Modelling reflected polarized light from exoplanetary atmospheres

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Abstract

I present numerical simulations of intensity and degree of polarization of light reflected by Earth-like exoplanets. The results are presented as a function of wavelength, and for a few different phase angles and a few different points on the planet. At this stage the aim is to show the working code and test a few different set-ups of the star-planet system in order to find preferable configurations for observations. Not surprisingly, phase angle 90° shows the largest degree of polarization. For beneficial wavelength regions, visual light shows a larger overall degree of polarization, while NIR shows very clear absorption patterns in the degree of polarization, making detection of the atmospheric composition possible.
1 Introduction

Exoplanetary science has previously mainly been about detecting the exoplanets and determining their size, mass and orbital parameters. The next step is to explore the exoplanets more in detail, especially the composition of the atmospheres and surface. As we are not able to directly detect the planets, one has to use clever methods to increase the contrast between the exoplanet and its host star. One such method is to observe in linearly polarized light. Light becomes polarized either by the presence of magnetic fields or by some asymmetry. For stars of solar type the tangential component of the magnetic field, which yields linear polarised light averages out over the stellar disk, and the line of sight component of the magnetic field produces only circular polarisation. As stars are also symmetric, there will be no linearly polarized light from stars of solar type. Illuminated planets will however introduce an asymmetry into the star - planet system, generating linearly polarized light. One therefore expect that light scattered by an atmosphere of an exoplanet will have a high degree of polarization. By removing the unpolarized light from an observation of a star - planet system, one should be able to greatly increase the contrast between the two. Light scattered by a rough surface (such as a rocky surface or a surface filled with vegetation) will generally not produce polarized light, so for a planet with a rough surface, the only contributing factor to the polarized light will be light scattered by small particles in the atmosphere, i.e. atoms, molecules and aerosols. By taking a spectrum of the polarized light, one will not only get the degree of polarization from the reflected light, but also how this varies with wavelength. The main contributor to the wavelength dependence is the atomic or molecular absorption. Because of the wavelength dependence and the fact that the atmosphere is the main contributor, the polarized signal of a star - exoplanet system is an excellent way to detect and examine the composition of an exoplanet’s atmosphere.

The relationship between degree of polarization and absorption is however not trivial, and in order to analyse observations and get reliable results, one will have to compare the observations with models. Modelling reflected light from exoplanetary atmospheres and surfaces therefore is of scientific interest when it comes to further exploration of exoplanets.

Figure 1 shows a sketch of a typical set up light reflected by a planet atmosphere and surface. A couple of different light paths are shown as examples of the many different path the light can take through that atmosphere - surface system until it reaches the observer.
The goal of this project is to create a code for modelling reflected light from exoplanetary atmospheres and surfaces. The following will be taken into account:

- Wavelength dependence
- State of polarization
- Phase angle of the planet

The modelling will be limited to visual and near infrared (NIR) light, since there are a number of advantages of using this wavelength range.

- Possible to use Earthbound telescopes
- Includes many molecular absorption lines
- Thermal radiation from the planet itself is diminishingly low
- Mie scattering can be used for scattering from aerosols and Rayleigh scattering can be used for scattering from molecules
The incoming light is assumed to be unpolarized, which is a reasonable assumption since main sequence stars generally have a very low degree of polarization, as was explained above.

The planet and its atmosphere must be simplified in order for this project to be feasible. The atmosphere is divided into number of homogeneous layers, stacked on top of each other, building up a horizontally homogeneous and vertically inhomogeneous atmosphere. The surface is considered to be homogeneous and rough enough to not produce any polarized reflections.

2 Previous work

Using polarized light to increase the contrast between an exoplanet and its host star is not a new concept. Saar and Seager (2003) discussed the advantages of observing exoplanets in linearly polarized light and concludes that the contrasts between the exoplanet and its star can be significantly enhanced.

This method has been used in the past for observations of gaseous giants. Berdyugina et al. (2008) made the first detection of polarized light scattered from an exoplanet and several other observations have been made since then.

An important part in analysing observations in linearly polarized light is modelling the reflected light. Detailed modelling of reflected light from Jupiter-like exoplanets were made by Stam, Hovenier & Waters (2004) and Stam (2008) then proceeded to model Earth-like planets. Zugger et al. (2010) modelled ocean and rocky planets. As expected, all results show that one can expect high degree of polarization in the reflected light, especially when the planet is close to quadrature.

Improvements on the previous studies include higher spectral resolution and better accuracy in vertical composition, temperature and pressure profiles of an Earth-like atmosphere, as well as presenting contributions on the reflected light from different points on the planetary surface.
3 Method - Theory

3.1 Scattering matrix

In order to fully include polarization in the computations, the intensity will have to be replaced by a vector of the Stokes parameters.

\[ I = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} \tag{1} \]

The scattering from small particles will be expressed by a 4 by 4 matrix, which multiplied with the incoming intensity and the single scattering albedo gives the reflected light’s intensity and polarization at reflection angle \( \theta \).

\[ I_{\text{sca}} = a F(\theta) I_{\text{in}} \tag{2} \]

Where \( F(\theta) \) is the scattering matrix and \( a \) is the albedo. \( F(\theta) \) is given in the following form.

\[ F(\theta) = \begin{bmatrix} a_1(\theta) & b_1(\theta) & 0 & 0 \\ b_1(\theta) & a_2(\theta) & 0 & 0 \\ 0 & 0 & a_3(\theta) & b_2(\theta) \\ 0 & 0 & -b_2(\theta) & a_4(\theta) \end{bmatrix} \tag{3} \]

one of the goals of this modelling is to estimate the degree of polarization of the reflected light. The degree of polarization is defined as.

\[ P = \frac{\sqrt{Q^2 + U^2 + V^2}}{I} \tag{4} \]

The first step in the modelling of reflected light is to create scattering matrices for small particles in the atmosphere. For molecules (denoted by the prefix \( m \)), Rayleigh scattering was used and for aerosols (denoted by the prefix \( a \)), Mie scattering was used.

