KTH Machine Design

Influence of gear surface roughness, lubricant viscosity and quality level on ISO 6336 calculation of surface durability

Technical Report

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Abstract

The International Organization for Standardization (ISO) provides a method by which different spur and helical gear designs can be compared in terms of the risk for surface fatigue. The calculation of surface durability is based on Hertzian contact theory modified by factors for making adjustments peculiar to gearing. These factors are mainly determined through experimentation, and include surface roughness and lubricant viscosity, as well as the gear quality level. The ISO standard has 13 quality levels, 0 to 12, where 0 describes a more or less theoretically perfect gear tooth. Ten or so tolerance parameters are specified for each gear quality level, for example profile variations and pitch (tooth spacing) variations.

In this study, a robustness analysis based on factorial design was performed for a helical gear. The idea of a robust design approach is to be able to find the design least sensitive to variations such as manufacturing variations and product use variations. A number of gear geometries were generated by varying the gear design parameters: pressure angle, helix angle, and face width. Each established gear design was then exposed to different combinations and levels of variations in surface roughness, lubricant viscosity, and quality level. The aim of the robustness procedure was to find out whether the gear designs produced are sensitive to variations in these noise factors; that is, whether the ISO standard can be used to simulate their effect on surface durability.

The analysis revealed that the different combinations and levels of variations, separately, influence the load capacity. However, it is not possible to find gear geometry that is more or less sensitive for variations in surface roughness, lubricant viscosity, and quality level. Also, the ISO standard calculation procedure is restricted to only a few tolerance parameters; pitch error, lead deviations (misalignment), and profile form deviation. The factors influenced by these parameters are currently the internal dynamic factor, the face load distribution factor, and the profile load distribution factor.

Contents

1 Introduction

Working life calculations for gears are quite a complex issue, since the failure mode of a gear element depends on numerous factors (e.g. loading, friction, lubrication, etc). Although the influence of these factors cannot be perfectly idealized, standards such as the ISO 6336 standard produced by the International Organization for Standardization (ISO) provide a method for comparing different gear designs.

ISO 6336 consists of several parts. The first part, ISO 6336-1 [1], provides the basic principles, an introduction, and general influence factors. ISO 6336-2 [2] and ISO 6336-3 [3] cover calculation of surface durability and bending strength calculation, respectively. These are used for prediction of the two prominent failure modes in gears: surface fatigue (i.e. pitting) and gear tooth bending fatigue. Pitting occurs when the surface durability of the meshing flanks is exceeded, and crack growth followed by particles break out from the flank.

Tolerances are one way of communicating in today's global market, in which designers and manufacturers are spread over the world. To assist in systemizing gear tooth quality, the ISO gear standards provide a number of tolerance tables [4]. The ISO standard has 13 quality accuracy levels, 0 to 12, where 0 describes a more or less theoretically perfect gear tooth. Ten or so tolerance parameters are specified for each gear quality level, for example helix (lead) deviations, profile deviations, and pitch (tooth spacing) deviations. In order to simulate the influence of tolerance with respect to a certain gear performance, designers either use standard tolerance tables or perform experiments, which are usually time-consuming and costly.

There are several national gear standards used in the gear industry. Besides the ISO standard, anyone involved in international gear trade should be familiar with the standards of the AGMA (American Gear Manufacturers Association), the DIN (Deutsches Institut für Normung), and the JIS (Japanese Industrial Standards). Currently, the most popular standards are ISO and AGMA, according to Kawalec et al. [5]. Manufacturers must also know how each standard compares to the others. The report "Gear Industry Vision in 2025" [6] has several strategic goals towards this pursuit, with special emphasis on two factors: the achievement by 2015 of a single global system of design and testing standards; and standardized quality and verification procedures. Today, ISO 6336 and DIN 3990 [7] are partially equivalent in revision and partially equivalent to AGMA 2101-D04 [8], according to EuroTrans [9].

Many parties have contributed to the comparison of different national standards, including the search for shortcomings and other aspects. Kawalec et al. [5] performed a comparative analysis of tooth-root strength using ISO and AGMA standards in spur and helical gear, using the finite element method (FEM) as verification. They found that in all studied cases the tooth-root stresses calculated in accordance with the ISO were bigger than the corresponding results according to AGMA. Additionally, the standards differed in their evaluation of the influence of the helix angle, pressure angle, and addendum modification coefficient on tooth-root strength. Pasta and Mariotti [9] used a FEM analysis for a spur gear with a corrected profile and found that the contact stress values obtained using the finite element method were generally lower than those obtained with the ISO rules.

Although gear standards have been updated since they were first introduced, they still include some assumptions based on tests of gears manufactured in earlier times, with precisions that differ from those produced by today's methods. Additionally, while the standards define failure by pitting to mean a substantial number of destructive pits, this is no longer considered to be an adequate failure criterion, since it is not safe to use the part if the cracks are above a certain size in a critical location [11]. Hence, the gear has to be removed even though it might run for another 1000 hours. The theoretical methods used today have led to further understanding of modifications which make the gear design insensitive to factors such as manufacturing errors, and which can help to attain a more uniform mesh engagement. Moreover, in order to achieve certain gear performance, for example lower noise emissions, the gear tooth thickness can be reduced, thus making them more slender and flexible, according to MackAldener [12]. Obviously, some of these new findings are not yet covered in the standard calculations procedures. Similarly, not all assumptions left from earlier times have been updated in line with current design and manufacturing practice.

The demand for more compact, less noisy gears and more environmentally friendly lubrication techniques etc is increasing due to customer environmental awareness and regulations. These demands cannot be achieved merely by using standard calculations. However, the standards are designed to facilitate the application of future knowledge and developments, and hence should be regularly updated. To achieve this, there is a need for solutions related to tribology, which is a complex interdisciplinary subject covering a challenging research area. The tribological influences are tied to aspects such as surface, lubricant, and material properties.

Before additional inputs are set for surface durability calculations, it is important to understand how tribological inputs are incorporated in the ISO standard today. The aim of this study was to use a robust design approach to find out to what extent the standard treats variations in surface roughness, lubricant viscosity, and gear quality level. An additional aim was to use the standard without the need for experimental testing, extensive knowledge, or advanced calculations. Thus, the simplest calculation method according to the ISO standard was used.

The following three chapters (2, 3 and 4) used together contain sufficient information to repeat the work in this study. Chapter 5 presents the results of the robustness analysis, while chapter 6 discusses these results as well as the standard ISO calculation procedure in chapter 3. The conclusions drawn from this study are provided in chapter 7.

2 Input to robustness analysis

This chapter presents the input, standard definitions, and background required for the ISO standard calculations and the robustness analysis.

