A Decentralized Stabilization Scheme for Large-scale Interconnected Systems

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Abstract

This thesis considers the problem of decentralized control of large-scale systems by means of a decomposition and decentralization approach. Having decomposed the system into a number of interconnected subsystems a set of local decentralized controllers is constructed for individual subsystems. An important issue is to adapt methods with existing information structure constraints. Namely, sometimes exchange of information is not possible among various subsystems, or the transmission can be costly, lossy or delayed, which the latter, is usually unavoidable due to the prevailing technological trends of present-day, in using shared wireless networks. So the best way is that individual agents utilize their own locally available information for the purpose of control and estimation. Understandably, this constraint could also make the class of stabilizable systems smaller.

Having described the basics of interconnected systems and decentralized control, a special class of stable systems is studied and an easy to follow algebraic solution for the stability of an interconnected system is put forward. Using the notion of diagonal dominance and some well known results from matrix theory, a decentralized stabilization scheme has been proposed for continuous and discrete time systems and the class of stabilizable systems has been identified using the linear programming technique. The design is reliable under structural perturbations by which subsystems are connected and disconnected again during the functioning of the system.

Finally some nice properties of the mentioned class of stable systems are shown, especially the problem of robust stability for interval matrix family is considered and it is shown that this problem admits a simple solution for the studied family of systems.

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Chapter 1

Introduction

One of the recent and most challenging problems in system theory and control is dealing with the ever growing size and complexity of mathematical models of real world processes. Indeed the amount of effort needed to analyse a large dynamical system increases more rapidly than the order of the corresponding model. This kind of problems may become unwise, costly or even impossible in practice to solve by simply using faster computers and larger memories. Typical motivating examples of this kind which are the essential parts of our modern society, arise, for instance, in control of large interconnected power distribution systems which have strong interactions, transportation and traffic systems with lots of external signals, water systems which are widely distributed in the environment, energy systems, communication systems and even socioeconomic systems. (see e.g. [2],[23], [6]). Therefore, for the purpose of stabilization analysis, control design, optimization and implementation of the strategies and algorithms, it has been widely accepted that newer and more efficient methods and ideas should be developed.

Therefore the notion of "large-scale" systems have been introduced for more than three decades as it became clear that there are practical control problems that cannot be tackled by classical on-shot methods. The general approach has been to divide the problem into simpler and smaller sub-problems which are only weakly dependent or can be treated completely independently and hence are easier to attack. However it should be noted that the notion of "largeness" in the literature of this field is subjective, and a system is usually referred as large-scale whenever it is more appropriate to be considered as a number of interconnected subsystems than dealing with the entire system as a whole, this is either for computational simplifications or for the purpose of easier treating the system conceptually (see [15]). So whenever we use the term large-scale it should not be necessarily regarded as high dimensionality of the corresponding system. In other words, instead of a normal definition of large-scale system, a more pragmatic definition has been given, meaning that when it is more convenient to deal with a portioned form of the system, we regard the system as large and we break it into smaller interactive subsystems.

After breaking the system down into smaller subsystems, a so-called *interconnected* system will be derived and as a result, the overall dynamical system is no longer controlled/estimated using a sole controller/estimator (which may also be impractical), instead, we design a number of independent local controllers/estimators that all together constitute a *decentralized* regulator for the entire plant. Since introducing decentralized control, it has always been an important control choice for interconnected systems. However, neglecting the cases in which decentralized scheme is an essential tool to construct controllers for large-scale systems, in other cases, this question may always arise that does this increase in complication end always in the improvement of the stability of the system or in the opposite direction. However this question does not have any clear general answer since the question arises in vastly wide and diverse fields such as engineering areas like electric power systems and spacecrafts and non-engineering fields like economics and environmental systems [15].

In order to make an overview of the previous works done in the field of decentralized control, we should start with mentioning about the problem of stabilization. Definitely one of the most fundamental and crucial problems in automatic control is stabilization. This is also the case in interconnected systems, in which it is required that the overall closed-loop system is stable as well as the closed-loop subsystems, by means of a decentralized scheme. Moreover, sometimes stability analysis is considered when perturbations affect the interactions among different subsystems. This was later called, the connective stability by Siljak in 1978. So, a large number of results have been published regarding connective stability.

The field of decentralized control of interconnected large-scale systems has been a topic of research since middle seventies. The first result on connective stability was based on state feedback in each subsystem by Davison [3]. Studying different time-domain methods continued until the middle of eighties, with efforts to extend the class of stabilizable systems, and it should be noted that those conditions on stabilizability are all sufficient ones. (Look at [17],[15], [20],[7], [13], [14] and references therein)

The methods which use state feedback, naturally assume that the states of each subsystem are ready to use. So, if we are not able to directly measure the states of the system, a state observer should be designed that only can use the local available information from the corresponding subsystem. However, due to the subsystem's interactions with a number of other subsystems, its dynamics is affected by the neighbours, making local observers construction not a straightforward task. Some efforts have been made in order to find non-restrictive observers for interconnected systems, so a number of authors have addressed this problem. In [18] and [21] the proposed local estimators need to be able to communicate with each other, meaning that there are some compensatory terms in the their model that stand for the interaction among individual subsystems. Constructing estimators that use neighbouring estimators' information may not be feasible when the exchange of information is not possible. A number of other authors, [22], [12] use a special class of observers called UIO, unknown input observer, that is, interaction terms are regarded as unknown inputs when designing local observers. This scheme may result into severe restrictions on the interconnection patterns and therefore reducing the range of applications.

The problem of controlling large-scale systems with uncertainties has also received considerable attention, see [23]. Hence, many researchers have proposed solutions to the problem of robust decentralized state or output feedback control of interconnected systems.

In fact, an important issue is to adapt methods with existing information structure constraints. That is, sometimes exchange of information is not possible among various subsystems, or the transmission can be costly, lossy or delayed, which the latter, for instance, is usually unavoidable due to the prevailing technological trends of present-day, in using shared wireless networks. So the best way is that individual agents utilize their own locally available information for control and estimation, however this constraint could also make the class of stabilizable systems smaller. In addition the robustness of the deigned closed-loop system under structural and external perturbations is another important issue which should be considered.

However, understandably, synthesis becomes harder in the presence of structural information constraints and stricter design requirements, and design methods are usually obtained at the expense of narrower class of systems.

Chapter 2

An Overview of Large-scale interconnected systems

The basic design problem is to find an appropriate control input $u(t)$ for a system represented by its model M based on a set of measurements $y(t)$ and command signal $r(t)$ so as to meet the design requirements.

In order to explain both classical and non-classical views to solve above problem, we should start by determining what we mean by classical information structure and classical model structures and the non-classical ones. The classical information structure corresponds with the classical centralized control; this has been demonstrated in Figure. 2.1, while the non-classical information structure corresponds to the decentralized control as illustrated in Figure. 2.2.

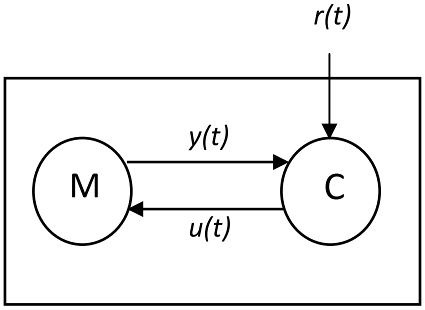

Figure 2.1. Classical information structure

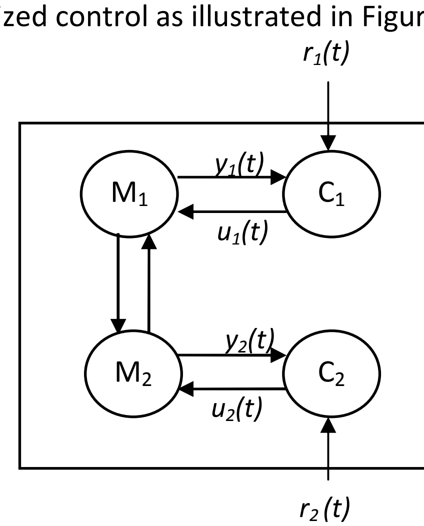

Figure 2.2. Non-classical information structure

From these figures one can observe that for the centralized case, the controller has the entire information about the outputs while in decentralized control, only partitions of the measurement information is available for each controller or decision maker [2].
As well as information structures, there are different model structures depending on the internal complexity of dynamics and largeness of the system; these structures are often called, centralized structure, *n*-channel system, and interconnected system shown in figures 2.3-2.5.

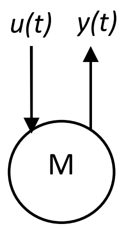

Figure 2.3. Centralized structure

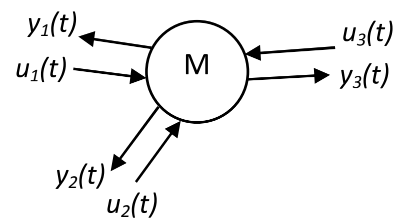

Figure. 2.4. Multi channel system

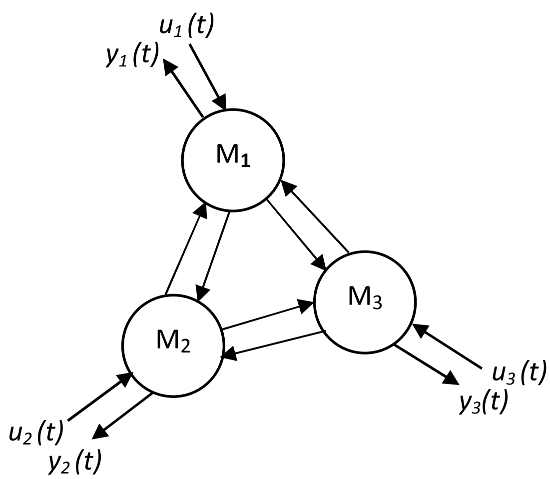

Figure. 2.5. Interconnected system

In multi-channel systems input and output vectors are partitioned into sub-vectors while the system is represented as a whole. In interconnected structure, system is decomposed into *n* different subsystems interacting with each other through the internal signals. Each subsystem has its own inputs and outputs, and its dynamics is affected by both the local corresponding inputs and interactions with adjacent neighbours.
Hence, a number of methodologies have been suggested to deal with the complication of the mentioned structures. The approach to handle these issues in large-scale coupled systems is

usually *decomposition* and *decentralization* schemes in order to partition the problem into almost separate problems and find independent solutions for each one.
Using decentralization enables us to implement separate control stations for a given dynamical process. As described by Bakule (2008) there are different motivations for decentralization like weak couplings among subsystems, having contradictory goals and the problem of high dimensionality. Basically, decentralizing the design task belongs to two categories [8]:

- Decentralized structure for weakly coupled systems
- Decentralized structure for strongly coupled systems

For the weakly coupled systems the coupling among the subsystems is possible to neglect while designing separate local controllers for individual subsystems, in this approach it should be determined how much weak should the couplings be. Whereas, internal interconnection models of all the adjacent subsystems should be considered while designing a set of decentralized control stations for a system with strong couplings.
There are also methods concerned with model simplifications, including model reduction methods and robust analysis which use the uncertain characteristics of interconnected systems in order to analyze stability issues.

