Wingbox Mass Prediction considering Quasi-Static Nonlinear Aeroelasticity

Diploma Thesis

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Affirmation

I hereby certify that the present work was made autonomously and without use of others than the indicated sources. All sources, which are literally or logically taken out of published and not-published writings, are marked as such. This work has not yet been presented in same or similar form to any other test authority.

Klaus Seywald, 04.11.2011
Abstract

Nonplanar wing configurations promise a significant improvement of aerodynamic efficiency and are therefore currently investigated for future aircraft configurations. A reliable mass prediction for a new wing configuration is of great importance in preliminary aircraft design in order to enable a holistic assessment of potential benefits and drawbacks. In this thesis a generic numerical modeling approach for arbitrary unconventional wing configurations is developed and a simulation tool for their evaluation and mass prediction is implemented. The wingbox is modeled with a nonlinear finite element beam which is coupled to different low-fidelity aerodynamic methods obtaining a quasi-static aeroelastic model that considers the redistribution of aerodynamic forces due to deformation. For the preliminary design of the wingbox various critical loading conditions according to the Federal Aviation Regulations are taken into account. The simulation tool is validated for a range of existing aircraft types. Additionally, two unconventional configurations, the C-wing and the box-wing, are analyzed. The outlook provides suggestions for extensions and further development of the simulation tool as well as possible model refinements.
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Contents

EAS  Equivalent air speed
f    Free nodes
fr   Front
flex Flexible
G    Geometric
glob Global coordinate system
i, 1, 2 Node index
j    Iteration index
k    Collocation point index
lo   Lower
loc  Local coordinate system
lim  Limit
M    Material
m    Midpoint
MO   Maximum operating
N    Normal
n    Nodal
P    Panel
re   Rear
root Wing root
s    Shear
st   Structural
sp   Spar, web
sk   Skin
t    Torsion
TAS  True air speed
up  Upper
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1 Introduction

"The development of aviation is a struggle against limitations imposed by nature upon man, created to live on the ground but nevertheless endeavoring to move in unlimited space surrounding our globe."

Theodore von Karman

The demand for mobility is steadily growing and this trend is expected to continue and even raise in future (Airbus, 2011). Transport and mobility make a significant portion of human made greenhouse gas emissions and since the issue of global warming has become increasingly important in recent years, there is a general demand in reducing the environmental impact of transportation, including aviation. In aviation the Advisory Council for Aeronautics Research in Europe (ACARE) has issued ambitious goals for the reduction of emissions in air traffic and air transportation (European Commission, 2011). In order to achieve these goals the development and implementation of new technologies is crucial.

Nonplanar wing configurations promise a significant rise of aerodynamic efficiency and are therefore investigated for future aircraft configurations. The most promising nonplanar configurations in terms of aerodynamic efficiency are the box-wing and the C-wing concept (Kroo, 2005).

1.1 Motivation and Overview

A reliable mass prediction for a new wing configuration is of significant importance in preliminary aircraft design in order to enable a holistic assessment of potential benefits and drawbacks of the investigated configuration. Existing handbook methods as presented in (LTH, 2006) or (Raymer, 1999) are only applicable for conventional configurations and can hardly be extended for unconventional wing configurations. The lack of available and accessible methods for the mass prediction of wings requires the development of a new and generally applicable method.

The throughput of today’s computers is sufficient to run numerical simulations already in the preliminary design phase which allows abandoning empirical estimation formulas, starting with low-fidelity numerical methods for predictions from...
1.2 Unconventional Wing Configurations

the beginning. The load-carrying structure in a wing is the wingbox, hence it is the critical part for the mass prediction of a wing. The mass of secondary wing components are primarily not load dependent and can still be estimated with empirical relations.

For the preliminary wingbox sizing and mass prediction of arbitrary wing configurations a low-fidelity numeric aeroelastic model appears suitable and still has a relatively low demand on computation power. An aeroelastic model is important since the wing deformation due to aerodynamic and inertial loading changes the original lift distribution of the wing which further influences the sizing of the wing. For future aircraft concepts nonlinear structural effects may be important, especially for very slender and flexible wings where deflections are likely to extend the valid range of linear approximations.

For the design of a wingbox critical wing loadings described in the Federal Aviations Regulations have to be taken into account. Based on these loads a preliminary wingbox sizing is performed which further enables a mass prediction. Since the box-wing and the C-wing configuration promise a number of advantages for future commercial transport aircraft these configurations serve as test scenarios. The software developed during this thesis is however not limited to these applications and can be used for other configurations accordingly.

1.2 Unconventional Wing Configurations

Since the investigation of unconventional wing configurations is a major motivation for this thesis a brief introduction in this topic is provided here. Figure 1 shows different nonplanar wing configurations along with their potential in induced drag reduction and their optimal lift distribution. The parameter $e$ in the figure denotes the induced drag of the planar configuration divided by the induced drag generated by the respective nonplanar configuration with the same span. It can be seen that the box-wing and the C-wing configurations on the left hand side have the highest potential for the reduction of induced drag (Kroo, 2005).

1.2.1 The Box-Wing

The box-wing configuration was introduced by Prandtl and is also referred to as best wing system (Prandtl, 1924). The best wing system is the theoretical solution
1.2 Unconventional Wing Configurations

Figure 1: Integrated lift forces for nonplanar wings (Kroo, 2005)

with minimal induced drag for a given span. In a practical point of view these benefits are possibly outweighed by the increased surface area and the resulting increase of friction drag. Figure 2 shows an aircraft concept with a box-wing configuration designed by Bauhaus Luftfahrt. The resulting weight of such a wing is investigated in Section 4 of this work.

Figure 2: The Bauhaus Luftfahrt Claire Liner box-wing configuration
1.3 Aeroelasticity

1.2.2 The C-Wing

The C-wing benefits from almost the same amount of drag reduction as the box-wing but at a lower cost of surface area increase. This configuration is found in many aircraft concept studies such as performed in (Kroo et al., 1995) or a current design project at Bauhaus Luftfahrt visualized in Figure 3. The resulting weight of this wing concept is evaluated in Section 4 of this thesis.

1.3 Aeroelasticity

Aeroelasticity is a multidisciplinary discipline dealing with the interaction between aerodynamic forces, elastic forces and inertial forces. The interactions between aerodynamics and structural deformation lead to several phenomena that are crucial for aircraft design. Often many trade-offs have to be made in order to overcome problems caused by aeroelastic effects. The most important consequence of aeroelasticity for aircraft design is that structural weight and stiffness often need to be redistributed or added in order to change the pressure distribution on aerodynamic surfaces created by bending and twisting motion (Luber, 2010). Aeroelastic phenomena can be separated into static and dynamic responses.

**Static aeroelasticity** neglects inertial forces and purely investigates the equilibrium between aerodynamic forces and elastic forces. Typical problems of static aeroelasticity are divergence, control reversal and control effectiveness.
Dynamic aeroelasticity additionally accounts for inertial forces causing dynamic responses which are also known as flutter phenomena. The most important examples are classical flutter, whirl flutter, transonic buzz and limit cycle oscillations (Bisplinghoff et al., 1996).

For the investigation of aeroelastic phenomena the structure as well as the aerodynamics have to be modeled accordingly. The fidelity of the modeling decides whether a certain phenomenon can be represented or not. Fundamental effects such as divergence can be captured by relatively simple methods whereas complex phenomena such as limit cycle oscillations are still poorly understood and are often only recognized during flight testing (Luber, 2010).

1.4 Previous Work

Various methods for the mass prediction of wings have been developed in the past since this is an essential task in preliminary aircraft design (Kelm et al., 1995). The range of methods for mass prediction goes from simple empirical methods such as found in (Raymer, 1999) or (LTH, 2006) to more refined ones such as (Torenbeek, 1992) where first analytical approaches are used. A tool based on a fully analytical method has been developed by (Kelm et al., 1995) but it is restricted to conventional configurations and linear structural analysis. High fidelity approaches such as performed in (Ainsworth et al., 2010) incorporating the software Abaqus, Nastran and Hypersizer or (Bindolino et al., 2010) incorporating Nastran are suitable for the intended purpose but are too computationally expensive and complex for the desired purpose. A quasi analytical mass estimation method is introduced in (Ajaj et al., 2011), developed at the University of Bristol. The code called UC-700 should serve as the basis for this thesis since it shows an appropriate level of model complexity and accuracy. The initial intention was an extension of this code for unconventional wings and nonlinear structures. After several fundamental errors were found and inappropriate key assumptions were revealed it was decided to redevelop a code incorporating the same level of fidelity but offering higher flexibility and focusing on unconventional, very slender and flexible wings as well as coupled wing structures including nonlinear structural analysis.
2 Theoretical Background and Modeling Approach

This section describes the modeling used for a generic aeroelastic wing configuration. Structural and aerodynamic models are selected and described in detail. The aeroelastic coupling approach is explained along with relevant phenomena expected to occur during simulations. Finally required fundamentals for the application of critical design load conditions specified by the aviation authorities are presented.

