y_u} = &- (Y_u)_{fg}\Big(\delta^{(j_1,k_1)(j_2,k_2)(j_3,k_3)}_{0,0,0}\varepsilon_{ba}\overline{Q^{(j_1,k_1)f}_{+\mathrm{L},b}}\phi^{(j_2,k_2)*}_a u^{(j_3,k_3)g}_{-\mathrm{R}} \\
&+ \varepsilon_{ba}\overline{Q^{(j,k)f}_{+\mathrm{R},b}}\phi^{(0,0)*}_a u^{(j,k)g}_{-\mathrm{L}} + \mathrm{h.c.}\Big). \qquad (2.46)
\end{aligned}$$

In comparison to the SM case we obtain an additional type of Yukawa couplings, which involves KK modes. The corresponding Feynman rules are given in appendix A.

2.5.4 The effective neutrino mass operator

One possibility for generation of neutrino masses is through a so-called Weinberg operator [56]. This is an operator of mass dimension $d = 5$, where two Higgs doublets are combined as an isospin triplet. The operator is consistent with the gauge symmetry of the SM and it will lead to a Majorana mass term after EWSB. In the SM, the effective term is given by

$$\mathcal{L}_\kappa = -\frac{1}{8}\kappa_{gf}(\overline{L_{\mathrm{L}}^C}^g \varepsilon\tau^i L_{\mathrm{L}}^f)(\phi^T\varepsilon\tau^i\phi) + \text{h.c.}, \tag{2.47}$$

where κ is a symmetric and complex matrix of mass dimension -1. Furthermore, L^C is the charge-conjugate term fulfilling $\overline{L^C} = -L^T C^{-1}$. This operation is linear, *i.e.* for $L_+ = L_{+,\mathrm{R}} + L_{+,\mathrm{L}}$, we obtain $L_+^C = L_{+,\mathrm{R}}^C + L_{+,\mathrm{L}}^C$. This can be generalized to 6 dimensions,

$$\mathcal{L}_\kappa = -\frac{1}{8}\hat{\kappa}_{gf}(\overline{L_+^C}^g \varepsilon\tau^i L_+^f)(\phi^T\varepsilon\tau^i\phi) + \text{h.c.} \tag{2.48}$$

We can use the fact that

$$\tau_{ab}^i\tau_{cd}^i = 2\delta_{ad}\delta_{bc} - \delta_{ab}\delta_{cd} \tag{2.49}$$

and rewrite (2.47) as

$$\mathcal{L}_{\kappa,6\mathrm{D}} = -\frac{1}{4}\hat{\kappa}_{gf}\overline{L_{+,c}^C}^g \varepsilon_{cd}\phi_d L_{+,b}^f\varepsilon_{ba}\phi_a + \text{h.c.} \tag{2.50}$$

We shall insert the KK expansions for the fermionic and scalar fields and integrate out the two dimensions on the chiral square, *i.e.*,

$$\mathcal{L}_\kappa = \int_0^L \mathrm{d}x^4 \int_0^L \mathrm{d}x^5 \mathcal{L}_{\kappa,6\mathrm{D}}. \tag{2.51}$$

The part of interest of the effective four-dimensional Lagrangian, *i.e.*, the terms with two KK particles and two SM particles, is given by

$$\begin{aligned} \mathcal{L}_\kappa &= -\frac{1}{4}\kappa_{gf}\Big(\delta_{0,0,0,0}^{(j_1,k_1)\ldots(j_4,k_4)} L_{+\mathrm{L},c}^{(j_1,k_1)g}\varepsilon_{cd}\phi_d^{(j_2,k_2)} L_{+\mathrm{L},b}^{(j_3,k_3)}\varepsilon_{ba}\phi_a^{(j_4,k_4)} \\ &+ L_{+\mathrm{R},c}^{(j,k)g}\varepsilon_{cd}\phi_d^{(0,0)} L_{+\mathrm{R},b}^{(j,k)}\varepsilon_{ba}\phi_a^{(0,0)} + \text{h.c.}\Big), \end{aligned} \tag{2.52}$$

with $\kappa = \hat{\kappa}/L^2$. Thus, compared to the SM, we obtain a new vertex involving KK modes and two SM Higgs fields. The Feynman rules are given in appendix A.

2.5.5 Lepton mixing and neutrino masses

Neutrino oscillations are due to the fact that a neutrino can change flavor over time. The mixing of the lepton flavors is described by a unitary matrix, which can be parametrized in terms of three mixing angles, θ_{12}, θ_{13}, and θ_{23}; and three CP-violating phases, δ, ρ, and σ. The parametrization is given by

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-\mathrm{i}\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{\mathrm{i}\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{\mathrm{i}\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{\mathrm{i}\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\mathrm{i}\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{\mathrm{i}\rho} & 0 & 0 \\ 0 & e^{\mathrm{i}\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

where $s_{ij} = \sin(\theta_{ij})$ and $c_{ij} = \cos_{ij}$, $ij = 12, 13, 23$. After EWSB, the neutrino mass matrix is given by

$$m_\nu = \kappa v^2, \tag{2.53}$$

where v is the vacuum expectation value of the Higgs field. The neutrino mass matrix can then be diagonalized by

$$U^\dagger m_\nu U^* = \mathrm{diag}(m_1, m_2, m_3), \tag{2.54}$$

where m_i, $i = 1, 2, 3$, are the neutrino masses. There are two possible orderings of the neutrino masses, consistent with experiment. Either normal ordering, where $m_1 < m_2 < m_3$, or inverted ordering, where $m_3 < m_1 < m_2$. From experiment, the neutrino masses have been bounded from above, the sum of the neutrino masses are of the order of 1 eV, which is very small compared to the other SM masses. The reason for the masses being so small is not yet known.

Chapter 3

Renormalization

When calculating quantities in QFT to loop-level, *i.e.*, to higher orders in perturbation theory, one finds that divergences appear. They are either ultraviolet (UV), from high-energy contributions, or infrared, from low-energy contributions. In order to make any physical sense of the theory, the infinities need to be handled. The method for handling these divergences is divided into two steps, *regularization* and *renormalization*. In the first step, the theory is regulated, which renders the integrals convergent. This can be done by several methods, *e.g.* Pauli-Villars regularization, lattice regularization and dimensional regularization. The integral will be convergent after the regularization, however dependent on the regularization and it might therefore still diverge in some limit. In the next step, the renormalization, this is handled by varying the regulating parameter and the parameters in the Lagrangian simultaneously in such a way that the integral remains convergent. Hence the integral converges and is independent on method of regularization. In the renormalization step we thus find a relation between the bare quantities in the Lagrangian and the physical quantities that can be experimentally measured. The infinities in the bare quantities before renormalization will be contained in *counterterms* after renormalization. There are several renormalization schemes, *e.g.* on-shell scheme and minimal subtraction (MS) scheme.

3.1 Dimensional Regularization and Minimal Subtraction

We are primarily concerned with UV divergences, and then, the most commonly used regularization method is dimensional regularization. Together with dimensional regularization, the MS is scheme often used, since the combination has several pleasant features, and it is this combination we shall use in this thesis.

3.1.1 Dimensional regularization

Dimensional regularization is due to Martinus Veltman and Gerardus 't Hooft and it is a particularly powerful method, since it respects gauge symmetries, and thus, will preserve Lorentz invariance, gauge invariance and unitarity [57]. In dimensional regularization, the energy-momentum integrals are formally performed in $d < 4$ dimensions instead of 4 dimensions, so that they can be determined in an unambiguous way, as the UV behavior will converge if d is sufficiently small. One should be careful in the use of the word dimension here, since d is a generic complex number and not an ordinary spacetime dimension. The integrals are extended according to

$$\int \frac{\mathrm{d}^4 k}{(2\pi)^4} \longrightarrow \int \frac{\mathrm{d}^d k}{(2\pi)^d}, \tag{3.1}$$

where we can define $d = 4-\varepsilon$, where ε is a complex number. As $\varepsilon \to 0$ our ordinary four-dimensional space-time is restored. In this model, ε is the cutoff and the limit $\varepsilon \to 0$ is equivalent to $M \longrightarrow \infty$. As we have not violated any physical law in extending the dimensions, the properties in the regularized theory are preserved, the only difference being that spacetime no longer is four-dimensional. In order to be able to perform the integrations, we need to define the Clifford algebra in d dimensions. The extension must be made in a consistent way, so that we retrieve the original integral in the limit $d \to 4$. One possible definition of the algebra in d dimensions is similar to that in 4 dimensions, *i.e.*,

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}, \tag{3.2}$$

where γ^μ are the Dirac gamma matrices and $\eta^{\mu\nu}$ is the Minkowski metric tensor. However, the trace changes and we thus obtain

$$\eta_{\mu\nu}\eta^{\mu\nu} = d, \tag{3.3}$$

$$\gamma_\mu\gamma^\mu = d. \tag{3.4}$$

The result of the various possible contractions of the gamma matrices will therefore differ from the four-dimensional case. The full set of relations in d dimensions are given in Ref. [31].

The calculations of the loop integrals arising from the Feynman diagrams are in principle rather complicated. The normal procedure begins with extending the integral to d dimensions and then Feynman parametrizing the integrand, which simplifies the subsequent calculations. Then, a Wick rotation of the integral is made, where either space or time is assumed to be imaginary. In principle, the integral is then rotated from Minkowski space to Euclidean space. Since we almost exclusively are interested in the divergent parts of the Feynman diagrams, we can skip the entire procedure. Instead can the integrals in practice be determined from a so-called *Passarino-Veltman decomposition*, and then, one can use the tabulated

values for the divergent parts of the integrals [58]. Hence, the divergent contributions can be determined without actually performing the entire, often tedious, calculation.
In the continuation to d dimensions, the dimensionality of the fields is altered. We require the action, given by

$$S = \int \mathrm{d}^d x \mathcal{L}, \tag{3.5}$$

to be dimensionless. In order for this to be fulfilled, we must require that the Lagrangian has mass dimension d and the dimensionality of the fields can be determined from terms in it. From the kinetic term of the Higgs field, $(\partial_\mu \phi)^\dagger (\partial^\mu \phi)$, we find that it must satisfy

$$[\phi] = \frac{d-2}{2}. \tag{3.6}$$

Similarly, we find for the fermionic and gauge fields

$$[\Psi] = \frac{d-1}{2}, \tag{3.7}$$

$$[A] = \frac{d-2}{2}. \tag{3.8}$$

As the dimensionality of the Lagrangian changes, the dimensionality of the coupling constants must also change. However, we want to keep the four-dimensional coupling constants dimensionless, and to do so, an arbitrary energy scale, the renormalization scale, μ, is introduced. Taking μ to relevant powers and then multiplying it with the coupling constants, we can keep the latter dimensionless. Hence, we make the replacements

$$g_i \longrightarrow \mu^{\frac{\varepsilon}{2}} g_i, \tag{3.9}$$

$$Y_j \longrightarrow \mu^{\frac{\varepsilon}{2}} Y_j, \tag{3.10}$$

$$\lambda \longrightarrow \mu^{\varepsilon} \lambda, \tag{3.11}$$

$$\kappa \longrightarrow \mu^{\varepsilon} \kappa, \tag{3.12}$$

where $i = 1, 2, 3$ and $j = e, u, d$.

3.1.2 Minimal subtraction

After dimensional regularization, it is convenient to express the Feynman diagrams as an expansion in powers of ε and then, it is practical to use the MS scheme, which is a renormalization scheme where the counterterms do not contain any finite parts and poles in different orders of $1/\varepsilon^n$ are removed. Contrary to most other renormalization schemes that work together with any regularization method the MS scheme only works with dimensional regularization. It was devised as a method for renormalization of electromagnetic and weak interactions by 't Hooft [59]. In the MS scheme, UV divergences are removed and the coefficients of the counterterms are c-numbers which do not contain any mass terms or even the renormalization scale.

3.2 Renormalization of the Four-Dimensional Effective Theory

In order to make sense of the Lagrangian we need to relate the bare quantities to the renormalized ones. This is done through *renormalization constants*, Z_i, which can be expressed in terms of the counterterms δZ_i as $Z_i = 1 + \delta Z_i$. The wavefunction renormalizations in the four-dimensional effective theory are then

$$\Psi_{\rm B}^{(j,k)f} = \left(Z_{\Psi_{j,k}}^{\frac{1}{2}}\right)_{fg} \Psi^{(j,k)g}, \tag{3.13}$$

for the left and right-handed fermions respectively,

$$\phi_{\rm B}^{(j,k)} = Z_{\phi_{j,k}}^{\frac{1}{2}} \phi^{(j,k)}, \tag{3.14}$$

for the Higgs scalar

$$A_{\rm B}^{(j,k)} = Z_{A_{j,k}}^{\frac{1}{2}} A^{(j,k)}, \tag{3.15}$$

for the gauge fields and

$$A_{\rm H,B}^{(j,k)} = Z_{A_{\rm H}(j,k)}^{\frac{1}{2}} A_{\rm H}^{(j,k)}, \tag{3.16}$$

$$A_{\rm G,B}^{(j,k)} = Z_{A_{\rm G}(j,k)}^{\frac{1}{2}} A_{\rm G}^{(j,k)}, \tag{3.17}$$

for the adjoint scalars. From the wavefunction renormalization and the Lagrangian we can determine the relation between bare and physical quantities for masses and the coupling constants which are given by

$$Z_\phi m_{\rm B}^2 = m^2 + \delta m^2, \tag{3.18}$$

$$Z_B^{\frac{1}{2}} Z_\phi g_{1\rm B} = \mu^{\frac{\epsilon}{2}} Z_{g_1} g_1, \tag{3.19}$$

$$Z_W^{\frac{1}{2}} Z_\phi g_{2\rm B} = \mu^{\frac{\epsilon}{2}} Z_{g_2} g_2, \tag{3.20}$$

$$Z_G^{\frac{1}{2}} Z_\phi g_{3\rm B} = \mu^{\frac{\epsilon}{2}} Z_{g_3} g_3, \tag{3.21}$$

$$Z_\phi^2 \lambda_{\rm B} = \mu^{\frac{\epsilon}{2}} Z_\lambda \lambda, \tag{3.22}$$

$$Z_{e_R}^{\frac{1}{2}} Y_{e\rm B} Z_\phi^{\frac{1}{2}} Z_{l_L}^{\frac{1}{2}} = \mu^{\frac{\epsilon}{2}} Y_e Z_{Y_e}, \tag{3.23}$$

$$Z_{u_R}^{\frac{1}{2}} Y_{u\rm B} Z_\phi^{\frac{1}{2}} Z_{q_L}^{\frac{1}{2}} = \mu^{\frac{\epsilon}{2}} Y_u Z_{Y_u}, \tag{3.24}$$

$$Z_{d_R}^{\frac{1}{2}} Y_{d\rm B} Z_\phi^{\frac{1}{2}} Z_{q_L}^{\frac{1}{2}} = \mu^{\frac{\epsilon}{2}} Y_d Z_{Y_d}. \tag{3.25}$$

Furthermore, the effective operator is renormalized by

$$Z_{L_{+\mathrm{L}}}^{T,\frac{1}{2}} Z_{\phi}^{\frac{1}{2}} \kappa_{\mathrm{B}} Z_{\phi}^{\frac{1}{2}} Z_{L_{+\mathrm{L}}}^{\frac{1}{2}} = \mu^{\epsilon}(\kappa + \delta\kappa). \tag{3.26}$$

3.2.1 Non-renormalizability

Contrary to the SM, which is a renormalizable theory, theories in more than four dimensions are non-renormalizable, independent of size and shape of the extra dimensions. A theory is not renormalizable if it requires an infinite number of counterterms in order to remove all the divergences. The six-dimensional UED should therefore be regarded as an effective field theory (EFT). An EFT is only valid in a certain energy range and is inherently approximative and the Lagrangian only contains the terms relevant for the energy range of interest. Outside of this range another theory has to step in, a more general UV completion. It is, however, not necessary to find a fundamental theory valid at all energies in order to describe interactions in a certain energy range. In order to keep the UED model within the energy range where it is valid, we need to introduce a UV cutoff, Λ, which will also restrict the number of contributing KK modes.