3.2 Rayleigh scattering

For scattering by atmospheric molecules, the scattering matrix for anisotropic Rayleigh scattering, as given by Hansen & Travis (1974), were used.
\[ F^m(\theta, \delta) = \Delta \begin{bmatrix}
\frac{3}{4}(1 + \cos^2 \theta) & -\frac{3}{4}(1 + \sin^2 \theta) & 0 & 0 \\
-\frac{3}{4}(1 + \sin^2 \theta) & \frac{3}{4}(1 + \cos^2 \theta) & 0 & 0 \\
0 & 0 & \frac{3}{2} \cos \theta & 0 \\
0 & 0 & 0 & \Delta' \frac{3}{2} \cos \theta
\end{bmatrix} + (1 + \Delta) \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \]

Where \( \Delta \) and \( \Delta' \) is given by.

\[ \Delta = \frac{1 - \delta}{1 - \delta/2} \]  
\[ \Delta' = \frac{1 - 2\delta}{1 - \delta} \]

\( \delta \) is the depolarization factor, which is a term that is introduced in order to account for the random orientation and anisotropy of the scattering particles. The depolarization factor for a specific molecule is a constant.

### 3.3 Mie scattering

For scattering by aerosols, Mie scattering were used. The computation of the scattering matrices were given by Hansen & Travis (1974).

\[ f^a(\theta, r) = \frac{1}{2} \begin{bmatrix}
S_1 S_1^* + S_2 S_2^* & S_1 S_1^* - S_2 S_2^* & 0 & 0 \\
S_1 S_1^* - S_2 S_2^* & S_1 S_1^* + S_2 S_2^* & 0 & 0 \\
0 & 0 & S_1 S_1^* - S_2 S_2^* & i(S_1 S_1^* - S_2 S_2^*) \\
0 & 0 & -i(S_1 S_1^* - S_2 S_2^*) & S_1 S_1^* - S_2 S_2^*
\end{bmatrix} \]

Here, the computation is not as straight forward as for Rayleigh scattering. For spherical particles the expression for \( S_1 \) and \( S_2 \) are given by.

\[ S_1 = \sum_{n=1}^{\infty} \frac{2n + 1}{n(n+1)} (a_n \pi_n + b_n \tau_n) \]  
\[ S_2 = \sum_{n=1}^{\infty} \frac{2n + 1}{n(n+1)} (b_n \pi_n + a_n \tau_n) \]
The functions $\pi_n$ and $\tau_n$ are functions of only the scattering angle and are given by the Legendre polynomials recurrence relations with the following starting conditions, given by Hansen & Travis (1974).

$$
\begin{align*}
\pi_1(\cos \theta) &= 1 \\
\pi_2(\cos \theta) &= 3 \cos \theta \\
\tau_1(\cos \theta) &= \cos \theta \\
\tau_2(\cos \theta) &= 3 \cos 2\theta
\end{align*}
$$

The recurrence relation is.

$$(n + 1) \cdot f_{n+1}(x) = x(2n + 1) \cdot f_n(x) - x \cdot f_{n-1}(x)$$

The coefficients $a_n$ and $b_n$ in equation (9) and (10) are function of the size parameters. The size parameters are:

$$x = \frac{2\pi r}{\lambda}$$

$$y = mx$$

Where $r$ is the radius of a spherical aerosol, $\lambda$ is the wavelength of the light and $m$ is the complex refraction index. The equations for calculating $a_n$ and $b_n$ were taken from Rooij & van der Stap (1984).

$$a_n = \frac{(D_n(y)/m + n/x)\psi_n(x) - \psi_{n-1}(x)}{(D_n(y)/m + n/x)\zeta_n(x) - \zeta_{n-1}(x)}$$

$$b_n = \frac{D_n(y) \cdot m + n/x\psi_n(x) - \psi_{n-1}(x)}{D_n(y) \cdot m + n/x\zeta_n(x) - \zeta_{n-1}(x)}$$

The functions $\psi_n$, $\zeta_n$ and $D_n$ are calculated by recurrence relations as given by Rooji & van der Stap (1983).

In practice the summation over infinity in the calculation of $S_1$ and $S_2$ was halted when

$$n > \frac{x - 1}{2}$$
since the recurrence relation for $\psi_n$ and $\zeta_n$ becomes unstable when $n$ becomes larger than this. As the size of the aerosols are larger the the wavelength of visual or NIR light, the summation will not be halted at a too early stage.

