Environmental Regime Shifts and Economic Activities
- the Shallow Lake Model

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Abstract
This thesis is about economic aspects of environmental regime shifts. Several models of a shallow lake and its related economic activities are explored. We find that complex behaviour is generated that deserves attention. With regime shifts, path dependence and recognizing uncertainty becomes important and valuation of alternate states, restoration costs and stability can be considered.
Contents

1  Introduction ........................................................................................................................................... 3
1.1 Background and Importance .................................................................................................................. 3
1.2 Purpose .................................................................................................................................................. 4
1.3 Research ............................................................................................................................................... 4
1.4 Outline .................................................................................................................................................. 4
2  The Convexity Assumption ..................................................................................................................... 5
3  The Shallow Lake Model ......................................................................................................................... 6
3.1 Regime Shifts ......................................................................................................................................... 7
3.2 Our Model ............................................................................................................................................. 10
4  Analysis .................................................................................................................................................. 12
4.1 Socially Optimal Static Equilibrium ....................................................................................................... 12
4.2 Private Static Equilibrium ....................................................................................................................... 14
4.3 Socially Optimal Dynamic Equilibrium .................................................................................................. 15
4.4 Dynamic Private Equilibrium ................................................................................................................ 18
5  Further Aspects ....................................................................................................................................... 21
5.1 Valuation ............................................................................................................................................... 22
5.2 Uncertainty .......................................................................................................................................... 22
5.3 Resilience ............................................................................................................................................ 24
6  Conclusion ............................................................................................................................................. 26
7  References .............................................................................................................................................. 27
1 Introduction

1.1 Background and Importance

There is a growing interest in studying environmental regime shifts, probably motivated by the globally increasing awareness of environmental problems. In the past, the scale of human economic activity was much different than the scale of nature’s processes. Therefore the environment could be considered as large, stable and with slow variations only and primarily a producer or production factor, yielding consumption goods and services (Drepper and Månsson 1993). The economic system could be seen as interacting with the environmental system through input and output only.

The scale of human activity has increased exponentially the last 100 years, with the consequence that the dynamic behaviour of both systems has to be considered in a way not needed in the past (Chave and Levin 2003). The human economic system could be seen as embedded in the environmental system and therefore dependent on the function of the surrounding ecosystem services (Common and Stagl 2005); in particular what could be called life-supporting services.

Ecosystem dynamics are non-linear and complex. As the interaction with the environment becomes more important, not accounting for this complexity can lead to severe surprises and great costs. Multiple steady states are frequently observed in ecosystems with completely different bundles of ecosystem goods and services as a consequence.

Deforestation of tropical forests is frequently debated. Tropical forests seem to be able to recover quite quickly from human disturbance (Wilkinson 2006), but modelling suggests that destruction of large areas of the Amazon forest could greatly reduce rainfall due to reduced transpiration and so lead to a climate unsuitable for tree growth. A large biomass of vegetation is therefore crucial to the continual existence of a climate able to support a large biomass of vegetation (Betts 2004). The top soils in wet tropics are among the poorest soils for farming and most nutrients that support the rain forest are locked up in the trees themselves. Therefore, when forests are cleared, most of the nutrients are removed as well. In only a few years, soils in a freshly cleared area may no longer be cultivable (Lutgens et al. 2011). This is an example of an ecosystem with alternate states and where the scale of human impact has greatly increased, making irreversible changes and damages possible.

Environmental regime shifts are caused by ecological thresholds where systems suddenly move into an alternate state once a threshold is passed. This behaviour is called hysteresis and leads to path dependence (which state you end up with, depends on where you start).

As our impact on the environment increases, scientists suggest that the frequency and amplitude of the shifts might become larger. Multiple steady states make factors like history dependence and stochastic behaviour more important to consider (Crépin, Norberg and Mäler 2010). There seem to be a need for modelling the ecological part of the economy in a more detailed way than before. Several authors, among them Dasgupta and Mäler (2003), discuss our limited understanding of the mechanisms for resource allocation in non-convex environments, and the need for more research in this area.
Natural resources are often owned collectively or exploited under the conditions of open access. Various problems can arise because of this, such as poorly defined property rights and asymmetric information. This thesis deals with how alternate states emerge and how the phenomenon can be described and managed from an economic standpoint.

1.2 Purpose
The purpose of this thesis is to examine a number of utility maximization models of a shallow lake and its related economic activities to see what light is shed on the problem of the economics of environmental regime shifts.

We start by examining the ecological part of the model and continue with studying socially optimal and private equilibria, both static and dynamic. We compare utility levels and structural differences between the different models and some aspects of valuation, uncertainty, stability and resilience concerning regime shifts are discussed. The question of valuating resilience is given some extra attention.

1.3 Research
Early important contributions of the economic consequences of ecological non-linearity was made by Lorenz (1989), Brock and Malliaris (1989) and Puu (1991). Environmental regime shifts were explored in Scheffer (1998) and Carpenter and Cottingham (1997) by their shallow lake model. Recently, Brock and Starrett (2003) and Mäler, Xepapadeas and de Zeeuw (2003) have given complete treatments of optimal management of shallow lakes and non-convex behaviour. For an extensive discussion of non-convexities and externalities, see Dasgupta and Heal (1978)

The shallow lake model is frequently used as an example of hysteresis behaviour in the environment, for example by Brock (2000), Brock, Mäler, Perrings (2000), Brock and Starrett (2003) and Mäler, Xepapadeas and de Zeeuw (2003). They view it as a metaphor for many current ecological problems. Lakes are usually common property and because of that often suffer from sub-optimal use when coordination is not used.

Critique has been made to this type of models, questioning their general applicability. Several recent articles have demonstrated that ecosystems like coral reefs (Crépin 2006), boreal forests (Crépin 2003), mixed tree-grass ecosystems (Scholes 2003), wetlands (Crépin 2002) and pest control (Crépin, Norberg and Mäler 2010) show the same behaviour.

Non-convex behaviour in combination with the pricing of resilience\(^1\) has been studied by Mäler, Li and Destouni (2007) and Mäler and Li (2010).

1.4 Outline
The outline of the thesis is as follows. We start by looking at the consequences of non-convexity for decentralized markets and the price system in section 2. In section 3, I present the shallow lake model together with a numerical example. Several types of equilibrium analysis are made in section 4 and in section 5 I discuss three economic aspects of regime shifts with the shallow lake example as illustration.

