Modelling of particle flows in the magnetosphere of Mercury

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Abstract

Mercury is the innermost planet of the solar system. In 1974 the NASA spacecraft Mariner 10 discovered that Mercury has an intrinsic magnetic field and a magnetosphere. The Mercury magnetic field may be approximated with a dipole which is offset by 0.2 Mercury radius from the centre.

The photons from the Sun that reach the Mercury surface will cause photoemission and the emitted electrons will be partially trapped in the Mercury magnetosphere. Because of the planet’s proximity to the Sun as well as the absence of a dense atmosphere the photoemission is significant. The shift of the dipole makes Mercury an interesting point of study because some of the electrons photoemitted from the south surface will mirror whereas all electrons from the north will reach the south surface.

This asymmetry will cause a small potential difference that accelerates the particles and changes the loss cone angle. The associated current circuit is assumed to be closed by an internal resistance inside the planet. The study of the values of the electric field, the current and the internal resistance in Mercury is the aim of this report.
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Chapter 1

Introduction

Mercury, with a radius of 2440 km, a mass of $3.30 \cdot 10^{23} kg$ and an average distance to the Sun of 0.38AU, is the innermost planet and the smallest planet of the solar system. It is a terrestrial body like the Earth (see a size comparison of the terrestrial planets in figure 1.1). Because it is very close to the Sun it can be seen for a short period of time during sunrise or sunset. The Romans named it Mercury after the Roman messenger god, maybe due to Mercury’s fast apparitions in the sky. But the first civilization to discover the planet was the Sumerians in the 3rd millennium BC.

Mercury has a rotation period of 59 Earth days and an equatorial magnetic field strength of 350nT (see page 100 in [1]). The solar wind at Mercury is denser than at the Earth because it is closer to the Sun. The surface temperature on Mercury is 700K during daylight and 90K in the night side because it has no atmosphere.

Mercury has a high density of $5.43 g/cm^3$, which is similar to those of the Earth ($5.5 g/cm^3$) and Venus ($5.2 g/cm^3$). However Earth and Venus are

Figure 1.1: Terrestrial planets size comparison (image created by NASA).
From left to the right: Mercury, Venus, Earth and Mars
large enough to compress their interior mass, but Mercury is not. The high density of Mercury can be explained by the composition of heavy elements (70% Iron-Nickel and 30% Silicate material). Most of the iron is believed to be concentrated in the core of the planet, and the abundance of iron explains the presence of Mercury’s magnetic field.

On November 2, 1973 Mariner 10, the first spacecraft to reach Mercury was launched. The spacecraft mapped 40-45% of the Mercury surface. It made three fly-bys near Mercury. The first and the third encounters with Mercury were very close to the planet, so it was possible to measure the magnetic field of the planet. The measurements revealed that Mercury has a magnetosphere but has a very unstable atmosphere.

The mission BepiColombo, a cooperation between the European Space Agency and the Japan Aerospace Exploration Agency will go to Mercury and will be equipped with scientific instruments to provide more images of the planet and will also measure the magnetic field. It will be launched in August 2013 and after a planetary cruise of six years it will be inserted into Mercury orbit in August 2019. Before BepiColombo, the NASA mission MESSENGER will be in orbit around Mercury in March 2011.

The magnetosphere is supported by its intrinsic magnetic field. The magnetic field has been modeled several times, with different assumptions. The dipole centred to the planet was not a good approximation because of the asymmetry of the magnetic field. A dipole shifted to the north by distance of 0.2 Mercury radius was a better adjustment to the measures (it is plotted in 1.2). But a quadrupole can also be a good approximation to the Mercury magnetosphere.

The shift of the dipole makes an interesting point of study because it creates magnetic mirrors so that some particles can reach the Mercury surface in the south but will never get the north surface.

The aim of this project is to make a model of the particle flow in the magnetosphere of Mercury. Will be considered the approximation of Mercury as a dipole shifted a distance of 0.2 Mercury radius. The electrons are photo-emitted from the surface, and because of the asymmetry of the magnetic field in the sunlit surface will lead to a flowing current. This current will continue flowing if it can form a closed circuit, or it will be impeded by an electric field.