Once we have all the terms in the calculation of the scattering matrix, the matrix needs to be normalized. Here $i$ and $j$ are indices for individual elements in the scattering matrix, since the normalization must be made for each element separately.

$$F^{aij}(\theta, r) = c f^{aij}(\theta, r)$$  \hspace{2cm} (17)

$$c = \frac{k^2 \sigma_{sca}}{4\pi}$$  \hspace{2cm} (18)

Where $k$ is the wave number and $\sigma_{sca}$ the scattering cross section, which is calculated from $a_n$ and $b_n$ as given by Hansen & Travis (1974).

$$\sigma_{sca}(r) = 2\pi r^2 \sum_{n=1}^{\infty} \left[ (2n + 1) \cdot \left( a_n a_n^* + b_n b_n^* \right) \right]$$  \hspace{2cm} (19)

### 3.4 Atmospheric layers

The next step is to divide the atmosphere into layers, and create scattering matrices, the single scattering albedo and the optical thickness for each layer. To combine the Rayleigh and Mie scattering matrices into a combined scattering matrix for an atmospheric layer as well as calculating the optical thickness and single scattering albedo, the method described by Stam, de Haan, Hovenier & Stammes (2000) was used. Here capital $Z$ denotes the index of the layer.

$$F(\theta, Z) = \frac{\sum_k b_{sca}^{m,k}(Z) \cdot F_{m,k}^{*}(\theta, \delta(k)) + \sum_l b_{sca}^{a,l}(Z) \cdot F_{a,l}^{*}(\theta, x, y)}{\sum_k b_{sca}^{m,k}(Z) + \sum_k b_{sca}^{a,l}(Z)}$$  \hspace{2cm} (20)

The summation over $k$ and $l$ are summation of different molecular species and different sizes of aerosols. Single scattering optical thickness for molecules ($b_{sca}^m$) are given by:

$$b_{sca}^m(Z) = \eta(Z) \frac{24\pi^3 N_{av}(n^2(\lambda) - 1)^2 (6 + 3\delta)}{\lambda^4 N_R^2 R(n^2(\lambda) - 2)^2 (6 + 7\delta)} \int_{Z_i}^{Z_j} \frac{p(z)}{T(z)} dz$$  \hspace{2cm} (21)
$N_{av}$ is Avagado’s constant, $N_L$ is Loschmidt’s constant, $R$ is the gas constant, $n$ is the refraction index, $\delta$ is the depolarization factor, $z$ is the altitude, $p(z)$ is the pressure as a function of altitude, $T(z)$ is temperature as a function of altitude. To incorporate the number density of different molecules, the expression is multiplied with the volume mixing ratio, $\eta(Z)$, of the molecular species (assumed to be constant though the layer). The integration over altitude was done assuming exponential decrease in $P/T$ through the layer, with $P(z_i)$, $T(z_i)$, $P(z_j)$, and $T(z_j)$ corresponding to the values at the top and the bottom of the layer.

The single scattering optical thickness for aerosols ($b_{sca}^a$) is given by:

$$b_{sca}^a(Z) = N(r, Z) \cdot (z_i - z_j) \cdot \sigma_{sca}(r)$$  \hspace{1cm} (22)$$

Here $N(r, Z)$ is the number density of particles with radius $r$ in layer $Z$, which is assumed to be constant through the layer.

The total optical thickness of a layer is given by.

$$b(Z) = \sum_k b_{sca}^{m,k}(Z) + \sum_k b_{abs}^{m,k}(Z) + \sum_l b_{sca}^{a,l}(Z) + \sum_l b_{abs}^{a,l}(Z)$$  \hspace{1cm} (23)$$

The molecular absorption optical thickness is given by.

$$b_{abs}^m(Z) = \eta(Z) \frac{N_{av}\sigma_{abs}^m(Z)}{R} \int_{Z_i}^{Z_j} \frac{p(z)}{T(z)} dz$$  \hspace{1cm} (24)$$

The integral is again evaluated assuming exponential decrease through the layer. The molecular absorption cross section $\sigma_{abs}$ are assumed to be constant through the layer.

The aerosols in the atmosphere of the planet is assumed to consist of only spherical droplets, which results in essentially no absorption.

### 3.5 Radiative transfer - The adding method

The next step is to combine the scattering matrices for the atmospheric layers into a single unified scattering matrix for the entire atmosphere. To do this, the adding method, as described by de Haan, Bosma & Hovenier (1987)
was used. The basic idea is to divide the atmosphere into homogeneous, plane-parallel layers and define the reflection and transmission matrices for each layer. The reflection matrix describes the light reflected back towards the direction of propagation and the transmission matrix describes incoming light that is scattered towards the layer below. With the reflection and transmission matrices for two immediate layers, one can combine them into a single reflection and a single transmission matrix describing the scattered light for the combined layer. This combined layer now defines a new layer, and the layer below this (a third layer) is added to this combined layer. The same process is repeated until all layers have been included into a combined layer, a single reflection and transmission matrix for the entire atmosphere.

Angles in the atmosphere are defined as.

$$\mu = |u| = |\cos \Theta|$$  \hspace{1cm} (25)

Where $\Theta$ is the angle between the direction of propagation of the light and the local zenith. $\mu$ for the outgoing light and $\mu_0$ for incoming light. The azimuth angle $\varphi$ (outgoing) and $\varphi_0$ (incoming) are measured from an arbitrary plane containing the local zenith and counter clockwise when looking downward. Because of rotational symmetry only the difference in azimuth angles are relevant.