3.3 Renormalization Group Equations

One interesting consequence of renormalization is that the coupling constants and masses depend on the energy scale at which they are measured. This phenomenon is called running, and since the running behavior is model-dependent, it provides a way to distinguish between different models. The running is determined from solving renormalization group equations (RGEs). The RGEs can be derived from the relation between the renormalized and the bare Green's function. The bare Green's function is defined as

$$G_{\mathrm{B}}^{(n)}(\{x_i\}; \mu, \lambda) := \langle 0|\mathcal{T}\phi_{\mathrm{B}}(p_1)...\phi_{\mathrm{B}}(p_n)|0\rangle. \tag{3.27}$$

The relation between this bare Green's function and the renormalized one is given by

$$G_{\mathrm{B}}^{(n)}(\{x_i\}; \mu, \lambda) = Z_{\mathrm{G}}^{-\frac{n}{2}} G^{(n)}(\{x_i\}; \mu, \lambda). \tag{3.28}$$

The bare quantities of a theory are per definition independent of the energy scale, μ. This is naturally valid for the bare Green's function as well and thus

$$\frac{\mathrm{d}G_{\mathrm{B}}}{\mathrm{d}\mu} = 0. \tag{3.29}$$

By explicitly performing the differentiation we obtain the *Callan-Symanzik* equation, discovered independently by Curtis Callan and Kurt Symanzik in 1970 [60, 61].

It describes the evolutions of the n-point correlation functions as the energy is varied. The structure of the equation is [31]

$$\left(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial\lambda} + n\gamma\right) G^{(n)}(\{x_i\};\mu,\lambda) = 0. \tag{3.30}$$

The coefficients of the equation are two functions, $\beta(\lambda)$ and $\gamma(\lambda)$, which will compensate for any shift in the renormalization scale. These functions are defined as

$$\beta = \mu\frac{\mathrm{d}\lambda}{\mathrm{d}\mu}, \qquad \gamma = \frac{1}{2Z_\phi}\mu\frac{\mathrm{d}Z_\phi}{\mathrm{d}\mu}. \tag{3.31}$$

In theories with multiple couplings and fields, there is one β term for each coupling and one γ term for each field.

3.3.1 Determining the β-function

The aim of the present work is to determine the running of neutrino parameters in the six-dimensional UED model. To do this we need to calculate the relevant RGEs to one-loop order. In particular, we are interested in the energy-dependence of the coupling constants, *i.e.*, the β-function. To determine the β-function, we need the relation between the bare quantity, Q_B, and its physical counterpart, Q, which can be put on the form

$$Q_\mathrm{B} = \left(\prod_{i=1}^{M} Z_{\phi_i}^{n_i}\right)[Q+\delta Q]\,\mu^{D_Q\epsilon}\left(\prod_{j=M+1}^{N} Z_{\phi_j}^{n_j}\right), \tag{3.32}$$

where D_Q is the mass dimension of Q. An expression for the β-function of Q can then be derived, using the fact that the bare quantities are independent of the renormalization scale, *i.e.*,

$$\frac{\mathrm{d}Q_\mathrm{B}}{\mathrm{d}\mu} = 0. \tag{3.33}$$

The renormalization constants do not depend explicitly on μ in an MS-like scheme, they do however depend on Q and an additional set of μ-dependent parameters, $\{V_A\}$, which must be taken into account upon performing the differentiation. The full derivation of a useful expression for the β-function together with expression itself is given in Ref. [62].

Chapter 4

Renormalization Group Equations

In this chapter, we calculate the RGEs for the Higgs self-coupling, the Yukawa couplings, and the gauge couplings in the six-dimensional UED model. The set of equations we thus obtain, are solved numerically in Matlab.

4.1 Renormalization of the Theory

In order to calculate the RGEs, we need to determine the corrections to propagators and vertices, and hence the counterterms. This is done to one-loop order. The Feynman diagrams in this chapter are calculated by means of the Feynman rules listed in appendix A. We begin with the wavefunction renormalization, which determines corrections to the propagators, and then continue on to the Yukawa vertices, the Higgs self-interaction and finally the effective neutrino mass operator. When the counterterms have been determined, we calculate the β-functions.
In the Feynman diagrams, SM fields are represented by a single line, whereas KK modes, with corresponding KK numbers, (j, k), are represented by a double line. The counterterms can be divided into two parts, which are calculated separately, one part from the SM fields and one from the KK modes. Only diagrams, which only contain fields corresponding to SM fields, contribute to the SM part of the counterterms. In each case, except for the renormalization of the Higgs mass, the contributions from each of the KK levels above the SM level are identical. Thus, we can write

$$\delta Z = \delta Z^{\mathrm{SM}} + s\delta Z^{\mathrm{UED}}. \tag{4.1}$$

The parameter s counts the number of KK levels below a given energy μ, and hence, s is defined as the number of KK modes that fulfill $\sqrt{j^2 + k^2} \leq \lfloor \frac{\mu}{\mu_0} \rfloor$, where

$\mu_0 = R^{-1}$ is the energy level in which the first KK modes are excited. In the subsequent section, the renormalization of the Higgs mass is considered, where the contributions from each KK level differ.
In general, when considering diagrams containing fermions, we need to keep track of the fermion-number flow [63]. We do not explicitly state in every Feynman diagram the direction of the flow, but the choice of direction is made consistently throughout the entire set of diagrams.

The self-energy of the Higgs field

We begin by calculating the one-loop corrections to the self-energy of the Higgs field. The one-loop contributions from SM particles, and the higher modes corresponding to them, are given in Fig. 4.1, and the contributions from the adjoint scalars are given in Fig. 4.2. In Fig. 4.3, the diagrams containing fermions which do not have a zero-mode component, are presented.

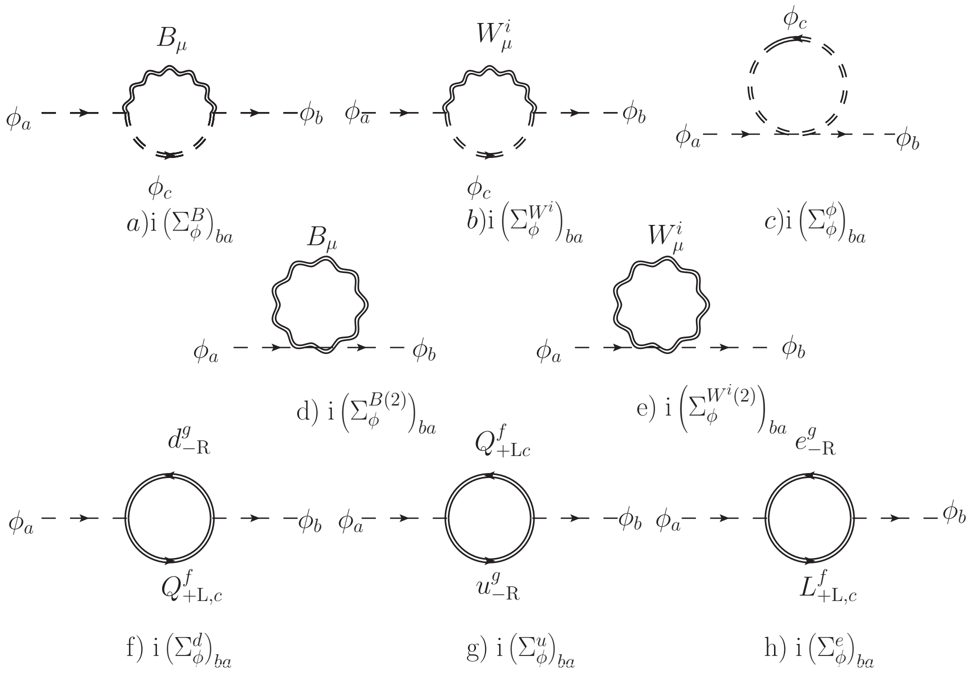


Figure 4.1. One-loop corrections to the Higgs self-energy from fields corresponding to SM particles.

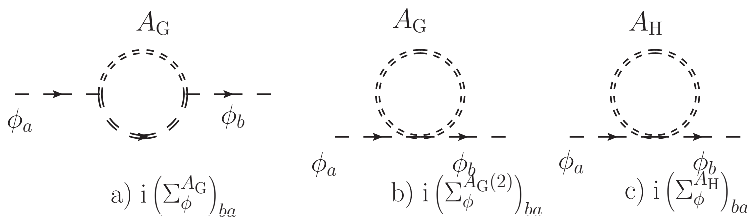


Figure 4.2. Contributions from adjoint scalars to the Higgs self-energy, with $A = B, W^i$.

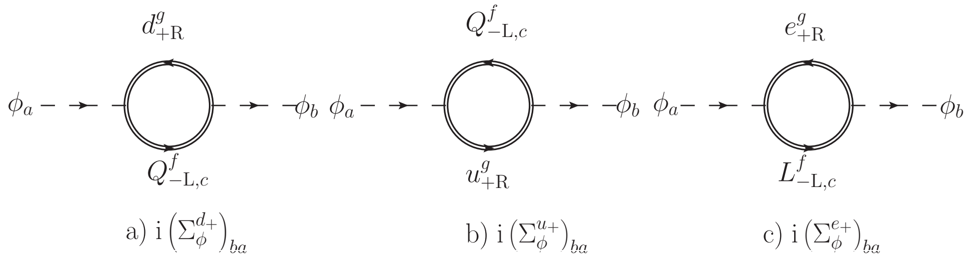


Figure 4.3. Contributions to the Higgs self-energy from fermions which do not have a zero-mode component.

As an example, we calculate diagram d) in Fig. 4.1. Note that we now introduce a factor μ^ε, in order for the gauge coupling to remain dimensionless. Thus, we obtain

$$
\begin{aligned}
&\mathrm{i}\left(\Sigma_\phi^{B(2)}\right) = \\
&= \int \frac{\mathrm{d}^d k}{(2\pi)^4}\left[\mathrm{i}\mu^\varepsilon \frac{1}{2} g_1^2 \eta_{\mu\nu}\right] \mathrm{i}\frac{-\eta^{\mu\nu} + (1-\xi_1)\frac{k^\mu k^\nu}{k^2-\xi_1 M_{j,k}^2}}{k^2 - M_{j,k}^2} \\
&= -\frac{\mathrm{i} g_1^2}{32\pi^2}\frac{\mu^\varepsilon}{\mathrm{i}\pi^2}\int \mathrm{d}^d k \eta_{\mu\nu}\frac{-\eta^{\mu\nu} + (1-\xi_1)\frac{k^\mu k^\nu}{k^2-\xi_1 M_{j,k}^2}}{k^2 - M_{j,k}^2} \\
&= -\frac{\mathrm{i} g_1^2}{32\pi^2}\frac{\mu^\varepsilon}{\mathrm{i}\pi^2}\int \mathrm{d}^d k\left(-\frac{4}{k^2 - M_{j,k}^2} + (1-\xi_1)\frac{k^2}{(k^2 - \xi M_{j,k}^2)(k^2 - M_{j,k}^2)}\right) \\
&= \frac{\mathrm{i} g_1^2}{16\pi^2}(3+\xi_1^2) M_{j,k}^2 \frac{1}{\varepsilon} + \text{UV finite.}
\end{aligned} \tag{4.2}
$$

Here, $M_{j,k}$ is the KK mass. This diagram is allowed in the SM but then, since $M_{j,k} = 0$ in the SM, it is exactly zero and thus, it gives no contribution to the wavefunction renormalization. The other diagrams are calculated in a similar way. Next, we want to determine the renormalization constant for the Higgs wavefunction and mass. In the MS scheme the counterterms cancel the divergences of the Higgs two-point function, but contain no other finite parts. Thus we have

$$
\begin{aligned}
&\mathrm{i}\Sigma_\phi^B + \mathrm{i}\sum_{i=1}^3 \Sigma_\phi^{W^i} + \mathrm{i}\Sigma_\phi^\phi + \mathrm{i}\Sigma_\phi^{B(2)} + \mathrm{i}\sum_{i=1}^3 \Sigma_\phi^{W^i(2)} + \mathrm{i}\Sigma_\phi^d + \mathrm{i}\Sigma_\phi^u + \mathrm{i}\Sigma_\phi^e \\
&+\mathrm{i}\Sigma_\phi^{B_\mathrm{G}} + \mathrm{i}\Sigma_\phi^{B_\mathrm{G}(2)} + \mathrm{i}\sum_{i=1}^3 \Sigma_\phi^{W_\mathrm{G}^i} + \mathrm{i}\sum_{i=1}^3 \Sigma_\phi^{W_\mathrm{G}^i(2)} + \mathrm{i}\Sigma_\phi^{B_\mathrm{H}} \\
&+\mathrm{i}\sum_{i=1}^3 \Sigma_\phi^{W_\mathrm{H}^i} + \mathrm{i}(p^2\delta Z_\phi - \delta m^2) \stackrel{!}{=} \text{UV finite.}
\end{aligned} \tag{4.3}
$$

This equation is solved and the SM and UED contributions to the counterterm for the Higgs wavefunction are given by

$$
\delta Z_\phi^{\mathrm{SM}} = -\frac{1}{16\pi^2}\left[2T - \frac{1}{2}(3-\xi_1)g_1^2 - \frac{3}{2}(3-\xi_2)g_2^2\right]\frac{1}{\varepsilon}, \tag{4.4}
$$

$$
\delta Z_\phi^{\mathrm{UED}} = -\frac{1}{16\pi^2}\left[4T - \frac{1}{2}(3-\xi_1)g_1^2 - \frac{3}{2}(3-\xi_2)g_2^2\right]\frac{1}{\varepsilon}, \tag{4.5}
$$

respectively, where $T = \mathrm{Tr}(Y_e^\dagger Y_e + 3Y_d^\dagger Y_d + 3Y_u^\dagger Y_u)$. The contributions to the counterterm for the Higgs mass are given by

$$(\delta m^2)^{\mathrm{SM}} = \frac{1}{16\pi^2}\left[6\lambda m_0^2 - \frac{1}{2}\xi_1 g_1^2 m_0^2 - \frac{3}{2}\xi_2 g_2^2 m_0^2\right]\frac{1}{\varepsilon}, \tag{4.6}$$

$$(\delta m^2)^{\mathrm{UED}}_{j,k} = \frac{1}{16\pi^2}\left[6\lambda(m_0^2 + M_{j,k}^2)\right. \tag{4.7}$$

$$+ \frac{1}{2}\left(5M_{j,k}^2 - (m_0^2 + M_{j,k}^2)\xi_1 - \xi_1^2 M_{j,k}^2\right) g_1^2 \tag{4.8}$$

$$\left. + \frac{3}{2}\left(5M_{j,k}^2 - (m_0^2 + M_{j,k}^2)\xi_2 - \xi_2^2 M_{j,k}^2\right) g_2^2 - 16 T M_{j,k}^2\right]\frac{1}{\varepsilon}, \tag{4.9}$$

where m_0 is the SM Higgs mass. Note that the contributions from each KK level are dependent on $M_{j,k}$, and thus on j and k. Hence, we will have different contributions from different KK levels, increasing with j and k. For a given energy μ, the counterterm is given by

$$\delta m^2 = (\delta m^2)^{\mathrm{SM}} + \sum_{\sqrt{j^2+k^2}\le\lfloor\frac{\mu}{\mu_0}\rfloor} (\delta m^2)^{\mathrm{UED}}_{j,k}. \tag{4.10}$$