2.1 Analytical object

The gear being analyzed (Table 1 and Figure 1) is a traditional gear design [12]. The material properties and working characteristics of the gear are shown in Table 2. The applied torque and tangential velocity are assumed values, unlike the other values which are taken from ISO 6336-5 [13].

Table 1. The nominal gear design.

	Pinion	Gear
Number of teeth z [–]	24	35
Profile shift coefficient x [–]	0.589	0.554
Reference diameter [mm]	125.72	183.35
Normal module m [mm]	5.06	
Nominal pressure angle α [°]	20	
Helix angle β [°]	15	
Centre distance a [mm]	160	
Face width b [mm]	37	

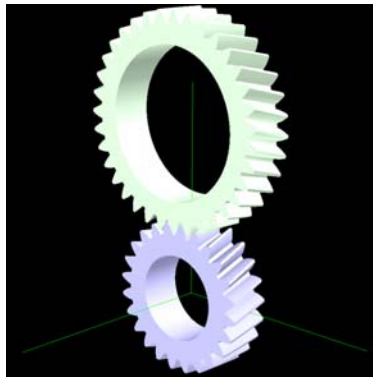

Figure 1. Illustration of the analyzed gear.

Table 2. Material properties of the pinion and gear and working characteristics.

	Pinion/Gear
Gear material [–]	Case-hardened steel
Allowance contact stress σ_{Hlim} [MPa]	1500
Poisson's ratio v [–]	0.3
Vickers hardness HV [–]	660
Modulus of elasticity E [MPa]	206000
Applied torque T [Nm]	1600
Tangential velocity v [m/s]	5

2.2 Standard reference test gears

The ISO standard is mainly based on large collections of results and empirical rules from practical experience. When the standard refers to standard reference test gears, these are spur gears in a limit range. For example, the allowance contact stress number is derived for spur gears in a limit range with a module between 3-5 mm, face width 10-20 mm, etc according to ISO 6336-5 [13]. Adjustments for gears other than the standard reference test gears are made using the (relative) influence factors. For helical gears, for example, differences in behaviour are compensated for by the contact ratio factor and the helix angle factor. When all influence factors are equal to 1, gears equal to standard reference test gears are used. The ISO standard reference test gear geometry is also manufactured using the rack cutter generation with standards pressure angle on the tool flank according to ISO 53 [14] followed by case-hardening and hard finishing (grinding).

2.3 Gear quality level definitions

The parameters describing the manufacturing accuracy (tolerances) of the gears are defined in ISO 1328-1. Figure 2 illustrates the transverse and single pitch deviation and profile form deviation.

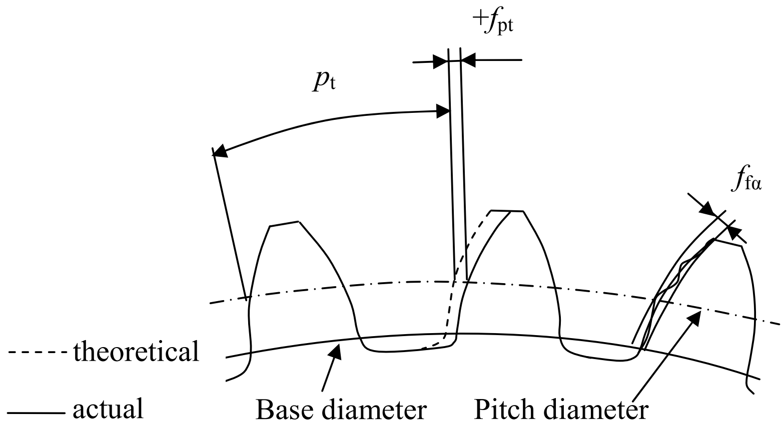

Figure 2. Schematic illustration of the transverse pitch deviation p_t, the single pitch deviation f_{pt}, and the profile form deviation $f_{f\alpha}$.

The single pitch deviation f_{pt} is the difference between the actual pitch and the corresponding theoretical pitch in the transverse plane at the pitch diameter (Figure 2). The base pitch deviation f_{pb} (not shown) is the same as the single pitch deviation except that it is located at the base diameter. The profile form deviation $f_{f\alpha}$ is defined as the distance between two facsimiles of the mean profile trace, which are each placed with constant separation from the mean profile, so as to enclose the actual profile trace over the evaluation length [4]. The helix (lead) slope deviation $f_{H\beta}$ (not shown) is the distance between two design helix (lead) traces which intersect the mean helix trace at the endpoints of the evaluation range. Helix slope deviations and assembly deviations have similar consequences. The single pitch deviation, profile form deviation and helix slope deviation values and calculated with the following formulae:

$$f_{pt} = 0.3\left(m + 0.4\sqrt{d}\right) + 4 \tag{1}$$

$$f_{f\alpha} = 2.5\sqrt{m} + 0.17\sqrt{d} + 0.5 \tag{2}$$

$$f_{H\beta} = 0.07\sqrt{d} + 0.45\sqrt{b} + 3 \tag{3}$$

where m is the module, d is the reference diameter, and b is the face width. These apply for quality level 5; the values for the succeeding higher (lower) levels are obtained by repeatedly multiplying (dividing) by a step factor of $\sqrt{2}$.

The deviation of the parameters f_{pt} and $f_{H\beta}$ can be either positive or negative.

3 Literature survey of ISO calculation procedure

This chapter presents the fundamental contact stress equations together with the modified factors which are peculiar to gearing. The ISO standards distinguish between factors which are determined by gear geometry or which have been established by convention (Z-factors) and factors (K-factors) which account for several influences.

