Low-dose phase-contrast X-ray imaging: a comparison of two methods

Tunhe Zhou

Master of Science Thesis
Biomedical and X-ray Physics
Department of Applied Physics
Royal Institute of Technology
Stockholm, Sweden 2012
This thesis summarizes the degree project work by Tunhe Zhou for the Master of Science degree in Engineering Physics. The work was performed during 2011-2012 at Biomedical and X-ray Physics, Royal Institute of Technology in Stockholm, Sweden. The supervisor was Anna Burvall and the examiner was Prof. Hans Hertz.

© Tunhe Zhou, March 2012

Tryck: Universitetsservice US AB
Abstract

Phase-contrast X-ray imaging is emerging as an alternative to the conventional X-ray radiography for materials with low absorption coefficients. For small features, phase-contrast imaging reduces dose compared to absorption-contrast imaging. There are a variety of phase-contrast imaging techniques, among which the grating-based and propagation-based methods are discussed in this thesis.

The principle of the two phase-contrast imaging methods is explained in this thesis. The image construction process of the grating-based method from detected images to the differential phase-contrast (DP) image, standard absorption image and dark-field image is introduced in detail. In order to make a theoretical comparison between the grating-based method and the propagation-based method, a model of the grating-based phase-contrast imaging method is built. Blood vessels with contrast agent CO₂ within 2-cm-thick soft tissue are simulated as objects. The doses of two methods are compared when SNR² equals 25 with diameters of the blood vessels as independent variables.
# Contents

Abstract........................................................................................................................................... i

Contents............................................................................................................................................... ii

Chapter 1 Introduction...................................................................................................................... 1

Chapter 2 Theory ............................................................................................................................... 3

2.1 X-Ray........................................................................................................................................... 3
2.2 Complex Refractive Index ........................................................................................................... 4
2.3 Wave Propagation ....................................................................................................................... 6
   2.3.1 Wave-Front Propagation ....................................................................................................... 7
   2.3.2 Talbot Effects ...................................................................................................................... 8
   2.3.3 Rescaling for Spherical Wave ............................................................................................. 9
2.4 Theory of SNR and Rose Model ................................................................................................. 10
2.5 Dose ............................................................................................................................................ 10

Chapter 3 Phase-Contrast Imaging Methods ..................................................................................... 13

3.1 In-Line Propagation Method ..................................................................................................... 13
3.2 Grating-Based Method ............................................................................................................... 14

Chapter 4 Simulation ........................................................................................................................ 19

4.1 Program Structure .................................................................................................................... 19
4.2 Simulation Assumptions ............................................................................................................ 21
   4.2.1 Source Specification ......................................................................................................... 21
   4.2.2 Gratings Simulation ........................................................................................................ 22
   4.2.3 Fresnel Diffraction .......................................................................................................... 23
   4.2.4 Image Detecting .............................................................................................................. 24

Chapter 5 Results and Discussion ................................................................................................... 25

5.1 Data Analysis ............................................................................................................................ 25
5.2 SNR Analysis ............................................................................................................................. 30
5.3 Comparison ............................................................................................................................... 32

Chapter 6 Conclusion ........................................................................................................................ 35

Acknowledgments ........................................................................................................................... 37
Bibliography ................................................................. 39
Chapter 1

Introduction

X-ray imaging is widely used in medical diagnostics, for example mammography, and chest X-ray. Conventional radiography utilizes the absorption-contrast imaging technique, which records the intensity attenuation due to absorption. This method is not very applicable for low absorption material, such as soft tissue or polymers. In recent years, phase-contrast techniques have been developed. Phase-contrast imaging makes use of the contrast from the phase shift of X-rays passing through objects. There are basically four different types of phase-sensitive methods: interferometry, propagation-based method, diffraction enhanced imaging and grating-based method [1].

This report focuses on the grating-based method and the propagation-based method. The former method utilizes the Talbot effect and uses a grating interferometer to produce a series of images by the phase stepping scan process. Three different images, i.e., standard absorption image, differential phase image and dark-field image, can be constructed from those detected images [2]. The latter method is basically Fresnel propagation using a coherent X-ray source. A comparison of those two from a dose perspective would be useful for further applications in medical or small-animal imaging. Our group has theoretically modeled and experimentally implemented the propagation-based phase-contrast imaging [3-4]. The aim of this project is to model a grating-based imaging system, and use it to compare the two methods theoretically with emphasis on dose.

In Chapter 2 the theoretical background of these phase-contrast methods and also the basic knowledge of SNR and dose are introduced. Principles and setups of the two phase-contrast imaging methods are described in Chapter 3. Simulation algorithm and assumptions are explained in Chapter 4. In Chapter 5 the simulation results are discussed: Data analysis processing from detected to constructed images and SNR calculation are explained, and the doses of two methods for blood vessels with different diameters are compared.
Chapter 2

Theory

In this chapter the basic physics of X-ray and X-ray imaging is introduced. When a wave passes through a material, it is both attenuated and phase shifted, which is the key to absorption- and phase-contrast imaging respectively. Also tools for evaluating the image quality are introduced, i.e., SNR and the Rose model.

2.1 X-Ray

Since Röntgen accidently discovered X-radiation in 1895, it has been studied and developed widely. The applications include medical diagnose, X-ray microscopy, products defects examination, etc.

![Electromagnetic wave spectrum](image)

Figure 2.1. Electromagnetic wave spectrum.

As shown in Figure 2.1, an electromagnetic wave with a wavelength from around 0.01 nm to 10 nm is called an X-ray. It is also divided into soft X-ray and hard X-ray, which separately corresponds to wavelength from about 0.1 nm to 10 nm and from about 0.01 nm to 0.1 nm [5]. Normally hard X-rays are used in medical imaging and photon energy is used to describe hard X-rays. The relationship between photon energy and wavelength is

\[ E = h\nu = \frac{hc}{\lambda}, \]

(2.1)
where $h$ is the Plank constant $6.626 \times 10^{34}$ J\cdot s, and $c$ is the light speed $2.998 \times 10^{8}$ m\cdot s$^{-1}$. It turns into an empirical formula:

$$E \cdot \lambda = 1239.8 \ nm \cdot eV .$$

Therefore hard X-ray can be described as radiation with photon energy from 10 keV to 100 keV.