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\(^1\) See section 5.3 for an explanation of resilience
2 The Convexity Assumption

An important economic assumption is convexity in the production set, on which the premise of efficiency of resource allocation rests. There are both theoretical and empirical reasons for this assumption. In some situations when we model ecosystem services in a more detailed way, we have to relax this assumption.

Convexity in production is associated with diminishing returns in factors of production. The production set is convex if the set of all feasible bundles is convex with the following definition of convexity: if for any two points in the set, the line connecting the points is also in the set, the set is convex (Gravelle 2004). See fig 1 below.

Assume an individual that is both consumer and producer. Two goods, x and y, are produced. In fig 2, II is the consumer preference, TT is the production possibilities frontier and PP is relative price of y in terms of x.
Beside perfect competition, convexity in production and preferences are what guarantees the tangency of II, PP and TT or in other words the foundation of the efficiency of prices in resource allocation. It means that we can decentralize decisions of consumption.

The first theorem of welfare economics says:

If there are markets for all commodities which enter into production and utility functions and all markets are competitive, then the equilibrium of the economy is Pareto efficient (Gravelle and Rees 2004).

If we let a competitive market operate, a Pareto optimum will prevail.

The second theorem of welfare economics says:

If all consumers have convex preferences and all firms have convex production possibility sets, any Pareto efficient allocation can be achieved as the equilibrium of a complete set of competitive markets after a suitable redistribution of initial endowments (Gravelle and Rees 2004)

A market can be relied on to give any Pareto optimal allocation if we just manage the initial distribution. If allocation mechanisms in the real world satisfy required conditions, policy makers can limit their interventions to redistributions.

If there is non-convexity in the production set, the second theorem of welfare does not hold and the possible range of Pareto optimums are limited.

Coase (1960) found that property rights are important for the price system to work properly, as long as the transaction costs are low. Brock, Mäler and Perrings (2000) shows that a one of the consequences of non-convexity is that transaction costs increases when feasibility sets are non-convex, because convergence at negotiations cannot be assumed any longer.

We can thus see the necessity of the convexity assumption for the resource allocation mechanism. If we introduce non-linear environmental inputs in the production function the convexity is disturbed and the price mechanism fails. Dasgupta and Mäler (2003) state, that for renewable resources we often have convexity in the production function at low values and concavity at high values, fisheries\(^2\) for example, and in this case the feasible set is a non-convex set. We will see that the shallow lake model is not convex in the production function.

3 The Shallow Lake Model
We will analyse economic activities related to a shallow lake model with regime shift behaviour. This model and variants of it have frequently been used by different authors to explore the relation between the economy and the environment. We start by examining the shallow lake in itself.

\(^2\) Denoting the biomass of a single-species fishery by \(K\), it is commonly assumed in fisheries economics that \(F(K)\) is convex at low values of \(K\), but concave beyond some value of \(K\) (Dasgupta, Mäler 2003)
The lake under study is polluted by phosphorous loading from fertilizers used in agriculture. As the level of phosphorous grows, the growth of phytoplankton is stimulated and the lake changes from clear (oligotrophic) to turbid (eutrophic) state. High phytoplankton productivity can sometimes be dominated by bloom-forming toxic species and in worst situations, leading to anoxia\(^3\) and fish kills (Begon et al 2006). Both nitrogen and phosphorous are important resources for plant growth. Models based on nitrogen is also possible, however, many studies show that phosphorous is the limiting factor of mineral nutrients for lakes and therefore we use phosphorus as the input resulting in eutrophication.

The change in the lake is gradual at first, but when a certain level is reached the turbidity process becomes self-reinforcing and the lake moves relatively fast to a turbid state. This non-linearity is what we will focus on.

From a pure ecological perspective the only concern might be how to prevent the lake from changing. When including the economic perspective, the problem becomes a trade-off between the utility of agriculture activities, which are responsible for the loading of phosphorous, and the utility of a clear lake. The outcome depends on the relative weight attached to the different welfare components.

As an example of the manifestation of this phenomenon, Gunderson and Holling (2002) mention a shift that happened in the early 1990’s in Florida Bay, a 2,200 km\(^2\) shallow estuary at the southern tip of Florida, which abruptly changed from clear to turbid water; from a regime dominated by sea grasses to a regime of dominated by phytoplankton blooms.

### 3.1 Regime Shifts

The simple model discussed here is deterministic and an approximation of the complicated food chain with phosphorous loading. It is discussed in Brock (2000), Brock, Mäler and Perrings (2000), Brock and Starrett (2003) and Mäler, Xepapadeas and de Zeeuw (2003).

The model is represented by the equation

\[
\frac{dP}{dt} = L - sP + h(P); \quad h(P) = r \frac{P^2}{(P^2 + m^2)} \tag{3.1.1}
\]

\(P\) = stock of phosphorous in algae,
\(L\) = inflow of phosphorous from agriculture,
\(s\) = outflow of phosphorous (sedimentation, sequestration in other biomass, outflow etc.),
\(m\) = oxygen level (exogenous constant),
\(r\) = max rate of internal recycling (exogenous constant).

The function \(h(P)\) is often used to model the internal recycling of phosphorous. Within the lake, phosphorous release from sediment is a major source of recycled phosphorous and the cause of the hysteresis phenomenon. When turbidity in the water increases some of the organisms take advantage of it and forces are created that increase the turbidity even more. This can be described as a positive (reinforcing) feedback loop that occurs

\(^3\) A total decrease in the level of oxygen
when $P$ becomes large enough. A positive feedback loop, once it gets triggered, feeds on itself and the level of $P$ increases on its own, until some limit is reached.

For low stocks of phosphorous, loadings are stored in the lake bed so there is little return to the water, for higher stocks the return increases and for high stocks it decreases when maximum suspension is reached. This creates an S-form (sigmoid-form) of the curve describing the relationship between phosphorous stock and loading.

