ELECTRODYNAMICS OF
THE IONOSPHERE

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7. ELECTRODYNAMICS OF THE IONOSPHERE

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7.1. Introduction

This chapter discusses the electric fields and some of their effects in the ionosphere. An electric field will set the charged particles into motion, so the ionization will start to drift and an electric current will flow. The relations between field, drift velocities, and current are not simple since the mobilities of the charged particles are different in different directions. The electric fields in the E and F layers are closely coupled to each other and to the fields in the outer magnetosphere. This coupling is caused by the high conductivity along the geomagnetic field lines which link the various regions. Thus processes in the outer magnetosphere (Chs. 8 to 12) affects, and are affected by processes in the ionosphere.

We will investigate the effects in the ionosphere assuming for convenience that the electric field is the primary, given quantity. In steady state situations we cannot tell whether it is the field that causes the current to flow through the ionosphere or whether the electric field is the result of the current flowing through the resistive ionosphere. Our knowledge of the electric fields has until recently been based solely upon observations of what is called the effects of the fields.
7.2. A Generalized Ohm's Law

If an electric field $\mathbf{E}$ is applied to the ionosphere the electrons and ions will start to drift. Due to collisions with the neutral particles, and the influence of the earth's magnetic field $\mathbf{B}$, the electrons and ions will move with different velocities and in different directions. Thus an electric current flows. The relation between the electric field and the current density may be written

$$ i = g \ (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (7.2.1) $$

where the conductivity $g$ is a tensor. The conductivity is different in different directions and an electric field in one direction may give rise to a current component in another direction. We may refer to equation (7.2.1) as a "generalized Ohm's law". The expression $\mathbf{E} + \mathbf{v} \times \mathbf{B}$ is the electric field measured in an appropriate frame of reference. Note that the electric field $\mathbf{E}$ is different in different frames of reference while $\mathbf{E} + \mathbf{v} \times \mathbf{B}$ is an invariant (Ch. 8) equal to the electric field in the frame of reference where the velocity $\mathbf{v} = 0$. In ionospheric physics $\mathbf{v}$ is generally taken to be the velocity of the neutral gas, $\mathbf{v}_n$. Thus the conductivity tensor relates the current density to the electric field measured in a frame of reference moving with the neutral gas.

7.3. Motion of Ions and Electrons

The drift motion of ions and electrons in the ionosphere differs from the motion of particles in the outer magnetosphere (cf. Ch. 8) in that the drift due to gradients of the magnetic field can be neglected but collisions with neutral gas particles are important.

Following a derivation by RATCLIFF (1959), we will study the motion of an ion for which collisions with neutrals occur at an average rate of $\nu$ per second. After the collision the ion has a randomly distributed velocity or on the average a zero velocity relative to the neutrals. The collision frequencies used in this chapter are not the actual number of encounters per second, but rather an effective rate of momentum
transfer taking the mechanics of the collision into account. Between the collisions the motion of the ion of charge $e$ is affected only by the electric and magnetic fields. Thus it is determined by the equation of motion

$$ m \frac{dv}{dt} = e(E + v \times B) \quad (7.3.1) $$

where $v$ is the ion velocity relative to the neutral gas. We assume that the fields are homogeneous with the magnetic field $B$ directed along the $-z$ axis and the electric field having an $x$ and $z$ component. Upon integrating Eq. 7.3.1 for an ion which at time $t = 0$ is at rest we find that the velocity components at time $t$ are

$$ v_x = \frac{F_x}{B} \sin \omega t \quad (7.3.2) $$

$$ v_y = \frac{F_x}{B} (1 - \cos \omega t) \quad (7.3.3) $$

$$ v_z = \frac{eE_z}{m} t \quad (7.3.4) $$

where $\omega$ is the gyro frequency (see Ch. 9). Between collisions the ion moves in a cycloid orbit (see Fig. 7.1) with an average velocity of $\frac{E_x}{B}$ along the $y$ axis and undergoes a steady acceleration in the $z$ direction. The interval between collisions will be statistically distributed so the probability that the interval will be in the range $t$ to $t + dt$ is $ve^{-\nu t} dt$. Thus when collisions are taken into account, the mean or drift velocity components become

$$ v_x = \int_0^\infty \frac{E_x}{B} \sin \omega t \ ve^{-\nu t} dt = \frac{ve^2}{v^2 + \omega^2} \frac{E_x}{B} \quad (7.3.5) $$

$$ v_y = \int_0^\infty \frac{E_x}{B} (1 - \cos \omega t) \ ve^{-\nu t} dt = \frac{\omega^2}{v^2 + \omega^2} \frac{E_x}{B} \quad (7.3.6) $$

$$ v_z = \int_0^\infty \frac{eE_z}{m} \ ve^{-\nu t} dt = \frac{eE_z}{mv} \quad (7.3.7) $$

Collisions reduce the average velocity $v_y$, introduce an average
velocity \( v_x \), and gives a steady mean value for \( v_z \). The different motion of the ion when collisions occur is illustrated in Fig. 7.1 for a case when the collision frequency is constant and equal to the gyro frequency.

