What does the hot-wire measure?

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Technical Report

This technical note investigates the heat loss characteristics from a hot-wire at high subsonic speeds. Classical works have demonstrated a square-root dependance between the heat loss in terms of the Nusselt number Nu, and the flow rate in terms of the flow Reynolds number Re. The hypothesis for the present work is that in compressible flow Nu is instead dependent of a Reynolds number based on the stagnation density. This hypothesis is then tested by means of experiments.

1. Introduction

Hot-wire anemometry is a velocity measurement technique based on forced convective heat transfer from a thin heated wire, immersed in a fluid flow[1]. The wire is made of a material with temperature dependent resistivity. When an electric current is passed through the wire, it heats the wire above the fluid temperature and the heat transfer from the wire depends on the flow rate it is exposed to. Hence if the temperature of the wire varies, so does also its resistance and consequently the Joule heating (Perry 1982).

If the hot-wire is operated in constant temperature anemometry (CTA) mode, the resistance of the wire is kept constant by a feedback loop. The forced convective heat transfer from the wire will then be balanced by the Joule heating (see e.g. Hultmark & Smits 2010), i.e.

$$\frac{E^2}{R_\mathrm{w}} = hA_\mathrm{w}(T_\mathrm{w} - T_\mathrm{a}), \tag{1}$$

where $R_\mathrm{w}, A_\mathrm{w}$ and T_w are the resistance, the projected area and the temperature of the wire respectively, T_a is the ambient fluid temperature, h is the convective heat transfer coefficient and finally, E is the voltage across the wire.

[1]Heat transfer due to radiation is for most applications negligible, and if the wire is sufficiently long, the heat conduction to the prongs are negligible as well.

Eq. (1) can be expressed in terms of the Nusselt number $\mathrm{Nu} = hd/k$, which is the ratio of convective to conductive heat transfer coefficients, as

$$\frac{E^2}{R_\mathrm{w}} = k\mathrm{Nu}\frac{A_\mathrm{w}}{d}(T_\mathrm{w} - T_\mathrm{a}) \tag{2}$$

where k is the thermal conductivity of the fluid and d a characteristic length (here the diameter of the wire). The Nusselt number depends on several parameters, and for a compressible fluid this functional relationship can according to Bruun (1995), be written as

$$\mathrm{Nu} = \mathrm{Nu}(\mathrm{Re}, \mathrm{Pr}, M, \tau, L/d) \tag{3}$$

where the dimensionless numbers are

$$\begin{aligned} \mathrm{Re} &= \text{Reynolds number} &&= \rho u d/\mu \\ \mathrm{Pr} &= \text{Prandtl number} &&= c_p\mu/k \\ M &= \text{Mach number} &&= u/a \end{aligned} \tag{4}$$

The included variables are in turn: velocity u, density ρ, wire length L, dynamic viscosity μ, specific heat at constant pressure c_p and speed of sound a. The so called temperature loading factor or overheat ratio $\tau = (T_\mathrm{w} - T_r)/T_0$, where T_0 is the stagnation temperature, T_r is the recovery temperature. For an unheated wire in a fluid flow, T_r is the temperature of the wire, which is greater than the static temperature but lower than the fluid temperature if it were brought to rest (Sandborn 1972). It can be defined through the the so called recovery factor r, namely

$$\frac{T_r}{T} = (1 + r\frac{\gamma - 1}{2}M^2) \tag{5}$$

where the recovery factor for laminar flow is assumed to be $\sqrt{\mathrm{P}r}$. A semi-empirical relationship for the Nusselt number (Smits *et al.* 1984) and the flow variables are

$$\mathrm{Nu} = A'(\tau) + B'(\tau)\mathrm{Re}^n \tag{6}$$

where n usually is in the range 0.4-0.55 and the above relation is known as King's Law. For calibration purposes, the above equation can be combined with Eq. (2) to yield

$$E^2 = A(\tau) + B(\tau)\mathrm{Re}^n \tag{7}$$

If the hot-wire is to be used merely to measure flow velocity, one has to compensate for the temperature dependance of the coefficients A and B, since the heat transfer from the wire is due to the ambient temperature as well. Such compensation techniques can be found in e.g. Kostka & Ram (1992), Bruun (1995) and Dijk & Nieuwstadt (2004).