Because Mariner 10 did not measure the electric field is not possible to know exactly the particles flow. The spacecraft BepiColombo will make more measurements of the Mercury magnetic and electric field, and may help to find out whether the assumptions done in this project are correct or not.

\[1\] For further explanation see [2]
Figure 1.2: 3 dimensional plot of a dipole centred 0.2 times the planet radius.
Chapter 2

Mercury as a dipole

According to the measurements of the Mariner 10, a dipole shifted 0.2 RM to the north of the center of Mercury is a good approximation for its magnetic field (a sketch is presented in figure 2.1).

The solar photons arrive unattenuated to the Mercury surface and emit electrons. The electrons follow the magnetic field lines, gyrating around them. If the electron has a bigger angle than the loss cone angle it will mirror and it may get trapped in the Mercury’s magnetosphere. Otherwise the electron will reach the conjugate surface.

The particles that will mirror will be the ones with a pitch angle greater than:

$$\alpha = \arcsin \sqrt{\frac{B_s}{B_n}}$$  \hspace{1cm} (2.1)

$\alpha$ is the loss cone angle, $B_s$ is the magnetic field in the south day surface and $B_n$ is the magnetic field in the north day surface same magnetic field line. Electrons that have a bigger angle than $\alpha$ will mirror and will not contribute to the current flow. Whereas the electrons with a pitch angle smaller than $\alpha$ will reach the conjugate surface, creating a current flow. This current depends on the magnetic field line, the incident angle of the sun light to the Mercury surface, and also the direction of the emitted electrons.

Because of this, a small potential difference will exist between the south and the north, and there will also be an electric field.

The electric field will accelerate the electrons and will change the the loss cone angle. But the exact value of the electric field is not known so we will work with a possible range of values for the electric field.

The following pages are a detailed study of particle flows in the Mercury magnetosphere, approximating the planetary magnetic field with an offset dipole.
Figure 2.1: Mercury surface and magnetic field lines. The magnetic field is represented by a dipole shifted 0.2 Mercury radius from the center of the planet. The angle is the colatitude from the centre of the dipole ($\theta'$) in degrees and the radius $r$ is the distance from the centre of the dipole in meters.
2.1 Dipole without an electric field

To calculate the magnetic field we consider a dipole in spherical coordinates:

\[ B_r = \frac{\mu_0 m \cos \theta'}{2\pi r^3} \]  \hspace{1cm} (2.2)

\[ B_\theta = \frac{\mu_0 m \sin \theta'}{4\pi r^3} \]  \hspace{1cm} (2.3)

\( \theta' \) is the angle from the axis of the dipole, \( r \) is the distance from the center of the dipole and \( m \) is the magnetic moment (see chapter 4.8 at [1]). A sketch of the coordinates is found in figure 2.2. The modulus of the magnetic field is:

\[ B = \frac{\mu_0 m}{4\pi} \frac{1}{r^3} \sqrt{3\cos^2 \theta' + 1} \]  \hspace{1cm} (2.4)

The distance from the centre of the dipole to the Mercury surface is:

\[ r^2 = (R_H \sin \theta)^2 + (R_H \cos \theta - y_0)^2 \]  \hspace{1cm} (2.5)
\( \theta \) is the colatitude centred in the planet, \( R_H \) is the Mercury radius and \( y_0 \) is the dipole offset. The next expression relates the Mercury coordinates with the dipole coordinates:

\[
r \cos \theta' = R_H \cos \theta - y_0
\]  

(2.6)

We now get for the magnetic field at the surface of the planet for a dipole shifted \( y_0 \) from the centre:

\[
B = \frac{\mu_0 m}{4\pi} \sqrt{\frac{3R_H^2 \cos^2 \theta - 8R_H \cos \theta y_0 + 4y_0^2 + R_H^2}{(R_H^2 - 2R_H \cos \theta y_0 + y_0^2)^2}}
\]  

(2.7)

Taking into account that the dipole is offset a distance 0.2 times the Mercury radius we get:

\[
B = \frac{\mu_0 m}{4\pi} \sqrt{3 \cos^2 \theta - 1.6 \cos \theta + 1.16} \frac{1}{R_H^2(1.04 - 0.4 \cos \theta)}
\]  

(2.8)

Where \( \theta \) is the colatitude of the planet and \( R_H \) is the Mercury radius. In the case of Mercury, \( \mu_0 m/(4\pi) \) is estimated to be \( 2 - 6 \cdot 10^{12} Tm^3 \) (see [3]). We choose the value \( 5 \cdot 10^{12} Tm^3 \) for our calculations. The magnetic field in the Mercury surface is represented in the plot 2.3.

