Modulation and noise properties of multi-element semiconductor lasers

Anders Karlsson, Richard Schatz and Olle Nilsson

Abstract

In this report a previously presented theory for the modulation and noise properties of multi-element lasers is revised. Explicit formulas for modulation response, amplitude and frequency noise are found. The theory is illustrated by numerical examples.

Contents

1 Introduction

In coherent optical communication systems severe requirements are put on the semiconductor lasers used as transmitters and local oscillators. This has lead to the advent of monolitic or hybrid structures where the characteristics of the laser more or less are tailored to meet a specific demand. Such demands could for example be a pure spectral behavior, large frequency tunability, a uniform and wideband FM-response with, at the same time, a low spurious AM modulation etc. Sometimes the laser should be optimised with respect to a pair or more of these characteristics, such as for a local oscillator in a multichannel FSK system, which should have both a pure spectral behaviour, ie at least a small linewidth, and at the same time be widely and preferably also rapidly tunable.

Much interest has focused on developing lasers with a small linewidths to be used as local oscillators. Indeed lasers with sub-Megahertz linewidths [1, 2] have been reported, this being a result of using long lasers with high Q-values, and by reduction of the gain-refractive index coupling enhancement of linewidth typical for semiconductor lasers.

Normally one should expect the laser linewidth to decrease inversely with output power, as predicted by the modified Shawlow-Townes linewidth formula [3, 5], but experimentally a linewidth floor or even a rebroadening of linewidth at high output powers has been found. This broadening has recently been the subject of intense research and debate and several models have been proposed to explain this behaviour [5].

In order to being able to study these multi-facetted aspects of laser noise, it would be preferable to have a desktop tool allowing for interactive simulation of the characteristics of complicated laser structures. We have developed a theory for prediction of modulation and spectral properties of multielement semiconductor lasers and implemented it in the form of a user friendly interactive simulation program on a Apple Macintosh computer. Here we present the theory, the frame of which has been presented earlier [4], and some results from computer calculations of modulation and spectral properties. The theory, at least partially, is shown to yield the same result as a theory independently developed by Tromborg et. al. [6]. This comparison is discussed in some detail in appendix D.

2 Theory

2.1 Linearized multiport description

Here we recapitulate the theory that was presented in Ref. [4]. This theory is based on ideas from [7, 8, 9], the steady state characteristics are calculated using a transfer matrix formalism [8], and the noise properties are calculated using an equivalent electrical circuit theory [7, 9]. However, in [7, 9] only a single active element was allowed, the present theory allows for multiple active elements or spatially varying noise sources.

Consider a multi-section laser structure, consisting of several active regions, gratings etc. The laser is assumed to operate in one single transverse and longitudinal mode. The oscillation frequency and the steady state field distribution is calculated from the transfer matrix method together with the carrier rate equations, as described in appendix E. The influence of modulation and noise, such as fluctuations in carrier numbers, is described in terms of small equivalent source currents inserted in the structure in a self-consistent way using a small signal linearization. The model we use is illustrated in Fig 1.

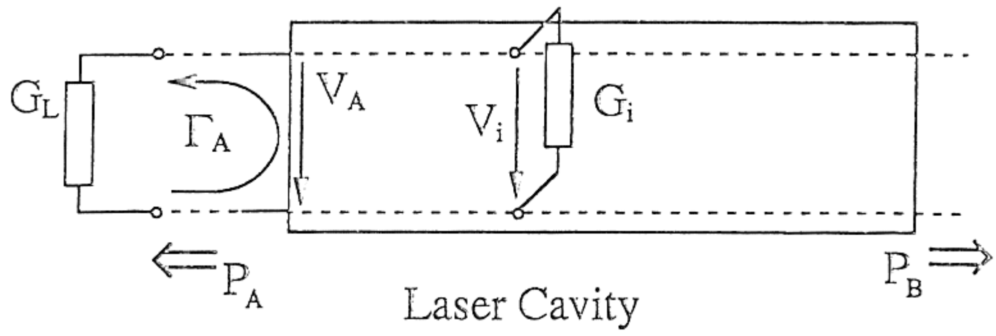


Fig. 1 : Schematic representation of the laser model. The laser cavity is treated as a transmission line. Conductances are inserted at appropriate locations.

The equivalent total admittance in plane A can be written as

$$Y = G_L + Y_A = G_L \frac{2}{1+\Gamma_A} \tag{1}$$

where G_L is the equivalent load conductance, Y_A the active admittance as seen looking into the laser structure and Γ_A is the reflection coefficient, which can be defined as

$$\Gamma_A \equiv \frac{V_{A,\text{out}}}{V_{A,\text{in}}} \quad . \tag{2}$$

At oscillation we have

$$Y(\omega_0) = 0 \quad . \tag{3}$$

In the transfer matrix method the oscillation frequency is found from the zeros of $1/\Gamma_A$ [8]. Since the reflection coefficient can be determined as a function of frequency, Y can be considered as a known function of frequency. The laser output field which usually is measured experimentally differs from the internal field which usually is calculated theoretically [10, 7]. In the language of electrical circuit theory the laser output field is represented by the outgoing voltage wave $V_{A,\text{out}}$, whereas the internal field (at the output mirror) is represented by the total voltage V_A. These are related by

$$V_{A,\text{out}} \equiv V_A - V_{A,\text{in}} = V_A - \frac{I_A}{2G_L} \quad . \tag{4}$$

Here I_A is the source current of the matched load G_L. To find the voltage amplitude V_A in the presence of small source currents in the active regions and at the outputs, a multiport description is used. The laser is divided in regions so small that the field can be considered as constant within each region. We also need the voltage amplitudes, V_i, at the locations of the source currents I_i. The multiport is linear as defined, so that one can write

$$\begin{aligned}
Y \cdot V_A &= I_A + \sum_i H_{Ai} I_i + H_{AB} I_B \\
Y \cdot V_i &= H_{iA} I_A + \sum_j H_{ij} I_i + H_{iB} I_B \\
Y \cdot V_B &= H_{BA} I_A + \sum_i H_{Bi} I_i + H_{BB} I_B
\end{aligned} \tag{5}$$

where, assuming reciprocity,

$$H_{Ai} = H_{iA} \quad \ldots \tag{6}$$

Here, both Y and the transfer functions H are functions of frequency. The calculation of the transfer functions H is discussed in appendix A.

2.2 The modulated field

The equivalent voltages describing the laser field can be written

$$V_{A,i,B}(t) = Re\{V_{A,i,B0}e^{j\omega_0 t + c_{A,i,B}(t)}\} \tag{7}$$

where

$$c(t) = a(t) + j\varphi(t) \tag{8}$$

describes the amplitude and phase modulation at the various locations. Here $a(t)$ and $\varphi(t)$ are real, small quantities except for a very slowly varying part of $\varphi(t)$ expressing the phase diffusion common to all fields in the laser, the difference of $c(t)$ for any two sections are therefore always small. We can take the source currents to be of the form

$$I_{A,i,B}(t) = Re\{I_{A,i,B\Omega}e^{j(\omega_0+\Omega)t + c_{A,i,B}(t)}\} \tag{9}$$

where $\Omega = \omega - \omega_0$. In eq. (9) we have only the upper sideband part, there will also be a lower sideband part, as it is discussed in appendix B. Eqs. (5) can then be approximated as

$$\begin{aligned}
\frac{1}{2}V_{A0}Y_\Delta(\Omega)\cdot(a_{A\Omega} + j\varphi_{A\Omega}) &= I_{A\Omega} + \sum_i H_{\Delta Ai}(\Omega)I_{i\Omega} + H_{\Delta AB}(\Omega)I_{B\Omega} \\
\frac{1}{2}V_{i0}Y_\Delta(\Omega)\cdot(a_{i\Omega} + j\varphi_{i\Omega}) &= H_{\Delta Ai}(\Omega)I_{A\Omega} + \sum_j H_{\Delta ij}(\Omega)I_{j\Omega} + H_{\Delta Bi}(\Omega)I_{B\Omega} \\
\frac{1}{2}V_{B0}Y_\Delta(\Omega)\cdot(a_{B\Omega} + j\varphi_{B\Omega}) &= H_{\Delta AB}(\Omega)I_{A\Omega} + \sum_i H_{\Delta Bi}(\Omega)I_{i\Omega} + H_{\Delta BB}(\Omega)I_{B\Omega} \quad (10)
\end{aligned}$$

and the corresponding equations with Ω replaced by $-\Omega$. We also used eq. (3) and the notation

$$Y_\Delta(\Omega) = Y(\omega), \qquad H_\Delta(\Omega) = H(\omega) \quad , \tag{11}$$

$$a(t), \varphi(t) = Re\{a_\Omega, \varphi_\Omega e^{j\Omega t}\} \quad . \tag{12}$$

Note that positive and negative frequency components will be coupled, as will be shown below, and that $Y_\Delta(-\Omega)$ does not equal $Y^*_\Delta(\Omega)$ in the general case.