This study does not represent the view of the ISO organization, though many of the descriptions of the factors in this chapter are more or less direct citations from the ISO standards 6336-1 and 6336-2. Some comments by other contributors are also described here. Since the calculations involve numerous analytical expressions, only a few have been written out. For detailed information, the standards used and required for calculation of surface durability are:

- ISO 6336-1: Basic principles, introduction and general influences
- ISO 6336-2: Calculation of surface durability (pitting)
- ISO 6336-5: Strength and quality of materials
- ISO 6336-6: Calculation of service life under variable load [15]
- ISO 1328-1: Cylindrical gears, ISO system of accuracy

The maximum contact stress and the permissible contact stress are based on the Hertzian contact theory (modified by factors), and consist of three fundamental stress equations:

$$\sigma_{H0} = Z_H \cdot Z_E \cdot Z_\varepsilon \cdot Z_\beta \cdot \sqrt{\frac{F_t}{d_1 \cdot b} \cdot \frac{u+1}{u}} \tag{4}$$

$$\sigma_H = Z_B \cdot \sigma_{H0} \cdot \sqrt{K_A \cdot K_V \cdot K_{H\beta} \cdot K_{H\alpha}} \leq \sigma_{HP} \tag{5}$$

$$\sigma_{HP} = \frac{\sigma_{H\,lim} \cdot Z_{NT}}{S_{H\,min}} \cdot Z_L \cdot Z_V \cdot Z_R \cdot Z_W \cdot Z_X \tag{6}$$

where σ_{H0} is the nominal contact stress, σ_H is the calculated contact stress, σ_{HP} is the permissible contact stress, and σ_{Hlim} is the allowance contact stress number. In this study, the allowance contact stress σ_{Hlim} was obtained from ISO 6336-5. It is derived from the contact pressure that may be sustained for 2 million load cycles without the occurrence of progressive pitting for a 1% probability of damage. S_{Hmin} is the minimum safety factor which is to be agreed on between designer and customer. The minimum safety factor was set to 1 in this study, since the analyzed gear pair has no specific target function. The contact stress σ_H should be less than the permissible contact stress σ_{HP} for preventing failure.

The contact stress is calculated at the pitch point (where almost pure rolling occurs) or at the inner point of single pair tooth contact. The higher of the two values obtained is used to determine the load capacity. For gears with transverse contact ratio and overlap (face) ratio equal or greater to 1, the contact stress is always calculated at the pitch point. For gears with overlap ratio less than 1, contact stress is determined by linear interpolation between two limit values, i.e. σ_H for spur gears and σ_H for helical gears with overlap ratio equal to 1. Helical gears with contact ratio less than 1 and a total contact ratio (sum of contact ratio and overlap ratio) larger than 1 are not covered by ISO 6336.

A mapping of the factors is also presented in Figure 3 in order to give an overview of the ISO 6336-2 (surface durability) calculations.

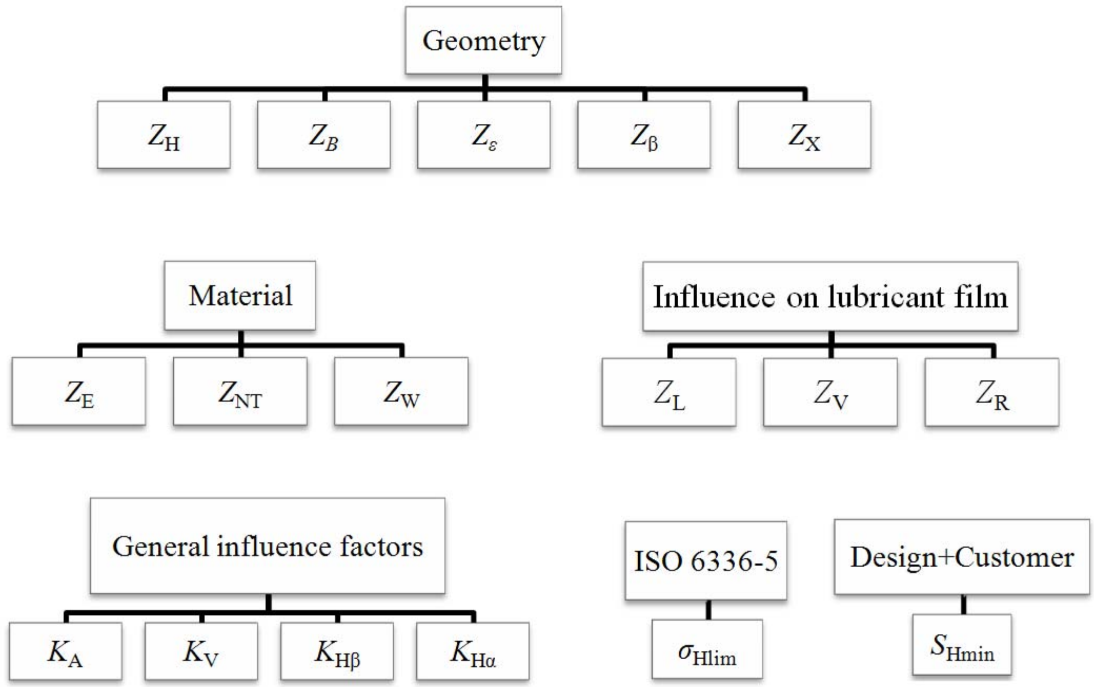

Figure 3. Mapping to establish the factors in ISO 6336-2.

The factors in the ISO standard can be determined by three methods, A, B, or C. Method A is superior to method B and method B is superior to method C. In method A, the factors are derived from full scale load tests; this method is therefore seldom used. In method B, the factors are derived with sufficient accuracy for most gear geometries. Method C is the same as method B except that simplified approximations are specified for some factors. In this study, method C is used when no experimental testing, extensive knowledge, or advanced calculation is required; and method B is used if no method C is specified in the standard.

In the following chapters, the separation of factors is as described in Figure 3 and all assumptions apply for the gear design presented in Table 1.

3.1 Geometric factors

The zone factor Z_H, the single pair tooth contact factor (pinion) Z_B, the contact ratio factor Z_ε, the helix angle factor Z_β, and the size factor Z_X are all determined by gear geometry and are introduced in ISO 6336-2.

The zone factor Z_H accounts for the influence on Hertzian pressure of tooth flank curvature at the pitch point. It transforms the tangential load at the reference cylinder to normal load at the pitch cylinder.

The single pair tooth contact factor (pinion) Z_B converts the contact stress at the pitch point to the contact stress at the inner point of single tooth contact on the pinion. If the single pair tooth contact factor is made equal to 1, the contact stress (equation 5) is the value at the pitch point. For helical gears with overlap ratio less than 1, the single pair tooth contact factor is determined by linear interpolation between the values for spur and helical gearing. Höhn et al. [16], who examined the pitting load capacity of helical and spur gears by experiments and standard calculation, found that for helical gears where there is insufficient tooth flank modification, adding a tooth flank correction factor to the single pair tooth contact factor gave a better correlation to experiments. This correction factor takes into consideration the higher contact stress at the beginning and the end of the path of contact for helical gears.

The contact ratio factor Z_ε accounts for the influence of the transverse contact and overlap ratios on the surface load capacity of cylindrical gears. As for the single pair tooth contact factor, the contact ratio factor is determined by linear interpolation between the values for spur and helical gearing for helical gears with overlap ratio less than 1.

The helix angle factor Z_β accounts for the influence of the helix angle on surface load capacity. According to McVittie [17], the helix angle factor tends to "run away" at high helix angles. However, ISO 6336 suggests that users confirm their results by experiment when the operating helix angle exceeds 30°.