### 2.2 Complex Refractive Index

An important factor to understand absorption- and phase-contrast is the complex index of refraction. In order to discuss it, X-ray is explained as an electromagnetic wave in this and also later sections on wave propagation.

When an electromagnetic wave passes through a medium, it is both attenuated and phase shifted. This can be explained by the effect of the complex index of refraction of the medium. For X-ray and EUV (extreme ultraviolet, see Figure 2.1), it is usually written as [6]

$$n = 1 - \delta + i\beta = 1 - \frac{r_e \lambda^2}{2\pi} \sum_i n_i f_i(0),$$

where $r_e = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{m_e c^2} = 2.82 \times 10^{-15}$ m is the classical electron radius and $\lambda$ is the wavelength. For a compound with element $i$, $n_i$ is the number of atoms per unit volume and $f_i(0)$ is the complex atomic scattering factor for forward scattering.

The propagation of a monochromatic, coherent wave in a medium can be described as [5]

$$E(r, t) = E_0 e^{i(k \cdot r - \omega t)},$$

where $r$ is the propagation distance vector, $k$ is the wave vector and $\omega$ is the angular frequency. With the dispersion relation $v = \frac{\omega}{k} = \frac{c}{n}$ and the complex refractive index (2.3), it can be expressed as

$$E(r, t) = E_0 e^{i(\omega r/c - \omega t)} e^{-i(2\pi\delta/\lambda)r} e^{-(2\pi\beta/\lambda)r}.$$
The first exponential term of Equation (2.5), $E_0 e^{i(\omega t/c - \omega t)}$, is the same as for wave propagation in vacuum where the refractive index is unity. The second exponential form indicates the phase shift and the third indicates the absorption.

Figure 2.2. Illustration of wave propagation through a medium with complex refractive index $n$. Red dashed line shows the phase shift and the blue shows the attenuation compared to free propagation in vacuum (the upper wave).

As shown in Figure 2.2, when a plane wave propagates from left to right in the $z$-direction through the gray square medium with a refractive index $n = 1 - \delta + i\beta$, the phase is shifted by $\Delta \phi = \frac{2\pi \Delta \delta}{\lambda} z$, where $\Delta \delta$ means the difference in $\delta$ between the background and the sample medium.

The intensity (or irradiance) is proportional to the square of the amplitude of the electrical field. Thus the intensity attenuation is

$$I / I_0 = (e^{-2\pi \Delta \delta / \lambda})^2 = e^{-\Delta \mu z},$$

(2.6)

where $\mu$ is the attenuation coefficient, which is dominated by photoelectric effect and Compton scattering for hard X-rays. When $\Delta \mu$ is very small, the absorption contrast is then

$$Contrast = \frac{|I_0 - I|}{I_0} = \left| 1 - e^{-\Delta \mu z} \right| \approx \Delta \mu z.$$

(2.7)

For light elements in the hard X-ray range, $\delta$ is usually much larger than $\beta$. As shown in Figure 2.3, for example when photon energy is 10 keV, for hydrogen and carbon, which are common in soft tissues, $\delta$ can be over 1000 times larger than $\beta$. It indicates that phase contrast can be more conspicuous than absorption contrast. An example is shown in Figure 2.4, from which we can see sharp edges and
structures in the phase-contrast image but only dark shadows in the absorption image.

![Graph showing δ/β of hydrogen, carbon, and calcium from 100 eV to 30 keV.]

**Figure 2.3.** For energy from 100 eV to 30 keV, δ/β of hydrogen, carbon and calcium are plotted. Their atomic numbers are 1, 6 and 20 respectively. Data are from X-ray database in CXRO (the center for X-ray optics) [7].

![Image of a spider showing absorption and phase-contrast images.]

**Figure 2.4.** An example of X-ray images of a spider. (a) is the absorption image and (b) is the phase-contrast image. (c) shows more details for the phase-contrast image. Image from Ref. [4], reprinted with permission.

### 2.3 Wave Propagation

When an X-ray passes through a medium, the wavefront is distorted because of the interaction with the matter. For X-ray projection radiography the detector is placed immediately after the object and records the intensity attenuation. If the detector is
placed in some distance away from the object, the wave propagates in free space and the wavefront changes, from which the detector can get phase information. In this section, the propagation process is described using a scalar approximation, in Cartesian coordinates.

2.3.1 Wave-Front Propagation

For simplicity, as in Section 2.2, consider a monochromatic wave propagating in the $z$ direction and time-independent. In near-field regime, the electrical field at position $z$ can be calculated using Huygens-Fresnel Principle as [8]

$$E(x, y) = -\frac{ie^{ikz}}{\lambda z} \iint E(x_0, y_0) \exp(ikr) dx_0 dy_0,$$  \hspace{1cm} (2.8)

where

$$r = \sqrt{x^2 + (x - x_0)^2 + (y - y_0)^2} \approx z \left[ 1 + \frac{1}{2} \left( \frac{x - x_0}{z} \right)^2 + \frac{1}{2} \left( \frac{y - y_0}{z} \right)^2 \right]$$  \hspace{1cm} (2.9)

in the paraxial approximation. Substitute it into Equation (2.8), we get the Fresnel diffraction integral [9]:

$$E(x, y) = -\frac{ie^{ikz}}{\lambda z} \iint E(x_0, y_0) \exp\left\{ i k \frac{1}{2z} \left[ (x - x_0)^2 + (y - y_0)^2 \right] \right\} dx_0 dy_0$$  \hspace{1cm} (2.10)

