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ON RELATIVISTIC DUST GRAINS
AND EXTENSIVE AIR SHOWERS

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Abstract

The hypothesis of dust grain origin of giant extensive air showers has been reexamined. We find that the electron to muon size gives a more decisive test than a study of lateral distribution functions. From the expected muon size and estimated electron size we conclude that dust grains must be accelerated to energies greater than 2000 GeV/N. Considering conventional grains with a density $\rho \sim 2-3 \text{ g cm}^{-3}$ it is very unlikely that these could be accelerated within a confinement region of heliospheric dimensions.
Introduction

The present knowledge of dust grains in interstellar space has been reviewed by Wesson (1974). However, our knowledge of relativistic dust grains in transplanetary and interstellar space is very sparse. The hypothesis that cosmic dust accelerated to relativistic energies is a source of extensive air showers (EAS) was first proposed by Richtmyer and Teller (1949). Herlofson (1956) investigated the idea, studying the lateral distribution function LDF. He found that a dust grain origin of EAS seemed unlikely, although it could not be excluded. (See also Alfvén (1954), reference 6).

The acceleration mechanism developed by Alfvén and Åström (1958), Alfvén (1959) known as 'magnetic pumping' has been further analysed by Fälthammar (1963). However, in this work we shall not deal with the question of acceleration. If cosmic dust is accreted from embryos in transplanetary or interstellar space it should be accelerated in these regions (Alfvén and Arrhenius, preprint 1974). Hayakawa (1972) has reconsidered the dust grain origin of giant EAS with a total energy $W > 10^{19}$ eV proposing that those may be produced by dust grains with a Lorentz factor $\gamma > 10^3$. The acceleration mechanism and the flux of high energy dust grains is discussed. Suga (see Hayakawa, 1972) finds that the lateral distribution near the center of the shower is much flatter than that of a nucleon induced shower. This is also our experience for two reasons. 1) The dust grain is disrupted in the first gcm$^{-2}$ in the atmosphere (Herlofson, 1956), 2) the central core structure of a cascade initiated by 50-500 GeV primaries is relatively flat. (See also the concluding remarks.) From the estimates of Hayakawa it is argued that the electron to muon size $N_e/N_\mu \gtrsim 10$ if $E_o \approx 10^3$ GeV/N. According to our estimates, considering primaries with the same energy it is found that $N_e/N_\mu < 6$. Actually it is probable that the electron content as an order of magnitude estimate is 60% at $E_o = 1000$ GeV/N. This value will be highly modified if we consider grains with energies $\sim 10^4 - 10^5$ GeV/N. The Durham group (Turver, private
communication) finds \( N_e/N_\mu \approx 0.5 \) and Hook and Turver (1974) get \( N_e/N_\mu \approx 0.3 \).

In this work we calculate the muon size as a function of nucleon energy. We consider the nucleon-meson cascade in successive generations (Aström, 1975, 1977) and estimate the contribution to the muon size from \( \pi N \)-collisions (section 2). In the estimates we replace the Markov process by a Markov chain and consider a leading pion model (LP) — section 2.2 and a compound constant energy model (CCE) — section 2.3. We also base our estimates on a heuristic approach (section 2.1).

The electron content is estimated from the LP-model such as to give \( \pi^0 \)'s at \( \sim 700 \) gcm\(^{-2}\). Hence the cascade maximum is reached near sea level. In another case we consider an analogy using the work of Thompson (1974) on \( \gamma \)-rays at sea level and our results on the increase of \( \pi N \) collision contribution to sea level muons \( N_\mu \) as function of \( E_0 \). In both estimates we reach the same conclusion \( N_e/N_\mu \approx 0.6 \) for \( E_0 = 10^3 \) GeV/N.