The ISO 6336 standard is thus both complex and flexible. Research by Höhn et al. [16] has led to adjustment (a technical corrigendum) of the helix angle factor which is in print at the moment and is calculated as follows:

$$Z_\beta = \frac{1}{\sqrt{\cos\beta}} \tag{7}$$

This adjustment was used in this study.

The size factor Z_X is set to be 1 for calculating contact stress in ISO 6336. This size factor enables the calculation to take into account the greater possibility of encountering a material defect in the larger stressed volume of gears.

3.2 Material factors

Material properties are covered by the work hardening factor Z_W, the life factor Z_{NT} for standards reference test gears and the elasticity factor Z_E, all of which are introduced in ISO 6336-2.

The work hardening factor Z_W takes into account the Brinell hardness of tooth flanks of the softer gear of the pair, as well as the equivalent roughness. It basically allows modelling of the beneficial effect of running a harder pinion against a softer gear. However, the effects of wear are not covered by ISO 6336. In this study, the value of Z_W was calculated for reference and long life stress.

The life factor Z_{NT} accounts for the higher contact stress, including static stress, which may be tolerable for a limited life (number of load cycles). In this study, the life factor is set to 1 since according to ISO 6336-2 the lubricant film is only fully effective at long life stress level. For higher limited-life stress levels) the life factor for static and reference stress may be taken from graphical values derived for standard reference test gears. The number of load cycles is defined as the number of mesh contacts, under load, of the gear tooth being analyzed. Wållgren [18], who compared the ISO standard calculation for tooth bending fatigue with FEM calculations, noted the disadvantage that the life factor does not take into account the fact that a gear wheel with less number of teeth will be in contact more frequently. The life factor also to some extent includes the tribological influences, since these are included in the empirical test curves for standard test gears.

Influences of material properties, such as the modulus of elasticity, are integrated in the elasticity factor Z_E.

3.3 Influence of lubricant film

The lubricant factor Z_L, the velocity (pitch line velocity) factor Z_V, and the surface roughness factor Z_R all influence the surface durability as characteristics of the lubricant film thickness. All are calculated according to ISO 6336-2 and based on tests using standard reference test gears. According to McVittie [17], the combined effects of these three factors is usually less than 10% for industrial gearing using ISO 6336. McVittie recommends additional inputs for surface finish and lubricant viscosity, but does not specify these requirements.

The lubricant factor Z_L accounts for the effects of the nominal lubricant viscosity at 40˚C and the allowable contact stress value. It applies for mineral oils with or without extreme pressure additives and is calculated as follows:

$$Z_L = C_{ZL} + \frac{4(1 - C_{ZL})}{\left(1.2 + {134}/{v_{40}}\right)^2} \tag{8}$$

where C_{ZL} is a correction factor for an allowable contact stress range. The viscosity v_{40} is adjusted for the different ISO viscosity classes (levels) specified in ISO 6336-2.

The velocity factor Z_V accounts for the influence on the lubricant film of the pitch line velocity and the allowable stress number.

The roughness factor Z_R accounts for the influence of surface roughness of the flanks after manufacturing, including running-in treatments. It is calculated as follows:

$$Z_R = \left(\frac{3}{R_{Z10}}\right)^{C_{ZR}} \tag{9}$$

where C_{ZR} is a correction factor for an allowable contact stress range and R_{Z10} is the mean relative peak-to-valley value roughness for the gear pair. R_{Z10} is defined as follows:

$$R_{Z10} = R_Z \sqrt[3]{\frac{10}{\rho_{red}}} \tag{10}$$

where R_z is the mean peak-to-valley roughness of the gear pair and ρ_{red} is the reduced radius of relative curvature at the pitch point.

3.4 General influence factors

The application factor K_A, internal dynamic factor K_V, face load factor $K_{H\beta}$, and transverse load factor $K_{H\alpha}$ are introduced in ISO 6336-1 and take into account physical considerations such as work speed, tolerances, how the machine works, and face load distributions. The factors also depend on the profile and helix (lead) modifications. These should only be taken into consideration when manufacturing accuracy is stated according to ISO 1328-1 for each factor, as in this study.

The application factor K_A is used to adjust the nominal load to take into account loads, additional to nominal loads, which are imposed on the gears from external sources; that is, the working characteristics of the driving and driven machines. In this study, the guideline values for the application factor as described in ISO 6336-6 were used according to method B (no method C exists). The application factor was taken to be 1 (for uniform loads).

The internal dynamic factor K_V is influenced by both design and manufacturing. It makes allowance for the effects of gear tooth quality level and modifications as related to speed and load, and is defined as follows:

$$K_V = \frac{\text{Total mesh torque at operating speed}}{\text{Mesh torque with perfect gears}}$$

Perfect gears are gears with perfect conjugate action. Perfect conjugate action is when there is no deviation from constant ratio in a gear.

The value of the internal dynamic factor K_V approaches 1 when optimum profile modification is applied appropriate to the loading.

For determining the internal dynamic factor using method B, simplifying assumptions are made that the gear pair consists of an elementary single mass and spring system comprising the combined masses of pinion and wheel, the stiffness being the mesh stiffness of the contacting teeth. Method C is derived from Method B by introducing further simplifying assumptions according to ISO 6336-1. These assumptions hold for all types of cylindrical gears when the following condition is satisfied:

$$v\frac{z_1}{100}\sqrt{\frac{u^2}{1+u^2}} < 3\text{m/s} \tag{11}$$

where v is the tangential velocity at the reference diameter, z_1 is the number of teeth (pinion), and u is the gear ratio. This condition is satisfied in the present study.

The internal dynamic factor K_V is determined by linear interpolation between two limit values, those for spur and for helical gears, where the overlap ratio is less than 0.9. If this is the case, the calculation includes factors that are specified in tables and graphs, which take into account the influence of load, gear quality level at a set specific loading, and the resonance ratio.

The face load factor $K_{H\beta}$ takes into account the effects of the non-uniform distribution of load over the gear face width on the surface stress, and is defined as follows:

$$K_{H\beta} = \frac{\text{Maximum load per unit face width}}{\text{Average load per unit face width}}$$

According to ISO 6336-1, the extent to which the load is unevenly distributed depends on several influences, such as the gear tooth manufacturing accuracy, alignment of the axes of rotation of the mating gear elements, elastic deflections of gear teeth, shaft, bearing, housing, and foundations which support the gear elements, bearing clearances, and so on.

