Modelling a Mineral Froth Flotation Process

Bilal Ur Rehman

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Umeå Institute of Technology
EL1202
Abstract

We present an approach to model the dynamic of a copper flotation process. The conventional approach of system identification is applied to model the dynamics. In this research, experiments are performed to collect process data of determined input and output variables.

It is followed by data pre-processing to handle outliers and to remove high frequency disturbances. Simulation and validation responses of linear estimated models, which captured the dynamic of the process, are presented. The long term goal is to use estimated models to design a models-based control system.
Acknowledgements

First of all, my special and humble thanks is only for Almighty Allah, Who gave me strength and opportunity to complete this research, by giving me knowledge and helping me in difficult times. Secondly, I am deeply thankful to my family for their all prayers, motivation and support during my studies. I would like to thanks my supervisors Shafiq Ur Rehman and Anders Sandberg for their support and advices during my thesis work. I am thankful to Pedro La Hera, who provide me all his help and knowledge to complete this difficult task. His quick feedback and professional suggestion really helped me to improve my work and skills. Finally, thanks to Daniel Ortiz Morales and Attayyab Khan for there support and suggestion during my thesis work.
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Part I

Froth Flotation Process
Chapter 1

Introduction

1.1 Froth Flotation Process

Froth flotation obtains improved concentration grade of valuable minerals from mined ore. By using differences in physical and chemical properties of valuable and non-valuable minerals particles, it allows to extract valuable minerals. The froth flotation is a complex method that has been investigated for a long time[1]. The basic unit for performing froth flotation is the flotation cell. This can be a tank or vessel with well designed dimensions, as shown in Fig.1.1. The flotation cell has one input stream of a slurry (mixture of water, minerals, and gangues

![Figure 1.1: Conventional Flotation Cell](image)

Figure 1.1: Conventional Flotation Cell
particles), two output streams of minerals usually referred as froth and tailing. Chemicals (frother and collector) are added into the flotation cell to make minerals particles hydrophobic (floatable) and to form a stable froth layer (containing minerals).

The flotation cell has a motor driven impeller used to agitate the slurry. Air flow is added near the impeller, which is designed to break the air into small bubbles. The hydrophobic (floatable) mineral particles attach to the surface of the bubbles as shown Fig.1.2(a), and they lift up the minerals to the top of the flotation cell, forming a mineral froth layer as shown in Fig.1.2(b). However, during this procedure, nonvaluable minerals can also float. The liquid volume in the flotation cell is usually controlled, so the upper most part of the froth layer falls over the edge, when more bubbles are added to the froth layer. The froth launder, at top of the tank, takes care of the froth form layer (containing minerals). The tailing grade has outflow of slurry to the next flotation cell in the flotation bank. Some of the pre-requirements to achieve the flotation of minerals are the following:

- The mineral particles size should be small enough, so bubbles can lift up the mineral particles to the froth layer. Otherwise, heavy particles attached to the bubbles will drop back to the pulp phase.
- The air bubbles must be able to form a froth layer on the top of the tank, otherwise bubbles will burst and drop the minerals back to pulp phase.
- The mineral particles should be hydrophobic (floatable). Otherwise they will not attach to the bubbles surface.

Figure 1.2: (a) Loaded Bubbles with minerals (b) Froth phase or froth layer and Pulp phase in flotation cell.
1.1.1 Flotation Chemicals

Chemicals are used to enhance the flotation process efficiency, since most of the valuable minerals do not float without a surface modification. The chemicals used for the surface modification are selected based on the type of mineral to be treated. Chemicals collector and frother, which are used in copper froth flotation process, are the following:

**Collector**

Collector modifies the surface of the mineral particles and bubbles to be hydrophobic. The main objective is to make minerals floatable and attachable to bubbles. The minerals have different surface properties, i.e. they can be polar or non-polar. The non-polar minerals easily float on water and only interact with non-polar substance (air bubble surface). Collector makes valuable mineral surface non-polar, increasing the floatability by easing the attachment of minerals to bubbles surface. The non-polar minerals are called hydrophobic. Water is a polar, substance which easily interacts with polar substances (non-valuable minerals). These polar substances are called hydrophilic.

**Frother**

Frother are added to keep the stability of the air bubbles containing mineral particles. Flotation cannot occur without a stable froth, to carry the valuable minerals particles from the cell.

1.1.2 Flotation Mechanics

The objective of the froth flotation method is to attain minerals recovery with high concentration grade of valuable minerals. To achieve this goal, the flotation cells are usually connected in series to form a flotation bank and different banks are connected to make a flotation circuit. This section will give a general overview about different configuration of flotation cells.

**Flotation Bank**

Flotation cells are connected in series to form a flotation bank, as shown in Fig.1.3(a). Initially the slurry is feed into the first cell of the flotation bank. A fractional amount of concentration grade of valuable minerals is extracted in the first cell, and slurry is sent to the next cell, where the more valuable minerals are extracted and so on. Slurry leaves the last flotation cell in the bank as tailing. The total concentration grade is the total of all collective concentration grade
from each flotation cell in a bank. The whole bank can be a series of tanks interconnected, or it can be one single tank with agitators in series.

(a) Flotation bank

(b) Flotation circuit

Figure 1.3: Flotation Mechanics

**Flotation circuits**

In industry, the flotation cells are connected in series of banks to form a flotation circuit. The flotation circuit is designed to achieve high concentration grade and recovery, by controlling flow rate and tank size, which give minerals enough time to be activated. Slurry is feed into the first series of cells, which make the rougher bank. The tailing of the rougher bank is feed into the next series of cells, which make the scavenger bank see Fig.1.3(b).

The minerals froth from the rougher and scavenger bank are feed into a cleaner bank. It represents another cycle of the froth flotation, and used to reduce non-valuable minerals concentration, by upgrading the content of the rougher and scavenger banks. Regrinding mills are also used to refine particles size, which help to increase and refine valuable minerals concentration.
1.2 Flotation Process Performance

Two fundamental terms are used to characterize the performance of the flotation process:

**Concentration grade:** An indication of the purity of the minerals recovered, and usually expressed as the concentration of valuable minerals.

**Recovery:** Defined as the fraction of minerals present infeed that is recovered in the concentrate.

\[
Recovery = \frac{C(I - A)}{I(C - A)} \% \tag{1.1}
\]

Where

- \(I\) = Percentage of the in-feed to the flotation cell.
- \(A\) = Percentage of the tailing from flotation cell.
- \(C\) = Percentage of the Concentration grade of valuable mineral.

The recovery and concentration grade of a single flotation cell are strongly correlated. When the infeed and mass flow are fixed, the relationship between them can be described by the curves shown in Fig.1.4. Changes in the operating conditions, i.e. changes in chemicals frother, or collector additions or air, moves the process performance along the curve of concentration grade and recovery. The same behaviour is shown by the tailing grade slurry, which also contains valuable minerals.

![Figure 1.4: Recovery vs Concentration grade](image-url)
1.3 Problem Formulation

Currently, industrial flotation process is operated manually. Experienced operators with their knowledge and skills try to maintain process performance, while dealing with different input chemical affects and complexity of the mined ore composition. The process performance is not fully optimized, to optimize the process performance with less human involvement, process should be controlled automatically.

To optimize the flotation process performance, it is useful to develop a mathematical model to simulate process dynamical behaviour. It is very hard to develop flotation process model based on the first principles. In order to develop flotation process model, system identification approach can be used.

1.4 Objective Of Thesis

The objective is to define a systematic procedure for modelling dynamics of the froth flotation process. Identified linear process model can be used to simulate the operation of industrial flotation circuit. The goals are:

- To determine suitable manipulating and target variables for the froth flotation process.
- To identify the process linear model, which describes system dynamics, and to validate its performance over different experiments.