Relations similar to Eqs. 7.3.5 to 7.3.7 hold for electrons but with a different sign for the \( x \) and \( z \) components. Noting that in the derivation above the neutral gas was assumed to be at rest, we may write more generally

\[
\begin{align*}
v_i - v_n &= \frac{v_{i\perp}}{v_{i\perp}} \frac{E_{\perp} + v_n x B}{B} + \frac{\omega_i^2}{v_{i\perp}} \frac{(E_{\perp} + v_n x B) x B}{B^2} + \\
&+ \frac{e}{m_i v_{i\perp}} E_{\parallel} \quad (7.3.8)
\end{align*}
\]

\[
\begin{align*}
v_e - v_n &= -\frac{v_{e\perp}}{v_{e\perp}} \frac{E_{\perp} + v_n x B}{B} + \frac{\omega_e^2}{v_{e\perp}} \frac{(E_{\perp} + v_n x B) x B}{B^2} - \\
&- \frac{e}{m_e v_{e\perp}} E_{\parallel} \quad (7.3.9)
\end{align*}
\]

The indices \( \parallel \) and \( \perp \) refer to components parallel and perpendicular to the magnetic field.

The directions and magnitudes of the velocity components transverse to the magnetic field may be represented graphically as in Fig. 7.2. The directions are given by the angles

\[
\alpha_i = \arctan \frac{\omega_i}{v_{i\perp}} \quad (7.3.10)
\]

\[
\alpha_e = \arctan \frac{\omega_e}{v_{e\perp}} \quad (7.3.11)
\]

and the magnitudes are

\[
v_i = \sin \alpha_i \frac{E_x}{B} \quad (7.3.12)
\]

\[
v_e = \sin \alpha_e \frac{E_x}{B} \quad (7.3.13)
\]
The locus of the arrowheads is a circle of diameter \( E_x/B \).

If the collision frequency is small compared to the gyro frequency for both electrons and ions (at high altitudes) they both move with the \( \mathbf{E} \times \mathbf{B}/B^2 \) velocity. If, on the other hand, collisions are very frequent ions move parallel and electrons antiparallel to the electric field but the velocity becomes small as the collision frequency increases. Fig. 7.3 shows \( \nu_i \) and \( \nu_e \) for three different heights in the ionosphere.

The collision frequencies will be different for different kinds of particles and depend also on the gas in which the particle moves (see DALGARNO, 1961). For the E layer rough numerical values for the collision frequencies may be estimated from the expressions

\[
v_{en} = 1.5 \times 10^{-17} n_n T \quad \text{(7.3.14)}
\]

\[
v_{in} = 4.2 \times 10^{-16} n_n \quad \text{(7.3.15)}
\]

\[
v_{ei} = \left[ 59 + 1.82 \times \ln \left( T^2/n_e \right) \right] \times 10^{-6} n_e T^{-3/2} \quad \text{(7.3.16)}
\]

where \( n_e \) and \( n_n \) are the electron and neutral particle number densities per cubic meter and \( T \) the absolute temperature. The electron-ion collision frequency is given for later reference.

A sometimes used alternative way of deriving the expressions for the drift velocities is to start from the equations

\[
\frac{d v_e}{d t} = - n_e (E + v_e \times B) - n_e m_e v_{en} (v_e - v_n) - n_e m_e v_{ei} (v_e - v_i) \quad \text{(7.3.17)}
\]

\[
\frac{d v_i}{d t} = n_i (E + v_i \times B) - n_i m_i v_{in} (v_i - v_n) - n_e m_e v_{ei} (v_e - v_i) \quad \text{(7.3.18)}
\]

\[
\frac{d v_n}{d t} = - n_e m_e v_{en} (v_n - v_e) - n_i m_i v_{in} (v_n - v_i) \quad \text{(7.3.19)}
\]

These relations express the momentum balance for the three components: electrons, ions, and neutrals. The effects of colli-
sions is now represented as a frictional force. Gravity forces and pressure gradients are neglected here, but may be important in some ionospheric problems. In Eqs. 7.3.17 to 7.3.19 the velocities are the average macroscopic velocities of the three components of the plasma so the equations give no insight into the detailed motion of the individual particles.

To derive the expressions 7.3.8 and 7.3.9 for the drift velocities a quasi-stationary state is considered where \( \frac{dv_e}{dt} = \frac{dv_i}{dt} = 0 \) but \( \frac{dv_n}{dt} \neq 0 \).

### 7.4. Ionospheric Conductivities

The difference between the electron and ion velocities gives rise to an electric current. Since there must be charge neutrality the ion number density \( n_i \) is equal to the electron number density \( n_e \) (assuming that there is only singly charged positive ions), and the current density is

\[
i = n_e \varepsilon (v_i - v_e)
\]

Using Eqs. 7.3.8 and 7.3.9 or Fig. 7.2 an expression for the conductivity tensor is obtained.

\[
\begin{pmatrix}
\sigma_P & \sigma_H & 0 \\
-\sigma_H & \sigma_P & 0 \\
0 & 0 & \sigma_i
\end{pmatrix}
\begin{pmatrix}
E_x - v_{ny}B \\
E_y + v_{nx}B \\
E_z
\end{pmatrix}
\]

\[
= \sigma_P (E + v_n \times B) + \sigma_H B \times (E + v_n \times B)/B + \sigma_i E_{\parallel}
\]

The three conductivities \( \sigma_P \), \( \sigma_H \), and \( \sigma_i \) are called respectively the Pederson, Hall, and parallel conductivity. We find

\[
\sigma_P = \left[ \frac{v_en_e \omega_e}{v_e^2 + \omega_e^2} + \frac{v_in_i \omega_i}{v_i^2 + \omega_i^2} \right] \frac{n_e \varepsilon}{B}
\]