2.3 Carrier-rate equations

The carrier-rate equation for region i can be written [11]:

$$\frac{dN_i}{dt} = J_i - R_i - \frac{P_{si}}{\hbar\omega} - \Gamma_{Ri} - \Gamma_{si} \tag{13}$$

where N_i is the number of carriers, J_i is the number of injected carriers per unit time including any pump noise, R_i is the spontaneous recombination rate, P_{si} is the net generated power through stimulated emission-absorption, Γ_{Ri} is the recombination fluctuation and finally Γ_{si} is the dipole fluctuation accompanying the stimulated emission-absorption. The noise sources will be discussed in Section 2.4. The carrier diffusion term included in [4] has been neglected. In order to find a small signal equation we define

$$s_{Ni} = \frac{1}{g_{si}} \cdot \frac{\partial g_{si}}{\partial N_i} \quad , \tag{14}$$

$$s_{Pi} = \frac{|V_i|^2}{g_{si}} \cdot \frac{\partial g_{si}}{\partial |V_i|^2} \tag{15}$$

where g_{si} is the gain constant, s_{Ni} express the differential gain and s_{Pi} express the non-linear gain. We also need the relative intensity variation

$$\frac{\Delta |V_i|^2}{|V_i|^2} = 2a_i(t) \quad . \tag{16}$$

Using

$$\Delta N_i(t) = Re\{\Delta N_{i\Omega} e^{j\Omega t}\} \quad , \tag{17}$$

we can do a small signal Fourier decomposition of the rate equation, finding

$$(j\Omega + \frac{P_{si}}{\hbar\omega} s_{Ni} + \frac{dR_i}{dN_i})\Delta N_{i\Omega} = \Delta J_{i\Omega} - 2a_\Omega \frac{P_{si}}{\hbar\omega}(1 + s_{Pi}) - \Gamma_{Ri\Omega} - \Gamma_{si\Omega} \quad . \tag{18}$$

2.4 Source currents and noise sources.

The total noise current to be used in eq. (10) can be written as a sum of four contributions

$$I_{i\Omega} = I_{Ni\Omega} + I_{Pi\Omega} + I_{Di\Omega} + I_{ri\Omega} \quad . \tag{19}$$

Here $I_{Ni\Omega}$ is proportional to the resulting carrier fluctuations, $I_{Pi\Omega}$ stems from the nonlinear gain, both of these could also be deterministic. $I_{Di\Omega}$ stems from the gain mechanism (dipole fluctuations) and finally $I_{ri\Omega}$ is a noise current accounting for optical losses others than those arising from the stimulated absorption, such as waveguide losses.

A fluctuation in carrier number ΔN_i gives rise to a fluctuation in conductivity that we account for by a source current

$$I_{Ni}(t) = Re\{V_{i0}e^{j\omega_0 t + c_i(t)} \cdot \Delta N_i(t) \cdot \frac{2P_{si}}{|V_{i0}|^2} s_{Ni} \cdot (1 - j\alpha_i)\} \tag{20}$$

where

$$\alpha = \frac{\partial \epsilon'}{\partial N} / \frac{\partial \epsilon''}{\partial N} \tag{21}$$

is the linewidth enhancement factor which is usually negative in semiconductor lasers [3][1]. Note that this current represent the gain clamping that acts as a restoring force on amplitude fluctuation. With Eq. (17) one obtains for the current to be used in Eqs. (10):

$$I_{Ni\Omega} = \frac{V_{i0}P_{si}}{|V_{i0}|^2} \cdot s_{Ni} \cdot (1 - j\alpha_i)\Delta N_{i\Omega} \quad . \tag{22}$$

The source current for the nonlinear gain is similarily obtained as

$$I_{Pi\Omega} = \frac{V_{i0}P_{si}}{|V_{i0}|^2} \cdot s_{Pi} 2a_{i\Omega} \quad . \tag{23}$$

Here we have neglected the nonlinearity in ϵ'.

Every stimulated recombination or excitation event is accompanied by noise stemming from the fluctuating dipoles. The noise sources can be assumed as spatially independent, and the single-sided power spectral density of I_{Di} is [9]

$$S_{fDi} = 2\hbar\omega \frac{2P_{si}}{|V_{i0}|^2}(2n_{\mathrm{sp}i} - 1) \tag{24}$$

where $n_{\mathrm{sp}i}$ is the spontaneous emission factor for region i. We also have a noise current I_{Ri} stemming from other losses than stimulated absorption, such as free carrier absorption and scattering losses. It has a power spectral density given by

$$S_{fri} = 2\hbar\omega \frac{2P_{Ri}}{|V_{i0}|^2} \tag{25}$$

where P_{Ri} is the total loss mentioned above.

Further, there are fluctuations due to the matched loads at A and B (ie. the zero-point fluctuations). The noise contribution to I_A and I_B will have spectral densities given by

$$S_{fA} = 2\hbar\omega G_L = 2\hbar\omega \frac{2P_A}{|V_A|^2} \qquad , S_{fB} = 2\hbar\omega \frac{2P_B}{|V_B|^2} \tag{26}$$

where P_A is the output power at facet A and P_B is the output power at facet B.

[1]In Henry's definition $\alpha \equiv \Delta n'/\Delta n''$ is positive, the negative α here is due to a different sign in the definition.

Three noise sources enters the carrier-rate equation. The first is the pump noise from the current injection, which can be directly included in J_i. This may or may not be suppressed depending on the pumping mechanism of the laser [12, 13]. The second is the recombination noise Γ_{Ri}, with power spectral density assumed to be

$$S_{fRi} = 2R_i \tag{27}$$

where R_i is the total recombination. At last we have the noise from the gain mechanism (dipole moment fluctuations) Γ_{si}, that must enter the rate equation since every stimulated emission or absorption event also involves a carrier. It therefore can be assumed that

$$\hbar\omega\Gamma_{si}(t) = \langle V_i(t) \cdot I_{Di}(t)\rangle \tag{28}$$

where the average is taken over several light periods. If the in-phase component of the fluctuation is defined to have the phase of V_i one finds, appendix C, eq. (C.4)

$$\Gamma_{si\Omega} = \frac{|V_{i0}|}{2\hbar\omega}[I_{Di\Omega} + I^*_{Di-\Omega}] \tag{29}$$

which shows the correlation between the gain (dipole) fluctuations entering the field equations, and Γ_{si} in the carrier rate equations.

2.5 Passive tuning region

For a passive tuning section the rate equations and the equivalent source current must be rewritten. For source current corresponding to eq. (20) we get

$$I_{Nti}(t) = Re\{V_{i0}e^{j\omega_0 t + c_i(t)} \cdot \Delta N_i(t) \cdot \frac{2\hbar\omega S_i}{|V_{i0}|^2} n_{ti} \cdot (1 - j\alpha_{ti})\} \tag{30}$$

where $\hbar\omega\Delta N_i \cdot S_i n_{ti}$ is the loss power when the absorbtion is changed from zero to $\Delta N_i n_{ti}$. Hence, n_{ti} is related to the change in waveguide loss with carrier density (ie. $n_{ti} = v_g d\gamma_i / dN_i$, where the absorbtion γ_i has the unit of inverse length). The change in real part of refractive index can be included as an equivalent α-parameter α_{ti}. The Fourier components of I_{Nti} may then be written as

$$I_{Nti\Omega} = \frac{V_{i0}\hbar\omega S_i}{|V_{i0}|^2} \cdot n_{ti} \cdot (1 - j\alpha_{ti})\Delta N_{i\Omega} \quad . \tag{31}$$

The carrier rate equation for a tuning section with carrier injection may be written as

$$\frac{dN_{ti}}{dt} = J_{ti} - R_{ti} - \Gamma_{Rti} \tag{32}$$

where R_{ti} and Γ_{Rti} are the deterministic recombination and its associated fluctuation. The recombination fluctuation is assumed to have a spectral density given by

$$S_{fRti} = 2R_{ti} \tag{33}$$

in analogy with Eq. (27) . With this the theory is complete and we will now apply it to noise spectrum calculations and illustrate with several examples.

3 Spectrum calculations

To calculate the noise spectra and the linewidth of a multielement laser we first treat a simplified case by assuming that the amplitude and phase can be taken as the same all over the laser, though one should note that this need not generally be true [14, 15, 6]. Very recently [16] it was suggested that such spatial fluctuations in photon distribution could explain the linewidth rebroadening observed in lasers at high output powers. Also one could expect that the approximation of common amplitude may be incorrect close to points of longitudinal instability [17]. The more general treatment is beyond the present report and will be given elsewhere. As a starting point we assume that we have low frequencies compared to any roundtrip times so that

$$Y_\Delta(\Omega) = \Omega \cdot \frac{\partial Y_\Delta(0)}{\partial \Omega}, \qquad H_\Delta(\Omega) \approx H_\Delta(0), \qquad V_{i0} = H_{\Delta Ai}(0) \cdot V_{A0} \quad . \tag{34}$$

We may also choose a reference plane such that

$$\frac{\partial Y_\Delta}{\partial \Omega} = j\frac{\partial B_\Delta}{\partial \Omega} \tag{35}$$

where B_Δ is the imaginary part of the admittance Y_Δ. We further assume that the in-phase component , ie. $I^I = I_\Omega + I^*_{-\Omega}$, of the current has the same phase as V_i and the quadrature phase is taken as $I^O = -j(I_\Omega - I^*_{-\Omega})$, (see appendix B for the definition of in-phase and quadrature phase components of I). Using this in the first of eqs. (10), again taking V_{A0} as real, and adding the complex conjugate of the corresponding equation for $-\Omega$, one obtains

$$\begin{aligned} j\Omega\frac{\partial B_\Delta}{\partial \Omega}V_{A0}^2 a_{A\Omega} &= V_{A0}I_A^I + V_{B0}I_B^I \\ +\sum_i [Re\{\frac{V_i^2}{|V_i|}\}(I_{Di}^I + I_{Pi}^I + I_{ri}^I) &- Im\{\frac{V_i^2}{|V_i|}\}(I_{Di}^O + I_{ri}^O)] \\ &+ \sum_i \frac{2P_{si}}{|V_i|^2} Re\{V_i^2(1-j\alpha_i)\} s_{Ni}\Delta N_{i\Omega} \quad , \end{aligned} \tag{36}$$

$$\begin{aligned} j\Omega\frac{\partial B_\Delta}{\partial \Omega}V_{A0}^2 \varphi_{A\Omega} &= V_{A0}I_A^O + V_{B0}I_B^O \\ \sum_i [Re\{\frac{V_i^2}{|V_i|}\}(I_{Di}^O + I_{ri}^O) &+ Im\{\frac{V_i^2}{|V_i|}\}(I_{Di}^I + I_{Pi}^I + I_{ri}^I)] \\ &+ \sum_i \frac{2P_{si}}{|V_i|^2} Im\{V_i^2(1-j\alpha_i)\} s_{Ni}\Delta N_{i\Omega} \quad . \end{aligned} \tag{37}$$

