CONCLUSIONS
This new textbook is a breath of fresh air in the market of books devoted to probability and random processes. The book lives up to its ambition of setting a new standard for a modern, computer-based treatment of the subject. Despite the issues discussed above, I fully recommend its use in undergraduate and first-year graduate courses.

REFERENCES

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REVIEWER INFORMATION
Osvaldo Simeone is currently an adjunct professor and post-doctoral researcher at the New Jersey Institute of Technology, Newark. He received his Ph.D. in information engineering from Politecnico di Milano, Milan, Italy, in 2005. His current research interests are in information theory and signal processing aspects of wireless systems with emphasis on cooperative communications, MIMO systems, ad hoc wireless networks, cognitive radio, and distributed synchronization.

A linear system with relay feedback can be described as
\[ \dot{x} = Ax + Bu, \]  
\[ y = Cx, \]  
\[ u = -\text{sgn}(y), \]  
where \( x \) is an \( n \)-dimensional vector, \( u \) and \( y \) are scalars, and \( A, B, \) and \( C \) are constant matrices. The relay is modeled as
\[ \text{sgn}(y) = \begin{cases} 1, & y > 0, \\ -1, & y < 0. \end{cases} \]

Since the sign function is discontinuous at \( y = 0 \), existence of solutions does not follow from the theory of ordinary differential equations. Instead, we rely on an abstract representation of (1)–(3) given by the differential inclusion
\[ \dot{x} \in F(x), \]
where the set-valued right-hand side is
\[ F(x) = \begin{cases} Ax - B, & Cx > 0, \\ Ax + B[-1, 1], & Cx = 0, \\ Ax + B, & Cx < 0. \end{cases} \]

The interpretation of \( F(x) \) is that when \( x \) belongs to the switching plane \( \{x : Cx = 0\} \), the time derivative of \( x \) can take any value in the set \( \{Ax + Bu : u \in [-1, 1]\} \). The particular choice of \( \dot{x} \) is made in such a way that the solution \( x : [0, \infty) \rightarrow \mathbb{R}^n \) has some desirable property, such as piecewise-continuous differentiability. There is an extensive literature on the relation between solutions of differential equations with discontinuous right-hand sides and their corresponding differential inclusions. A classical reference on these generalized solutions is [6].
If the solutions to the differential equation always traverse the switching plane, the solutions can, in many cases, be considered in the classical sense. However, if the solutions approach the switching plane tangentially, more care needs to be taken in the definition of the solution [7], [8]. For example, the classical solution of \( \dot{x} = -\text{sgn}(x) \), \( x(0) = 1 \), does not extend beyond the time instant \( t \) when \( x(t) = 0 \). Furthermore, the two-dimensional example

\[
\begin{align*}
\dot{x}_1 &= -\text{sgn}(x_1) + 2\text{sgn}(x_2), \\
\dot{x}_2 &= -2\text{sgn}(x_1) - \text{sgn}(x_2)
\end{align*}
\]

has a classical solution that spirals toward the origin in finite time [6].

Relay feedback systems often give rise to limit cycles. The traditional approach to analyzing oscillations in relay feedback systems is through frequency-domain or state-space methods [9]. The describing function approach is a frequency-domain method that in many cases gives approximate conditions for stable limit cycles. Rigorous results can be obtained by considering the Poincaré map, which describes the state evolution of the system between two consecutive intersections of the switching plane. However, it is well known that relay feedback systems can exhibit complex limit cycles that require alternative mathematical tools [8], [10]–[12].

The frequency and amplitude of the output of a relay feedback experiment reflects the dynamics of the plant \( P(s) = C(sI - A)^{-1}B \). For a stable plant \( P \) with positive steady-state gain and damped frequency response, the oscillation corresponds typically to the first intersection point of the Nyquist curve \( P(j\omega) \) with the negative real axis [2]. Although this single point is a crude estimate of \( P \), it gives information about the system in a frequency range important for control design. For simple control laws such as PI and PID controllers, this information is often sufficient to tune the controller parameters to obtain adequate closed-loop performance. The auto-tuner is consequently based on a scheme in which the controller is first replaced with a relay, the amplitude and frequency of the oscillation is measured, the controller parameters are derived from these measurements, and, finally, the controller replaces the relay in the control loop. By inserting a filter in series with the plant, additional oscillation frequencies can be obtained. In this way, multiple points on the Nyquist curve can be identified, and thus a more accurate model of \( P(j\omega) \) is found. The cost of obtaining a higher fidelity model is longer experiment time, which may have implications in practice. For more elaborate control-design schemes, the excitation signal should be optimized to maximize the benefit of each experiment. However, many process control loops in industry can be improved considerably by the simple auto-tuner with a single relay experiment.