1. Muons from nucleons

Consider a primary nucleon with energy \( E_0 \) (GeV) and a nucleon-meson cascade with a narrow opening angle. The number of muons \( N_1 (r) \) reaching sea level from the first generation (collision) with momentum \( > p_{SO} \) (GeV/c) is a given function of the generating parameter \( r \) (Aström, 1975). \( N_1 (1) \) is identically called the fundamental specific yield function \( S_1 (E_0', p_{SO}) \). Let \( n(p_s, E_0, r) \) be the number of muons per momentum interval having the momentum \( p_s \) at sea level, emitted by a single primary \( (E_0) \) in the first generation. This quantity is basically important for the evaluation of muon spectra, response functions of muon monitors and muon size of a shower (see also Aström, 1977). By definition

\[
N_1 (r) = \int_{p_{SO}}^{\infty} n(p_s, E_0, r) dp_s
\]

Direct calculations show

\[
N_1 (r) = N_1 (1) \cdot r^{-\alpha (E_0', p_{SO})}
\]
To a good approximation (typically within ±3%) we have \( a=a(E_o) \) for \( E_o > 10 \text{ GeV} \) and \( 0.2 \leq p_{so} \leq 2 \text{ GeV/c} \) (Figure 1). \( S_1(E_o, p_{so}) \) is given in Figure 2. Without further justification (Aström, 1977) we neglect the small changes in the results introduced by using an inelasticity distribution. For the energies of the \( i \)th and \( i+1 \)st generations \( E_i \) and \( E_{i+1} \), we get with the elasticity \( f \)

\[
E_{i+1} = fE_i \quad i \geq 0
\]  

(3)

As the Poisson distribution applies for the collision probabilities, suppressing \( p_{so} \) in the arguments, we have

\[
N_{i+1}(E_o', r) = \frac{1}{i!} \frac{\partial^n N_i(E_o', r)}{\partial x} \quad i \geq 1
\]

(4)

where \( N_i \) is the contribution to the sea level number of nucleons (muon size) of the \( i \)th generation (Aström, 1975, 1977). The total number of muons \( N_\mu(E_o) \) is

\[
N_\mu(E_o) = \sum_{i=1}^{n} N_i(E_o, 1)
\]

(5)

where \( n \) denotes the number of generations that are essential for the cascade. Here \( n \leq 6 \) for \( E_o \leq 1000 \text{ GeV/N} \). In calculations of muon spectra and coupling functions we could neglect \( \pi N \) collision contributions. We shall estimate the influence of such collisions in section 2. The absolute value of \( S_1(E_o, p_{so}) \equiv N_1(1) \) is derived from a normalisation of the calculated sea level muon spectrum at zenith to a value at 1 GeV/c that is 10\% above Rossi's standard value (Rossi, 1948, Crookes and Rastin, 1972). Using the recent primary spectrum including heavy particles (Erlykin et al., 1974) the normalisation factor was 1.0. The uncertainties (in theory and experiments) make this agreement fortuitous. In Figure 3 the number of muons in \( NN...N\mu \) collisions at sea level is given for different values of the elasticity in \( NN \)-collisions, \( f \), as function of \( E_o \). To first order \( N_\mu \propto f \). Hence, the normalised muon size is independent of the true value of \( f \). If we consider
a distributed elasticity, we can use appropriate averages of f and moments of f such that the calculations are correct to second order. For our purpose, first order results are sufficient.

2. Estimates of the contribution from πN collisions to the muon size

2.1. Heuristic estimate based on nucleon production of secondaries.

In this approach the following facts are essential:

1) The multiplicity of pions in πN-collisions is approximately the same as that of NN-collisions in the 50-500 GeV/N range as shown in ISR and high energy AGS experiments.

2) The hadrons (pions and nucleons) have similar interaction properties at high energies (≥ 50 GeV). The relation between the inelastic interaction lengths \( \lambda_{\text{NN}} \) and \( \lambda_{\pi N} \) (in gcm\(^{-2} \)) is roughly given by

\[
\lambda_{\pi N} \approx \frac{3}{2} \lambda_{\text{NN}}
\]

3) In the actual energy range \( E_0 \geq 50 \text{ GeV/N} \) we have \( 1 < \alpha < 1.3 \), Figure 1. From our theory we find that the calculated value \( N_\mu(E_0, \lambda) \) — cf equation (5) as function of the interaction length \( \lambda \) is slowly varying

\[
N_\mu(E_0, \lambda) \propto \lambda^\alpha(E_0)^{-1}
\]

4) The pions are unstable.