Carpenter et al (1999), Brock, Mäler and Perrings 2000, Mäler et al (2003) and others make the problem scale invariant through the following substitution

\[ t = \text{time} \]
\[ x = \frac{P}{m} \quad \text{(stock of phosphorous)}, \]
\[ A = \frac{L}{r} \quad \text{(inflow of phosphorous)}, \]
\[ B = Sm/r \quad \text{(outflow of phosphorous)}. \]

We change the time scale to $rt/m$ and simplify and we get

\[ \frac{dx}{dt} = \frac{dx}{d(t)} = \frac{A(t) - Bx(t) + \frac{x(t)^2}{x(t)^2 + 1}}{x(t)^2 + 1} \quad (3.1.2) \]

If we assume steady states where the stock of phosphorous is constant ($dx/dt = 0$) we get

\[ A(t) = Bx(t) - \frac{x(t)^2}{x(t)^2 + 1} \quad (3.1.3) \]

This can be interpreted as external loading as a function of the stock of phosphorous while maintaining steady-state. In fig. 3 we see a plot of $A$ (inflow) on the horizontal axis and $x$ (stock) on vertical axis.

If we start with a low phosphorus load ($A$) and slowly increase it, the level of phosphorus ($x$) suddenly jumps from level $x_a$ to a much higher level when we reach $A**$. The system
flips from one state to another. If this high level is undesirable and we would like to revert the change, we have to decrease $A$ all the way down to $A^*$, which is lower than $A^{**}$. $A^*$ and $A^{**}$ are called bifurcation points; where the properties of the system drastically change. This system behaviour is called hysteresis.

We have two regions or “regimes”, where behaviour is similar within each region. The structure of the system appears to change at the bifurcation points. We have two basins or domains of attraction that attract the current state to one or the other regime.

It is clear from the above reasoning, that the state of the system is path dependent, which means that the history of the stock $(x)$ matters. Whether we have passed the threshold level and entered into an undesired state makes a lot of difference when trying to regulate the system.

Several authors mentions example of this kind of dynamics in nature; spruce budworms, the Baltic Sea, boreal forests, competitive grazing (Ludwig, Walker and Holling 1997), shifts from grass- to shrub-dominated communities in the Chihuahuan Desert (Brown et 1997), wet and dry Sahel$^4$ regimes that persist for decades at a time (Foley et al 2003). The transition time depends on the size of the object. Shallow lakes can change from clear to turbid state in a few months and garden ponds are known to change in a few hours (Brock 2000).

**Non-reversible System**

The above system is reversible; it is possible to move back to the previous state. Consider now the following diagram (fig. 4) that shows irreversible behaviour.

![Diagram showing irreversible system](image)

Fig. 4. Source: Dasgupta and Mäler (2003)

Once we reach the critical level of inflow $A^{**}$, a bifurcation is triggered, we move suddenly to a higher level of phosphorous and there is no way to return back to the original state, even if you decrease loading to zero. It is too late for a change.

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$^4$ Region in northern Africa
Research indicates that regime shifts seems to be more common than previously thought. If the loading of economic activity is low compared to natural processes, we might assume a simple, even linear behaviour of ecosystem services. As the scale of human impact increases, these threshold effects must affect how we value and manage resources. We cannot afford to ignore them, because the costs can be great. If the shifts are irreversible, even more concern is needed.

3.2 Our Model

Numerical solutions are presented throughout to make the example more illustrative. Let us look at the shallow lake model again.

\[
\frac{dx}{dt} = A(t) - Bx(t) + \frac{x(t)^2}{x(t)^2 + 1} \quad (3.2.1)
\]

The value of B, the outflow of phosphorous through sedimentation, sequestration in other biomass, is critical for the character of the lake. In our model (3.2.1) we have three different types of behaviour depending on the numeric value of B: irreversible and reversible hysteresis and no hysteresis at all.

**Irreversible Hysteresis: \( B \leq 1/2 \)**

If the lake is in clear state (fig 5) and A is gradually increased past 0.0614 there is a jump in the level of phosphorous \( x \) from 0.276 to 1.65 resulting in a turbid state. After that, even if A is reduced to zero, we can never get back to the original value or even below 1.33. We are trapped in the turbid state. In reality to get down past the threshold, we must influence the parameter B by for example affecting the fish stock, use biomanipulation or oxygenation. Local maximum for x is 0.279 and local minimum is 1.04.

**Reversible Hysteresis: \( 1/2 < B < 3\sqrt{3}/8 \approx 0.6495 \)**

If the lake is in the clear state (fig 6) and A is increased gradually, when we reach A = 0.1021 there is a jump in the level of phosphorous \( x \) from 0.4084 to 1.0, causing a
sudden flip to a turbid state. To get back to our original position we have to decrease A to 0.0897 and then up to 0.1021

![Graph](image)

**Fig 6, reversible hysteresis**

**No Hysteresis:** $B \geq 3\sqrt{3}/8 \approx 0.6495$

We have just one state for all values of external loading (A). A marginal change of loading does not give any surprise. The internal recycling rates of the lake is such that the lake could principally be used as a waste-sink without regard to the level of pollution (A)

![Graph](image)

**Fig 7, no hysteresis**

In the rest of this thesis we focus on reversible hysteresis and therefore choose $B = 0.6$ as the basis for our study, because then the lake can be controlled by the level of phosphorus.
4 Analysis
We will now extend the shallow lake model and put it in an economic context. Both static and dynamic behaviour will be explored.

4.1 Socially Optimal Static Equilibrium
In our first approach we have a social planner who acts for the best of society. He can measure the stock and control the discharge and chooses the loadings to best trade off the conflicting interests of the different stake holders. Dynamic adjustment effects are not taken into account. Agents are committed to strategies once they are decided.

There are \( n = 2 \) communities situated around the lake. The flow of phosphorous \( (a_i) \) from a community into the lake is a by-product of agriculture and is a good; the lake has a value as a waste sink for agriculture We choose the logarithmic function because it is convenient for the technicalities of the analysis. The lake also provides ecological services and we define the stock of phosphorous \( (x) \) in the water as a bad. The parameter \( c \) defines the relative weight of welfare and we use \( c = 1 \) meaning that enough weight is put on the ecological services for the communities to aim for a clear state. In all the following examples we use \( B = 0.6 \), meaning that the lake has reversible hysteresis.