(7.4.3)
\[ \sigma_H = \left[ \frac{\omega_i^2}{v_{en} + \omega_e^2} - \frac{\omega_i^2}{v_{in} + \omega_i^2} \right] \frac{n_e e}{B} \]  \hspace{1cm} (7.4.4)

\[ \sigma_{\parallel} = \left[ \frac{1}{m_i v_{in}} + \frac{1}{m_e v_{en}} \right] n_e e^2 \] \hspace{1cm} (7.4.5)

The conductivity \( \sigma_{\parallel} \) is the one that would apply to all directions if there was no magnetic field present. As collisions with neutrals become more numerous, then \( \sigma_{\parallel} \) decreases. On the other hand \( \sigma_P \) and \( \sigma_H \) are different from zero just because collisions occur. Above 100 km \( \omega_e >> v_{en} \) and the electrons drift very nearly with the velocity \( \mathbf{E} \times \mathbf{B}/B^2 \) as they would do in the absence of any collisions. The ion-neutral collisions prevent the ions from attaining this velocity and gives rise to the current. At these heights \( \sigma_P \) and \( \sigma_H \) depend only on \( v_{in}/\omega_i \) and \( n_e \). In the expression for \( \sigma_{\parallel} \), on the other hand, the term containing \( v_{en} \) dominates over the other one. Along the field lines the electrons have a much greater mobility than the ions. At great heights the electron-ion collision frequency \( v_{ei} \) also becomes of importance for evaluating \( \sigma_{\parallel} \), but it may still be neglected in the expressions for \( \sigma_P \) and \( \sigma_H \). Neglecting \( 1/m_i v_{in} \) compared to \( 1/m_e v_{en} \), a more accurate expression for \( \sigma_{\parallel} \) would be

\[ \sigma_{\parallel} = \frac{n_e e^2}{m_e (v_{en} + v_{ei})} \] \hspace{1cm} (7.4.6)

Fig. 7.4 shows some typical curves for the variation of \( \sigma_P \), \( \sigma_H \), and \( \sigma_{\parallel} \) with altitude. Except for the very lowest part of the ionosphere the conductivity along the geomagnetic field lines, \( \sigma_{\parallel} \), is much higher than the perpendicular components, \( \sigma_P \) and \( \sigma_H \). Thus, electric field components \( E_{\parallel} \) along the geomagnetic field lines will generally be much smaller than the perpendicular field components \( E_{\perp} \). Often the geomagnetic field lines may be considered perfectly conducting. At high latitudes, where the geomagnetic field lines are nearly vertical, ionospheric layers at different heights are effectively coupled together, and the horizontal electric field becomes almost height-independent. One may then use the concept height-integrated conductivities.
\[ \Sigma_p = \int \sigma_p dh \quad (7.4.7) \]
\[ \Sigma_H = \int \sigma_H dh \quad (7.4.8) \]
and height-integrated current density (amperes/meter)

\[ \mathbf{I} = \int \mathbf{i} dh \quad (7.4.9) \]

Provided \( \nu_n \) is height-independent or negligible these are related by

\[ \mathbf{I} = \Sigma_p \mathbf{(E}_\perp + \nu_n \times \mathbf{B}) + \Sigma_H \mathbf{B} \times (\mathbf{E}_\perp + \nu_n \times \mathbf{B})/B \quad (7.4.10) \]

The relation is obtained upon integrating the horizontal component of Eq. 7.4.2. At low altitudes \( \sigma_H \) dominates over \( \sigma_p \) but in higher layers the reverse is true and \( \Sigma_p \) and \( \Sigma_H \) are in general of the same magnitude; about 20 mho at daytime and 1 mho at nighttime.

At high altitudes the conductivity \( \sigma_H \) given by Eq. 7.4.6 approaches the conductivity of a fully ionized gas which is approximately independent of the particle density and only dependent on the temperature which is almost constant. Thus \( \sigma_H \) becomes approximately constant.

Another conductivity sometimes used is the Cowling conductivity. It relates to the power dissipation or Joule heating due to the currents transverse to the magnetic field. Per unit volume this is

\[ P = \mathbf{i}_\perp \times (\mathbf{E}_\perp + \nu_n \times \mathbf{B}) = \sigma_p (\mathbf{E}_\perp + \nu_n \times \mathbf{B})^2 = \frac{\mathbf{i}_\perp^2}{\sigma_C} \quad (7.4.11) \]

where the Cowling conductivity

\[ \sigma_C = \sigma_p + \frac{\sigma_H^2}{\sigma_p} \quad (7.4.12) \]

7.5. Electric Field Spreading

The assumption, made in the previous section that the horizontal electric fields are height-independent holds true
for large-scale fields. A field of small spatial extent produced at one height spreads to other heights but is damped. We will consider this spreading in some detail; of special importance is to find the conditions under which the assumption of perfect coupling between different ionospheric layers can be used.

If we assume that the magnetic field is directed along the \(-z\) axis and that the electric field has only \(x\) and \(z\) components and the neutral gas is at rest the electric current density

\[
\mathbf{i} = \sigma_p E_x \mathbf{\hat{x}} - \sigma_H E_x \mathbf{\hat{y}} + \sigma_\parallel E_z \mathbf{\hat{z}} \tag{7.5.1}
\]

We consider a two dimensional problem with no variations in the \(y\) direction and assume that the conductivities are functions only of the vertical coordinate \(z\). The current must be divergence free so

\[
\text{div} \mathbf{i} = \sigma_p \frac{\partial E_x}{\partial x} + \frac{\partial}{\partial z} (\sigma_\parallel E_z) = 0 \tag{7.5.2}
\]