Consider the number of muons reaching sea level \( N_{\text{NN}} \) and \( N_{\pi N} \) in cascades initiated by NN and πN collisions, having the same initial energy E. From the arguments above we conclude

\[
N_{\pi N} \ll N_{\text{NN}}
\]

The approximation sign arises because \( M_{\pi N} \sim 1.1 M_{\text{NN}} \) (multiplicities) and \( \alpha(E_0) > 1 \).
Thus, the relation (8) is non-trivial. Considering point 4) above only one might think that it is obvious. In this context we may neglect production of kaons and other particles (except in the partitioning of energy).

It is reasonable to assume that the energy partition in NN-collisions is roughly 0.5:0.4:0.1, where the figures denote the part of energy going into a leading secondary nucleon, pions and other particles resp. (see also Brooke et al. 1964 a, b, Jabs 1972). We will here assume that there is a leading pion that carries on the average $\leq 0.3$ of the initial nucleon energy. The probability that the leading pion is charged is $\approx 2/3$. Thus 20% of the initial energy, averaged over many events, goes into a leading charged pion. Hence the estimate of the $\pi N$-contribution is

$$N_{\pi N} \leq N_{NN} \left( 0.2 \ E_0 \right)$$

(see Table 1).

2.3. Estimate from a simple leading pion (LP) model

Consider pion collisions in a Markov chain such that each collision occurs at the average distance between collisions $\lambda_{\pi N}$ (in gcm$^{-2}$). The NN-collision generating the first leading pion starts around $x \sim 100$ gcm$^{-2}$. We use the inelasticity $K_{\pi N}$ as a function of pion energy given by Brooke et al. (1964 b).

Let $n_{\pi} \frac{dE}{dx}$ be the number of colliding and $\pi_{d} \frac{dE}{dx}$ those decaying having energies between $(E, E+dE)$ in the depth interval $(x, x+dx)$ (gcm$^{-2}$). Let $p$ be the momentum, $\pi(E, \bar{x})$ the differential pion spectrum at the average level $\bar{x}$ and $B$ a constant $\approx 120$ GeV/c. $\bar{x}$ is given by the total number of collisions considered in a given chain. The interplay between pion decay and interaction is found from the transport equation of pions in the atmosphere. We get roughly

$$\pi \frac{C}{d} \approx \frac{R_{x}}{\lambda_{\pi N}} \cdot \frac{\pi(E/K_{\pi N}, \bar{x})}{\pi(E, \bar{x}) \cdot K_{\pi N}}$$
Assuming separability in depth and energy an approximate solution gives $\pi(E, x) \propto E^{-1}$. Equation (10) reduces to equation (12), see section 2.3 below.

From knowledge of the survival probability of muons we can now estimate the number of muons reaching sea level above a given energy, taking into account neutrino and muon energy losses. At $E_0 = 500$ GeV we also consider the low energy pion contribution. These "target" pions were given equal energies. The relative contribution to the number of muons at $E_0 = 50$ and 500 GeV are summarised in Table 1. It is interesting to note that at $E_0 = 500$ GeV the dominating contribution is distributed throughout the entire 100-1030 gcm$^{-2}$ layer, when the "leading" pion has collided up to $\approx 6$ times. Of course, to talk about a leading pion as if it were the same before and after a collision is only a loose way of speaking. However, the concept is useful and should not give rise to any misunderstanding. At $E_0 = 50$ GeV the contribution mentioned comes from decaying pions in the 50-200 gcm$^{-2}$ layer. Thus the physics of the LP-model changes rapidly in the atmosphere for $50 < E_0 < 500$ GeV. This model can be regarded as representing isobar production and subsequent decay.