We define the utility for each community as

\[
U(a_i, x) = \ln a_i - cx^2, \quad c > 0
\]

The social welfare function of the model is defined as the sum of the community’s function and the social planner’s problem is to

\[
\max_{a_1 \ldots a_n} \sum_{i=1}^{n} \ln a_i - ncx^2 \quad s.t. \quad A - Bx + \frac{x^2}{x^2 + 1} = 0, \quad A = \sum_{i=1}^{n} a_i \quad (4.1.1)
\]

We use a Lagrangian to solve this problem.

\[
L(x, a_i, \lambda) = \sum_{i=1}^{n} \ln a_i - ncx^2 + \lambda \left( \sum_{i=1}^{n} a_i - Bx + \frac{x^2}{x^2 + 1} \right) \quad (4.1.2)
\]

First order conditions

\[
\frac{\partial L}{\partial x} = -2ncx - \lambda B + \lambda \left( \frac{2x}{(x^2 + 1)^2} \right) = 0 \quad (4.1.3)
\]

\[
\frac{\partial L}{\partial \lambda} = A - Bx + \frac{x^2}{x^2 + 1} = 0 \quad (4.1.4)
\]

\[
\frac{\partial L}{\partial a_i} = \frac{1}{a_i} + \lambda = 0 \quad (4.1.5)
\]

\[
\frac{1}{a_i} + \lambda = 0 \Rightarrow a_i = -\frac{1}{\lambda} \Rightarrow a_1 = a_2 = \cdots = a_n = -\frac{1}{\lambda} \cdot \sum_{i=1}^{n} a_i = na_i = A
\]

\[
\Rightarrow A = -\frac{n}{\lambda} \quad (4.1.6)
\]
We substitute 4.1.6 into 4.1.5 and we get

\[ n = -a\lambda = \lambda \left( \frac{x^2}{x^2 + 1} - Bx \right) \quad (4.1.7) \]

Above is substituted into 4.1.4

\[-2cx\lambda \left( \frac{x^2}{x^2 + 1} - Bx \right) - \lambda B + \lambda \left( \frac{2x}{(x^2 + 1)^2} \right) = 0 \quad (4.1.8)\]

We divide by \(-\lambda\)

\[ B = \frac{2x}{(x^2 + 1)^2} - 2cx \left( Bx - \frac{x^2}{x^2 + 1} \right) = 0 \quad (4.1.9) \]

We solve for \(x\) and among several solutions we get one positive and real valued \(x = 0.3279\), which we interpret as our pareto-optimal solution.

We compute total \(A\) by

\[ A = Bx - \frac{x^2}{x^2 + 1} \quad (4.1.10) \]

which gives us an external loading of \(A = 0.09966\). Utility for each community at this point is -3.10, and we choose this as our socially optimal reference level. Remember that the lake flips when \(A > 0.1021\). The optimal point is at the edge of hysteresis.

Our model is deterministic which means that we do not have random variation of one or more parameters. If there were a random element in the model it is easy to see the need of a safety margin to the critical threshold. Likewise we could need a safety margin if we suspect that there are important unobservable parameters.

With lower values for \(c\) (ecological services valued less) we would get higher values for external loading \(A\) and accordingly higher values for phosphorous stock \(x\) and we would perhaps pass the critical threshold.

A plot of the marginal disutility would show a sharp rise as we approach the critical threshold. Therefore welfare drops by a large amount at the same point.

A phase plot of the lake behaviour reveals the following if the external loading is fixed at \(A = 0.09966\). In the diagram below, \(x\) is horizontal axis and \(\dot{x}\) is vertical axis.
We find two stable steady-states, $x = 0.3275$ and $x = 0.9934$; which one we end up in depends on the original value of phosphorous stock $x$. We have path dependence.

### 4.2 Private Static Equilibrium

In our second approach we look at the case when each community manage its own use of the lake independently, giving Nash equilibria. We would like to examine the difference in utility between a socially optimal solution and a decentralized market solution. It is also interesting to see if there is a structural difference between the cases.

The problem is defined as

$$\max_{a_i} (\ln a - cx^2) \quad s.t. \quad A - Bx + \frac{x^2}{x^2 + 1} = 0, \quad A = \sum_{i=1}^{n} a_i = na_i \quad (4.2.1)$$

Again we use a Lagrangian to solve the problem

$$L_i(x, a_i, \lambda) = (\ln a_i - cx^2) + \lambda_i \left( na_i - Bx + \frac{x^2}{x^2 + 1} \right) \quad (4.2.2)$$

First order conditions

$$\frac{\partial L_i}{\partial x} = -2cx - \lambda_i B + \lambda_i \left( \frac{2x}{(x^2 + 1)^2} \right) = 0 \quad (4.2.3)$$

$$\frac{\partial L_i}{\partial \lambda_i} = na_i - Bx + \frac{x^2}{x^2 + 1} = 0 \quad (4.2.4)$$

$$\frac{\partial L_i}{\partial a_i} = \frac{1}{a_i} + \lambda_i = 0 \quad (4.2.5)$$

$$\frac{1}{a_i} + \lambda_i = 0 \Rightarrow a_i = -\frac{1}{\lambda_i} \quad (4.2.6)$$
We combine 4.2.4 and 4.2.6 and we get

\[
\frac{n}{\lambda_i} = \left( \frac{x^2}{x^2 + 1} - Bx \right) \quad (4.2.7)
\]

Above is substituted into 4.2.3 and \( \lambda_i \) is eliminated

\[
B - \frac{2x}{(x^2 + 1)^2} - 2\frac{c}{n} x \left( Bx - \frac{x^2}{x^2 + 1} \right) = 0 \quad (4.2.8)
\]

If we have two communities and parameters \( c = 1 \) and \( B = 0.6 \) as before, we get three steady state solutions: \( x_1 = 0.3592, x_2 = 1, x_3 = 1.5125 \). The first solution leads to the clear state with a utility -3.11 for each community, comparable to the socially optimal solution (reference level); the second is very close to the critical threshold (unstable) and the last solution leads to the turbid state with a utility of -4.53.

It is interesting to note that despite the fact that each community aims for the clear state, with a non-cooperative model they might end up in the turbid state, because of the local optimum. This externality could be handled with taxation in order to reach the socially optimal outcome. We could design a tax on phosphorous loading to make the cost to each community the same as the cost to society and in this way induce the communities to decrease their loading to societally optimum level.

### 4.3 Socially Optimal Dynamic Equilibrium

We now turn to a dynamic scenario. Again we have a social planner that acts for the best of society. He is able to continuously measure the phosphorous stock over time and control the discharge in the interests of different stake holders.

Here we have an infinite time horizon, which means that the lake is used for all future. Loading, \( A \), can now be changed over time. The level of phosphorous does not adjust instantaneously, but successively to its steady-state. \( \rho \) is the discount factor, \( x \) is the state variable and \( A \) is the control variable.