Stability and complexity

There is an interplay between the stability and complexity of dynamical systems. As it was mentioned before there are lots of mathematical models of various large systems in diverse engineering fields consisting of several agents (subsystems) interacting with each other through communication or physical links. These subsystems influence each other through competition or cooperation or both. For the case where subsystems compete with which other, their interaction is stable if decoupled subsystems are all stable and the magnitude of their interconnections is sufficiently small. However, cooperation among subsystems can lead to the stability of the overall system at the price of an increase in the interaction among subsystems.
Another issue is that it may happen for a large-scale system that subsystems lose their connection with each other or in other words they may be subject to structural perturbations. It means that during the operation of the system, different subsystems may disconnect and connect again due to faults in the operation or on purpose. This disables us to exploit the advantageous interactions in a cooperative manner which may end up in instability of the overall system.
Therefore, in order to maintain the reliability of the system besides stability, we may require having competitive subsystems while having sufficiently weak interactions among them. It is a common framework for the synthesis of stable large-scale systems [19].

2.1. Competition

Let us consider a linear time-invariant system equation,

$$\dot{x} = Ax + e \qquad (2.1)$$

This can be used to describe the dynamical interactive behaviour of a number of agents coupled to one another. x Is the state of the system, and may represent various things due to the context of study. x is an *n* column vector, $x \in \mathbb{R}^{\mathrm{n}}$.

In (2.1), matrix *A* is a constant $n \times n$ matrix representing the dynamics and e is a column vector describing how the external world affects the system. Matrix *A* consists of constants elements a_{ij} that represent how the jth agent influences ith agent.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \quad , \quad e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix} \qquad (2.2)$$

This system describes the interactive connection among several agents. In economics this can be used to model multiple markets of services which compete for consumers, in chemistry this equation can be used to describe a chemical reaction involving *n* reactants, and it can be of interest in population dynamics where the states represent species competing with one another in the same environment.

In the aforementioned examples, there is a common situation which can be of interest to mention here. That is *A* matrix is such that,

$$A_{ij} \begin{cases} < 0 & i = j \\ \geq 0 & i \neq j \end{cases}$$

This means that diagonal elements are negative and off-diagonals are non-negative. This kind of sign pattern is called *Metzler* in economics [15]. Metzler matrices describe the competitive behaviour among agents.

2.2. Systems over graph

Now, we will remove the effect of the input signals from outside and focus on the autonomous system represented by the following,

$$\dot{x} = Ax$$

Let us study the structural characteristics of the above system, where usually a graph theoretical approach is convenient. Where, the system structure is represented by directed graphs.

Consider a coupled system composed of two agents where the dynamics can be represented by the following,

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 \tag{2.3}$$
$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2$$

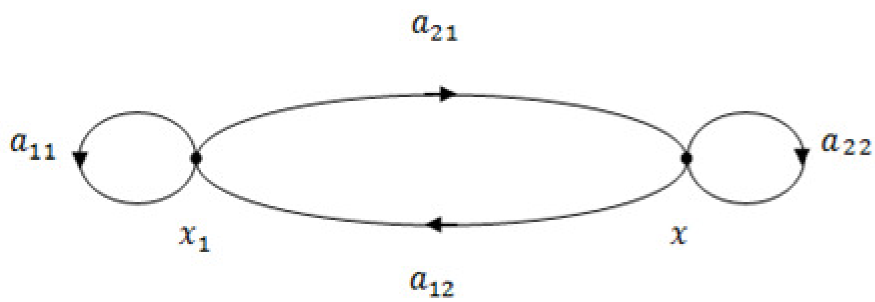

Figure. 2.6. Interconnection graph of two agents

This is the same system (2.1) where n=2 and $e_1 = e_2 = 0$. The interconnection of these two agents can be shown by a graph shown in Figure. 2.6.

This directed graph, or digraph is composed of two nodes (or vertices) representing the states of the system connected by directed lines (or edges). The nodes are labelled x_1 and x_2 and the directed lines are labelled $a_{11}, \ a_{12}, \ a_{21} \ a_{22}$.

The values of $a_{11}, \ a_{12}, \ a_{21} \ a_{22}$ can be called the parameters of the system. Now we ignore these values and focus on the structure of the system, which means that instead of considering the magnitudes, we simplify the presentation by just considering the zero, non-zeros values of the lines. In this viewpoint, the lines of the graph are either present or absent. We call the matrix obtained from the structured pattern, the interconnection matrix $\bar{A}$;

$$\bar{A}_{ij} \begin{cases} 1 & A_{ij} \neq 0 \\ 0 & A_{ij} = 0 \end{cases}$$

Hence, $A_{ij} = 0$ it shows that jth node does not acts on ith and $A_{ij} \neq 0$ shows that jth does not act on ith .

Now we are ready to explain the notion of structural perturbation. As previously noted, it is common in many system types that the agents disconnect and connect again during the operation of the system. This may be caused, on purpose or by fault. Either ways, in order to make a reliable system it should be made robust to structural perturbations or be *connectively stable* (see [15]). In the case of the system (2.3) the interconnection matrix may get different structures, for instance,

If $\bar{A}_1 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, x_1 acts on itself and on x_2 , while x_2 acts on itself but not on x_1 (see Figure.2.7), or

If $\bar{A}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, x_1 and x_2 just act on themselves, (see Figure.2.8.)

Look at the digraphs in Figures 2.7 and 2.8 showing these structures,

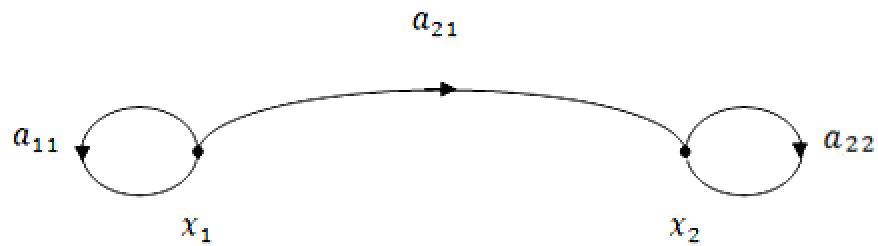

Figure. 2.7.

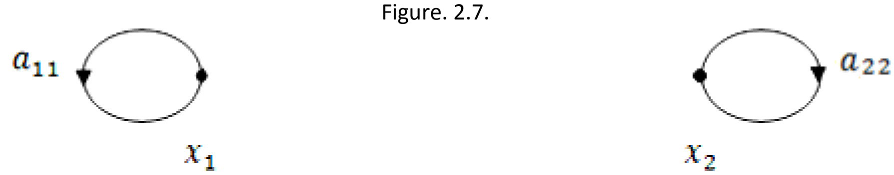

Figure. 2.8.

Therefore, basically a structurally robust system should be stable for all $\bar{A}$ in order to obtain connective stability for the overall system.

2.3. Cooperation

Considering again the system equations 2.3,

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2$$

This interconnection is called competitive, if the system matrix $A \in \mathbb{R}^{2\times 2}$ is Metzler, that is diagonal entries are negative and non-diagonal elements are non-negative. From matrix algebra we know that the necessary and sufficient conditions for overall connective stability of the equilibrium $x^* = 0$ are,

$$a_{11} < 0 \quad , \quad \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} > 0$$

These conditions are known as Hicks conditions. From these conditions we conclude the following necessary conditions,

$$a_{11} < 0 \quad , \quad a_{22} < 0$$

These conditions guarantee that the perturbed structure below is still asymptotically stable,

$$\dot{x}_1 = a_{11}x_1$$

$$\dot{x}_2 = a_{22}x_2$$

That is when two subsystems are decoupled the disjoint system remains stable. Since the conditions for disjoint stability were necessary conditions for overall connective stability of the system, we can conclude that disjoint stability of the system is a necessary requirement for the connective stability in a competitive manner.
However, when the off-diagonal elements a_{12} and a_{21} are not negative we cannot use Hicks conditions for connective stability any more. In this way, the stability of the whole system may be obtained by cooperation among subsystems. This means that after breaking the system apart, the decoupled system may not remain stable, although the connected system is. So we can say that the stability of the system has been obtained cooperatively when different agents are connected and have beneficial interaction with each other. This structure will not end in, what is called, structurally reliable system, where the stability of the equilibrium under structural perturbations is guaranteed. To make it more clear we derive the necessary and sufficient conditions for a having stable *2*by*2* non-symmetric matrix as follows,

$$a_{11} + a_{22} < 0 \quad , \quad a_{11}.a_{22} > a_{12}.a_{21}$$

The above mentioned conditions show that connecting two disjoint unstable subsystems together may result in an overall stable system. That is, for instance; $a_{11} > 0 > a_{22}$, while $|a_{22}| > |a_{11}|$, which makes the first subsystem unstable after being disconnected from the second one. Therefore in this case the cooperation is beneficial to the overall stability of the system.

Decomposition

Large systems comprised of large number of states are usually hard to deal with by one-shot approaches. Several techniques have been elaborated in order to tear complex systems into several simpler dynamical elements with lower dimensions. The *decomposition principle* has been introduced for more than a half-century, originally in order to solve mathematical systems of equations, or for the analysis of electrical networks.
While this approach can be helpful in finding simpler solutions for a given large problem, how to perform an appropriate decomposition for a particular system is not an easy task. This is due

to the fact that after breaking down the equations, we need to end up in suitable subproblems which are solvable with the methods at hand and the overall solution for the system should be possible to get by bringing the these solutions together [15].

For this difficulty a physical insight to the system under study can be helpful. So, generally we can recognize two types of decomposition approaches, *physical* and *numerical.* For some types of large-scale systems, the physical modeling of the plant ends up in several interconnected dynamical partitions, a famous example of this category is power distribution networks. This, not only can result in simplifications in the analysis and design but also can bring some insight into the structure of the system at hand. However, sometimes the decomposition approach is done only due to numerical reasons, although tearing the given model, may take the physical interpretation of the system completely away. For the case of a physical modeling, we are given an interconnected system of equations, and we may start by looking at the connective properties of the system. While for a given large-scale one-piece system we should develop some decomposition methods where finally we can consider the problem of stabilization locally while considering the interactions between different agents.