The only thing left is to elimate ΔN from the equations by using the carrier rate equations, which can be written

$$s_{Ni}\Delta N_{i\Omega} = \frac{\hbar\omega}{P_{si}(1+\epsilon_i)}[\Delta J_{i\Omega} - 2a_\Omega \frac{P_{si}}{\hbar\omega}(1+s_{Pi}) - \Gamma_{Ri\Omega} - \Gamma_{si\Omega}] \quad . \tag{38}$$

Here we have put

$$\epsilon_i = (\frac{dR_i}{dN_i} + j\Omega)/\frac{P_{si}}{\hbar\omega}s_{Ni} \quad , \tag{39}$$

which tends to zero for high pumping and low frequencies. For a passive tuning region we introduce

$$\epsilon_{ti} = \frac{j\Omega}{\frac{dR_i}{dN_i}} \equiv j\Omega\tau_{spti} \tag{40}$$

where the spontaneous lifetime of the passive section also has been introduced. To simplify the notation we introduce

$$V_{\alpha,i}^2 \equiv V_i^2(1-j\alpha_i) \tag{41}$$

and

$$V_{\alpha,ti}^2 \equiv V_i^2(1-j\alpha_{ti}) \quad . \tag{42}$$

We can introduce the contributions from the passive regions by inserting Eq. (38) and Eq. (29) into the amplitude equation Eq. (36). This gives

$$\begin{aligned}
2a_\Omega[\sum_i \frac{2P_{si}}{(1+\epsilon_i)}[(1+s_{Pi})Re\{\frac{V_{\alpha,i}^2}{|V_i|^2}\} - s_{Pi}(1+\epsilon_i)Re\{\frac{V_i^2}{|V_i|^2}\} \quad + \quad j\frac{1}{2}V_{A0}^2\Omega\frac{\partial B_\Delta}{\partial\Omega}] = \\
V_{A0}I_A^I + V_{B0}I_B^I + \sum_i \frac{2P_{si}}{(1+\epsilon_i)}Re\{\frac{V_{\alpha,i}^2}{|V_i|^2}\}(\frac{\hbar\omega\Delta J_{i\Omega}}{P_{si}} \quad - \quad \frac{\hbar\omega\Gamma_{Ri}}{P_{si}}) \\
+\sum_i I_{Di}^I \frac{|V_i|}{(1+\epsilon_i)}[(1+\epsilon_i)Re\{\frac{V_i^2}{|V_i|^2}\} - Re\{\frac{V_{\alpha,i}^2}{|V_i|^2}\}] \quad - \quad \sum_i Im\{\frac{V_i^2}{|V_i|^2}\}I_{Di}^O \\
+\sum_i \frac{Re\{V_i^2\}}{|V_i|}I_{ri}^I \quad - \quad \frac{Im\{V_i^2\}}{|V_i|}I_{ri}^O \\
+\sum_i^t \frac{2\hbar\omega S_{ti}n_{ti}\tau_{sp,i}}{(1+\epsilon_{ti})}Re\{\frac{V_{\alpha,ti}^2}{|V_i|^2}\}(\Delta J_{ti\Omega} \quad - \quad \Gamma_{Rti}) \quad .
\end{aligned} \tag{43}$$

By index t on the summation sign we indicate that this summation is to be taken over tuning regions only. For the phase equation we introduce

$$\alpha_{ave} = -\frac{\sum_i \frac{2P_{si}}{(1+\epsilon_i)}[(1+s_{Pi})Im\{\frac{V_{\alpha,i}^2}{|V_i|^2}\} - s_{Pi}(1+\epsilon_i)Im\{\frac{V_i^2}{|V_i|^2}\}]}{\sum_i \frac{2P_{si}}{(1+\epsilon_i)}[(1+s_{Pi})Re\{\frac{V_{\alpha,i}^2}{|V_i|^2}\} - s_{Pi}(1+\epsilon_i)Re\{\frac{V_i^2}{|V_i|^2}\}] + j\frac{1}{2}V_{A0}^2\Omega\frac{\partial B_\Delta}{\partial\Omega}}, \quad (44)$$

which for low frequencies is an average α-parameter for the entire cavity. Note that α_{ave} is a frequency-dependent term containing also the enhancement at the relaxation oscillation peak. Further, we then insert the resulting amplitude in the phase equation, and get for the low frequency fluctuations

$$\begin{aligned}
j\Omega\frac{\partial B_\Delta}{\partial\Omega}V_{A0}^2\varphi_{A\Omega} &= V_{A0}I_A^O + V_{A0}I_B^O + \alpha_{ave}(V_{A0}I_A^I + V_{B0}I_B^I) \\
&+ \sum_i \frac{2P_{si}}{(1+\epsilon_i)}(\frac{\hbar\omega\Delta J_{i\Omega}}{P_{si}} - \frac{\hbar\omega\Gamma_{Ri}}{P_{si}})[Im\{\frac{V_{\alpha,i}^2}{|V_i|^2}\} + \alpha_{ave}Re\{\frac{V_{\alpha,i}^2}{|V_i|^2}\}] \\
&+ \sum_i I_{ri}^O(\frac{Re\{V_i^2\}}{|V_i|} - \alpha_{ave}\frac{Im\{V_i^2\}}{|V_i|}) + I_{ri}^I(\frac{Im\{V_i^2\}}{|V_i|} + \alpha_{ave}\frac{Re\{V_i^2\}}{|V_i|}) \\
&+ \sum_i \frac{I_{Di}^I|V_i|}{(1+\epsilon_i)}[(1+\epsilon_i)(\alpha_{ave}Re\{\frac{V_i^2}{|V_i|^2}\} + Im\{\frac{V_i^2}{|V_i|^2}\}) - (Im\{\frac{V_{\alpha,i}^2}{|V_i|^2}\} + \alpha_{ave}Re\{\frac{V_{\alpha,i}^2}{|V_i|^2}\})] \\
&+ I_{Di}^O|V_i|(Re\{\frac{V_i^2}{|V_i|^2}\} - \alpha_{ave}Im\{\frac{V_i^2}{|V_i|^2}\}) \\
&+ \sum_i^t \frac{2\hbar\omega S_{ti}n_{ti}\tau_{sp,ti}}{(1+\epsilon_{ti})}[Im\{\frac{V_{\alpha,ti}^2}{|V_i|^2}\} + \alpha_{ave}Re\{\frac{V_{\alpha,ti}^2}{|V_i|^2}\}][\Delta J_{ti\Omega} - \Gamma_{Rti}] \quad . \quad (45)
\end{aligned}$$

In this expression, we can see how the influence of the dipole fluctuations via both the field equation and the carrier rate equation affects the phase noise. The first terms with I_D^I stems from the field equation, and the second, via α_{ave}, from the carrier rate equation.

The frequency noise spectrum is a sum of three contributions (four if there are contributions from passive tuning regions)

$$S_{\frac{d\phi}{dt}}(\Omega) = S_{f,ord}(\Omega) + S_{f,NN}(\Omega) + S_{f,NP}(\Omega) \quad , \quad (46)$$

where

$$S_{f,ord}(\Omega) = 4\hbar\omega[\frac{G_L}{\frac{\partial B_\Delta}{\partial\Omega}}]^2\frac{1+|\alpha_{ave}|^2}{P_A^2}\sum_i P_{s,i}n_{sp,i} \quad , \quad (47)$$

$$\begin{aligned}
S_{f,NN}(\Omega) = \frac{2\hbar\omega}{P_A^2}[\frac{G_L}{\frac{\partial B_\Delta}{\partial\Omega}}]^2 \cdot \sum_i \frac{(\hbar\omega\xi J_i + \hbar\omega R_i + P_{si}(2n_{sp,i}-1))}{|1+\epsilon_i'|^2} \\
\cdot|Im\{\frac{V_{\alpha,i}^2}{|V_i|^2}\} + \alpha_{ave}Re\{\frac{V_{\alpha,i}^2}{|V_i|^2}\}|^2 \quad , \quad (48)
\end{aligned}$$

$$S_{f,NP}(\Omega) = -\frac{4\hbar\omega}{P_A^2}[\frac{G_L}{\frac{\partial B_\Delta}{\partial\Omega}}]^2 \quad \cdot \quad \sum_i P_{si}(2n_{sp,i}-1)$$

$$\cdot Re\{[\alpha_{ave}Re\{\frac{V_i^2}{|V_i|^2}\} + Im\{\frac{V_i^2}{|V_i|^2}\} \quad][\quad \frac{Im\{\frac{V_{\alpha,i}^2}{|V_i|^2}\} + \alpha_{ave}Re\{\frac{V_{\alpha,i}^2}{|V_i|^2}\}}{1+\epsilon_i'}]^*\}, \qquad . \qquad (49)$$