The goal of the social planner is to maximize the sum of objective functions \( W_i \) s.t. \( \dot{x}(t) = 0 \) with \( A = \sum a_i \).

This means

\[
\max_{\sum_{i=1}^{n} a_i} \sum_{i=1}^{n} W_i = \max_{\sum_{i=1}^{n} a_i} \sum_{i=1}^{n} \int_{0}^{\infty} e^{-\rho t} [\ln a_i(t) - cx^2(t)] dt =
\]

\[
= \max_{\sum_{i=1}^{n} a_i} \int_{0}^{\infty} e^{-\rho t} \left[ \sum_{i=1}^{n} \ln a_i(t) - n cx^2(t) \right] dt \quad (4.3.1)
\]

s. t. \( \dot{x}(t) = A(t) - Bx(t) + \frac{x^2(t)}{x^2(t) + 1} = 0 \), \( (4.3.2) \)

and \( A = \sum_{i=1}^{n} a_i \)
We use current value Hamiltonian, which gives us
\[
H^c = \sum_{i=1}^{n} \ln a_i(t) + ncx^2 + \lambda(t)[A(t) - Bx(t) + \frac{x^2(t)}{x^2(t) + 1}]
\]
\[
\lambda(t) = e^{\rho t} \mu \quad (4.3.3)
\]
The Pontryagin Maximum Principle and first order conditions gives
\[
\frac{\partial H^c}{\partial a_i} = \frac{1}{a_i(t)} + \lambda(t) = 0 \quad (4.3.4)
\]
\[
\frac{d\lambda(t)}{dt} - \rho \lambda = -\frac{\partial H^c}{\partial x} = 2n cx(t) + \lambda(t) \left[ B - \frac{2x(t)}{(x^2(t) + 1)^2} \right] \quad (4.3.5)
\]
\[
\frac{\partial H^c}{\partial \lambda} = A(t) - Bx(t) + \frac{x^2(t)}{x^2(t) + 1} = 0 \quad (4.3.6)
\]
From 4.3.5
\[
\frac{d\lambda(t)}{dt} = 2n cx(t) + \lambda(t) \left[ B - \frac{2x(t)}{(x^2(t) + 1)^2} \right] + \rho \lambda =
\]
\[
= 2n cx(t) + \lambda(t) \left[ B + \rho - \frac{2x(t)}{(x^2(t) + 1)^2} \right] \quad (4.3.7)
\]
From 4.3.4
\[
\lambda(t) = -\frac{1}{a_i(t)} \Leftrightarrow \frac{d}{dt} \lambda(t) = \frac{d}{dt} \left( -\frac{1}{a_i(t)} \right) = \frac{d}{dt} \left( \frac{a_i(t)}{a_i^2(t)} \right) = \frac{\dot{a}_i(t)}{a_i^2(t)} \quad (4.3.8)
\]
In Pontryagin’s maximum principle the costate (adjoint) variable \(\lambda(t)\) can be interpreted as the shadow price of the state variable. It is equal to the marginal economic value of the asset \(x\) at time \(t\), assuming that \(x\) will be optimally managed for the time period (Clark 2010). So \(\lambda(t)\) is the marginal economic value of phosphorous at time \(t\) in our case.
\[
A(t) = \sum_{i=1}^{n} a_i(t) = na_i(t) \quad (4.3.9)
\]
We combine 4.3.7, 4.3.8 and 4.3.9 and get
\[
\frac{\dot{a}_i(t)}{a_i^2(t)} = 2n cx(t) + \lambda(t) \left[ B + \rho - \frac{2x(t)}{(x^2(t) + 1)^2} \right] =
\]
\[
= 2n cx(t) - \frac{1}{a_i(t)} \left[ B + \rho - \frac{2x(t)}{(x^2(t) + 1)^2} \right] \quad (4.3.10)
\]
We derive $\dot{A}(t)$

$$\dot{a}_i(t) = 2ncx(t)a_i^2(t) - a_i(t) \left[ B + \rho - \frac{2x(t)}{(x^2(t) + 1)^2} \right] =$$

$$= 2A(t)cx(t)a_i(t) - a_i(t) \left[ B + \rho - \frac{2x(t)}{(x^2(t) + 1)^2} \right]$$

$$n\dot{a}_i(t) = 2A^2(t)cx(t) - A(t) \left[ B + \rho - \frac{2x(t)}{(x^2(t) + 1)^2} \right] =$$

$$\dot{A}(t) = 2A^2(t)cx(t) - A(t) \left[ B + \rho - \frac{2x(t)}{(x^2(t) + 1)^2} \right] \quad (4.3.11)$$

The system is determined by the two differential equations 4.3.2 and 4.3.11.

Let us now plot (fig 9) the steady-state curve for the lake where phosphorous level is stable $dx/dt = 0$ and the steady-state curve for optimal control where loading is stable $da/dt = 0$ and find out where they intersect. As parameters we use: $B = 0.6, \rho = 0.03$ and $c = 1$.

![Fig 9](image)

If we solve the system we get an intersection point $p_1 = (0.3534, 0.1010)$, which is a saddle point. Eigenvalues are -0.3202 and 0.3502. This is an intertemporal equilibrium were neither $A$ nor $x$ will change over time. This point, $A = 0.1010$ is lower than the critical bifurcation point where the lake flips, which means that the dynamic optimum exists in the clear regime. Utility at this point is -3.11 for a community, which is comparable to the reference level.
With the help of Matlab\(^5\) we can construct a phase plot (fig 10). We have \(\dot{x}\) on the horizontal axis and \(\dot{A}\) on the vertical axis. In the background of the diagram you can see small arrows symbolizing the direction fields which indicate the direction of trajectories.

Above we can see the separatrices that separate the basins of attraction. If we have an initial point that lies on one of the two stable branches, the dynamics of the system will lead us to the saddle point. In other cases, the level of \(x\) will move into a high state or a low state. In fig 10 we can also see the direction fields in the background as small grey arrows.

We actually also have a critical point at \((0, 0)\), which is an asymptotically stable improper node as the direction fields indicate.

### 4.4 Dynamic Private Equilibrium

We will now look at the dynamic open-loop Nash equilibrium. This is a non-cooperative situation and each community maximizes independently of the others.