The scattering matrices were previously defined with respect to the plane of scattering, but since there are many planes of scattering in a multiple scattering problem, another plane of reference is needed. This reference plane is chosen to be the local meridian, and hence the scattering matrix needs to be rotated.

$$Z(\mu_0, \mu, \varphi_0 - \varphi) = L(\pi - i_2)F(\theta)L(i_1)$$  \hspace{1cm} (26)

Where $L$ is a rotational matrix and the relation between the angles $i_1, i_2, \theta$, $\mu, \mu_0, \varphi$ and $\varphi_0$ are given by Hovenier (1969) and Hovenier (1971).

When multiple scattering is neglected and no internal light sources are present the radiative transfer in the atmosphere is calculated as given below in equation (27) and equation (28). It is reasonable to neglect internal light sources as we using visual and NIR light. Multiple scattering will be reintroduced at
a later stage.

\[
I_{\text{down}}(\tau, \mu, \varphi) = \left( \frac{1}{\pi} \int_0^1 \mu_0 d\mu_0 \int_0^{2\pi} d\varphi_0 \cdot \left[ \frac{1}{4\mu\mu_0} \int_0^\tau d\tau' \cdot e^{\frac{\mu_0}{\mu} \cdot \frac{a(\tau')}{\tau'}} \cdot Z(\tau', \mu_0, \mu, \varphi_0 - \varphi) \right] + e^{\frac{\mu}{\pi}} \right) \cdot I_0(0, \mu_0, \varphi_0)
\]

(27)

\[
I_{\text{up}}(\tau, -\mu, \varphi) = \left( \frac{1}{\pi} \int_0^1 \mu_0 d\mu_0 \int_0^{2\pi} d\varphi_0 \cdot \left[ \frac{1}{4\mu\mu_0} \int_0^b d\tau' \cdot e^{\frac{\mu_0}{\mu} \cdot \frac{a(\tau')}{\tau'}} \cdot Z(\tau', \mu_0, \mu, \varphi_0 - \varphi) \right] \cdot I_0(0, \mu_0, \varphi_0) \right)
\]

(28)

Here \( I_{\text{down}} \) is the intensity of light moving downwards at optical depth \( \tau \) and at an angle given by \( \mu \) and \( \varphi \), and likewise for \( I_{\text{up}} \). The integration over \( \tau' \) is to account for reflections along the depth of the layer adding to the intensity and the exponentials gives the attenuation when the light moves through the atmosphere at the angle given by \( \mu \) (before scattering) or \( \mu_0 \) (after scattering).

The integration within the square brackets can be evaluated analytically for homogeneous atmospheres or homogeneous layers, where \( b \) is the total optical thickness of the layer. We call these matrices reflection matrices, \( R \), for light being scattered 0 - 90° from the upwards vertical and transmission matrices, \( T \), for light being scattered 90 - 180°.

\[
R(\mu_0, \mu, \varphi_0 - \varphi) = \frac{a}{4(\mu + \mu_0)} \left( 1 + e^{\frac{\mu}{\mu_0} \cdot \frac{a}{\mu_0}} \right) \times Z(b, \mu_0, \mu, \varphi_0 - \varphi)
\]

(29)

If \( \mu \neq \mu_0 \).

\[
T(\mu_0, \mu, \varphi_0 - \varphi) = \frac{a}{4(\mu - \mu_0)} \left( e^{\frac{-\mu}{\mu_0} \cdot \frac{a}{\mu_0}} \right) \times Z(b, \mu_0, \mu, \varphi_0 - \varphi)
\]

(30)

If \( \mu = \mu_0 \).

\[
T(\mu_0, \mu, \varphi_0 - \varphi) = \frac{a}{4\mu^2} \cdot e^{\frac{-\mu}{\mu_0}} \times Z(b, \mu_0, \mu, \varphi_0 - \varphi)
\]

(31)
For light incident on the lower boundary of the layer, the reflection and transmission matrices are denoted by $\ast$ and can be calculated from symmetry relations as given by Hovenier (1969) since we assume homogeneous layers.

\[
R^\ast(\mu_0, \mu, \varphi_0 - \varphi) = R(\mu_0, \mu, \varphi - \varphi_0) \\
T^\ast(\mu_0, \mu, \varphi_0 - \varphi) = T(\mu_0, \mu, \varphi - \varphi_0)
\] (32)

These reflection and transmittance matrices are used in the adding method. First, the reflection and transmission matrices of two layers are combined into a single reflection and a single transmission matrix describing the scattered light from the combined layer. In the following section, the subscript $a$ refers to the upper layer, the subscript $b$ refers to the lower layer and no subscript refers to the combined layer.