The Hall current term is found to give no contribution. The electric field components can be expressed in terms of the potential \(V\) defined by

\[
\mathbf{E} = -\text{grad} \, V \tag{7.5.3}
\]

Introducing this in Eq. 7.5.2 a differential equation for \(V\) is obtained

\[
\sigma_p \frac{\partial^2 V}{\partial x^2} + \frac{\partial}{\partial z} (\sigma_\parallel \frac{\partial V}{\partial z}) = 0 \tag{7.5.4}
\]

The anisotropy of the conductivity is taken into account by introducing a new scale for the vertical coordinate defined by

\[
d\xi = \sqrt{\frac{\sigma_p}{\sigma_\parallel}} \, dz \tag{7.5.5}
\]

Then Eq. 7.5.4 takes the form
\[ \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial \zeta^2} + \frac{1}{\sigma_m} \frac{d\sigma_m}{d\zeta} \frac{\partial V}{\partial \zeta} = 0 \] (7.5.6)

where

\[ \sigma_m = \sqrt{\sigma_i \sigma_p} \] (7.5.7)

To solve Eq. 7.5.6 analytically it is necessary to make some assumption regarding the variation of the mean conductivity \( \sigma_m \). Most simple is to assume that it varies exponentially with the height coordinate \( \zeta \). Looking at Fig. 7.4 we see that in the upper parts of the ionosphere, where \( \sigma_1 \) is constant, \( \sigma_p \) decays roughly exponentially, so \( \sigma_m \) may be approximated by an exponential function of \( \zeta \). Also for the atmosphere below 70 km, where the conductivity is isotropic and thus \( \zeta = z \), an assumption of exponential variation of \( \sigma_m \) is useful although \( \sigma_m \) now increases with height. For the rest of the ionosphere one way to obtain a solution is to divide it into several layers with different rates of exponential variation. The solution for the different layers are matched to each other so that \( V \) and \( E_z \) are continuous across the interfaces. By introducing in Eq. 7.5.6

\[ \sigma_m = \sigma_0 e^{-\zeta/\zeta_0} \] (7.5.8)

where \( \zeta_0 \) is the scale height for \( \sigma_m \), the final form of this equation is obtained

\[ \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial \zeta^2} - \frac{1}{\zeta_0} \frac{\partial V}{\partial \zeta} = 0 \] (7.5.9)

This equation is separable and has solutions of the form

\[ V_\lambda = (A_\lambda e^{-k_1 \zeta} + B_\lambda e^{-k_2 \zeta}) \cos \frac{2\pi x}{\lambda} \] (7.5.10)

where \( \lambda \) is the wavelength for the horizontal variations and \( k_1 \) and \( k_2 \) the two roots of a quadratic equation obtained upon inserting Eq. 7.5.10 into Eq. 7.5.9.

\[ k_{1,2} = -\frac{1}{2z_0} \pm \sqrt{\left(-\frac{1}{2z_0}\right)^2 + \left(\frac{2\pi}{\lambda}\right)^2} \] (7.5.11)
A general solution may be built up by adding Fourier components of the form given by Eq. 7.5.10. The electric field components are obtained by forming derivatives of \( V \) with respect to \( x \) and \( z \). The constants \( k_1 \) and \( k_2 \) determine how fast the Fourier components of the field are damped in the vertical direction. They are damped to \( 1/e \) on a distance (measured with the \( z \) coordinate) of \( 1/k_1^2 \). The damping constants depend on the spatial extent of the field \( \lambda \) and the scale height \( r_0 \) for \( \sigma_m \). The coefficients \( A_\lambda \) and \( B_\lambda \) of Eq. 7.5.10 are determined by the boundary conditions of the problem. Going downwards from a source at one level the first term of Eq. 7.5.10 represents a field increasing exponentially while the latter term represents a decaying field. For an ionosphere that extends infinitely in the \(-z\) direction we would then have to put \( A_\lambda = 0 \) to obtain a field that is finite everywhere. This would also be approximately correct for an ionosphere of finite extent provided the field is of such a small horizontal extent (small \( \lambda \)) that it decays to approximately zero before one leaves the ionosphere. The other constant \( B_\lambda \) is determined at the source level. For the spreading upwards of the field it is the first term that represents the decaying field.

The damping depends on the particular ionospheric model used and varies between day and night. Fig. 7.5 shows numerical results according to SPREITER and BRIGGS (1961) for summer noon at middle latitudes and sunspot maximum. The figure shows how a horizontal electric field produced by a source at 130 km is damped when it spreads upwards. A field of a horizontal scale of more than a few kilometers spreads to great heights without significant damping while a field of a horizontal scale of 1 km or less is drastically damped.

The good coupling along the field lines holds in general also in the magnetosphere. Thus for closed magnetic field lines the two hemispheres are coupled together, and the fields in the ionosphere are images of the magnetospheric fields. Processes that decrease the conductivity along the field lines have been suggested to be operative on the high-latitude field lines to the auroral ovals. Such processes would increase the
limiting scale for fields that are undamped, but fields extending over a large scale will still be transferred between the outer magnetosphere and the ionosphere.

Fields extending over more than 100 km will penetrate far down into the atmosphere below the ionosphere and may be measured at balloon altitudes (20-40 km) and may possibly even affect the (vertical) field at the surface of the earth.

7.6. Polarization Electric Fields

During polar magnetic substorms intense localized currents, known as auroral electrojets, flow from east to west along the midnight section of the auroral oval (cf. Ch. 12). Along the magnetic dip equator the Sq variations show a strong enhancement caused by an equatorial electrojet (cf. Ch. 3). Both phenomena may be ascribed to the effect of secondary polarization electric fields produced when the Hall current is prevented from flowing in the direction associated with the primary electric field.