The maximisation problem for each community looks like this:

\[
\max_{a_i} \sum_{i=1}^{n} W_i = \max_{a_i} \int_0^\infty e^{-\rho t}[\ln a_i(t) - cx^2(t)]dt \tag{4.4.1}
\]

s.t. \(\dot{x}(t) = A(t) - Bx(t) + \frac{x^2(t)}{x^2(t) + 1} = 0\), \tag{4.4.2}

and \(A = \sum_{i=1}^{n} a_i = na_i\)

\(^5\)Matlab software from MathWorks
The Hamiltonian gives us

\[ H^c = \ln a_i(t) + cx^2 + \lambda_i(t)[na_i(t) - Bx(t) + \frac{x^2(t)}{x^2(t) + 1}], \quad i = 1 \ldots n, \]
\[ \lambda_i(t) = e^{\mu t} \mu \quad (4.4.3) \]

The Pontryagin Maximum Principle and first order conditions gives

\[ \frac{\partial H^c}{\partial a_i} = \frac{1}{a_i(t)} + \lambda_i(t) = 0 \quad (4.4.4) \]
\[ \frac{d\lambda_i(t)}{dt} - \rho \lambda_i = -\frac{\partial H^c}{\partial x} = 2cx(t) + \lambda_i(t) \left[ B - \frac{2x(t)}{(x^2(t) + 1)^2} \right] \quad (4.4.5) \]
\[ \frac{\partial H^c}{\partial \lambda_i} = na_i(t) - Bx(t) + \frac{x^2(t)}{x^2(t) + 1} = 0 \quad (4.4.6) \]

From 4.4.5

\[ \frac{\partial \lambda_i(t)}{\partial t} = 2cx(t) + \lambda_i(t) \left[ B - \frac{2x(t)}{(x^2(t) + 1)^2} \right] + \rho \lambda_i = \]
\[ = 2cx(t) + \lambda_i(t) \left[ B + \rho - \frac{2x(t)}{(x^2(t) + 1)^2} \right] \quad (4.4.7) \]

From 4.4.4

\[ \lambda_i(t) = -\frac{1}{a_i(t)} \iff \frac{d}{dt} \lambda_i(t) = \frac{d}{dt} \left( -\frac{1}{a_i(t)} \right) = \frac{d}{dt} \left( \frac{a_i(t)}{a_i^2(t)} \right) = \frac{\dot{a}_i(t)}{a_i^2(t)} \quad (4.4.8) \]

\[ A(t) = \sum_{i=1}^{n} a_i(t) = na_i(t) \quad (4.4.9) \]
\[ a_i(t) = \frac{A(t)}{n}; \quad a_i^2(t) = \frac{A^2(t)}{n^2} \]

We combine 4.4.7, 4.4.8 and 4.4.9 and get

\[ \frac{\dot{a}_i(t)}{a_i^2(t)} = \lambda_i(t) = 2cx(t) + \lambda_i(t) \left[ B + \rho - \frac{2x(t)}{(x^2(t) + 1)^2} \right] = \]
\[ = 2cx(t) - \frac{1}{a_i(t)} \left[ B + \rho - \frac{2x(t)}{(x^2(t) + 1)^2} \right] \quad (4.4.10) \]

We derive \( \dot{A}(t) \)

\[ \dot{a}_i(t) = 2cx(t)a_i^2(t) - a_i(t) \left[ B + \rho - \frac{2x(t)}{(x^2(t) + 1)^2} \right] = \]
\[ n\dot{a}_i(t) = 2cx(t)na_i(t)a_i(t) - na_i(t) \left[ B + \rho - \frac{2x(t)}{(x^2(t) + 1)^2} \right] = \]
\[ A(t) = 2cx(t)A^2(t) \frac{1}{n} - A(t) \left[ B + \rho - \frac{2x(t)}{(x^2(t) + 1)^2} \right] \quad (4.4.11) \]

The system is determined by the two differential equations 4.4.2 and 4.4.11.

We plot (fig 11) the steady-state curve for the lake where phosphorous level is stable \( \frac{dx}{dt} = 0 \) and the steady-state curve for optimal control where loading is stable \( \frac{da}{dt} = 0 \). We use \( B = 0.6, \rho = 0.03, n = 2 \) and \( c = 1 \).

![Plot of A vs x showing steady-state curves for lake and control](image)

**Fig 11**

We have three real valued solutions \( p_1 = (0.3931, 0.1020), p_2 = (0.9078, 0.0929) \) and \( p_3 = (1.5838, 0.2353) \) for this system, resulting in utility values -3.13, -3.89, -4.65 respectively.

Eigenvalues for \( p_1 = -0.2716 \) and 0.3016, indicating that this is a saddle point.

Eigenvalues for \( p_2 = 0.015+0.1779i \) and 0.015-0.1779i, a complex conjugate showing that this is an unstable spiral point.

Eigenvalues for \( p_3 = -0.3200 \) and 0.3500 and we conclude that this is a saddle point.

As in the previous case we have a critical point at \((0, 0)\), which is an asymptotically stable improper node.
Let us take a look at the phase plot of $\dot{x}$ and $\dot{A}$ (fig 12):

![Phase plot of $\dot{x}$ and $\dot{A}$](image)

Now the diagram is more complex. When the initial value of $x_0$ lies to the right of the set of curls, the equilibrium follows the upper trajectory to the higher critical point on the right, which represents a turbid state. When $x_0$ lies to the left of the curls, the equilibrium follows the lower trajectory to the critical point to the left, the clear steady-state. The curl or spiral at $x = 0.9078$ is an indifference (Skiba) point at which both trajectories are possible.

Thus, our model confirms our suspicion of path dependence. The same level of loading can lead to a turbid state if the initial level of phosphorous $x$ is in the higher basin of attraction or to a clear state if $x$ is in the lower basin of attraction. In the region between the saddle points, the outcome could be either state.

Another important insight when considering the alternate state behaviour is that the stability of the lake might have a value of its own. Depending on how the communities valuate the clear state, it is obvious that the preservation of desired stability is worth something, measured against the restoration costs of the lake. The time perspective is important to consider here, because the flip to an alternate state might be distant from now.