Eq. (7) implies that at a given temperature, the anemometer output voltage $E^2 \sim (\rho u)^n$, which has been confirmed also at lower subsonic speeds, see for instance Durst *et al.* (2008). Hot-wire measurements in high speed flows have been conducted as well, where it has been shown for supersonic flows that the Reynolds number is the predominant parameter that affects the heat loss in terms of the Nusselt number (Laufer & McClellan 1956). Since a bow shock forms in front of the hot-wire at supersonic speeds, the situation is quite different from subsonic flow and the Reynolds number behind the shock, is the controlling variable. Since the Mach number behind the shock converges slowly to a constant value the higher the upstream Mach number, its impact on the heat transfer is small as compared to Re.

A number of authors (e.g. Kovasznay 1953; Spangenberg 1955; Sandborn 1972; Dewey 2002) have favored to describe the heat transfer loss from the cylinder in terms of

$$\begin{aligned} \mathrm{Nu}_0 &= \frac{hd}{k_0}; \\ \mathrm{Re}_0 &= \frac{\rho u d}{\mu_0} \end{aligned}$$

where the fluid properties, heat conductivity and dynamic viscosity, are evaluated at the stagnation temperature (denoted by subscript 0), whereas the density still is evaluated as the density of the flow. This offers advantages in flows with non-uniform flow fields. With these definitions, the asymptotic trend $\mathrm{Nu}_0 \sim \sqrt{\mathrm{Re}_0}$ has been demonstrated.

The purpose of the present work is to investigate if the hot-wire is sensitive to ρu even at Mach numbers M, approaching unity. A hypothesis for the present work is that the hot-wire is sensitive to $\rho_0 u$ i.e. the product of stagnation density and velocity, rather than ρu. The difference between these variables is given by

$$\rho_0 u = \left(\frac{\rho_0}{\rho}\right) \rho u = \beta(M) \rho u \tag{8}$$

where β is the isentropic relation for the densities

$$\beta = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/(\gamma-1)} \tag{9}$$

The Mach numbers in the present study range from $M = 0.3$ to $M = 1$, where the corresponding β from Eq. (9) are 1.045 and 1.58, respectively. Hence, with this difference between ρu and $\rho_0 u$ at Mach number close to unity, the discrimination of the heat loss dependancies would be clearly noticeable. The value of β increases towards $M = 1$ and it starts to decrease for Mach numbers beyond 1, due to the shock in front of the wire.

2. Experimental Set-up

The experiments were performed in the CICERO Laboratory of KTH CCGEx, in a flow rig as described in Laurantzon *et al.* (2012) and the equipment and instrumentation is similar to that employed in Laurantzon *et al.* (2010). For convenience the most important details will be repeated here.

In the present investigation a hot-wire calibration facility consisting of an electrical heater, a stagnation chamber and a convergent nozzle, was connected to the main pipe system of the laboratory. Two compressors can provide up to 0.5 kg/s at 6 bars, however in the present study only a small fraction of the capacity is needed. The pipe system has a high quality mass flow meter (ABB Thermal Mass Flowmeter FMT500-IG) that gives the flow rate. The nozzle, schematically shown in Fig. 1, has an inlet and exit diameter of 110 and 14 mm, respectively. A digital thermometer (FLUKE) was connected to the stagnation chamber to assess the stagnation temperature. The stagnation pressure was measured at the inlet of the nozzle, where the flow is nearly stagnant and the Mach number almost zero. The hot-wire probe used has a long probe body