2.1 Model Requirements Specification

The system to be modeled is an arbitrary aircraft wing or lifting surface. This wing should be assessed in a preliminary design phase. Consideration of aeroelasticity implies that models of both disciplines, structural mechanics and aerodynamics, are required to describe the interactions and their effects. The models must be able to resolve desired effects and influences but should as well be kept as simple as possible to minimize the required computational power. The restriction to static aeroelasticity implies that no dynamic behavior like flutter is covered by the developed method. For the preliminary design phase of an aircraft low-fidelity aerodynamic and structural models provide sufficient accuracy (Luber, 2010).

2.1.1 Requirements for the Structural Model

For the structural part a load-deformation model with bending and twist is required. A special requirement for this thesis is to cover structural nonlinearities since the developed tool is especially designed for unconventional wing configurations where large deflections and other nonlinear effects are expected to occur.

2.1.2 Requirements for the Aerodynamic Model

The aerodynamic model must provide a good prediction of the main aerodynamic forces namely lift and pitching moment since the acting aerodynamic forces are crucial for the sizing of the wing. An accurate prediction of drag is not required since compared to lift and pitching moment this force does not have a significant influence on the wing sizing. For all forces the respective span-wise distribution is required. The requirements are met by common low-fidelity aerodynamic methods such as the lifting line method or the vortex lattice method. These methods
produce adequate results for the lift and moment distributions but lack accuracy for drag predictions (Breitsamter, 2008).

### 2.2 Structural Modeling

A wing can be approximated as a beam or a shell structure. An extensive study has been made in (Dorbath et al., 2010) comparing the validity and accuracy of shell and beam theory for wing preliminary design. The outcome is that a beam model provides sufficient accuracy for the given purpose and therefore is the model of choice for this thesis. Figure 4 sketches the used structural modeling approach for a wing. The wingbox is modeled by means of a three-dimensional finite element beam concentrated in the elastic axis of the wing, the circles show the element nodes. Each element is described by a local coordinate system, where the respective y-axis is oriented along the beam element. The transformation between the global coordinate system and the element’s local coordinate system is done by rotation of the global system using the wing sweep, $\varphi$, the wing dihedral, $\nu$, and the wing twist, $\epsilon$, as shown in Figure 5.

![Figure 4: Sketch of the structural wing model](image)

For a beam model the cross-section of the structure needs to be modeled and expressed in terms of cross-sectional area, moments of inertia and material properties. The following section explains the assumptions and simplifications made to obtain the cross-sectional parameters and defines sizing rules resulting from external loads. Thereafter the equations for linear and nonlinear beam bending
and torsion are presented and the most important steps in their derivation are explained. Then the linear set of differential equations is discretized using the finite element method. Afterwards the step from linear to a nonlinear finite element approach is explained in more detail and the derivation for the nonlinear beam elements is sketched.

2.2.1 Cross-Section Modeling

Most commonly the load carrying structure in a wing is the wingbox. As shown by the section cut in Figure 6 it usually consists of a front spar and a rear spar, also called webs, and an upper and lower skin that is stiffened by stringers. The shear center or elastic axis is the axis in which the beam is concentrated. To simplify the model, the cross-section of a wingbox is reduced to a rectangular shape as shown in Figure 7. The skin and stringers are approximated by an equivalent thickness for the upper and lower side. A number of different approaches for the modeling of a wingbox cross-section have been tested by (Bindolino et al., 2010) where better refined cross-sectional models did not show a significant improvement for the mass prediction of the wingbox. All models however only incorporated one material for the cross-section. Usually the wingbox consists of a number of different materials, hence a multi-material cross-section model is likely to increase prediction accuracy but is not considered further for this work.
2.2 Structural Modeling

Sizing rules  Based on the loads acting on the cross-section a structural layout can be performed. The main forces acting on the wing cross-section are the shear force due to lift, the bending moment due to lift and the torsional moment due to the aerodynamic pitching moment. Drag causes a shear force and a bending moment accordingly, but the magnitude is roughly one twentieth compared to lift forces and hence is neglected. Figure 8 shows the main components of a wing
2.2 Structural Modeling

and their major load carrying function. For the layout of the simplified wingbox cross-section it is assumed that the bending moment about the x-axis is only carried by the upper and lower skin represented by the respective equivalent thickness. Moreover the front and rear spar are the only load bearing parts for the shear force along the z-axis. The torsional moment however is carried by the entire closed cross-section. This load distribution for the wingbox parts is also suggested by (Torenbeek, 1992). The bending moments of inertia assuming thin-wall approximations are then given by

\[ I_z = \frac{hw^3 - (h - t_{eq,up} - t_{eq,lo})(w - t_{sp,fr} - t_{sp,re})^3}{12}, \]

and

\[ I_x = \frac{h^3w - (h - t_{eq,up} - t_{eq,lo})^3(w - t_{sp,fr} - t_{sp,re})}{12} \]

The torsional moment of inertia is found by applying the Breth Bartho theory for thin-walled cross-sections (Gross et al., 2007) and results in

\[ I_p = \frac{4A_{en}^2}{h - \frac{t_{eq,up} - t_{eq,lo}}{t_{sp,fr}} + \frac{t_{eq,up} - t_{eq,lo}}{t_{sp,re}} + \frac{w - t_{sp,fr} - t_{sp,re}}{t_{eq,up}} + \frac{w - t_{sp,fr} - t_{sp,re}}{t_{eq,lo}}} \]

where

\[ A_{en} = (h - \frac{t_{eq,up}}{2} - \frac{t_{eq,lo}}{2})(w - \frac{t_{sp,fr}}{2} - \frac{t_{sp,re}}{2}) \]

is the enclosed area. For simplification reasons it is assumed that the front and rear spar thickness is equal. Furthermore the equivalent upper and lower skin thickness are assumed to be identical. The simplified equations with \( t_{sp,fr} = t_{sp,re} \) and \( t_{sk,up} = t_{sk,lo} \) are then given by

\[ I_z = \frac{hw^3 - (h - 2t_{sk})(w - 2t_{sp})^3}{12} \]

and

\[ I_x = \frac{h^3w - (h - 2t_{sk})^3(w - 2t_{sp})}{12} \]

The torsional moment of inertia is reduced to

\[ I_p = \frac{4A_{en}^2}{2h - \frac{t_{sk}}{t_{sp}} + \frac{w - t_{sp}}{t_{sk}}} \]

with

\[ A_{en} = (h - t_{sk})(w - t_{sp}) \]
2.2 Structural Modeling

**Material properties** The structural layout is commonly performed based on the yield strength of the considered material. The yield strength can be found in standard material tables and is usually provided for a permanent deformation of 0.2%. For nonferrous alloys as aluminum the yield strength does not coincide with the proportional limit stress which is the stress at which plastic deformation is still zero. Since a permanent deformation is not allowed for the limit load (see Section 2.5) the yield strength of the material along with a safety factor, $FoS$, is used for the structural layout process in order to cover the 0.2% of deformation and other design uncertainties. This provides the allowable stress at limit load

$$\sigma_{alw,lim} = \frac{\sigma_{yield}}{FoS}$$

(9)

For a layout in shear the yield shear stress is required. For aluminum alloys which are most commonly used in wing design the yield shear stress can be approximated from the yield tensile stress according to (Beardmore, 2011):

$$\tau_{alw,lim} = 0.55 \frac{\sigma_{yield}}{FoS}$$

(10)

**Sizing of the upper and lower skin** The equivalent upper and lower skin thickness is determined by

$$t_{sk} = \frac{M_b}{h \cdot w \cdot \sigma_{alw,lim} \cdot k_e}$$

(11)

where $M_b$ is the acting bending moment. In order to compensate for the simplifications a correction factor $k_e$ according to (Reynolds, 1960) is introduced. For most current wing profiles $k_e = 0.8$ is a good approximation.