The formulae for the calculations of elastic deflections of the pinion and pinion shaft are simplified and based on assumptions. It is assumed that neither deflections of the wheel and wheel shaft nor deformations of the gear case and bearings are included, since these are normally sufficiently stiff that their deflections can be ignored.

For accuracy gearing, the face load factor approaches 1.1...1.4 (indicative range only), according to van Beek [19]. The face load factor would be equal to 1 at the design loading of gear pairs having optimum helix modification.

In this study, the face load factor was determined where the contact is calculated to extend across the full face width:

$$K_{H\beta} = 1 + \frac{F_{\beta y} c_{\gamma\beta}}{2F_t K_A K_V / b} \tag{12}$$

where $c_{\gamma\beta}$ is the mesh stiffness and $F_{\beta y}$ is the effective equivalent mesh misalignment derived, which is set equal to the initial equivalent misalignment $F_{\beta x}$ in this study. The initial equivalent misalignment is defined as the absolute value of the sum of deformations, displacements, and manufacturing deviations of pinion and wheel, measured in the plane of action as follows:

$$F_{\beta x} = 1.33 B_1 f_{sh} + B_2 f_{ma} \tag{13}$$

where the constant 1.33 corrects the error arising from the assumption that the elastic deformation is linear, and B_1 and B_2 are constants taking into account the helix modification. In this study, these are set for central crowning only. The equivalent misalignment f_{sh} was set equal to the allowable helix slope deviation $f_{H\beta}$. The mesh misalignment f_{ma} is the maximum separation between the tooth flanks of the meshing teeth of mating gears, when the teeth are held in contact without significant load, the shaft being in their working attitudes. A useful formulation for use under most circumstances in cases of an average quality control regime is:

$$f_{\mathrm{ma}} = \sqrt{f_{\mathrm{H}\beta 1}^{2} + f_{\mathrm{H}\beta 2}^{2}} \tag{14}$$

where $f_{\mathrm{H}\beta1}$ and $f_{\mathrm{H}\beta2}$ are the helix slope deviations for pinion and gear respectively.

The transverse load factor $K_{\mathrm{H}\alpha}$ accounts for the effects of the non-uniform distribution of transverse load between several pairs of simultaneously contacting gear teeth. The main influences are deflections under load, profile modifications, tooth manufacturing accuracy, and running-in effects. For accurate gearing the transverse load factor is equal to 1 and for lesser quality levels the transverse load factor is equal to 1.2 (indicative range only), according to van Beek [19].

The transverse load factor $K_{\mathrm{H}\alpha}$ was determined according to method B (no method C exists). Method B involves the assumption that the average difference between the base pitches of the pinion and wheel is the major parameter in determining the distribution of load between several pairs of teeth in the mesh zone, and is suitable for all types of gearing (spur or helical with any basic rack profile and any accuracy) on the condition that equation 11 holds true. If the total contact ratio (sum of the transverse and overlap contact ratios) is greater than 2, the transverse load factor is determined as follows:

$$K_{\mathrm{H}\alpha} = \frac{\varepsilon_{\gamma}}{2}\left(0.9 + 0.4\frac{c_{\gamma\alpha}\left(f_{\mathrm{pb}} - y_{\alpha}\right)}{F_{\mathrm{t}}K_{A}K_{V}K_{H\beta}/b}\right) \tag{15}$$

where ε_{γ} is the total contact ratio, $c_{\gamma\alpha}$ is the mesh stiffness, f_{pb} is the base pitch deviation, and y_{α} is the running in factor. Adjustments are made in equation 15 when the total contact ratio is less than 2.

The base pitch deviation accounts for the total effect of all gear tooth deviations which affect the transverse load factor. Thus, using method B for determining the transverse load factor, as in this study, the base pitch deviations f_{pb} are set equal to the transverse pitch deviation f_{pt}. If, nevertheless, the profile form deviation $f_{\mathrm{f}\alpha}$ is greater than the base pitch deviation, the profile form deviation should be taken instead of the base pitch deviation.

4 Robustness analysis

According to Andersson and Söderberg [20], the difference between robust design and traditional optimization is related to insensitivity to uncontrollable perturbations (accidents or noise). A design that is classified as robust will deliver target performance regardless of uncontrollable perturbations (noise).

The procedure of achieving robustness is based on factorial design and measurement of the quality. Factorial design, also called design of experiments, is a method for structuring a series of experiments to be able to gain as much information as possible about the studied system. The method proposed by Taguchi [21], and described in more detail by Bergman [22] and Phadke [23], is often associated with robust design.

In this study, the experiments are replaced by numerical calculations. Therefore, the results will not be influenced by any noise, which has to be explicitly added. Usually, the goal is to find a combination of product parameter values that gives the smallest variation in the value of the quality characteristics around the desired target value [23]. That is, if the response is adversely affected close to the optimal value, it is better to choose another value and thus obtain a solution which is not optimal but is insensitive (i.e. robust) to variation [20]. However, in this study no explicit target value exists, and so the procedure is used to find out whether or not the generated gear deigns are sensitive to variation in surface roughness, lubricant viscosity, and quality level.

A number of test gear designs were produced by varying selected gear design parameters (control design factors) which are set during the design process and which are known to influence the robustness. The normal pressure angle, the normal helix angle, and the face width were chosen as control design factors (Table 3). Neither the pressure angle nor helix angle reaches above 25° since the standard is not making allowance for that. In order to get unambiguous test gear designs, since several gear parameters are linked, the centre distance and backlash were held constant. With these constraints, the metric modulus and profile shift coefficient were calculated for each combination of levels of the control design factors.

Table 3. Control design factors and their levels. Nominal values are underlined.

Control design factor	Description	Level		
		0	1	2
a	Pressure angle a [°]	17.5	<u>20</u>	22.5
β	Helix angle β [°]	<u>15</u>	20	25
b	Face width b [mm]	34	<u>37</u>	40

Each gear design is then exposed to a combination of levels of noise factors, presented in Table 4, which is known to influence the gear performance in terms of manufacturing variations and product use. The noise factors were set in order to analyze the influence of gear quality level (Q) along with tribological factors in the form of surface roughness (R) and lubricant viscosity (L). The gear quality levels (tolerances) were calculated according to equations 1-3 which holds definitions and allowable values of deviations relevant to corresponding flanks of gear tooth. The calculated gear quality levels and their deviations for the nominal gear design are presented in Table 5; they include single pitch deviation, profile form deviation, and helix slope deviation. The selected quality levels 5, 7 and 9, as well as the selected roughness values, can be seen as general limit values in gears for heavy trucks. The selected viscosities are applicable for the working characteristics of the analyzed gear. Lower viscosities can be used for high speed transmissions.