where

$$h(x, y) = \frac{ie^{ikz}}{\lambda z} \exp\left[ \frac{ik}{2z} (x^2 + y^2) \right]$$  \hspace{1cm} (2.11)

is the propagator function and $\ast$ denotes convolution. The Fourier transform of the propagator function (2.11) is [10]

$$H(u, v) = \mathcal{F}\left\{ h(x, y) \right\} = e^{ikz} \exp[-i\pi \lambda z (u^2 + v^2)]$$  \hspace{1cm} (2.12)

where $u$ and $v$ are the spatial frequencies. According to the convolution theorem and Eq. (2.10), the electrical field at distance $z$ can be calculated as [11]:

$$E(x, y) = E_0 * h$$
\[ E_z = \mathcal{F}^{-1}[\mathcal{F}(E_0) \cdot H], \quad (2.13) \]

which is numerically realized by the Fast Fourier Transform algorithm (FFT), and inverse Fourier transform (IFFT) in the simulation.

### 2.3.2 Talbot Effects

A periodic wave repeats its pattern after certain distances within Fresnel regime, and this phenomenon is called Talbot effect after the name of who discovered it [12]. The interest of this thesis focuses on the Talbot effect after phase gratings, which are also called as Talbot array illuminators (TAI) [13].

Let us consider a one-dimensional binary infinite phase grating at position \( z = 0 \) illuminated by a plane wave with unit amplitude, as Figure 2.5. The wave field immediately after the grating can be described with Fourier series as [14]:

\[
E(0) = \exp(i\phi) = \sum_{m} C_m \exp(i2\pi m x / p), \quad (2.14)
\]

where \( \phi \) is the phase step, \( p \) is the period of the grating, and \( C_m \) is the \( m \):th Fourier coefficient:

\[
C_m = \begin{cases} 
1 + \frac{w}{p} \left[ \exp(i\phi) - 1 \right], & \text{for } m = 0, \\
\frac{w}{p} \sin c \left( m \frac{w}{p} \right) \left[ \exp(i\phi) - 1 \right], & \text{for } m \neq 0. 
\end{cases} \quad (2.15)
\]

With the paraxial approximation, propagation in free space for a distance \( z \) leads to

\[
E(z) = \sum_{m} D_m \exp(i2\pi m x / p), \quad (2.16)
\]

where \( D_m \) is the new Fourier coefficient [15]:

\[
D_m = C_m \exp(-i\pi \lambda z m^2 / p^2) = C_m \exp \left\{ -i2\pi m^2 \frac{z}{z_T} \right\}, \quad (2.17)
\]

where \( \lambda \) is the wavelength of the light, and \( z_T \) is the Talbot distance:

\[
z_T = 2p^2 / \lambda. \quad (2.18)
\]
Obviously, when \( z = N z_T \), \( E(N z_T) = E(0) \), where \( N = 0, 1, 2, \ldots \) is just an integer. Therefore at these planes are exactly revivals of the original periodic wave-front. When \( z = (N + 1 / 2)z_T \), which is half Talbot distance, the wave-front is also revived except a \( \pi \) phase shift. Thus it is called inverse Talbot image. The intensity is unity in Talbot or inverse Talbot images since here a phase grating is used, therefore fractional Talbot distances are used instead in order to obtain variation in intensity. There are more details discussed about the relations between image intensity and period with grating duty cycle, phase step and the image position for binary TAI in Suleski’s paper [16].

**Figure 2.5.** The intensity section profile of the wave propagation after a phase grating with phase step \( \phi \) and duty cycle \( w/p \). For a phase grating, the intensity for self-image is unity at Talbot and half Talbot image planes. At fractional Talbot image planes, there can be sub images with intensity variances. This specific example shown here is the intensity pattern of a phase grating with \( \phi = \pi \), and \( w/p = 0.5 \). Note that the binary intensity pattern revives at position \( z = (2N-1)z_T/16 \), where \( N=1, 2, \ldots, 8 \).

### 2.3.3 Rescaling for Spherical Wave

The dimensions discussed in Section 2.3.2 are assumed to be under plane waves, which is not true in reality since it means the source should be placed infinitely far away. They should be rescaled to match a spherical wave. Assume the source-object distance is \( L \) and the object-detector distance is \( d_o \). Under a spherical wave a point source introduces a magnification of

\[
M = 1 + \frac{d_o}{L}.
\]  

(2.19)

Under a plane wave the same intensity distribution can be got at distance [17]
The image at this plane is demagnified by $M$ compared to the one at the $d_s$ under a spherical wave. This relation can be used to simplify the calculation along the propagation process, which applies to the simulation introduced later on.

2.4 Theory of SNR and Rose Model

When evaluating the quality of detected images, one can consider many factors, such as contrast, spatial resolution and noise. For medical imaging, there are no specific requirements for these parameters and the final destination is to correctly diagnose.

The noise that directly affects an observer of an image is usually referred as relative noise, or the coefficient of variation (COV) as follows [18]:

$$COV = \frac{\sigma}{N},$$

(2.21)

where $N$ is the average number of photons detected in each pixel for the photon counting system, and $\sigma$ is the standard deviation or the noise. The inverse of relative noise is SNR (signal-to-noise ratio):

$$SNR = \frac{N}{\sigma} = \frac{\mu}{\sigma},$$

(2.22)

where $\mu$ is the signal mean for a digital or film imaging system.

In order to identify the object to 100%, SNR should be larger than 5, according to Albert Rose [19], and this requirement is called Rose’s criterion.

2.5 Dose

As deduced from Section 2.4 it seems that in order to increase the SNR we only need to increase the radiating time. However, in X-ray imaging for medical diagnose the dose should be controlled. Risk estimation of radiation is a controversial topic, but currently the best model agreed by most experts is linear extrapolation, which means the lower dose the better.