Here the $*$ denotes complex conjugate and the vertical bars denote modulus. If we have passive tuning sections, we get an additional term equal to

$$S_{f,tNN}(\Omega) \;=\; \frac{2}{P_A^2}[\frac{G_L}{\frac{\partial B_\Delta}{\partial\Omega}}]^2$$

$$\cdot\sum_i^t \frac{(\hbar\omega S_{ti} n_{ti}\tau_{sp,ti})^2(\xi J_{ti}+R_{ti})}{|1+\epsilon_{ti}'|^2} \quad \cdot \quad |Im\{\frac{V_{\alpha,ti}^2}{|V_i|^2}\} + \alpha_{ave}Re\{\frac{V_{\alpha,ti}^2}{|V_i|^2}\}|^2 \qquad . \qquad (50)$$

The semiconductor laser linewidth is given from expression Eq. (46) by looking at the frequency noise for low frequencies, using

$$\Delta\nu = \frac{1}{4\pi} S_{\frac{d\phi}{dt}}(\Omega \to 0) \qquad . \qquad (51)$$

Note that the modulus and complex conjugate in the equations above may be omitted for linewidth calculations. For the linewidth, this result generalizes the previous results of Ujihara [18], Henry [19], Arnaud [20], Björk [9], Tromborg [21], Wang [22] and Duan [23]. The first term is the ordinary linewidth formula, the second is carrier induced frequency noise, the ξ term is to account for an eventual pump noise suppression. Finally the third term arise from the correlation between carrier number and photon number through the gain mechanism. This result for the linewidth is very similar to results recently given by Tromborg, Pan and Olesen [25, 26, 6], who used a Green function formalism. The similarities and differences between these two theories are described in appendix D. However, we can conclude that the phenomena of linewidth rebroadening is not expected to occur in the present approximation of the theory, since it does not occur for the same approximations in the theory of [6]. If we now concentrate on the passive sections only, we find by inspecting eq. (45) that the frequency tuning may be written as

$$\Delta f = \frac{1}{2\pi}[\frac{G_L}{P_A\frac{\partial B_\Delta}{\partial\Omega}}] \cdot \sum_i^t \hbar\omega S_{ti} n_{ti}\tau_{sp,ti}[Im\{\frac{V_{\alpha,ti}^2}{|V_i|^2}\} + \alpha_{ave}Re\{\frac{V_{\alpha,ti}^2}{|V_i|^2}\}]\Delta J_{ti\Omega} \equiv \sum_i^t[\frac{\partial f}{\partial J_{ti}}]\Delta J_{ti\Omega}, \qquad (52)$$

where this equation also defines the local tuning efficiency $\partial f/\partial J_{ti}$ In appendix D we show that this result is in agreement with the result of [6]. If we assume full shot noise of the injection current of the tuning region we have that $J_{ti} = R_{ti}$ and $S_{J_{ti}} = S_{R_{ti}} = 2J_{ti}$ which gives

$$S_{f,tNN}(\Omega) = 4\pi^2 S_{\Delta f} = S_{f,tNN}(\Omega) = 16\pi^2 \sum_i^t [\frac{\partial f}{\partial J_{ti}}]^2 J_{ti} \quad . \tag{53}$$

For a case where the local tuning efficiency is constant ($\partial f/\partial J_{ti}$ is constant) e.g. a Fabry Perot section [6], we get the linewidth contribution from the tuning region as

$$\Delta\nu_{tNN} = 4\pi[\frac{\partial f}{\partial J_t}]^2 J_t \quad . \tag{54}$$

This is the result obtained by Amann et. al. [27, 28].

Now, turning to the calculation of amplitude noise, the amplitude of the outgoing field can be expressed as

$$a_{\Omega,out} = a_\Omega - \frac{|V_{A0}|}{4P_A} I_A^I \quad . \tag{55}$$

Putting

$$P_c = \frac{1}{2}[\sum_i \frac{2P_{si}}{(1+\epsilon_i')}[(1+s_{Pi})Re\{\frac{V_{\alpha,i}^2}{|V_i|^2}\} - s_{Pi}(1+\epsilon_i')Re\{\frac{V_i^2}{|V_i|^2}\}] + j\frac{1}{2}V_{A0}^2\Omega\frac{\partial B_\Delta}{\partial\Omega}] \quad , \tag{56}$$

we get the amplitude noise spectrum as

$$\begin{aligned} S_{a,out}(\Omega) = \frac{\hbar\omega}{2|P_c|^2}[|1-\frac{P_c}{P_A}|^2 P_A + P_B + \sum_i P_r \quad &+ \quad \sum_i P_{si}(2n_{sp,i}-1) \\ +\sum_i \frac{(\hbar\omega\xi J_i + \hbar\omega R_i + P_{si}(2n_{sp,i}-1))}{|1+\epsilon_i'|^2} \quad & \quad Re\{\frac{V_{\alpha,i}^2}{|V_i|^2}\}^2 \\ -2\sum_i P_{si}(2n_{sp,i}-1)\cdot Re\{\frac{V_i^2}{|V_i|^2}\} \quad &\cdot \quad Re[\frac{Re\{\frac{V_{\alpha,i}^2}{|V_i|^2}\}}{1+\epsilon_i'}]\}] \quad . \end{aligned} \tag{57}$$

We can now make a test of the validity of the approximation of common amplitude. If we assume output coupling only on one side , ie. $P_B \to 0$, no internal losses $\sum_i P_r \to 0$, then if the laser is pumped very high all injected electrons will sooner or later be converted to photons that are coupled out from the laser. This implies that for timescales longer than any storage time, the statisics of the output light should follow that of the total pump current. If the pump current statistics were sub-Poissonian, an amplitude squeezed light output results. In terms of Eq. (57) this means that only the current terms J_i

should remain. However, from Eq. (57) it looks like that a low-reflectivity cavity could have residual noise, since the dipole moment and vacuum fluctuations do not cancel out. This leads to our conclusion that the formula cannot be correct for this case since the squeezing property follows purely from energy conservation.

4 Modulation properties

The AM and FM modulation properties of a multisection laser simply follows by only considering the deterministic currents, thus from Eqs. (43) and (45) we have

$$2a_\Omega[\sum_i \frac{2P_{si}}{(1+\epsilon_i)}[(1+s_{Pi})Re\{\frac{V_{\alpha,i}^2}{|V_i|^2}\} - s_{Pi}(1+\epsilon_i)Re\{\frac{V_i^2}{|V_i|^2}\}] + j\frac{1}{2}V_{A0}^2\Omega\frac{\partial B_\Delta}{\partial\Omega}]$$
$$= \sum_i \frac{2P_{si}}{(1+\epsilon_i)}Re\{\frac{V_{\alpha,i}^2}{|V_i|^2}\}\frac{\hbar\omega\Delta J_{i\Omega}}{P_{si}} + \sum_i^t \frac{2\hbar\omega S_{ti}n_{ti}\tau_{sp,i}}{(1+\epsilon_{ti})}Re\{\frac{V_{\alpha,ti}^2}{|V_i|^2}\}\Delta J_{ti\Omega} \quad , \quad (58)$$

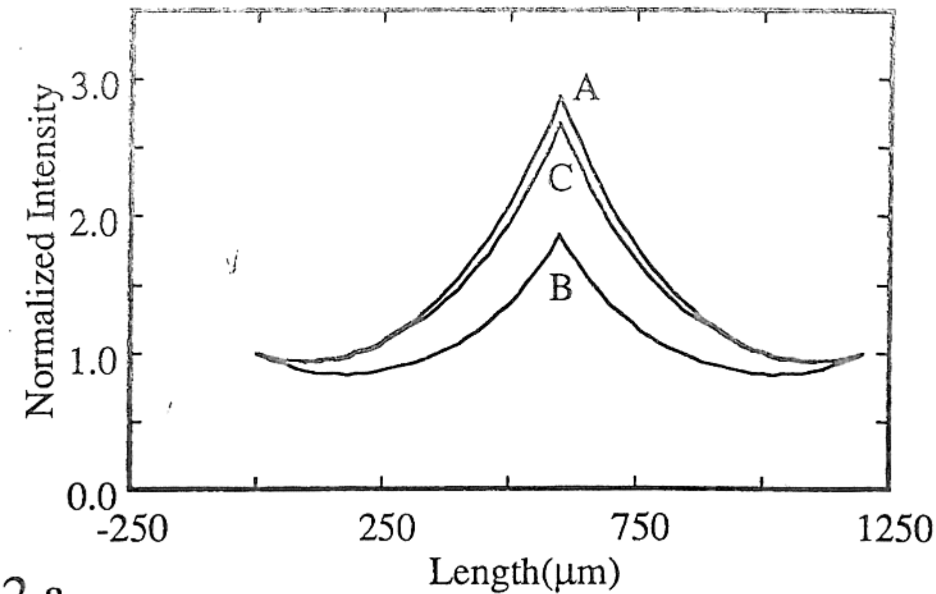
$$j\Omega\frac{\partial B_\Delta}{\partial\Omega}V_{A0}^2\varphi_{A\Omega} = \sum_i \frac{2P_{si}}{(1+\epsilon_i)}\frac{\hbar\omega\Delta J_{i\Omega}}{P_{si}}[Im\{\frac{V_{\alpha,i}^2}{|V_i|^2}\} + \alpha_{ave}Re\{\frac{V_{\alpha,i}^2}{|V_i|^2}\}]$$
$$\sum_i^t \frac{2\hbar\omega S_{ti}n_{ti}\tau_{sp,ti}}{(1+\epsilon_{ti})}[Im\{\frac{V_{\alpha,ti}^2}{|V_i|^2}\} + \alpha_{ave}Re\{\frac{V_{\alpha,ti}^2}{|V_i|^2}\}]\Delta J_{ti\Omega} \quad . \quad (59)$$

This result is a generalisation of the previously obtained result for the AM and FM response of a two section laser with inhomogenous $\alpha-$parameter [29]. It also includes the "grating factors" used by Kuznetsov [30]. However, as was previously mentioned, this result does not include the fill factor redistribution that was included in Ref. [14] and Ref. [6].