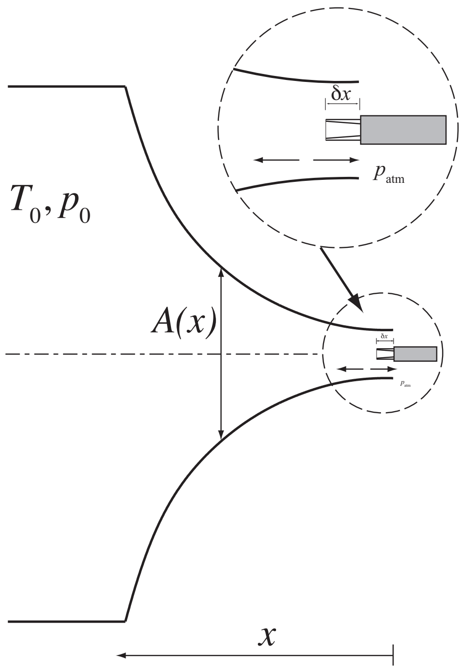

Figure 1. The geometry of the nozzle. The probe can be traversed in the x-direction along the centerline. The probe itself causes a blockage of about 8 % of the outlet cross section area.

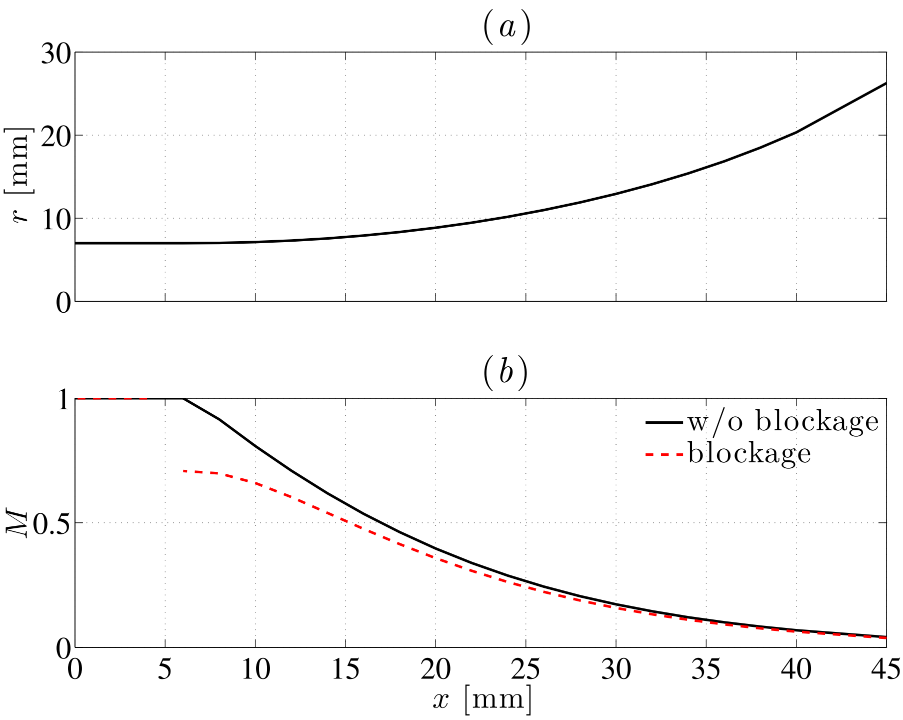

FIGURE 2. (a) The nozzle radius r as function of axial distance x. (b) Theoretical Mach number distribution at choked conditions, with and without the blockage introduced by the hot-wire probe itself, with diameter $D = 4$ mm.

with a diameter of 4 mm. The probe body is always inserted into the nozzle creating the same critical area at the nozzle exit for all different positions of the probe inside the nozzle. The sensing element consists of a 5 micron Tungsten wire of length, $L = 1$ mm, giving a length-to-diameter ratio of around 200. The hot-wire was operated by means of an AA-Labs AA-1003 anemometry system in CTA mode. The hot-wire was operated at an overheat ratio of 60%. The hot-wire probe was mounted on a micrometer screw which could be manually traversed along the centerline, i.e. the x-axis.