For the adding equations a shorter way of writing the matrices and integrations will be used. All matrices are written without the angle dependency, $X$ is an arbitrary scattering matrix.

\[
X = X(\mu_0, \mu, \varphi_0 - \varphi)
\] (33)
Each multiplication of two matrices includes an integration over the adjoining angles. X and Y are two arbitrary scattering matrices.

\[
XY = \frac{1}{\pi} \int_0^1 \mu' d\mu' \int_0^{2\pi} d\varphi' \cdot X(\mu_0, \mu', \varphi_0 - \varphi')Y(\mu', \mu, \varphi' - \varphi)
\] (34)

\[R\] and \[T\] are defined as the matrix that when multiplied with the incoming light gives the outgoing light. From figure 2 one can now derive the adding equations.

\[
R_{up} = RI_0 = RaI_0 + e^{-\frac{a}{\rho_0}}R^b e^{-\frac{h_a}{\mu}}I_0 + e^{-\frac{h_a}{\rho_0}}R^b T^{as}I_0 + T^a R^b e^{-\frac{h_a}{\mu}}I_0 + T^a R^b T^{as}I_0 + \\
+ e^{-\frac{h_a}{\rho_0}}R^b R^{as} R^b e^{-\frac{h_a}{\mu}}I_0 + e^{-\frac{h_a}{\rho_0}}R^b R^{as} R^b T^{as}I_0 + T^a R^b R^{as} R^b e^{-\frac{h_a}{\mu}}I_0 + ... \\
\] (35)

\[
I_{down} = TI_0 = T^a T^b I_0 + e^{-\frac{h_a}{\rho_0}}T^b I_0 + T^a e^{-\frac{h_b}{\rho}}I_0 + e^{-\frac{h_b}{\rho_0}}R^a R^{as} e^{-\frac{h_b}{\mu}}I_0 + \\
+ e^{-\frac{h_b}{\rho_0}}R^b R^{as} T^b I_0 + T^a R^b R^{as} e^{-\frac{h_b}{\mu}}I_0 + T^a R^b R^{as} T^b I_0 + ... \\
\] (36)

Each time the light is scattered only a fraction of the light is scattered, and as the light is scattered more and more, the optical path and therefore the attenuation will increase, removing most of the light. The light scattered many times make little contribution to the total reflected light, and the iteration is therefore stopped at four scatterings between two immediate layers. With the terms that include more than 4 scatterings removed and the factor \(I_0\) omitted in each term the equations can be written in a more easily handled form.

\[
R = Ra + e^{-\frac{h_a}{\rho_0}}R^b e^{-\frac{h_a}{\mu}} + e^{-\frac{h_a}{\rho_0}}R^b T^{as} + T^a R^b e^{-\frac{h_a}{\mu}} + T^a R^b T^{as} + \\
+ e^{-\frac{h_a}{\rho_0}}R^b R^{as} R^b e^{-\frac{h_a}{\mu}} + e^{-\frac{h_a}{\rho_0}}R^b R^{as} R^b T^{as} + T^a R^b R^{as} R^b e^{-\frac{h_a}{\mu}} \\
\] (37)

\[
T = T^a T^b + e^{-\frac{h_a}{\rho_0}}T^b + T^a e^{-\frac{h_b}{\rho}} + e^{-\frac{h_b}{\rho_0}}R^a R^{as} e^{-\frac{h_b}{\mu}} + \\
+ e^{-\frac{h_b}{\rho_0}}R^b R^{as} T^b + T^a R^b R^{as} e^{-\frac{h_b}{\mu}} + T^a R^b R^{as} T^b \\
\] (38)

These equations gives the reflection and transmission matrices for the combined layer. The next step is to use this combined layer as the upper layer, layer a, and add a new layer to this, using the layer below as layer b. By
doing this through the entire atmosphere, adding new layers to the combined layer, we will eventually build up the entire atmosphere from all the sublayers the atmosphere was divided into.

The surface of the planet is included into the this reflection matrix in the same way as an atmospheric layer, and is added to the reflection matrix using the same adding equations. The reflection matrix of the surface is considered totally depolarizing, with all elements of the matrix equal to 0 with exception of the first element (usually called the phase function) which is equal to the (wavelength but not angle dependant) surface albedo of the planet, \( a_s \). This is a reasonable approximation for a rough rocky or plant filled surface, where the light will be spread evenly in all directions. There is no need for a transmission matrix for the surface, as no light will pass through it.

\[
\begin{bmatrix}
a_s(\lambda) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]  

(39)

### 3.6 Disk integration

For the disk integration, the reflection matrix from a vertical column of the atmosphere, calculated with the adding method was multiplied with the incoming light. This yields the outgoing intensity, and since the intensity only has 4 elements (where the reflection matrix has 16 elements) the computational time for calculating the integrations, which has to be done separately for each element, can be reduced by a factor of four.

\[
I^{up}(\mu_0, \mu, \varphi_0 - \varphi) = R(\mu_0, \mu, \varphi_0 - \varphi) \cdot I(\mu_0, \varphi_0) \tag{40}
\]

Integration over the planetary disk was then performed in the following way.

\[
I^{\text{planet}}(\alpha) = \int_0^1 \mu' d\mu' \int_0^{2\pi} d\varphi' \cdot L(i_2 - \pi)I^{\text{out}}(\mu_0, \mu, \varphi_0 - \varphi) L(-i_1) \tag{41}
\]

Where \( \alpha \) is the phase angle of the planet (the angle star - planet - observer), which is related to the scattering angle as \( \pi - \theta = \alpha \). \( L \) is the same rotational matrices used earlier and is now used to rotate the Stokes parameters of the outgoing light back to the original plane of reference (the planet-star-observer plane). For values of the angle of the outgoing light to the local
vertical, $\mu < 0$, corresponding to the scattered light travelling through the surface of the planet to reach us, the outgoing light will be to 0.