**Sizing of the spars** For the sizing of the spars the shear flow due to the shear force and the shear flow due to torsion have to be added. Shear flow due to shear force is given by

$$\zeta_{shear} = \frac{3Q_z}{4h}$$

(12)

and the shear flow due to torsion is determined by

$$\zeta_{torsion} = \frac{M_t}{2 \cdot h \cdot w}$$

(13)

Then the total shear flow is calculated by the addition of both

$$\zeta_{spar,max} = \zeta_{torsion} + \zeta_{shear}$$

(14)

The spar thickness is finally obtained by

$$t_{sp} = \frac{\zeta_{sp,max}}{\tau_{alw,lim}}$$

(15)
2.2 Structural Modeling

2.2.2 Wing Ribs

The wing ribs mainly serve to transfer the distributed aerodynamic pressure loads from the skin into the spars and to prevent buckling of the skin. Since buckling is not considered and an analytical sizing of these complex components would require too much effort, an empirical formula found in (Torenbeek, 1992) is applied to account only for the mass contribution of the wing ribs.

\[ W_{\text{rib}} = k_r \rho \left( 1 + \frac{c_{\text{root}} + c_{\text{tip}}}{2} \right) \] (16)

where \( \rho \) is the density of the wing rib material, \( c_{\text{root}} \) and \( c_{\text{tip}} \) are the root and tip chord of the wing. The correction factor \( k_r \) is assumed with \( k_r = 0.5 \cdot 10^{-3} \).

2.2.3 Linear Beam Bending

Now the equation for linear beam bending is derived using the principle of minimum potential energy. Note that the derivation is limited to two dimensions since uncoupled bending is considered because of the symmetric cross-section. According to the local coordinate system introduced in Section 2.2, \( y \) is the beam longitudinal axis, \( x \) and \( z \) are the perpendicular bending axis. The total potential energy of a system is written as

\[ \Pi = U - P \] (17)

where \( U \) is the elastic strain energy stored in the system and \( P \) is the potential energy of the applied forces. The strain energy of a linear structure is given by

\[ U = \int_V \int_0^e \sigma \cdot \epsilon \cdot dV \] (18)

where \( \sigma \) is the stress and \( \epsilon \) is the strain. \( V \) denotes the volume of the beam. Applying linear elastic material properties, \( \sigma = \epsilon \cdot E \), Equation 18 simplifies to

\[ U = \int_V \int_0^e E \epsilon d\epsilon dV = \frac{1}{2} \int_V E \epsilon^2 dV \] (19)

Thereafter the strain measure used for an Euler Bernoulli beam \( \epsilon_{yy} = z \frac{d^2 u_z}{dy^2} \) (Gross et al., 2007) is applied which leads to

\[ U = \frac{1}{2} \int_V E z^2 \left( \frac{d^2 u_z}{dy^2} \right)^2 dV = \frac{1}{2} E \int \int \int z^2 \left( \frac{d^2 u_z}{dy^2} \right)^2 dz dx dy \] (20)

The second moment of area, \( I_z = \int \int z^2 dz dx \), is assumed to be constant along the beam segment. Thus, Equation 20 simplifies to

\[ U = \frac{1}{2} EI_z \int_0^l \left( \frac{d^2 u_z}{dy^2} \right)^2 dy \] (21)
The potential of the applied external forces for a beam is given by

\[ P = \int_0^l q_z(y)u_z dy \]  \hspace{1cm} (22)

where \( q_z \) is the distributed load. The total energy is now given by

\[ \Pi(u) = \frac{1}{2} EI_z \int_0^l \left( \frac{d^2 u_z}{dy^2} \right)^2 dy - \int_0^l q_z(y)u_z dy \]  \hspace{1cm} (23)

Making a variational formulation, \( u_z + \delta u_z \), and minimizing the potential energy the beam equation in the weak formulation is obtained

\[ \int_0^l q_z(y)\delta u_z dy = EI_z \int_0^l \frac{d^2}{dy^2} \left( \frac{d^2 u_z}{dy^2} \right) \delta u_z dy \]  \hspace{1cm} (24)

Applying the fundamental theorem of calculus the strong form is obtained

\[ q_z(y) = EI_z \frac{d^2}{dy^2} \left( \frac{d^2 u_z}{dy^2} \right) = EI_z \frac{d^4 u_z}{dy^4} \]  \hspace{1cm} (25)

The beam equation for the perpendicular x-axis is found by replacing the index \( z \) by \( x \).

### 2.2.4 Nonlinear Beam Bending

Linear formulations are based on certain assumptions and simplifications which limit their validity within a certain range (Rust, 2009). These assumptions are:

1) Equilibrium of the undeflected system,

2) Small rotations, hence linear kinematics as shown in Figure 9 \((\sin \Theta \approx \Theta \text{ and } \cos \Theta \approx 1)\),

3) Small strains.

To find a suitable nonlinear formulation it has to be investigated for which assumption the range of validity is extended. For the given application of very flexible wings with significant bending deformation assumptions 1) and 2) are extended whereas strains can still be assumed small.

The nonlinear beam structure is described in a Total Lagrangian fashion. The derivation is essentially based on the procedure suggested in (Felippa, 2010). For the nonlinear formulation the elastic strain energy \( U \) stored in the beam has to be formulated in a suitable way. At first the coordinates used for the derivation...
2.2 Structural Modeling

![Figure 9: Linear vs. nonlinear kinematics](image)

Figure 9: Linear vs. nonlinear kinematics

have to be defined. As shown in Figure 10 the coordinates \( y \) and \( z \) denote an arbitrary point of the beam in the reference configuration, \( y' \) and \( z' \) denote the coordinates of this point in the displaced configuration.

![Figure 10: Displacements of a beam element](image)

Figure 10: Displacements of a beam element

The Cauchy Stress Tensor as used for linear analysis

\[
\sigma = \begin{bmatrix}
\sigma_{yy} & \sigma_{yz} \\
\sigma_{zy} & \sigma_{zz}
\end{bmatrix}
\]  

is not suitable anymore for large deformations. Therefore, the Second Piola Kirchhoff Stress (Wall, 2010), which expresses the stress relative to the reference configuration, is a more appropriate stress measure for large deformations. The Cauchy Stress Tensor can be transformed into the Second Piola Kirchhoff Stress Tensor, \( S \), by

\[
S = \det(D)D^{-1}\sigma D^{-T} = E : \varepsilon
\]  

(27)
2.2 Structural Modeling

where \( \mathbf{D} \) is the displacement gradient tensor and \( \mathbf{\varepsilon} \) a strain measure. For large deformations the Green-Lagrange strain tensor is a commonly used strain measure (Wall, 2010) and is given by

\[
\mathbf{\varepsilon} = \frac{1}{2}(\mathbf{D}^T \mathbf{D} - \mathbf{I})
\]

Hence the strain energy equation 18, can be written as

\[
U = \int_V \int_0^L \mathbf{S} : \mathbf{\varepsilon} \cdot dV = \frac{1}{2} \int_V \mathbf{E} : \mathbf{\varepsilon} \cdot dV = \frac{1}{2} \int_0^L A_0(y) \left[ \mathbf{\varepsilon}^T \cdot \mathbf{E} \cdot \mathbf{\varepsilon} \right] \cdot dy
\]

After introducing the element kinematics:

\[
\begin{bmatrix}
  y' \\
  z'
\end{bmatrix} = \begin{bmatrix}
  y + u_y - z \sin \Theta \\
  u_z + z \cos \Theta
\end{bmatrix}
\]

the deformation gradient tensor

\[
\mathbf{D} = \begin{bmatrix}
  \frac{\partial y'}{\partial y} & \frac{\partial y'}{\partial z} \\
  \frac{\partial z'}{\partial y} & \frac{\partial z'}{\partial z}
\end{bmatrix}
\]

can be written in the form

\[
\mathbf{D} = \begin{bmatrix}
  1 + \frac{\partial u_y}{\partial y} \cos \Theta + \frac{\partial u_z}{\partial y} \sin \Theta & -\frac{\partial \Theta}{\partial y} \\
  \frac{\partial u_z}{\partial y} - \frac{\partial \Theta}{\partial y} \sin \Theta & \cos \Theta
\end{bmatrix}
\]

This enables to determine the Green-Lagrange strain tensor

\[
\mathbf{\varepsilon} = \begin{bmatrix}
  e_{yy} & e_{yz} \\
  e_{zy} & e_{zz}
\end{bmatrix} = \frac{1}{2}(\mathbf{D}^T \mathbf{D} - \mathbf{I})
\]