Experiments need to be designed and performed with process optimal operating points, to collect process data and estimate the model.
Chapter 2

Copper Flotation process at Boliden Aitik

Aitik site has the biggest open copper mine of Boliden AB. The ore deposit consists of pyrite and chalcopyrite\(^1\), which yield copper, gold and silver. Copper (Cu) is the most important metal extracted from this facility. The copper grade in the ore is very low, and it must be compensated with a highly efficient production concentrator. The production capacity increased from 18 to 36 million tonnes of ore after completion of a new concentrator in 2010.

2.1 Ore Composition

Since 1968, Aitik has mined 529.6 million tons of ore, delivered 6.46 million tons of copper concentrate. The ore composition is given in the next table.

<table>
<thead>
<tr>
<th>Minerals</th>
<th>Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chalcopyrite</td>
<td>CuFeS(_2)</td>
</tr>
<tr>
<td>Galena</td>
<td>PbS</td>
</tr>
<tr>
<td>Sphalerite</td>
<td>ZnS, ZnS(Fe)S</td>
</tr>
<tr>
<td>Pyrites</td>
<td>FeS(_2)</td>
</tr>
<tr>
<td>Pyrrhotite</td>
<td>Fe(_{0.8-0.9})S</td>
</tr>
<tr>
<td>Arsenopyrite</td>
<td>FeAsS</td>
</tr>
</tbody>
</table>

\(^1\)Chalcopyrite contains copper, whereas pyrite is heavier and heavier particles and it does not contain copper.
In the year 2010, the production of ore was 27.6 Million tons, waste rock was 26.7 Mton, and copper concentrate was 262.6 Kton. Metal Content of copper was 64,542 ton, gold 1945 kg, and silver 28,592 kg. Feeding grade was 0.2g gold/ton, 2g silver/ton and 0.29% copper/ton.

2.2 Copper Froth Flotation Circuit

The copper froth flotation circuit is unique in size and dynamics, schematic diagram of the process is shown in Fig.2.1. The flotation circuit consists of 46 flotation cells, and the slurry flows between cells under the gravitational force. There are two mixing tanks used for conditioning the slurry with chemicals, which provides the slurry to the froth flotation circuitry. Eighteen flotation cells are placed in two parallel lines that make two rougher and scavenger banks. Furthermore, they are connected with two parallel lines, which make the rougher and scavenger cleaner banks. The flotation circuit also contains cleaning and re-cleaning stages to further purification.

2.2.1 Manipulating Variables

The state of the art behind the froth flotation process consists of controlling the tread-off curve between concentration and recovery. The flotation variables are manipulated to increase the performance curve. These manipulated variables have local and global effects. Local affects are produced by air addition, change in forth thickness, etc. Global affects are caused by addition of chemicals.

Addition of Chemicals

Collector and frother are two important chemicals involved in the froth flotation process. Choosing the right amount of collector and frother addition will help to get balance concentration and recovery point.

- **Collector** is usually added in small amounts. Initially, it leads to an increment in the recovery of the valuable minerals. After it reaches its saturation point, further addition of collector would increase the recovery of gangue or non-valuable minerals.

- **Frother** addition is a complex action. An excessive dosage of frother at one point has global affect on the process. The froth must be stable, so that bubbles do not burst.
Figure 2.1: Flotation Process Schematic Diagram
Air Addition

Air flow is directly provided in the pulp phase of the flotation cell, near the impeller. This control variable has very quick response and affects the process locally. Rational increment in air flow rate reduces the froth residence time, which leads gangue particles to drain back to slurry from froth. Higher air flow can blow away the froth, which will increase the concentrate grade rapidly, but recovery will be reduced. Air addition should be chosen to keep the concentration and recovery ratio at an optimal point.

Froth Phase Thickness

Froth phase thickness is the difference between cell height and pulp phase height (as shown in Fig.2.2). A rational incremental of the froth phase thickness will increase the froth residence time, leading the gangue particles to drain back to slurry, and increasing the mineral recovery rate.

2.2.2 Flotation Process Flow

Ideally, the chemical process infeed needs to be smooth and stable enough to work in normal conditions. In reality, it is not possible to have ore with constant concentrate grade. Infeed to the flotation circuit depends upon different type and quality of the ore coming from the mining ground. Disturbance can also be caused by changes infeed rate to the circuit, pulp or slurry
density, size of particles distribution, and composition of ore feed. The flotation circuit is shown in Fig.2.1. A brief description of the flotation circuit is given below.

**The Mixing Tank**

The mixing tank (BL4001) has infeed of mineral particles coming from the grinding mills. Before feeding pulp or slurry to flotation cells, pulp gets mixed with chemicals, fine minerals particles, and water, which form the slurry for flotation process. Chemicals mixing tank have dosing points. After mixing, the slurry is pumped to the rougher bank.

**The Rougher Bank**

The rougher bank (FA4010 to FA4013) contains four flotation cells (See Fig: 2.1). Each cell has 160 m$^3$ of volume capacity. A high volume capacity gives minerals enough time to get activated. It is the first bank of the flotation circuit and it is designed to float the main recovery containing the lighter particles. The infeed of slurry comes from the mixing tank. The tailing from rougher bank (y2) is fed to the scavenger bank for further processing. The concentration grade (y1) of the rougher bank is fed to the rougher re-grind mill.

The rougher re-grinding mill (KV4210) is used to refine size of particles. The slurry is then feed to cyclone (CY4234), where lighter particles are blown towards cleaners. Cleaners help to increase and refine valuable minerals concentration grade.

The rougher bank has two chemicals dosing points for frother and collector in flotation cells (FA4011 and FA4013). The rougher bank has air flow addition and froth thickness control in each flotation cell.

**The Scavenger Bank**

The scavenger bank (FA4014 to FA4018) contains five flotation cells, each cell has 160 m$^3$ volume capacity. It is designed to extract heavier mineral particles, it is aimed to maximize the recovery of the process. The infeed of slurry comes from the rougher bank, the tailing from scavenger (y3) bank is process waste, and the concentration grade (y4) is feed to re-grinding mill. The scavenger re-grinding mill (KV4240) is used to refine particles size of scavenger concentration grade. The output of scavenger re-grinding mill is fed to the rougher cleaner bank for further processing.

The scavenger bank has two chemical dosing points for frother and collector in flotation cells.
(FA4015 and FA4017). The scavenger bank has air flow addition and froth thickness control in each flotation cell.

**Cleaner Cells**

The cleaner cells represent another cycle of the froth flotation process. They are used to clean the concentration from the rougher and scavenger. Each cleaner cells have air flow addition and froth thickness control. The cleaners floats valuable minerals with high concentration because, cleaners contain high froth thickness and low airflow.

- The Rougher Cleaner 1 contains three flotation cells (FA4320, FA4321 and FA4322). Each cell has 60 m$^3$ volume capacity. In this cleaning stage, the concentration of copper grade extracted from the scavenger is further purified for valuable minerals. Tailing form the rougher cleaner 1 is infeed to the scavenger cleaner 1. Concentration grade from the rougher cleaner 1 is sent to cleaner 2 for further processing.

- Scavenger Cleaner 1 (FA4323 to FA4326) contains four flotation cells. Each cell has 60 m$^3$ volume capacity. Tailing from the rougher cleaner 1 is the infeed to this cleaning stage. This cleaner extracted concentration grade of copper and sends it back to mixing tank. The back flow of this concentrate grade of copper is very low, it has ignorable effect on process. The tailing from this scavenger cleaner 1 is process waste.