2.3. Estimate from a compound constant energy (CCE) model

Consider a NN-collision producing a leading pion. Let this pion collide with a nucleon, producing $M$ pions with equal energy in the CM-frame. We assume here that the angular distribution in this frame is peaked, such that most of the pions are produced in the forward and backward cones with an opening angle of $\lesssim 90^\circ$. It can be shown that the forward pions carry all the significant energy. Furthermore the variation of energy of these pions in the LS-frame is $\lesssim 15\%$. Thus, for the purpose of order of magnitude estimates we may also use constant energy in the LS. As in section 2.2 we take into account the interplay between decay and interaction of pions, neutrino energy loss etc. However, we assume that the inelasticity $K_{\pi N}$ is constant. For special assumptions it can be shown that the pion spectrum produced by a single pion $\pi(E, x)$
has the property

$$\pi(E, x) = E^{-1} f(x)$$

(11)

Counting the forward pions (in the CMS) only, equation (10) reduces to

$$\frac{\pi_c}{\pi_d} \approx \frac{M}{2} \cdot \frac{p_x}{\lambda_{\pi NN} B}$$

(12)

We consider two cases $K_{\pi NN} = 0.5$ and 1.0 (see Table 1 and 2). For an energy $E = 100$ GeV of the leading pion we include an estimate from the contribution of target nucleons, carrying a total energy of 50 GeV (if $K_{\pi NN} = 0.5$). If only one target nucleon is knocked out, then we may from Figure 3 put the contribution $\sim 10\%$. This figure is slightly biased since we have used a value from a cascade that is on the average initiated around 80 gcm$^{-2}$ whereas in the actual case it should be initiated around $\sim 200$ gcm$^{-2}$. Thus the nucleon-meson cascade takes place in a more dense an atmosphere. If, however, 5 target nucleons with an energy of 10 GeV are produced by spallation, the contribution to the muon number is practically zero. As an estimate we take the average of these two extreme situations. In Table 1 the results from different considerations are summarised. Averaging the results from the LP and CCE models as being a more realistic situation we find that the $\pi NN$ contribution is $\sim 2\%$ at $E_0 = 50$ GeV and $\sim 16\%$ at 500 GeV. Extrapolation to $E_0 = 1000$ GeV on a logarithmic scale gives a $\pi NN$ contribution of $\sim 30\%$. The muon size per nucleon is $\sim 21$ (Figure 4), which can be directly compared with the results of the Durham group (Turver, private communication). It may be of interest to note that in the CCE model the dominating contribution to the sea level muon number comes from the 50-300 gcm$^{-2}$ layer both at $E_0 = 50$ and 500 GeV. Thus the physics of the LP and CCE models differ considerably for 500 GeV nucleons. The CCE model may be regarded as a representation (very rough) of the pionisation process.
Thus, the relation (8) is non-trivial. Considering point 4) above only one might think that it is obvious. In this context we may neglect production of kaons and other particles (except in the partitioning of energy).

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$$N_{\pi N} \approx N_{NN} \left(0.2 \ E_0\right)$$ (9)

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Let $\pi_c \, dE dx$ be the number of colliding and $\pi_d \, dE dx$ those decaying having energies between $(E, E+dE)$ in the depth interval $(x, x+dx)$ - (gcm$^{-2}$). Let $p$ be the momentum, $\pi(E, \bar{x})$ the differential pion spectrum at the average level $\bar{x}$ and $B$ a constant $\approx 120$ GeV/c. $\bar{x}$ is given by the total number of collisions considered in a given chain. The interplay between pion decay and interaction is found from the transport equation of pions in the atmosphere. We get roughly

$$\frac{\pi_c}{\pi_d} \approx \frac{\partial x}{\lambda_{\pi N} \cdot B} \cdot \frac{\pi(E/K_{\pi N}, \bar{x})}{\pi(E, \bar{x}) \cdot K_{\pi N}}$$ (10)
Assuming separability in depth and energy an approximate solution gives $\pi(E, x) = E^{-1}$. Equation (10) reduces to equation (12), see section 2.3 below.