A basic example of how we perform the decomposition on a linear system is given. Let us consider again the linear system,

$$\dot{x} = Ax$$

The state vector x is composed of *n* variables, $x \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n\times n}$ is a constant matrix. We divide the state vector into two sub-vectors x_1 , x_2 such that $x_1 \in \mathbb{R}^{n_1}$ and $x_2 \in \mathbb{R}^{n_2}$ and $n_1 + n_1 = n$, this partition the system into the following interconnected form,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

Where $A_{11} \in \mathbb{R}^{n_1\times n_1}$, $A_{12} \in \mathbb{R}^{n_1\times n_2}$, $A_{21} \in \mathbb{R}^{n_2\times n_1}$ and $A_{22} \in \mathbb{R}^{n_2\times n_2}$. A_{12} and A_{21} illustrate the interaction between two subsystems, and if they are equal to zero matrices with proper dimensions then the decoupled system will be derived.

In the next and following sections, we introduce how the decomposition is done for typical dynamical systems with multiple inputs and outputs, in a unified way that will make the problem of control design feasible.

Decentralization

In section 2.2 we introduced a system with external influence from environment,

$$\dot{x} = Ax + e$$

Where *'e'* stands for the external inputs to the system. A typical representation of the input signal is

$$\dot{x} = Ax + Bu$$

Where $B \in \mathbb{R}^{n\times m}$ is a constant matrix, x is the n-vector representing the states of the system and $u \in \mathbb{R}^{m}$ is the vector of inputs. u(t) is what we will call control input when we are going to force the system to have a desirable behaviour. This is fundamentally done using a suitable linear or nonlinear function of the states of the system, called state-feedback control.

Let us consider the following system $\mathfrak{S}$,

$$\dot{x} = \begin{bmatrix} -3 & -1 \\ -4 & 2 \end{bmatrix} x + \begin{bmatrix} 2 & -3 \\ 2 & 3 \end{bmatrix} u$$

We are going to decompose the system into two interconnected subsystems each with one scalar input.
A digraph that shows the dynamic flow among states is shown in Figure 2.9.

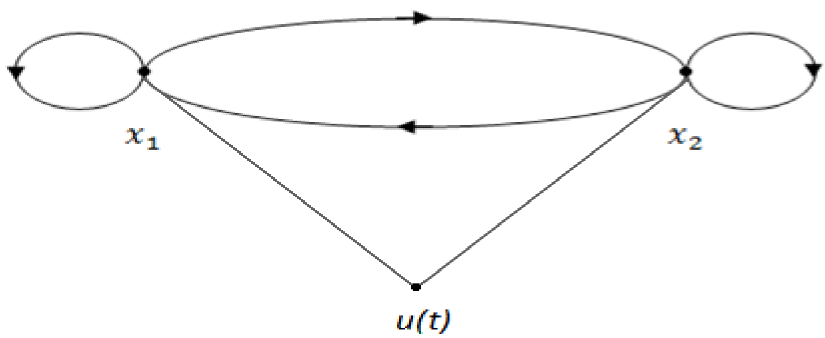

Figure.2.9. Input-centralized scheme

This scheme is called *input-centralized* system [15]. Now, if we use the transformation T to get,

$$\bar{x} = T^{-1}x ,$$

This results in a transformed system $\bar{\mathfrak{S}}$,

$$\bar{A} = T^{-1}AT \ , \bar{B} = T^{-1}B$$

Taking $T = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ gives us the following differential equations

$$\dot{\bar{x}} = \begin{bmatrix} -3 & 4 \\ 1 & 2 \end{bmatrix} \bar{x} + \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \bar{u} \ , \qquad \bar{u} = (\bar{u}_1, \bar{u}_2)^T$$

This representation makes the subsystems decoupled in terms of inputs, so individual subsystems have distinct inputs now. This results in a completely decentralized system composed of two subsystems, $\bar{\mathfrak{s}}_1, \bar{\mathfrak{s}}_2$

$$\dot{\bar{x}}_1 = -3\bar{x}_1 + 4\bar{x}_2 + 2\bar{u}_1$$

$$\dot{\bar{x}}_2 = \bar{x}_1 + 2\bar{x}_2 + 3\bar{u}_2$$

The associated digraph is shown in Figure 2.10.

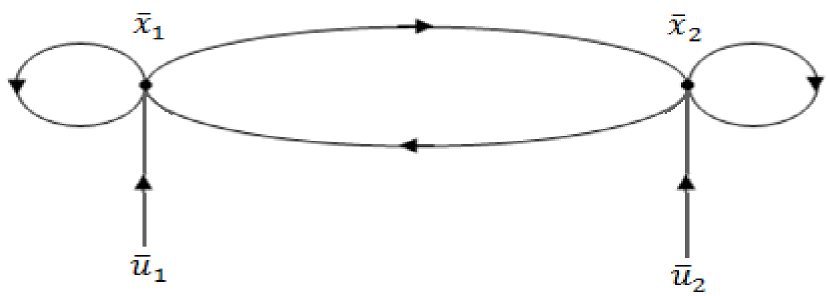

Figure.2.10. Input decentralized scheme

The difference between the input-decentralized and input-centralized schemes, is easy to see from Figures 2.9 and 2.10.
Figure 2.10 shows two distinct inputs of the interconnected system which are the input of the input-decoupled subsystems. The number of inputs is equal to the number of subsystems.
Besides input decentralized pattern, output decentralization scheme is also of interest since we may require a state estimator for the large-scale system at hand. The states of each decoupled system should be available for feedback and this requires either the availability of the states or constructing decentralized estimators for the interconnected system. Therefore this is the dual problem of input-decentralized scheme in which we produce agents with distinct inputs.
Now, let us illustrate how to transform a given system into output-decentralized scheme. Consider the previous example but this time including output equations,

$$\dot{x} = \begin{bmatrix} -3 & -1 \\ -4 & 2 \end{bmatrix} x + \begin{bmatrix} 2 & -3 \\ 2 & 3 \end{bmatrix} u \,,$$

$$y = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} x$$

The Figure 2.11 shows the corresponding digraph. We easily can derive the transformation that results in output-decentralized scheme. Let us take the following transformation matrix,

$$T = \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}$$

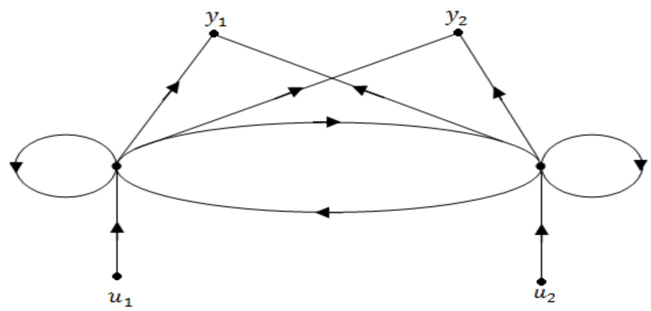

Figure.2.11. Output-centralized scheme

We get the transformed system $\bar{\mathfrak{S}}$ as,

$$\dot{\bar{x}} = \begin{bmatrix} 9 & 8 \\ 10 & -10 \end{bmatrix} \bar{x} + \begin{bmatrix} 4 & 0 \\ 6 & -3 \end{bmatrix} \bar{u} \,,$$

$$\bar{y} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x$$

Compared to the input-decentralized form, the dual problem, $\bar{\mathfrak{s}}$, is called output-decentralized representation. The associated graph is also shown in Figure 2.12. It can be seen that in this case, outputs are associated with distinct inputs.

It should be noted that after we perform an output-decentralization transformation on the input-decentralized form, the result probably wouldn't be input-decentralized anymore; however this is not the case in physical modeling of interconnected systems, where we usually derive the input-output decentralized representation of the whole system.

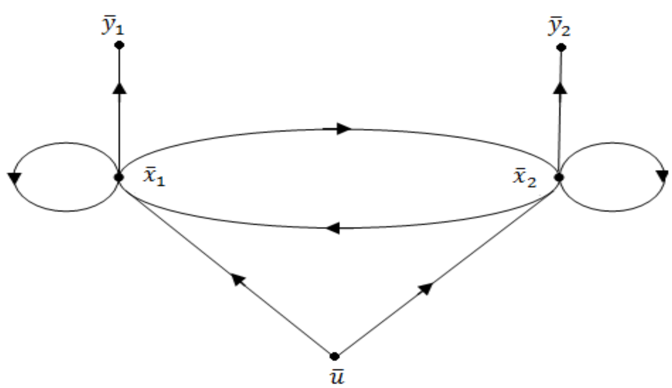

Figure.2.12. Output-decentralized scheme

2.4. Decentralization Method

So far, we have mentioned about the problem of decentralization from input and output aspects and its importance in decentralized control problems, especially in solving problems with so many variables. Decentralized control (estimation) methods can be used for the problem of control of large-scale systems and control of interconnected dynamical systems, which for the former; we firstly decentralized the given system with effective methods. Then design a set of decentralized controllers (estimators) for the subsystems. Therefore, as a preliminary step, a procedure to decentralize a system from input (output) views should be developed.

Now we are ready to briefly explain a mathematical method for decentralization. A procedure has been proposed (see [15]) which produces a number of interconnected subsystems with distributed inputs or outputs. This procedure mainly consists of three stages; *transformation, decomposition* and *recollection.*

Consider a linear system,

$$\dot{x} = Ax + Bu$$

Where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the input vector and A , B are constant matrices with proper dimensions.

This equation can always be decomposed into k partitions,

$$\dot{x}_p = A_p x_p + \sum_{\substack{q=1,\\ q\neq p}}^{k} A_{pq} x_q + B_p u \qquad p = 1,..,k$$

Where $x_p \in \mathbb{R}^{d_p}$, $n = \sum_{p=1}^{k} d_p$ and $x = (x_1^T x_2^T \dots x_k^T)^T$.

By means of a linear transformation, T, we can compute the transformed system such that,

$$\bar{x}_p = {T_p}^{-1} x_p$$

$$\dot{\bar{x}}_p = \bar{A}_p \bar{x}_p + \sum_{\substack{q=1,\\ q\neq p}}^{k} \bar{A}_{pq} \bar{x}_q + \bar{B}_p \bar{u} \qquad p = 1,2,..,k$$

$$\bar{B}_p = \begin{bmatrix} \bar{b}_1^p & & & \mathbf{0} \\ & \bar{b}_2^p & & \\ & & \ddots & \\ \mathbf{0} & & & \bar{b}_m^p \end{bmatrix}, \bar{b}_i^p = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad (2.4)$$

Where $\bar{A}_p = {T_p}^{-1} A_p T_p$, $\bar{A}_{pq} = {T_p}^{-1} A_{pq} T_q$, $\bar{B}_p = {T_p}^{-1} B_p$. Looking at equation (2.4) we can decompose each state of the transformed subsystems such that, $\bar{x}_p = \left(\bar{x}_{p1}^T \ \bar{x}_{p2}^T \dots \bar{x}_{pm}^T\right)^T$ so that each component $\bar{x}_{pi}^T$ is associated with $u_i \in \mathbb{R}$ from the input vector u.