Table 4. Noise factors and their levels.

Noise factor	Description	Level		
		0	1	2
Q	Quality level [–]	5	7	9
R	Roughness [µm]	1	2	3
L	Viscosity [mm^2/s]	100	150	200

Table 5. The quality levels 5, 7, and 9 and the coupled allowable values of deviations for the nominal gear design.

Deviations	Quality level Q		
	5	7	9
Single pitch deviation f_{pt} [µm]	6.9	9.7	13.7
Profile form deviation $f_{f\alpha}$ [µm]	8.0	11.4	16.1
Helix form/slope deviation $f_{H\beta}$ [µm]	6.5	9.2	13.0

Each combination of the noise and control design factors will give a certain response. A three-level reduced factorial design was used to produce different gear geometries in a structural manner. A three-level reduced design is written as a 3^{4-1} factorial design. This means that three (4-1) factors are considered, each at three levels, which gives nine tests (27 tests are required for a complete factorial design). The three levels are referred to as low, intermediate, and high level and are numerically expressed as 0, 1, and 2.

The same factorial design was used for the control design factors and noise factors. These two are then combined so that each design factor level combination (D1-D9) occurs with each noise factor level combination (N1-N9). Taguchi [21] referred to these two components designs as inner array and outer array, respectively. The inner array sets the combinations (D1-D9) of the control design factors (a, β, b). Then noise analyses (N1-N9) are performed in the inner array to estimate the sensitivity to noise factors (Q, R, L). The test lay-up is presented in Table 6. The mean value m and standard deviation s is calculated for each test design (D1-D9). For each test design and set of noise analyses the response is computed and entered into the appropriate box (shaded area in Table 7). This results in a total of 81 subanalyses.

Table 6. The test lay-up. The inner and outer arrays have the same factorial design. Each row (D1-D9) in the inner array is one test design for which nine noise analyses (N1-N9) are performed.

				Outer array										
				N1	N2	N3	N4	N5	N6	N7	N8	N9		
				0	0	0	1	1	1	2	2	2	Q	
	Inner array			2	1	0	2	1	0	2	1	0	R	
				2	1	0	1	0	2	0	2	1	L	
	a	β	b	*Responses*									m	s
D1	0	2	2											
D2	0	1	1											
D3	0	0	0											
D4	1	2	1											
D5	1	1	0											
D6	1	0	2											
D7	2	2	0											
D8	2	1	2											
D9	2	0	1											

4.1 Responses

Responses are chosen in order to study the sensitivity of the standard calculation procedure to surface roughness, lubricant viscosity, and quality level. The calculated safety factor includes the calculated contact stress and the permissible contact stress (equations 5 and 6), which are dependent on a correction factor and a tooth load factor respectively. These three factors are chosen as responses and are defined as follows:

$$\text{Safety factor} = \frac{\sigma_{HP}}{\sigma_H} \tag{16}$$

$$\text{Correction factor} = \frac{Z_{NT} Z_L Z_V Z_R Z_W Z_X}{S_{H\,min}} \tag{17}$$

$$\text{Tooth load factor} = \sqrt{K_A K_V K_{H\beta} K_{H\alpha}} \tag{18}$$

The safety factor indicates whether the design will suffer from failure. The correction factor gives indications of the influence of the tribological factors. A high value of the correction factor increases the permissible contact stress (equation 6). The tooth load factor gives an indication of the influence of quality level. A low value of the tooth load factor will lower the calculated contact stress (equation 5).

5 Results

The calculated factors for test gear design D1 (see Table 6) are presented in Table 7 in order to give a general idea of the factors calculated in this study. Minimum and maximum values are presented for factors influenced by noise (valid for test design D1).

Table 7. The calculated factors (min/max values) for the test gear design D1.

Zone factor	Z_H=2.21
Elasticity factor [(N/mm^2)$^{1/2}$]	Z_E=190.27
Contact ratio factor	Z_ε=0.84
Helix angle factor	Z_β=0.95
Single pair tooth contact factor (pinion)	Z_B=1
Application factor	K_A=1
Dynamic factor	K_V=1.03/1.14
Face load factor for contact stress	$K_{H\beta}$=1.65/3.38
Transverse load factor for contact stress	$K_{H\alpha}$=0.96/1.01
Life factor	Z_{NT} (long life reference stress level) = 1
Lubricant factor	Z_L = 0.97/1.01
Velocity factor	Z_V=0.98
Roughness factor	Z_R=1.13/1.23
Work hardening factor	Z_W=0.97/1.18
Size factor	Z_X=1

Table 8 presents the calculated safety factors for all tests (shaded area in the test lay-up). As can be seen, the safety factor is the highest for noise combination in column N3 for all test gear designs (D1-D9). The highest and lowest mean safety factors for all noise combinations occur for test design D8 and D3 respectively. For test design D8, all control factors except the helix angle β factor are at their high control design factor level (Table 3). Similarly, for the test design D3, all control factors are at their low level. Column N7 has the lowest safety factors, nearly all below 1.

Table 8. The safety factors shown in each row have similar control design factors but different test conditions with respect to noise factors.

				N1	N2	N3	N4	N5	N6	N7	N8	N9		
				0	0	0	1	1	1	2	2	2	*Q*	
				2	1	0	2	1	0	2	1	0	R	
				2	1	0	1	0	2	0	2	1	*L*	
	a	*β*	*b*	*Safety factor*									*m*	*s*
D1	0	2	2	1.35	1.43	1.60	1.18	1.24	1.58	1.02	1.21	1.37	1.33	0.19
D2	0	1	1	1.14	1.21	1.35	1.01	1.06	1.35	0.87	1.04	1.18	1.13	0.16
D3	0	0	0	0.99	1.05	1.18	0.89	0.93	1.18	0.77	0.92	1.04	0.99	0.14
D4	1	2	1	1.24	1.31	1.47	1.09	1.15	1.46	0.94	1.12	1.27	1.23	0.17
D5	1	1	0	1.09	1.16	1.29	0.97	1.02	1.29	0.84	1.00	1.13	1.09	0.15
D6	1	0	2	1.33	1.41	1.58	1.18	1.23	1.57	1.01	1.21	1.37	1.32	0.18
D7	2	2	0	1.16	1.23	1.38	1.03	1.08	1.37	0.89	1.06	1.20	1.15	0.16
D8	2	1	2	1.38	1.46	1.64	1.22	1.27	1.62	1.05	1.25	1.41	1.37	0.19
D9	2	0	1	1.17	1.24	1.39	1.04	1.09	1.38	0.90	1.07	1.21	1.16	0.16

Table 9 presents the calculated correction factor for all tests (shaded area). As expected, only the surface roughness (*R*) and lubricant viscosity (*L*) had any influence on the correction factor, since none of the factors in equation 17 are related to gear quality level. The gear test designs (D1-D9) have almost identical correction factors.