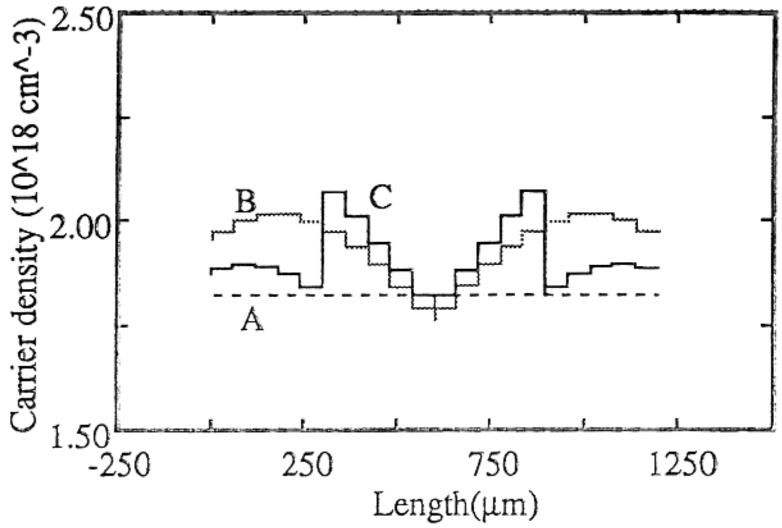
5 Some numerical examples

First, we made some calculations on a laser with one active section and no nonlinear gain to compare our results with the linewidth calculation method described by Björk and Nilsson [9]. The present method gave, as it should, exact agreement with these calculations. Furthermore, we also found, for the laser types used in the calculations below, a good agreement between the low-frequency modulation response calculated by Eqs. (58) and (59) and the change of output power and frequency of the steady state solution when the input currents were varied. As the steady state solution includes the fill factor redistributions, this comparison implies that our approximation of a uniform change in amplitude and phase is valid for these cases.

Furthermore we made a comparison with the linewidth theory presented in Ref. [6]. Accordingly, we performed calculations on the same three section phase-shifted DFB ($\kappa L = 2$) using the same material parameter values as described in [6]. However, in [6] the gain spectral dependence is included as a parabolic gain model whereas we have neglected the spectral dependence of the gain. The laser consists of two 300 μm long outer electrodes with a 600 μm center electrode inbetween. A $\pi/4$ phase-shift is inserted to avoid the mode-degeneracy of ordinary DFB lasers. However, the sharp intensity peak at the phase-shift causes a depletion in the carrier density as is illustrated in Figs. 2 a and b for the same biasing conditions in [6] (Fig. 3 and 5).



2 a.



2 b.

Fig. 2 a: Intensity distribution in a phase-shifted DFB laser. Curve A is at threshold, curve B for $J/J_{th} = 3.08$ for all electrodes and finally curve C is with $J/J_{th} = 3.08$ in the outer electrodes and $J/J_{th} = 4.8$ in the center electrode. b: the corresponding carrier distributions, A (dashed): at threshold (no spatial holeburning), curve B (dotted) for $J/J_{th} = 3.08$ for all electrodes and finally curve C (solid) $J/J_{th} = 3.08$ in the outer electrodes and $J/J_{th} = 4.8$ in the center electrode. The steplike shape is to indicate the that the carrier density is taken as constant within that region. The homogeneous threshold current was 52 mA.

In Fig.3 we illustrate the effect of spatial holeburning on linewidth. For pumprates up to $I/I_{th} = 3.08$ the laser is homogenously pumped and above only the center-electrode is pumped. This curve is similar to the curve given in [6] (Fig. 6 b). In the curves we have marked out the different linewidth contributions $\Delta\nu_{ord}$, $\Delta\nu_{NN}$ and $\Delta\nu_{NP}$ (in [6] these terms are denoted $\Delta\nu_{sp}$, $\Delta\nu_{NN}$ and $\Delta\nu_{NS}$). In the present formalism we get a negative $\Delta\nu_{NP}$ quite opposite to the result of [6], where $\Delta\nu_{NP}$ was positive. By closer inspection we found that this difference could be attributed to different assumptions on the mechanisms responsible for that term (see appendix D). This, together with the use of a longitudinally constant population inversion factor n_{sp} in [6], contributes to give a change in sign of $\Delta\nu_{NP}$. The different size of $\Delta\nu_{ord}$ and $\Delta\nu_{sp}$ in [6] is explained by the difference in the mean magnitude of n_{sp}.

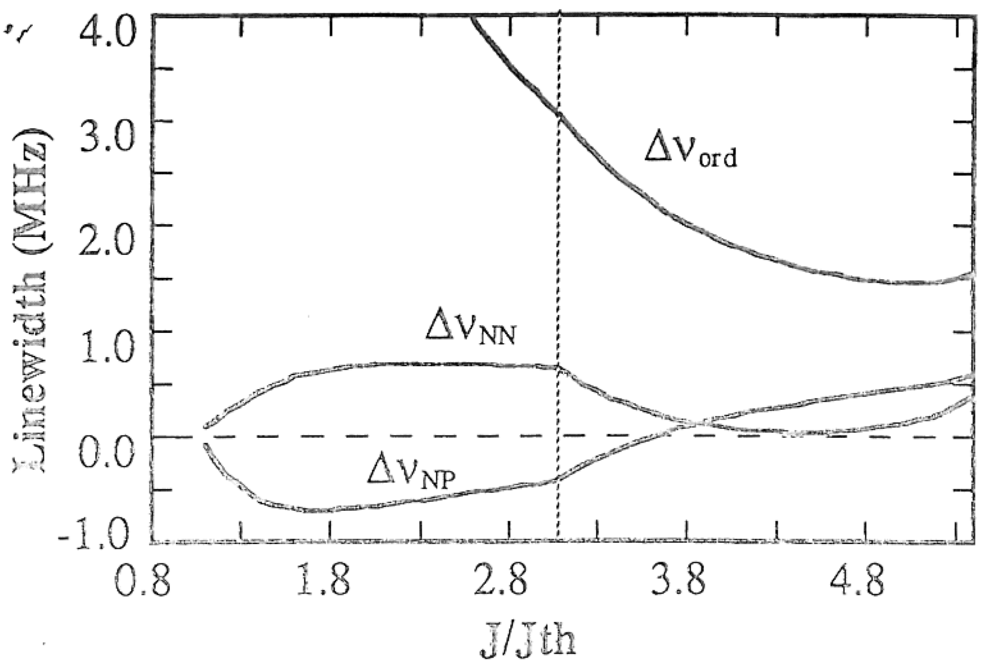
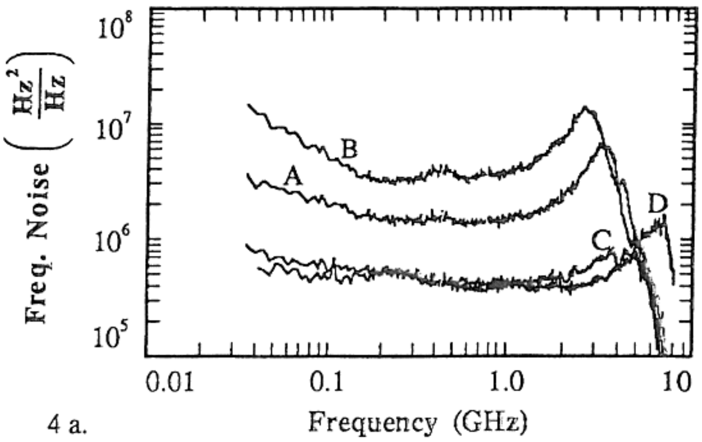


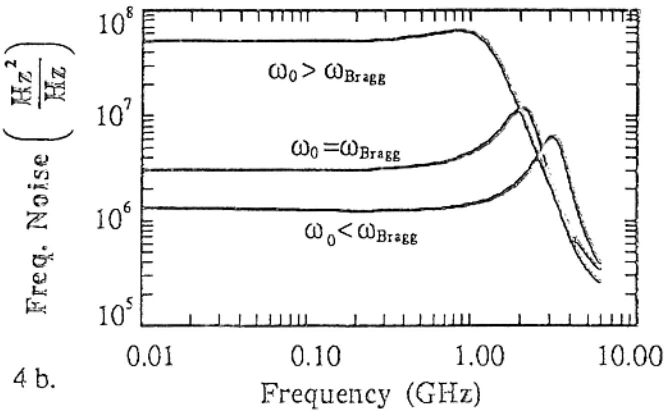
Fig. 3 : Calculated linewidth of a three section phase-shifted DFB laser. In the left half of the curve, from $J = 1.1J_{th}$ to $J = 3.1J_{th}$ the laser is pumped homogenously. In the right half of the curve only the center electrode is pumped and the outer electrodes are kept at constant current densities $J = 3.1J_{th}$.