From knowledge of the survival probability of muons we can now estimate the number of muons reaching sea level above a given energy, taking into account neutrino and muon energy losses. At $E_0 = 500$ GeV we also consider the low energy pion contribution. These "target" pions were given equal energies. The relative contribution to the number of muons at $E_0 = 50$ and 500 GeV are summarised in Table 1. It is interesting to note that at $E_0 = 500$ GeV the dominating contribution is distributed throughout the entire 100-1030 gcm$^{-2}$ layer, when the "leading" pion has collided up to ~6 times. Of course, to talk about a leading pion as if it were the same before and after a collision is only a loose way of speaking. However, the concept is useful and should not give rise to any misunderstanding. At $E_0 = 50$ GeV the contribution mentioned comes from decaying pions in the 50-200 gcm$^{-2}$ layer. Thus the physics of the LF-model changes rapidly in the atmosphere for $50 < E_0 < 500$ GeV. This model can be regarded as representing isobar production and subsequent decay.

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$$\pi(E, x) \propto E^{-1} f(x)$$

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We consider two cases $k_{\pi N} = 0.5$ and 1.0 (see Table 1 and 2). For an energy $E = 100$ GeV of the leading pion we include an estimate from the contribution of target nucleons, carrying a total energy of 50 GeV (if $k_{\pi N} = 0.5$). If only one target nucleon is knocked out, then we may from Figure 3 put the contribution $\sim 10\%$. This figure is slightly biased since we have used a value from a cascade that is on the average initiated around $80 \text{ gcm}^{-2}$ whereas in the actual case it should be initiated around $\sim 200 \text{ gcm}^{-2}$. Thus the nucleon-meson cascade takes place in a more dense an atmosphere. If, however, 5 target nucleons with an energy of 10 GeV are produced by spallation, the contribution to the muon number is practically zero. As an estimate we take the average of these two extreme situations. In Table 1 the results from different considerations are summarised. Averaging the results from the LP and CCE models as being a more realistic situation we find that the $\pi N$ contribution is $\approx 2\%$ at $E_0 = 50$ GeV and $\approx 16\%$ at 500 GeV. Extrapolation to $E_0 = 1000$ GeV on a logarithmic scale gives a $\pi N$ contribution of $\sim 30\%$. The muon size per nucleon is $\approx 21$ (Figure 4), which can be directly compared with the results of the Durham group (Tunver, private communication). It may be of interest to note that in the CCE model the dominating contribution to the sea level muon number comes from the 50-300 gcm$^{-2}$ layer both at $E_0 = 50$ and 500 GeV. Thus the physics of the LP and CCE models differ considerably for 500 GeV nucleons. The CCE model may be regarded as a representation (very rough) of the pionisation process.
The chains

\[ \begin{align*}
N(N, \pi) & \to X \\
N(N, \pi) & \to \mu \\
N(N, \pi) & \to \mu \\
\vdots & \\
\text{etc} & + \mu
\end{align*} \]

where \( X \) stands for anything, are considered in our original calculations (Aström 1975, 1977) using a real temperature distribution \( T(x) \). In Table 2 we illustrate the reactions considered in our estimates. Thus, the contribution from chains of the kind

\[ \begin{align*}
\pi(N, \pi) & \to X \\
\to \pi(N, \pi) & \to X \\
\to N(N, \pi) & \to \mu \\
\vdots & \\
\text{etc} & + \mu
\end{align*} \]

are neglected. Also kaon production and other particles giving muons are neglected (except for the target nucleons mentioned earlier). We see that the CCE model produced muons with greater efficiency than the LP model. Part of this difference is due to the fact that they represent physically two kinds of events (isobar production and pionisation). Hence the layers of dominating contribution to muon size may differ considerably. A small part is due to the difference in inelasticities \( K_{\pi N} \) although the averages are almost equal. Evidently the production of muons is more efficient if the cascade maximum is at a high level in the atmosphere. The muon size \( N_\mu (E_0) \) per nucleon, Figure 4, may now be used to calculate the expected muon size \( N_\mu \) in a cascade initiated by a dust grain of given atomic number (or size).
3. The muon size of a cascade initiated by dust grains