After making a polar decomposition and simplifying the expression assuming small strains \( \mathbf{\varepsilon} \) becomes:

\[
\mathbf{\varepsilon} = \begin{bmatrix}
  (1 + \frac{\partial u_y}{\partial y}) \cos \Theta + \frac{\partial u_z}{\partial y} \sin \Theta - z \frac{\partial \Theta}{\partial y} - 1 & -\frac{1}{2}(1 + \frac{\partial u_y}{\partial y}) \sin \Theta + \frac{1}{2} \frac{\partial u_z}{\partial y} \cos \Theta \\
  -\frac{1}{2}(1 + \frac{\partial u_z}{\partial y}) \sin \Theta + \frac{1}{2} \frac{u_z}{\partial y} \cos \Theta & 0
\end{bmatrix}
\]

It can be seen that the normal strain along the \( z \) coordinate, \( e_{zz} \), is zero as expected and that \( e_{yz} = e_{zy} \). The strains can be decomposed into an axial, a bending and a shear strain component where

\[
e_a = (1 + \frac{\partial u_y}{\partial y}) \cos \Theta + \frac{\partial u_z}{\partial y} \sin \Theta - 1
\]

is the axial strain component,

\[
\gamma = -(1 + \frac{\partial u_y}{\partial y}) \sin \Theta + \frac{\partial u_z}{\partial y} \cos \Theta
\]
2.2 Structural Modeling

is the shear strain component and

\[ z \frac{\partial \Theta}{\partial y} \]  

(37)

is the strain component due to curvature. Hence \( \varepsilon \) can be written as

\[
\begin{bmatrix}
\varepsilon_{yy} & \varepsilon_{zy} \\
\varepsilon_{yz} & 0 \\
\end{bmatrix}
= \begin{bmatrix}
e_a - z \frac{\partial \Theta}{\partial y} & \gamma \\
\gamma & 0 \\
\end{bmatrix}
\]

(38)

Because only the total strain energy is of interest \( \varepsilon \) can be merged into a vector

\[
\varepsilon = \begin{bmatrix}
e_{yy} \\
e_{yz} \\
2e_{yz} \\
\end{bmatrix}
= \begin{bmatrix}
e_1 \\
e_2 \\
\end{bmatrix}
= \begin{bmatrix}
e_a - z \frac{\partial \Theta}{\partial y} \\
\gamma \\
\end{bmatrix}
\]

(39)

Now constitutive laws are applied and the SECOND PIOLA KIRCHHOFF STRESS is found by

\[
S = \begin{bmatrix}
Ee_1 \\
Ge_2 \\
\end{bmatrix}
= \begin{bmatrix}
E & 0 \\
0 & G \\
\end{bmatrix}
\begin{bmatrix}
e_1 \\
e_2 \\
\end{bmatrix}
= E \cdot \varepsilon
\]

(40)

At this point the strain energy equation is fully determined and can be written as the sum of three integrals

\[
U = \frac{1}{2} \int_{l_0} A_0 E e_a^2 dy + \frac{1}{2} \int_{l_0} A_0 G \gamma^2 dy + \frac{1}{2} \int_{l_0} E I_z \left( \frac{\partial \Theta}{\partial y} \right)^2 dy
\]

(41)

This equation is the basis for the following finite element discretization.

2.2.5 Classical Torsion Theory

The modeling of torsional deformation is essential for aeroelasticity since the lift distribution is very sensible with respect to wing twist. The relation between torsional moment and twist deformation is given by

\[
m_t(y) + \frac{\partial}{\partial y} \left( GI_p \frac{\partial \phi}{\partial y} \right) = 0
\]

(42)

where \( \phi \) is the twist angle, \( m_t \) is the distributed torsional moment, \( G \) is the shear modulus and \( I_p \) is the torsional constant. For closed thin-walled cross-sections the Bredt-Batho theory can be used to determine \( I_p \) (Gross et al., 2007) as conducted in Section 2.2.1.
2.2.6 Finite Element Discretization

For the wing beam model a fully three-dimensional beam element is selected, which results in six degrees of freedom (DOFs) per node, three translational DOFs and three rotational DOFs. Bending about the $x$ and $z$-axis is assumed to be uncoupled, since coupled bending only occurs for asymmetric cross-sections whereas the considered cross-section model is fully symmetric. For a possible asymmetric extension of the cross-section model this fact should be kept in mind. For swept wings however the effect due to sweep is expected to outrange the effect of skew bending due to asymmetric cross-sections. The displacement vector for an element is

$$u_e = (u_{x1}, u_{y1}, u_{z1}, \theta_{x1}, \phi_1, \theta_{z1}, u_{x2}, u_{y2}, u_{z2}, \theta_{x2}, \phi_2, \theta_{z2})$$ (43)

where the indices 1 and 2 denote the two element nodes, $u$ denotes displacements, $\Theta$ denotes bending twist angles and $\phi$ denotes the torsion twist angle, cp. Figure 10. The force vector for one element is

$$f_e = (Q_{x1}, N_{y1}, Q_{z1}, M_{x1}, M_{T1}, M_{z1}, Q_{x2}, N_{y2}, Q_{z2}, M_{x2}, M_{T2}, M_{z2})$$ (44)

where $Q$ denotes transverse forces, $N$ denotes the normal forces and $M$ denotes moments. The structural analysis is implemented as a classical stiffness method. This implies that for the entire structure the nodal forces and nodal displacements are related through a stiffness matrix $K$ (Zenkert, 2009). The global system is determined by the relationship

$$f = K \cdot u$$ (45)

where $f$ is the global force vector and $u$ is the global displacement vector. The global stiffness matrix $K$ is obtained by assembly of single element stiffness matrices $K_e$ in the following scheme

$$K = \begin{bmatrix}
1 & \cdots & i \\
\vdots & \ddots & \vdots \\
i & \cdots & n
\end{bmatrix}$$ (46)

where one block represents a 12x12 element stiffness matrix for the element in between node $i$ and node $i + 1$. 
The element stiffness matrix can be derived by discretization of the weak form of the differential equations. Using the presented linear theories stated in Sections 2.2.3 and 2.2.5 the stiffness matrix $K_{e}^{loc}$ for the local beam element becomes:

$$
K_{e}^{loc} = \begin{bmatrix}
\frac{12EI}{l^3} & 0 & 0 & 0 & 0 & -\frac{6EI}{l^2} & -\frac{12EI}{l^3} & 0 & 0 & 0 & 0 & -\frac{6EI}{l^2} \\
0 & \frac{EA}{l} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{12EI}{l^3} & -\frac{6EI}{l^2} & 0 & 0 & 0 & -\frac{12EI}{l^3} & -\frac{6EI}{l^2} & 0 & 0 & 0 \\
0 & 0 & -\frac{6EI}{l^2} & \frac{4EI}{l} & 0 & 0 & 0 & \frac{6EI}{l^2} & \frac{2EI}{l} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{GJ}{l^3} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{GJ}{l^3} & 0 \\
-\frac{6EI}{l^2} & 0 & 0 & 0 & 0 & \frac{4EI}{l} & \frac{6EI}{l^2} & 0 & 0 & 0 & 0 & 0 & \frac{2EI}{l} \\
-\frac{12EI}{l^3} & 0 & 0 & 0 & 0 & \frac{6EI}{l^2} & \frac{12EI}{l^3} & 0 & 0 & 0 & 0 & \frac{6EI}{l^2} & 0 \\
0 & -\frac{EA}{l} & 0 & 0 & 0 & 0 & \frac{EA}{l} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{12EI}{l^3} & \frac{6EI}{l^2} & 0 & 0 & 0 & \frac{12EI}{l^3} & \frac{6EI}{l^2} & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{6EI}{l^2} & \frac{2EI}{l} & 0 & 0 & 0 & \frac{6EI}{l^2} & \frac{4EI}{l} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{GJ}{l^3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{GJ}{l^3} & 0 \\
-\frac{6EI}{l^2} & 0 & 0 & 0 & 0 & \frac{2EI}{l} & \frac{6EI}{l^2} & 0 & 0 & 0 & 0 & \frac{4EI}{l} & 0 \\
\end{bmatrix}
$$

The element stiffness matrix is given in element local coordinates, hence if the local element coordinates deviate from the global system coordinates a transformation has to be performed. Before $K$ and $f$ can be assembled all matrices and forces must be transformed into one global coordinate system. The rotation matrix $T$ transforms the element stiffness matrix into global coordinates.