- Cleaner 2 contains five flotation cells (FA4510 to FA4514). Each cell has 40 m$^3$ volume capacity. In this cleaning stage, extracted concentration grade of copper send to Cleaner 3 as in-feed and tailing grade is sent to rougher cleaner 1.

- Cleaner 3 contains four flotation cells (FA4530 to FA4533). Each cell has 40 m$^3$ volume capacity. This cleaning stage has infeed of concentration grade of copper from cleaner 2. Concentration from cleaner 3 is sent to cleaner 4 for further process and tailing grade is fed back to the in-feed of cleaner 2.

- Cleaner 4 contains three flotation cells (FA4550 to FA4552). Each cell has 40 m$^3$ volume capacity. Cleaner 4 is the final cleaning stage concentration grade of copper is sent further to the drying process while tailing grade is sent back to the process as infeet to Cleaner 2. The final concentration grade of flotation circuit at Aitik site is reached from 0.3% Cu to 23% - 35%Cu and recovery reaches 88% - 92%Cu.
Part II

System Identification
Chapter 3

System Identification

In this chapter we discuss the data-driven modelling approach adopted in this work. This is done by applying well known concepts of system identification theory. The aim of system identification is (a) to model systems dynamics that cannot easily be described using first principles, or (b) to ease the task of control design. The system identification iterative work flow is shown in Fig.3.1 and key steps are summarized below.

Steps:
1. We perform experimental tests to the process, for recording data of the input and output variables.
2. This data follows a data pre-processing, in order to remove outliers and high frequency disturbances, to reconstruct missing data, etc.
3. Parameters of a model in the form of difference equations is identified by least-square methods.
4. The selected model is used in validation tests, by verifying its performance under other experimental conditions, that is using other experimental data. In case of failure, one has to go back to one of previous steps.

3.1 Experiment Design

Performing experiments for flotation process is expensive and time consuming. To obtain informative experimental data for system identification, we need to consider:

- What are the manipulating inputs and measurable output variables?
Figure 3.1: System Identification Cycle
• How to design persistently exciting signals that reveal the system dynamics, without altering the cost efficiency of the process.

3.1.1 Manipulating inputs signals

The main objective of flotation process, consists on the design of control strategies that can influence the tread-off curve between concentration copper (Cu) grade and recovery. The flotation process is controlled by influencing the following four input variables,

• Frother is a chemical dose to the rougher and the scavenger banks. The rougher bank have two dosing points, and they are coupled together to be controlled by one input signal. Same arrangement is present for the scavenger frother dosing points.

• Collector is a chemical dose to the rougher and the scavenger banks. The rougher bank have two dosing points, and they are coupled together to be controlled by one input signal. Same arrangement is present for the scavenger collector dosing points.

• Air flow has addition points in each flotation cell for the rougher bank, scavenger bank, cleaner 2 bank, cleaner 3 bank, and cleaner 4 bank. Each addition point in the flotation cells are combined as one input signal for each flotation bank.

• Froth thickness is controlled for each flotation cell of the rougher bank, scavenger bank, cleaner 2 bank, cleaner 3 bank, and cleaner 4 bank. The froth thickness manipulating points in the flotation cells are combined as one input signal.

3.1.2 Measurable output signals

The measurable output signals are shown in Fig.2.1, Details about rougher, scavenger, cleaner 2, cleaner 3, and cleaner 4 are summarized below1:

• Concentration Copper Grade: from rougher bank, scavenger bank, cleaner 2 bank, cleaner 3 bank, and cleaner 4 bank.

• Tailing Copper Grade: from rougher bank, scavenger bank, cleaner 2 bank, cleaner 3 bank, and cleaner 4 bank.

1See also section 2.2.2
3.1.3 Output Signals

The flotation process outputs signals are measured on streams with an X-ray analyser. This device has the capacity to update the measurement each 7 to 8 minutes. During these time the last measured value is kept by a zero order hold (zoh). The X-ray analyser output signals names and descriptions are given in Table. 3.1.

<table>
<thead>
<tr>
<th>X-Ray Analysis Points</th>
<th>Object Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concentrate Cu Grade Rougher</td>
<td>Konc RaFlot</td>
<td>%</td>
</tr>
<tr>
<td>Tailing Cu grade from Rougher</td>
<td>Mp RaFlot</td>
<td>%</td>
</tr>
<tr>
<td>Concentrate Cu grade from Scavenger</td>
<td>KoncScav</td>
<td>%</td>
</tr>
<tr>
<td>Tailing Cu grade from Scavenger</td>
<td>Mp Scav</td>
<td>%</td>
</tr>
<tr>
<td>Concentrate Cu grade from Cleaner 2</td>
<td>KoncRep 2</td>
<td>%</td>
</tr>
<tr>
<td>Tailing Cu grade from Cleaner 2</td>
<td>MPRep 2</td>
<td>%</td>
</tr>
<tr>
<td>Concentrate Cu grade from Cleaner 3</td>
<td>KoncRep 3</td>
<td>%</td>
</tr>
<tr>
<td>Tailing Cu grade from Cleaner 3</td>
<td>MPRep 3</td>
<td>%</td>
</tr>
<tr>
<td>Concentrate Cu grade from Cleaner 4</td>
<td>KoncRep 4</td>
<td>%</td>
</tr>
<tr>
<td>Tailing Cu grade from Cleaner 4</td>
<td>MPRep 4</td>
<td>%</td>
</tr>
</tbody>
</table>

3.1.4 Design of Input Signals

Experimental input signals need to be designed with few basic properties, to extract informative dynamical response from the process. The main idea for performing qualitative experiment tests is to design input signals which:

- are rich enough to reveal the system dynamics,
- let the system perform around nominal operating points, and
- are chosen according to the bandwidth of the expected dynamics of the process.

The desired property of the input waveform is defined as a crest factor. A good waveform is one that has a small crest factor. Theoretically, the lowest achievable crest factor is '1', which is achievable with binary signals [2]. Crest factor or peak-to-power ratio, can be calculated from
the peak amplitude of the waveform divided by the root mean square (RMS) value of waveform.

$$CrestFactor = \frac{|U|_{\text{peak}}}{U_{\text{RMS}}}$$

(3.1)

Input signals have limitation only for the amplitude that should remain between certain maximum and minimum values, see Table 3.2. These values are chosen to remain within nominal operating points and cost efficiency of the process. We can generate input signals with desired waveform as, Random Binary signals 'rbs' or Random Gaussian signals 'rgs' with Matlab$^2$. This is done as follows:

$$u = \text{idinput}(N,\text{type},\text{band},\text{levels})$$

where

$N = [L \ nu]$ gives an input with nu channels each of length L.

$\text{type} = \text{'rgs'}$: Gives a random, Gaussian signal.

$\text{type} = \text{'rbs'}$: Gives a random, binary signal

$^2$ See “Way to Obtain identification data”, page 2-4 under reference [3]
Figure 3.2: Example of generated input signals.

\[ \text{band} = [\text{wlow}, \text{whigh}] \] : This argument defines low and high frequency bounds for passband levels = [\text{minu}, \text{maxu}] : This argument defines the input level, \([-1 1]\) is the default value

The change in input levels should not be faster than the process dynamics. The flotation process have quite slow dynamics and therefore, the switching interval is chosen to be between 20-40 minutes. One example of input signals, is shown in Fig.3.2.

### 3.2 Data Preprocessing

Data preprocessing involves data polishing, handling missing data, removing outliers and offsets, etc. If it is necessary, filtering and re-sampling should also be performed.