The magnetic field line follows the relation 2.9.

\[
r = A \sin^2 \theta'
\]  

(2.9)

where \( r \) is the distance from the center of the dipole to the magnetic field line, \( A \) is constant for a magnetic field line and \( \theta' \) is the colatitude centered in the dipole.

The solar photons will reach the Mercury surface and cause photoemission. The points in a magnetic field line where the photoemission is triggered are the ones where the magnetic field line crosses the Mercury surface. These points are calculated equating the expression of the magnetic field line in a dipole (2.9) and the expression of the Mercury surface (both in the same coordinate system, centered in the dipole).

The Mercury surface written in cartesian coordinates is:

\[
R_H^2 = x^2 + (y - y_0)^2
\]  

(2.10)

\( R_H \) is the Mercury radius, \( x \) and \( y \) are represented in 2.2, and \( y_0 \) is the dipole offset. Rewriting 2.10 in polar coordinates the result is:

\[
R_H^2 = r^2 - 2r \cos \theta' y_0 + y_0^2
\]  

(2.11)
Figure 2.3: Magnetic field in the Mercury surface against the colatitude dependence.

Then, using the expression for $r$ of equation 2.9, and substituting into 2.11 the result is:

$$R_H^2 = A^2 \sin^4 \theta' - 2A \sin^2 \theta' \cos \theta' y_0 + y_0^2$$  \hspace{1cm} (2.12)

Which for $y_0 = 0.2R_H$ is:

$$0 = A^2 \sin^4 \theta' + 0.4A \sin^2 \theta' \cos \theta' R_H - 0.96R_H^2$$  \hspace{1cm} (2.13)

This expression can be solved with numerical methods, and the results are two angles (centred in the dipole) that give the coordinates where the magnetic field line goes through the Mercury surface.

Using that result we can calculate the magnetic field in the points where the flux tube reaches the Mercury surface. The magnetic field values in the north and in the south are needed to calculate the loss cone angle in the expression 2.1.

The loss cone angle calculated in that way is around $32^\circ - 55^\circ$ (it depends on the chosen colatitude). The particles with a bigger angle than the loss cone angle will mirror, and the particles with an angle smaller than the loss cone angle will reach the conjugate surface.

The photo electron current is about 4.5 $\mu A/m^2$ at Earth orbit, and we assume that it will be 5-11 times stronger at Mercury (see [4]).
The sun light reach the Mercury surface with an angle $\varphi$ respect to the Mercury surface normal, so the current from the photoemission depends on the incidence angle with $\cos \varphi$. The angle of the sun light is related to the Mercury colatitude as $\varphi = \pi/2 - \theta$, where $\theta$ is the colatitude.

The current of the southern photoemitted particles and the northern photoemitted particles are:

$$j_S = j_0 \cdot \cos \left(\frac{\pi}{2} - \theta_S\right)$$ \hspace{1cm} (2.14)

$$j_N = j_0 \cdot \cos \left(\frac{\pi}{2} - \theta_N\right)$$ \hspace{1cm} (2.15)

$\theta_S$ and $\theta_N$ are the colatitude of the surface points in the southern part of the magnetic field line and in the northern part of the magnetic field line respectively. $j_0$ is the photo electron current (it will take the values of 22.5-49.5 $\mu A/m^2$).

Then with the known loss cone angle we can calculate the probability of the electron to mirrors or not. For an isotropic distribution, the probability that the electron mirrors is:\footnote{see page 378 in [5]}

$$\frac{\Delta \Omega}{2\pi} = 1 - \cos \alpha$$ \hspace{1cm} (2.16)

$\Delta \Omega/(2\pi)$ is the probability of the particle to have the angle $\alpha$ or bigger.