The work of Wesson (1974) on abundance and size spectrum of interstellar dust indicates that the grain sizes of interest are those with radii $10^{-7} < r < 3 \times 10^{-6}$ cm. Also the possible energies/N suggest that we should look in that range. However, to our knowledge the size spectrum of relativistic grains - if they exist - is unknown. In Table 3 we summarise the expected shower size $N^H (>1$ GeV/c) initiated by a grain with a density $\sim 2.5 \text{ gcm}^{-3}$ and a total energy $10^{17} < W < 10^{20}$ ev. The values corresponding to $E_0 > 500$ GeV/N are approximate because of deviations of our modified CKP-model (Aström, 1977) from ISR-data. However, they may serve as order of magnitude estimates. We shall not discuss the confinement regions until we have some idea of the characteristics such as size, density and energy per nucleon that a dust grain must have in order to produce the observed giant EAS. In Table 4 we show the result of our calculations in comparison with the cascade calculations made by the Durham group (Turver, private communication) for a dust grain consisting of $10^6$ nucleons with a total energy $W = 10^{18}$ ev. The characteristics of a typical nucleon induced shower is also given. From our calculations - compared with measurements (Lindgren, private communication) - of the electron content (electrons and positrons) of a muon telescope (Aström, 1977) having a median energy of 50 GeV we know that the electron content of a 1000 GeV/N shower is $>30\%$ of the muon size. However, in order to satisfy the conditions for a typical $10^{18}$ ev shower, the electron size $N_e$ must rise a factor $\sim$ 2500 from 50 to 1000 GeV/N. (The relative increase in electron size, compared with that of the muon size must be greater than a factor 100). However, it is not probable that $\pi^0$:s will completely dominate the production of the electromagnetic component. This statement expresses our (strong) belief rather than being based on stringent upper limit proofs. From the work of Thompson (1974) on $\gamma$-rays in the atmosphere and the increase in the contribution of $\pi^\pm$-collisions from 20-30 GeV (the average energy of the primary cosmic rays) up to 1000 Gev, noting that the source of $\pi^0$:s should not be very
different from that of charged pions, we may give an estimate of the $\pi^0$ contribution to the electromagnetic component at 1000 GeV. It is of the same order of magnitude as that from charged pions in the chains

$$\pi^\pm \rightarrow \mu^\pm \rightarrow \begin{cases} e^+ \text{ (knock-on, charge exchange)} \\ e^+ \text{ (other processes)} \end{cases}$$

This result is consistent with the Monte Carlo calculations made at Durham (see Table 4). A rough direct estimate of the electron size $N_e$ gives the value $3 \times 10^7$ electrons and positrons. This number has been used in the last column of Table 4. We find also the same number of muons as the Durham group at sea level (using different methods).

The primary energy based on electron size may be overestimated if the primary particle is a dust grain (Hayakawa, 1972, Alfvén, private communication). An estimate, based on the experimental lateral distribution function (LDF) assuming it to be known at distances $> 100$ m from the shower axis (Turver, 1973), shows that $N_e$ and hence $w$ can be overestimated by a factor $\xi 10$. From the muon LDF, however, we find ceteris pari-bus that the overestimate cannot be more than a factor $\sim 2$ (Armitage et al. 1974, Bell et al. 1974). Using the muon size as a measure of $w$ and considering a grain with $E_o = 1000$ GeV/N, there is a gap greater than a factor 50 in the estimated and measured electron size (Table 4, normalised values to a typical nucleon induced shower). Hence it is very unlikely that a grain with energy $< 2000$ GeV/N can produce giant EAS with a total energy $W > 10^{17}$ eV. Below that energy the conditions require grain sizes $r < 2 \times 10^{-7}$ cm. Such grains carry a positive charge $q < 80$ electron charges and offer practically no advantages in comparison with heavy ions. The possible sizes (Figure 5) are given as a function of total energy. We arbitrarily put the lower limit of the grain radius to $10^{-7}$ cm, corresponding to (conventional) grains with a maximum charge
~ 20 electron charges. If dust grains are the origin of EAS, we should look for grains with an energy \( \sim 10^4 - 10^5 \) GeV/N. We have here used the maximum attainable charge for a spherical dust grain withstand an electric field at the surface of \( 3 \times 10^8 \) V/cm (Alfven, private communication).