The self-energy of the left-handed lepton doublet, $L_{+\mathrm{L}}$

The one-loop corrections to the self-energy of the left-handed lepton singlet are given in Fig. 4.4. We shall explicitly calculate diagram d) in Fig. 4.4 for B_{H}. We

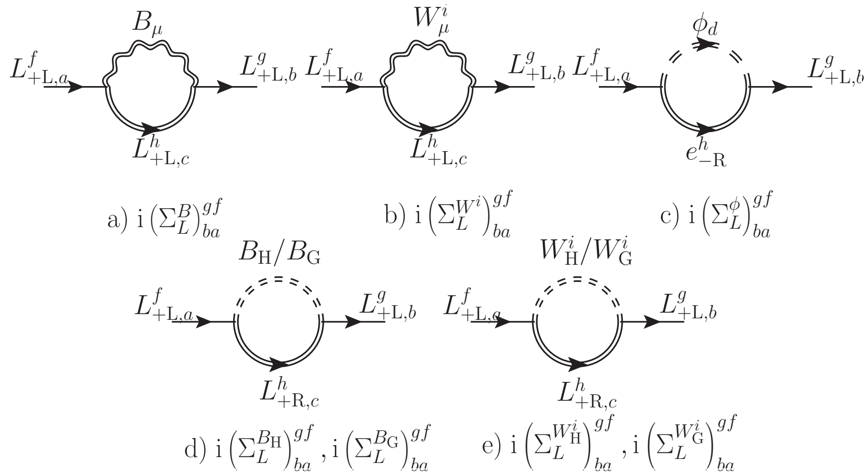


Figure 4.4. The one-loop corrections to the self-energy of the left-handed lepton doublet, $L_{+\mathrm{L}}$.

denote the momenta of the incoming and outgoing fermions p, and the momentum of B_{H} in the loop is denoted k, and hence the momentum for the fermion in the loop is $p+k$. We find that

$$\begin{aligned}
&\mathrm{i}\left(\Sigma_L^{B_{\mathrm{H}}}\right) = \\
&\int \frac{\mathrm{d}^d k}{(2\pi)^d}\left[\frac{1}{2}g_1\delta_{gh}\delta_{bc}P_{\mathrm{R}}\right]\frac{\mathrm{i}(\not p+\not k)}{(p+k)^2}\left[-\frac{1}{2}g_1\delta_{hf}\delta_{ca}P_{\mathrm{L}}\right]\frac{\mathrm{i}}{k^2-M_{j,k}^2} \qquad (4.11)\\
&= \frac{\mathrm{i}}{16\pi^2}g_1^2\delta_{ba}\int \mathrm{d}^d k\frac{\not p+\not k}{(p+k)^2(k^2-M_{j,k}^2)}P_{\mathrm{L}}\\
&= \frac{\mathrm{i}}{64\pi^2}g_1^2\not p\delta_{gf}\delta_{ba}P_{\mathrm{L}}\frac{1}{\varepsilon}+\text{UV finite.}
\end{aligned}$$

The other diagrams are calculated in a similar fashion. We calculate the SM and the UED contributions to the counterterm separately, and find that these contributions are given by

$$\delta Z_{L_{\mathrm{L}}}^{\mathrm{SM}} = -\frac{1}{16\pi^2}\left[Y_e^\dagger Y_e + \frac{1}{2}\xi_1 g_1^2 + \frac{3}{2}\xi_2 g_2^2\right]\frac{1}{\varepsilon}, \qquad (4.12)$$

$$\delta Z_{L_{\mathrm{L}}}^{\mathrm{UED}} = -\frac{1}{16\pi^2}\left[Y_e^\dagger Y_e + \frac{1}{2}(1+\xi_1) g_1^2 + \frac{3}{2}(1+\xi_2) g_2^2\right]\frac{1}{\varepsilon}. \qquad (4.13)$$

The self-energy of the left-handed quark doublet, $Q_{+\mathrm{L}}$

The one-loop corrections to the self-energy of the left-handed quark doublet are given in Fig. 4.5. The diagrams are calculated, and the counterterm is determined, as in the previous section. The SM and the UED contributions to the counterterms are given by

$$\delta Z_{Q_{\mathrm{L}}}^{\mathrm{SM}} = -\frac{1}{16\pi^2}\left[Y_u^\dagger Y_u + Y_d^\dagger Y_d + \frac{1}{18}\xi_1 g_1^2 + \frac{3}{2}\xi_2 g_2^2 + \frac{8}{3}\xi_3 g_3^3\right]\frac{1}{\varepsilon}, \qquad (4.14)$$

$$\begin{aligned}
\delta Z_{Q_{\mathrm{L}}}^{\mathrm{UED}} = &-\frac{1}{16\pi^2}\left[Y_u^\dagger Y_u + Y_d^\dagger Y_d + \frac{1}{18}(1+\xi_1) g_1^2 + \frac{3}{2}(1+\xi_2) g_2^2\right.\\
&\left.+\frac{8}{3}(1+\xi_3) g_3^3\right]\frac{1}{\varepsilon}. \qquad (4.15)
\end{aligned}$$

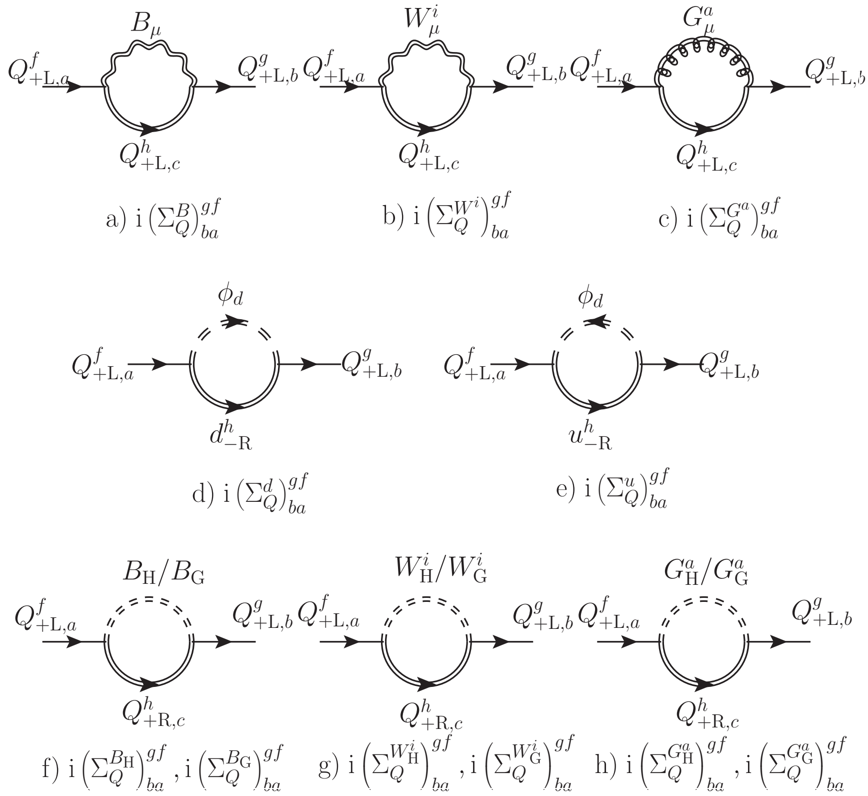


Figure 4.5. The one-loop corrections to the self-energy of the left-handed quark doublet, $Q_{+\mathrm{L}}$.

The self-energy of the right-handed lepton singlet, $e_{-\mathrm{R}}$

The Feynman diagrams to one-loop order contributing to the self-energy of the right-handed lepton singlet are given in Fig. 4.6. The diagrams are calculated as previously described, and hence, the contributions to the counterterm are given by

$$\delta Z_{e_\mathrm{R}}^{\mathrm{SM}} = -\frac{1}{16\pi^2}\left[2Y_e^\dagger Y_e + 2\xi_1 g_1^2\right]\frac{1}{\varepsilon}, \tag{4.16}$$

$$\delta Z_{e_\mathrm{R}}^{\mathrm{UED}} = -\frac{1}{16\pi^2}\left[2Y_e^\dagger Y_e + 2(1+\xi_1) g_1^2\right]\frac{1}{\varepsilon}. \tag{4.17}$$

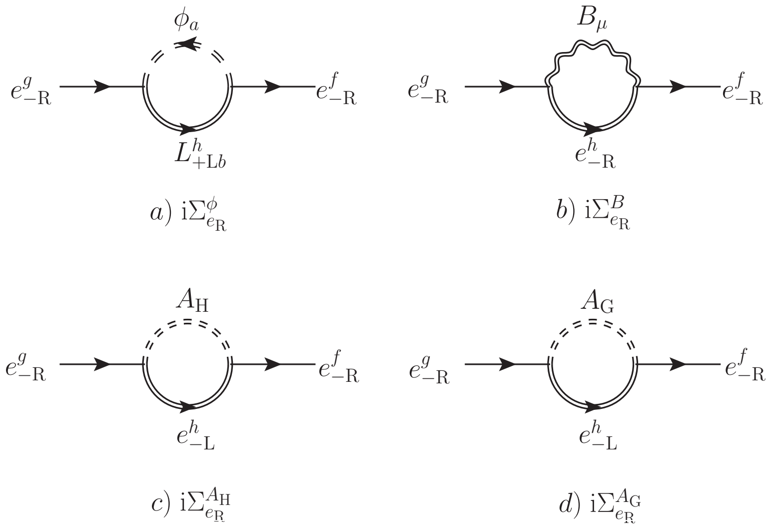


Figure 4.6. The one-loop correction to the right-handed lepton singlet, $e_{-\mathrm{R}}$.

The self-energy of the right-handed down-quark singlet, $d_{-\mathrm{R}}$

The contributions, to one-loop order, to the correction of the self-energy are listed in Fig. 4.7. The SM and UED parts of the counterterm are given by

$$\delta Z^{\mathrm{SM}}_{d_\mathrm{R}} = -\frac{1}{16\pi^2}\left[2Y_d^\dagger Y_d + \frac{2}{9}\xi_1 g_1^2 + \frac{8}{3}\xi_3 g_3^2\right]\frac{1}{\varepsilon}, \tag{4.18}$$

$$\delta Z^{\mathrm{UED}}_{d_\mathrm{R}} = -\frac{1}{16\pi^2}\left[2Y_d^\dagger Y_d + \frac{2}{9}(1+\xi_1) g_1^2 + \frac{8}{3}(1+\xi_3) g_3^2\right]\frac{1}{\varepsilon}. \tag{4.19}$$

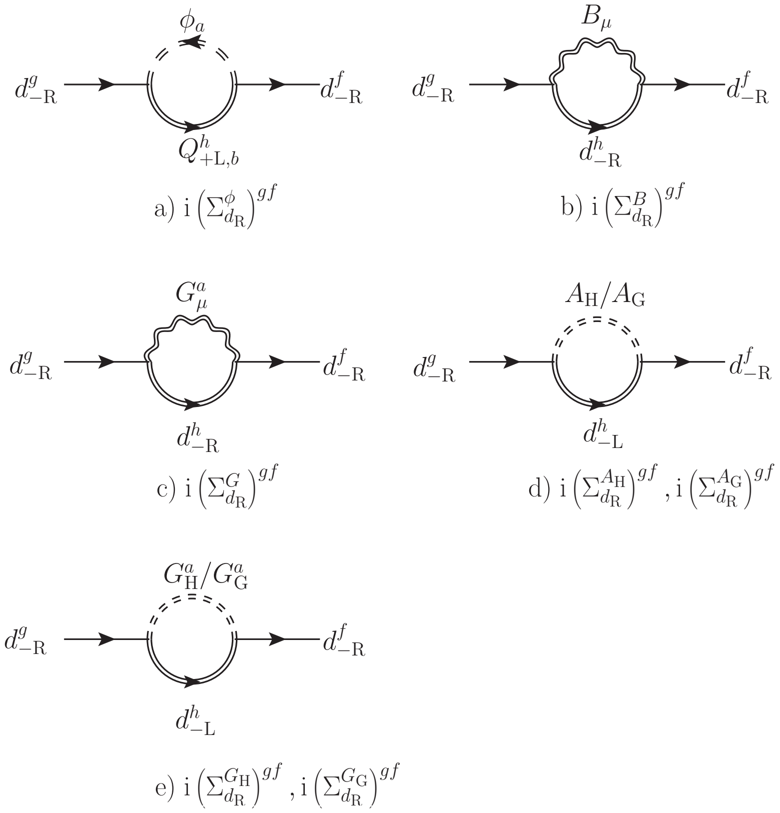


Figure 4.7. The contribution to one loop order to the self-energy och the down-quark singlet, $d_{-\mathrm{R}}$.

The self-energy of the right-handed up quark singlet, $u_{-\mathrm{R}}$

The contributions, to one-loop order, to the self-energy of the up quark, are similar to those for the down quark, and they are presented in Fig. 4.8. The contributions to the counterterm are

$$\delta Z^{\mathrm{SM}}_{u_\mathrm{R}} = -\frac{1}{16\pi^2}\left[2Y_u^\dagger Y_u + \frac{8}{9}\xi_1 g_1^2 + \frac{8}{3}\xi_3 g_3^2\right]\frac{1}{\varepsilon}, \tag{4.20}$$

$$\delta Z^{\mathrm{UED}}_{u_\mathrm{R}} = -\frac{1}{16\pi^2}\left[2Y_u^\dagger Y_u + \frac{8}{9}(1+\xi_1) g_1^2 + \frac{8}{3}(1+\xi_3) g_3^2\right]\frac{1}{\varepsilon}. \tag{4.21}$$

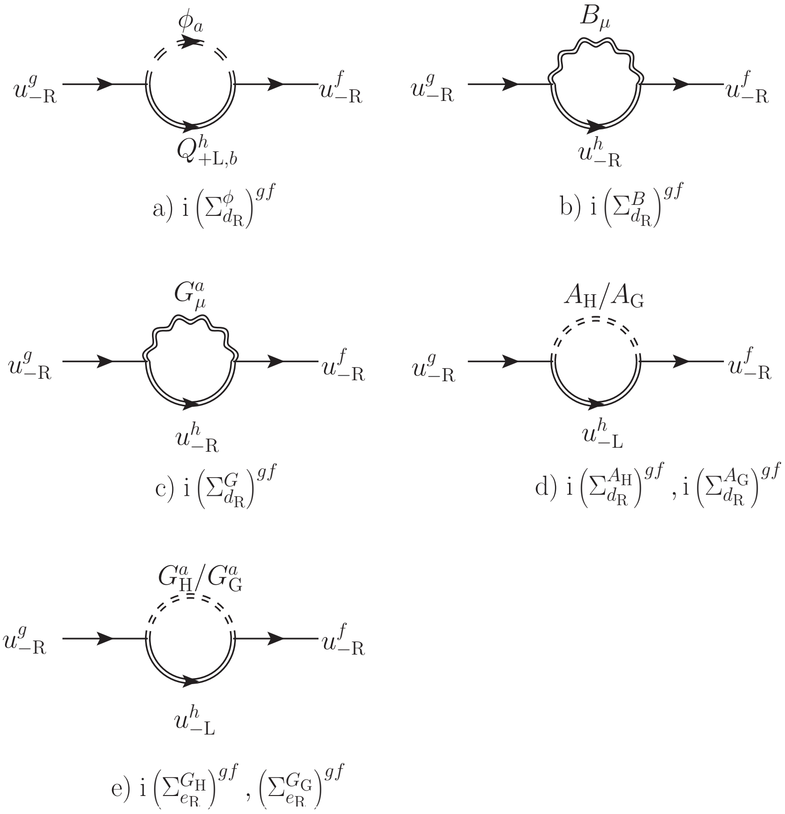


Figure 4.8. One-loop corrections to the self energy of the right-handed up-quark singlet, $u_{-\mathrm{R}}$.