$$K_{e}^{glob} = T^T K_{e}^{loc} T \quad (48)$$

The same transformation matrix is valid for the transformation from the local element force vector into the global element force vector.

$$f_{e}^{glob} = T^T f_{e}^{loc} \quad (49)$$

Before the assembled system can be solved all desired boundary conditions need to be incorporated. For simple boundary conditions the nodal deflection $u_i$ is set to zero. This is done by deleting the respective row $i$ and column $i$ from $K$ to obtain the structural stiffness matrix $K_f$ and deleting row $i$ from $f$ to get $f_f$. The linear system for the free nodes is then solved by matrix inversion of $K_f$:

$$u_f = K_f^{-1} f_f \quad (50)$$
2.2 Structural Modeling

2.2.7 Nonlinear Finite Element Approach

For nonlinear finite elements the same basic principle as used for linear finite elements is valid. The system is determined by a stiffness matrix, $K$, and the system loading, $f$. The equation system however becomes nonlinear since $K$ is now a function of the discrete displacement field $u$. The stiffness matrix $K$ is called tangent stiffness matrix and is denoted as $K_T$ for nonlinear systems. For the external forces a distinction between non-follower loads and follower loads has to be made as shown in Figure 11. The latter results in a further dependence of $f$ on the discrete displacement field $u$:

$$f(u) = K_T(u)u$$  \hspace{1cm} (51)$$

For the given application aerodynamic forces would be of the type follower load (surface pressure distribution) whereas gravitational forces are non-follower loads.

![Figure 11: Load distinction for nonlinear approach](image)

Due to the nonlinear nature of the equation system an iterative scheme is required. Different solution schemes are available for nonlinear finite elements (Rust, 2009). For the given application a simple Newton-Raphson scheme is sufficient, since no extensive nonlinearities such as snapping need to be investigated. The solution of the nonlinear system is found once an equilibrium between internal and external forces is obtained, hence the residual for the Newton iteration scheme is given by

$$r = f_{int}(u_j) - f_{ext}(u_j) = p - f.$$  \hspace{1cm} (52)$$

For the initial solution the internal force vector, $p$, and the displacement field, $u$, are set to zero. One step $\Delta u$ of the iterative solution scheme for the displacement field is determined by

$$\Delta u = K_T(u_j)^{-1}(-r).$$  \hspace{1cm} (53)$$

The new system displacement field is then found by

$$u_{j+1} = u_j + \Delta u$$  \hspace{1cm} (54)$$
2.2 Structural Modeling

where \( u_j \) is the displacement field from the previous step. After each step the internal force vector \( p \) needs to be recalculated. The same applies for the external force vector \( f \) if forces follow the displacement field (follower loads).

**Hybrid Finite Element Matrix**  For the given problem, i.e. very flexible non-planar wings, only the bending about the x-axis is expected to be nonlinear. A nonlinear deflection about the z-axis is not expected and neither desired from a current point of view on aircraft design. Hence the nonlinear model is only required to cover for nonlinearities about the x-axis. For this purpose the six DOF element stiffness matrix has nonlinear entries for only three DOFs, the rest is assumed to be linear and hence equal to the linear element stiffness matrix. The resulting hybrid, linear-nonlinear matrix is

\[
K_{loc} = \begin{pmatrix}
\frac{12EI}{l^2} & 0 & 0 & 0 & 0 & -\frac{6EI}{l^2} & 0 & 0 & 0 & 0 & -\frac{6EI}{l^2} \\
0 & K_T & K_T & K_T & 0 & 0 & 0 & K_T & K_T & K_T & 0 \\
0 & K_T & K_T & K_T & 0 & 0 & 0 & K_T & K_T & K_T & 0 \\
0 & K_T & K_T & K_T & 0 & 0 & 0 & K_T & K_T & K_T & 0 \\
0 & 0 & 0 & 0 & GJ & 0 & 0 & 0 & 0 & 0 & -GJ & 0 \\
-\frac{6EI}{l^2} & 0 & 0 & 0 & 0 & \frac{4EI}{l^2} & \frac{6EI}{l^2} & 0 & 0 & 0 & 0 & \frac{2EI}{l^2} \\
-\frac{12EI}{l^2} & 0 & 0 & 0 & 0 & \frac{6EI}{l^2} & \frac{12EI}{l^2} & 0 & 0 & 0 & 0 & \frac{6EI}{l^2} \\
0 & K_T & K_T & K_T & 0 & 0 & 0 & K_T & K_T & K_T & 0 \\
0 & K_T & K_T & K_T & 0 & 0 & 0 & K_T & K_T & K_T & 0 \\
0 & K_T & K_T & K_T & 0 & 0 & 0 & K_T & K_T & K_T & 0 \\
0 & 0 & 0 & 0 & -GJ & 0 & 0 & 0 & 0 & 0 & GJ & 0 \\
-\frac{6EI}{l^2} & 0 & 0 & 0 & 0 & \frac{4EI}{l^2} & \frac{6EI}{l^2} & 0 & 0 & 0 & 0 & \frac{4EI}{l^2}
\end{pmatrix}
\]

(55)

where the elements \( K_T = f(u) \) denote the nonlinear entries which are a function of the displacement field. These coefficients are relatively complex to determine, their derivation is sketched in the next subsection. For this derivation the nonlinear strain energy formulation derived in Section 2.2.4 is used.

### 2.2.8 Derivation of the Nonlinear Finite Element Matrix

For a finite element discretization the displacement field must be expressed by means of shape functions and nodal displacement values. The order of the shape function is determined by the highest derivative of the respective variable occurring in the differential equation. For an Euler-Bernoulli beam model as used for the
2.2 Structural Modeling

linear equations $\Theta$ is the derivative of the displacement $u_z$, hence second order shape functions would be required. This is avoided by using a Timoshenko beam model, where $\Theta$ is independent of the displacement. In this case only first derivatives appear and linear shape functions are suitable. The displacement vector for a node $n$ is then given by

$$u_n = \begin{bmatrix} u_{yn} \\ u_{zn} \\ \Theta_n \end{bmatrix} \quad (56)$$

Applying linear shape functions the displacements in between two nodes can be written as follows, where $\xi$ is the normalized element coordinate which ranges from $-1$ at node 1 to $+1$ at node 2.

$$u_e = \begin{bmatrix} u_y(y) \\ u_z(y) \\ \Theta(y) \end{bmatrix} = \frac{1}{L} \begin{bmatrix} 1 - \xi & 0 & 0 & 1 + \xi & 0 & 0 \\ 0 & 1 - \xi & 0 & 0 & 1 + \xi & 0 \\ 0 & 0 & 1 - \xi & 0 & 0 & 1 + \xi \end{bmatrix} \begin{bmatrix} u_{y1} \\ u_{z1} \\ \Theta_1 \\ u_{y2} \\ u_{z2} \\ \Theta_2 \end{bmatrix} \quad (57)$$

The first derivative of the displacement is then given by

$$\frac{\partial u_e}{\partial y} = \begin{bmatrix} \frac{\partial u_y(y)}{\partial y} \\ \frac{\partial u_z(y)}{\partial y} \\ \frac{\partial \Theta(y)}{\partial y} \end{bmatrix} = \frac{1}{L_0} \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{y1} \\ u_{z1} \\ \Theta_1 \\ u_{y2} \\ u_{z2} \\ \Theta_2 \end{bmatrix} \quad (58)$$

Next, the strains and displacements need to be related to obtain the strain displacement matrix $B$:

$$\delta h = B \delta u \quad (59)$$

with $h$ as generalized strain vector

$$h = \begin{bmatrix} e_a \\ \gamma \\ z \frac{\partial \Theta}{\partial y} \end{bmatrix} \quad (60)$$
2.2 Structural Modeling

The components of $\mathbf{B}$ can be found by component-wise partial derivation, for example

$$B_{1,1} = \frac{\partial e_a}{\partial u_{y1}}$$

(61)

The internal force can now be found by the first variation of the internal energy with respect to the nodal displacements:

$$p = \int_{l_0} B \tilde{z} dy$$

(62)

with

$$\tilde{z} = \begin{bmatrix} EA_0 \\ GA_0 \gamma \\ EI \frac{\partial \Theta}{\partial y} \end{bmatrix}$$

(63)

Using a one-point gauss rule for integration this results in

$$p = l_0 \mathbf{B}^T \mathbf{m} \tilde{z} = \begin{bmatrix} -c_m & -s_m & -\frac{1}{2} l_0 \gamma_m & c_m & s_m & \frac{1}{2} l_0 \gamma_m \\ s_m & -c_m & -\frac{1}{2} l_0 (1 + e_m) & -s_m & c_m & -\frac{1}{2} l_0 (1 + e_m) \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \tilde{z}$$