In Fig 2(a) the radius $r(x)$, of the nozzle is shown and in Fig 2(b) the corresponding theoretical Mach number distribution based on choked conditions (i.e. the exit Mach number $M_e = 1$), is shown.

3. Experimental results

3.1. *Calibration procedures*

The hot-wire response can be obtained in two principally different ways. One is to place the hot-wire at a specific position inside the nozzle and then change the mass flow rate from zero up to the point when the flow is choked. In this way both ρu and the Mach number change simultaneously. A second possibility is to run the nozzle under choked conditions and varying the stagnation pressure thereby changing the density at the sensor but not the velocity and Mach number.

An example of the former is shown in Fig. 3. Here the hot-wire was placed at $x = 8$ mm, which is the position where $M = 0.7$ at choked conditions. The facility reference mass flow meter, was used to obtain $\dot{m}_{\rm ref}$, i.e. the total mass flow through the nozzle. Since the cross sectional variation with x is known, the mass flux ρu is obtained from

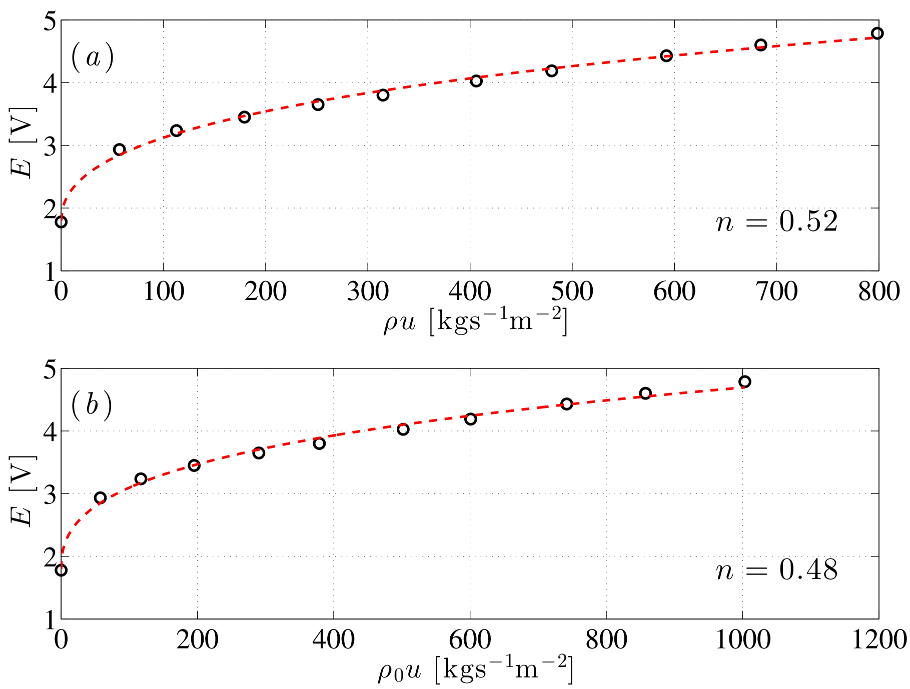

FIGURE 3. Calibration curves for the hot-wire sensor with least square fits to the measured points. (*a*) $E^2 = A + B(\rho u)^n$. (*b*) $E^2 = A + B(\rho_0 u)^n$.

$$\dot{m}_{\text{ref}} = (\rho u)A|_{x=8\text{mm}} \tag{10}$$

and $\rho_0 u$ is obtained from Eq. (8), where M is obtained from the so called Area-Mach (A-M) number relationship (Anderson 2004). The exit Mach number M_e is based on p_0 and p_{atm}. The anemometer output is plotted vs. both ρu and $\rho_0 u$ in Fig. 3(a) and (b) respectively. Least square fits to the calibration points are also provided where A, B and n all are fitted. As can be seen from the figures the least square fit of the calibration data gives values of the exponent n in King's law close to the theoretical value of 0.5 in both cases and also that A becomes close to the measured voltage squared at no flow.