5  Further Aspects

With the previous models and examples as background, I will discuss some further economic aspects of regime shifts, valuation, uncertainty and in particular stability/resilience.
5.1 Valuation
Economic theory traditionally uses the willingness-to-pay and willingness-to-accept concepts for valuation of environmental resources. These measures and the analytical methods used to compute them are useful in situations where ecosystems are stable and operate far from above discussed bifurcations. However, as we have seen, there are cases where marginal changes might not result in predictable effects. The marginal willingness-to-pay is not well-defined where there is a regime shift and a jump in utility level.

In these situations we have to move beyond marginal valuation and consider the following:
- the possibility of sudden shifts,
- the possibility of unproportionally large restoration costs,
- the possibility of irreversible change,
- uncertainty in observation, processes and model.

Regime shifts are non-convexity behaviour and the price mechanism and market valuation have difficulties and markets might not be efficient in allocation. As we have seen in the shallow lake example, optimal management could be close to the bifurcation point, which means that a small mistake will result in large costs. We can ignore the thresholds if risk of passing is low or cost of passing is low.

If the alternate state is evaluated as undesirable, the estimated cost of the undesirable state, together with valuation of the four above mentioned possibilities have to be considered. The stream of goods and services from both regimes and possible restoration costs need to be incorporated when calculating costs and benefits.

Even more complex is when we have several optimum strategy paths that are possible; some of them suboptimal in a larger context. The best management path depends on previous actions, so, to avoid mistakes, decision makers must calculate all future costs and benefits of each alternative path (Crépin 2006).

As in more linear cases, there is also an intertemporal concern of future generations carrying the cost of today’s decisions. For an extensive discussion of discount rates see Dasgupta and Heal (1979).

When management is evaluated, it is important to have both the economic perspective and the ecological perspective; to consider both possible market failures and also potential hysteresis effects in the environment. Both economic and ecological theory is needed (Brock 2000).

5.2 Uncertainty
We conclude that the nonlinear behaviour at the tipping point makes uncertainty an important factor to consider. In reality we always have uncertainty at multiple levels and therefore it would be preferably to model the system as a stochastic process (Crépin, Norberg and Mäler 2010). This means that we think of the system’s flip into an undesired state in terms of probabilities. Clark (2010) highlights the importance of recognizing uncertainty and treating it explicitly in models.
The shallow lake model presented earlier is a deterministic model, which means that there is no random element involved. What difference would an inclusion of random elements make?

We would like to control discharges so that

$$\Pr(A < A^{**}) \leq \varepsilon \quad (5.2.1)$$

Where $A$ is pollution level, $A^{**}$ is critical threshold and $\varepsilon$ is some predetermined very small tolerance level. The problem is that $A^{**}$ does not have a constant value over time, nor is it usually known with certainty. To better describe the situation we could instead write

$$\Pr(A < Y_{\varepsilon} \cdot A^{**}) \leq \varepsilon \quad (5.2.2)$$

where $Y_{\varepsilon}$ is a random process, independent and identically distributed. We would have to approximate the probability distribution of the stochastic process in order to compute the cumulative distribution or probability of flip. For an interesting discussion about deterministic and stochastic models for systems with thresholds see Clark (2010). Equilibrium solutions might differ between a deterministic and a stochastic model, especially if wide stock fluctuations are possible (Clark 2010).

The implications of the uncertainty of the threshold could be highly asymmetric. The consequences might be very different between an underestimation by 10% and an overestimation by 10%.

Where there is uncertainty, a feedback strategy becomes very important and even necessary. In the shallow lake example, this could mean that parameters are observed at regular intervals, in order to learn about the system and adjust management if necessary. A good strategy for managers could be to revise their tactics at more or less regular intervals to see if they need to adapt their plans or not. Prescriptions are constantly re-evaluated based on actual ecosystem response to management (Holling 1978). It is critical to find important high-stakes thresholds and receive empirical information from these as feedback.

Brock (2000), Walters (1986) and Holling (1978) discuss the importance of dynamic environmental regulation and adaptive management. Here operations are directed not merely to maximize yield, but also to generate information useful for future management (Conrad and Clark 1987).

Doing experiments to learn is another management approach. Gathering new information is costly. In this case there is a trade-off between doing something now to the best of knowledge and doing experiments that might be costly at the moment but leads to a deeper understanding of the system and perhaps better management in the long run. Balancing the benefits from adapting and the information cost will indicate how often the revision should be made (Crépin 2003).
Brock, Carpenter and Scheffer (2008) say “The policy design problem is to identify patterns of evidence that should prompt us to choose actions to avert unwanted and impending regime shifts. Growing evidence for environmental thresholds and regime shifts suggests that this policy design problem will become more prominent in coming decades.”

5.3 Resilience

The last aspect I would like to discuss is that of stability and resilience. The stability of a system, in our case the shallow lake, might have a value of its own. To explore this aspect we use the resilience framework developed by Holling and others. We begin by their definition of stability and resilience.

Holling (1973) defines resilience as the propensity of a system to retain its organizational structure following a perturbation (disturbance). Resilience refers to the stability of the system parameters (organizational principles), but accepts the possibility of multiple equilibria. Stability on the other hand, is defined as the propensity of the populations within an ecosystem to return to the equilibrium conditions. You can have resilience without stability, but not the other way around (Common and Perrings 1992). Resilience is structural stability. We discussed in earlier sections about different possible regimes in an ecosystem. Resilience is the capacity to remain in a given regime.

To use a metaphor, imagine a landscape with a valley and a ball at the bottom. A small perturbation makes the ball move from the equilibrium at the bottom temporarily, but it soon settles at its original position (Scheffer, Carpenter, Foley, Folke and Walker 2001). This is a picture of a single state system.

Now imagine that there is another valley nearby and the ball is perturbed enough to move over the pass and into the other valley. Here we have a system with two states. The pass separating the two valleys marks the separate two basins of attraction, the two possible basins (domains) of the ball. This could be a picture of hysteresis behaviour and in particular a picture of the earlier mentioned shallow lake system.

To continue with the landscape metaphor, local resilience refer to the size of the valley or basin of attraction around a state, which corresponds to the maximum perturbation that can be taken without causing a shift to an alternative stable state (Scheffer, Carpenter, Foley, Folke and Walker 2001).

In more applied terms, the resilience of an ecosystem is its capacity to absorb stress and shocks without fundamental change. This property is closely associated with biomass and the diversity of species in an ecosystem. Diversity means redundancy in the sense that species could act as substitutes and therefore may prevent a flip in case of a shock (Levin et al 1998). Modern agriculture could be described as a mono-culture and is therefore sensitive to perturbation, for example like an attack of insect pest (Brock 2000). This is an example of how specialization in farming might cause a loss of resilience.