Consider the following model (Fig. 7.6): In the region A, that is for |y|<a, there is a field aligned slab of ionospheric plasma with enhanced conductivity relative to the surrounding plasma C. For the auroral case this slab would be vertical and the enhanced conductivity caused by precipitating auroral electrons. For the equatorial case the slab A is horizontal and represents the most conductive part of the ionosphere.

A primary homogeneous electric field $E_x$ is applied in the x direction. $E_x$ is the same in both regions A and C since it must be continuous at the interfaces (curl $E=0$). This electric field will drive a Pedersen current in the x direction and a Hall current in the -y direction. The currents will be stronger in region A than in C due to the higher conductivity, but the normal (y) component of the current must be continuous at the interfaces (y=±a). Thus in region A a secondary polarization electric field $E_y$ is built up which gives rise to a Pedersen current in the +y direction that balances the excess of Hall current in the -y direction in this region. However, this secondary field also drives a
Hall current along the slab in the +x direction which adds to the Pedersen current of the primary field. The net result is that a more intense current flows in the +x direction within the slab. In the y direction the net current is only the one given by the primary field \( E_x \) and the Hall conductivity in region C. From Eq. 7.4.10 we find (neglecting the effect of \( v_n \))

\[
I^A_x = \Sigma_P^A E_x + \Sigma_H^A E_y \\
I^A_y = -\Sigma_H^A E_x + \Sigma_P^A E_y \\
I^C_y = -\Sigma_H^C E_x
\]

Equating \( I^A_y \) and \( I^C_y \) we can express \( E_y \) in terms of \( E_x \) and we find

\[
I^A_x = \left( \Sigma_P^A + \frac{\Sigma_H^A (\Sigma_H^A - \Sigma_H^C)}{\Sigma_P} \right) E_x
\]

We have used the height-integrated conductivities, which is appropriate for the auroral case with a vertical slab with a width of more than one kilometer. For the equatorial case the integration should be done along the almost horizontal magnetic field lines in the most conductive part of the ionosphere.

Eq. 7.6.4 shows that the effective conductivity along the slab is enhanced. In the limiting case of very high conductivity ratio between region A and C we find that the effective conductivity approaches \( \Sigma_P^A + (\Sigma_H^A)^2/\Sigma_P^A \). This is an expression of a similar structure as for the Cowling conductivity \( \sigma_C \) (Eq. 7.4.12), but it is not equal to the integral over height of \( \sigma_C \).

For the auroral case we assumed that the Pedersen current flowing in the y direction is the only current that tends to short-circuit the polarization field. That is, in the model no currents flow from the interfaces between regions A and C to the outer magnetosphere. We may thus have overestimated the intensity of the polarization field.
7.7. Drift Motion of Ionization Irregularities

Under the action of an electric field the ions and electrons drift with the velocities \( v_i \) and \( v_e \) which are given by Eqs. 7.3.12 and 7.3.13 and shown again in Fig. 7.7. The components of \( v_i \) and \( v_e \) are of equal magnitude in a direction forming the angle \( \gamma \) with the electric field vector. This common component is denoted \( v_d \) and called the neutral ionization drift velocity. It is obviously perpendicular to the electric current vector. Using Fig. 7.7 and Eq. 7.3.13 we find

\[
v_d = \sin \alpha_i \sin \alpha_e \frac{E_x}{B} \quad \text{(7.7.1)}
\]

\[
\gamma = \frac{\pi}{2} + \alpha_i - \alpha_e \quad \text{(7.7.2)}
\]

where \( \alpha_i \) and \( \alpha_e \) are given by Eqs. 7.3.10 and 7.3.11.

This drift of the uniformly distributed ionization is not readily detectable. The motion of an ionization irregularity (a region of enhanced or decreased ionization density) would be easier to trace. However, such a region moves in quite a different way, because the boundary of the irregularity is not defined by some particular group of particles but by a density gradient. The interface between two regions of different ionization density will propagate like a wave, similar to a shock front in gas dynamics, in which the particles at the front all the time are interchanged.

To understand the motion of irregularities we consider a plane interface between two regions A and C of different ionization density and conductivity, as shown in Fig. 7.8. The interface is assumed to be aligned with the geomagnetic field lines. For ions as well as electrons the flux through the interface must be continuous, thus

\[
n_i^A (v_i^A - V_\eta) = n_i^C (v_i^C - V_\eta) \quad \text{(7.7.3)}
\]

\[
n_e^A (v_e^A - V_\eta) = n_e^C (v_e^C - V_\eta) \quad \text{(7.7.4)}
\]

Here \( V \) is the propagation velocity of the interface, and \( \eta \)
denotes the normal component. All quantities are assumed to be height independent. For a more accurate treatment Eqs. 7.7.3 and 7.7.4 may be substituted by integrals over the height (assuming perfect coupling along the magnetic field lines).

Eliminating \( V_n \) between the two equations gives a relation between the particle velocities or electric fields in the two regions A and C. Thus, it is only for certain electric field configurations that a steady state solution is possible with no charges accumulating at the interface.

Since \( V \) depends linearly on the particle velocities which are proportional to the electric field, the velocity \( V \) may be determined by a superposition of two cases: one with only electric field components \( E_\xi \) parallel to the interface, and another with only perpendicular field components \( E_\eta \), see Fig. 7.6.