The last stage is regrouping the states $\bar{x}_{pi}^T$ which are affected by the same input and form the state of the ith subsystem $z_i \in \mathbb{R}^{n_i}$, $z_i = (\bar{x}_{1i}^T\ \bar{x}_{2i}^T \dots \bar{x}_{ki}^T)^T$, this also can be done by a linear transformation which is explained in detail in [15].
Finally we get the representation of m interconnected subsystem by the following equation,

$$\dot{\bar{z}}_i = \bar{A}_i \bar{z}_i + \sum_{\substack{j=1, \\ j\neq i}}^{m} \bar{A}_{ij} \bar{z}_j + \bar{b}_i \bar{u}_i \qquad i = 1,..,m$$

The overall system equations can be written as,

$$\dot{z} = \bar{A}z + \bar{B}u$$

$$\bar{B} = \begin{bmatrix} b_1 & & & \mathbf{0} \\ & b_2 & & \\ & & \ddots & \\ \mathbf{0} & & & b_m \end{bmatrix}, b_i = \begin{bmatrix} \bar{b}_i^1 \\ \bar{b}_i^2 \\ \vdots \\ \bar{b}_i^r \end{bmatrix}$$

Example 2.1
Now, let us give a numerical example to show how the decentralization algorithm works,
Consider the system,

$$\dot{x} = \begin{bmatrix} 1 & 2 & 0 & 3 & 5 & 1 \\ 0 & 3 & 0 & 1 & 2 & 0 \\ 2 & 4 & 1 & 3 & 2 & 2 \\ 5 & 0 & 1 & 1 & 1 & 3 \\ 4 & 2 & 2 & 0 & 0 & 6 \\ 3 & 1 & 0 & 1 & 2 & 4 \end{bmatrix} x + \begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 2 \\ 3 & 1 \end{bmatrix} u$$

With $n = 6$, $m = 2$, the system can be decomposed into two dynamical elements ($r = 2$) as follows,

$$\dot{x}_1 = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & 0 \\ 2 & 4 & 1 \end{bmatrix} x_1 + \begin{bmatrix} 3 & 5 & 1 \\ 1 & 2 & 0 \\ 3 & 2 & 2 \end{bmatrix} x_2 + \begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} u,$$

$$\dot{x}_2 = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 0 & 6 \\ 1 & 2 & 4 \end{bmatrix} x_2 + \begin{bmatrix} 5 & 0 & 1 \\ 4 & 2 & 2 \\ 3 & 1 & 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 3 & 1 \end{bmatrix} u$$

Where $d_1 = 3$, $d_2 = 3$ are the dimensions of the subsystems. By means of the following transformations T_1 , T_2 we derive the following representation of the subsystems,

$$\dot{\bar{x}}_1 = \begin{bmatrix} 0 & -1 & -1.5 \\ 1 & 2 & 1.5 \\ 0 & 0 & 3 \end{bmatrix} \bar{x}_1 + \begin{bmatrix} 1.12 & 1.625 & 1.875 \\ 1.37 & 2.875 & 0.625 \\ 2 & 2 & 3 \end{bmatrix} \bar{x}_2 + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} u\,,$$

$$\dot{\bar{x}}_2 = \begin{bmatrix} 0 & -16 & -1.8 \\ 1 & 20 & 2.1 \\ 0 & -130 & -15 \end{bmatrix} \bar{x}_2 + \begin{bmatrix} 0.4 & -12.8 & -1 \\ -0.8 & 5.6 & 0.5 \\ 11 & 8 & 1 \end{bmatrix} \bar{x}_1 + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} u$$

The vectors $\bar{b}_i^p$ for the previous representation looks like the following,

$$\bar{b}_1^1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \ , \ \bar{b}_2^1 = 1 \ , \ \bar{b}_1^2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \ , \ \bar{b}_2^2 = 1$$

We take $\bar{x}_1 = (\bar{x}_{11}^T \bar{x}_{12}^T)^T$ and $\bar{x}_2 = (\bar{x}_{21}^T \bar{x}_{22}^T)^T$, hence the final state elements will be,

$$z_1 = (\bar{x}_{12}^T \bar{x}_{22}^T)^T, z_2 = (\bar{x}_{11}^T \bar{x}_{21}^T)^T$$

And the final state equations become,

$$z_1 = \begin{bmatrix} 3 & 3 \\ 1 & -15 \end{bmatrix} z_1 + \begin{bmatrix} 0 & 0 & 2 & 2 \\ 11 & 8 & -130 & -15 \end{bmatrix} z_2 + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_2$$

$$z_2 = \begin{bmatrix} 0 & -1 & 1.12 & 1.625 \\ 1 & 2 & 1.37 & 2.875 \\ 0.4 & -12.8 & 0 & -16 \\ -0.8 & 5.6 & 1 & 20 \end{bmatrix} z_2 + \begin{bmatrix} -1.5 & 1.875 \\ 1.5 & 0.625 \\ -1 & -1.8 \\ 0.5 & 2.1 \end{bmatrix} z_1 + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} u_1$$

It should be noted that this method always ends up in decentralized systems with distinct scalar inputs, this means that the blocks of the B matrix will be vectors. However it is not necessary to have scalar inputs on individual subsystems to call an interconnected system input-decentralized. To generalize, we may group some of these inputs to get matrices instead of vectors in the blocks, hence a system with m inputs can be decomposed into h interconnected subsystems where the B matrix looks like the following,

$$B = \begin{bmatrix} B_1 & & & \mathbf{0} \\ & B_2 & & \\ & & \ddots & \\ \mathbf{0} & & & B_h \end{bmatrix} \ , \ B_i \in \mathbb{R}^{n_i \times m_i} \ , \ \sum_{i=1}^{h} n_i = n \ , \ \sum_{i=1}^{h} m_i = m \qquad (2.5)$$

Where n is the total number of states, m is the total number of inputs, m_i is the number of inputs of the ith subsystem and n_i is the number of states of the ith subsystem.
It is also obvious that the B matrix derived from physical modelling of the plant generally have the (2.5) representation in which individual subsystems have multiple inputs.

Therefore, we will use this more general structure of an interconnected system in this study.

Chapter 3

A Decentralized Stabilization Scheme

In this chapter first we shall start with a description of decentralized control problem, and then we will study a special class of stable linear systems which will be tailored in designing a stabilization method for interconnected systems, this method is the main contribution of this study.

3.1. Problem statement

Consider a linear system which is categorized as a large-scale system base on our definition, so we would prefer to avoid centralized schemes to control the whole system and hence we start our design by decomposing the system into input-decentralized representation which was introduced in chapter 2. The final result of a decomposition algorithm will generally be a set of h interconnected subsystems,

$$\dot{x}_i = A_i x_i + B_i u_i + \sum_{\substack{j=1 \\ j \neq i}}^{h} A_{ij} x_j, \qquad i = 1, \dots, h \tag{3.1}$$

Where $x_i(t) \in \mathbb{R}^{n_i}$ and $x_j(t) \in \mathbb{R}^{n_j}$ are the states of ith and jth subsystems. The total state vector is $x = ({x_1}^T {x_2}^T \dots {x_h}^T)^T$ and the total number of states is $n = \sum_{i=1}^{h} n_i$.

It should be noted that this representation may also be derived by physical modeling of the plant. Hence the overall matrices of the transformed system can be constructed in a partitioned form as follows:

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$A=\begin{bmatrix} A_1 & A_{12} & \dots & A_{1h} \\ A_{21} & A_2 & & \vdots \\ \vdots & \vdots & \ddots & \\ A_{h1} & \dots & \dots & A_h \end{bmatrix},\ B=\begin{bmatrix} B_1 & & & \mathbf{0} \\ & B_2 & & \\ & & \ddots & \\ \mathbf{0} & & & B_h \end{bmatrix}$$

We are aiming at stabilizing the whole system. The basic assumption is that all pairs (A_i, b_i) are controllable, which is equal to say that:

For every $A_i \in R^{n_i \times n_i}, B_i \in R^{n_i \times m_i}$, $(\lambda I - A_i \ \vdots\ B_i)$ has full row rank at every λ in $\mathbb{C}$, where $\mathbb{C}$ stands for the set of complex numbers, and $i = 1, \dots, h$.

Solving the stabilization problem will also introduce a method to construct stable estimators for the systems by duality.

Consider again the linear time invariant system equation

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{3.2}$$

$$y(t) = Cx(t)$$

Decomposition:

Forming the dual of (3.2) we get

$$\dot{x}(t) = A^T x(t) + C^T u(t)$$

$$y(t) = B^T x(t)$$

Where we have used the same variable names for the states and measurements for consistency, but note that they are different variables with different dimensions. Now if we apply the input decentralized scheme,

$$\dot{x}_i = A_i^T x_i + c_i u_i + \sum_{\substack{j=1 \\ j \neq i}}^{p} A^T{}_{ij} x_j \quad , \quad i = 1, \dots, p \tag{3.3}$$

$$y(t) = \sum_{i=1}^{p} B_i{}^T x_i$$

And finally forming the dual of (3.3) gives us the output decentralized version of the plant,

$$\dot{x}_i = A_i x_i + B_i u + \sum_{\substack{j=1 \\ j \neq i}}^{p} A_{ij} x_j, \qquad i = 1, \dots, p \tag{3.4}$$

$$y_i(t) = C_i^T x_i$$

Where $y_i(t)$ is the measurement of the *i*th subsystem.

In order to construct state estimators, we will form the dynamic equation of the subsystem observers as follows,

$$\hat{\dot{x}}_i = F_i\hat{x}_i + g_i y_i + \sum_{\substack{j=1 \\ i\neq j}}^{p} F_{ij}\,\hat{x}_j + B_i u \qquad i = 1,\ldots,p \tag{3.5}$$

Where the unknown matrices F_i, g_i, F_{ij} are to be determined.