The Mach number distribution can also be obtained in the following way

$$\rho u = \frac{\rho}{\rho_0}\rho_0 M\sqrt{\gamma R\frac{T}{T_0}T_0} \tag{11}$$

After some algebra (using $\gamma = 1.4$) we get the following equation for M

$$\frac{\gamma-1}{2}M^2 - \left(\frac{\gamma}{RT_0}\right)^{1/6}\left(\frac{p_0}{\rho u}\right)^{1/3} M^{1/3} + 1 = 0 \tag{12}$$

where all other quantities are known for a given measurement point. Both methods gave similar results, which gives confidence that the procedures are correct: The former method is based on the area distribution and pressure measurements and the latter on the mass flow rate. When $\rho_0 u$ is known the following equation can be used to find M:

$$\frac{\gamma-1}{2}M^2 - \frac{\gamma}{RT_0}\left(\frac{p_0}{\rho_0 u}\right)^2 M^2 + 1 = 0 \tag{13}$$

The second approach, keeping M constant and varying the stagnation pressure p_0 is illustrated in Fig. 4. Here M and hence u are fix for a given measurement point (each specific line in Fig. 4) and M, p_0 and T_0 are known, therefore ρu and $\rho_0 u$ can readily be determined. The squared output voltage, plotted vs. the square root of the mass flux, shows the approximately linear relation as expected ($E^2 - E_0^2 \sim (\rho u)^{0.48}$ and $E^2 - E_0^2 \sim (\rho_0 u)^{0.52}$). However it is clear from Fig. 4 that the heat transfer from the sensor decreases with increasing Mach number for a given value of ρu (or equivalently $\rho_0 u$).

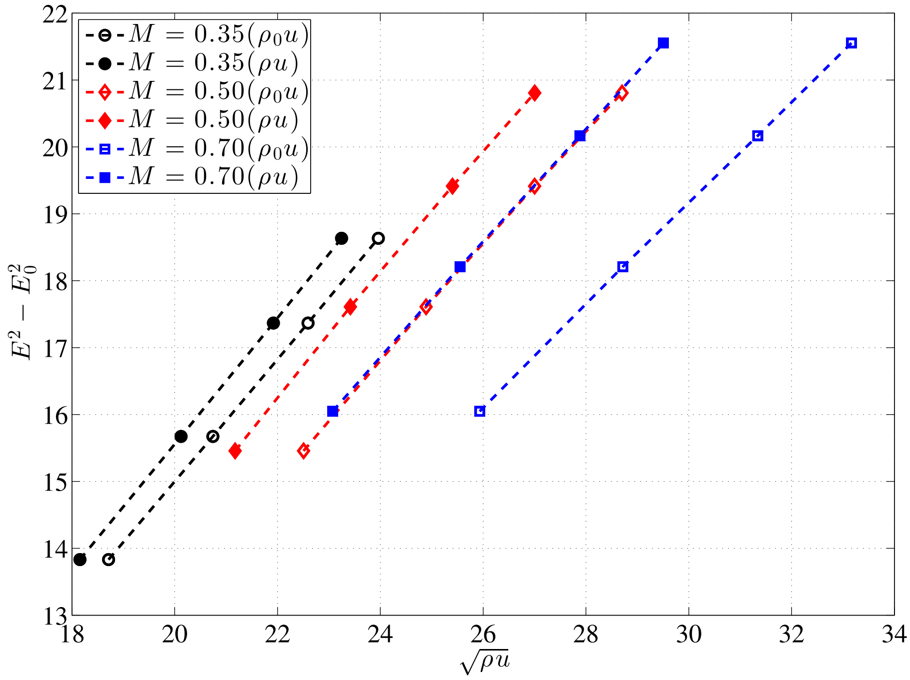

FIGURE 4. E^2 as function of $\sqrt{\rho u}$ and $\sqrt{\rho_0 u}$. The hot-wire is at a fix position for a given M, but the stagnation pressure is changed and hence also the density. Numbers are in SI-units.