1. Since \( E_\xi \) must be continuous across the interface, the particle velocities must in the first case be the same in region A as in region C. Then Eqs. 7.7.3 and 7.7.4 gives

\[
V'_\eta = v_{i\eta} = v_{iy} \quad (7.7.5)
\]
\[
V''_\eta = v_{e\eta} = v_{ey} \quad (7.7.6)
\]

since the \( \eta \) components correspond to the \( y \) components of Fig. 7.7 and \( E_\xi = E_\chi \). Because these velocities are different the electric field cannot have only a \( \xi \) component or charges would accumulate at the interface giving rise to a secondary electric field.

2. In the second case with only electric field components perpendicular to the interface, \( E_\eta \) and the particle velocities are different in region A and C. However, the ions as well as the electrons move in the same direction in region A and C and they move with velocities which are related as the electric fields. Then upon solving Eqs. 7.7.3 and 7.7.4 for \( V_\eta \) we obtain
\[ \nu'_{\eta} = \nu_{\eta}^A = \lambda \nu_{ix} \quad (7.7.7) \]

\[ \nu''_{\eta} = \kappa \nu_{\eta}^A = \lambda \nu_{ex} \quad (7.7.8) \]

The \( \eta \) components now correspond to the \( x \) components of the velocities of Fig. 7.7. The constant \( \kappa \) is the same for ions and electrons and is determined by the ratio \( \frac{n_{i}^A}{n_{i}^C} = \frac{n_{e}^A}{n_{e}^C} \). The constant \( \lambda \) takes into account the fact that the electric field \( E_{\eta} \) may be of a different magnitude than the field \( E_{x} = E_{x} \) to which Fig. 7.7 applies. In general the velocities given by Eq. 7.7.7 and 7.7.8 are different so this particular solution is also physically unrealistic. (In case there were only perpendicular field components present we would have to require \( \kappa \) to be zero, which corresponds to \( n_{e}^A E_{\eta} = n_{e}^C E_{\eta} \).)

Let us now superpose case 1 and 2. Then

\[ \nu'_{\eta} = \nu_{iy} + \lambda \nu_{ix} \quad (7.7.9) \]

\[ \nu''_{\eta} = \nu_{ey} + \lambda \nu_{ex} \quad (7.7.10) \]

where the velocities \( \nu_{iy}, \nu_{ix}, \nu_{ey}, \) and \( \nu_{ex} \) are those of Fig. 7.7.

The constant \( \lambda \) should be chosen so that \( \nu'_{\eta} = \nu''_{\eta} \). To the \( y \) components of the velocities we should add terms proportional to the \( x \) components. In Fig. 7.9 this is done graphically. Using the angle \( \gamma \) of Fig. 7.7 and Eqs. 7.7.1 and 7.7.2 we find

\[ \nu_{\eta} = \frac{\nu_{d}}{\sin \gamma} = \frac{\sin \alpha_{i} \sin \alpha_{e}}{\cos(\alpha_{e} - \alpha_{i})} \frac{E_{y}}{B} = \]

\[ = \frac{1}{1 + \cot \alpha_{e} \cot \alpha_{i}} \frac{E_{y}}{B} \quad (7.7.11) \]

Using a frame of reference that is not at rest relative to the neutral gas and Eqs. 7.3.10 and 7.3.11 for \( \alpha_{i} \) and \( \alpha_{e} \) we get
\[ V_n - V_{nn} = \omega \frac{\omega_1}{\omega_1 + \omega_1 B B_n} \left( \frac{E_B}{B} - V_{nn} \right) \]  

(7.7.12)

In Eqs. 7.7.11 and 7.7.12 \( E_B \) is the tangential component of the electric field which is the same on both sides of the interface. The expressions seem to be independent of the change in ionization density at the interface; however, this affects the electric field and thus \( V_n \).

The coefficient in front of \( E_B / B - V_{nn} \) in Eq. 7.7.12 is close to unity at altitudes above 100 km. The coefficient is about 0.02 at 50 km, 0.97 at 100 km, and 0.9994 at 120 km. Above 100 km the terms of Eq. 7.7.12 containing \( V_{nn} \) then cancel and the velocity of the interface is determined only by the electric field, which generally is independent of height (cf. sec. 7.5). The velocity of the interface will then be the same at all heights above 100 km, and the previously made assumption of height independence is reasonable. The neutral wind may still affect the propagation velocity because in an ionization irregularity the wind may produce a polarization electric field (cf. sec. 7.6).

Only the normal component of the propagation velocity \( V \) has been used. This is the only component which is measurable and physically meaningful. For a slab of enhanced ionization such as the one shown in Fig. 7.6, the propagation velocity (above 100 km) is given by \( E_x / B \). In this case the velocity depends only on the primary electric field and not on the secondary polarization fields, but in general this will not be the case.

For a cylindrical field-aligned irregularity (see Exs. 10 and 11) the tangential component of the electric field \( E_\parallel \) will vary sinusoidally along the interface. Thus \( V_n \) varies in a way such that the cylinder will move bodily in one direction without changing its shape. This is the case because the internal electric field is uniform. The propagation velocity (above 100 km) is \( E_{\text{internal}} \times B / B^2 \). For a more general form of the irregularity the relations are more complex and the shape of the irregularity may change during the motion.
7.8. Electric Field Measurements

The first direct measurements of the ionospheric electric fields were made in the middle of the sixties. Earlier our knowledge of the fields was based on observed effects of the fields: motion of electrons and ions, electric currents, and drift of ionization irregularities. Some motions can be studied by radio and radar methods (see Ch. ) and the electric currents by their magnetic effects at ground level (see Ch. ). The drifts may be of the order of a few hundred meters per second corresponding to fields of some tens of millivolts per meter. The currents are more difficult to use since the geometry of the current system cannot be uniquely inferred from only ground measurements. However, fields of a few millivolts per meter may be typical for the Sq currents, while in the auroral electrojets the field may be of the order of 100 m volt/meter. Some drift motions of visual auroras may also be interpreted in terms of electric fields, and they indicate fields of the same order of magnitude.