Forming the estimation error equation,

$$e_i(t) = x_i(t) - \hat{x}_i\,(t) \tag{3.6}$$

Getting the derivative of both sides and using (3.6) and (3.4) we get

$$\dot{e}_i(t) = (A_i - g_i C_i^T)x_i(t) - F_i\hat{x}_i(t) + \sum_{\substack{j=1 \\ i\neq j}}^{p} A_{ij}\,x_j - \sum_{\substack{j=1 \\ i\neq j}}^{p} F_{ij}\hat{x}_j$$

We choose

$$A_i - g_i {C_i}^T = F_i \quad , \quad A_{ij} = F_{ij}$$

So the error dynamics becomes,

$$\dot{e}_i(t) = F_i e_i(t) + \sum_{\substack{j=1 \\ i\neq j}}^{p} F_{ij}\,e_j \tag{3.7}$$

Having an asymptotic stable observer we need to stabilize (3.7), or equivalently stabilize the dual form,

$$\dot{\tilde{e}}_i(t) = {F_i}^T\tilde{e}_i(t) + \sum_{\substack{j=1 \\ i\neq j}}^{p} {F_{ij}}^T\,\tilde{e}_j$$

we get,

$$\dot{\tilde{e}}_i(t) = (A_i^T - C_i g_i^T)\tilde{e}_i(t) + \sum_{\substack{j=1 \\ i\neq j}}^{p} {A_{ij}}^T\,\tilde{e}_j$$

Finally we obtain the final decentralized observer equations as

$$\hat{\dot{x}}_i = A_i\hat{x}_i + g_i(y_i - C_i^T\hat{x}_i) + \sum_{\substack{j=1 \\ i\neq j}}^{p} A_{ij}\,\hat{x}_j + B_i u \qquad i = 1,\ldots,p$$

Connection of controller and state estimator

Now it is of interest to analyze the regulator system composed of estimator and controller together and verify if the separation property holds. That is whether design of state feedback and estimators can be carried out independently, and where the eigenvalues of the regulator system will lie.

The control law using estimated state is,

$$u_i = r_i - k_i^T \hat{x}_i \tag{3.7}$$

Now, by substituting (3.8) in input-output decentralized system equations

$$\begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_m \\ \cdots \\ \dot{\hat{x}}_1 \\ \vdots \\ \dot{\hat{x}}_m \end{bmatrix} = \begin{bmatrix} A_1 & \dots & A_{1m} & -B_1K_1 & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ A_{m1} & \dots & A_m & 0 & \dots & -B_mK_m \\ g_1C_1^T & \dots & 0 & A_1 - g_1C_1^T - B_1K_1 & \cdots & A_{1m} \\ \vdots & & \vdots & \vdots & & \vdots \\ 0 & \dots & g_1C_1^T & A_{m1} & \dots & A_m - g_mC_m^T - B_mK_m \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_m \\ \cdots \\ \hat{x}_1 \\ \vdots \\ \hat{x}_m \end{bmatrix} + \begin{bmatrix} B_1r_1 \\ \vdots \\ B_mr_m \\ \cdots \\ B_1r_1 \\ \vdots \\ B_mr_m \end{bmatrix}$$

Applying the transformation

$$\begin{bmatrix} x_1 \\ \vdots \\ x_m \\ \cdots \\ e_1 \\ \vdots \\ e_m \end{bmatrix} = \begin{bmatrix} I_m & 0 \\ I_m & -I_m \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_m \\ \cdots \\ \hat{x}_1 \\ \vdots \\ \hat{x}_m \end{bmatrix}$$

We get,

$$\begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_m \\ \cdots \\ \dot{e}_1 \\ \vdots \\ \dot{e}_m \end{bmatrix} = \begin{bmatrix} A_1 - B_1K_1 & \dots & A_{1m} & B_1K_1 & \dots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ A_{m1} & \dots & A_m - B_mK_m & 0 & \dots & B_mK_m \\ & & & A_1 - g_1C_1^T & \cdots & A_{1m} \\ & \mathbf{0} & & \vdots & & \vdots \\ & & & A_{m1} & \dots & A_m - g_mC_m^T \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_m \\ \cdots \\ e_1 \\ \vdots \\ e_m \end{bmatrix} + \begin{bmatrix} B_1r_1 \\ \vdots \\ B_mr_m \\ \cdots \\ \\ 0 \\ \\ \end{bmatrix} \tag{3.8}$$

Therefore we are able to split the equations into two independent sets and separation property holds. So the stabilization method introduced can be used twice to stabilize the diagonal blocks in (3.8).

3.2. A sufficient Condition for Stability

Naturally, for a diagonal matrix the stability conditions are easily described based on diagonal entries, but this task is not straightforward for non-diagonal matrices. However, since all the eigenvalues are continuous functions of the entries, some useful studies about the location of eigenvalues based on matrix entries have been done, [5].

Theorem (Geršgorin)

$$\text{Let } A = [a_{ij}] \in M_n\,(\mathbb{C})$$

Where $M_n\,(\mathbb{C})$ denotes the set of $n \times n$ matrices over the field $\mathbb{C}$ and let

$$R_i(A) \equiv \sum_{\substack{j=1 \\ j\neq i}}^{n} |a_{ij}|, \qquad 1 \leq i \leq n$$

Denote the deleted absolute row sum of A. Then all the eigenvalues of A are located in the union of n disks:

$$\bigcup_{i=1}^{n} \{z \in C : |z - a_{ii}| \leq R_i(A)\} \equiv G(A)$$

Also, the off-diagonal entries are sometimes regarded as perturbations in a so called *perturbation theorem* [5], and result is expressed as follows,

Let $D = diag(\lambda_1, \lambda_2, \dots \lambda_n)$ and let $E = [e_{ij}]$ and consider the perturbed matrix $D + E$, the eigenvalues of $D + E$ are contained in the disks ([15]),

$$\left\{ z \in C : |z - \lambda_i| \leq R_i(E) = \sum_{j=1}^{n} |e_{ij}| \right\}, \qquad i = 1,2 \dots, n$$

Example 3.1.

If $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -4 & 2 \\ -3 & 3 & 4 \end{bmatrix}$, the set $G(A)$ is shown in figure (3.1) ,

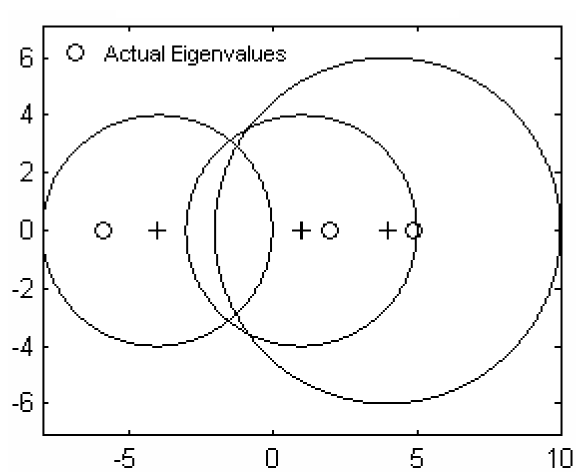

Figure 3.1. Geršgorin disks

Here the Geršgorin set consists of three disks centered on 1, -4, 4 with radii 4,4 and 6 respectively. It can be seen that the eigenvalues are located inside the union of these three discs.

Definition 3.1.
Let $A = [a_{ij}] \in M_n(\mathbb{C})$, the matrix is said to be diagonally dominant if

$$|a_{ii}| \geq \sum_{\substack{j=1 \\ j\neq i}}^{n} |a_{ij}| = R_i \text{ for all } i = 1, \dots, n$$

Is said to be strictly diagonally dominant if

$$|a_{ii}| > \sum_{\substack{j=1 \\ j\neq i}}^{n} |a_{ij}| = R_i \text{ for all } i = 1, \dots, n$$

Corollary 3.1.

Let $A = [a_{ij}] \in M_n(C)$ be strictly diagonally dominant. Then if all main diagonal entries of A are negative, all the eigenvalues of A will have negative real part and hence A will be Hurwitz. This is a special family of stable matrices and was called *superstable* first time in [10].

Stability

Having derived the corollary 3.1, we are going to use it in our problem. The general idea is to dominate the diagonal entries of the total closed-loop matrix while stabilizing each subsystem. To show the framework of the procedure we start by an illustrative system composed of controllable subsystems.

Suppose that a set of h state feedback local controllers are to be designed as follows,

$$u_i = -k_i x_i, \quad i = 1,2, \dots, h \quad , \quad K = \begin{bmatrix} k_1 & & & \mathbf{0} \\ & k_2 & & \\ & & \ddots & \\ \mathbf{0} & & & k_h \end{bmatrix}$$

$$\dot{x}_i = (A_i - B_i k_i) x_i + \sum_{\substack{j=1 \\ j\neq i}}^{h} A_{ij} x_j, \qquad i = 1,2, \dots, h$$

$$A_i^{cl} = A_i - B_i k_i, \qquad i = 1,2, \dots, h$$

Hence, the closed-loop equation for the dynamical interconnected system can be re-written as,

$$\dot{x} = (A - BK)x = \begin{bmatrix} A_1 - B_1k_1 & A_{12} & \dots & A_{1h} \\ A_{21} & A_2 - B_2k_2 & & \vdots \\ \vdots & \vdots & & \\ A_{h1} & \dots & \dots & A_h - B_hk_h \end{bmatrix} x$$

Since we have assumed that all pairs $(A_i,\ B_i)$ are fully controllable, there always exists k_i to place the poles of $A_i - B_ik_i$ in n_i distinct and negative locations $({\lambda^i}_1,\ {\lambda^i}_2, \dots, {\lambda^i}_{n_i})$.

In other words, we first disconnect all the subsystems and stabilize them by conventional pole placement methods to bring the poles of each subsystem into desired stable locations. This ensures the stability of the decoupled subsystems.

Then all $A_i - B_ik_i$ will be diagonalizable and there would be a nonsingular permutation matrix P_i such that,

$$\overline{x}_i = {P_i}^{-1}x_i$$

$$\dot{\overline{x}}_i = \overline{A}_ix_i + \overline{B}_iu_i + \sum_{\substack{j=1, \\ i \neq j}}^{h} \overline{A}_{ij}\,\overline{x}_j\,, \qquad i = 1, \dots, h$$

$$\overline{A}_i = {P_i}^{-1}A_iP_i\ ,\ \ \overline{A}_{ij} = {P_i}^{-1}A_{ij}P_j\ ,\ \ \overline{B}_i = {P_i}^{-1}B_i$$

$$\overline{A}_i^{\,cl} = {P_i}^{-1}A_i^{cl}\,P_i$$

$$\overline{A}_i^{\,cl} = \begin{bmatrix} {\lambda^i}_1 & & & \mathbf{0} \\ & {\lambda^i}_2 & & \\ & & \ddots & \\ \mathbf{0} & & & {\lambda^i}_{n_i} \end{bmatrix} \quad,\ \ i = 1, \dots, h$$

And finally the overall closed-loop matrix of the system has the following form:

$$\overline{A}_{cl} = \begin{bmatrix} \overline{A}_1^{\,cl} & \overline{A}_{12} & \dots & \overline{A}_{1h} \\ \overline{A}_{21} & \overline{A}_2^{\,cl} & & \vdots \\ \vdots & \vdots & \ddots & \\ \overline{A}_{h1} & \dots & \dots & \overline{A}_h^{\,cl} \end{bmatrix}$$

The conditions to guarantee stability can be expressed by the following h inequalities,

$$\begin{cases} \left|\lambda_i\left(\overline{A}_1^{cl}\right)\right| < \sum_{k=1}^{h}\sum_{q=1}^{n_k}\left|\left[\overline{A}_{1k}\right]_{iq}\right| \quad , \ k \neq 1 \\ i = 1,2,\dots n_1 \\ \vdots \\ \left|\lambda_i\left(\overline{A}_h^{cl}\right)\right| < \sum_{k=1}^{h}\sum_{q=1}^{n_k}\left|\left[\overline{A}_{hk}\right]_{iq}\right| \quad , \ k \neq h \\ i = 1,2,\dots n_h \end{cases} \rightarrow Re\left\{\lambda_i\{\overline{A}_{cl}\}\right\} < 0 \ ; \ i = 1,2\dots,n \qquad (3.7)$$

Fulfilling the above inequalities guarantee the stability of the entire system based on corollary 3.1. The numerical example shows the procedure;

Example 3.2.