The absorbed dose is defined as the energy absorbed per unit mass of material [18]:

$$d = \frac{d_s}{M}.$$
\[
D = \frac{\Delta E}{\Delta m} = \frac{E(1 - e^{-\frac{\mu_{en}/\rho}{\rho V}})}{\rho V},
\tag{2.23}
\]

where \(\mu_{en}/\rho\) is the mass energy-absorption coefficient, \(\rho\) is the density of the object material, \(z\) is the length X-rays pass through in the material, and \(V\) is the volume of the object. As can be seen from Equation (2.23), the unit of the absorbed dose is J/kg; 1 J/kg = 1 gray (Gy). Gray is the SI unit, and the traditional unit is rad (radiation absorbed dose): 1 rad = 0.01 J/kg = 10 mGy.

To measure the damage potential to biological tissues by different types of radiation, the equivalent dose is defined as

\[
H = W_R \cdot D,
\tag{2.24}
\]

where \(W_R\) is the weighting factor. For example \(W_R = 1\) for photons and electrons, \(W_R = 2\) for protons and \(W_R = 20\) for alpha particles [20]. The SI unit for equivalent dose is Sievert (Sv). 1 Sv is defined as the dose of absorbed radiation that has the same biological effect as 1 Gy of absorbed gamma rays. For X-rays, \(W_R = 1\), 1 Sv = 1 Gy.
Chapter 3

Phase-Contrast Imaging Methods

In this chapter two specific phase-contrast methods are introduced: the in-line free-space propagation method and the method using grating interferometer. They are of interest since they have relatively high resolution, low dose and are potentially applicable for living animals.

3.1 In-Line Propagation Method

In conventional X-ray radiography, in order to avoid blurring, the distance between the object and the detector should be as small as possible, as shown in Figure 3.1 (a). While for in-line propagation phase-contrast imaging, it is in the opposite situation, which means the distance between the object and the detector should be large. Principally it is the only difference in arrangement between them since in-line propagation phase-contrast imaging requires no other optics. Simplicity is definitely one big advantage of this method.

In principle, propagation-based phase-contrast imaging can be simply regarded as the incident beam being refracted by the object, and after propagating for some distance the emerging beam get some intensive parts and sparse parts by edges, which lead to brighter and darker fringes, as shown in Figure 3.1 (b).

![Figure 3.1](image)

Figure 3.1. Illustration of (a) absorption–contrast imaging (b) in-line phase–contrast imaging, and the detected intensity section profile.

This method requires a highly coherent and bright X-ray source. Therefore most of the work needs to be done with a synchrotron radiation source. Apart from that,
extreme-brightness microfocus X-ray sources realize in-line propagation phase-contrast imaging in normal labs [4, 21].

3.2 Grating-Based Method

The method introduced in this section involves a grating interferometer, thus the experiment setup is more complicated than for the free-space propagation method. The most important principle of this method is the Talbot effect, which is introduced in Section 2.3.2.

![Figure 3.2. Illustration of the experiment setup of the method using grating interferometer. G0 is the source grating, G1 is the phase grating and G2 is the absorption grating.](image)

The experimental setup is illustrated in Figure 3.2. The X-ray source here can be a conventional X-ray tube instead of a microfocus X-ray source or synchrotron radiation source as in other phase-contrast imaging techniques, which makes it applicable to medical or industrial imaging [22]. This is due to the use of the source grating, which is an absorption mask that makes the source into a series of individual line sources. Thus the coherence length of the source can be large enough even for a conventional X-ray tube. In order to contribute constructively in the imaging process by matching the period of absorption grating G2, the period of the source $p_0$ grating should satisfy

$$p_0 = p_2 \times \frac{L}{d},$$  \hspace{1cm} (3.1)

where $p_2$ is the period of G2. $L$ and $d$ are the source-object, and the object-detector distance respectively. This geometrical relation can be seen from Figure 3.3. The
width of transmission slits $s$ should be small enough to satisfy the requirement for the transverse coherence length $l_t$ [23]:

$$l_t = \lambda \frac{L}{s}$$  \hspace{1cm} (3.2)

The source grating should be as close to the source as possible and thus they can be regarded as lying approximately in the same plane. The detector and G2 are placed with the same spatial proximity.

![Figure 3.3. Sketch of the grating interferometer.](image)

The grating interferometer consists of two gratings. The phase grating G1 here can be chosen as a $\pi$ phase shift binary grating with duty cycle of 0.5, the Talbot effect of which is shown in Figure 2.5. There are also other choices, but then $d$, $p_2$ and the intensity distribution of the patterns will also be changed correspondingly, and besides, most existing experiments use this kind of grating [24-25]. When it is illuminated by a plane wave, the absorption grating G2 should have a period of

$$p_2 = p_1 / 2$$  \hspace{1cm} (3.3)

in order to match the intensity pattern of the “Talbot image” of G1, and the distance between G1 and G2 $d = \frac{(2N - 1)z_T}{16}$, where $N=1,2,\ldots,7$. An example of the gratings used in an experiment is shown in Figure 3.4. The dimensions in reality are rescaled to spherical wave as discussed in Section 2.3.3 as $p_{2s} = p_2M$ and $d_s = dM$. 
The object is placed right in front of G1. The wave propagates through the object and is refracted, which leads to distortion of the wave-front after G1. The angular phase deviation $\alpha$ is [26]

$$\alpha \approx \frac{\lambda}{2\pi} \frac{\partial \phi(x)}{\partial x},$$

(3.4)

where $\phi(x)$ is the phase of a plane wave at a given position $x$. The approximation is valid when $\alpha$ is small, which is usually in the order of a few microradians for X-rays. Thus in the distance $d$ a transverse shift $S(x)$ of the periodic pattern is introduced as

$$S(x) = \alpha d = \frac{\lambda}{2\pi} \frac{\partial \phi(x)}{\partial x} d.$$ (3.5)

Then with the use of G2, which is also called the analyzer grating, images containing information about the phase shift can be obtained. When G2 is placed in different transverse positions, which means some parts of the wave which are absorbed by G2 for one image may pass through G2 for another image. In this thesis only one-dimensional gratings are considered, so G2 is only scanned in the $x$ direction within one period as shown in Figure 3.5.