If $\mu > 0$:

$$I_{\text{out}}(\mu_0, \mu, \varphi_0 - \varphi) = I_{\text{up}}(\mu_0, \mu, \varphi_0 - \varphi)$$  \hspace{1cm} (42)

If $\mu \leq 0$:

$$I_{\text{out}}(\mu_0, \mu, \varphi_0 - \varphi) = 0$$  \hspace{1cm} (43)

Once the integration over the entire illuminated surface has been made, and the part of the planet outside the observer’s line of sight was removed, the result will be the Stokes parameters of the reflected light from the entire planet.

4 Method - code

The choice of programming language was IDL (www.idl.com) partly because of its efficiency in handling large arrays. The code is divided into four main parts,

I Creation of scattering matrices for small particles

   a Molecules - Rayleigh scattering
   b Aerosols - Mie scattering

II Creation of reflection and transmission matrices for each atmospheric layer

III Radiative transfer in vertical atmospheric column

IV Integration over planetary disk

I Scattering matrices for small particles

The first step in calculating the reflected light is to calculate the reflection matrices for the individual particles in the atmosphere; atoms, molecules and aerosols. In section 3.2 and 3.3. the equations used for this are given. The Rayleigh scattering matrices are not wavelength dependent, at least not in a rather narrow wavelength band, so the computational time to calculate them are very short and was because of this not precalculated. For the Mie scattering, the computation is more demanding, but since only $\pi_n$ and $\tau_n$
are functions of the scattering angle, $a_n$ and $b_n$ could be precalculated for each aerosol size and when the Mie scattering matrix for a given angle was needed, the computational time could be greatly reduced by calculating the angle dependant part separately and then combining it with the precalculated $a_n$ and $b_n$ to create the full Mie scattering matrix.

II Reflection and transmission matrices

The biggest difficulty to overcome when writing the code was the radiative transfer, and how to do it in a practical way with a reasonable computational time. Calculating each reflection and transmission matrix for given angles during each step proved to be very time consuming and led to multiple calculations of the same matrices for the some combinations of angles. To overcome this, the reflection and transmission matrices for the two layers that were to be added together with the adding equations needed to be precalculated. To do this, most of the terms in the equations for calculating reflection and transmission matrices was precalculated (section and 3.4 and 3.5) as well as the single scattering albedo and optical depth. The scattering matrices and the rotational matrices were also precalculated and all these terms were then combined to create the reflection and transmission matrices for each layer for all combinations of angles and wavelengths.

III The adding method

In the adding method calculations, the precalculated reflection and transmission matrices for given angles and wavelengths were called for when needed in the adding equations.

For the first two layers that were to be combined, the code used the uppermost and next uppermost layer, and calculates $R$ and $T$ for all combinations of angles and combines them with the adding equations. The combined layer is calculated for all combinations of angles and this new combined layers is now used as the upper layer in the next iteration of the radiative transfer code. For the new layer to be added (the layer right below the combined layer) $R$ and $T$ were precalculated for all combinations of angles and was then added to the combined layer. This new layer, consisting of three combined layers, is then used in the next iteration as the upper layer. This process is repeated until all atmospheric layers have been added together and then finally the surface is also added in a similar fashion.

As there are many of precalculated matrices, and they are quite large, es-
especially the wavelength dependant ones, the wavelength grid was broken down into smaller fractions that were handled separately. These fractions were then recombined into a single spectrum after all the radiative transfer and disk integrations have been performed for each part separately. This made it possible to keep all matrices in the computer’s memory and therefore speed up the computations. This was also very convenient for reducing the wavelength resolution of the smooth functions. Any functions not including molecular absorption were considered smooth and is only calculated once for each part of the wavelength grid. In practice, if the resolution of the smooth functions was reduced by factor of 10 - 15 the effects on the end results were not noticeable.

IV Disk integration

Since we are considering planets at distances where we normally can not resolve a small part of the planetary disk, we need to integrate over the entire illuminated disk. To do this, the reflection matrix for vertical atmospheric columns is precalculated for all combinations of in and outgoing angles (with the adding method), the task in the disk integration is to use the correct precalculated matrices and combine them in the correct way. The integration is made over the entire illuminated half of the planet, i.e. with \( \mu_0 = [0, 1] \) and \( \varphi_0 - \varphi = [0, 2\pi] \). The phase angle \( \alpha \) of the planet is predetermined, and since the scattering angle is \( \theta = \pi - \alpha \), the angle of the outgoing light to the local vertical (\( \mu \)) is calculated for each point in the atmosphere. With \( \mu \) given, the reflection matrix with the corresponding \( \mu_0, \mu, \varphi_0 - \varphi \) could be used. For values of the angles between precalculated values, a linear interpolation between the immediate points was used, which is a small approximation since the matrices are smooth function of the scattering angle. Once the correct reflection matrix for a specified point in the atmosphere had been determined, it was multiplied with the incoming (unpolarized) light. For points in the atmosphere that for a given phase angle gave \( \mu < 0 \), the light needs to travel through the surface in order to reach us, and was therefore simply set to 0. This process was repeated for 5000 points on the illuminated part of the planet and the integrations were carried out over the angles.