In the present formalism we calculate the population inversion factor locally from $n_{sp} = N_c/(N_c - N_0)$, where N_c is the local carrier density (dependant on spatial holeburning) and N_0 is the carrier density required for inversion, which is a material parameter.

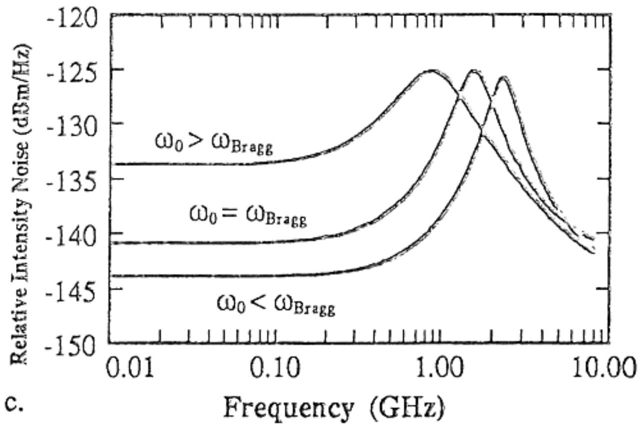
As a next example we will compare theory with measured FM-noise spectra of a two-section DBR laser by Goobar and Schatz [31], Fig. 5. These measured frequency noise spectra are reproduced in Fig. 4 a. An interesting feature observed in these spectra was that the entire frequency noise spectrum at a given output power could be increased or decreased depending on the injection current to the DBR region. Also one could observe a change in the relaxation oscillation peak. The reason for this behaviour, we believe, is that the Bragg region constitutes a frequency dependent loss that will modify the lasing dynamics. This mechanism, detuned loading, is the well known cause of linewidth reduction in external-cavity lasers.



4 a.



4 b.



4 c.

Fig. 4 a: Measured frequency noise of a two section DBR laser. Trace A is at 1.75 mW output power and $I_B = 0$ mA, trace B is at the same output power but with $I_B = 5$ mA, trace C ($I_B = 0$ mA) and D ($I_B = 5$ mA) are at 6.5 mW output power. b: Calculated frequency noise spectra of a two-section DBR laser. c: Calculated relative intensity noise (RIN) for the same parameters as in b.

Its effects on modulation properties and noise has also been investigated [32, 33]. In Fig. 4 b. we show the corresponding calculated spectras using representative parameter values. Depending on which side of the Bragg wavelength the lasing mode is, the spectras are either enhanced or decreased due to detuned loading and the non-zero α-parameter of the active region. In Fig. 4. c. we have calculated the relative intensity noise spectra for the same set of parameters.

We also made a comparison between the results of [34] and our formalism method for the tuning properties of a phase tunable DFB laser and found almost identical result for the tuning, output power and linewidth, this will not be shown. A result of our calculation and of the calculation of [34], is that the linewidth decreases with tuning current and that it is high after a mode-jump toward shorter wavelengths has occured. This is contrary to what has been experimentally observed [35, 36]. The reason for this descripancy is not known, but could perhaps be related to instabilities occuring before a mode-jump.

6 Conclusions

In this report a theory for spectral properties of lasers has been developed and implemented on a desktop computer. The theory has been shown to reproduce several established results and has been illustrated with calculated examples of modulation response, AM- and FM noise, for some specific laser structures. All this can be made interactively by the user, the calculation time is modest despite the fact that only a desktop computer is used. However, so far the theory has only been implemented in an approximate form, where a uniform change of amplitude has been assumed. This approximation seems to be good for high reflectivity lasers operating at moderate pump levels, but at high pump levels, other authors have found that similar theories are not sufficient. In particular the phenomena of linewidth rebroadening often observed in experiments does not occur in the present theory, but other authors has found that it can occur in theories going beyond the approximation of a uniform change in amplitude. To include such phenomena the theory should be implemented in its general form. This seems, however, to be rather difficult and is therefore left as a future task.

Acknowledgement

The authors would like to acknowledge Mr Jean Luc Vey of Telecom Paris, for useful discussions and for helping with computer programming.

A Determination of the transfer functions $H(\omega)$

The transfer functions H_{Ai}, H_{ii}, H_{ij} can be determined by introducing source currents $I_{A,i,B}$ at the individual locations and then calculating the resulting voltage distributions. For example, H_{iA} (and H_{Ai} due to reciprocity) is obtained from V_i/V_A with all currents except I_A equal to zero. Instead of using this direct approach one may study how the admittance $Y \equiv I_A/V_A$ changes when we connect a small additional conductance δG_i at the location i such that I_i becomes $-V_i \cdot \delta G_i$. One then finds that

$$H_{Ai}^2 = \frac{\partial Y}{\partial G_i} \quad , \tag{A.1}$$

$$H_{ii} = H_{Ai}^2 - \frac{1}{2} \cdot \frac{Y}{H_{Ai}^2} \cdot \frac{\partial^2 Y}{\partial G_i^2} \quad , \tag{A.2}$$

$$H_{ij} = H_{Ai} H_{Aj} - \frac{1}{2} \cdot \frac{Y}{H_{Ai} H_{Aj}} \cdot \frac{\partial^2 Y}{\partial G_i \partial G_j} \quad . \tag{A.3}$$

These quantities can be readily found from a transfer matrix analysis of the laser structure. They are suitable if one wants to approximate the real laser with a lumped model.

B The complex notation used in this chapter

In eq. (9) we defined the current as

$$I(t) = Re\{I_\Omega e^{j(\omega_0+\Omega)t+\Phi}\} \quad , \tag{B.1}$$

which is actually only the upper sideband. The complete current reads

$$I(t) = Re\{(I_\Omega e^{j\Omega t} + I_{-\Omega} e^{-j\Omega t}) e^{j(\omega_0 t+\Phi)}\} \quad , \tag{B.2}$$

which we can separate in quadrature components as

$$\begin{aligned} I(t) = Re\{(I_\Omega + I^*_{-\Omega})e^{j\Omega t}\}cos(\omega_0 t + \Phi) \quad &- \quad Re\{-j(I_\Omega - I^*_{-\Omega})e^{j\Omega t}\}sin(\omega_0 t + \Phi) \\ = a(t)cos(\omega_0 t + \Phi) \quad &- \quad b(t)sin(\omega_0 t + \Phi) \quad . \end{aligned} \tag{B.3}$$

Writing

$$a(t), b(t) = Re\{a_\Omega, b_\Omega e^{j\Omega t}\} \quad , \tag{B.4}$$

yields

$$a_\Omega = I_\Omega + I^*_{-\Omega} \equiv I^I, \qquad b_\Omega = \frac{1}{j}(I_\Omega - I^*_{-\Omega}) \equiv I^O \quad . \tag{B.5}$$

This equation defines the (real) in-phase and quadrature phase components of the fluctuations, which were denoted I^I and I^O in the previous paragraphs. The noise spectral densities of these components are easily calculated. The noise current $I(t)$ of the conductance G has a white, single sided, spectral density of

$$S_I(\Omega) = 2\hbar\omega|G| \quad . \tag{B.6}$$

Here ω is the carrier frequency, ie. the frequency of the lasing mode. The spectrum of $I(t)$ is symmetrical around ω, this implies that the spectral density of the narrowband components will be given by

$$S_a(\Omega) = S_b(\Omega) = 4\hbar\omega|G|, S_{a,b}(\Omega) = 0 \quad . \tag{B.7}$$

That is, their spectral densities are equal, plus that a and b are uncorrelated. Note that using the relationship between the conductance and the power $|G| = 2P/|V|^2$, we get

$$S_a(\Omega) = S_b(\Omega) = 8\hbar\omega\frac{P}{|V|^2} \quad , \tag{B.8}$$

which is the result used in the calculations.

C Calculation of noise from stimulated emission-absorption

In Eq. (28) it was stated that the dipole fluctuation that enters the carrier rate equation is given by

$$\hbar\omega\Gamma_{si}(t) = \langle V_i(t) I_{Di}(t)\rangle \tag{C.1}$$

where I_{Di} is the optical field fluctuation from the fluctuating dipoles, and the average is taken over one or several lightperiods. Writing V_i as

$$V_i(t) = |V_{i0}|cos(\omega_0 t + \Phi) \qquad , \tag{C.2}$$

where Φ is the same phase as in eq. $(B.1)$, so that the in-phase fluctuation of I_D hase the same phase as V_i. Performing the integration we have

$$\hbar\omega\Gamma_{si}(t) = \frac{1}{T}\int_t^{T+t} |V_{i0}|cos(\omega_0 t'+\Phi)[a(t')cos(\omega_0 t'+\Phi) - b(t')sin(\omega_0 t'+\Phi)]dt' = \frac{1}{2}|V_{i0}|a(t), \tag{C.3}$$

where $a(t)$ and $b(t)$ are slowly varying over a light field period T. From $(B.3)$ and $(A.5)$ we find the Fourier component of Γ_{si} as

$$\Gamma_{si\Omega} = \frac{1}{2\hbar\omega}|V_{i0}|[I_{Di\Omega} + I^*_{Di-\Omega}] \qquad , \tag{C.4}$$

which gives the result used in eq. (29).