#### 3.2.1 Data Smoothing

As mentioned earlier, the measured data from the X-ray analyser up-dates within 7 to 8 minutes. Measured output signal represent discrete data with zero order hold. A typical measured data from the X-ray analyser is shown in Fig.3.4(a).
To create a smooth data set, we apply curve fitting, which involves either interpolation (to obtain the exact fit to the data), and smoothing functions are used to create a data set that is at least twice continuous differentiable. This allows to approximate missing dynamics and reduce sensitivity to the measured values from the X-ray analyser. The work flow towards data smoothing is shown in Fig.3.3 and each step results are shown in fig.3.4, 3.5.

In Matlab this can be accomplished in the following form:

\[ f_o = \text{fitoptions('method','SmoothingSpline','SmoothingParam',factor)} \]

Where

\[ \text{fitoptions} = \text{Create or modify fit options structure}. \]
\[ \text{method} = \text{curve fitting method used is 'SmoothingSpline'}. \]
SmoothingParam = smoothing factor

\[ ft = \text{fittype}('\text{smoothingspline}') \]

Where

fittype = Fit-type object for curve and surface fitting.

method = curve fitting method used is 'SmoothingSpline'.

\[ cf = \text{fit}(\text{time}, \text{data}, ft, fo); \]

Where

fit = Fit model to data.

cf = Fitted Object Final smooth fitted data

3.2.2 Data De-trending

Data de-trending is done to remove offsets, which appear due to non-linear effects that are ignored. This is done in order to identify a linear approximation. The mean of the output value is considered as the nominal operating condition. In Matlab, this can be done by use of:
3.2.3 Merging Experiments

Merging consists on the tasks of treating various experiments at once. We can merge data sets that, could be different parts of one experiment, or could be series of different experiments. We can accomplish this by using of the command “merge” in Matlab, i.e.

\[ \text{Merge} \_\text{data}=\text{merge}(\text{data1, data2,...}) \]

where

\[ \text{constant} = 0, \text{removes the mean value.} \]

In this case studies, model are estimated with merged experimental and regular process data sets around different nominal operating values. Merged data sets with different operating nominal values give understanding about process behaviour at different operating nominal values to estimated model.
3.3 Model Structure and Estimation

Due to complexity of dynamics, we divide the process into small sub-processes. In this work we consider the case of multiple inputs and single output models (MISO) see fig.3.6 in which dynamics of the concentration copper grade corresponds to one output and the tailing copper grade to other output. However, the input is similar for both dynamics.

Selecting best model structure for estimation is a difficult step in modelling due to availability of wide range of model structures. MISO system can be estimate with ARX, ARMAX, OE and state-space(ss) model structures. During this case study different model structured are evaluated, ARX and state-space(ss) model structures shown good results and their result are compared.

ARX model with high and low orders are estimated, high order models shown good estimation for few experimental data. As compare to ARX model with state-space model low order have better estimation (Fig.C.1) and validation (Fig.C.2) results. State-space models (Eq.3.2) are estimated with predictive error method for black box MISO models. For example, scavenger tailing model matrices are given as following:

\[ x(t + Ts) = Ax(t) + Bu(t) + Ke(t) \]
\[ y(t) = Cx(t) + Du(t) + e(t) \]  \hspace{1cm} (3.2)

Where:

A is a matrix of order 2x2;

\[
A = \begin{bmatrix}
x_1 & x_2 \\
x_1 & 0.93013 & 0.005168 \\
x_2 & 0.02326 & 0.99622 \\
\end{bmatrix}
\]

B is a matrix of order 2x5;

\[
B = \begin{bmatrix}
Rougher & Tailing & Frother & Collector & Air & FrothThick \\
0.23424 & -1.902e-005 & 1.1728e-005 & 0.00026839 & -4.1798e-006 \\
-0.037411 & -0.00071157 & 0.00016456 & 0.00053641 & -3.3846e-005 \\
\end{bmatrix}
\]

C is row matrix of order 1x2;

\[
C = \begin{bmatrix}
x_1 \quad x_2 \\
Rougher & Tailing \\
0.29075 & -0.036658 \\
\end{bmatrix}
\]

\(^3\)See section 2.2.2 for more details

\(^4\)Concentration and tailing grade have different dynamical behaviours, that is the reason to divide each sub-processes into two different MISO systems.
D is row matrix of order 1x5:

\[ D = \begin{bmatrix}
RougherTailing & Frother & Collector & Air & FrothThickn \\
0 & 0 & 0 & 0 & 0
\end{bmatrix} \]

K is row column of order 2x1:

\[ K = \begin{bmatrix}
RougherTailing \\
x_1 & -286.48 \\
x_2 & -2345.6
\end{bmatrix} \]

### 3.3.1 Model Estimation Example

We present an example of modelling the scavenger tailing grade. To this end, we consider a model with 5 inputs and 1 output, as following:

**Inputs**: In-feed of slurry Cu (%), Frother (g/t) chemical dosing, Collector (g/t) chemical dosing, Air (m³/min) flow addition, and Froth thickness (cm).

**Output**: Tailing grade from Scavenger (%).

**Merge data sets**: We consider four data sets: experiment 2 (see fig.A.1), experiment 3 (see fig.A.3), process data 2 (see fig.B.3), and process data 3 (see fig.B.5).

Matlab command:

\[ \text{Merged} \ ST = \text{merge}(ST_{Exp2}, ST_{Exp3}, ST_{var2}, ST_{var3}); \]

**Model Estimation**: We consider a PEM model of second order as follows:

Matlab command:

\[ \text{Est} \ ST \ Model = \text{pem}(\text{Merged} \ ST, 2, 'nk', nk, 'Focus', 'Sim'); \]

**Model Simulation**: the output responses of the estimated model, are shown in Fig.3.7. This is done with the help of the “compare” command from Matlab, i.e.

\[ \text{compare} (\text{Merged} \ ST, \text{Est} \ ST \ Model); \]

### 3.4 Model Validation

We validate the estimated model with an independent data set. The validation data set is pre-processed in the same way as the estimated data set.
Figure 3.6: Sub-process Models: Multi Inputs Single Output
Figure 3.7: Measured and Simulated model output of Scavenger Tailing Model: (a) Experiment 2 (b) Experiment 3 (c) Process Data 2 (d) Process Data 3
Figure 3.8: Scavenger Tailing Model Validation

3.4.1 Model Simulation

The model simulation of the output response of the estimated model, over validation data set, is shown in fig.3.8. The validation data set is shown in Fig.B.1(e to h) and fig.B.2(c). The estimated model validate the data set with 60.65% fit.

3.5 Step Responses

Step responses of each input is shown in Fig.3.9.
Figure 3.9: Scavenger Tailing Step response
Chapter 4

Results and Discussion

4.1 Results

4.1.1 Rougher Bank

The rougher bank is divided into two multi input and single output (MISO) models. The used input signals are shown in Fig.4.1.

Rougher Concentration grade model

Black box (state-space) linearly stable model of 2nd order, is used to approximate process dynamic. Four merged data sets are considered for model estimation. Simulated output responses of estimated model are shown in Fig.4.2. For validation of estimated model, two independent data sets are used and simulated output responses are shown in Fig.4.3.

Figure 4.1: Rougher or Scavenger Bank can be described as four inputs and two single outputs
Figure 4.2: Output measured and simulated of the rougher concentration grade model

Figure 4.3: Validation of the rougher concentration grade model
Figure 4.4: Rougher Concentration Step response

**Rougher Tailing grade model**

Rougher tailing grade model dynamic is approximated with 3rd order black-box (state-space) linear model. Estimated model simulation output responses are shown in Fig.4.5. Estimated model is validated with process regular data set, simulated output response is shown in Fig.4.6. Rougher tailing grade model quality can be improve for validation over independent data set, suggestion is to re-perform experiments to improve the quality (for more detail see section 4.2.2)

**4.1.2 Scavenger Bank**

The scavenger bank is also divide into two MISO system models as rougher bank.