Loop corrections to the $L_{\mathrm{L}} e_{\mathrm{R}} \phi$ Yukawa vertex

Next, we continue to the renormalization of Yukawa vertices. The renormalization process is basically the same for the vertices as for the wavefunctions. The contributing diagrams to the correction to the $L_{\mathrm{L}} e_{\mathrm{R}} \phi$ Yukawa vertex, are given in Fig. 4.9. We shall explicitly calculate diagram f) in Fig. 4.9. The momentum of the incoming fermion is p, the momentum of the outgoing fermion is q and the momentum of the adjoint scalar in the loop is k. Thus, the diagram is calculated as

$$
\begin{aligned}
&\mathrm{i}\mu^{\frac{\varepsilon}{2}} \left(\Sigma_{Y_e}^{B_{\mathrm{H}}} \right) = \\
&\int \frac{\mathrm{d}^d k}{(2\pi)^d} \left[\mu^{\frac{\varepsilon}{2}} g_1 \delta_{jg} P_{\mathrm{L}} \right] \frac{\mathrm{i}(\not{q} + \not{k})}{(q+k)^2} \left[-\mathrm{i}\mu^{\frac{\varepsilon}{2}} (Y_e)_{jh} \delta_{ca} P_{\mathrm{R}} \right] \frac{\mathrm{i}(\not{p} + \not{k})}{(p+k)^2} \\
&\times \left[-\frac{1}{2} \mu^{\frac{\varepsilon}{2}} g_1 \delta_{cb} \delta_{hf} P_{\mathrm{L}} \right] \frac{\mathrm{i}}{k^2 - M_{j,k}^2} \\
&= \mathrm{i}\mu^{\frac{\varepsilon}{2}} \frac{g_1^2}{16\pi^2} (Y_e)_{gf} \delta_{ba} \frac{\mu^{\varepsilon}}{\mathrm{i}\pi^2} \int \mathrm{d}^d k \frac{(\not{q} + \not{k})(\not{p} + \not{k})}{(q+k)^2 (p+k)^2 (k^2 - M_{j,k}^2)} P_{\mathrm{L}} \\
&= \mathrm{i}\mu^{\frac{\varepsilon}{2}} \left(\frac{1}{16\pi^2} g_1^2 (Y_e)_{gf} \delta_{ba} P_{\mathrm{L}} \right) \frac{1}{\varepsilon} + \text{UV finite.}
\end{aligned}
\tag{4.22}
$$

The other diagrams are calculated in a similar fashion and the counterterm is given by

$$
\delta_{Y_e}^{\mathrm{SM}} = -\frac{1}{16\pi^2} \left[3(1 + \frac{1}{2}\xi_1) g_1^2 + \frac{3}{2} \xi_2 g_2^2 \right] \frac{1}{\varepsilon}, \tag{4.23}
$$

$$
\delta_{Y_e}^{\mathrm{UED}} = -\frac{1}{16\pi^2} \left[(1 + \frac{3}{2}\xi_1) g_1^2 + \frac{3}{2} \xi_2 g_2^2 \right] \frac{1}{\varepsilon}. \tag{4.24}
$$

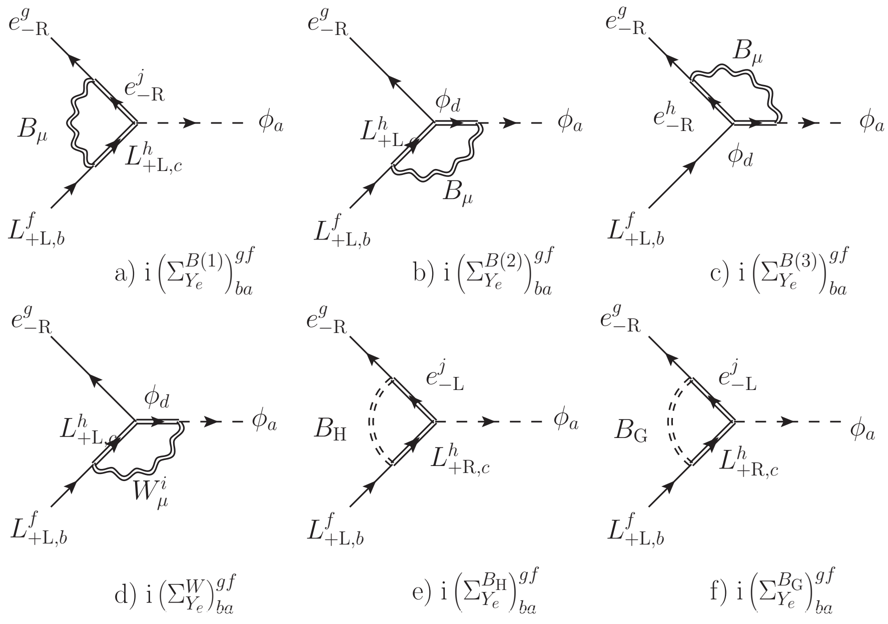


Figure 4.9. The one-loop corrections to the $L_\mathrm{L}e_\mathrm{R}\phi$-Yukawa vertex.

Corrections to the $Q_\mathrm{L}d_\mathrm{R}\phi$-Yukawa vertex

The diagrams contributing to the down-quark Yukawa vertex is similar to those contributing to the lepton Yukawa vertex. The main difference is that quarks carry color charge, and thus we will obtain diagrams, which contain gluons. The one-loop corrections are given in Fig. 4.10. Hence, the counterterm is given by

$$\begin{aligned}\delta Z^{\mathrm{SM}}_{Y_d} = \quad & -\frac{1}{16\pi^2}\left[2Y_u^\dagger Y_u - \frac{1}{3}g_1^2 + \frac{7}{18}\xi_1 g_1^2\right. \\ & \left. + \frac{3}{2}\xi_2 g_2^2 + 8g_3^2 + \frac{8}{3}\xi_3 g_3^2\right]\frac{1}{\varepsilon},\end{aligned} \tag{4.25}$$

$$\begin{aligned}\delta Z^{\mathrm{UED}}_{Y_d} = \quad & -\frac{1}{16\pi^2}\left[2Y_u^\dagger Y_u - \frac{1}{9}g_1^2 + \frac{7}{18}\xi_1 g_1^2\right. \\ & \left. + \frac{3}{2}\xi_2 g_2^2 + \frac{8}{3}(1+\xi_3)g_3^2\right]\frac{1}{\varepsilon}.\end{aligned} \tag{4.26}$$

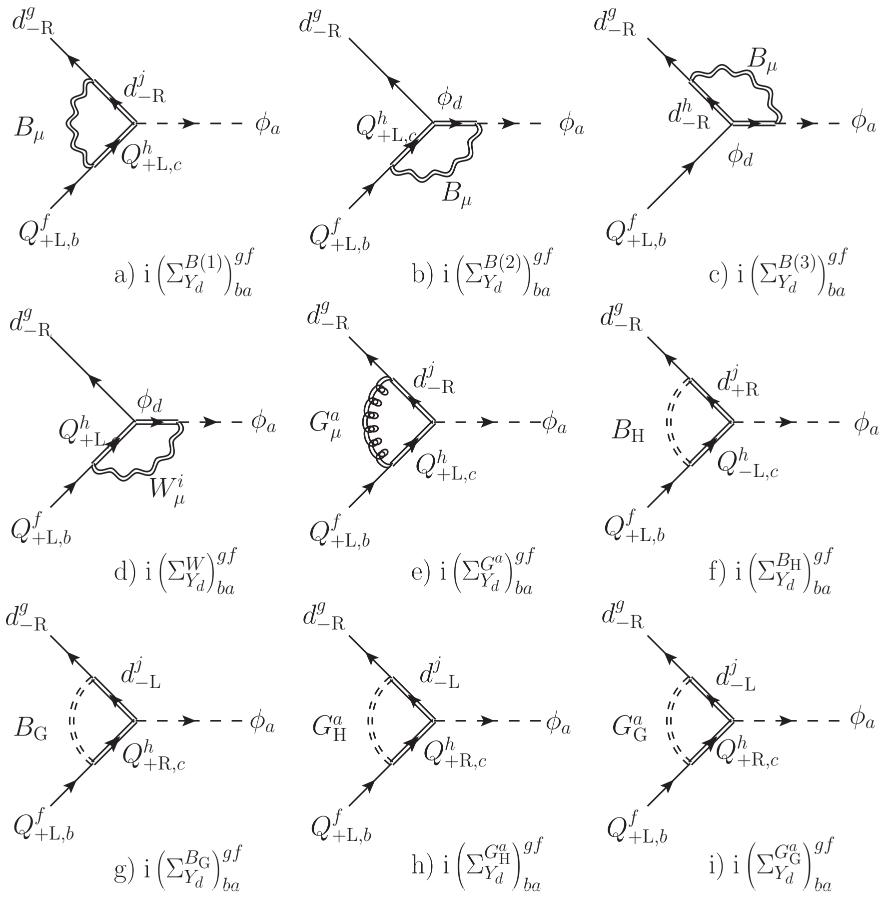


Figure 4.10. The one-loop corrections to the $Q_{\rm L}d_{\rm R}\phi$ Yukawa vertex.

Corrections to the $Q_{\rm L}u_{\rm R}\phi$-Yukawa vertex

The corrections to the $Q_{\rm L}u_{\rm R}\phi$-vertex are given in Fig. 4.11. The SM and UED contributions to the counterterm are given by

$$\begin{aligned}\delta Z^{\rm SM}_{Y_u} = \quad & -\frac{1}{16\pi^2}\left[2Y_d^\dagger Y_d + \frac{2}{3}g_1^2 + \frac{13}{18}\xi_1 g_1^2\right.\\ & \left. + \frac{3}{2}\xi_2 g_2^2 + 8g_3^2 + \frac{8}{3}\xi_3 g_3^2\right]\frac{1}{\varepsilon},\end{aligned} \tag{4.27}$$

$$\begin{aligned}\delta Z^{\rm UED}_{Y_u} = \quad & -\frac{1}{16\pi^2}\left[2Y_u^\dagger Y_u + \frac{2}{9}g_1^2 + \frac{7}{18}\xi_1 g_1^2\right.\\ & \left. + \frac{3}{2}\xi_2 g_2^2 + \frac{8}{3}(1+\xi_3)g_3^2\right]\frac{1}{\varepsilon}.\end{aligned} \tag{4.28}$$

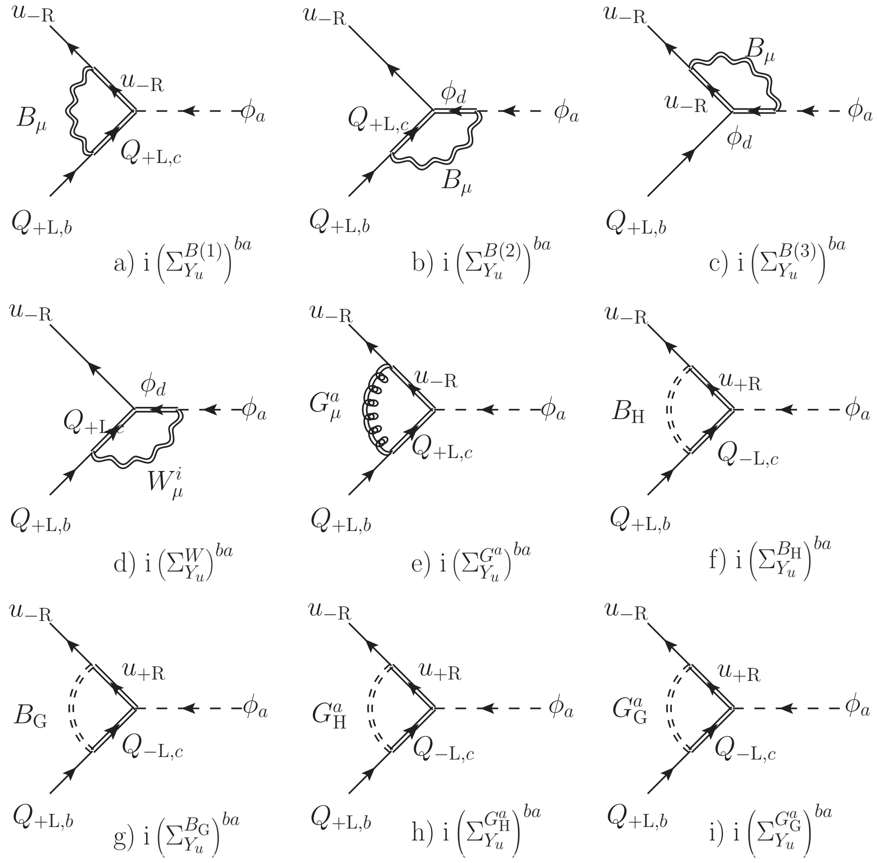


Figure 4.11. The one-loop corrections to the $Q_\mathrm{L} u_\mathrm{R} \phi$ Yukawa vertex.

Corrections to the Higgs self-coupling

The corrections to one-loop order to the Higgs self-coupling are given in Figs. 4.12-4.16. The counterterm for the Higgs self-coupling is given by

$$\begin{aligned}\delta Z_\lambda^{\rm SM} &= \frac{1}{16\pi^2}\Big[12\lambda^2+\frac{3}{4}(g_1^4+2g_1^2g_2^2+3g_2^4)-\lambda(\xi_1 g_1^2+3\xi_2 g_2^2)\\ &\quad -4{\rm Tr}\left((Y_e^\dagger Y_e)^2+3(Y_u^\dagger Y_u)^2+3(Y_d^\dagger Y_d)^2\right)\Big]\frac{1}{\varepsilon}, \qquad (4.29)\\ \delta Z_\lambda^{\rm UED} &= \frac{1}{16\pi^2}\Big[12\lambda^2+\frac{5}{4}(g_1^4+2g_1^2g_2^2+3g_2^4)-\lambda(\xi_1 g_1^2+3\xi_2 g_2^2)\\ &\quad -8{\rm Tr}\left((Y_e^\dagger Y_e)^2+3(Y_u^\dagger Y_u)^2+3(Y_d^\dagger Y_d)^2\right)\Big]\frac{1}{\varepsilon}. \qquad (4.30)\end{aligned}$$

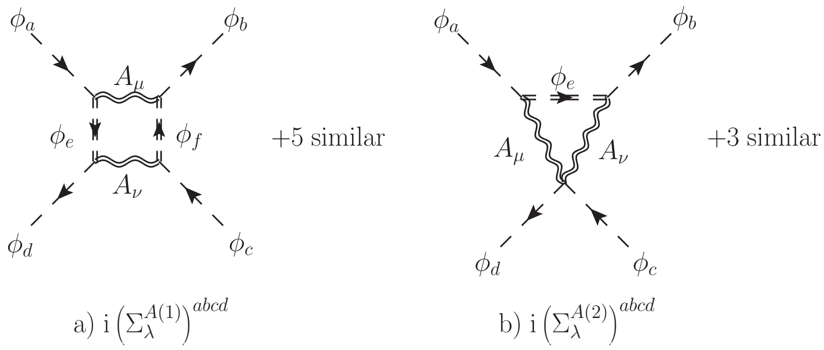
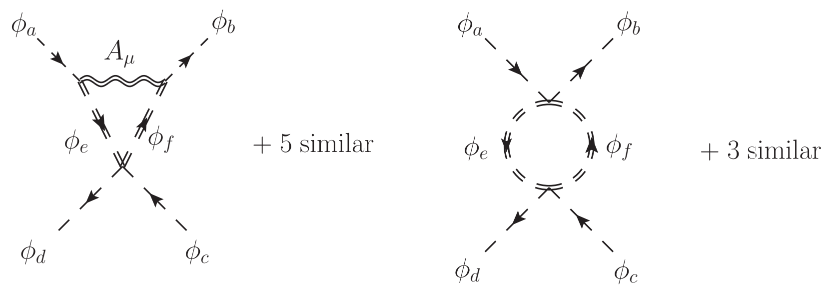
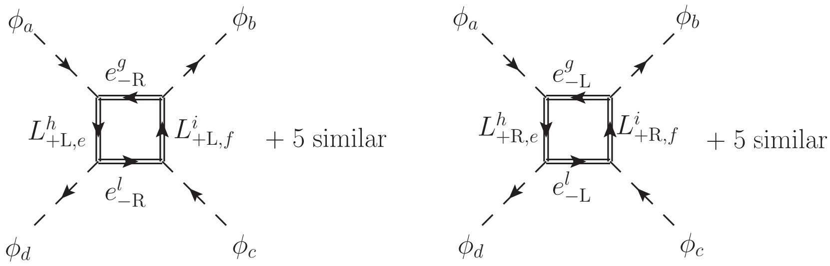


Figure 4.12. Diagrams, with gauge bosons and scalars in the loop, contributing to the Higgs self-interaction. The gauge bosons are $A_\mu = B_\mu, W_\mu^i$.