So, taking into account the current distribution and the angle of the sun rays, the current density for the particles that mirror is estimated with the expression:

$$j_{SM} = j_0 \cos \left(\frac{\pi}{2} - \theta_S\right) \left[1 - \cos \alpha\right]$$ \hspace{1cm} (2.17)

The current density is related to the current by expression:

$$I = \int_S j \cdot d\vec{A}$$ \hspace{1cm} (2.18)

$A$ is the surface of the flux tube. Because $\vec{j}$ is parallel to $d\vec{A}$ in a field line the equation 2.18 becomes $I = j \cdot A$.

We consider for a flux tube to have an area of $1m^2$ in the north. So the area of the conjugate surface will be bigger than $1m^2$ and it can be calculated with the conservation of the magnetic flux in a flux tube:

$$\Phi = A_S \cdot Bs = A_N \cdot Bn$$ \hspace{1cm} (2.19)

Then, with 2.14, 2.15, 2.17 and 2.19 the total current is:
Mercury as a dipole

Figure 2.4: Plot of the current for several angles of Mercury without an electric field. The blue line is for the density current of 49.5 $\mu A/m^2$ and the red line is for the density current of 22.5 $\mu A/m^2$.

$$I = j_S A_S - j_N \cdot 1 - j_{SM} A_S$$  \hspace{1cm} (2.20)

The current for particles from the north surface that mirrors is zero, because the magnetic field in the north is bigger than the magnetic field in the south.

The result for several colatitudes is presented in the figure 2.4.

2.2 Dipole with an electric field

Because some particles originating in the south will mirror but no particles originating from the north, there will be a net current flow from the south towards the north (because the particles are electrons and the current flows in opposite direction than the negative charges), resulting in a larger concentration of electrons in the south than in the north, inducing a potential difference. So there will exist an electric field accelerating the particles, and modifying the loss cone angle.

In order to simplify the problem we assume that the electric field will be constant along the magnetic field line, and will always have the direction of the magnetic field line (hence $\vec{E}$ is always parallel to the magnetic field
2.3 Loss cone angle in an electric field

The potential difference between the north and the south surface of the planet in the same magnetic field line is:

\[ V = \vec{E} \cdot \vec{L} = EL \] (2.21)

\( V \) is the potential difference between the north and the south surface. \( L \) is the length of the magnetic field line between the north surface and the south surface. To calculate \( L \) we consider a differential of length in polar coordinates:

\[ dl^2 = dx^2 + dy^2 = dr^2 + r^2 d\theta'^2 \] (2.22)

\( \theta' \) is the colatitude angle from the center of the dipole. Then, to put all the equation in the same differential we make use of relation 2.9:

\[ dr = \frac{\partial r}{\partial \theta'} d\theta' = 2A \sin \theta' \cos \theta' d\theta' \] (2.23)

So to know the length it is necessary to solve the integration:

\[ L = \int_{\theta'_0}^{\theta'_1} dl = A \int_{\theta'_0}^{\theta'_1} \sin \theta' d\theta' \sqrt{3 \cos^2 \theta' + 1} \] (2.24)

\( \theta' \) is the colatitude from the center of the dipole, analyzed from \( \theta'_0 \) (the initial angle) to \( \theta'_1 \) (the final angle). Thus, the length in a magnetic field line is:

\[ L = \frac{A}{2} \left[ \cos \theta' \sqrt{3 \cos^2 \theta' + 1} + \frac{1}{\sqrt{3}} \ln \left( 2 \sqrt{9 \cos^2 \theta' + 3} + 6 \cos \theta' \right) \right]_{\theta'_0}^{\theta'_1} \] (2.25)

Mariner 10 did not measure Mercury’s electric field, so the value of the electric field is unknown. In this study we shall use a range of values of the electric field.

2.3 Loss cone angle in an electric field

The energy is conserved, and for each particle it is calculated as:

\[ W = K_\parallel + K_\perp + qEl \] (2.26)

\(^2\)In order to solve the integral it is useful to change the variable to \( t = \cos \theta \) and use the integrals 14.285 and 14.280 in [6]
$K$ is the kinetic energy and $W$ is the total energy, and because of energy conservation the energy will remain constant. $l$ is the length of the dipole magnetic field line calculated with the expression 2.25 from the point considered to the reference point.