This phase stepping process produces a series of images, from which the differential phase contrast image, absorption image and dark-field image can be obtained by data processing discussed in Chapter 5.
Figure 3.5. Illustration of the phase-stepping process. G1 is a phase grating with a $\pi$ phase step and 0.5 duty cycle. G2 is an absorption grating with a period of half of G1’s. The propagation distance is $\frac{\pi r}{16} = \frac{p_2}{8\lambda}$ as discussed in Section 2.3.2. G2 is scanned along $x$ axis per $p_2/4$. The position shift of G2 is $x_g$. The corresponding detected images are in the right column. The object used here is a 100 $\mu$m blood vessel with CO$_2$ as contrast agent within a 2-cm-thick soft tissue as discussed in Section 5.3.
Chapter 4

Simulation

The simulation of the grating-based method is done during this degree project. The basic process and assumptions of the simulation are introduced in this chapter. The simulation of the in-line propagation method is done with the software developed by Lundström as part of his master thesis [3]. The simulation tool of this thesis is Matlab.

4.1 Program Structure

As shown in Figure 4.1, the start point is to assume a plane wave with unit amplitude. The size of the plane wave should match the period of G1 in order to avoid edge effects during propagation since the field of view is not very large. There should be enough points within one period of G2 for enough phase-stepping scan numbers.

Then multiply the plane wave by the electrical field that transmits through the object and G1, which is calculated with the projection approximation:
where $\mu$ is the mass attenuation coefficient which can be acquired from NIST [27], and $\delta$ from X-ray database [7]. The shapes, positions and material compositions of each object can be defined respectively. The setting of the object part is mostly based on Lundtröm’s program where more details are discussed [3]. The phase step of G1 is fixed to be $\pi$ while the thickness changes automatically according to different photon energies and index of refraction. At the same time, repeating the process without multiplying by the function of the object is necessary to obtain a reference image.

After G1, the electrical field propagates in free space for a fractional Talbot distance and can be calculated at G2 as discussed in Section 2.3.1. The same algorithm is applied to the calculation of the convolution of the source intensity distribution and the wavefront intensity distribution later.

Since from then on the image is only considered in intensity, the complex electrical field can be replaced by photon number instead according to the source property. If the source is not a point source but has some geometrical brightness distribution, such as a Gaussian distribution or uniformly distributed over a surface, and assumed to be transversely incoherent, the image can be convolved with the radiance distribution of the source. When an extended source is used, a source grating G0 is also needed, of which the transmission function should also be convolved with the image.

Then the blurred image should be multiplied by a series of transmission functions of G2, which are shifted in x direction within one period. At last, these images are detected by the detector with the pixel size of at least two times of the period of G2 to resolve the dark-bright fringes. Noises can be added to these detected images. So intensity variances can be obtained from these images for each pixel and more analysis can be made from them, which is discussed later in Section 5.1.
4.2 Simulation Assumptions

Since the final purpose is to compare the propagation-based method and grating-based method, some specific assumptions can be made to simplify the situation and to realize the comparison. Details of each part are discussed in this section.

4.2.1 Source Specification

Parameters that determine the source property are size, intensity distribution, emission spectrum and power. The power of the source together with the exposure time determines the number of photons per solid angle which is important for calculating the quantum noise later on:

\[
\text{photon number} = \frac{\text{power} \cdot \text{time}}{4\pi \cdot q_e \cdot \text{photon energy}}. \tag{4.2}
\]

To realize an ideal case, a monochromatic point source can be simulated. A synchrotron source can be approximately regarded as a point source since the demagnified source size at the image plane is very small because of the large distance between the source and the setup, which is over 100 m.

![Figure 4.2](image)

**Figure 4.2.** An example that a spectrum is parted and represents by five discontinuous energies. This spectrum is generated from an X-ray tube with a tungsten anode, under 40 kV, 30 mA.

A standard X-ray tube has a source size of around 0.5 mm which requires a source grating in the setup. A standard X-ray tube is not monochromatic but has an emission spectrum, which can be simulated into several energies as shown in Figure 4.2 and the whole simulation process is undertaken under these energies and summed at last [3]. When a polychromatic source is used, a center energy has to be chosen to determine the thickness of the gratings and the fractional Talbot distance.
It means the gratings and propagation distance only fit for the center energy and the mismatches with other energies decrease the visibility.

In order to approach the liquid-jet-target microfocus X-ray source in the lab of BIOX at KTH for in-line propagation method, which can also applied to the grating-based method, a source can be simulated to have a Gaussian distribution with 8 μm FWHM.

### 4.2.2 Gratings Simulation

The gratings are designed to be made of gold for G0, silicon for G1, and silicon wafer filled with gold for G2 according to existing experiments as in Figure 3.4. The thickness is adjusted according to the source photon energy. For instance, for Si under X-ray with photon energy 20 keV, δ = 1.211, \( \frac{\mu}{\rho} = 4.464 \text{ cm}^2/\text{g} \), \( \rho = 2.330 \text{ g/cm}^3 \). Thus in order to have π phase shift, the thickness should be 25.60 μm. The corresponding amplitude transmission is thus 0.9868, which is almost 1.

The period of G1 is set to 4 μm and G2 to 2 μm since it is simulated under plane wave. The period of G0 should match the distance \( L \) and \( d \) as in Eq. (3.1) which should be the same as \( p_2 \) at the image plane. The duty cycle is assumed to be 0.25. The thickness should be large enough to ensure absorption close to 100%. Figure 4.3 shows the properties of the phase grating and the absorption grating that are used in simulation.
Figure 4.3. (a) Amplitude transmission distribution of G0 at the image plane. (b) Phase transmission of the phase grating G1, of which the phase step is designed to be $\pi$. The period is 4 $\mu$m. (c) The transmission of amplitude of the absorption grating G2. The bright strip is transmission of the silicon part of the grating, which is close to 1 as can be seen from the sidebar, while the dark stripe, which is approaching 0, comes from the gold fill. The thickness of G2 is 20 $\mu$m here for 20 keV X-ray. The period is 2 $\mu$m.