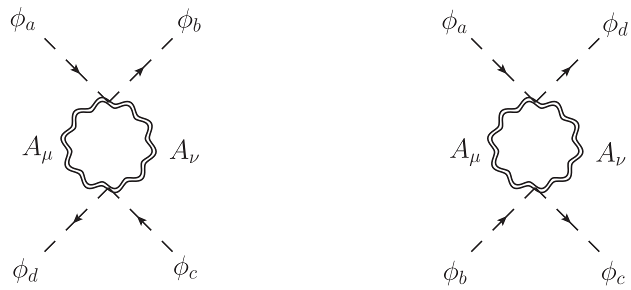
c) $\mathrm{i}\left(\Sigma_\lambda^{A(3)}\right)^{abcd}$ d) $\mathrm{i}\left(\Sigma_\lambda^{\phi}\right)^{abcd}$

Figure 4.13. Diagrams, with gauge bosons and scalars in the loop, contributing to the Higgs self-interaction. The gauge bosons are $A_\mu = B_\mu, W_\mu^i$.



a) $\mathrm{i}\left(\Sigma_\lambda^{\Psi}\right)^{abcd}$ b) $\mathrm{i}\left(\Sigma_\lambda^{\Psi(2)}\right)^{abcd}$

Figure 4.14. The contributions from fermion loops. We obtain new contributions from the KK modes, which do not have a zero-mode component.



a) $\mathrm{i}\left(\Sigma_\lambda^{A(4)}\right)^{abcd}$ a) $\mathrm{i}\left(\Sigma_\lambda^{A(5)}\right)^{abcd}$

Figure 4.15. Contributions from the gauge bosons, where we have $A_\mu = B_\mu, W_\mu^i$.

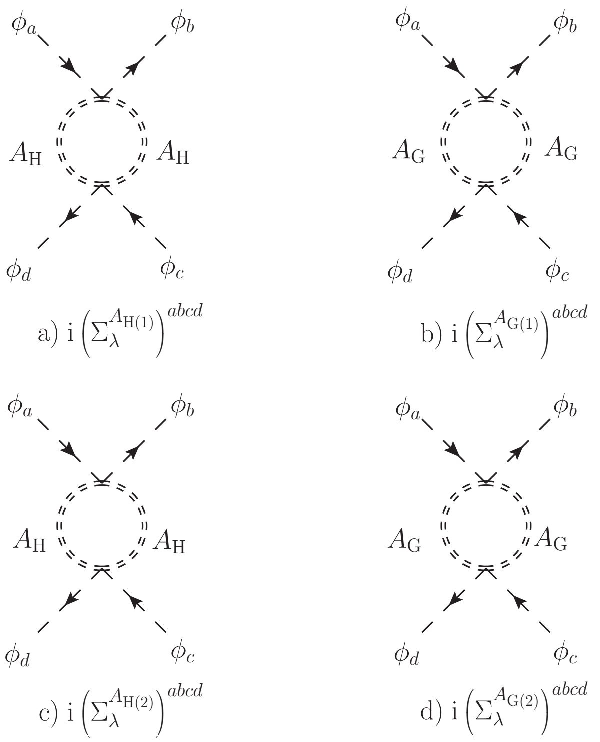


Figure 4.16. The contributions from the adjoint scalars, where $A_\mu = B_\mu, W_\mu^i$.

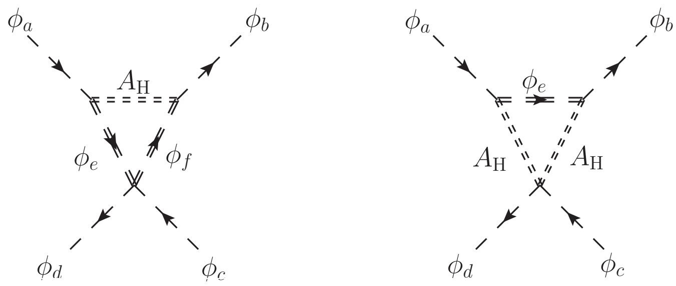


Figure 4.17. Diagrams of this type are in principle allowed. They do, however, only give a finite contribution to the one-loop correction.

The effective neutrino mass operator

Finally, we renormalize the effective neutrino mass operator. The one-loop contributions are given in Figs. 4.18-4.20. We shall calculate the diagram e) in Fig. 4.18,

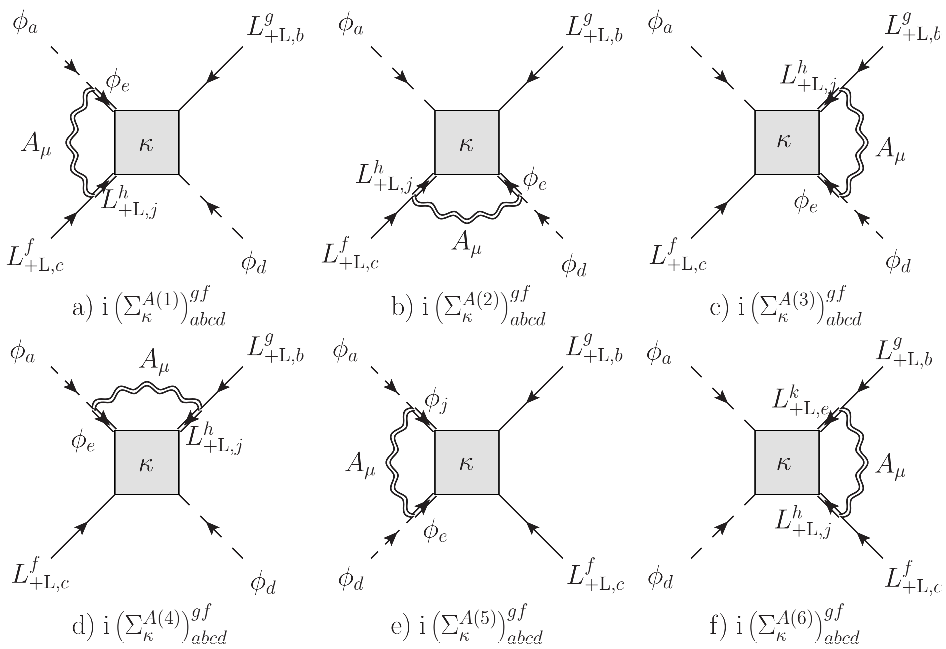


Figure 4.18. The Feynman diagrams containing gauge bosons contributing to the one-loop corrections to the Weinberg operator, here $A_\mu = B_\mu, W^i_\mu$.

for both the B and the W^i bosons. The incoming momenta of the Higgs particles are denoted p' and q', and the momentum of the gauge boson in the loop is denoted

k. Thus, we obtain

$$
\begin{aligned}
& \mathrm{i}\mu^{\epsilon}(\Sigma_{\kappa}^{B(1)}) = \\
= \quad & \int \frac{\mathrm{d}^d k}{(2\pi)^2}\left[\mathrm{i}\mu^{\epsilon}\kappa_{gf}\frac{1}{2}(\varepsilon_{ce}\varepsilon_{bj}+\varepsilon_{cj}\varepsilon_{be})P_{\mathrm{L}}\right]\frac{\mathrm{i}}{(q'-k)^2-m_1^2} \\
\times \quad & \left[\frac{\mathrm{i}}{2}\mu^{\epsilon}g_1(-2q'+k)_{\mu}\delta_{ja}\right]\mathrm{i}\frac{-\eta^{\mu\nu}+(1-\xi_1)\frac{k^{\mu}k^{\nu}}{k^2-\xi_1 M^2}}{k^2-M^2} \\
\times \quad & \left[\frac{\mathrm{i}}{2}\mu^{\epsilon}g_1(-2p'-k)_{\nu}\delta_{de}\right]\frac{\mathrm{i}}{(p'+k)^2-m_2^2} \\
= \quad & \mu^{\epsilon}\frac{\mathrm{i}}{64\pi^2}g_1^2\kappa_{gf}\frac{1}{2}(\varepsilon_{cd}\varepsilon_{ba}+\varepsilon_{ca}\varepsilon_{bd})\frac{\mu^{\epsilon}}{\mathrm{i}\pi^2} \\
\times \quad & \int \mathrm{d}^d k\frac{(2q'-k)_{\nu}}{(q'-k)^2-m_1^2}\frac{-\eta^{\mu\nu}+(1-\xi_1)\frac{k^{\mu}k^{\nu}}{k^2-\xi_1 M^2}}{k^2-M^2}\frac{(2p'+k)_{\nu}}{(p'+k)^2-m_2^2}P_{\mathrm{L}} \\
= \quad & -\mu^{\epsilon}\frac{\mathrm{i}}{32\pi^2}g_1^2\kappa_{gf}\frac{1}{2}(\varepsilon_{cd}\varepsilon_{ba}+\varepsilon_{ca}\varepsilon_{bd})\xi_1 P_{\mathrm{L}}\frac{1}{\varepsilon}+\text{UV finite.} \qquad (4.31)
\end{aligned}
$$

For the corresponding diagram with W bosons, we shall use the relation

$$
\tau^i_{ab}\tau^i_{cd} = 2\delta_{ad}\delta_{bc}-\delta_{ab}\delta_{cd} \qquad (4.32)
$$

and from this we obtain

$$
\begin{aligned}
(\varepsilon_{ce}\varepsilon_{bj}+\varepsilon_{cj}\varepsilon_{be})\tau^i_{ja}\tau^i_{ed} &= (\varepsilon_{ce}\varepsilon_{bj}+\varepsilon_{cj}\varepsilon_{be})(2\delta_{jd}\delta_{ae}-\delta_{ja}\delta_{ed}) \\
&= \varepsilon_{ca}\varepsilon_{bd}+\varepsilon_{cd}\varepsilon_{ba}. \qquad (4.33)
\end{aligned}
$$

Thus, for the W boson we have

$$
(\Sigma_{\kappa}^{W(1)}) = -\frac{1}{32\pi^2}g_2^2\kappa_{gf}\frac{1}{2}(\varepsilon_{cd}\varepsilon_{ba}+\varepsilon_{ca}\varepsilon_{bd})\xi_2 P_{\mathrm{L}}\frac{1}{\varepsilon}+\text{UV finite.} \qquad (4.34)
$$

The other diagrams are calculated in a similar way. Finally, we obtain the counterterm for the Weinberg operator to be

$$
\begin{aligned}
\delta\kappa^{\mathrm{SM}} &= -\frac{1}{16\pi^2}\Big[2\kappa(Y_e^{\dagger}Y_e)+2(Y_e^{\dagger}Y_e)^{\mathrm{T}}\kappa-\lambda\kappa \qquad (4.35) \\
&\quad -\left(\frac{3}{2}-\xi_1\right)g_1^2\kappa-\left(\frac{3}{2}-3\xi_2\right)g_2^2\kappa\Big]\frac{1}{\varepsilon}, \\
\delta\kappa^{\mathrm{UED}} &= -\frac{1}{16\pi^2}\Big[2\kappa(Y_e^{\dagger}Y_e)+2(Y_e^{\dagger}Y_e)^{\mathrm{T}}\kappa-\lambda\kappa. \qquad (4.36) \\
&\quad -\left(\frac{1}{2}-\xi_1\right)g_1^2\kappa-\left(\frac{1}{2}-3\xi_2\right)g_2^2\kappa\Big]\frac{1}{\varepsilon}.
\end{aligned}
$$

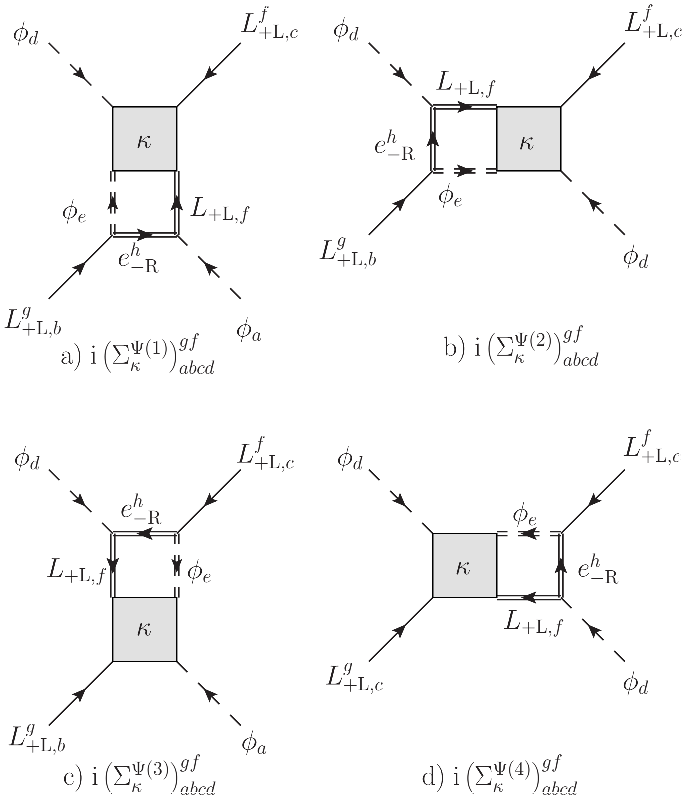


Figure 4.19. The Feynman diagrams, with fermions in the loop, contributing to the one-loop corrections to the Weinberg operator.