$K$ can be separated into perpendicular and parallel components. Because of the conservation of the first adiabatic invariant (relation 2.27), the perpendicular component of the kinetic energy is known.

$$\mu = \frac{mv_\perp^2}{2B} = \frac{K_\perp}{B} = \text{constant} \quad (2.27)$$

To calculate the new loss cone angle (the one modified by an electric field) it is necessary to take into account the first adiabatic invariant 2.27 and the energy conservation 2.26.

We choose the potential origin in the north surface, then $qEL_N$ is zero. The energy at the north and the south surfaces are:

$$W_{\text{total north}} = K_{\parallel N} + K_{\perp N} \quad (2.28)$$

$$W_{\text{total south}} = K_{\parallel S} + K_{\perp S} + qEL \quad (2.29)$$

$K_{\parallel S} = 0$ because the aim is to calculate the loss cone angle (the pitch angle for the particle will mirror exactly on the south surface). $W_{\perp N}$ and $W_{\perp S}$ are given by 2.27. And for the energy conservation $W_{\text{total north}} = W_{\text{total south}} = W$. Then by isolating $\mu$:

$$W - K_{\parallel N} = \frac{B_N}{B_S} [W - qEL] \quad (2.30)$$

Because the potential origin is at the north, then all the energy is kinetic energy $K_{\parallel N} = W \cos^2 \alpha_{NS}$, where $\alpha_{NS}$ is the loss cone angle of the electrons that follow the field line from the north towards the south. The particles that mirrors will have a pitch angle bigger than the loss cone angle:

$$\sin^2 \alpha_{NS} \geq \frac{B_N}{B_S} \frac{W - qEL}{W} \quad (2.31)$$

With a similar treatment we get the expression for the loss cone angle of the particles from the south that will mirror:

$$\sin^2 \alpha_{SN} \geq \frac{B_S}{B_N} \frac{W + qEL}{W} \quad (2.32)$$
Figure 2.5: Plot of the loss cone angle against the electric field for a magnetic field line that coincides with the colatitude of $\theta = 135^\circ$ and $W = 3\,\text{eV}$. The loss cone angles are calculated with the relations 2.31 and 2.32.

With different values of $E$ the loss cone angle will change. We see from expression 2.32 that for $E=0$ the loss cone angle is the same as in 2.1. And for a bigger $E$, electrons from the north will also mirror. And for a larger $E$, just the particles from the north will mirror.

2.4 Maxwellian distribution

We assume that the photoemitted particles have a temperature of $2\,\text{eV}$ ($k_B \cdot T = 2\,\text{eV}$, where $k_B$ is the Boltzmann constant), and they are emitted isotropically following a Maxwellian distribution. So the particles will be emitted with all velocities, and the velocity that more particles will be photoemitted (the average energy of random motion) is $3\,\text{eV}$.$^3$ It is represented the loss cone angle against the electric field for the average random energy and for a given colatitude in 2.5.

$^3$The average energy of random motion is calculated with:

$$\left\langle \frac{1}{2}m (\vec{v} - \vec{u})^2 \right\rangle = \frac{N k_B T}{2}$$

$N$ is the number of spatial degrees of freedom of the particle distribution. In our case $N=3$ and $\vec{u} = 0$. 
To calculate the distribution of the particles, we need to take into account the distribution function for particles that follow a Maxwellian distribution:

$$f(\vec{r}, \vec{v}) = n \left( \frac{m}{2\pi kT} \right)^{3/2} \exp \left[ -\frac{m(\vec{v} - \vec{u})^2}{2kT} \right]$$ \hspace{1cm} (2.33)

\(\vec{r}\) is the space position, \(\vec{v}\) is the velocity of individual particle, \(n\) is the number density (number per unit volume), \(m\) is the electron mass and \(\vec{u}\) is the average velocity of a collection of particles. The integral of 2.33 for all possible directions is:

$$\int_{\text{directions}} f(\vec{r}, \vec{v}, t) \, d\Omega v^2 \, dv = \left[ 4\pi f(\vec{r}, |v|, t) \, v^2 \right] \, dv = g(\vec{r}, v) \, dv$$ \hspace{1cm} (2.34)

Here \(g(\vec{r}, v) \, dv\) is the number of particles per unit velocity with velocities between \(v\) and \(v + dv\). \(g(\vec{r}, v) \, dv\) against the velocity is plotted in figure 2.6.