4.2.3 Fresnel Diffraction

The object-detector distance should be chosen to be a fractional Talbot distance $z = (2N - 1)z_T / 16$ as discussed in Section 2.3.2 to realize the Talbot effect. With longer distance, the transverse shift is larger which can increase the signal of the differential phase-contrast image. But a longer distance can also lead to a smaller angular range which leads to a smaller field of view. Besides this, if the source is incoherent with a broad spectrum such as a standard X-ray tube, the visibility is lower for a longer distance. For synchrotron source with high coherence monochromatic radiation it is possible to take a longer distance such as 9th fractional Talbot order distance. In the simulation here the distance between G1 and G2 here is set to be the fifth Talbot distance of the phase grating with phase shift $\pi$:

$$d = 5p_1^2 / 8\lambda,$$  \hspace{1cm} (4.3)

which is a common value in experiments [26] and a compromise in the trade-off of the visibility and field of view with the differential phase-contrast signal. This is calculated under a plane wave, thus this distance will be modified as in Equation (2.20). However, this real distance is only needed for calculating the photon number in detector.
4.2.4 Image Detecting

As G2 is shifted along the x-axis within one period, at each step, one image is recorded. The stepping process is assumed to be ideally stable without noise from jitter. The detector is assumed to be an ideal photon counting detector which has a 100% quantum efficiency without readout noise or other imperfections. I assume there is only detector quantum noise since it is the dominant limitation [28]. The noise is assumed to have a Gaussian distribution and it is added to the detected images as a normal distribution with zero mean and a standard deviation of the square root of the photon number of each pixel.

The detector resolution for the grating-based method cannot be smaller than twice the period of the absorption grating \( p_2 \) and should be an even multiple of \( p_2 \) in order to avoid Moiré effects in the detected images, though Moiré fringes can be cancelled out by subtracting the reference images. In existing experiments the detectors have much larger pixel size than the grating period [22]. But a small field of view is simulated in this thesis because the simulated object is in a small scale (starting from 10 \( \mu \)m) and it requires less running time for simulation. The pixel size \( r \) is set to 8 \( \mu \)m for plane waves corresponding to approximately \( r_s = 9 \mu m \) when \( L = 1.5 \text{ m} \) and \( d \) is the 5\textsuperscript{th} Talbot distance for spherical waves as

\[
r_s = rM = r \frac{L}{L - d} \approx 8.96 \mu m ,
\]

as discussed in Section 2.3.3.

9-\( \mu \)m pixel size is realizable for detectors at BIOX KTH and is used for the propagation-based imaging experiments, which require high resolution detector to resolve fine edge enhancements. It is also for fair to use similar detector for both methods later in the comparison.
Chapter 5

Results and Discussion

In this chapter simulation results are shown. From series of simulated images, standard absorption, differential phase-contrast and dark-field images can be obtained. SNR of results from method using grating interferometer and in-line propagation are analyzed. Also doses are compared between these two methods in accordance with Rose’s criterion.

5.1 Data Analysis

As G2 is scanned over one period, the intensity of one pixel has a relation as a periodic triangular function for an ideal point source as illustrated in Figure 5.1 because it is actually the convolution of two top-hat functions - the transmission function of G2 and intensity distribution of the Talbot image [29].

![Figure 5.1](image)

*Figure 5.1.* Illustration of the intensity periodic triangular function of one pixel during phase stepping process under a point source. The x axis means the shift of G2 along phase-stepping scan. The period is 2 μm as the same as p2, and the average intensity is normalized as one.
For an extended source, the triangular function should be convolved by the distribution function that source projects to the detector plane, and then the shape of the intensity variance is more like a sinusoidal curve. The sinusoidal curve function can be approximately expressed as

\[
I(m, n, x_g) = I_0(m, n) + a(m, n) \cos\left(\frac{2\pi x_g}{p_2} + \varphi(m, n)\right),
\]  
(5.1)

where \( m \) and \( n \) are the coordinates in the detector plane and \( x_g \) is the position shift of \( G2 \). In the simulation \texttt{lsqcurvefit()} is used for this fitting, and it can also be done by inverse discrete Fourier transform [30].

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.2.png}
\caption{Schematic of a series of the detected image intensities and their fittings to sinusoidal curves of one pixel. The blue solid line and circles correspond to the reference image while the red dashed line and squares correspond to the image containing object at the same pixel. Three parameters \( I_0, a \) and \( \varphi \) can be determined from the fitting result. In this graph there are two periods, but in fact scanning only needs to be undertaken in one period since they are periodic.}
\end{figure}

From Equation (5.1) the relative phase shift, relative absorption and relative visibility can be defined [26]:

\[
\Delta \varphi \equiv \varphi_s - \varphi_r, \tag{5.2}
\]

\[
I_0^s \equiv \frac{I_0^s}{I_0^r}, \tag{5.3}
\]

\[
V \equiv \frac{V_s}{V_r} = \frac{a_r I_0^r}{a_s I_0^s}, \tag{5.4}
\]

where the subscript \( s \) refers to the image containing the sample, while \( r \) refers to the image without the sample used as a reference as exemplified in Figure 5.3. For
every pixel in the reference image and the sample image three parameters can be deduced, as illustrated in Figure 5.2. To prevent aliasing, the number of scan steps should be larger than three. These three parameters for all pixels together constitute three images: the standard absorption, differential phase-contrast and dark-field image as shown in Figure 5.4.