5 Model planet

In order to test the code for a realistic case, the model planet was created as similar as possible to the Earth, which has the advantages that we know its atmosphere very well. The Earth is also an interesting case since it would
be very exiting to find an Earth-like exoplanet, and polarization signatures from exoplanets might help us find that.

However, the code has no build in assumption about the planet and its atmosphere, and changing the planet is as straight forward as changing the input parameters given below.

5.1 Data sources

For data on the atmosphere and surface of the Earth, input data were gathered from a couple of different sources. For the atmosphere temperature and pressure profile, as well as molecular volume mixing ratio, the US standard atmosphere 1976 was used (COESA 1976), see figure 3.

Molecular absorption data was taken from the HITRAN database (Rothman et. al 2009), and was calculated for 20 different altitudes.

For molecular scattering, the depolarization factor was taken from Penndorf (1957) and the index of refraction from Cox (1999).

For aerosol scattering, the size distribution equation was taken from Hansen & Travis (1974) and the parameters to the equation and number density (corresponding to typical Earth size distribution) was taken from Stam (2008). The complex index of refraction was taken from Irvine & Pollack (1968).

The surface albedo was taken from the ASTER spectral library. For this, a vegetation (grass) filled surface was chosen. See figure 4.
Figure 3: The temperature and pressure profile in the Earth atmosphere as function of altitude.

Figure 4: The surface albedo of grass as function of wavelength.
6 Results

In this section some results of calculations using the code will be presented. Two cases are examined, how the flux and polarization depends on the phase angle of the planet and on the different latitude points on the planet. When using points along the midday longitude ($\varphi_0 = 0$), the latitude corresponds to $\arccos \mu_0$ (if the planet has no axial tilt). The tests are however made a few degrees off the midday longitude.

In both section, the results will be presented as the total intensity (Stokes I), given as the fraction of the incoming light that is reflected. The polarization will be presented as the degree of polarization, i.e. the fraction of the light that is polarized, see equation 4. As there is no circular polarization, only the stokes Q and U will be included in the calculations and the results will be the degree of linear polarization. The polarization will also be presented as the intensity of the linearly polarized light. Because of the choice of reference system, the Stokes Q will be used for amplitude of linear polarization, which for positive values are the amplitude of light polarized in the star-planet-observer plane and perpendicular to this for negative values.

6.1 Phase angle

For the phase angle of the planet, three cases were examined, $\alpha = 45^\circ$, $\alpha = 90^\circ$ and $\alpha = 135^\circ$, see figure 5. For the intensity, we expect the light reflected when $\alpha = 45^\circ$ to be higher due to more of the illuminated part of the planet being visible and likewise we expect light reflected when $\alpha = 135^\circ$ to have a lower intensity due to less of the illuminated part being visible. The results are showed in figure 6, and shows the expected angular dependency, but also shows an interesting increase in the difference between the three cases in the NIR. This increase is due to the high albedo of the surface (see figure 4) coupled with the decreasing Rayleigh scattering in the NIR, and thus the amount of visible surface becomes increasingly important. In the visual part of the wavelength range, the surface albedo is lower and the Rayleigh scattering higher, and thus the intensity becomes more complicated as several terms are important.

The degree of polarization for the three cases are presented in figure 7 and the amplitude of linear polarization in figure 8. The degree of polarization is generally highest for $\alpha = 90^\circ$. Spectral features are visible in both the degree and amplitude of the polarization, and for both cases are more prominent in the NIR. The absorption band are however weaker in the degree of
Figure 5: A sketch of the three cases, phase angle $\alpha = 45^\circ$, $\alpha = 90^\circ$ and $\alpha = 135^\circ$. The star light illuminates half of the planet with nearly parallel beams. Part of this illuminated side reflects light towards the observer.

polarization and some of the weak bands are therefore hard to observe. In figure 7, the continuum is essentially Rayleigh scattering over surface albedo, since Rayleigh scattering is the main contributor to amplitude to linear polarization and the surface the main contributor to the unpolarized intensity (see equation 4). It is thus possible to say something about the surface of the planet, especially if one also have access to the amplitude of linear polarization.
Figure 6: The reflected intensity from the planet at three different phase angles.
Figure 7: The degree of polarization of the reflected light, given at three different phase angles.
Figure 8: The reflected polarized light perpendicular to the star-planet-Earth plane, given at three different phase angles.
6.2 Latitude

For these tests, the disk integration was left out in order to see how different points on the planet contribute to the polarization. The radiative transfer code calculates the reflected light for given angles of the incoming and outgoing light in a plane-parallel atmosphere. In order to calculate light reflected from a small part of a planet, the incoming light was set to a fixed value. If the radius of the planet is large compared to the height of the atmosphere, the contribution from light incoming at different angles is very small and the plane-parallel atmosphere model with light incoming in parallel beams at the top of the atmosphere is a reasonable approximation. See figure 9 for a visualisation of when the plane-parallel model is a good approximation and when it is not. Fortunately, the Earth has a very thin atmosphere, where the height of the atmosphere is only about 1% of the radius.

For the latitude tests, three cases were examined, with incoming light at 10°, 45° and 80°, see figure 10. These angles correspond to points on the planet at different latitudes, see figure 11. All cases are calculated for phase angle $\alpha = 90^\circ$.