(64)

where $c_m = \cos(\Theta_m)$, $s_m = \sin(\Theta_m)$, $\Theta_m = (\Theta_1 + \Theta_2)/2$ and

$$e_m = \frac{\cos(\Theta_m)(l_0 + u_{y2} - u_{y1}) + \sin(\Theta_m)(l_0 + u_{z2} - u_{z1})}{l_0} - 1$$

(65)

The shear distortion is given by

$$\gamma_m = -\frac{\sin(\Theta_m)(l_0 + u_{y2} - u_{y1}) + \cos(\Theta_m)(l_0 + u_{z2} - u_{z1})}{l_0}$$

(66)

The tangent stiffness matrix is the first variation of the internal force vector with regard to the nodal displacements:

$$\delta p = \int_{l_0} (B \delta \tilde{z} + \delta B \tilde{z}) dy = (K_M + K_G) \delta \mathbf{u} = \mathbf{K} \delta \mathbf{u}$$

(67)

The tangent stiffness matrix consists of a material stiffness matrix and a geometric stiffness matrix:

$$K_T = K_M + K_G$$

(68)

The material stiffness matrix also consists of three components:

$$K_M = K_M^a + K_M^s + K_M^b$$

(69)
where $K^a_M$ is the axial component, $K^s_M$ is the shear component and $K^b_M$ is the bending component. In detail, they are defined by

$$K^a_M = \frac{E A_0}{l_0} \begin{bmatrix} c_m^2 & c_m s_m & -c_m \gamma_m \frac{l_0}{2} & -c_m^2 & -c_m s_m & -c_m \gamma_m \frac{l_0}{2} \\ c_m s_m & s_m^2 & -\gamma_m s_m \frac{l_0}{2} & -c_m s_m & -s_m^2 & \gamma_m s_m \frac{l_0}{2} \\ -c_m \gamma_m \frac{l_0}{2} & -\gamma_m s_m \frac{l_0}{2} & \gamma_m^2 \frac{l_0^2}{4} & c_m \gamma_m \frac{l_0}{2} & c_m s_m & \gamma_m \gamma_m \frac{l_0}{2} \gamma_m \frac{l_0^2}{4} \\ -c_m^2 & -c_m s_m & c_m \gamma_m \frac{l_0}{2} & c_m^2 & c_m s_m & c_m \gamma_m \frac{l_0}{2} \\ -c_m \gamma_m \frac{l_0}{2} & -s_m^2 & \gamma_m s_m \frac{l_0}{2} & c_m s_m & s_m^2 & \gamma_m s_m \frac{l_0}{2} \\ -c_m \gamma_m \frac{l_0}{2} & -c_m s_m \frac{l_0}{2} & \gamma_m^2 \frac{l_0^2}{4} & c_m \gamma_m \frac{l_0}{2} & c_m s_m & \gamma_m \gamma_m \frac{l_0}{2} \gamma_m \frac{l_0^2}{4} \end{bmatrix}$$

(70)

$$K^s_M = \frac{G A_0}{l_0} \begin{bmatrix} s_m^2 & c_m s_m & -a_m l_0 s_m \frac{l_0}{2} & -s_m^2 & c_m s_m & -a_m l_0 \frac{s_m}{2} \\ -c_m s_m & c_m^2 & c_m a_m \frac{l_0}{2} & c_m s_m & -c_m^2 & c_m a_m l_0 \frac{l_0}{2} \\ -a_m l_0 s_m \frac{l_0}{2} & c_m a_m l_0 \frac{l_0}{2} & a_m l_0 \frac{s_m}{2} & a_m l_0 \frac{s_m}{2} & a_m^2 \frac{l_0^2}{4} & a_m^2 \frac{l_0^2}{4} \\ -s_m^2 & c_m s_m & a_m l_0 s_m \frac{l_0}{2} & s_m^2 & -a_m s_m & a_m \frac{l_0 s_m}{2} \\ c_m s_m & -c_m^2 & -a_m a_m l_0 \frac{l_0}{2} & c_m^2 & -c_m a_m l_0 \frac{l_0}{2} & c_m^2 \frac{l_0^2}{4} \\ -a_m l_0 s_m \frac{l_0}{2} & c_m a_m l_0 \frac{l_0}{2} & a_m l_0 \frac{s_m}{2} & -a_m l_0 \frac{s_m}{2} & -a_m^2 \frac{l_0^2}{4} & a_m^2 \frac{l_0^2}{4} \end{bmatrix}$$

(71)

where $a_m = 1 + c_m$, and

$$K^b_M = \frac{E I_Z}{l_0} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \ -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \ 1 \end{bmatrix}$$

(72)

The shear locking effect is removed by a residual bending flexibility correction as suggested by (MacNeal, 1978):

$$\frac{1}{G A_0} = \frac{l_0^2}{12 E I_0} + \frac{1}{G A_s}$$

(73)

where $G A_s$ is the shear rigidity which is assumed to be infinite. Next the geometric stiffness matrix is determined which describes the effect of normal and transverse forces on the stiffness:

$$K_G = K_{GN} + K_G^Q$$

(74)
2.3 Aerodynamic Modeling

with

\[
K^N = \frac{N_m}{2} \begin{bmatrix}
0 & 0 & s_m & 0 & 0 \\
0 & 0 & -c_m & 0 & 0 \\
s_m & -c_m & -\frac{1}{2}l_0(1 + e_m) & -c_m & s_m \\
0 & 0 & -s_m & 0 & 0 \\
0 & 0 & c_m & 0 & 0 \\
s_m & -c_m & -\frac{1}{2}l_0(1 + e_m) & -s_m & c_m \\
\end{bmatrix}
\]  

(75)

and

\[
K^Q = \frac{Q_m}{2} \begin{bmatrix}
0 & 0 & c_m & 0 & 0 \\
0 & 0 & s_m & 0 & 0 \\
c_m & s_m & -\frac{1}{2}l_0\gamma_m & -c_m & -s_m \\
0 & 0 & -c_m & 0 & 0 \\
0 & 0 & -s_m & 0 & 0 \\
c_m & s_m & -\frac{1}{2}l_0\gamma_m & -c_m & -s_m \\
\end{bmatrix}
\]  

(76)

For a detailed step by step derivation of a nonlinear element matrix it is referred to (Felippa, 2010).

2.3 Aerodynamic Modeling

The aerodynamics are modeled with two different approaches, a vortex lattice method and a nonlinear lifting line method. Two existing aerodynamic codes are incorporated for this work. The vortex lattice code used is TORNADO (Melin, 2000) and the nonlinear lifting line tool used is PAWA T (Propeller and Wing Aerodynamics Tool) (Steiner, 2010). The theories on which these tools are based on are explained shortly in the following section. Moreover restrictions of each method are stated, since it is very important to be aware of them to avoid unreasonable results.

2.3.1 Vortex Lattice Method (TORNADO)

The vortex lattice method (VLM) is a method commonly used in aircraft preliminary design. It is essentially an extension of Prandtl’s lifting line theory which is based on potential flow theory. The vortex lattice method divides a lifting surface into separate panels. Each of these panels is modeled by means of a horseshoe vortex. Chord-wise panels are oriented along the camber line of the airfoil which
allows to model cambered airfoils. The horseshoe vortex is fixed on the quarter-chord line of the panel. The most common way is to divide the horseshoe vortex into three vortex filaments, starting at infinity going to the quarter-chord line following the freestream velocity, going over the panel along the quarter-chord line and then going to infinity again as shown in Figure 12. For numerical calculations a reasonable large number is used for infinity. A better refinement is reached by a seven filament horseshoe vortex which allows to model rudder deflections additionally. This approach is also called vortex sling (Melin, 2000). A vortex sling, also shown in Figure 12, starts at infinity, reaches the trailing edge, goes upstream to the hinge line of the control surface and moves forward to the quarter-chord line of the panel. Then the panel is crossed and the filament goes downstream in the same way as upstream. If no control surface is modeled the trailing edge is assumed to be the hinge line.

\[ v_{N,P} \cdot \hat{n}_P = 0 \]  (77)

The collocation points are located in the middle of each panel, three quarters from the front, cp. Figure 12. The vorticity of each horseshoe vortex is unknown initially. For the solution of the system an influence coefficient matrix, \[ \mathbf{V}_s \], is determined that contains the downwash induced by each of the horseshoe vortices with a
2.3 Aerodynamic Modeling

unit vorticity on all collocation points. Knowing \( \mathbf{V} \) a linear equation system can be solved to obtain the vorticity of each panel.

\[
\begin{bmatrix}
V_{11} & V_{12} & \cdots & V_{1k} \\
V_{21} & \ddots & & \\
\vdots & & \ddots & \\
V_{k1} & V_{12} & \cdots & V_{kk}
\end{bmatrix}
\begin{bmatrix}
\Gamma_1 \\
\Gamma_2 \\
\vdots \\
\Gamma_k
\end{bmatrix}
= 
\begin{bmatrix}
v_{N1} \\
v_{N2} \\
\vdots \\
v_{Nk}
\end{bmatrix}
\tag{78}
\]

Once the vorticities are known, the aerodynamic forces and coefficients can be calculated using the Kutta-Joukowski law.

\[ L_P = \rho \Gamma_P v_P s_P \tag{79} \]

**Restrictions** The restrictions of this method are essentially based on the linear theory that is used for VLM. The method is limited to small angles of attack, cannot detect stall and only applies a simple Prandtl Glauert compressibility correction, which works well for Mach numbers up to 0.7.