4.2 β-Functions

We now want to calculate the β-functions from the counterterms that have been determined. Since the counterterms can be split into one SM and one UED part, the β-functions can be divided into one SM part, corresponding to $j = 0, k = 0$ and one UED part, which corresponds to higher KK modes [64, 65]. This can be written for any quanitity Q as

$$\frac{\mathrm{d}Q}{\mathrm{d}\ln\mu} = \beta^{\mathrm{SM}}_{\mathrm{Q}} + s\beta^{\mathrm{UED}}_{\mathrm{Q}}, \tag{4.37}$$

where $\beta^{\mathrm{SM}}_{\mathrm{Q}}$ denotes the SM part and $\beta^{\mathrm{UED}}_{\mathrm{Q}}$ denotes the contributions from one KK level. We have used the fact that the particle content at each KK level above the SM is identical.

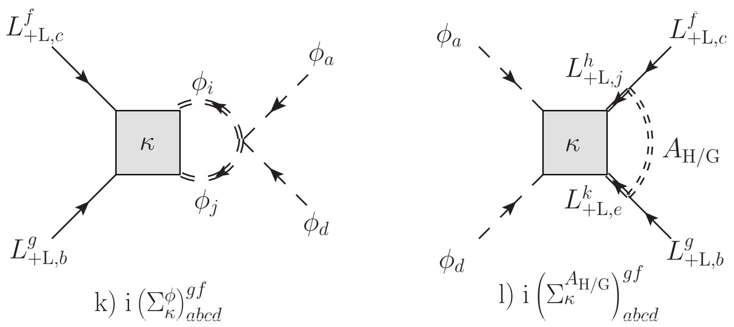


Figure 4.20. The Feynman diagrams, contributing to the one-loop correction, containing the Higgs and adjoint scalars, with $A_\mu = B_\mu, W^i_\mu$.

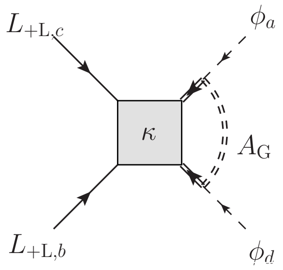


Figure 4.21. This Feynman diagram is in principle allowed, it does, however, only give a finite contribution.

β-function for the Higgs self-coupling

The relation between the Higgs self-coupling and its bare counterpart is

$$Z^2_\phi \lambda_B \quad = \quad \mu^{\frac{\epsilon}{2}} Z_\lambda \lambda. \tag{4.38}$$

Thus we need δZ_λ and δZ_ϕ, given in Eqs. (4.4), (4.5), (4.29), and (4.30), in order to calculate the β-function, which is done by the expression given in Ref. [66]. The UED contribution to the β-function can be calculated as

$$\beta^{\mathrm{UED}}_\lambda \quad = \quad \frac{1}{16\pi^2}\Big[12\lambda^2 + \frac{5}{4}(g_1^4 + 2g_1^2 g_2^2 + 3g_2^4) - 3\lambda(g_1^2 + 3g_2^2) \tag{4.39}$$

$$+ \quad 8\lambda T - 8\mathrm{Tr}\left((Y_e^\dagger Y_e)^2 + 3(Y_u^\dagger Y_u)^2 + 3(Y_d^\dagger Y_d)^2\right)\Big]. \tag{4.40}$$

The SM contribution to the β-function is given in Ref. [67]

$$\beta_\lambda^{\rm SM} = \frac{1}{16\pi^2}\Big[12\lambda^2 + \frac{3}{4}(g_1^4 + 2g_1^2g_2^2 + 3g_2^4) - 3\lambda(g_1^2 + 3g_2^2) \tag{4.41}$$

$$+4\lambda T - 4{\rm Tr}\left((Y_e^\dagger Y_e)^2 + 3(Y_u^\dagger Y_u)^2 + 3(Y_d^\dagger Y_d)^2\right)\Big]. \tag{4.42}$$

The total β-function at a given energy is then given by

$$\beta_\lambda = \beta_\lambda^{\rm SM} + s\beta_\lambda^{\rm UED}. \tag{4.43}$$

β-functions for the Yukawa couplings

The SM beta functions for the Yukawa couplings can be found in, *e.g.*, Ref. [68]. The UED contributions to the β-function have been calculated using Ref. [66]. Thus, the β-functions for the Yukawa couplings are given by

$$\beta_{Y_e}^{\rm SM} = \frac{1}{16\pi^2}Y_e\left(T + \frac{3}{2}Y_e^\dagger Y_e - \frac{15}{4}g_1^2 - \frac{9}{4}g_2^2\right), \tag{4.44}$$

$$\beta_{Y_e}^{\rm UED} = \frac{1}{16\pi^2}Y_e\left(2T + \frac{3}{2}Y_e^\dagger Y_e - \frac{1}{2}g_1^2 - \frac{3}{2}g_2^2\right), \tag{4.45}$$

$$\beta_{Y_u}^{\rm SM} = \frac{1}{16\pi^2}Y_u\left(T + \frac{3}{2}Y_u^\dagger Y_u - \frac{3}{2}Y_d^\dagger Y_d - \frac{17}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right), \tag{4.46}$$

$$\beta_{Y_u}^{\rm UED} = \frac{1}{16\pi^2}Y_u\left(2T + \frac{3}{2}Y_u^\dagger Y_u - \frac{3}{2}Y_d^\dagger Y_d - \frac{1}{2}g_1^2 - \frac{3}{2}g_2^2\right), \tag{4.47}$$

$$\beta_{Y_d}^{\rm SM} = \frac{1}{16\pi^2}Y_d\left(T + \frac{3}{2}Y_d^\dagger Y_d - \frac{3}{2}Y_u^\dagger Y_u - \frac{17}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right), \tag{4.48}$$

$$\beta_{Y_d}^{\rm UED} = \frac{1}{16\pi^2}Y_d\left(2T + \frac{3}{2}Y_d^\dagger Y_d - \frac{3}{2}Y_u^\dagger Y_u - \frac{1}{2}g_1^2 - \frac{3}{2}g_2^2\right). \tag{4.49}$$

β-function for the effective neutrino mass operator

We also calculate the β-function for the effective neutrino mass operator. The SM contribution has been calculated elsewhere, see *e.g.*, Refs. [69, 70, 66], and it is given by

$$\beta_\kappa^{\rm SM} = \frac{1}{16\pi^2}\left(2T\kappa + \lambda\kappa - \frac{3}{2}\kappa(Y_e^\dagger Y_e) - \frac{3}{2}(Y_e^\dagger Y_e)^T\kappa - 3g_2^2\kappa\right). \tag{4.50}$$

$$\tag{4.51}$$

The UED contribution is calculated using Ref. [66], and we thus obtain

$$\beta_\kappa^{\mathrm{UED}} = \frac{1}{16\pi^2}\left(4T\kappa + \lambda\kappa - \frac{3}{2}\kappa(Y_e^\dagger Y_e) - \frac{3}{2}(Y_e^\dagger Y_e)^T\kappa - \frac{3}{2}g_1^2\kappa - \frac{5}{2}g_2^2\kappa\right). \tag{4.52}$$

β-function for the coupling constants

In order to solve the RGEs, we need the β-functions for the gauge couplings. These have previously been calculated and are given by

$$16\pi^2\frac{\mathrm{d}g_i}{\mathrm{d}\ln\mu} = (b_k + s\bar{b}_k)g_i^2, \tag{4.53}$$

where $b_1 = 41/6, b_2 = -19/6, b_3 = -7$ and $\bar{b}_1 = 27/2, \bar{b}_2 = 3/2, \bar{b} = -2$ [55].

4.3 Numerical Results

We want to solve the RGEs in order to determine the running of the neutrino parameters. Since it is impossible to do so analytically, we shall solve them numerically using Matlab. The particle data that have been used are taken from Refs. [71, 72, 73]. We have defined the energy where the KK modes begin to contribute to the β-function to be $\mu_0 = R^{-1} = 1$ TeV. In addition to solving the equations, we can determine the cutoff scale, Λ, where the effective theory no longer is valid. The cutoff energy is then the energy where the running of the couplings begins to diverge, and the energy can be defined in terms of μ_0. We find the cutoff scale to be $\Lambda \approx 6.5\mu_0$. This cutoff is rather low and it restricts the number of contributing KK modes heavily. We have assumed normal mass hierarchy in the calculations, *i.e.*, $m_1 < m_2 < m_3$, and the CP-violating phases have been taken to be equal to zero. We study the RG evolution of the parameters in the region from M_{Z}, *i.e.*, the Z boson mass, to Λ.

In Figs. 4.22 and 4.23, we present the RG evolution of the lepton mixing angles. At M_{Z} are $\theta_{23} \approx 45°$, *i.e.*, maximal, and $\theta_{12} \approx 34°$. Until recently, the mixing angle θ_{13} has been assumed to be zero. New data, however, from especially the Daya Bay experiment, but also the T2K, MINOS, and Double Chooz experiments show that it is in fact non-zero, and actually comparatively large [23, 74, 75, 76, 77]. As these experiments are currently retreiving more data, it is to be expected that a value of θ_{13} with smaller error margins soon ought to be known. We have therefore made calculations both with $\theta_{13} = 0$ and with $\theta_{13} = 6.5°$, which is the value determined in Ref. [77]. Furthermore, we have taken the lowest neutrino mass to be 0.5 eV. We observe that the running of the mixing angle θ_{12} is considerable, especially above 1 TeV, *i.e.*, the energy where the first KK modes are excited. This parameter is at the cutoff energy almost maximal. The mixing angle θ_{23} exhibits a marginal running behavior that can be neglected. As for the parameter θ_{13}, it remains constantly zero in the case when $\theta_{13}(M_{\rm Z}) = 0$, whereas it in the case when $\theta_{13}(M_{\rm Z}) = 6.5^\circ$ exhibits a very small running near the cutoff energy, which can be completely neglected. The RG running of the mixing angles lies in line with other models, where the RG running of θ_{12} normally is the only significant one and both θ_{23} and θ_{13} are stable under RG running, see *e.g.*, [78, 79].

As expected, the extra dimensions enhance the RG running, which now has a power-law dependence. The results in the six-dimensional UED model are relevant to discuss in comparison to the ones in the five-dimensional UED model. The RG running of the mixing angles and the neutrino masses have been investigated in the context of a five-dimensional UED model in Ref. [65], and the running of the gauge coupling is discussed in Ref. [80]. First, we can observe that the cutoff energy in the five-dimensional model is taken to be $\Lambda = 50$ TeV, which is significantly higher than for the six-dimensional case. In accordance with the six-dimensional model, the only mixing angle which exhibit a significant running behavior in the five-dimensional model is θ_{12}, which then becomes nearly maximal and satisfies $\theta_{12}(\Lambda) > \theta_{23}(\Lambda)$, due to the higher cutoff energy. In neither of the models is the running of any of the other mixing angles considerable.

In both UED models is the RG contribution to the mass significant. In Fig. 4.24, we present the running of the quantity $m_i(\mu)/m_i(M_{\rm Z})$, for both the six-dimensional and the five-dimensional UED model. This quantity is universal within the model and thus equal for $i = 1, 2, 3$. The RG evolution has been plotted for m_1, which fulfills $m_1(M_{\rm Z}) = 0.5$ eV. There is a significant contribution to the mass, especially at energies above μ_0. In the six-dimensional model we find at the cutoff that $m_i(\Lambda)/m_i(M_{\rm Z}) \approx 6.5$. At this energy the mass seems to run rather fast. In the five-dimensional model, we find that $m_i(\Lambda)/m_i(M_{\rm Z}) \approx 3.8$. The five-dimensional and the six-dimensional models run in a similar ways up to an energy slightly above μ_0. Then the six-dimensional model run faster but is, however, limited by the cutoff energy. Both UED models exhibit a power-law running, and as expected, we find the running of θ_{12} and the masses near the cutoff energy to be more considerable in the six-dimensional model, due to the higher number of excited KK modes.

We present the RG evolution of the gauge couplings in Fig. 4.25. For the gauge couplings, the running in both the models is quite similar. All the gauge couplings exhibit significant RG running, the running of g_3 is considerable already below μ_0, whereas the running of g_1 and g_2 only is considerable above μ_0. In the six-dimensional model the gauge couplings seem to approach each other as the energy increases, but they do not reach each other in the valid energy range and thus do not attain the energy needed for a unification. In the five-dimensional model, it is found that the couplings unify at a higher energy, however this occurs in an energy region forbidden by a vacuum stability criterion.

We can conclude that the RG running in the five-dimensional and six-dimensional UED model is similar. There are, however, differences which hopefully are sufficient to distinguish them. Contrary to the power-law behavior of the UED models, the running of the SM is logarithmic. Thus, it ought to be possible to, based on the

RG running, rather easily distinguish between the UED model and the SM.

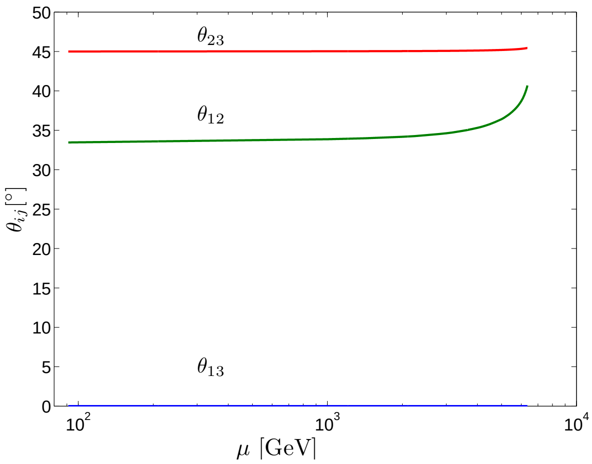


Figure 4.22. The RG evolution of the leptonic mixing angles in the six-dimensional UED model, from M_Z to the cutoff, Λ, dependent on the energy scale, μ for $\theta_{13}(M_Z) = 0$.

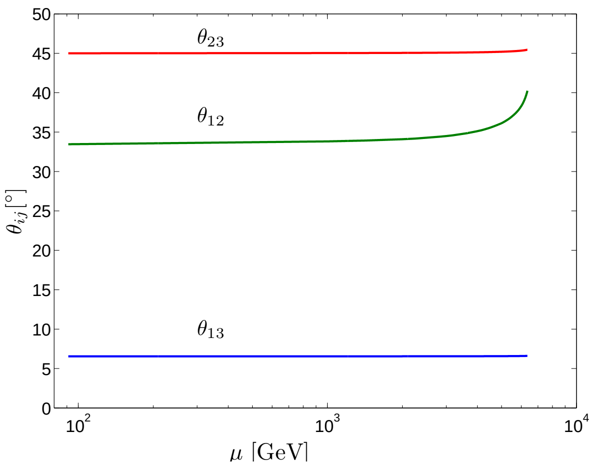


Figure 4.23. The RG evolution of the leptonic mixing angles in the six-dimensional UED model, from M_Z to the cutoff, Λ, dependent on the energy scale, μ, for $\theta_{13}(M_Z) = 6.5°$.

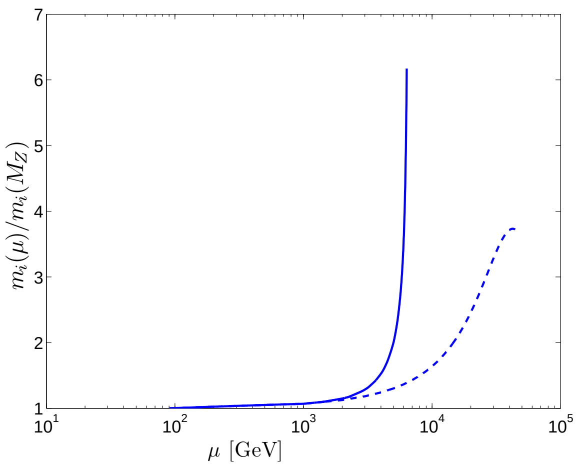


Figure 4.24. The RG evolutions of $m_i(\mu)/m_i(M_Z)$, in the UED models. The solid line corresponds to the six-dimensional model and the dashed line corresponds to the five-dimensional model. Here $\mu_0 = 1$ TeV, and normal mass hierarchy. The five-dimensional data are taken from Ref. [65].