Consider the system

$$\dot{x} = \begin{bmatrix} -3 & 1 & 1 & 0.2 \\ 1 & 3 & -0.1 & 0 \\ 1 & 0.2 & 2 & 1 \\ 1 & 0 & 0 & -4 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 2 \\ 0 & 4 \end{bmatrix} u$$

Poles of the system are,

$$\lambda_1 = -4.0437, \ \ \lambda_2 = -3.3284\,, \ \ \lambda_3 = 3.1864, \ \lambda_4 = 2.1857$$

It has two unstable poles.
Firstly, we shall decompose the system into two interconnected subsystems,

$$\dot{x}_1 = \begin{bmatrix} -3 & 1 \\ 1 & 3 \end{bmatrix} x_1 + \begin{bmatrix} 1 & 0.2 \\ -0.1 & 0 \end{bmatrix} x_2 + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_1$$

$$\dot{x}_2 = \begin{bmatrix} 2 & 1 \\ 0 & -4 \end{bmatrix} x_2 + \begin{bmatrix} 1 & 0.2 \\ 1 & 0 \end{bmatrix} x_1 + \begin{bmatrix} 2 \\ 4 \end{bmatrix} u_2$$

The poles of the decoupled systems are

$$\lambda_1^1 = -3.1623, \ \lambda_2^1 = 3.1623\,, \ \lambda_1^2 = 2, \ \ \lambda_2^2 = -4$$

And both (A_1, B_1) and (A_2, B_2) are controllable. Hence we can move their poles to any desired locations, we will shift the poles to

$$\lambda_1^1 = -5, \ \lambda_2^1 = -8\,, \ \lambda_1^2 = -5, \ \ \lambda_2^2 = -10$$

For this we calculate the local gains using conventional pole placement methods,

$$k_1 = [0.3333 \quad 12.6667]$$

$$k_2 = [5.2500 \quad 0.6250]$$

By these feedback gain the poles of the entire system move to

$$\lambda_1 = -8.5961,\ \ \lambda_2 = -9.9211\ ,\ \ \lambda_{3,4} = -4.7414\ \pm\ 0.6792i$$

And the transformed matrix of the closed system becomes

$$\dot{\bar{x}} = \begin{bmatrix} -8 & 0 & 2.03 & -0.33 \\ 0 & -5 & 1.71 & -0.46 \\ -0.63 & 1.71 & -10 & 0 \\ -0.5 & 1.34 & 0 & -5 \end{bmatrix} \bar{x}$$

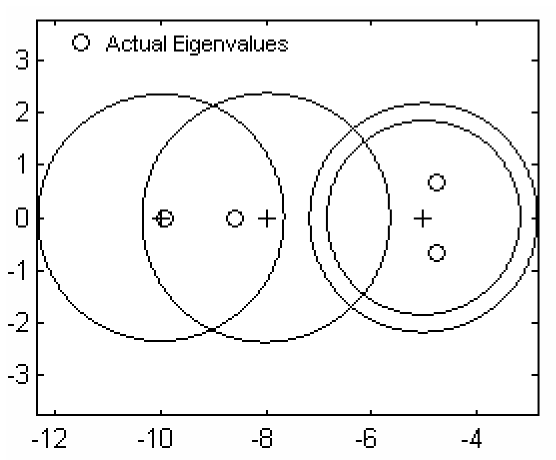

Figure 3.2. Geršgorin disks

The closed loop system matrix shows that the inequalities 3.7 are fulfilled and hence the stability of the system is established. Looking at the Geršgorin disks of the system admits the result. (Figure.3.2).

However this is important to note that, although we have assumed controllable subsystems and thus we have secured the ability to place subsystem poles arbitrarily, these poles cannot always dominate interactions. Looking at the inequalities 3.7, shows that a new selection of poles for individual subsystems introduces new permutations and thus changes the transformed interactions ($\overline{A}_{ij}$). Roughly speaking, shifting the poles of subsystems to the left half plain which stabilizes each subsystem individually, may on the other hand increase the norm of associated interactions. And hence the attempt to dominate the diagonal entries over off-diagonals may fail due to unboundedness of interactions. Numerical examples admit this observation as well.

In addition, some earlier results in the literature (e.g. Unstable fixed modes in [4]) show that increasing the degree of stability in each closed loop subsystem may not always result in the stability of total system.

This implies that to fulfill the mentioned sufficient condition for stability we need more degrees of freedom than controllability of subsystems. So it will be shown that the conditions will cover a narrower class of systems.

We will show that the conditions can be found by means of linear programming technique. In fact, the conditions can be expressed as linear constraints on system matrix entries. It should be noted that recently this ideology has been studied in some works and appeared to be useful for centralized control problems, [10], [11]. This study mainly apply this ideology to decentralized schemes.

3.3. Stabilization of continuous-time systems

Let us consider the linear continuous system equation 3.1.

The closed loop system has the form,

$$\dot{x} = A_{cl}\, x \quad , \quad A_{cl} = A - BK \quad , \quad K = \begin{bmatrix} k_1 & & & \mathbf{0} \\ & k_2 & & \\ & & \ddots & \\ \mathbf{0} & & & k_h \end{bmatrix}$$

$$A_{cl} = \left[\rho_{ij}\right]$$

The entries s_{ij} are linear functions of the entries of $K = \left[k_{ij}\right]$

$$\rho_{ij}(K) = a_{ij} + b_i K_j$$

Where b_i is the ith row of B and K_j is the jth column of K.

Based on the earlier definition of superstable matrices, we call K superstabilizing if A_{cl} is superstable, i.e. the closed loop matrix satisfy the following condition,

$$\mu = \mu(A_{cl}) \doteq \min\left(-\rho_{ii} - \sum_{i \neq j} \left|\rho_{ij}\right|\right) > 0, \quad i = 1, .., n$$

In which the parameter μ will be later used to describe the properties of the system.

Or based on corollary 3.1.

$$-\rho_{ii} > \sum_{i \neq j} \left|\rho_{ij}\right| , \quad i = 1, .., n \qquad (3.8)$$

If matrices are in interconnected form, the closed loop system takes the form,

$$A_{cl} = \begin{bmatrix} A_1 - B_1k_1 & A_{12} & \dots & A_{1h} \\ A_{21} & A_2 - B_2k_2 & & \vdots \\ \vdots & \vdots & \ddots & \\ A_{h1} & \dots & \dots & A_h - B_hk_h \end{bmatrix}$$

Now, let us call the rectangular blocks of this matrix as follows,

$$P^1 = [A_1 - B_1k_1 \quad A_{12} \quad \cdots \quad A_{1h}] = [\rho_{ij}^1],$$
$$P^2 = [A_2 - B_2k_2 \quad A_{21} \quad \cdots \quad A_{2h}] = [\rho_{ij}^2],$$
$$\vdots$$
$$P^h = [A_h - B_hk_h \quad A_{h1} \quad \cdots \quad A_{(h-1)\,h}] = [\rho_{ij}^h].$$

Where we have rearranged the sequence of blocks in some places in order to simplify the formulation of problem.

Introducing the slack variables μ , q_{ij}, for each ρ^i, conditions to obtain strict diagonal dominance as in corollary 3.1 can be expressed. This is shown for instance for P^1 :

$$\mu^1 > 0$$
$$-\rho^1{}_{ii} - \sum_{j\neq i} q_{ij}^1 \geq \mu^1 \quad , \quad i = 1, \dots, n_1 \,, j = 1, \dots, n \tag{3.9}$$
$$-q_{ij}^1 \leq \rho_{ij}^1 \leq q_{ij}^1 \quad , \quad i = 1, \dots, n_1 \,, j = 1, \dots, n \ , i \neq j$$

And similar conditions hold for $P^2, \dots, P^h$.

If the linear inequalities in 3.9 have a solution k_{ij}^1 , q_{ij}^1 for some $\mu^1 > 0$ then the conditions 3.8 are fulfilled for the first block row P^1, these conditions should be satisfied for the other block rows accordingly in order to make the total A_{cl} superstable. To test the existence of a solution the following LP can be considered for P^1,

$$max \ \mu^1$$
$$-\rho^1{}_{ii} - \sum_{j\neq i} q_{ij}^1 \geq \mu^1 \quad , \quad i = 1, \dots, n_1 \,, j = 1, \dots, n \tag{3.10}$$
$$-q_{ij}^1 \leq \rho_{ij}^1 \leq q_{ij}^1 \quad , \quad i = 1, \dots, n_1 \,, j = 1, \dots, n \ , i \neq j.$$

Similarly separate LPs should be considered for $P^2, \dots, P^h$.

Provided that a solution k_1 exists, it gives the best estimate over all possible ones, i.e. the solution to the LP 3.10 is the maximal value of μ^1 over all k_1. It will be shown in chapter 4 that these values impose the strictest constraint on system's state variables.
Also for $P^2, \dots, P^h$ similar LPs can be solved to find $k_2, \dots, k_h$, using these feedback gains altogether satisfy the following set of inequalities

$$\begin{gathered} -\rho^1{}_{ii} > \sum_{i \neq j} \left|\rho^1{}_{ij}\right|, \quad i = 1,2,..,n_1, j = 1, \dots, n \\ \vdots \\ -\rho^h{}_{ii} > \sum_{i \neq j} \left|\rho^h{}_{ij}\right|, \quad i = 1,2,..,n_h, j = 1, \dots, n \end{gathered} \tag{3.11}$$

And hence the static feedback K satisfies the inequality in 3.11 and decentrally stabilizes the total system. In addition, the parameter μ for the overall system is defined $\mu = \inf\{\mu^1, \dots, \mu^h\}$.