**4.1.3 Scavenger Concentration grade model**

Scavenger concentration model is estimated with 2nd order black-box (state-space) linearly stable model. The model estimated and validation fit are good enough to approximate process dynamics. Model estimation and validation simulation responses are shown in Fig.4.8 and Fig.4.9 respectively.
4.1.4 Scavenger Tailing grade model

The scavenger tailing grade model is the best estimated model, it is a 2nd order black box (state-space) linearly stable model. The estimation and validation fit prove that the dynamics of system is well captured with estimated model. Model simulation output response are shown in Fig.3.7. Model is validated over independent data set and its simulated output response is shown in Fig.3.8.

4.2 Discussion

4.2.1 Process Output Measurement (X-ray analyser)

The copper concentration and tailing grade are measured by X-ray analyser. Currently this device has capacity to update measurements after 7 to 8 minutes. In the future, this device will be able to measure copper grade within 3 to 4 minutes, it will reduce the uncertainty and outliers. This improvement will help to capture better approximation of process dynamics.
4.2.2 Experiment redesigning

Estimated models are not good enough to implement model predictive controller (MPC). Further work is needed to implement MPC, which requires better quality on the estimated models. However, some suggestions are given below to perform experiments for data collection:

- Perform two experiments, one for model estimation and other for model validation. For linear model estimation, design both experiments around same nominal operating values, with different input waveform. A linear model can only validate a process data around same nominal value, which are used to estimate the model.

- Bandwidth of input signals for experiments (estimation and validation) need to be same. Therefore, input signals can excite process dynamics in same manner.

4.3 Future work

The long term goal is to provide linear process model, which can be use to implement a model predictive controller (study will be continued after this thesis work). The expectation from model-based controller that will increase the performance of process recovery and small increment in process recovery will have significant improvement to make process more cost efficient.

Figure 4.6: Validation of the rougher tailing grade model
Figure 4.7: Rougher Tailing Step response

Figure 4.8: Output measured and simulated of the scavenger concentration grade model
Figure 4.9: Validation of the scavenger concentration grade model

Figure 4.10: Scavenger Concentration Step response
Appendix A

Experiments

A.1 Experiment 1

Experiment Start Date and Time: 2011/05/26 22:10:00.
Experiment End Date and Time: 2011/05/27 06:20:00.
Duration of Experiment: 8 hours, 10 minutes.

A.1.1 Input Signals

• Frother to the rougher bank, • Collector to the rougher bank, • Air flow to the rougher bank,
• Froth thickness in the rougher bank, • Frother to the scavenger bank, • Collector to the scavenger bank, • Air flow to the scavenger bank, • Froth thickness in the scavenger bank, See Fig.A.1. • In feed to the rougher bank.

A.1.2 Output signals

• Concentration grade from the rougher bank, • Tailing from the rougher bank, • Concentration grade from the scavenger bank, • Tailing from the scavenger bank, See Fig.A.2.
Figure A.1: Inputs Experiment 1

Figure A.2: Outputs Experiment 1
A.2 Experiment 2

Experiment Start Date and Time: 2011/06/30 22:00:00.
Experiment End Date and Time: 2011/07/01 06:00:00.
Duration of Experiment: 8 hours.

A.2.1 Input Signals

- Frother to the rougher bank, • Collector to the rougher bank, • Air flow to the rougher bank,
- Froth thickness in the rougher bank, • Frother to the scavenger bank, • Collector to the scavenger bank, • Air flow to the scavenger bank, • Froth thickness in the scavenger bank, See Fig.A.3. • In feed to the rougher bank.

A.2.2 Output signals

- Concentration grade from the rougher bank, • Tailing from the rougher bank, • Concentration grade from the scavenger bank, • Tailing from the scavenger bank, See Figure: A.4.
A.3 Experiment 3

Experiment Start Date and Time: 2011/06/28 22:00:00.
Experiment End Date and Time: 2011/06/29 16:00:00.
Duration of Experiment: 18 hours.

A.3.1 Input Signals

• Frother to the rougher bank, • Collector to the rougher bank, • Air flow to the rougher bank,
• Froth thickness in the rougher bank, • Frother to the scavenger bank, • Collector to the scavenger bank, • Air flow to the scavenger bank, • Froth thickness in the scavenger bank, See Fig.A.5. • In feed to the rougher bank.

A.3.2 Output signals

• Concentration grade from the rougher bank, • Tailing from the rougher bank, • Concentration grade from the scavenger bank, • Tailing from the scavenger bank, See Fig.A.6.
Figure A.5: Inputs Experiment 3

Figure A.6: Outputs Experiment 3
Appendix B

Regular Process Data

B.1 Process Data 1

Process Data Start Date and Time: Line 1 2011/05/12 06:00:00.
Process Data End Date and Time: Line 1 2011/05/14 06:00:00.

B.1.1 Input Signals

- Frother to the rougher bank, • Collector to the rougher bank, • Air flow to the rougher bank,
- Froth thickness in the rougher bank, • Frother to the scavenger bank, • Collector to the scavenger bank, • Air flow to the scavenger bank, • Froth thickness in the scavenger bank, See Fig.B.1. • In feed to the rougher bank.

B.1.2 Output signals

- Concentration grade from the rougher bank, • Tailing from the rougher bank, • Concentration grade from the scavenger bank, • Tailing from the scavenger bank, See Figure: B.2.

B.2 Process Data 2

Process Data Start Date and Time: Line 1 2011/05/14 06:00:00.
Process Data End Date and Time: Line 1 2011/05/16 06:00:00.
Figure B.1: Inputs Process Data 1: (a) Frother to Rougher (b) Collector to Rougher (c) Air flow to Rougher (d) Froth thickness in Rougher (e) Frother to Scavenger (f) Collector to Scavenger (g) Air flow to Scavenger (h) Froth thickness in Scavenger

Figure B.2: Process Data 1 Outputs: (a) Concentration grade from Rougher (b) Tailing from Rougher (c) Tailing from Scavenger (d) Concentration grade from Scavenger
B.2.1 Input Signals

- Frother to the rougher bank,
- Collector to the rougher bank,
- Air flow to the rougher bank,
- Froth thickness in the rougher bank,
- Frother to the scavenger bank,
- Collector to the scavenger bank,
- Air flow to the scavenger bank,
- Froth thickness in the scavenger bank,
- In feed to the rougher bank.

B.2.2 Output signals

- Concentration grade from the rougher bank,
- Tailing from the rougher bank,
- Concentration grade from the scavenger bank,
- Tailing from the scavenger bank.

B.3 Process Data 3

Process Data Start Date and Time: Line 1 2011/05/16 06:00:00.
Process Data End Date and Time: Line 1 2011/05/18 06:00:00.
Figure B.4: Process Data 2 Outputs: (a) Concentration grade from Rougher (b) Tailing from Rougher (c) Tailing from Scavenger (d) Concentration grade from Scavenger

### B.3.1 Input Signals
- Frother to the rougher bank,
- Collector to the rougher bank,
- Air flow to the rougher bank,
- Froth thickness in the rougher bank,
- Frother to the scavenger bank,
- Collector to the scavenger bank,
- Air flow to the scavenger bank,
- Froth thickness in the scavenger bank, See Fig.B.5,
- In feed to the rougher bank.