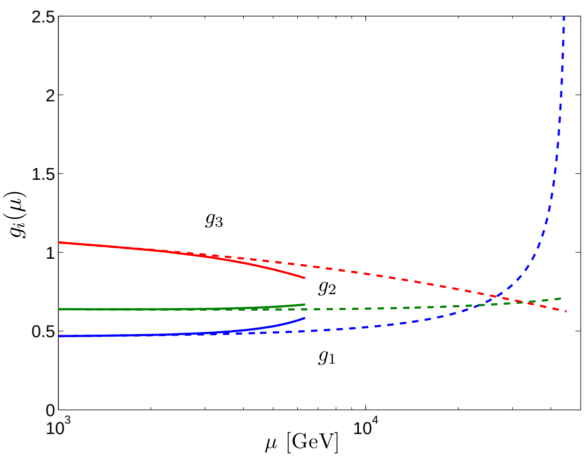


Figure 4.25. The RG evolution of the three gauge couplings, as functions of the energy scale, where $\mu_0 = 1$ TeV. The solid lines correpsond to the six-dimensional model and the dashed lines correspond to the five-dimensional model. The five-dimensional data are taken from ref. [80].

Chapter 5

Summary and Conclusions

In this thesis, we have studied the subject of neutrinos in a six-dimensional model with UEDs. The model has several pleasant features; it provides a mechanism for EWSB, it contains a natural dark matter candidate, and it requires the number of fermion families to be a multiple of three. The two extra, spatial, dimensions are, in this model, compactified on the so-called chiral square, and all SM fields probe them. In our effectively four-dimensional world, we perceive the extra dimensions as infinite towers of heavy particles, so-called KK towers. From the six-dimensional gauge fields, we effectively obtain two new, real, scalar particles, known as the adjoint scalars. The KK modes and the adjoint scalars can interact with the SM particles, and at high energies these interactions give rise to new contributions to the renormalization of wavefunctions and vertices, which affects the RGEs.
In Ch. 1, we gave an introduction to the subject of physics, especially particle physics, neutrinos, and extra dimensions. In Ch. 2, the concept of QFT was introduced, as well as a more detailed discussion on the SM and its shortcomings. We have focused on the existence of neutrino masses, which are assumed to be zero in the SM. In that chapter, we also introduced the six-dimensional UED model, which is the model studied in this thesis, together with the concept of KK decomposition. We described how interactions in six dimensions are manifested in an effective four-dimensional Lagrangian, which describes the interactions we can observe in our four-dimensional world. The Feynman rules for the effective Lagrangian were derived and they are presented in appendix A. We also introduced an effective neutrino mass operator, the Weinberg operator, used to generate neutrino masses in the model. Furthermore, the concept of neutrino oscillations and lepton mixing was discussed. In Ch. 3, we introduced the concept of renormalization, especially dimensional regularization and the MS scheme, which have been used in the thesis. We also discussed the derivation of RGEs. In Ch. 4, we calculated the RGEs for the Higgs self-coupling, the Yukawa couplings, and the effective neutrino mass operator. The RGEs were solved numerically, and the RG evolution of the mixing angles, neutrino masses, and gauge couplings was determined. We could conclude that the

extra dimensions enhance the running, as each KK level that comes into play gives an additional contribution to the RGEs. The RG running was significant for θ_{12}, but could be neglected for θ_{23} and θ_{13}. The RG contribution to the neutrino masses are significant in this model. Furthermore, the gauge coupling run and seem to be approaching each other. They do not, however, attain the energy where a unification possibly could occur. We also determined the cutoff scale, where the model is no longer valid, which was relatively low. This heavily restricts the number of contributing KK levels.

The main results of this thesis are the RGEs for the Higgs self-coupling, the Yukawa couplings, and the Weinberg operator, in the six-dimensional UED model. These are presented in Eqs. (4.39), (4.44)-(4.49), and (4.52), the RG evolutions of the leptonic mixing angles, the neutrino masses, and the gauge couplings, plotted in Figs. 4.22, 4.24, and 4.25, as well as the derivations of the Feynman rules presented in appendix A.

Extra-dimensional theories are popular at present, especially in the context of string theory, and extra dimensions provide a mathematically rather natural extension to the SM. The types of models studied in this thesis, with one or two extra dimensions, are of particular interest since they, contrary to string theory, have interesting features at comparatively low energies, which can be investigated at the LHC. At present, the results from the LHC already limits the size of the extra dimensions, and as the energies investigated increase, the possible sizes decrease. Hopefully, it will give some indications on whether extra-dimensional theories is the right way to go.

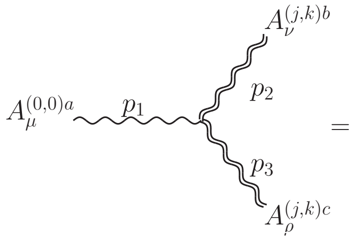
Appendix A

Feynman Rules

In this appendix, we present the relevant Feynman rules, derived from the effective four-dimensional Lagrangian, which are the rules for vertices containing at least one SM particle. In the diagrams, the SM particles are denoted by single lines, and the KK modes by double lines.
In principle, the diagrams containing fermions should include the direction of fermion-number flow [63]. However, in general, we find that the Feynman rules are identical for both flow directions, and the only difference occurs for the fermionic propagator where a fermion-number flow opposite to the direction of the propagator will yield a change of sign.
There are multiple possible ways to choose two SM particles in vertices corresponding to terms in the Lagrangian containing a $\delta_{0,0,0}^{(j_1,k_1),(j_2,k_2),(j_3,k_3)}$ or a $\delta_{0,0,0,0}^{(j_1,k_1),..,(j_4,k_4)}$, defined in Eq. (2.27), all with the same Feynman rule. In some of these cases, the delta-function has been kept in the Feynman rule for simplicity. We shall denote a four-dimensional gauge coupling g_4. Specifically, g_1 is the gauge coupling for $U(1)_{\mathrm{Y}}$, g_2 the one for $SU(2)_{\mathrm{L}}$, and g_3 the one for $SU(3)_{\mathrm{c}}$. The Higgs self-coupling is denoted λ and the Yukawa couplings are Y_e, Y_d, and Y_u. The four-dimensional couplings must be multiplied by some power of μ, in order for them to be dimensionless in d dimensions.
In the cases when there are similar diagrams for several gauge bosons, we generically denote them by A_μ, where, in general, $A_\mu = B_\mu, W_\mu^i, G_\mu^a$. However, only the quarks interact strongly, *i.e.*, couple to G_μ^a, and, as the right-handed fermions do not interact via the weak interaction, they do not couple to W_μ^i.

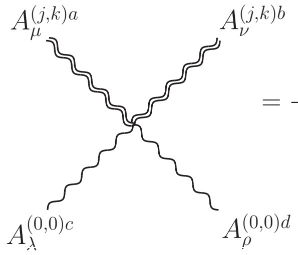
Gauge boson self-interactions

There are several possible self-interactions of the different types of gauge bosons, and we only state the generic case, $A_\mu = B_\mu, W_\mu^i, G_\mu^a$. The structure factors of the gauge group, corresponding to the gauge bosons, are denoted f^{abc}. Note that the KK numbers are conserved separately at each vertex.



$$= g_4 f^{abc} \left[\eta^{\mu\nu}(p_2 - p_1)^\rho + \eta^{\nu\rho}(p_3 - p_2)^\mu + \eta^{\rho\mu}(p_1 - p_3)^\nu\right]$$

Figure A.1. The self-interaction of three gauge bosons.



$$= -\mathrm{i} g_4^2 \left[f^{abc} f^{ade}(\eta^{\mu\lambda}\eta^{\nu\rho} - \eta^{\mu\rho}\eta^{\nu\lambda}) + f^{abd} f^{ace}(\eta^{\mu\nu}\eta^{\lambda\rho} - \eta^{\mu\rho}\eta^{\nu\lambda}) + f^{acd} f^{abe}(\eta^{\mu\nu}\eta^{\lambda\rho} - \eta^{\mu\lambda}\eta^{\nu\rho})\right]$$

Figure A.2. The self-interaction of four gauge bosons.

Gauge interactions of fermions

For the gauge interactions of fermions, we have vertices corresponding to vertices in the SM, as well as an additional set with the new fermions present at KK levels above the SM. In these interactions only the quarks interact via the strong force. The factor C_1 is defined in table A.1.

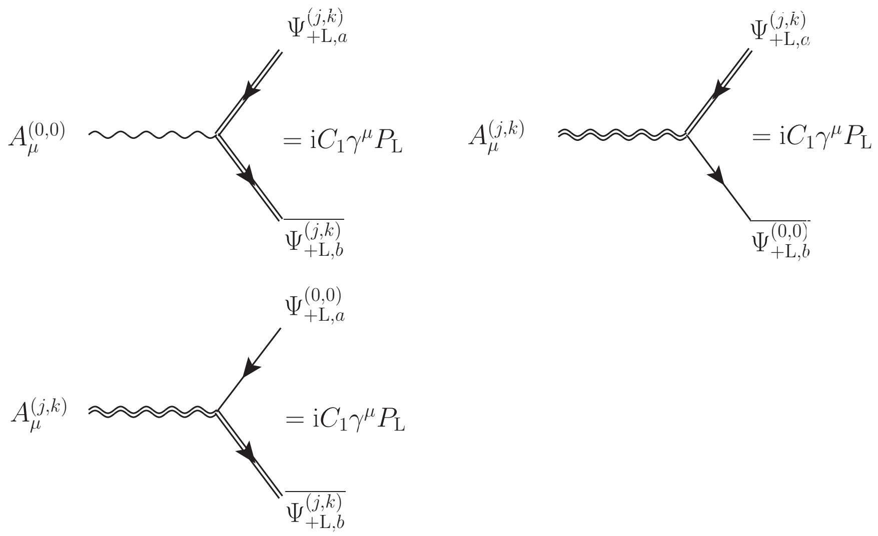


Figure A.3. The fermion-gauge boson vertices corresponding to the interactions of left-handed SM particles, $A_\mu = B_\mu, W^i_\mu, G^a_\mu$ and C_1 is given in table A.1.

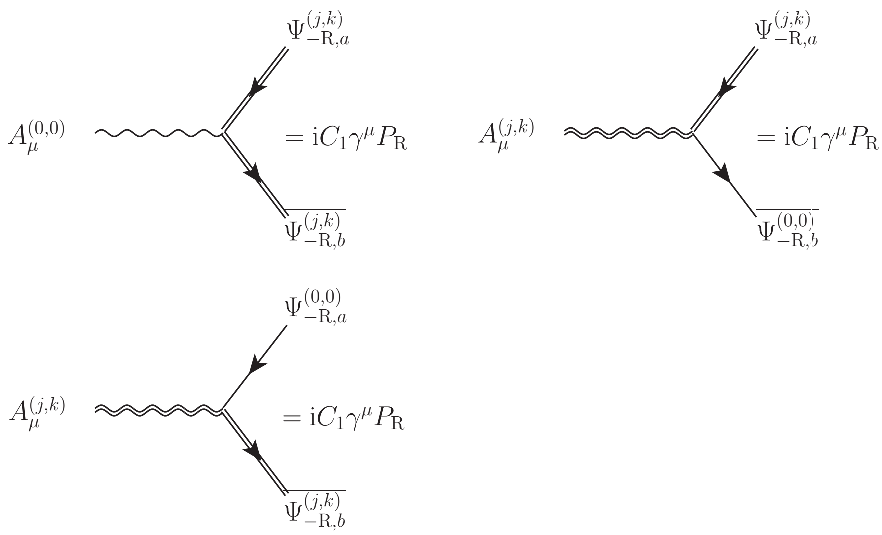


Figure A.4. The fermion-gauge boson vertices corresponding to the interactions of right-handed SM particles, $A_\mu = B_\mu, G^a_\mu$ and C_1 is given in table A.1.

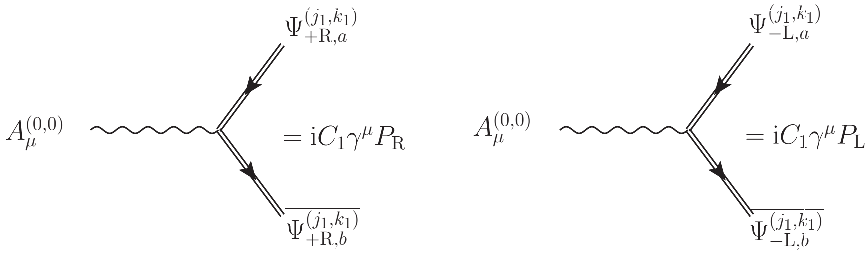


Figure A.5. The fermion-gauge boson vertices with fermions only present at KK levels above the SM level. The fermions in the rightmost diagram do not interact weakly and C_1 is given in table A.1.

A_μ	C_1
B_μ	$g_1 Y^f \delta_{ba}$
W^i_μ	$g_2 \frac{\tau^i_{ba}}{2}$
G^a_μ	$g_3 \frac{\lambda^a_{ba}}{2}$

Table A.1. The values of the parameter C_1 for the different gauge bosons.

Scalar gauge interactions

In the diagrams for the interaction of gauge bosons and the scalar Higgs boson, a factor $\partial_\mu\phi$ in the Lagrangian give rise to a factor $-\mathrm{i}p_\mu$ in the Feynman rules. The parameter C_2 is defined in table A.2.

A_μ	C_2
$BB\phi\phi$	$\frac{1}{2}\delta_{ba}g_1^2$
$BW^i\phi\phi$	$\frac{\tau^i_{ba}}{2}g_1g_2$
$W^iW^j\phi\phi$	$\frac{1}{2}\delta^{ij}\delta_{ba}g_2^2$

Table A.2. The values of the parameter C_2 for the possible combinations of gauge bosons.

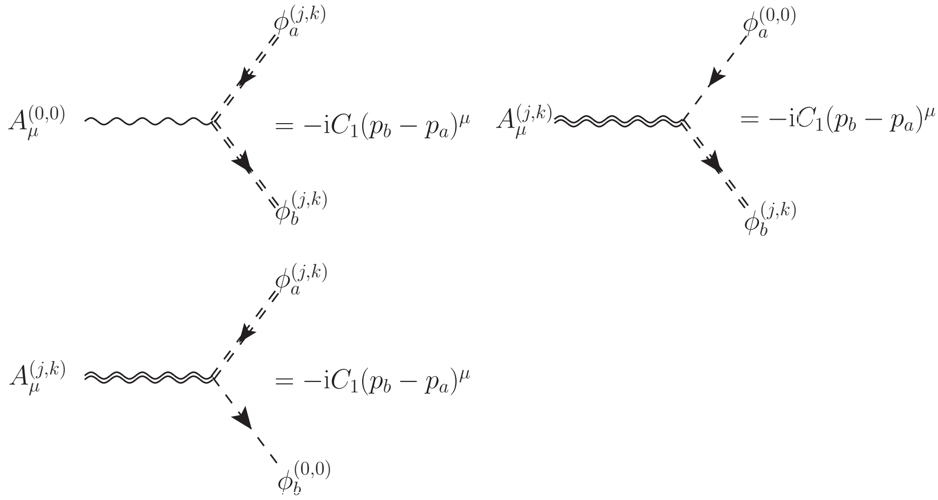


Figure A.6. Gauge interactions involving two Higgs and one gauge boson, where C_1 is given in table A.1.