Example 3.3
Let us consider the following system composed of two subsystems each one affected by a scalar input,

$$\dot{x} = Ax + Bu, \quad A = [a_{ij}], \qquad i,j = 1, \dots, 4 \;, B = \begin{bmatrix} b_1 & 0 \\ b_2 & 0 \\ 0 & b_3 \\ 0 & b_4 \end{bmatrix},$$

$$u = Kx \;, K = \begin{bmatrix} k_1 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & k_4 \end{bmatrix}$$

Then

$$A_{cl} = A + BK = \begin{bmatrix} a_{11} + b_1 k_1 & a_{12} + b_1 k_2 & a_{13} & a_{14} \\ a_{21} + b_2 k_1 & a_{22} + b_2 k_2 & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} + b_3 k_3 & a_{34} + b_3 k_4 \\ a_{41} & a_{42} & a_{43} + b_4 k_3 & a_{44} + b_4 k_4 \end{bmatrix}$$

And the conditions 3.11 for the first subsystem (P^1) become,

$$a_{11} + b_1 k_1 < -|a_{12} + b_1 k_2| - |a_{13}| - |a_{14}| \;\;, \; a_{22} + b_2 k_2 < -|a_{21} + b_2 k_1| - |a_{23}| - |a_{24}|$$

We continue with P^1 (the same procedure can be done for P^2).
For simplicity assume that $b_1 = b_2 = 1$, the task is find the appropriate k_1 and k_2 to satisfy the following inequalities

$$a_{11} + k_1 < -|a_{12} + k_2| - |a_{13}| - |a_{14}| \quad , \quad a_{22} + k_2 < -|a_{21} + k_1| - |a_{23}| - |a_{24}|$$

The solution to these inequalities lies in the common intersection of two regions specified on the k1-k2 plane. (Figure 3.3)

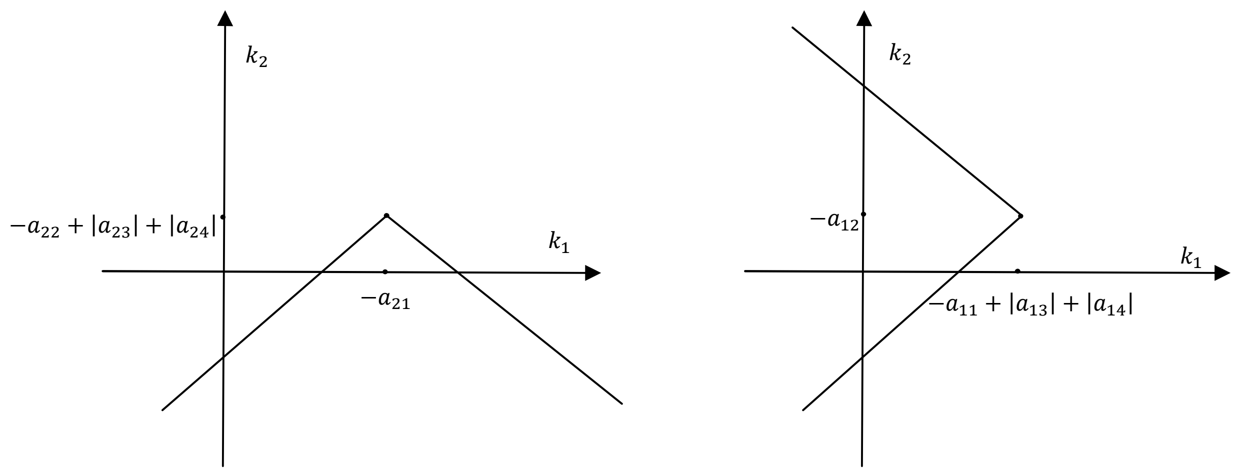

Figure 3.3. The right-angle sectors specified by entries

A solution exists if and only if a vertex of one angle belongs to the other angle i.e. if and only if one of the following inequalities holds,

$$a_{11} - a_{21} < -\big|a_{12} - a_{22} - |a_{23}| - |a_{24}|\big| - |a_{13}| - |a_{14}| \quad ,$$

$$a_{22} - a_{12} < -\big|a_{21} - a_{11} - |a_{13}| - |a_{14}|\big| - |a_{23}| - |a_{24}| \qquad (3.12)$$

Similarly for P^2, system matrix entries should satisfy one of the following inequalities if $b_3 = b_4 = 1$,

$$a_{33} - a_{43} < -\big|a_{34} - a_{44} - |a_{41}| - |a_{42}|\big| - |a_{31}| - |a_{32}| \quad ,$$

$$a_{44} - a_{34} < -\big|a_{34} - a_{33} + |a_{31}| + |a_{32}|\big| - |a_{41}| - |a_{42}| \qquad (3.13)$$

Therefore checking the conditions 3.11 reduces to check the inequalities 3.12 and 3.13 , and provided that a solution exists, decentralized stability of the total system is attainable by dominating the main diagonal of the total system matrix A_{cl}.

In addition, trivially, if the number of inputs of each subsystem is equal to the number of its states, superstability of the total system is always attainable due to the possibility of modifying each entry of the local closed loop subsystem matrix freely and hence satisfying the conditions (3.11).

In addition, if It is necessary to add limitations on the magnitude of gain matrix to bound the magnitude of control input, it suffices to include these constraints in the corresponding LP, for instance,

$$K = ((k_{ij})) \ , \ \big|k_{ij}\big| \le \overline{k}_{ij} \quad i,j = 1, \dots, n.$$

For the special case of no interactions among subsystems (completely decoupled) i.e. $a_{13} = a_{14} = \cdots = 0$, the conditions 3.12 ,3.13 reduce to the following:

$$a_{11} - a_{21} + a_{22} - a_{12} < 0\ ,\ a_{33} - a_{43} + a_{44} - a_{34} < 0$$

This shows that for the decoupled structure, controllability of the subsystems is still not sufficient and the stabilizable family of systems by this method reduces to a narrower class.
It should be noted that, although this restricts the family of stabilizable systems in the centralized scheme where only stabilizability (having no unstable uncontrollable mode) of the system is necessary and sufficient for stabilization, for the case of interconnected systems it is not much restrictive, since the stabilizability of subsystems is not generally sufficient for the task of total system stabilization due to the effects of internal interactions on the dynamics of the system.

3.4. Stabilization of discrete-time systems

So far, we have studied the stabilization problem for continuous-time interconnected systems. A similar study can be done for discrete-time systems. Given the continuous A_c and B_c matrices of the system, and taking T as the sampling time, using the transformation

$$x(k+1) = e^{A_c T} x(k) + \left(\int_0^T e^{A_c \theta} d\theta \right) B_c u(k)$$

the discrete-time state equation take the form

$$x(k+1) = Ax(k) + Bu(k)$$

Where $x(k) \in R^n$ is the state of the system and $u(k) \in R^m$ is the input to the system. Both *A* and *B* are constant portioned matrices with proper dimensions. Similar to the continuous case, system can be decomposed into several dynamical partitions:

$$x_i(k+1) = A_i x_i(k) + b_i u(k) + \sum_{\substack{j=1 \\ j \neq i}}^{h} A_{ij} x_j(k), \qquad i = 1, \dots, h$$

Where $x_i(k) \in R^{n_i}$ and $x_j(k) \in R^{n_j}$ and $x = ({x_1}^T {x_2}^T \dots h^T)^T$, $n = \sum_{i=1}^{h} n_i$.
Here the control algorithm which guarantees the stability, should keep the eigenvalues of the closed-loop system inside the unit circle.
The procedure is very similar to continuous-time. We use *h* local state-feedback controllers,

$$u_i = -k_i x_i, \quad i = 1,2,\dots,h \quad , \quad K = \begin{bmatrix} k_1 & & & \mathbf{0} \\ & k_2 & & \\ & & \ddots & \\ \mathbf{0} & & & k_h \end{bmatrix}$$

$$x_i(k+1) = (A_i - b_i k_i) x_i(k) + \sum_{\substack{j=1 \\ j \neq i}}^{h} A_{ij} x_j(k), \qquad i = 1,\dots,h$$

Hence, the closed-loop equation for the dynamical interconnected system can be written,

$$x(k+1) = A_{cl} x(k) = \begin{bmatrix} A_1 - b_1 k_1 & A_{12} & \dots & A_{1h} \\ A_{21} & A_2 - b_2 k_2 & & \vdots \\ \vdots & \vdots & & \\ A_{h1} & \dots & \dots & A_h - b_h k_h \end{bmatrix},$$

$$A_{cl} = [\rho_{ij}], \qquad i,j = 1,\dots,n$$

The condition for discrete superstability of A_{cl} takes the form,

$$\sum_{j=1}^{n} |\rho_{ij}| < 1 \quad , \quad i = 1,\dots,n \tag{3.13}$$

This inequality is derived based on Geršgorin disks, i.e. satisfying the condition guarantees that all the disks lie inside the unit circle.
Based on the definition of ∞-norm for matrices, the condition 3.13 can be rewritten

$$\|A_{cl}\| = \max_{1 \le i \le n} \left(\sum_{j=1}^{n} |\rho_{ij}| \right) < 1$$

Similar partitions $P^1, \dots, P^h$ are considered, and for each subsystem a linear inequality should be satisfied that altogether satisfy the inequality in 3.13.
In order to find the $min_K \|A_{cl}\|$ based on a block diagonal K in for each subsystem an LP can be considered.

$$min \ \mu^1$$

$$\sum_j q_{ij}^1 \le \mu^1 \qquad , \quad i = 1,\dots,n_1, j = 1,\dots,n$$

$$-q_{ij}^1 \le \rho_{ij}^1 \le q_{ij}^1 \quad , \quad i = 1,\dots,n_1, j = 1,\dots,n .$$

If this LP and similar LPs for $P^2, \dots, P^h$ possess a solution $k_1, \dots, k_h$ for $\mu^1, \dots, \mu^h < 1$ then $\|A_{cl}\| < 1$ and hence all the closed loop poles are guaranteed to lie inside the unit circle.

It should also be noted that both the continuous and discrete systems, obtain the so-called connective stability using the mentioned methods. Namely, individual decoupled systems obtain stability in addition to the whole interconnected system, and hence the closed-loop plant will be reliable under structural perturbations. Furthermore this distributed method reduces the complexity and dimensionality for stabilization of the overall system, and only requires having local states available which the latter is especially important when controllers don't have access to the global state variable, or transmission is costly, lossy or delayed.
Finally, for the case when states of adjacent subsystems are available for local controllers, the LPs can be modified to include feedbacks from other agents as well. This naturally makes the class stabilizable systems broader.

Chapter 4

Performance Properties of diagonally dominant stable systems

This chapter gives some insight into the useful features of diagonally dominant stable or superstable systems. These properties like monotonic decrease of state norm or the robustness issues are discussed in some works, (e.g. [5]). Very similar statements hold for large-scale or interconnected systems where the whole close-loop system matrix has been made superstable by means of a decentralized controller.

4.1. Monotonic Convergence and BIBO Stability

Consider the system $\dot{x} = Ax + Bu$. Where matrix A is superstable,

- If $u(t) \equiv 0$

$$\|x(t)\| \leq \|x_0\| e^{-\mu t} \quad , \; t \geq 0 \tag{4.1}$$

where $\mu = \mu(A)$ and the infinity norm for vector x has been used,

$$x \in \mathbb{R}^n \colon \|x\| = \max_{1 \leq i \leq n} |x_i|$$

This shows that the infinity norm of the state of the autonomous system converges monotonically to the origin.