Figure 9: To the left, in a thinner atmosphere, all light that is reflected comes from light incoming at the top of the atmosphere at approximately the same angle. The plane-parallel atmosphere model is therefore a good approximation. To the right, in a thicker atmosphere, contributions to the reflected light comes from a light incoming at the top of the atmosphere at different angles. In this case, the plane-parallel model is not a very good approximation. In the calculations, only the light reflected from one point is taken into account, showed by the darker arrow.
Figure 10: A sketch of the three examined angles of the incoming light. The light arrives in parallel beam at the top of the atmosphere, and is scattered towards the observer. Only contributions from light arriving at the given angle is taken into account. Note that only the light path of single scattering is considered here.
Figure 11: A sketch of the three examined points on the planet, close to the limb, $\mu_0 = \cos 10^\circ$, close to the terminator, $\mu_0 = \cos 80^\circ$ and in between $\mu_0 = \cos 45^\circ$. In all cases the phase angle of the planet is set to $\alpha = 90^\circ$.

The intensities of the reflected light from the different points are shown in figure 12. The effects of the surface on the reflected light can be seen in the different shapes of the spectra. When $\mu_0 = \cos 45^\circ$, light travels through less atmosphere, making the surface profile clearly visible in the continuum. For $\mu_0 = \cos 10^\circ$, the light that reaches the surface has a high intensity, but the light leaving the surface travelling towards the observer passes through more atmosphere, leading to more absorption and scattering, making the surface profile less clear. For $\mu_0 = \cos 80^\circ$, the reverse is happening, little light reaches the surface, but the light that reaches the surface and is reflected towards the observer then travels through little atmosphere. The light at $\mu_0 = \cos 45^\circ$ travels through less atmosphere overall, which also can be seen in the depths of the absorption bands, especially in the visual region.

The degree of polarization is showed in figure 13 and the amplitude of polarization is showed in figure 14. Light from $\mu_0 = \cos 45^\circ$ shows a small amplitude of linear polarization. This can be explained by the fact that the light overall travels through less atmosphere compared to the other cases. However, due to the low intensity in the visual part of the spectrum (figure 12), the degree of polarization is high in visual but low in the NIR, where the
intensity is high. The other two cases have higher amplitude of polarization, which is explained by the longer path through the atmosphere. Due to the higher intensity when $\mu_0 = \cos 10^\circ$ and $\mu_0 = \cos 80^\circ$, the degree of polarization is not much higher than when $\mu_0 = \cos 45^\circ$, and the difference between the continuum and the absorption bands is not as strong.
Figure 12: The reflected intensity from the planet at three different latitudes.
Figure 13: The degree of polarization of the reflected light, given at three different latitudes.
Figure 14: The reflected polarized light in the planet-star-Earth plane, given at three different latitudes.
7 Summary and conclusions

I have studied the theory and methods for modelling reflected light from planetary atmosphere and surface systems. From this, a code for modelling spectra of intensity and level of polarization of the reflected light from the illuminated and visible part of a planet have been written in IDL. For given input parameters, the code will calculate the wavelength dependent spectrum for each of the four Stokes parameters of the reflected light.

While Stokes V (the circular polarized light) always will be zero for reflected light (given unpolarized incoming light) the other three Stokes parameters, especially the light polarized perpendicular to the star-planet-observer plane, contain valuable information about the presence and properties of the atmosphere. With an overall degree of polarization of about 10%, the contrast between an exoplanet and its host star can be greatly increased by using linearly polarized light.

The results shows some preferable set-ups for observations. Observing exoplanets close to quadrature seems favourable. The amplitude of linear polarization (Stokes Q) and the degree of polarization both gives valuable information. The amplitude of linear polarization in visual wavelengths suits better for detecting molecular absorption bands and are therefore better for deriving the composition of the atmosphere. The continuum of the degree of polarization can be used to derive properties of the surface. The degree of polarization is essentially $Q/I$ (see equation 4), and the main contributor to the continuum of I is light reflected from the surface, and the main contributor to the continuum of Q is Rayleigh scattering. The degree of polarization might therefore be usable for observing features of the surface, while simultaneously be used for detection of most of the molecular absorption bands.

Visual light shows an overall higher degree of polarization than in the NIR, the continuum is about twice as large in visual compared to NIR. This is due to the depolarizing surface being brighter in NIR and Rayleigh scattering, the most important scattering process in the atmosphere, being weaker in NIR. While this makes visual light preferable for detection of linearly polarized light, NIR contains many more molecular absorption bands. In these absorption bands, the degree of polarization is in many cases as high or even higher than the in the visual continuum.

The level of polarization varies greatly with latitude. The observations needed for detecting this difference are however not easy to obtain. If mea-
surements with sufficient S/N would be possible, one might be able to study latitude stratification in the atmosphere.

For future work, the code can easily be used to examine more types of planets, by simply changing input parameters to match a planet of the user’s choice. Another interesting test would be to add stellar light and noise to the reflected light, creating an simulated observation of a star - exoplanet system. By adding instrumental profiles and resolution of a particular instrument, one would be able to estimate the capabilities of the instrument in this regard. One could also use this to develop and test methodology for handling similar observations as well as further examining the ideal set-up for actual observations.
References