### B.3.2 Output signals
- Concentration grade from the rougher bank,
- Tailing from the rougher bank,
- Concentration grade from the scavenger bank,
- Tailing from the scavenger bank, See Fig.B.6

### B.4 Process Data 4

Process Data Start Date and Time: Line 2 2011/05/16 06:00:00.
Process Data End Date and Time: Line 2 2011/05/18 06:00:00.
Figure B.5: Inputs Process Data 3: (a) Frother to Rougher (b) Collector to Rougher (c) Air flow to Rougher (d) Froth thickness in Rougher (e) Frother to Scavenger (f) Collector to Scavenger (g) Air flow to Scavenger (h) Froth thickness in Scavenger

Figure B.6: Process Data 3 Outputs: (a) Concentration grade from Rougher (b) Tailing from Rougher (c) Tailing from Scavenger (d) Concentration grade from Scavenger
Figure B.7: Inputs Process Data 4: (a) Frother to Rougher (b) Collector to Rougher (c) Air flow to Rougher (d) Froth thickness in Rougher (e) Frother to Scavenger (f) Collector to Scavenger (g) Air flow to Scavenger (h) Froth thickness in Scavenger

B.4.1 Input Signals

- Frother to the rougher bank, • Collector to the rougher bank, • Air flow to the rougher bank,
- Froth thickness in the rougher bank, • Frother to the scavenger bank, • Collector to the scavenger bank, • Air flow to the scavenger bank, • Froth thickness in the scavenger bank, See Fig.B.7. • In feed to the rougher bank.

B.4.2 Output signals

- Concentration grade from the rougher bank, • Tailing from the rougher bank, • Concentration grade from the scavenger bank, • Tailing from the scavenger bank, See Fig.B.8.
Figure B.8: Process Data 3 Outputs: (a) Concentration grade from Rougher (b) Tailing from Rougher (c) Tailing from Scavenger (d) Concentration grade from Scavenger
Appendix C

Model structures Comparison

C.1 Scavenger Tailing Model

Figure C.1: Output measured and simulated of the Scavenger tailing model with 2nd order ARX and State-space(ss) model
Figure C.2: Validation of the Scavenger tailing model with 2nd order ARX and State-space(ss) model
Appendix D

MATLAB CODE

All files used in this case study are given in this chapter.

D.1 Data Collection

This file collect the data from excel files, all files used in this case study with names are mentioned in comments.

```matlab
[u_Exp2 u_text_Exp2 y_Exp2 y_text_Exp2 d_Exp2 d_text_Exp2 Time_Exp2]...
```
=Data_Collection_Rearrange; % This Function will read selected excel file and return input, output variables

save('Dynamic_Flotation_Exp2','u_Exp2','u_text_Exp2', 'y_Exp2', ...
   'y_text_Exp2', 'd_Exp2', 'd_text_Exp2', 'Time_Exp2')

%%

%-----------------Experiment 3 Data-------------------------------
%Data File Name: DATA_L1_20110629a.xlsx (It contain 4 data sheets)
%Input File Name: Input_Signals.xlsx
%Output File Name: Output_Signals.xlsx
%Distrubance File Name: disturbance_Signals
%Data start row: Start=187
%Data End row: End=1207
%---------------------------------------------------------------

[u_Exp3 u_text_Exp3 y_Exp3 y_text_Exp3 d_Exp3 d_text_Exp3 Time_Exp3]...
 =Data_Collection_Rearrange;% This Function will read selected excel file and return input, output variables

save('Dynamic_Flotation_Exp3','u_Exp3', 'u_text_Exp3', 'y_Exp3', ...
   'y_text_Exp3', 'd_Exp3', 'd_text_Exp3', 'Time_Exp3')

%%

%-----------------Regular process Data 1-------------------------------
%Data File Name: DATA_L1_20110512_Verification.xlsx (It contain 4 data sheets)
%Input File Name: Input_Signals.xlsx
%Output File Name: Output_Signals.xlsx
%Distrubance File Name: disturbance_Signals
%Data start row: Start=7
%Data End row: End= 2800
%---------------------------------------------------------------
[u\_var1 u\_text\_var1 y\_var1 y\_text\_var1 d\_var1 d\_text\_var1 Time\_var1]...
   =Data\_Collection\_Rearrange; \% This Function will read selected excel
   \% file and return input, output variables

save('Dynamic\_Flotation\_var1','u\_var1', 'u\_text\_var1', 'y\_var1', ...
   'y\_text\_var1', 'd\_var1', 'd\_text\_var1', 'Time\_var1')

\%}
\%-----------------Regular process Data-------------------------------
\%Data File Name: DATA\_L1\_20110514\_Verification.xlsx(It contain 4 data sheets)
\%Input File Name: Input\_Signals.xlsx
\%Output File Name: Output\_Signals.xlsx
\%Distrubance File Name: disturbance\_Signals
\%Data start row: Start=7
\%Data End row: End= 2800
\%---------------------------------------------------------------

[u\_var2 u\_text\_var2 y\_var2 y\_text\_var2 d\_var2 d\_text\_var2 Time\_var2]...
   =Data\_Collection\_Rearrange; \% This Function will read selected excel
   \% file and return input, output variables

save('Dynamic\_Flotation\_var2','u\_var2', 'u\_text\_var2', 'y\_var2', ...
   'y\_text\_var2', 'd\_var2', 'd\_text\_var2', 'Time\_var2')

\%}
\%-----------------Regular process Data-------------------------------
\%Data File Name: DATA\_L1\_20110516\_Verification.xlsx(It contain 4 data sheets)
\%Input File Name: Input\_Signals.xlsx
\%Output File Name: Output\_Signals.xlsx
\%Distrubance File Name: disturbance\_Signals
\%Data start row: Start=7
\%Data End row: End= 2800
\%---------------------------------------------------------------
D.1.1 Data Collection Rearrange.m

This is a sub function for collecting data and arranging as input, output and disturbances. It return all objects names and values.

```matlab
function [u_Exp u_text_Exp y_Exp y_text_Exp d_Exp d_text_Exp Time_Exp]=...
    Data_Collection_Rearrange

%*******************************************************************************
%                Data Collection
%*******************************************************************************

[Data1 Data2 Data3 Data4 textdata1 textdata2 textdata3]...
    =Read_Data_excel_file; %Calling Read_Data_excel_file

%Read Excel File, which contain Input Objects names
text_Input=Read_text_excel_file;

%Read Excel File, which contain Output Objects names
text_Output=Read_text_excel_file;

%Read Excel File, which contain Disturbances Objects names
text_Dist=Read_text_excel_file;

Start= input(’Enter starting row for data=’); %Starting row for data= 7
```
%Ending row depend upon length of excel file
End = input('Enter Ending row for data=');

[u_Exp u_text_Exp y_Exp y_text_Exp d_Exp d_text_Exp Time_Exp]...
= Data_Rearrange(Data1, Data2, Data3, Data4, textdata1, textdata2...
 ,textdata3, text_Inf, text_Out, text_Dist, Start, End);
end

D.1.2  Read Data excel file.m

function [Data1 Data2 Data3 Data4 textdata1 textdata2 textdata3]=Read_Data_excel_file

[FileName, PathName] = uigetfile('*.xlsx', 'select the .xlsx-file for identification');

% Read data from sheet 1 using num = xlsread(filename, sheet, range)
[Data1, textdata1] = xlsread(FileName, 1);

% Read data from sheet 1 using num = xlsread(filename, sheet, range)
[Data2, textdata2] = xlsread(FileName, 2);

% Read data from sheet 1 using num = xlsread(filename, sheet, range)
[Data3, textdata3] = xlsread(FileName, 3);

% Read data from sheet 1 using num = xlsread(filename, sheet, range)
[Data4, textdata4] = xlsread(FileName, 4);
end
D.2  Data Processing

This section contain all files used for data processing. Curve fitting, smoothing, removing mean values and creating data objects.