The drift of artificially created ion clouds can be used for electric field measurements (HAERENDEL et al., 1967). Barium clouds are used because the sunlight both ionizes and excites the barium, so the ion cloud is visible in the sunset or sunrise. A neutral cloud is also created that can be used to study the wind in the neutral gas. The drift velocity of the cloud is given by Eq. 7.7.12. Since the cloud is injected at altitudes above 100 km it moves with the \( E \times B/B^2 \) velocity. If the cloud is injected at 200 km, say, the extra ionization may not increase the height-integrated conductivities much, and the electric field is essentially undisturbed. Since the diameter of the ion cloud is many kilometers we have a good coupling to the E-layer (sec. 7.5) where the transverse conductivities are highest. The development of polarization electric fields depend on the height-integrated conductivities and not on the local change in conductivity.

Barium ion cloud experiments have shown that the ionospheric electric fields are of the order of a few millivolts per meter at low latitudes, while at high latitudes they in-
crease during magnetic disturbances. In the region of the westward auroral electrojet the field is directed equatorward and may reach a magnitude of 50-100 m volt/m.

Direct measurements of the fields have been done using double probe systems. The potential difference between two spherical probes separated a few meters is measured (FAHLESON, 1967). If the probes are identical (geometrically and electrically) the disturbances in the potential distribution introduced by each probe will be equal, and the voltage difference between the probes is the same as in the undisturbed plasma. To obtain the field in the earth's frame of reference the measurements must be corrected for the induced $v \times B$ field, due to the rocket or satellite motion.

Double probe systems may also be used on balloons to measure the large-scale electric fields of ionospheric origin which spreads down to balloon altitudes (see sec. 7.5).

Fig. 7.10 shows results of electric field measurements ($E + v \times B$) on the polar orbiting satellite OGO-6 (MAYNARD and HEPPNER, 1970). At low latitudes the field is very smooth and derives almost exclusively from the induced $v \times B$ field. At a latitude of about 60° variations of the order of 10 m volt/m occur, and in the auroral regions (≈70°) the field is very irregular and of the order 100 m volt/m.
Exercises

1. Prove that the graphical construction of Fig. 7.2 and Eqs. 7.3.10 to 7.3.13 give the same result for \( v_i \) and \( v_e \) as Eqs. 7.3.8 and 7.3.9, and derive expressions for \( \sigma_p \) and \( \sigma_H \) from this figure.

   (Answer: \( \sigma_p = (\sin a_1 \cos a_1 + \sin a_c \cos a_c) n_e e/B \))

   \( \sigma_H = (\sin^2 a_c - \sin^2 a_1) n_e e/B \))

2. The Eqs. 7.4.3 and 7.4.4 for \( \sigma_p \) and \( \sigma_H \) are valid only for processes with a time scale in a certain range \( t_1 < T < t_2 \). What determines the limits \( t_1 \) and \( t_2 \)? Estimate numerical values for these for the altitude 120 km. What are the conductivities \( \sigma_p \) and \( \sigma_H \) in the limit \( T >> t_2 \)?

   (Answer: \( t_1 = \frac{1}{v_{in}} = 5 \times 10^{-3} \) sec, \( t_2 = \frac{n_i n_e}{\sigma_p B^2} = 10 \) hours = time to accelerate the neutral gas. \( \sigma_p = \sigma_H = 0 \) in the limit \( T >> t_2 \).)

3. Discuss the effect on \( g \) of choosing a different \( v \) in Eq. 7.2.1. In treatments of fully ionized gases this

\[ v = \frac{m_i v_i + m_e v_e}{m_i + m_e} \]

Show that by choosing a particular \( v \), \( g \) can be made isotropic, and equal to \( \sigma_H \).

4. At what altitude is \( \sigma_p = \sigma_H \)? (Answer: Close to the altitudes where \( \omega_i = v_{in} \) (120 km) and where \( \omega_e = v_{en} \) (70 km), cf. Fig. 7.3)

5. Evaluate \( \sigma_H \) at heights where collisions with neutrals are no longer important assuming a temperature of 1500°K.

   (Answer: \( \sigma_H = \frac{(e^2 T_{\text{3/2}} \times 10^6)}{m_e [59 + 1.82 ln T^3 / n_e]} = 30 \) mho/m for \( h > 300 \) km)

6. Find the distance over which a field of the wavelength 100 km will be damped to \( 1/e \) in an infinitely extended
atmosphere with parameters typical for the atmosphere below 70 km. Assume that the scale height for the conductivity is 8 km. (Answer: 7 km for spreading upwards, 40 km for spreading downwards)

7. Assume for the top of the ionosphere

\[ \sigma_p = 2 \times 10^{-10} e^{-\frac{z - 1000}{70}} \]

where \( z \) is the real height in kilometers, \( \sigma_p = 30 \) mho/m, and a source field at 1000 km \( E_x = E_o \cos \left( \frac{2 \pi x}{\lambda} \right) \) where \( \lambda = 100 \) meters. Find the relation between the \( \xi \) and \( z \) coordinates and the height where the downward spreading field has been damped by a factor \( e \), using the boundary condition \( E_x = 0 \) for \( z = -\infty \). Discuss also the spreading upwards. (Answer: 470 km. For \( z = +\infty \) \( \xi \) is finite. Assuming that at some great, but finite, height corresponding to the top of the field line \( E_z = 0 \) we find that \( E_x \) is not damped to \( 1/e \) before this point is reached.)