In Figure 5 we have given a border corresponding to grains accelerated to rigidities \( P_0 = 5 \times 10^{15} \) V. It is seen that the hypothesis of dust grain origin of giant EAS is ruled out for \( P < P_0 \) and \( E_0 \geq 1 \times 10^{13} \) eV/N. Thus, it does not seem possible to have the grains contained within regions of heliospheric dimensions (which is the case if we consider \( P < 1 \times 10^{14} \) V particles (Alfven, 1975).

We do not believe that a study of the LDF for muons can give a decisive test of the hypothesis. According to our preliminary studies, an \( E_0 = 500 \) GeV/N induced shower seems to be possible (although the central core structure of the LDF is then flatter than earlier believed). Hence, such a test is not strong enough. If our estimates on nN collisions are correct within one order of magnitude, dust grains as a dominating source of giant EAS are ruled out if \( E_0 < 2000 \) GeV/N.
<table>
<thead>
<tr>
<th>E0 (GeV)</th>
<th>Contribution in % of the uncorrected production of muons in NN-collisions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heuristic approach</td>
<td>CCE model</td>
</tr>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>50</td>
<td>&lt; 3</td>
</tr>
<tr>
<td>500</td>
<td>&lt; 23</td>
</tr>
</tbody>
</table>
TABLE 2. The chains in \( \pi N \) collisions that are considered in the estimates. The cases A, B and C refer to the chains for which estimates have been made (see the headings of Table 1).

A: \( N(N, \pi) \ X \)
\[ \Downarrow \]
\( \pi(N, \pi) \ X \)
\[ \Downarrow \]
\( \pi(N, \pi) \ X \)
\( \vdots \)
\( \text{etc} \rightarrow \mu \)

B: \( N(N, \pi) \ X \)
\[ \Downarrow \]
\( \pi(N, \pi) \ X \)
\[ \Downarrow \]
\( \pi(N, \pi) \ X \)
\( \vdots \)
\( \text{etc} \rightarrow \mu \)

Case C is the same as A considering multiple chains denoted by fat arrows \( \Rightarrow \):

\( N(N, \pi) \ X \)
\[ \Downarrow \]
\( \pi(N, \pi) \ X \)
\[ \Downarrow \]
\( \pi(N, \pi) \ X \)
\( \vdots \)
\( \text{etc} \rightarrow \mu \)
TABLE 3. Muon shower size $N^\mu (> 1 \text{ GeV/c})$ as a function of grain radius $r(\rho = 2.5 \text{ g cm}^{-3})$ and total energy $W$. $E_o$ is the energy/N. (See also Table 4).

<table>
<thead>
<tr>
<th>$r$ (cm)</th>
<th>$E_0$ (GeV)</th>
<th>$W$ (eV)</th>
<th>$3 \cdot 10^{-7}$</th>
<th>$10^{-6}$</th>
<th>$3 \cdot 10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1$0^{-17}$</td>
<td>500</td>
<td>15</td>
<td>0.5</td>
<td>$2.4 \cdot 10^6$</td>
<td>$7 \cdot 10^5$</td>
</tr>
<tr>
<td>1$0^{-18}$</td>
<td>5000</td>
<td>150</td>
<td>5</td>
<td>($\sim 10^7$)</td>
<td>$2.7 \cdot 10^7$</td>
</tr>
<tr>
<td>1$0^{-19}$</td>
<td>-</td>
<td>1500</td>
<td>50</td>
<td>-</td>
<td>$2 \cdot 10^8$</td>
</tr>
<tr>
<td>1$0^{-20}$</td>
<td>-</td>
<td>-</td>
<td>500</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
TABLE 4. Characteristics of a dust grain EAS and a nucleon induced shower at sea level. $W = 10^{18}$ eV and $E_0 = 10^3$ GeV. In the last column the muon size is normalised to a typical shower. $x_{\text{max}}$ denotes the level of cascade maximum for electrons.