### 2.3.2 Nonlinear Lifting Line Theory (PAWAT)

The nonlinear lifting line theory is based on the method described by Phillips and Snyder (Phillips and Snyder, 2000) and constitutes an adaptation of the classical lifting line theory applying a three-dimensional vortex lifting law instead of the two-dimensional Kutta-Joukowski law used for the classical theory. This enables the method to be used for systems of lifting surfaces with arbitrary camber, sweep and dihedral. Further the described method accounts for nonlinear airfoil data by solving a nonlinear equation system. The equation system is given by the equation for the lift force on each panel

\[ L_P = C_L(\alpha, Re) q S_P \tag{80} \]

where \( \alpha \) and \( Re \) are a function of the local flow velocity, which is derived from the circulation \( \Gamma \) on all other panels applying the Biot-Savart law. For \( C_L \) nonlinear airfoil data can be used as function of \( \alpha \) and the Reynolds number \( Re \). Even stall regions can be calculated if the respective profile data is available. The three-dimensional vortex lifting law is then given by

\[ \vec{L}_P = \Gamma_P \rho s_P \times \vec{v}_P \tag{81} \]

A sketch of the resulting discretization is shown in Figure 13. In comparison to the vortex lattice method no chord-wise discretization is possible.
2.4 Static Aeroelastic Modeling

Restrictions  The lifting line theory is restricted to lifting surfaces with aspect ratios, $AR$, greater than 4 since no chord-wise discretization is made.

2.4 Static Aeroelastic Modeling

An elastic wing deflects if subjected to aerodynamic loading and this in return influences the aerodynamic forces. Static aeroelasticity neglects forces of inertia and only investigates the effects occurring from the static aerodynamic and structural forces. In order to obtain an aeroelastic model the structural model and the aerodynamic model are coupled. Multiple methods are available for the solution of fluid-structure interaction problems, for this application a standard surface coupling approach is applied. The coupling conditions for surface coupling are the equality of forces and displacements in between the fluid mechanic and the structural solution. The coupling approach as demonstrated in Figure 14 is also known as Dirichlet-Neumann coupling (Mehl, 2011). This solution method requires a staggered iteration procedure starting by solving for the fluid mechanic forces, then solving the structural problem, displacing the fluid mesh and reiterating until a converged solution is obtained. A converged solution implies a static equilibrium condition between external aerodynamic loads and internal elastic restoring forces.

2.4.1 Mesh Coupling

The surface coupling approach requires the coupling of the aerodynamic and the structural meshes. The coupling rules must convert the displacements of the beam axis (elastic axis) to the quasi two-dimensional aerodynamic mesh of the respective aerodynamic method. An example for the aerodynamic and structural mesh is shown in Figure 15. Displacing the aerodynamic mesh is done by first
2.4 Static Aeroelastic Modeling

![Fluid-Structure Coupling](image)

**Figure 14:** Dirichlet-Neumann fluid-structure coupling

**Figure 15:** Example for an aerodynamic and a structural mesh

determining the displacements on the center of each spanwise panel by linear interpolation. Then each panel is shifted by $u_x$ and $u_z$ and rotated by the twist deformation $\phi$ as sketched in Figure 16. Further the panel has to be rotated by the rotational displacement, $\Theta$, which describes the bending deformation of the wing. The aerodynamic forces are computed for the quarter-chord line of the profile in local panel coordinates and must be projected to an equivalent force on the elastic axis of the beam. This is done by the relations

\[
F_{z,\text{st}} = F_{z,\text{aer}} \quad (82)
\]

\[
F_{x,\text{st}} = F_{x,\text{aer}} \quad (83)
\]

and

\[
M_{y,\text{st}} = M_{y,\text{aer}} - F_{z,\text{st}}e_x \quad (84)
\]
2.4 Static Aeroelastic Modeling

where $ex$ is the distance in between the elastic axis and the quarter-chord line, also called excentricity.

2.4.2 Divergence

In case the structure does not have the required stiffness for a certain flight state a static equilibrium condition is not existent. The aerodynamic forces due to wing deflections overcome the elastic restoring forces and the wing will deflect until deterioration. This instability is called aeroelastic divergence. The occurrence of this phenomenon mainly depends on the dynamic pressure and the torsional stiffness of the wing. Forward swept wings are particularly sensitive to this instability since the forward sweep couples the bending deformation to torsional deformations in an unfavorable way, increasing the local angle of attack with increasing wing bending. The dynamic pressure at which divergence occurs is called divergence dynamic pressure. The general relation for the divergence condition is given by

$$Ku = \bar{q}_{\text{crit}} Qu$$  \hspace{1cm} (85)

where $Q$ is the static aerodynamic matrix and $\bar{q}_{\text{crit}}$ is the divergence dynamic pressure. Solving the eigenvalue problem

$$[Q - \lambda K]u = 0$$  \hspace{1cm} (86)

with $\lambda = \frac{1}{\bar{q}_{\text{crit}}}$ and taking the largest real value of $\lambda$ the divergence dynamic pressure can be found (Borglund and Eller, 2009). For a computational mesh as applied by the presented aerodynamic methods $Q$ is possibly not definable. To estimate the divergence condition a representative cross-section of the wing, usually at 75% span for a conventional wing can be investigated instead.

Figure 16: Mesh transformations
2.4.3 Aileron Reversal

Aileron reversal is another undesired phenomenon caused by the elasticity of a wing. The effect is caused by an aileron deflection, $\delta$, which causes a torsional moment that results in an elastic twist deformation of the wing. This twist deformation can reduce the efficiency of the rudder deflection and even reverse its effect. The aileron efficiency, $\eta$, and reversal depends on the dynamic pressure and the structural stiffness of the wing. The dynamic pressure at zero aileron efficiency is called reversal dynamic pressure. The aileron efficiency is determined by

$$\eta = \frac{C_{L\delta,\text{flex}}}{C_{L\delta,\text{rigid}}}$$

where $C_{L\delta}$ is the rolling moment derivative with respect to the aileron deflection $\delta$. Forward swept wings are less sensible to aileron reversal due to the beneficial coupling between bending and twist (Luber, 2010). This effect has to be kept in mind for the wing design since a certain aileron efficiency is required to keep the aircraft maneuverable.

2.5 Critical Design Loads

In order to perform the sizing of any load carrying structure the design loads have to be defined. Usually different load cases are distinguished where each of them represents a loading condition that the structure has to withstand. The definitions of structural strength requirements for the certification of commercial aircraft are found in (EASA, 2003), CS-25 Subpart C, issued by the European Aviation Safety Agency or in Part 25 of the Federal Aviation Regulations (FAR) defined by the Federal Aviation Administration (FAA). The requirements issued by both authorities are essentially the same.

The structural strength requirements can be separated into static and dynamic loading conditions. In the scope of this thesis the latter are disregarded since the model is purely static. Furthermore different loads are distinguished in the specification requirements, i.e. limit loads and ultimate loads. Limit Loads, also called maximum service loads, are the maximum loads expected in service. It is specified that no permanent deformation of the structure is allowed at limit load. At any load up to the limit loads, the elastic deformation may not interfere with safe operation.
2.5 Critical Design Loads

Ultimate loads are defined as the limit loads multiplied by a safety factor. The safety factor is specified as 1.5. The structure must be able to withstand the ultimate load for at least 3 seconds without failure. For ultimate loads permanent deformations of the structure are accepted.