3.2. *Nozzle measurements*

This section shows data from the hot-wire sensor where the sensor is traversed through the nozzle. The stagnation pressure p_0 is kept constant and the flow is choked. In this case the sensor is exposed to both a varying mass flux (ρu) and a varying Mach number. In Fig. 5 the anemometer output (voltage) is shown and the corresponding mass fluxes when M is varied by means of traversing the probe along the nozzle.

Now we can calculate the Mach number distribution along the nozzle obtained by the hot-wire and the calibration function given in Fig. 5 and compare it with the distribution obtained from the area distribution. For the latter case two possibillities exist:

- From the A-M relation, see Fig. 1.
- From the reference flow rate $\dot{m}_{\mathrm{ref}}$ and the cross section area $A(x)$ where the measurement is performed, which gives ρu, and M is obtained from Eq. (12).

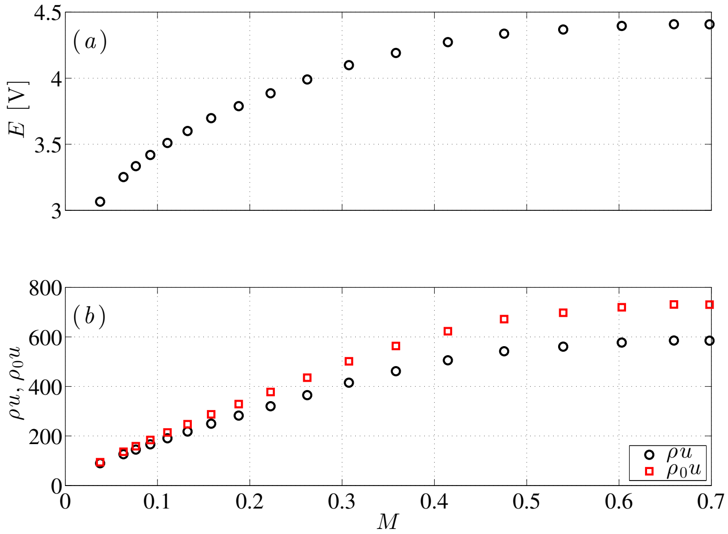

FIGURE 5. Traversing with hot-wire along the nozzle with p_0 kept constant. (a) Anemometer output. (b) The mass fluxes obtained from the calibration, Fig. 3.

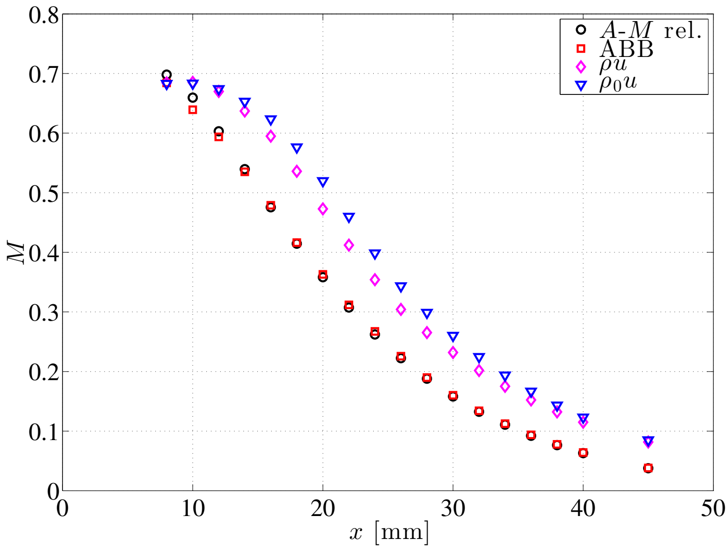

FIGURE 6. M distribution in the nozzle obtained in four independent ways.