D.2.1  Sys Data processing.m

```matlab
% close all;clear all;clc;
close all
global para
global h
para = 0.000004; %smoothing parameter
h=60; %sample time

load Dynamic_Flotation_Exp1
[RC_Exp1 RT_Exp1 SC_Exp1 ST_Exp1]=...
    Data_Processing(u_Exp1,y_Exp1,y_text_Exp1,d_Exp1,d_text_Exp1,Time_Exp1);

%-----------------Experiment 2 Data-------------------------------
%Data File Name: DATA_L1_20110701a.xlsx (It contain 4 data sheets)
%Input File Name: Input_Signals.xlsx
%Output File Name: Output_Signals.xlsx
%Distrubance File Name: disturbance_Signals
%Data start row: Start=1717
%Data End row: End=2066
%---------------------------------------------------------------

load Dynamic_Flotation_Exp2
[RC_Exp2 RT_Exp2 SC_Exp2 ST_Exp2]=...
    Data_Processing(u_Exp2(1:2500,:),y_Exp2(1:2500,:),...
    y_text_Exp2,d_Exp2(1:2500,:),d_text_Exp2,Time_Exp2(1:2500,:));

%-----------------Experiment 3 Data-------------------------------
%Data File Name: DATA_L1_20110629a.xlsx (It contain 4 data sheets)
```
28 \%Input File Name: Input_Signals.xlsx
29 \%Output File Name: Output_Signals.xlsx
30 \%Distrubance File Name: disturbance_Signals
31 \%Data start row: Start=187
32 \%Data End row: End=1207
33 %---------------------------------------------------------------
34 close all
35 load Dynamic_Flotation_Exp3
36 [RC_Exp3 RT_Exp3 SC_Exp3 ST_Exp3]=...
37 Data_Processing(u_Exp3,y_Exp3,y_text_Exp3,...
38 d_Exp3,d_text_Exp3,Time_Exp3);
39 %-----------------Regular process Data 1-------------------------------
40 %Data File Name: DATA_L1_20110512_Verification.xlsx (It contain 4 data sheets)
41 %Input File Name: Input_Signals.xlsx
42 %Output File Name: Output_Signals.xlsx
43 %Distrubance File Name: disturbance_Signals
44 %Data start row: Start=7
45 %Data End row: End= 2800
46 %---------------------------------------------------------------
47 close all
48 load Dynamic_Flotation_var1
49 [RC_var1 RT_var1 SC_var1 ST_var1]=...
50 Data_Processing(u_var1,y_var1,y_text_var1,...
51 d_var1,d_text_var1,Time_var1);
52 %-----------------Regular process Data-------------------------------
53 %Data File Name: DATA_L1_20110514_Verification.xlsx (It contain 4 data sheets)
54 %Input File Name: Input_Signals.xlsx
55 %Output File Name: Output_Signals.xlsx
56 %Distrubance File Name: disturbance_Signals
close all
load Dynamic_Flotation_var2
[RC_var2 RT_var2 SC_var2 ST_var2]=...
    Data_Processing(u_var2,y_var2,y_text_var2,...
    d_var2,d_text_var2,Time_var2);

%-----------------Regular process Data-------------------------------
%Data File Name: DATA_L1_20110516_Verification.xlsx (It contain 4 data sheets)
%Input File Name: Input_Signals.xlsx
%Output File Name: Output_Signals.xlsx
%Distrubance File Name: disturbance_Signals
%Data start row: Start=7
%Data End row: End= 2800
%---------------------------------------------------------------

close all
load Dynamic_Flotation_var3
[RC_var3 RT_var3 SC_var3 ST_var3]=...
    Data_Processing(u_var3,y_var3,y_text_var3,...
    d_var3,d_text_var3,Time_var3);

close all
load Dynamics_Exp_Ini1
[RC_Ini1 RT_Ini1 SC_Ini1 ST_Ini1]=...
    Data_Processing(u,y,y_text1,d,d_text3,Time);

close all
load Dynamics_Exp_Ini2
[RC_Ini2 RT_Ini2 SC_Ini2 ST_Ini2]=...
Data_Processing(u_out,y_out,y_text,d_out,d_text,Time);

close all
load Dynamic_Flotation_var7
[RC_var4 RT_var4 SC_var4 ST_var4]=...
    Data_Processing(u_var7,y_var7,y_text_var7,d_var7,d_text_var7,Time_var7);

% Saving Mat-Files
save('Rougher_concentration_grade_iddata',...
    'RC_Exp1','RC_Exp2','RC_Exp3','RC_var1',...
    'RC_var2','RC_var3','RC_var4','RC_Ini1','RC_Ini2')

save('Rougher_tailing_grade_iddata'...
    'RT_Exp1','RT_Exp2','RT_Exp3','RT_var1',...
    'RT_var2','RT_var3','RT_var4','RT_Ini1','RT_Ini2')

save('Scavenger_concentration_grade_iddata',...
    'SC_Exp1','SC_Exp2','SC_Exp3','SC_var1','SC_var2',...
    'SC_var3','SC_var4','SC_Ini1','SC_Ini2')

save('Scavenger_tailing_grade_iddata',...
    'ST_Exp1','ST_Exp2','ST_Exp3','ST_var1','ST_var2',...
    'ST_var3','ST_var4','ST_Ini1','ST_Ini2')

D.2.2 Data Processing

function [me_RC_Exp me_RT_Exp me_SC_Exp me_ST_Exp] = Data_Processing(u,y,y_text,y_text,...
global h

%**************************************************************************
%----------------------------Data Pre-processing-------------------------------
%**************************************************************************
%***********************************************************************
[su_Exp, sy_Exp, sd_Exp] = Data_Polish(u_Exp, y_Exp, y_text_Exp, d_Exp, d_text_Exp, Time_Exp);

%***********************************************************************

u_\_R = [sd_Exp(:,2) su_Exp(:,3) su_Exp(:,4) su_Exp(:,5) su_Exp(:,6)];

u_\_S = [sy_Exp(:,2) su_Exp(:,7) su_Exp(:,8) su_Exp(:,9) su_Exp(:,10)];

y_\_RC = sy_Exp(:,1);  \% Concentrate Grade from Rougher

y_\_RT = sy_Exp(:,2);  \% Tailing from Rougher

y_\_ST = sy_Exp(:,3);  \% Tailing from Scavenger

y_\_SC = sy_Exp(:,4);  \% Concentrate Grade from Scavenger

RC_\_Exp = iddata(y_\_RC, u_\_R, h, 'Name','Rouger\_C','InputName',...
                 {'Infeed to Rougher', 'Frother Rougher', 'Collector Rougher', 'Air Rougher'...
                  , 'Froth Thickness Rougher'}, 'InputUnit', {'...'
                  , 'g/ton', 'g/ton', 'm3/min', 'cm'}, 'OutputName', {'...'.Concentrate Grade from Rougher'},'OutputUnit', {''}, 'Tstart', 0, 'TimeUnit', 's');

RT_\_Exp = iddata(y_\_RT, u_\_R, h, 'Name','Rouger\_T','InputName'...
                 , {'Infeed to Rougher', 'Frother Rougher', 'Collector Rougher', 'Air Rougher'...
                  , 'Froth Thickness Rougher'}, 'InputUnit', {'%','g/ton'...
                  , 'g/ton', 'm3/min', 'cm'}, 'OutputName'...
                  , {'Tailing from Rougher'},'OutputUnit', {'%'}, 'Tstart', 0, 'TimeUnit', 's');