![Figure 2.6: Plot of the normalized Maxwellian distribution.](image)

For the particles from the north surface the total energy is given by the kinetic energy (because the north point is the potential origin):

$$W = \frac{1}{2} m v_{NS}^2$$ \hspace{1cm} (2.35)

\(^4\text{It is explained in pages 34-38 in:}[7]\)
2.4 Maxwellian distribution

And for particles from the south the total energy is given by the kinetic energy and the electric energy:

\[ W = \frac{1}{2}mv_{SN}^2 + qEL \]  \hspace{1cm} (2.36)

In order to know the number of particles that have a smaller velocity than the velocity given for the loss cone angle, 2.34 is integrated from zero to loss cone angle velocity. The velocity for the loss cone angle is calculated isolating the energy in the equations 2.31, and 2.32, and using the total energy equations 2.35 and 2.36. Then the velocity for a particle in the north surface for a given loss cone angle is:

\[ v_{NS} \leq \sqrt{\frac{-2qEL}{m(1 - \sin^2 \alpha_{NS} \frac{B_N}{B_S})}} \]  \hspace{1cm} (2.37)

\( q \) is the electron charge \((q = -1.602 \cdot 10^{-19}C)\), \( E \) is the electric field, \( L \) is the length of the magnetic field line from the north to the south, \( m \) is the electron mass, \( \alpha_{SN} \) and \( \alpha_{NS} \) are the loss cone angles at the south and at the north surface respectively, \( B_N \) is the magnetic field in the north surface of the magnetic field line and \( B_S \) is the magnetic field in the south surface of the magnetic field line. And for the south velocity, if \( B_N/B_S \sin^2 \alpha_{SN} \geq 1 \) is:

\[ v_{SN} \geq \sqrt{\frac{4qEL}{m(1 - \sin^2 \alpha_{SN} \frac{B_N}{B_S})}} \]  \hspace{1cm} (2.38)

And for \( B_N/B_S \sin^2 \alpha_{SN} \leq 1 \) all the particles will reach the conjugate surface. \( v_{NS} \) and \( v_{NS} \) are plotted in figures 2.7 and 2.8 respectively.

\( v_{NS} \) is always positive \((q \) is always negative), but \( v_{NS} \) for several loss cone angle has a negative square root. It is necessary to consider carefully the sign of the function in order to know the number of particles that mirrors.

Using the Maxwellian distribution the ratio of the mirroring particles is:

\[ p = \frac{\int_0^{v_{lc}} g(\vec{r}, v) \, dv}{\int_0^{\infty} g(\vec{r}, v) \, dv} \]  \hspace{1cm} (2.39)

where \( v_{lc} \) is the velocity for a given loss cone angle (if the particles from the north are considered then \( v_{lc} \) is 2.37 and for particles from the south \( v_{lc} \) is 2.38). The numerator of 2.39 can be evaluated with numerical methods. The denominator is a known integral and the result is:\(^5\)

\(^5\)It is solved in the book [6] integral 15.77
Figure 2.7: Plot of $v_{NS}$ against the $\alpha_{NS}$ for the colatitude $\theta = 135^\circ$. The particles that will mirror will be the ones with a velocity below the line.

\[ \int_0^\infty g(\vec{r}, v) \, dv = n \]  

Next we calculate the current coming from the north (or from the south) that mirrors for a specific loss cone angle, colatitude and electric field.