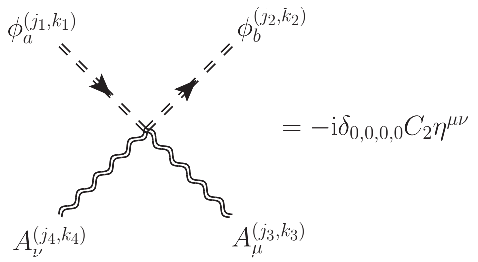
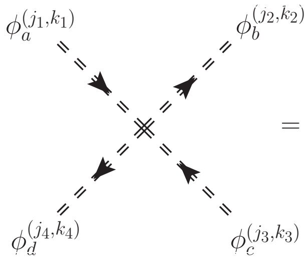


Figure A.7. Gauge interactions involving two Higgs bosons and two gauge bosons. We have abbreviated $\delta_{0,0,0,0} = \delta_{0,0,0,0}^{(j_1,k_1),\ldots,(j_4,k_4)}$ and C_2 is defined in table A.2.

Yukawa interactions

We obtain two sets of Yukawa interactions, one corresponding to the SM fields, and one involving fermions only present at KK levels above the SM.



$$= -\mathrm{i}\delta_{0,0,0,0}\lambda\left(\delta_{ad}\delta_{cb} + \delta_{ab}\delta_{cd}\right)$$

Figure A.8. The self-interactions of the Higgs boson. We have abbreviated $\delta_{0,0,0} = \delta_{0,0,0,0}^{(j_1,k_1),\ldots,(j_4,k_4)}$.

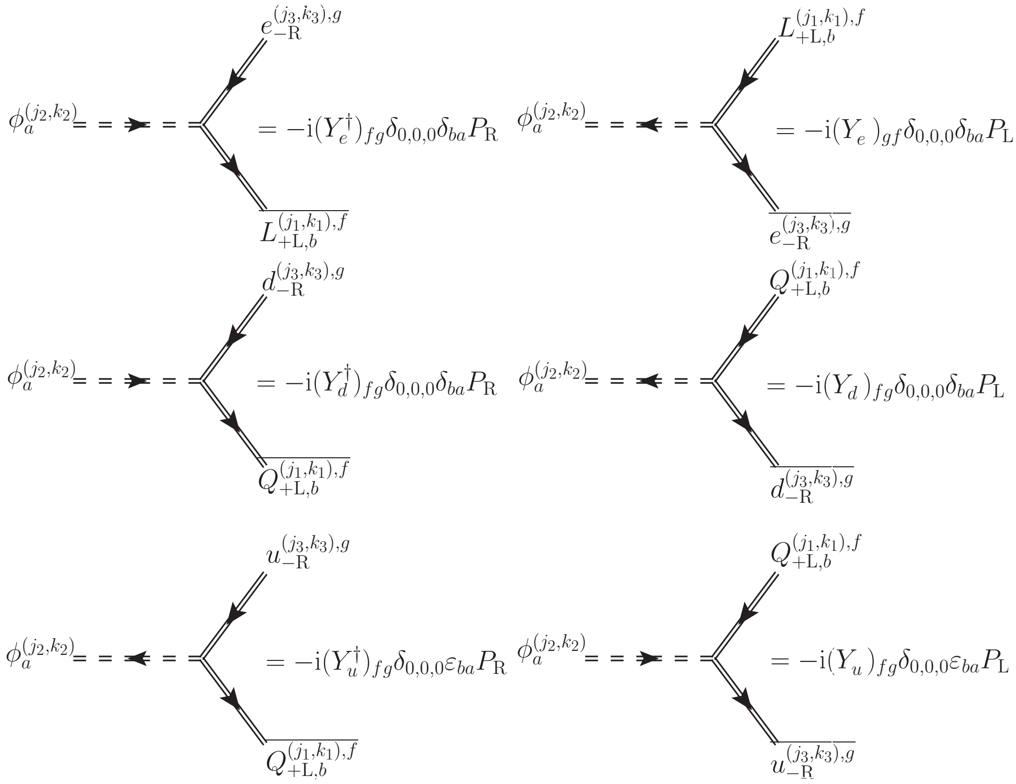


Figure A.9. The Yukawa vertices corresponding to the SM couplings. We have abbreviated $\delta_{0,0,0} = \delta_{0,0,0}^{(j_1,k_1)(j_2,k_2)(j_3,k_3)}$.

Interactions involving adjoint scalars

In the six-dimensional UED model we obtain two adjoint scalar particles, which are not present in the SM. Hence, we obtain a set of new interactions involving these

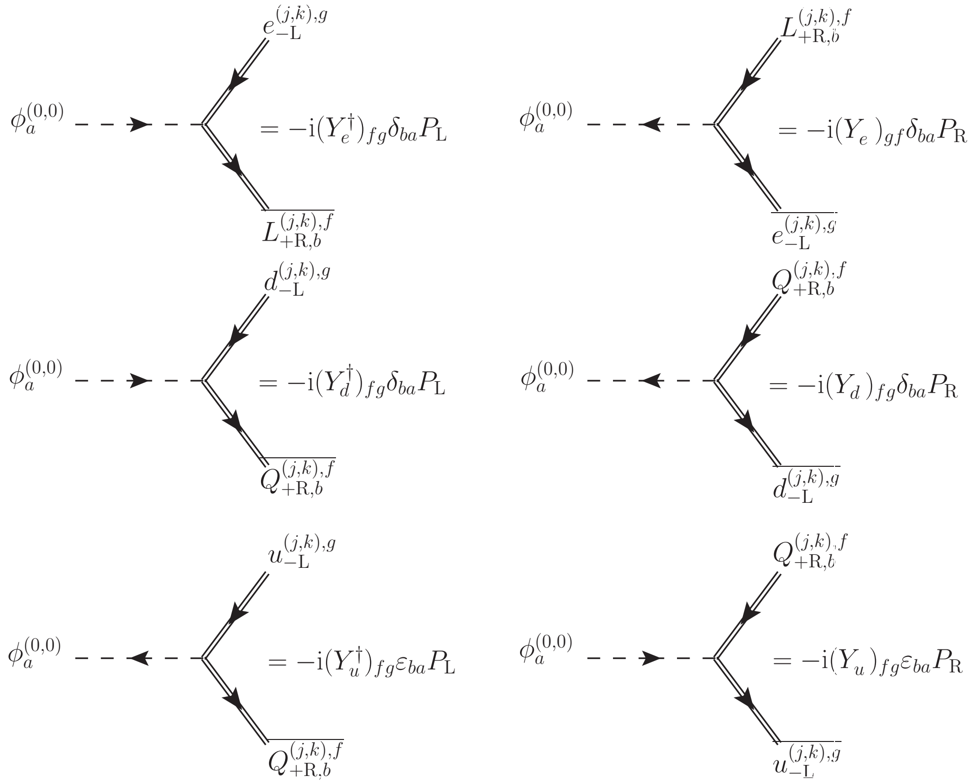
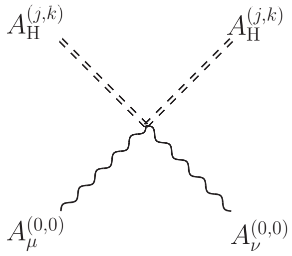
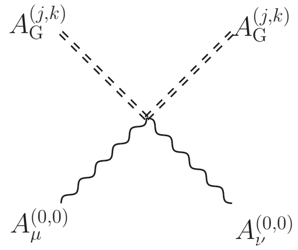


Figure A.10. The Yukawa vertices, which are new in the six-dimensional UED model.

particles.

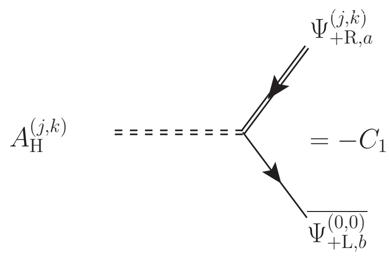


$$= \mathrm{i}g_4^2(f^{abc}f^{ade} + f^{adc}f^{abe})\eta^{\mu\nu}$$

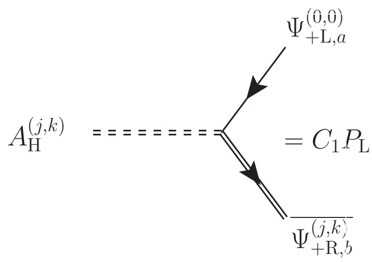


$$= \mathrm{i}g_4^2(f^{abc}f^{ade} + f^{adc}f^{abe})\eta^{\mu\nu}$$

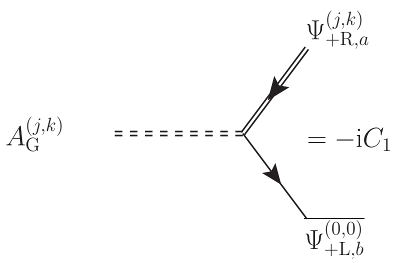
Figure A.11. Vertices with gauge bosons and adjoint scalars.



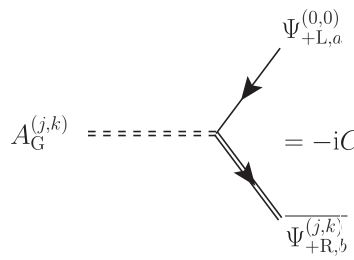
$$= -C_1 P_{\mathrm{R}}$$



$$= C_1 P_{\mathrm{L}}$$



$$= -\mathrm{i}C_1 P_{\mathrm{R}}$$



$$= -\mathrm{i}C_1 P_{\mathrm{L}}$$

Figure A.12. Interactions of left-handed fermions with adjoint scalars, where C_1 is defined in table A.1.

Effective neutrino mass operator

In the six-dimensional UED model, there are two vertices involving the effective operator, one of which involves fields only present at KK levels above the SM.

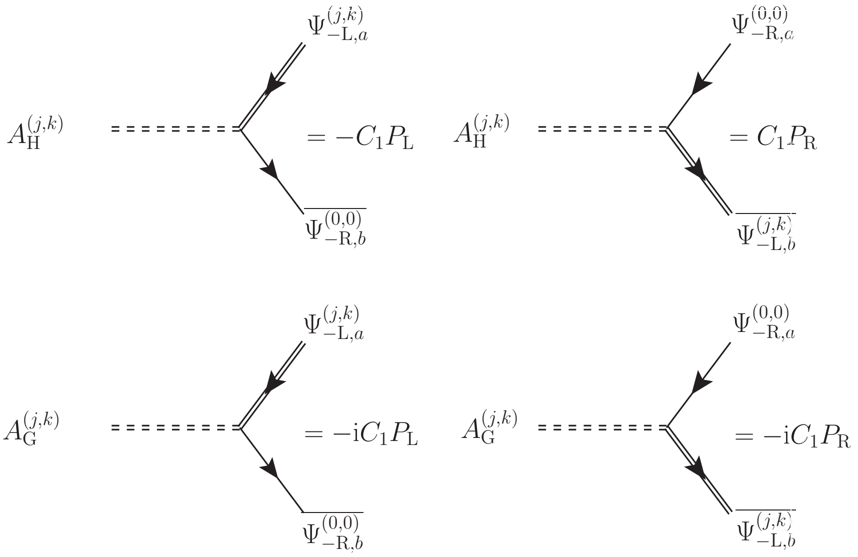


Figure A.13. Interactions of right-handed fermions with adjoint scalars, where C_1 is defined in table A.1.

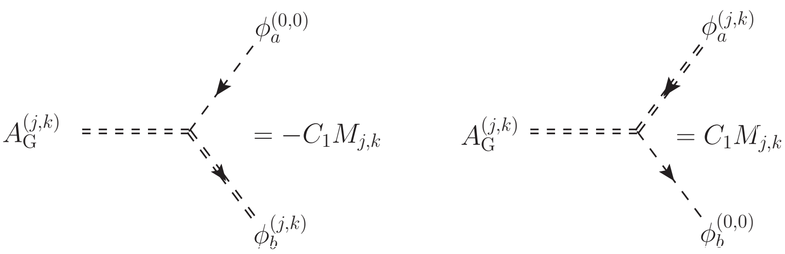


Figure A.14. Vertices with two Higgs scalars and one adjoint scalar, where C_1 is defined in table A.1.

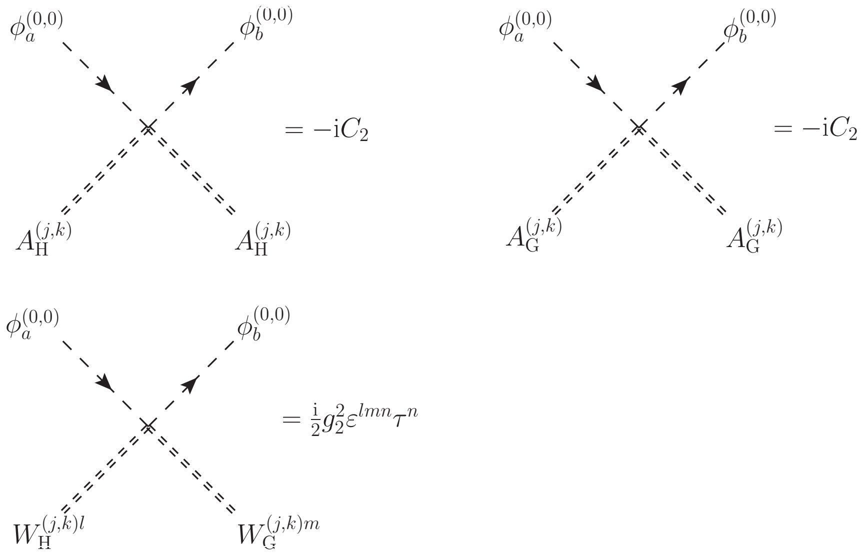


Figure A.15. Vertices with two Higgs bosons and two adjoint scalars, where C_2 is defined in table A.2.

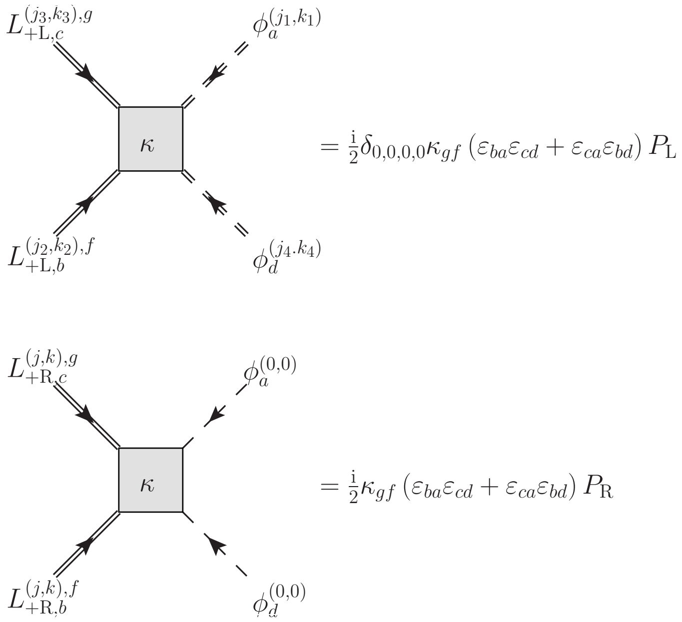


Figure A.16. The vertices involving the effective neutrino mass operator, with $\delta_{0,0,0,0} = \delta_{0,0,0,0}^{(j_1,k_1),\ldots,(j_4,k_4)}$.

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