D Correspondence to Green function formalism for calculation of linewidth in lasers

Several authors [19, 21, 22, 6, 23] have used a Green function formalism in order to asses the spectral properties of semiconductor lasers. We will therefore now show that the connection between the Green function formalism and the formalism of this paper. We will especially connect to the recent results of Tromborg et al. [6] which in the authors opinion represent the most complete formulation to date. The linewidth formulation of [6] Eqs. (52)-(53c) is more complete than the low frequency limit of Eqs. (46) -(49) here, since the spatial redistribution of photons is included. However, to compare our results with those of [6] we use the approximations as they were used in the numerical calculations in [6], that is, we put $\mathcal{M}_S = \mathcal{M} = 0$, $\bar{H} = 0$. We then get according to Eqs. (57)- (58b) of [6]

$$\alpha_{eff} = \frac{\mathrm{Im}\{\bar{C}_N \cdot \bar{S}_\alpha - \bar{C}_S \cdot \bar{S}_0\}}{\mathrm{Re}\{\bar{C}_N \cdot \bar{S}_\alpha - \bar{C}_S \cdot \bar{S}_0\}}, \tag{D.1}$$

where

$$\bar{C}_N \cdot \bar{S}_\alpha = \int_0^L C_N(z)\tau_R(z)\frac{\partial R_{st}}{\partial S}(z)S_0(z)dz, \tag{D.2}$$

$$\bar{C}_S \cdot \bar{S}_0 = \int_0^L C_S(z)S_0(z)dz, \tag{D.3}$$

are the spatially averaged quantities. The weight functions C_N, C_S are given by [6] Eq. (22)

$$C_X(z) \equiv -j\frac{\delta W}{\delta k(z)}\frac{\partial k}{\partial X}(z)/\frac{\partial W}{\partial \omega}, \tag{D.4}$$

where $X = N, S$ and W is the Wronskian of the laser. The linewidth is then given by [6] Eqs. (52)-(53c)

$$\Delta\nu = \Delta\nu_{sp} + \Delta\nu_{NN} + \Delta\nu_{NS}, \tag{D.5}$$

$$\Delta\nu_{sp} = \frac{R_{sp}}{4\pi I_0}(1 + \alpha_{eff}^2), \tag{D.6}$$

$$\Delta\nu_{NN} = \frac{1}{\pi}\int_0^L K(z)^2 D_{NN}(z)dz, \tag{D.7}$$

$$\Delta\nu_{NS} = -\frac{2\alpha_{eff}}{\pi}\int_0^L K(z)D_{NS}(z)dz, \tag{D.8}$$

where $\eta = 1$ under the approximation given previously and $K(z)$ is defined as

$$K(z) = \tau_R(z)\{C_{Ni}(z) - \alpha_{eff}C_{Nr}(z)\} \quad . \tag{D.9}$$

Here τ_R is the total recombination time. To connect to the formalism used in the present paper we can put in a fictive (or physical) reflectance $r_1 = r_R$ to find that the derivates of the Wronskian W can be written $(Z_L^-(0) = 1, Z_R^+(0) = r_1)$

$$\frac{\partial W}{\partial X} = 2jk(0)r_1\frac{\partial}{\partial X}ln r_L \quad . \tag{D.10}$$

However, this can also be written as an admittance derivate

$$\frac{\partial Y}{\partial X} = -2G_L\frac{r_1}{1-r_1^2}\frac{\partial}{\partial X}ln r_L \quad , \tag{D.11}$$

which can be found from the reflexion coefficient. Here $X = N, S, \Omega$ and the derivate with respect to X and S is taken as a functional derivate. If we now consider a case with a discrete number of elements, the functional derivate can be replaced by derivates with respect to variations in the discrete elements. Using this and the fact that the derivate of the Wronskian and the admittance are proportional we can write C_X as

$$C_X(z) = -j\frac{\partial Y}{\partial X}/\frac{\partial Y}{\partial \omega}. \tag{D.12}$$

The next step is to relate C_X to the field distribution. This is done by using

$$\frac{\partial Y}{\partial N_i} = \frac{\partial Y}{\partial g_i}\frac{\partial g_i}{\partial N_i}(1-j\alpha_i) = -\frac{G_L P_{si}}{P_A}\{\frac{V_i^2}{|V_i|^2}\}\frac{1}{g_i}\frac{\partial g_i}{\partial N_i}(1-j\alpha_i) \quad , \tag{D.13}$$

$$\frac{\partial Y}{\partial S_i} = \frac{\partial Y}{\partial g_i}\frac{\partial g_i}{\partial S_i} = \frac{\partial Y}{\partial g_i}\frac{g_i}{S_i}s_{Pi} = -\frac{G_L P_{si}}{P_A}\{\frac{V_i^2}{|V_i|^2}\}\frac{g_i}{S_i}s_{Pi} \tag{D.14}$$

where

$$\frac{\partial Y}{\partial g_i} = \frac{G_i}{g_i}\frac{\partial Y}{\partial G_i} = -\frac{G_i}{g_i}\frac{\partial Y}{\partial\{-G_i\}} = -\frac{2P_{si}}{V_A^2}\frac{1}{g_i}\{\frac{V_i^2}{|V_i|^2}\} = -\frac{G_L P_{si}}{P_A}\frac{1}{g_i}\{\frac{V_i^2}{|V_i|^2}\} \quad , \tag{D.15}$$

has been used. The minus sign appears because G_i in Eq. (A.1) is to be interpretated as a negative conductance. Note also that α of [6] is defined to be positive whereas α is negative in the notation used here. If we now use the specification of the reference plane of Eq. (35) i.e. that

$$\frac{\partial Y_\Delta}{\partial \Omega} = j\frac{\partial B_\Delta}{\partial \Omega} \quad . \tag{D.16}$$

Using this we find the final expression for C_N and C_S as

$$C_N = [\frac{G_L}{P_A \frac{\partial B_\Delta}{\partial \Omega}}] \frac{P_{si}}{g_i} \frac{\partial g_i}{\partial N_i} \{ \frac{V_{\alpha,i}^2}{|V_i|^2} \} \quad , \tag{D.17}$$

$$C_S = [\frac{G_L}{P_A \frac{\partial B_\Delta}{\partial \Omega}}] \frac{g_i}{S_i} s_{Pi} \{ \frac{V_i^2}{|V_i|^2} \} \quad , \tag{D.18}$$

Using this we can write Eqs. (D.2) and (D.3) as

$$\bar{C}_N \cdot \bar{S}_\alpha = \frac{1}{\frac{\partial B_\Delta}{\partial \Omega}} \sum_i \frac{2P_{si}}{V_A^2} \{ \frac{V_i^2}{|V_i|^2} (1 - j\alpha_i) \} \frac{1 + s_{Pi}}{1 + \epsilon_i}, \tag{D.19}$$

$$\bar{C}_S \cdot \bar{S}_0 = \frac{1}{\frac{\partial B_\Delta}{\partial \Omega}} \sum_i \frac{2P_{si}}{V_A^2} \{ \frac{V_i^2}{|V_i|^2} \} s_{Pi} \tag{D.20}$$

where we have used that $1 + \epsilon_i = \tau_{st,i}/\tau_{R,i}$. If Eqs. (D.19) and (D.20) are inserted in Eq. (D.1) one finds that α_{eff} of [6] is identical to α_{ave} of eq. (44), (apart from the minus sign due to our sign convention). We can now check the equivalence of Eq. (47) and Eq.(D.6). To do this we use that the admittance derivate $\partial B_\Delta / \partial \Omega = 2G_L \partial \{1/\Gamma_A\} / \partial \Omega$ can be written as according to Eq. (30) of [24]

$$P_A \cdot \frac{\partial}{\partial \Omega} \{ \frac{1}{\Gamma_A} \} = \frac{\epsilon_0}{2} | \int_{\text{Vol}} n n_g E^2 dV | \quad , \tag{D.21}$$

where $\epsilon_0 n n_g = \epsilon + 1/2 \omega \partial \epsilon / \partial \omega$ has been used to define the group index n_g. Further we can use the fact that the dissipated power to stimulated emission (stimulated absorption not included) can be written

$$\sum_i P_{si} n_{sp,i} = \frac{\epsilon_0}{2} \int_{\text{Vol}} n n_g v_g g n_{sp} |E|^2 dV \quad , \tag{D.22}$$

where g is the modal gain and v_g the group velocity. Further we can define a photon number I_0 related to the field energy W [2] by

$$\hbar \omega I_0 = W = \frac{\epsilon_0}{2} \int_{\text{Vol}} n n_g |E|^2 dV \quad , \tag{D.23}$$

Using this we find that

$$\sum_i P_{si} n_{sp,i} / [P_A \cdot \frac{\partial}{\partial \Omega} \{ \frac{1}{\Gamma_A} \}] = \frac{R_{sp}}{\hbar \omega I_0} \quad , \tag{D.24}$$

[2] The external Q-value Q_e can be defined through $P_A = \omega / Q_e \cdot W$ where W is the field energy. In this case it would be tempting to use an admittance derivate directly to define the Q-value. These are related by: $Q_e = K^{-1/2} \omega \partial / \partial \omega \{1/\Gamma_A\}$ where $K = [\int_{\text{Vol}} n n_g |E|^2 dV]^2 / [| \int_{\text{Vol}} n n_g E^2 dV |]^2$ is the Petermann or Arnaud K-factor. That the correction term $K^{-1/2}$ occurs simply reflects the fact that the field energy cannot be directly related to the admittance derivate in a lossy system.