The current for a particle that mirrors is:

\[ j_M = \int_0^{\pi/2} j_0 \cdot p \cdot \cos \varphi (1 - \cos \alpha_{lc}) \cdot d\alpha_{lc} \]  

The total current flow for a surface of 1$m^2$ flux tube in the north is:

\[ I = j_{NM} \cdot 1 + j_S \frac{B_N}{B_S} - j_N \cdot 1 - j_{SM} \frac{B_N}{B_S} \]  

Where $j_{NM}$, $j_S$, $j_N$ and $j_{SM}$ are the current densities of the particles from the north that will mirror, the particles from the south (calculated with 2.14), the particles from the north (calculated with 2.15) and the particles
2.4 Maxwellian distribution

Figure 2.8: Plot of $v_{SN}$ against the $\alpha_{SN}$ for the colatitude $\theta = 135^\circ$. For the pitch angles from zero till approximately 40 degrees all the particles will reach the conjugate surface. And for a bigger pitch angle will mirror the particles above the line.

from the south that will mirror respectively (see figure 3.3). The surface in the south is calculated using conservation of magnetic flux in a flux tube (equation 2.19).
Chapter 3

Results

What is obtained is a relation between the current and the electric field. For a given latitude of the planet we get different representations, as the ones in figures 3.1. They are the representation of the current against the potential difference (the potential difference is calculated with the relation 2.21). Each plot has two lines, the blue one is for the maximum photo electron current \((j = 22.5 \mu A/m^2)\) and the black one is for the minimum photo electron current \((j = 49.5 \mu A/m^2)\).

For an \(E\) near zero (and \(V\) near zero), the current is northwards. For a larger \(E\), the current is southwards the planet. But the electric field is supposed to reach stationary state in a value where the current is northwards.

A zoom at the positive part of the potential is plot in 3.2 for several colatitudes and for a density current of \(j = 49.5 \mu A/m^2\).

3.1 Internal resistance

Finally, with the Ohm Law 3.1 and knowing that the current inside the planet is equal to the current outside the planet, we can estimate the behavior of the resistance inside Mercury. We imagine the planet with an internal resistance as is sketched in the figure 3.3.

\[
I_{\text{inside}} = \frac{V}{R} = \frac{EL}{R} \quad (3.1)
\]

\(V\) is the potential difference between the north surface and the south surface (calculated by the expression 2.21), \(L\) is the length of the magnetic field line between the north and the south surface and \(R\) is the effective resistance through the planet between the footpoints of the field line considered.

It is possible to plot the resistance against the electric field and against the current. The resistance is a function of the electric field and the current, and it’s related with the Ohm’s Law 3.1. At figure 3.4 and 3.5 is plotted the resistance against the electric field and the current for a given latitude.
### 3.1 Internal resistance

Figure 3.1: Plot of the current against the electric field and the current against the potential for a Mercury colatitude of $\theta = 135^\circ$.

Figure 3.2: Zoom of the plot of the current against the potential for different colatitudes. The current is calculated with the maximum value of the photo electron current.
Figure 3.3: Sketch of the Mercury model as a dipole offset by $0.2 \ R_H$ and with an internal resistance $R$.

Figure 3.4: Plot of the resistance against the electric field and the current for the Mercury colatitude of $\theta = 100^\circ$).
It’s difficult to know the exact value of the electric field and the resistance. The lack of data from the Mariner 10 of the electric field makes it difficult to know the exact value.

The electric field associated with the effect studied here will likely be below the sensitivity threshold of future instruments. So the next Mercury spacecraft will not probably be able to measure the electric field.
Chapter 4

Total current flow

To know the total current flow it is necessary to integrate all the current for the entire Mercury surface. At the previous chapters the calculations have been done for \(1m^2\). Now the aim is to determine the current for the whole Mercury surface.

In order to do that the integration must be done for the colatitude and the longitude for all the sun lit Mercury surface.

It is done for small parts of the surface and then it is added up for the entire surface. It depends on the longitude with \(\cos \lambda\) where \(\lambda\) is the longitude coordinate.

The integral is:

\[
I_{total} = \int_{-\pi/2}^{\pi/2} \int_{\arccos 0.3}^{\pi} I d\theta \, d\lambda = 2 \cdot \int_{0}^{\arccos 0.3} \int_{0}^{\arccos 0.3} I d\theta 
\]

\(I_{total}\) is the total current, \(\lambda\) is the longitude coordinate, \(I\) is the current calculated in 2.42 but that depends on \(\lambda\) and \(\theta\) and \(\theta\) is the colatitude coordinate.