<table>
<thead>
<tr>
<th></th>
<th>Dust grain</th>
<th></th>
<th>Nucleon</th>
<th>Normalised</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Durham</td>
<td>This work</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_\mu$</td>
<td>$2 \times 10^7$</td>
<td>$2.1 \times 10^7$</td>
<td>$6 \times 10^6$</td>
<td>$6 \times 10^6$</td>
</tr>
<tr>
<td>$N_e$</td>
<td>$10^7$</td>
<td>$6 \times 10^6 &lt; N_e &lt; 6 \times 10^7$</td>
<td>$4 \times 10^8$</td>
<td>$\sim 9 \times 10^6$</td>
</tr>
<tr>
<td>$x_{\text{max}}$</td>
<td>300</td>
<td>-</td>
<td>700</td>
<td>-</td>
</tr>
</tbody>
</table>

(gcm$^{-2}$)
Captions to the Figures:

Figure 1. The generating index $\alpha(E_0, p_{SO})$ as a function of primary energy $E_0/m_Nc^2$ and the sea level muon momentum $p_{SO}$.

Figure 2. Fundamental specific yield of sea level muons, equal to the number of muons with momentum $> p_{SO}$ (GeV/c) produced by a primary nucleon with energy $E_0$ (GeV) in the first collision.

Figure 3. Relative yield of muons in NN-collisions $Y_R = N_\mu / N_e (> 1 \text{ GeV/c})/cE_0$, $c = 0.02/m_Nc^2$, as function of primary energy for three values of the elasticity $f$. The calculated yield for $f = 0.5$ is normalized to the sea-level differential muon spectrum at 1 GeV/c. See also text.

Figure 4. The number of sea level muons $> 1 \text{ GeV/c}$ expected in NN and $\pi N$ collisions as a function of primary energy $E_0/m_Nc^2$, for $f = 0.5$. For $E_0 < 20 \text{ GeV}$ see scale to the right.

Figure 5. Possible dust grain radii as a function of total energy $W$. We have also for comparison shown the limit if the energy per nucleon must be $> 10^{13}$ eV. Considering the local acceleration hypothesis we may put an upper limit of the rigidity $P < 5 \cdot 10^{15}$ V corresponding to a radius of gyration $\rho_G < 15 \text{ LY}$ (light years) in an interstellar magnetic field $B \sim 0.1 \text{ G}$. The dust grains are assumed to be spherical, having a density $\sim 2.5 \text{ g cm}^{-3}$ and a maximum charge corresponding to an electric field $\sim 3 \cdot 10^8 \text{ V/cm}$ at the surface. (See also text).
References:


TURVER K E, 1973, Cosmic Rays at Ground Level, ed. A W Wolpendale, London and Bristol, 159-90, pp 244.


Fig. 3

Muon Yield $Y_R = \frac{N_{I>1 \text{ GeV/c}}}{m_N c^2}$

\[ \frac{E_0}{m_N c^2} \]

$(E_0/m_N c^2, Y_R) = (10, 0.15)$
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ON RELATIVISTIC DUST GRAINS AND EXTENSIVE AIR SHOWERS

K. Aström

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Abstract

The hypothesis of dust grain origin of giant extensive air showers has been reexamined. We find that the electron to muon size gives a more decisive test than a study of lateral distribution functions. From the expected muon size and estimated electron size we conclude that dust grains must be accelerated to energies greater than 2000 GeV/N. Considering conventional grains with a density \( \rho \sim 2-3 \text{ g cm}^{-3} \) it is very unlikely that these could be accelerated within a confinement region of heliospheric dimensions.

Keywords: High energy dust grains, Giant extensive air showers (EAS), Nucleon-meson cascade.