**Figure 5.3.** The reference image and sample image for each phase stepping scan. In this example the period is divided into 4 steps and no noise is considered. The sample is simulated as a blood vessel with CO₂ as contrast agent inside a soft tissue with a thickness of 2 cm as discussed in 5.3. For reference images there is only the soft tissue. As we can see from Equation (5.1), there are three unknown parameters to find, and therefore at least three measurements over a period are needed [31].

$I₀$ tells the mean value of the sinusoidal curve which is the same as the intensity in the in-line propagation method in the same distance without using the grating interferometer – absorption and edge enhancements, depending on the distance $d$ and the detector resolution.
The phase $\Delta \varphi$ presenting the shift of the interference pattern which has a range of $[-\pi, \pi]$ for a period $p_2$ leads to a relation with transverse shift $S(m,n)$ as

$$S(m,n) = \Delta \varphi(m,n) \frac{p_2}{2\pi}. \quad (5.5)$$

From Equation (3.5), the differential phase shift can be calculated as

$$\frac{\partial \phi(m,n)}{\partial x} = \frac{p_2 \Delta \varphi(m,n)}{\lambda d}. \quad (5.6)$$

Thus the image made of $\Delta \varphi$ can reveal the differential phase shift of the object with only a constant factor difference, and it is called the differential phase-contrast image. From $\frac{\partial \phi}{\partial x}$ the real phase shift $\phi$ can be calculated by integration as shown in Figure 5.5, which also makes it easier to verify the correction by comparing it with the true phase shift. Because one-dimensional integration is undertaken here, it is easily affected by noises, as shown in Figure 5.5, where noise lines and asymmetric section profiles are obvious. Different methods are used to get better looking integration results such as 2-D integration [32].

The normalized visibility $V \equiv \frac{V_s}{V_r}$ can be regarded as a measurement of incoherent scattering which reduces the visibility of the interference pattern. This effect is significant at the edges of the object as can be noticed in the third part of Figure 5.4.
Figure 5.4. Standard absorption, differential phase-contrast and dark-field image that represent the three parameters from fitting. In the right column are their section profiles in $x$ direction. The object is a CO$_2$ cylinder with 100 $\mu$m diameter surrounded by soft tissue with thickness of 2 cm to represent a small animal blood vessel with CO$_2$ as contrast agent. Therefore the cylinder has lower absorption than the background as in the top right figure.

Figure 5.5. Simulation results (a) of the test sample containing one polytetrafluoroethylene (PTFE) and two polymethylmethacrylate (PMMA) spheres compared to experimental results (b), from Ref. [22], reprinted with permission. Compared are the phase gradient images $d\phi/dx$, the object phase shift $\phi$, and their section profiles. In Ref. [22], a standard X-ray tube with a mean energy value 22.4 keV is used while in simulation a monochromatic X-ray is used, which leads to a different resolution. A smaller size of sample used in the simulation leads to a smaller phase shift. Details of the detector and noise level are also not given and cause differences in result.
5.2 SNR Analysis

The noise is assumed to be uniform variance, uncorrelated Gaussian noise, thus the SNR can be calculated as [33]

\[
SNR^2 = \sum_{m} \left| \frac{\Delta s_m}{\sigma_m} \right|^2, \tag{5.7}
\]

where \( \Delta s_m \) is the signal difference of the reference image and the sample image, and \( \sigma_m \) is the standard deviation of each pixel. The noise distribution of the reference image and the sample image is approximately the same, thus we can use the \( \sigma \) of the reference image instead. Thus Equation (5.7) is simplified to

\[
SNR^2 = \frac{1}{\sigma^2} \sum_{m} \left| \Delta s_m \right|^2 = \frac{\left| \Delta s \right|^2}{\sigma^2}. \tag{5.8}
\]

The variance \( \sigma^2 \) is inversely proportional to the number of scan steps \( N \) as shown in Figure 5.6, which means \( SNR^2 \) is proportional to \( N \) [28]. The absorbed dose is also proportional to \( N \) if the exposure time of each step keeps the same. Therefore \( N \) has no effect on the dose at a specific \( SNR^2 \) level.

![Graph showing the relationship between inverse of sigma squared and step number.](image)

**Figure 5.6.** The variance \( \sigma^2 \) is inversely proportional to the number of scan steps \( N \). \( N \) is taken from 3 to 16 as a normal range for experiments. \( \sigma^2 \) is calculated from the DP images. The exposure time is the same for every step.
Figure 5.7. Using different exposure times while keeping the other parameters consistent, images of different noise level can be obtained. (a) Exposure time of 600 s with a source having 10 W power and 0.1% efficiency, (b) Exposure time 30 s. It is very obvious that the differential phase-shift (DP) image has the much higher SNR than the other two images.

As shown in Figure 5.7, the differential phase shift has the highest SNR under different exposure time, therefore it is used for the comparison in Section 5.3. Since the standard deviation needs to be calculated here, the total number of pixel cannot be too small in order to make it statistically significant.

From Figure 5.8 we can see \( \text{SNR}^2 \) is proportional to the photon number, as well as the dose, thus \( \text{SNR}^2 \) is proportional to dose, which can be used to calculate the dose when \( \text{SNR}^2 \) is known after simulating with a random photon number (large enough to have Gaussian noise).
Figure 5.8. The relation between $\text{SNR}^2$ and the exposure time. With other assumptions constant, exposure time is proportional to the photon number detected. $I_0$, $\Delta \varphi$ and $a$ are presented in square, circle and triangle respectively. $\Delta \varphi$ has the highest $\text{SNR}^2$ obviously. The red line is the linear fitting.

5.3 Comparison

In this section, comparisons between the grating-based and the propagation-based phase-contrast method are made in view of the required dose when $\text{SNR}^2 = 25$.

Because of the interest in angiography, the object in comparison is specified to be a blood vessel inside a sample of soft tissue. Inside the vessel there supposed to be carbon dioxide (CO$_2$) as contrast agent [34]. To simplify, the soft tissue is set as a cube with thickness 2 cm, which is to simulate a small animal. The vessel is modeled as a cylinder with different diameters [35]. Meanwhile, in order to get rid of the influence of the length of the vessel on the SNR, the cylinder is specified to have the same length as its width.