These two methods give similar results as is shown in Fig. 6. However if the Mach number is determined from from the hot-wire measurements using Eq. (12) or Eq. (13) it is clear that for both ρu and $\rho_0 u$ the Mach number is overestimated. This is however no surprise when reviewing Fig. 4, for a given ρu the anemometer output voltage depends on the Mach number, and hence it will no be possible to obtain a perfect match with the real Mach number distribution in this way. However as expected the agreement is good at $x = 8$ mm, since this was the position where the hot-wire was calibrated.

3.3. *Measurements with constant ρu*

As a final investigation to determine what the hot-wire is sensitive to, M and p_0 are adjusted in such a way that ρu remains constant and such that $\rho_0 u$ increases with increasing Mach number (see Fig. 7). The hypothesis that the hot-wire is sensitive to $\rho_0 u$ would then imply that the output voltage should also increase with increasing M, but if it instead is sensitive to ρu, then the output voltage should be unaltered. However, in Fig. 7 one can note that the

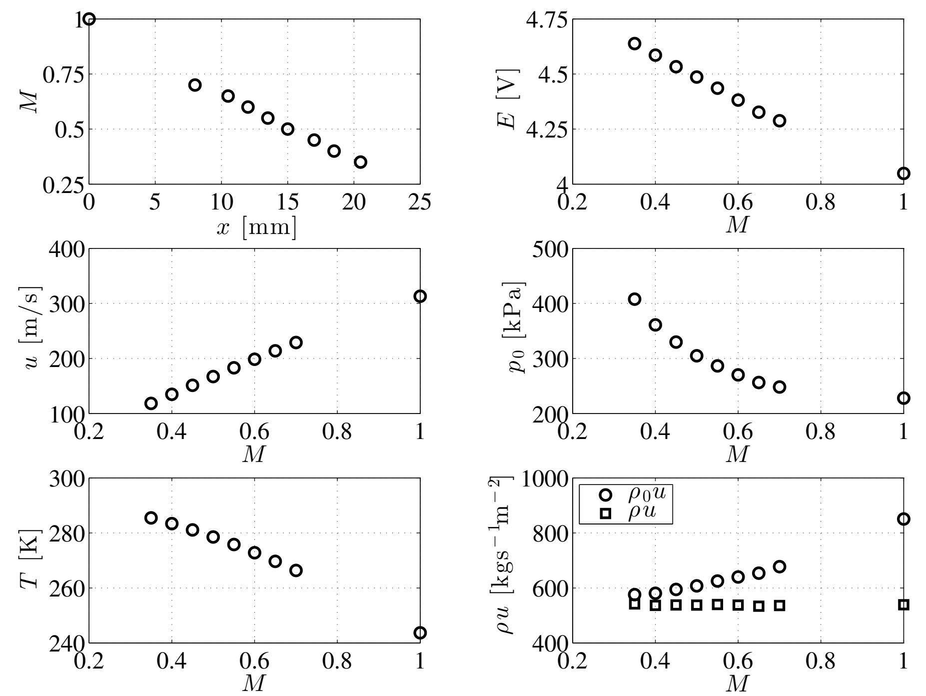

FIGURE 7. For all these measurement points M and p_0 are adjusted such that ρu is constant. T_0 is constant throughout the measurement series.

signal from the anemometer decreases despite that ρu is constant and that $\rho_0 u$ increases.

4. Summary and conclusions

In the present work we have tried to establish how the various flow variables affect the heat transfer and thereby hot-wire anemometer output in compressible flows. In all experiments we keep the stagnation temperature constant, but both the Mach number and the mass flux will affect the anemometer output. The experiments show clearly that for a given mass flux the anemometer output, i.e. heat transfer, decreases with increasing Mach number. This behaviour was not unexpected and have been observed earlier (Sigfrids 2003).