The highest correction factor value can be seen in column N6 and the lowest in column N7. Here, the surface roughness and lubricant viscosity are at their extremes. In N6 the surface roughness is low and the lubricant viscosity high, and vice versa in column N7. The combination of noise levels in column N6 will contribute to the highest permissible contact stress. It can be seen that the response follows the outline of the roughness factor level (R) for respective level of quality level.

Table 9. The correction factors shown in each row have similar control design factors but different test conditions with respect to noise factors.

				N1	N2	N3	N4	N5	N6	N7	N8	N9		
				0	0	0	1	1	1	2	2	2	*Q*	
				2	1	0	2	1	0	2	1	0	R	
				2	1	0	1	0	2	0	2	1	*L*	
	a	*β*	*b*	*Correction factor*									*m*	*s*
D1	0	2	2	1.12	1.19	1.33	1.08	1.13	1.44	1.03	1.23	1.39	1.22	0.14
D2	0	1	1	1.12	1.18	1.33	1.08	1.13	1.44	1.03	1.23	1.39	1.21	0.14
D3	0	0	0	1.13	1.19	1.33	1.09	1.14	1.45	1.04	1.23	1.40	1.22	0.14
D4	1	2	1	1.13	1.20	1.34	1.09	1.14	1.46	1.04	1.24	1.41	1.23	0.14
D5	1	1	0	1.14	1.20	1.35	1.10	1.15	1.46	1.05	1.25	1.41	1.23	0.15
D6	1	0	2	1.14	1.21	1.35	1.10	1.15	1.46	1.05	1.25	1.41	1.23	0.15
D7	2	2	0	1.14	1.21	1.36	1.10	1.16	1.47	1.05	1.26	1.42	1.24	0.15
D8	2	1	2	1.15	1.22	1.36	1.11	1.16	1.48	1.06	1.26	1.43	1.25	0.15
D9	2	0	1	1.14	1.21	1.36	1.10	1.16	1.47	1.05	1.25	1.42	1.24	0.15

Table 10 presents the calculated tooth load factor for all tests. As expected, only the quality had any influence on the tooth load factor, since none of the factors in equation 18 are related to surface roughness or lubricant viscosity. Test gear design D6 gives the lowest mean value at 1.37, which is the lowest for all designs. D6 also has level combinations closest to the nominal gear design (Table 3). Test gear design D3, which has all its control design factors at the low level, gives the highest mean value. The lowest tooth load factor values can be seen in column N1-N3 and the highest in column N7-N9, which corresponds to the best and less precise quality level tested.

Table 10. The tooth load factors shown in each row have similar control design factors but different test conditions with respect to noise factors.

				N1	N2	N3	N4	N5	N6	N7	N8	N9		
				0	0	0	1	1	1	2	2	2	*Q*	
				2	1	0	2	1	0	2	1	0	*R*	
				2	1	0	1	0	2	0	2	1	*L*	
	a	β	*b*	*Tooth load factor*									*m*	*s*
D1	0	2	2	1.28	1.28	1.28	1.41	1.41	1.41	1.57	1.57	1.57	1.42	0.12
D2	0	1	1	1.42	1.42	1.42	1.55	1.55	1.55	1.71	1.71	1.71	1.56	0.13
D3	0	0	0	1.55	1.55	1.55	1.68	1.68	1.68	1.84	1.84	1.84	1.69	0.13
D4	1	2	1	1.37	1.37	1.37	1.50	1.50	1.50	1.66	1.66	1.66	1.51	0.13
D5	1	1	0	1.49	1.49	1.49	1.61	1.61	1.61	1.78	1.78	1.78	1.63	0.13
D6	1	0	2	1.25	1.25	1.25	1.36	1.36	1.36	1.51	1.51	1.51	1.37	0.11
D7	2	2	0	1.47	1.47	1.47	1.61	1.61	1.61	1.78	1.78	1.78	1.62	0.13
D8	2	1	2	1.26	1.26	1.26	1.38	1.38	1.38	1.54	1.54	1.54	1.39	0.12
D9	2	0	1	1.36	1.36	1.36	1.48	1.48	1.48	1.63	1.63	1.63	1.49	0.12

6 Discussion

In this study a robustness analysis based on factorial design was performed for a helical gear (Table 1), calculating the surface durability using the ISO standard. A number of gear geometries (gear designs) were used, in which a number of chosen gear parameters (control design factors) which are known to affect the robustness were varied in a structural manner. For each test design, uncontrollable perturbations (noise factors) which are known to affect the surface durability, but which to some degree may be beyond the control of the designer, were explicitly added. Face width, pressure angle, and helix angle were defined as control design factors, while surface roughness, lubricant viscosity, and quality level were defined as noise factors. Each combination of the control design and noise factors will give a certain response.

Three responses were analyzed; the safety factor (equation 16), the correction factor (equation 17), and the tooth load factor (equation 18).

The case that the safety factor sometimes reaches below zero might be influenced by the fact that the working characteristics are made up for the nominal gear design in this study. Nevertheless, the safety factor is the most interesting response in this study, as it is influenced by all noise factors (Table 4). However, it is not possible to find a set of combinations of control design factors (Table 3) that is more or less sensitive for variations in surface roughness, lubricant viscosity, and quality level. This can be seen by studying the standard deviations in Table 8. On the other hand, the combinations of noise factors in column N7 (Table 8) demonstrate that the standard do result in the lowest safety factor when the least accurate quality level, the roughest surface, and the lowest viscosity are used in combination, which is reasonable.

The correction factor (Table 9) takes into account the influence of surface roughness and lubricant viscosity. The control design factors were not significantly affected by variation in surface roughness and lubricant viscosity. However, when comparing level combinations in column N6 and N7 in Table 9, it can be seen that the smoothest surface in combination with the highest viscosity gives the highest correction factor value and the roughest surface in combination with the lowest viscosity gives the lowest correction factor value, which is sensible.

The tooth load factor takes into account the influence from quality level. It was found that the lowest (most accurate) quality level combinations (N1-N3) in Table 10 resulted in the lowest tooth load factor, which decreases the contact stress. In addition, the test gear design (D6 in table 10) most related to the nominal gear design (Table 1) gave the lowest mean value of the tooth load factor. This suggests that the proposed traditional gear design is properly designed.