SC_\_Exp = iddata(y_\_SC, u_\_S, h, 'Name','Scavenger\_C','InputName'...
                 , {'Tailing from Rougher', 'Frother Scavenger', 'Collector Scavenger', 'Air Scavenger'...
                  , 'Froth Thickness Scavenger'}, 'InputUnit', {'%','g/ton'...
                  , 'g/ton', 'm3/min', 'cm'}...}
                 , 'OutputName', {...
                  'Concentrate Grade from Scavenger'},'OutputUnit', {'%'}, 'Tstart', 0, 'TimeUnit', 's');
D.2.3 Data Polish.m

function [su_Exp,sy_Exp,sd_Exp]=Data_Polish(u_Exp,y_Exp,y_text_Exp,d_Exp, ... d_text_Exp,Time_Exp)
global para

%************************************************************
% Data Pre-processing
%************************************************************

oy_Exp=y_Exp;
od_Exp=d_Exp;
para=0.000004;
sy_Exp= ReShaping(oy_Exp,y_text_Exp,Time_Exp,para);
sd_Exp= ReShaping(od_Exp,d_text_Exp,Time_Exp,para);
u=u_Exp;

[Lu,c]=size(u(:,1));
u1=u(:,1);u2=u(:,2);u3=u(:,3);u4=u(:,4);
u5=C_Fu(Time_Exp,u(:,5),Lu);u6=C_Fu(Time_Exp,u(:,6),Lu);
u7=u(:,7);u8=u(:,8);u9=C_Fu(Time_Exp,u(:,9),Lu);u10=C_Fu(Time_Exp,u(:,10),Lu);
su_Exp=[u1 u2 u3 u4 u5 u6 u7 u8 u9 u10];
end

D.2.4 Reshaping.m

function [y_out]= ReShaping(y,y_text,Time,parameter)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Data Interpolation and Curve Fitting
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
[Ly,c]=size(y);
for i=1:c
    dout=Data_points_interpolation(y(:,i));
    dy=dout(:,1);
    dt=dout(:,2);
    figure,plot(dt,dy,'-r','LineWidth',2),hold on,plot(Time,y(:,i),'LineWidth',2)
    legend('Data Point Interpolation')
    title(y_text(i))
    Dy_S(:,i)=C_Fy(dt,dy,Ly,parameter);
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Curve Fitting
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
[row,col]=size(y);
for i=1:col
    y_S(:,i)=C_Fy(Time,y(:,i),Ly,parameter);
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Final Average data
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for j=1:col
    for i=1:row
        Y(i,j)= (y_S(i,j)+Dy_S(i,j))/2;
    end
end
D.2.5  C Fy

function out=C_Fy(time,data,L,factor)

time = time(:);
data = data(:);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Data smoothing
%SMOOTH(Y,SPAN,METHOD) smooths data Y with specified METHOD
%    'moving' - Moving average (default)
%    'lowess' - Lowess (linear fit)
%    'loess' - Loess (quadratic fit)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% sm_.y2 = smooth(time,data,0.25,'loess',0);
% sm_.y3 = smooth(time,data,5,'moving',0);
% sm_.y4 = smooth(time,data,3,'moving',0);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% --- Create fit "fit 2"

fo_ = fitoptions('method','SmoothingSpline','SmoothingParam',factor);
ok_ = isfinite(time) & isfinite(data); %ISFINITE True for finite elements
if ~all( ok_ )  % ALL True if all elements of a vector are nonzero.
    warning( 'GenerateMFile:IgnoringNansAndInfs', ...
D.3 Rougher Model Estimation

This file is for rougher concentration and tailing model estimation. Also plot poles and zeros of the model.

%--------------------------------------------------------------------------
% Rougher Concentration Grade Modelling
%--------------------------------------------------------------------------
clear all
load Rougher_concentration_grade_iddata

"Ignoring NaNs and Infs in data"; end

ft_ = fittype('smoothingspline'); % FITTYPE Fit-type object for curve and surface fitting

% CFLIBHELP check it for more detail

% Fit this model using new data

cf_ = fit(time(ok_), data(ok_), ft_, fo_);

% done Fitting this model using new data now saving for output,

for i=1:L
    temp(i) = cf_(i);
end
out = temp';
% Estimation Rougher Concentration Model with Merged Multiexperiments Data
%---------------------------------------------------------------

Est_RC=merge(RC_Exp2(1700:2500),RC_Exp3,RC_var1,RC_var2,RC_var3); %Merged iddata Object
Est_RC_Model = pem(Est_RC,2,'Focus','Sim'); %Estimating Rougher
compare(Est_RC,Est_RC_Model) %Estimated Model com

%---------------------------------------------------------------------
% Validation Rougher Concentration Model with Merged Multiexperiments Data
%---------------------------------------------------------------------

load RC_Ini1 %Data set for validation
load RC_Ini2 %Data set for validation
figure
compare(Val_RC_Ini1,Est_RC_Model) %Model Validation
figure
compare(Val_RC_Ini2,Est_RC_Model) %Model Validation

%---------------------------------------------------------------------
%------------------------- Poles and Zeros Plot --------------------------
%---------------------------------------------------------------------

figure
pzmap(Est_RC_Model) %Poles and Zeros Plots

%%
%---------------------------------------------------------------------
% Rougher Tailing Grade Modelling
%---------------------------------------------------------------------

load Rougher_tailing_grade_iddata
Estimation Rougher Tailing Model with Merged Multiexperiments Data

Est_RT=merge(RT_Exp2,RT_Exp3,RT_var2,RT_var3); %Merged iddata Objects for
Est_RT_Model = pem(Est_RT,3,'Focus','Sim','MaxIter',15); %Estimating Rougher Tailin
compare(Est_RT,Est_RT_Model,'--r') %Estimated Model compare

Validation Rougher Tailing Model

figure
compare(RT_var4,Est_RT_Model,'--r') %Model Validation

Poles and Zeros Plot

figure
pzmap(Est_RT_Model) %Poles and Zeros Plot

figure
compare(RT_Exp2,Est_RT_Model,'--r')
figure
compare(RT_Exp3,Est_RT_Model,'--r')
figure
compare(RT_var2,Est_RT_Model,'--r')
figure
compare(RT_var3,Est_RT_Model,'--r')
D.4 Scavenger Model Estimation

Scavenger concentration and tailing model estimation. Also poles and zeros plots.

```matlab
% Scavenger Concentration Grade Modelling

clear all
load Scavenger_concentration_grade_iddata

Est_SC=merge(SC_Exp1,SC_Exp3,SC_var1(1:2600,:,:),SC_var2(1:2640,:,:)); % Merged iddata
Est_SC_Model = pem(Est_SC,2,'Focus','Sim'); % Estimating Ro
compare(Est_SC,Est_SC_Model) % Estimated Mod

% Validation Scavenger Concentration Model with Merged Multiexperiments Data

Val_SC=SC_var3(1:2600,:,:);
figure
compare(Val_SC,Est_SC_Model) % Model Validation

% Poles and Zeros Plot

figure
pzmap(Est_SC_Model) % Poles and Zeros Plots
```

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% Scavenger Tailing Grade Modelling

load Scavenger_tailing_grade_iddata

% Estimation Scavenger Tailing Model with Merged Multiexperiments Data

Est_ST=merge(ST_Exp2(1700:2500,:,:),ST_Exp3,ST_var2,ST_var3); %Merged iddata Object
Est_ST_Model = pem(Est_ST,2,'Focus','Sim'); %Estimating Rougher T
figure
compare(Est_ST,Est_ST_Model) %Estimated Model comp

% Validation Scavenger Tailing Model with Merged Multiexperiments Data

Val_ST_var1=ST_var1(1:2600,:,:);
figure
compare(Val_ST_var1,Est_ST_Model) %Model Validation

% Poles and Zeros Plot

figure
pzmap(Est_ST_Model) %Poles and Zeros Plot
Bibliography