We propose the following hypothesis for this behaviour. In compressible subsonic flow the streamlines are moving away from the body with increasing Mach number, according to the so called Prandtl-Glauert rule. It can be shown that this effect is proportional to

$$s \sim \frac{s_0}{\sqrt{1 - M_\infty^2}} \tag{14}$$

where s is the distance normal to the flow direction and s_0 is the distance at zero Mach number. This will also mean that velocity gradients become smaller normal to the surface of the body (a well-known phenomenon in transonic flow) and our hypothesis is based on the idea that a similar scaling would affect the temperature field as well and hence result in a lower heat transfer. In Fig. 8 we have plotted the same data as in Fig. 7b, using the Prandtl-Glauert transformation directly on the heat transfer term in order to account for smaller gradients. As can be seen the resulting transformed heat transfer now increase with Mach number instead of decreasing. In addition we have normalized these values with $\sqrt{\rho_0 u}$, that is the square root of the stagnation density and flow velocity (also plotted separately in Fig. 7f). Doing so the variation of the anemometer output is $\pm 2\%$ over the Mach number range 0.35-0.7. It is also shown that the anemometer output is a function of the stagnation density of the gas rather than the gas density *per se*.

The results in this study shows that to use hot-wire anemometry in compressible flows it is important not only calibrate the hot wire against the mass flux, but also to have a knowledge of the Mach number. This makes the use of hot-wire anemometry complicated at high subsonic Mach numbers and this will studied in more detail in the future.

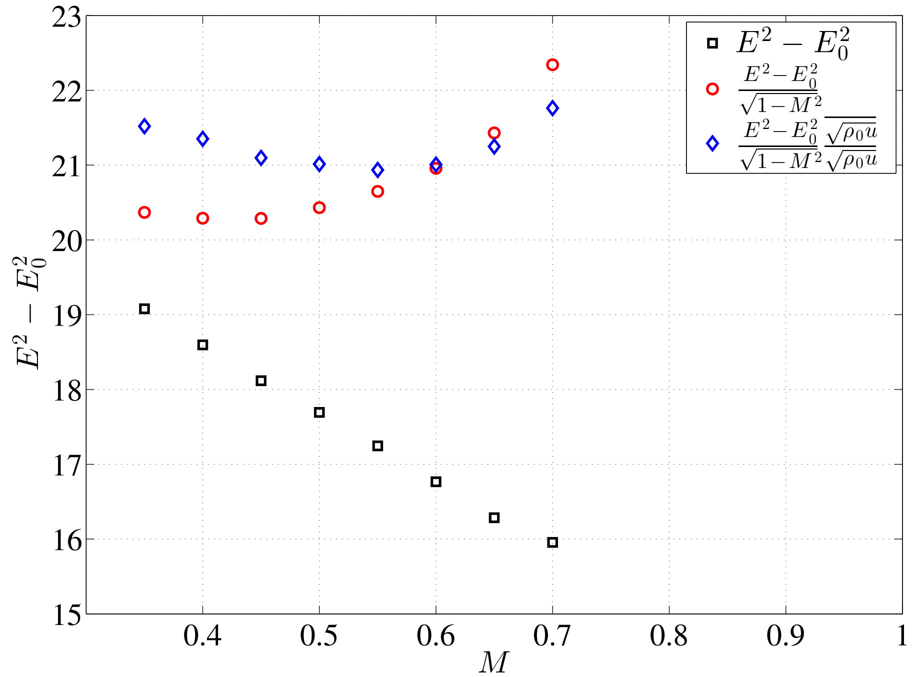

FIGURE 8. $E^2 - E_0^2$ as well as the Prandtl-Glauert transformed value, as function of Mach number. Same data as in Fig. 7b. Furthermore the Prandtl-Glauert transformed values are normalized with $\sqrt{\rho_o u}$ giving an almost constant value.

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