A monochromatic point source is simulated in order to approach an ideal situation. The parameters are summarized in Table 5.1. The energy is chosen to be 20 keV within the range of hard X-ray, which is normal for small-animal imaging. For in-line propagation method the object-detector distance is set to 1.5 m since it is used in experiments [35] and for grating-based method the distance is the 5th Talbot distance as discussed in Section 4.2.3.
5.3 COMPARISON

<table>
<thead>
<tr>
<th></th>
<th>Propagation-based</th>
<th>Grating-based</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source</td>
<td>Point source (Gaussian, FWHM 8 mm)</td>
<td></td>
</tr>
<tr>
<td>Photon energy</td>
<td>20 keV monochromatic</td>
<td></td>
</tr>
<tr>
<td>Object</td>
<td>2 cm soft tissue; 10-2000 μm blood vessel (CO₂ as contrast agent)</td>
<td></td>
</tr>
<tr>
<td>Source-object distance</td>
<td>1.5 m</td>
<td></td>
</tr>
<tr>
<td>Object-detector distance</td>
<td>1.5 m</td>
<td>5\textsuperscript{th} Talbot distance (0.16 m)</td>
</tr>
<tr>
<td>Detector resolution</td>
<td>9 μm</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1. A summary of parameters used in the simulations.

The result is illustrated in Figure 5.9, which indicates that the differences in dose of these two methods are small. For grating-based method, which is the red dashed line, when the diameter of the vessel is small, 10-50 μm, compared to the detector pixel size, the differential phase gradient cannot be fully resolved which drags down the SNR. While for propagation-based method, when diameters are small, since the object-detector distance is much larger than $d$ in grating-based method, the magnification is much larger, which means with the same detector pixel size the image resolution is higher for propagation-based method, so more fine edge enhancements can be detected which leads to larger SNR. And both of them approximately have a $d^2$ dependency and require much lower dose than absorption-contrast imaging for small diameter [35].

Simulation under a monochromatic Gaussian distribution source with FWHM 8 μm is also used to compare these two methods and the result is illustrated in Figure 5.10. The doses increase by approximately a factor of 1.7 for propagation-based method and 6.6 for grating-based. The doses of grating-based method are all higher than the propagation-based method. The grating-based method seems to be more sensitive to the source blurring than the propagation-based method.
Figure 5.9. Dose-diameter relation of the in-line propagation method and the grating-based method (Talbot imaging). Compared under a point source with photon energy 20 keV. The diameter of the blood vessel is from 10 to 2000 \( \mu \text{m} \). The detector resolution is 9 \( \mu \text{m/pixel} \). The distance from the source to the object is 1.5 \( \text{m} \) for both. For propagation-based method, the distance between the object to the detector is 1.5 \( \text{m} \), while for grating-based the 5\(^{th} \) fractional Talbot distance (0.16 \( \text{m} \)). Red dashed line is for grating-based method while the black solid line for propagation-based method.

Figure 5.10. Comparison under a 20 keV monochromatic Gaussian distribution X-ray source.
Chapter 6

Conclusion

In this thesis the principle of phase-contrast imaging and simulation of the grating-based method is described. Simulation results are presented and comparisons between the propagation-based method and the grating-based method are made.

Theoretical description of the wave propagation process is given, which includes the Fresnel diffraction algorithm and the Talbot effect. The Fractional Talbot distance is used corresponding to the phase grating. Experimental setups are explained. Simulation of the grating-based method is done during the thesis project. The object is simulated to be small animal blood vessels with carbon dioxide as contrast agent.

Grating-based phase-contrast imaging technique can yield a series of images from phase-step scanning process, from which three parameters can be fitted out, therefore standard absorption, differential phase-contrast and dark-field images can be constructed. Analysis of the results is illustrated. To evaluate image quality, SNR is used. Comparisons of doses when SNR^2 equals 25 are made between the propagation-based method and grating-based method. The result indicates that the latter one requires slightly lower doses for vessels with large diameters but more for small diameters under a point source, and under an extended source the grating-based method requires higher doses.

It indicates that both of these two phase-contrast methods are applicable for small animal imaging and works much better than conventional absorption-contrast imaging for viewing fine details. The grating-based method can also work with a standard X-ray tube though the high resolution is lost, which makes it more applicable for medical imaging at local hospitals. But the dose is always needed to be considered and more work will be done on this.
Acknowledgments

Here I would like to express my gratitude to those who have helped and supported me during my master thesis work.

First of all, I would like to thank my supervisor Anna Burvall for her kindness as a good friend or big sister, patience in teaching and helping me, and encouragement when I lack confidence. Without her offer of this thesis work, I could be totally lost in last summer.

Thank Ulf Lundström for his XRaySimulator, his previous work which teaches me a lot from the principle to simulation, and his patience in helping me with programming. Thank Daniel Larsson for his help during this thesis project.

Thank my examiner Hans Hertz for his encouragement and big support. Thank our cute master students’ coordinator Kjell Carlsson, for his patience and help since the first week I came to Sweden. Thank Göran Manneberg for his brilliant teaching and help in many courses and his encouragement when I faced problems in the first semester here. Thank Ulrich Vogt for his X-ray course, and Björn Cederström for his medical imaging course that give me the background knowledge I need for this thesis. And thank all my teachers that helped me.

I would also like to thank all the students and staff at BIOX who make here such a warm place. Thank Goldis Hadialhejazi, Emelie Hansson, Ann-Katrin Batzer and other students for their companion in the cold office.

Thank my dear friends in Sweden, in China or in other countries now for keeping me far away from lonely. Special thanks to Fei Yang who gives me many good suggestions and recommended me to choose the course Optical System Design where I met my supervisor.

At last, thank my dearest families for their support and love forever.
Bibliography