The integration is done for the south surface because the current calculated in 2.42 takes into account the particles from the north and from the south (so the north surface must not taken into account because will be considered twice).

\(\theta\) goes from \(\arccos 0.3\) till \(\pi\). \(\arccos 0.3\) is the minimum value for the \(\theta\) for the south surface (where the \(r\), the radius from the center of the dipole, is equal to \(R_H\), and it can be calculated with 2.6 and 2.11).

The integration is done for a constant resistance. Because of that it is need to get a relation between the electric field with the colatitude for a given resistance. It is done approximating the 3.2 with lines, and using the value of the resistance and the equation 3.1 we got a relation between the electric field and the colatitude.
4.1 For a given resistance

We considered two resistances, $R_1 = 10^3\Omega$ and $R_2 = 10^4\Omega$, and a current of $j = 49.5\mu A/m^2$. The correlation is a fourth degree function, because a line gave a singularity in the boundary point near $\arccos 0.3$. The second degree function was a good approximation, but we considered that the forth degree function was even better. For the first one we got:

$$V_1 = -0.02120 \cdot \theta^4 + 0.1951 \cdot \theta^3 - 0.6569 \cdot \theta^2 + 0.9498 \cdot \theta - 0.4869 \quad (4.2)$$

And for the second one:

$$V_2 = -0.1285 \cdot \theta^4 + 1.1907 \cdot \theta^3 - 4.0477 \cdot \theta^2 + 5.9255 \cdot \theta - 3.0864 \quad (4.3)$$

$\theta$ is the colatitude and for the potential should be considered with radians. The functions are plotted at 4.1 and 4.2.

The sum of all the current of the Mercury sun lit surface for the resistance $R_1 = 10^3\Omega$ is $I_{total} = 7.9669 \cdot 10^7A$. And in the 4.3 is plotted the current density in the different colatitudes and longitudes in the Mercury sun lit surface. The sum of all the current for the $R_2 = 10^4\Omega$ is $I_{total} = 5.6988 \cdot 10^7A$ and the current density for the Mercury sun lit surface is plot at 4.4.

**Figure 4.1:** Plot of the colatitude against the potential difference for the given resistance $R_1 = 10^3\Omega$. 
Figure 4.2: Plot of the colatitude against the potential difference for the given resistance $R_2 = 10^4 \Omega$.

Figure 4.3: Plot of the current for the Mercury sun lit surface for $R_1 = 10^3 \Omega$. The current density is written in $A/m^2$. 
4.1 For a given resistance

Figure 4.4: Plot of the current for the Mercury sun lit surface for $R_2 = 10^4 \Omega$. The current density is written in $A/m^2$. 
Chapter 5

Summary and conclusions

Considering the model of the Mercury magnetosphere as a dipole centred 0.2 times the Mercury radius we wanted to calculate the currents in the Mercury surface caused by the photoemission of the electrons. The electrons after being photoemitted they will follow a magnetic field line.

Due to the offset of the Mercury field line the electrons photoemitted in the north will reach the conjugate surface but no all the electrons emitted from the south will reach the north surface.

It makes a potential difference between the north and the south surface, and hence an electric field.

The electric field will change the loss cone angle of the electrons, and will change the probability of the particle to mirror. The current flow will change with the value of the electric field.

Because we don’t know the exact value of the electric field we worked with a possible range of values. We supposed that the electric field is closed in a circuit, and because of that we considered the planet to have an internal resistance. With the Ohm law we could correlate the current with the resistance and the electric field with the resistance.

After considering a magnetic field line we did a steep beyond and integrated all the current for the entire mercury surface, estimating the total current that flows in Mercury.

All the results given in the thesis are obtained from a Matlab program using the values that are written in the thesis. But they are an approximation to the possible values that Mercury can have.

Regarding the results obtained in that report the potential difference and the electric field are very small. The electric field associated with the effect studied is likely to be below the threshold of some spacecraft devices. For future missions it will be very difficult to detect the electric field associated with the photoemission of the electrons.

But besides the current originated by the photoemission the ionosphere also has electrons and positive ions flowing. These charged particles will be
affected by the electric field and will contribute to the total current. In this report we didn’t take into account these particles but their contribution can be important for the value of the total current.
Bibliography