where R_{sp} is the spontaneous emission rate into the lasing mode as given by Eq. (E.7) of [6]. With this it is shown that Eq. (47) and Eq.(D.6) are exactly the same. For a one-section laser without any nonlinear gain Eq.(D.6) is the only linewidth term appearing. It is worth to repeat that the result here, which was first presented in [4] and [9], as was argued in [4] includes the Petermann [22, 37] and Arnaud [20] K-factor automatically, an argument for this was also given in [24]. The next step is to compare the second and the third term in the linewidth expressions. For the second term we can use Eq. (55) of [6] that the local tuning efficiency is given as

$$\frac{\partial f}{\partial J_i} = \frac{1}{2\pi} K(z) \quad . \tag{D.25}$$

By using equation Eqs. (D.12)- (D.15) we find that this can be written as

$$\frac{\partial f}{\partial J_i} = \frac{1}{2\pi} [\frac{G_L}{P_A \frac{\partial B_\Delta}{\partial \Omega}}] \frac{P_{si}}{g_i} \frac{\partial g_i}{\partial N_i} \tau_{Ri} [Im\{\frac{V_{\alpha,i}^2}{|V_i|^2}\} + \alpha_{ave} Re\{\frac{V_{\alpha,i}^2}{|V_i|^2}\}] \quad . \tag{D.26}$$

By inspecting Eq.(59) one finds that the same modulation response or tuning efficiency is obtained. As a paranthesis one may compare Eq. (D.26) with Eq. (52), it is seen that the term $P_{si}/g_i \partial g_i/\partial N_i \tau_{Ri}$ is replaced by $\hbar\omega S_{ti} n_{ti} \tau_{sp,ti}$ which physically seems correct. However, returning to the problem of showing the equivalence, if one finally uses Eq. (E.14) or (E.15) for $D_{NN}(z)$ one finds that the end result is that the two linewidth terms are identical. For the third term the two formalisms do not give identical results. The difference is that in [6] an integrated Langevin source for the field is used when finding the correlation to the local carrier fluctuations, whereas in the present formalism both local field fluctuations and local carrier fluctuations are assumed. In the formalism of [6] the integrated Langevin source gives rise to the term α_{eff} in front of the integral in Eq. (D.8) whereas in the present formalism a term $\alpha_{ave} Re\{V_i^2/|V_i|^2\} + Im\{V_i^2/|V_i|^2\}$ inside the summation sign as in Eq. (49) appears. Further in [6] the correlation is assumed to be induced by stimulated emission only (spontaneous emission into guided mode) yielding a factor $2P_s n_{sp}$ whereas in the present formalism the correlation is induced by both stimulated emission and stimulated absorbtion yielding a factor $P_s(2n_{sp}-1)$. This discrepancy can be traced back to an assumption of the diffusion coefficient entering the rate equations. Compare, for instance, the diffusion coefficients in [11, 19] with those of [38, 39, 40]. In [39, 40] these issues are discussed in detail, however, a more exact discussion on why the diffusion coefficients are different is not given.

E Steady state calculations

The selfconsistent solution of the steady state properties of laser diodes using the transfer matrix method [8], together with carrier rate equations has been given by several authors [41, 42]. Here we briefly describe the method used by the present authors.

The optical field in the laser can be described as a wave with amplitude $u(z)$, going in the right direction, and a wave with amplitude $v(z)$ going in the left direction. The wave amplitudes on the left side of a segment i can be calculated from the wave amplitudes of the right side by using the transfer matrix $\overline{\overline{A_i}}$ of the segment.

$$\begin{pmatrix} u_i^l \\ v_i^l \end{pmatrix} = \overline{\overline{A_i}} \begin{pmatrix} u_i^r \\ v_i^r \end{pmatrix} \tag{E.1}$$

The transfer matrix can be easily calculated for segments with a homogeneous or a periodically perturbated complex propagation constant as is described in [8]. Accordingly, the laser is divided into M segments (numbered from right to left) so that, within each segment, the carrier and the photon densities, and hence also the propagation constant, can be regarded as constant (except for a possible periodic perturbation). The complex propagation constant k_i used in the calculation of the transfer matrix of segment i can be divided in a real and imaginary part

$$k_i = \beta_i' + i\beta_i'' \tag{E.2}$$

where

$$\beta_i' = \beta_{gp} + \frac{d\beta}{d\omega}(\omega_0 - \omega_{gp}) + \Delta\beta_i. \tag{E.3}$$

Here $\beta_{gp} = (\omega_{gp}/c_0)n_{th}$ is the real part of propagation constant determined by the nominal refractive index, n_{th}, defined at the gain peak frequency, ω_{gp}, and with a carrier density level, N_{th}, that yields the homogeneous threshold gain g_{th}. Furthermore, ω_0 is the lasing frequency, $d\beta/d\omega = n_g/c_0$, where n_g is the group refractive index and finally $\Delta\beta_i = (\omega_0/c_0)\Delta n_i$ is the small change of the propagation constant due to the perturbation of the refractive index, Δn_i, caused by the local carrier density variation i.e.

$$\Delta n_i = \alpha \frac{c_0}{2\omega_0} \Gamma a (N_i - N_{th}) \tag{E.4}$$

where α is the linewidth enhancement factor, Γ is the lateral confinement factor and a is the gain/carrier density slope. The imaginary part of complex propagation constant is written as

$$\beta_i'' = \frac{1}{2}(\Gamma g_i - g_{\text{loss}}) \tag{E.5}$$

where g_{loss} is the waveguide loss. The gain in each segment g_i depends not only on the local carrier density N_i, but also on the confined photon number S_i, through nonlinear gain.

$$g_i = \frac{a}{1 + \epsilon\Gamma S_i/V_i}(N - N_0) \tag{E.6}$$

where V_i denotes the volume of segment i.

To have a normalized gain distribution we define a mean gain g_0 as the homogenous gain yielding the same total stimulated emission rate as the actual gain distribution and write $g_i = \eta_i g_0$. The same type of normalization is done with the photon number distribution, i.e. $S_i = \gamma_i S$, where S is the total number of photons in the cavity. At start, N_i is put to N_{th} and S is put to zero, which gives $\eta_i = 1$ and $\Delta n_i = 0$, so that the below calculated g_0 will equal the threshold gain g_{th}.

For given η_i and Δn_i we calculate ω_0, g_0 and γ_i with the help of the transfer matrix method by finding the frequency and mean gain that gives no right-going wave at the left side of the laser when no left-going wave at the right side is applied i.e.

$$u_{M+1}(\omega_0, g_0) = 0 \tag{E.7}$$

whith $v_0 = 0$. Here subscript $M+1$ and 0 of the field amplitudes denotes that they are taken outside the lasers left and right mirror, respectively.

The fill factors γ_i are given from

$$\gamma_i = C_i / \sum_j C_j \tag{E.8}$$

where

$$C_i = n_g n_i \int_{z_i^l}^{z_i^r} (|u(z)|^2 + |v(z)|^2) dz = \frac{n_g n_i}{\Gamma g_i - g_{\text{loss}}}(|u_i^r|^2 + |v_i^l|^2 - |u_i^l|^2 - |v_i^r|^2) \tag{E.9}$$

The last equality is derived by observing that, at steady state, the generated power in each segment (which is proportional to the confined energy) must equal the power that escapes the segment minus the power that goes into it.

The carrier density in each segment N_i and the total photon number S can then be calculated using the carrier rate equation for each segment

$$J_i = R_i(N_i) + \frac{c_0}{n_g}\gamma_i \Gamma g_i(N_i, S) S \tag{E.10}$$

together with the total photon rate equation

$$\sum_i \frac{C_i}{n_g}(\Gamma g_i(N_i, S) - g_{\text{loss}}) = |u_0|^2 + |v_{M+1}|^2 \tag{E.11}$$

Eq. (E.11) simply states that the total net generated power must equal the power lost from right and left mirrors. J_i is the injected number of carriers per second and R_i is the number of spontaneously recombinated carriers per second. S is adjusted until the values of N_i calculated from Eq. (E.10), are consistent with Eq. (E.11).

In order to find a consistent steady state solution, we repeatively use Eq. (E.2) to Eq. (E.11) until convergence is reached.

F Laser followed by traveling wave laser amplifier

In this appendix we will derive a result for the linewidth of a laser followed by a traveling wave laser amplifier, previously given by Berglind and Nilsson in[43]. We stick to the uniform amplitude distribution case. The laser amplifier is assumed to have a spontaneous emission factor $n_{sp,A}$, power gain G, internal efficiency η_{iA} (defined as total generated stimulated emission power minus internal loss power divided by the total stimulated emission power. Assuming that only the first term $\Delta\nu_{ord}$ is dominant we get

$$\Delta\nu = \frac{\hbar\omega}{\pi}[\frac{G_L}{\frac{\partial B_\Delta}{\partial\Omega}}]^2 \frac{1+\alpha_{ave}^2}{P_A^2}(P_{s,L}n_{sp,L} + P_{s,A}n_{sp,A}) \quad . \tag{E.1}$$

Now we must study how the terms are modified by the amplifier. The output power P_A is the laser output power multiplied by a factor G, but the admittance derivate after the amplifier is the admittance derivate at the laser divided by G (the reflection factor is multiplied by G, thus the product of the admittance derivate and the output power remains unchanged. This may also be inferred from Eq. D.23), since the integral of E^2 over a traveling wave field is zero. The effective $\alpha-$ parameter α_{ave} equals α_{laser} since the quotient the integral over a traveling wave field will be zero. Thus we can write the formula for the linewidth as

$$\Delta\nu = \Delta\nu_0 \cdot [1 + \frac{\eta_r}{n_{sp}} \cdot \frac{n_{sp,A}}{\eta_{iA}} \cdot G - 1] \quad . \tag{E.2}$$

Here we have used $\eta_{iA}P_{S,A} = (G-1)P_A$ and $\eta_r = P_A/P_{s,las}$. Eq. (E.2) is exactly the same as [43] Eq. (13).

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