None of the responses showed any influence on sensitivity to variation in the proposed noise factors. The reason for this certainly depends on the affected numerical factors, which are set for adjusting mathematical models to reality, and how they are connected and set in relation to each other. It may also depend on the fact that the least accurate calculation method (method C) has been used. For careful tolerance design, the most accurate calculation method (method A) in the ISO standard should be used, which requires a numerical or analytical approach and/or testing with the actual components. However, this method also requires extensive previous knowledge of gear design.

In this study, the quality level were the same for all tolerances used (helix slope deviation, pitch deviation, and profile form deviation). Normally, this is not the case. A gear element can have narrow tolerances for a specific parameter and wide tolerances for another. This is due to manufacturing possibilities (i.e. accuracy for a specific machine operation), company-based experiences, and the desired gear performance. However, this would most likely not affect the outcome of the results in this study.

The factors influenced by the gear quality level belong to the general influence factors (K-factors), which are the internal dynamic load factor, the face load factor, and the transverse load factor. When calculating the internal dynamic load factor, tables and graphs had to be used in this study where only the quality level (0-12) is specified, not specific tolerance parameters. This complicates the understanding how the current noise parameters are embedded in the calculation procedure. Further, the face load factor (equation 12) was only linked to the helix slope deviation and the quality level inherent from the internal dynamic factor. The transverse load factor (equation 15) was coupled to the base pitch deviation, profile form deviation, and the deviations inherent from the face load factor and the internal dynamic factor.

The standard reference test applies to spur gears, and hence adjustments were applied in order to compensate for differences in behaviour for helical gears. However, Höhn et al. [16], who examined the pitting load capacity of helical and spur gears, found that the calculation methods according to ISO 6336-2 determine the pitting load capacity of spur gears more accurately than for helical gears, when comparing experimental results and standard calculations. Therefore, the fact that a helical gear pair was analyzed may have influenced the results.

The Hertz formula which the ISO 6336-2 calculations are based on can only calculate the tooth contact stress approximately. The finite element method (FEM) can be used to conduct more precise analysis of the tooth contact stress, and is a good way of confirming or disproving the standard calculations. Li [24] used FEM to study the influence of tooth modifications, manufacturing errors, and assembly errors. Furthermore, Litvin and Fuentes [25] used FEM to perform stress analysis for positions of pinion and gear obtained from tooth contact analysis. However, there are still challenges associated with modelling the gear geometrical deviations and the gear tolerances which represent standard tolerance practices.

Today, many programs exist which have been specially developed for gear calculations. The Load Distribution Program (LDP) [26] from Ohio State University is used by several researchers and gear companies in order to compare different gear design or to improve already existing designs. Another gear program is CALYX [27] developed by Vijayakar [28], which combines finite element and boundary element solutions. These programs can incorporate a limited range of geometrical deviations, such as pitch deviations, runout, and tooth modifications under static load conditions. Additionally, computer programs which offer the standard calculations in software form may also be helpful if the manual ISO calculations seem too time consuming (see for example, KiSSsoft [29]). Nevertheless, if the aim is to reach the goal of stronger, more compact, less noisy gears and more environmentally friendly lubrication techniques etc, there is a need for a simulation tool that include effects of form deviations, surface roughness effects, and lubricant properties. The benefits of such tool are that more of the gear contact potential can be used.

Generally, the ISO standard surface durability calculations are time consuming, and it can be difficult and sometimes confusing to get a grip on the factors, assumptions, and advice given in footnotes. Berzal et al. [30] have developed an auto-learning program aimed at helping the inexperienced gear designer to understand the physical meaning of the modified factors, acquire practice in gear design, and obtain further information for specific applications. This is of great importance, since misunderstanding of the many factors can lead to misleading results.

7 Conclusions

Using ISO 6336 calculation (Method C) of contact load capacity, it is not possible to find a gear design that is sensitive to variations in surface roughness, lubricant viscosity and quality level.

The tolerance tables in ISO 1328-1 should mainly be used for guidance and communication purposes, since these quality levels are poorly incorporated in the ISO 6336 surface durability calculations.

The ISO 6336 standard is restricted to using only a few tolerance parameters, as it focuses on the pitch error, lead deviations (misalignment) and profile form deviation.

8 References

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Notation

a	Centre distance [mm]
b	Face width [mm]
B1, B2	Constants taking into account helix modifications [–]
C_{ZL}	Correction factor (lubricant) for allowable contact stress range [–]
C_{ZR}	Correction factor (roughness) for allowable contact stress range [–]
d_1	Reference diameter (pinion) [mm]
E	Modulus of elasticity [MPa]
$f_{f\alpha}$	Profile form deviation [µm]
$f_{H\beta}$	Helix (lead) slope deviation [µm]
f_{ma}	Mesh misalignment [µm]
f_{pb}	Base pitch deviation [µm]
f_{pt}	Single pitch deviation [µm]
f_{sh}	Component of equivalent misalignment [µm]
$F_{\beta y}$	Effective equivalent mesh misalignment [µm]
F_t	Nominal tangential load [N]
HV	Vickers hardness [–]
K_A	Application factor [–]
$K_{H\alpha}$	Transverse load factor for contact stress [–]
$K_{H\beta}$	Face load factor for contact stress [–]
K_V	Dynamic factor [–]
m	Normal module [mm]
R_{Z10}	Mean relative peak-to-valley roughness [µm]
R_Z	Mean peak-to-valley roughness [µm]
S_{Hmin}	Minimum safety factor [–]
s	Standard deviation [–]
u	Gear ratio [–]
v	Tangential velocity [m/s]
x	Profile shift coefficient [–]
y_α	Running in allowance for a gear pair [µm]

$z_{1,2}$	Number of teeth for gear and pinion [–]
Z_B	Single pair tooth contact factor (pinion) [–]
Z_β	Helix angle factor [–]
Z_E	Elasticity factor $[(N/mm^2)^{1/2}]$
Z_ε	Contact ratio factor [–]
Z_H	Zone factor [–]
Z_L	Lubricant factor [–]
Z_{NT}	Life factor [–]
Z_R	Roughness factor [–]
Z_V	Velocity factor [–]
Z_W	Work hardening factor [–]
Z_X	Size factor [–]

a	Nominal pressure angle [°]
β	Helix angle [°]
ε_γ	Total contact ratio [–]
ρ_{red}	Reduced radius of curvature [mm]
σ_{Hlim}	Allowable contact stress $[N/mm^2]$
σ_{HP}	Permissible contact stress $[N/mm^2]$
σ_{H0}	Nominal contact stress $[N/mm^2]$
ν_{40}	Nominal viscosity at 40°C $[mm^2/s]$
ν	Poisson's ratio [–]