8. Consider the model of a polarized slab discussed in sec. 7.6. What would happen if we applied the primary electric field in the \( y \) direction in the model? What would be the effects of neutral winds blowing in the \( x \) and \( y \) direction? For the case of a primary field in the \( x \) direction discuss the current flow at various altitudes. Since \( \sigma_p \) and \( \sigma_H \) varies differently with altitude the Hall current produced by \( E_x \) will not be balanced by the Pedersen current produced by \( E_y \) at each altitude, but only in the integrated sense. Find an expression for the maximum current flow in the \( z \) direction at the boundaries and the altitude at which this maximum occurs assuming that \( \xi^A_H = \xi^A_P \) and \( \xi^C_H \) negligible.

(Answer: Maximum I at height \( h \) where \( \sigma_p^A = \sigma_H^A \); \( I_{\text{max}} = \int_h^0 E_x \left( \sigma_H^A - \sigma_P^A \right) dh \).)
9. Prove that \( \mathbf{i} \times \mathbf{B} = n_e \left[ m_i v_{in} (\mathbf{v}_i - \mathbf{v}_n) + m_e v_{en} (\mathbf{v}_e - \mathbf{v}_n) \right] = n_e (m_i v_{in} + m_e v_{en}) (\mathbf{v}_d - \mathbf{v}_n) \). The \( \mathbf{i} \times \mathbf{B} \) force is balanced by collisions with the neutral gas. This is the direction in which the neutral gas is being accelerated.

10. Derive an expression for the polarization field in a cylindrical field aligned region of enhanced conductivity. The applied external electric field \( \mathbf{E} \) is homogeneous. Assume that the height-integrated conductivities are \( \lambda \cdot \Sigma_P \) and \( \lambda \cdot \Sigma_H \) within the cylinder and \( \Sigma_P \) and \( \Sigma_H \) outside. What will the disturbed external electric field look like? (Answer: \( E_{\text{internal}} = \frac{2(\lambda+1) \Sigma_P^2 \mathbf{E} + 2(\lambda-1) \Sigma_P \Sigma_H \mathbf{E} \times \mathbf{B}/\mathbf{B}}{\lambda+1)^2 \Sigma_P^2 + (\lambda-1)^2 \Sigma_H^2} \))

where \( \mathbf{E} \) is the applied homogeneous field. \( E_{\text{external}} = E + \) a dipole field.)

11. Consider the cylindrical irregularity of Exercise 10 at a height \( >100 \) km. In which direction and with what velocity will the irregularity propagate if (a) \( \lambda \) is very close to unity (b) \( \lambda \) is very great. (Answer:

a) \( E_{\text{primary}} \times \mathbf{B}/\mathbf{B}^2 \)

b) In a direction in between \( E_{\text{primary}} \times \mathbf{B}/\mathbf{B}^2 \) and a vector antiparallel to \( E_{\text{primary}} \) at an angle of \( \arctan (\Sigma_P/\Sigma_H) \) to the latter direction. The velocity will be \( \frac{2 \Sigma_P}{\lambda \sqrt{\Sigma_P^2 + \Sigma_H^2}} E_{\text{primary}}/\mathbf{B} \).
References

The topics of this chapter are discussed in more detail in the following papers


Figure captions

Fig. 7.1. Motion of an ion in the absence of collisions (v=0) and when the collision and gyro frequencies are equal (v=ω). The orbit is shown for a case when there is no electric field component Ez along the magnetic field.

Fig. 7.2. Graphical construction used to depict the directions and magnitudes of the drift velocities v_i and v_e for ions and electrons due to an electric field E_x.

Fig. 7.3. The direction of the drift velocities and the electric current i relative to the electric field E at three different heights in the ionosphere.

Fig. 7.4. Typical variation of the ionospheric conductivities with height for a night-time ionosphere (BOSTRÖM, 1964).

Fig. 7.5. The damping of a horizontal electric field produced by a source at 130 km when the field spreads upward for various wavelengths (λ) of the horizontal variation (SPREITER and BRIGGS, 1961).

Fig. 7.6. Model of a polarized region.

Fig. 7.7. Graphical construction used to derive the neutral ionization drift velocity v_d.

Fig. 7.8. The geometry of the two cases used to derive the propagation velocity V_n of an interface between two regions A and C of different ionization density.

Fig. 7.9. Graphical construction used to derive the propagation velocity V_n of an ionization irregularity.

Fig. 7.10. OGO 6 DC electric field data showing a latitude profile of the north-south electric field component on a dawn to dusk pass across the southern auroral and polar cap regions (MAYNARD and HEPPNER, 1970).
Fig. 7.1
primary: 

\[ i_H \rightarrow i_P \rightarrow E_x \rightarrow \hat{y} \rightarrow C \]

\[ y = +\varepsilon \]

secondary:

\[ i_H \rightarrow i_P \rightarrow E_x \rightarrow \hat{x} \rightarrow E_y \rightarrow \hat{i}_P \rightarrow \hat{i}_H \]

\[ y = -a \]

Fig 7.6
Fig. 7.7

Fig. 7.8

case 1:  
case 2:
Fig. 7.9

Fig. 7.10