- If $\|u(t)\| \leq 1$, $t \geq 0$ with the initial condition x_0,

$$\|x(t)\| \leq \gamma + e^{-\mu t}(\|x_0\| - \gamma) \quad , \qquad t \geq 0$$

Where $\gamma \doteq \|B\|/\mu$. This result relates to the BIBO (Bounded Input Bounded Output) stability and also implies that the trajectories originate and stay within an invariant set around the origin.
The proofs can be found in [10].
However, for a stable system the equation 4.1 is replaced with the following,

$$\|x(t)\| \leq C(A, \nu)\|x_0\|e^{-\nu t} \quad , \qquad 0 < \nu < \min_i\{-Re\lambda_i\}$$

Where C is a constant parameter and in general can be large. So in this case the norm does not monotonically decreases and may have an undesirable increase within the first time instants. Such overshoot does not occur in superstable systems as expressed by equation 4.1.

4.2. Robustness

Thus far, we have assumed to have the exact model of the plant, where the exact system matrices A and B are given. However, in practice control design should be reliable in the presence of model uncertainties. In fact model uncertainty has been an important issue in the evolution of automatic control and as a result this problem has received substantial attention [1]. One of the classes of robustness problem is the robust stability for interval matrix families.

Robustness of interval matrix family

Consider the state-space representation

$$\dot{x} = A(q)x$$

Where q is a vector of real uncertain parameters in a set Q. A family of interval matrices A_I can be defined,

$$A_I = \{A | a_{ij} = q_{ij} \ , \ q_{ij}^- \leq q_{ij} \leq q_{ij}^+\}$$

This shows that the system is described by a dynamic matrix where the entries take value in the interval $[q_{ij}^-, q_{ij}^+]$.
An uncertain matrix belonging to this family can be defined by a nominal matrix A_0 and uncertainties Δ_{ij} [10] ;

$$A = [a_{ij}] \ , \ a_{ij} = a_{ij}^0 + \gamma \Delta_{ij} \ , \qquad |\Delta_{ij}| \leq m_{ij} \ , \qquad i,j = 1, \dots, n$$

Where $\gamma \geq 0$ is a numerical parameter and $m_{ij} \geq 0$ are the entries of the matrix $M = [m_{ij}]$. Assuming A_0 is Hurwitz we aim to find the largest stability radius, γ_{max}, implying that the family will be stable for $\gamma < \gamma_{max}$. For the case where A_0 is stable in a general sense the problem has been proved to be NP-hard [9]. However for diagonally dominant stable matrices or the so-called superstable matrices the problem is simple. [10]

If the nominal matrix A_0 is superstable,

$$\mu(A_0) \doteq \min_i (-a_{ii}^0 - \sum_{i \neq j} |a_{ij}^0|) > 0$$

This condition should be satisfied for the interval family that is,

$$-(a_{ii}^0 + \gamma \Delta_{ii}) - \sum_{i \neq j} |a_{ij}^0 + \gamma \Delta_{ij}| > 0 \ , \qquad i = 1, \dots, n$$

This inequality will be satisfied if,

$$-(a_{ii}^0 + \gamma m_{ii}) - \sum_{i \neq j} (|a_{ij}^0| + \gamma m_{ij}) > 0 \ , \qquad i = 1, \dots, n \qquad (4.1)$$

And hence,

$$\gamma < \gamma_{max} = \min_i \frac{-a_{ii}^0 - \sum_{i \neq j} |a_{ij}^0|}{\sum_j m_{ij}}$$

Specially if $m_{ij} \equiv 1$, then,

$$\gamma_{max} = \frac{\mu(A_0)}{n}$$

Similarly for discrete case,

$$\gamma_{max} = \frac{1 - \|A_0\|}{n}$$

Robust stabilization

Consider an uncertain system model

$$\dot{x} = Ax + Bu$$

Assuming uncertainty is collocated in matrix A, we can consider it to be in interval form. The problem is find a decentralized state-feedback controller that stabilizes the entire family.
The closed-loop system take the form,

$$A_{cl} = A + BK = A_0 + \gamma\Delta + BK = A_{cl}^0(K) + \gamma\Delta$$

Where $\Delta = [\Delta_{ij}]$ and A_{cl}^0 is the nominal closed-loop matrix.
Similar to 4.1 the uncertain family will be stabilized if,

$$-(a_{ii}^0(K) + \gamma m_{ii}) - \sum_{i \neq j} \left(\left| a_{ij}^0(K) \right| + \gamma m_{ij} \right) > 0 \ , \qquad i = 1, \dots, n$$

If the decentralized superstabilization problem for nominal matrix A_{cl}^0 admits a solution K and μ, then

$$\gamma < \gamma_{max} = \frac{\mu(A_{cl}^0)}{n}$$

(For $m_{ij} \equiv 1$)
This gives the maximal radius of robustness because the parameter μ has been optimized over controllers K. Similar results hold for the discrete case.
To conclude, the problem of robust stability of interval matrix family admits a simple solution for the case of diagonally dominant stable systems, and the maximum robustness radius is found based on the parameter μ which can be optimized in the process of decentralized controller design.

Chapter 5

Conclusions

This thesis considers the problem of stability of large-scale interconnected systems. In order to deal with the growing size and complexity of the mathematical models of real processes a common method is to break the system down into a set of interconnected subsystems. The considered information structure assumes that each so-called agent or subsystem has only information about its own states variables.

A brief overview on the field of large-scale interconnected systems was given. Having introduced the classical and non-classical information and model structures, the problem of decomposition and decentralization has been considered with a graph theoretical view. In particular a common method for decomposition has been described which can be utilized to transform a large-scale system into an interconnected system.

The problem of decentralized control of an interconnected system with locally available states was formulated. Using some well-known results from matrix theory a sufficient condition for the stability of total system was derived. The so-called superstability condition was tailored thereafter for the stabilization problem at hand.

Stabilization of both continuous and discrete time interconnected systems was studied and sufficient conditions for the stability of both models were derived. Using the linear programming technique the stabilizable class of systems was found. Provided that the LPs are feasible the best estimates of stability degree could be calculated which imposes the strictest constraints on the system states.

The proposed method obtains the so-called connective stability, namely, individual decoupled systems obtain stability in addition to the whole interconnected system, and hence the closed-loop plant will be reliable under structural perturbations. Furthermore this distributed method reduces the complexity and dimensionality for stabilization of the overall system, and only requires having local states available which the latter is especially important when controllers don’t have access to the global state variable, or transmission is costly, lossy or delayed. Furthermore no check for unstable fixed modes or controllability of individual subsystems is needed for this scheme, i.e. having fulfilled the corresponding linear constraints, the stabilizability of system is guaranteed.

Some performance properties of the designed controllers were discussed such as monotonic convergence of state infinity norms to the origin and robustness in the interval matrix family. It was shown that these problems admit simple solutions if the superstability ideology is employed.

The study will further be extended for the case when states of adjacent subsystems are available for local controllers, hence the LPs should be modified to include feedbacks from other agents as well. This attempt may broaden the class of stabilizable systems.

Bibliography

[1] K.J.Åström., *Model Uncertainty and Robust Control*, Lecture Notes on Iterative Identification and Control , 2000.

[2] L. Bakule., *Decentralized Control: An overview*, Annual Reviews in Control, 32 (2008) 87-98

[3] E.J.Davison., *The decentralized Stabilization and Control of a Class of Unknown Nonlinear Time Varying Systems*, Automatica, 10, 309-316.

[4] E.J.Davison, U.Ozguner., *Characterization of Decentralized Fixed modes for Interconnected Systems*, Automatica, Vol. 19, No.2, pp. 169-182, (1983)

[5] R.A.Horn, C.R.Johnson, *Matrix Analysis*, Cambridge university press, 1985.

[6] M.Ikeda., *Decentralized Control of Large-scale Systems*, (1989), Three decades of Mathematical system theory, (1989) Springer

[7] M.Ikeda, and D.D.Siljak, *On Decentrally Stabilizable Large-scale Systems*, Automatica, 16, 331-334.

[8] J.Lunze , 1992, *Feedback Control of Large scale Systems*, Prentice Hall PTR

[9] A.A.Nemirovskii, *Several NP-hard Problems Arising in Robust Stability Analysis*, Math. Control, Sign., Syst., 1994, vol. 6,pp. 99-105

[10]. B.T. Polyak. And P.S. Shcherbakov., *Superstable Linear Control Systems. 1.Analysis*, Avtom. Telemekh., 2002, no.8,pp.37-53.

[11]. B.T. Polyak. And P.S. Shcherbakov., *Superstable Linear Control Systems. 2.Design*, Avtom. Telemekh., 2002, no.11,pp.56-75.

[12] M.Saif, Y. Guan., *Decentralized State Estimation in Large-scale Interconnected Dynamical Systems*, 1992 – Elsevier

[13] M.E. Sezer, and D.D.Siljak. *On Decentralized Stabilization and Structure of Linear Large-scale Systems*, Automatica, 17, 641-644 (1981)

[14] Z.C.Shi., W.B. Gao, *Stabilization by Decentralized Control for Large-scale Interconnected Systems*, Large-scale Systems, 10, 147-155

[15] D.D.Siljak, *Large-scale Dynamic Systems*, North Holland (1978)

[16] D.D.Siljak , A.I.Zecevic. *Control of Large-scale Systems: Beyond decentralized feedback*, Annual reviews in control 29 (2005) 169-179

[17] D.D.Siljak, and M.B. Vukcevic (1977). *Decentrally Stabilizable Linear and Bilinear Large-scale Systems*, Int.J.Control, 26, 289-305

[18] D.D. Siljak, M.B. Vukcevic., *Decentralization, Stabilization and Estimation of Large-scale Linear Systems,* IEEE Transactions on Automatic Control. Vol. AC-21, pp. 363-366. June 1976

[19] D.D.Šiljak., *Decentralized Control of Complex Systems*, Academic Press, 1991.

[20] M.G.Singh and A.Title Systems: *Decomposition , Optimization, and Control*, Pergamon Press (1978)

[21] M.K. Sundareshan, R.M. Elbanna., *Design of Decentralized Observation Schemes for Large-scale Interconnected Systems: Some new results*, Automatica, 1990 – Elsevier

[22] N.Viswanadham, and A.Ramakrishna (1982). *Decentralized Estimation and Control for Interconnected Systems*, large-scale systems, 3, 255-266

[23] H.Wu., *Decentralized Adaptive Robust Control for a class of Large-scale Systems with Uncertainties in the Interconnections*, International Journal of Control, 67(3), 253-265.