### CONTENTS OF THE BOOK

*Relay Feedback: Analysis, Identification and Control* is an extensive text covering the analysis of oscillations in relay feedback systems, system identification based on relay feedback experiments, and controller design based on the identified models. The book is divided into three parts: (I) analysis of relay feedback systems, (II) process identification from relay feedback tests, and (III) controller design. Each part is divided into four or five chapters.

Part I presents fundamental properties of single-input, single-output (SISO) linear systems with relay feedback. Relay feedback systems that include time delays and relay hysteresis are also treated. As a result, the model structure given in (1)–(3) is only a special case. Chapter 1 discusses the existence of solutions. This topic is important since relay feedback systems do not always have solutions in the classical sense. The book avoids complications resulting from solutions converging to the switching surface by making appropriate technical assumptions.

The remaining three chapters of Part I deal with limit cycles. Specifically, Chapter 2 presents conditions on the existence of limit cycles, while chapters 3 and 4 give results on local and global stability of limit cycles. Local stability is derived through linearization of the Poincaré map. A global stability result is obtained by applying the contraction mapping theorem. The presentation in Part I is well written and easy to follow. Although an overview is given at the beginning of each chapter, the rest of the material is quite technical and consists of a collection of recent results reported by the authors in various papers. Several examples and figures are also used to help illustrate the development.

Part II discusses system identification based on relay feedback experiments. In Chapter 5 the authors review the basic relay feedback experiment and some of its variants. The introduction of an extra relay in the feedback loop to enhance the excitation of the closed-loop dynamics is discussed. A decentralized relay experiment for multivariable plants is briefly introduced as well. Instead of computing a single point on the Nyquist curve, a frequency-response analysis is executed directly on the input and output plant data. The authors discuss the advantages of this technique in Chapter 6 and also discuss experimental results on two pilot plants, namely, a water tank laboratory process and a heat exchanger. To utilize design methods based on parametric models of the plant, Chapter 7 discusses methods for approximating frequency responses with low-order transfer functions. Chapter 8, the last chapter of Part II, presents an alternative method for identifying the closed-loop plant model. The material in Part II is less technical compared to Part I and should be straightforward for the practitioner to apply. Some more
discussion on the relation to the system identification literature would have been desirable. For example, how does the presented MIMO identification scheme compare to existing techniques for identification based on frequency-domain or time-domain data?

Part III focuses on the design of linear controllers. As pointed out by the authors, there is a vast domain of applicable techniques, and thus there is no attempt in the book to cover them all. The text reviews internal model control (IMC) for SISO systems in Chapter 9 and MIMO systems in Chapter 10. Chapter 11 discusses a variant of IMC for unstable plants. Chapter 12 is on decentralized control. There is no specific link between Part III and Parts I or II other than the fact that decentralized relay experiments are briefly mentioned at the end of the last chapter. In fact, the focus on IMC is motivated by the general opinion that it is a popular control architecture in the process control industry. It would have been nice to see some further connection with the material of previous chapters. For example, can the model uncertainty introduced by a simpler relay experiment be compensated for by closed-loop design? Is there a tradeoff between experiment time and achievable control performance?

WHO SHOULD READ THIS BOOK
This book is suitable for both researchers and workers interested in obtaining an in-depth understanding of relay feedback systems and their application to automatic tuning of controllers. The book is a research monograph and consists of a detailed survey of recent papers by the authors on the analysis of relay feedback systems and their application to identification and control design. The book does not compare alternative approaches but rather is focused along a particular line of research. As a consequence, the book is probably not suitable as a stand-alone textbook for a graduate control course. Instead it is better suited as a complement to a nonlinear control course, a system identification course, or a course on control design.

For the next edition of this book, it would be desirable to include some discussion on the limitations of the approach taken. Although a few related methodologies are listed in the introduction of each chapter, little discussion of the relative pros and cons of various methodologies is presented. For example, the extensive recent literature on iterative identification and control design [13] is not mentioned. Additional questions can also be raised, such as why relay experiments for MIMO plants, or SISO systems with multiple relays, are not treated in Part I. For example, systems with more than one relay can lead to interesting extensions of the results presented in Part I, with possible connections to the literature on hybrid systems.

To conclude, the authors have produced a well-written and detailed text on relay feedback with applications to system identification and control design. The book presents an ambitious project that takes the reader from advanced mathematical theory on nonsmooth dynamical systems to tuning techniques directly applicable to industrial control systems. The book appeals to a diverse audience, from researchers in nonlinear control to practicing control engineers.

REFERENCES

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JUNE 2007 << IEEE CONTROL SYSTEMS MAGAZINE 103