Loss of resilience means that it takes longer for the system to recover after shock. Loss of resilience can also result in a smaller supply of ecosystem services.

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6 A perturbation of system is an alteration of function, induced by external or internal mechanisms
In natural systems we could have slow changing unobserved variables that eventually change the structure of the system. In the landscape metaphor, the slow change is like a slow deformation of the landscape until a hill sinks lower than a valley. The basin of attraction shrinks and this time a smaller perturbation can cause a system change; the ball moves and the level of turbidity changes. As new valleys emerge in this dynamic landscape, decision makers must find new methods to manage the resource.

When considering the shallow lake model, we could easily think of the need to stabilize the phosphorous level. Holling argues that the stability-oriented approach is overly restrictive as a management idea in an evolutionary environment. Resilience instead allows the evolution of systems within bounds allowed by the organization of the system. The focus of analysis moves from the stability of detached populations within an ecosystem, to stability of the particular ecosystem itself, a broader view, congruent with system dynamics.

There are multiple levels of meaning for resilience: as a metaphor related to sustainability, as a property of dynamic models and as a measurable quantity that can be assessed in field studies of socio-ecological systems (Carpenter, Walker, Anderies and Abel 2001). Resilience could be seen as a service of the ecosystem, a service that keeps ecological subsystems in desirable states to deliver goods and services to us. As a valuable service, it should be recognized and properly valued.

We saw earlier in the welfare function of the shallow lake model that the phosphorous stock entered the welfare function as a constraint. We could expand this approach and let the broader concept of resilience be regarded as a capital stock together with the ordinary capital stocks and allow it to be measured and priced (Mäler, Li and Destouni 2007). Due to the complexity, to cast resilience in the framework of social cost-benefit analysis is not a trivial issue.

The opportunity cost of capital begins with the assumption that capital is productive in the sense that one unit today will generate more than one unit in the future. A thing to consider is that the environment is not necessarily productive in the above sense.

Even though it is not directly involved in production, the resilience stock may have a value on its own. The level of the resilience stock variable might not matter for ecosystem services and wellbeing under normal circumstances, but when the ecosystem flips into a qualitatively different state, the production potential and other ecosystem services might be very different.

Mäler, Li and Destouni (2007) give an example: Keystone species in a tropical ecosystem may play a vital role for the system’s functioning and stability. Decrease of these species below a threshold level, may cause an undesirable structural change. The biomass over a minimum level could be seen as an asset, a resilience stock variable. The more resilience in the system, the less likely that an external shock drive the species to extinction with structural change and degradation in ecosystem services as a consequence.

Often a “preferred state” is the reference when resilience is discussed, but in principle the cases of desirable and undesirable are perfectly symmetric. A decline in resilience is seen as a decline in wealth. Resilience could be seen as a kind of insurance against reaching an
undesired state (Mäler and Li 2010). An interesting aspect is to consider if we could “invest” and thereby enhance the resilience of a system.

Resilience is typically not traded in a market and we have therefore no market price information that indicates its value. Resilience can still be seen as having a shadow price (accounting price) that corresponds to the change in the expected net present value of future ecosystem services resulting from a marginal increase in resilience stock today. This definition is developed by Mäler, Li and Destouni (2007) and Mäler, Li (2010) where they expand it to an interesting resilience pricing model. Resilience could thus be seen as a stability enhancing capital stock variable with appropriate accounting price. It could be identified as a single or composite variable that can be quantified, priced and used in measures of wealth.

Mäler, Li and Destouni (2007) and Walker et al (2009) discuss how resilience valuation could be used to assess ecosystem and welfare dynamics in the Goulburn-Broken Catchment (GBC) in south-east Australia. In this system we have strong hysteresis behaviour.

When the system is regarded as a stochastic process because of uncertainty, there is a probability of regime change. If resilience decreases this probability increases. The shadow price of the resilience stock changes as the likelihood of changing into an alternative regime increases. The value of regime change is affected by its reversibility.

Management is often stability-oriented and seeks to minimize the variance of some target variable, but these attempts to stabilize ecosystems, while often successful in the short run, may lead to qualitative changes in the nature of the wider system with negative consequences for the resilience (Holling 1986).

6 Conclusion
The purpose of this thesis was to study the economic consequences of alternate regimes in a shallow lake by examining a number of utility maximation models of the lake and its related economic activities. We have now explored four models, socially optimal and private equilibrium; both static and dynamic and we have found that hysteresis in the models creates path dependence and complex behaviour, just like previous research have mentioned.

Path dependence means that if you compare two lakes; the same treatment can lead to quite different results. Standard solutions might fail, because the initial value of the governing stock is important.

Generally, if we have multiple optima, if a coordinator knew the curve in its entirety he could simply pick the local optimum which is the also the global optimum. The regulator needs global information or some kind of adaptive strategy.

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7 Agricultural expansion has led to declines in the native vegetation cover resulting in rising water tables. There are salt deposits in the ground and when the water tables reach 2 meters below the surface, the water with dissolved salt is drawn to the surface by capillary action and the stock of soil is radically changed. There is a regime shift to degraded, salinized soil and the change is practically irreversible.

8 See section 1.3
Concerning the cases of private equilibrium, if the communities get stuck in the turbid state and decide to coordinate, it will be hard to find agreement for a decrease of pollution because of the cost to get out of the turbid state. A situation with hysteresis makes negotiation and cooperation difficult.

We have also discussed the consequences of regime shifts on valuation and how the stream of goods and services from both regimes and possible restoration costs need to be incorporated when calculating costs and benefits. Likewise, the possibility of a sudden transition to an undesirable state highlights the importance of recognizing uncertainty and treating it explicitly.

When the possibility of a shift is perceived and fully acknowledged, the question of valuating stability becomes interesting. If one state is desirable, we might be able to enhance the resilience: the capacity of the system to remain in this particular state by finding suitable leverage points that bring great effect with minimal cost. There is interesting on-going research on how to valuate enhancement or loss of resilience.

As the scale of human impact on the earth continues to increase and the concern for sustainability also increases, the joint economic-ecological study of environmental regime shifts will get much attention in the future.

7 References


Holling, C. S. *Adaptive Environmental Assessment and Management*. The Blackburn Press 2005