In this thesis the limit loads are incorporated along with the allowable yield stress introduced in Section 2.2.1 which includes a factor of safety. As mentioned before, this factor of safety is set to $FoS = 1.5$, hence this is equivalent to a layout for the ultimate load with the yield tensile strength.

Each part of the aircraft is subject to many different loading conditions. In the final design of an aircraft structure, one might examine tens of thousands of loading conditions of which several hundred may be critical for some parts of the airplane. In addition to the obvious loads such as wing bending moments due to aerodynamic lift many other loads must be considered, this includes inertial relief due wing weight, fueling or engines that tend to reduce wing bending moments (Kroo, 2011).

For wing design the expected maneuver loads and gust loads are most crucial for sizing. Three loading conditions are taken into account for this work since they are considered to be the most critical for wing design. These are gust loads, symmetric maneuver loads and maximum aileron roll maneuver loads (Federal Aviation Authorities, 2011). Depending on the aircraft type these maneuvers can all contribute to the wingbox design but usually most parts of the wingbox are designed for 2.5g maneuvers. The outboard wing tip is generally designed for roll maneuvers. If winglets are added, the high winglet loads during sideslip maneuvers cause additional loadings on the wing tip area. Therefore, sideslip maneuvers can become the critical design case for the wing tip and winglet (Faye et al., 2011).

2.5.1 Design Speeds and the Placard Diagram

For aircraft design several design speeds are introduced (EASA, 2003), the most important are:

- Design cruising speed, $v_C$
- Design cruise Mach number, $Ma_C$
- Design dive speed, $v_D$
2.5 Critical Design Loads

- Design dive Mach number, $M_aD$

Additional design-relevant speeds are the design maneuvering speed, the design speed for maximum gust intensity and the design flap speed, but these are not required in the scope of this thesis.

The placard diagram as depicted by Figure 17 is an envelope that shows the structural design airspeeds as a function of altitude. The construction of the diagram is typically based on the aircraft cruise Mach number and the design altitude. The design cruise Mach number, $M_aC$, is typically 6% higher than the typical cruise Mach number, $M_{acruise}$, of the aircraft and is specified by the designer. $M_aC$ is equivalent to the maximum operating Mach number $M_{aMO}$. According to (Kroo, 2011) the design dive Mach number can be approximated as

$$M_aD = 1.07 M_aC$$  \hspace{1cm} (88)$$

The design airspeed $v_C$ is determined by the structural design altitude which is the minimum altitude at which the aircraft is able to fly at $M_aC$. This altitude must be specified by the designer and is determined by operational requirements. The example of a placard diagram shown in Figure 17 has a design altitude of 26000 ft. The diagram demonstrates that this altitude also defines the maximum dynamic pressure that occurs for this aircraft. The maximum dynamic pressure is crucial for the identification of critical structural design states. Descending from the critical design altitude the dynamic pressure is kept constant, hence the true airspeed is declining. According to (Kroo, 2011) the design dive speed $v_D$ can be approximated from $v_C$ by

$$v_D = 1.15 v_C$$  \hspace{1cm} (89)$$
2.5 Critical Design Loads

2.5.2 Maneuver Loads and the v-n Diagram

The v-n diagram is an envelope that plots the load factor versus airspeed, see Figure 18. Since many of the load requirements on aircraft are defined in terms of the load factor $n$, it is an important diagram for the structural layout of aircraft. The load factor is defined as the component of aerodynamic force perpendicular to the longitudinal axis divided by the aircraft weight. Assuming the angle of attack is not large, this results in $n = L/(W \cdot g)$ which is the effective perpendicular acceleration of the aircraft in units of $g$.

The v-n plot facilitates the identification of critical loads. The states found on each corner or kink of the v-n plot are usually critical cases, e.g. for maximum drag force and for maximum lift force. In the scope of this thesis, only the latter is considered. A v-n plot is however only valid for one altitude. Therefore the critical altitude given by the maximum dynamic pressure for the respective maneuver has to be found first in the placard diagram. Usually the crucial cases are the symmetric pull up maneuver and the rolling maneuver at maximum aileron deflection.

![Placard Diagram](image)

Figure 17: Design speeds vs. altitude (Kroo, 2011)
2.5 Critical Design Loads

Symmetric Maneuver Loads The symmetric maneuver is conducted at the design dive speed, hence \( v_D \) is the critical speed for the pull-up maneuver. The load factor is defined as \( 2.5g \) in (EASA, 2003).

Rolling Maneuver Loads The requirements for the rolling maneuver are more complex and given in terms of the rate of roll. The CS-25 states that at \( v_C \) the same rate of roll as the one produced at \( v_A \) with full aileron deflection has to be reached. At \( v_D \) only a third of this value has to be reached. For simplification it is assumed that the critical condition for the aileron maneuver is a maximum aileron deflection at \( v_C \). The required load factor for the maximum aileron maneuver is two thirds of the value used for the symmetric maneuver, hence \( 1.667g \) (EASA, 2003).

2.5.3 Gust Loads and Gust Envelope

For the gust load investigation a discrete sharp edged gust is assumed, dynamic gust responses are not regarded. Vertical as well as lateral gust load cases have to be considered for the structural layout. A vertical gust essentially results in an additional angle of attack whereas a lateral gust induces a side-slip angle. In both cases the result is a rapid change of the aerodynamic forces which accelerates the aircraft in the respective direction. The magnitude of the gust velocities can
be found in (EASA, 2003) and is given as a function of altitude and the aircraft design weights. The gust load factor is then found by

\[ n = 1 \pm \frac{1/2 \rho \cdot C_{L\alpha} \cdot v \cdot v_{gust} \cdot S}{W \cdot g} \]  

(90)

where \( v \) is the flight speed, \( v_{\text{gust}} \) is the gust velocity, \( W \) is the aircraft mass, \( C_{L\alpha} \) is the lift slope, \( S \) the wing reference area and \( \rho \) the air density (Megson, 2007). According to (Torenbeek, 1992) the maximum zero fuel weight (MZFW) is the critical condition for gust loads. The design gust velocity is given for the design cruise speed, \( v_C \). This gust velocity is then reduced linearly to fifty percent of the original value while increasing the flight speed to the design dive speed, \( v_D \). Therefore the maximum load factor for gust cases does not appear at \( v_D \) but rather at \( v_C \).

The gust envelope is a v-n diagram visualizing the load factor due to gust velocities vs. flight speed occurring at the respective flight states. An example is shown in Figure 19.

![Gust envelope diagram](image)
3 Implementation of a Tool for Aeroelastic Preliminary Wing Design

This section details the implementation of the aeroelastic design tool named dAEDalus developed during this thesis. At first, the tool requirements and features are listed briefly, then a general overview on the implementation approach is provided. Thereafter, single modules are explained in more detail.

3.1 Tool Features and Requirements

The focus of this thesis was to develop a tool for the aeroelastic preliminary design of unconventional wing configurations. The tool should be applicable for arbitrary planar and nonplanar wing configurations. Since not all future wing configurations are completely foreseeable, the method must be kept as generic as possible allowing flexibility for easy modifications and later extension. Depending on the given problem, linear or nonlinear structural models can be applied for computations. In addition, the aerodynamic solver can be selected, too. The wingbox design can either be performed by defining critical design cases or manually by user input. Flight state calculations on a previously sized wing must be possible allowing computation of deflections, lift distribution and internal forces for any given steady flight state. All this should require as little user input and information as possible.

3.2 Required Input Information

The required input for the computations is described qualitatively, for more details it is referred to (Seywald, 2011). The input information is divided into a general and a structural part.

3.2.1 General Input Information

This subsection explains the general input that needs to be provided for a computation.

**Wing geometry** The wing geometry has to be defined including the wing planform, profiles and the twist distribution. The wing can have multiple partitions, for
3.2 Required Input Information

each of them the sweep, dihedral, chord, twist and airfoil can be set separately. Depending on the aerodynamic method parameters for the aerodynamic mesh and solver settings need to be defined additionally.

Aircraft state The aircraft state is fully defined by the listed parameters:

- Speed
- Altitude
- Weight
- Fueling condition
- Deflection of control surfaces
- Load factor

Since only steady level flight conditions are assumed the angle of attack is determined by the aircraft weight multiplied by the load factor. For the critical design process a reference state needs to be provided, here the design cruise Mach number and design altitude must be set as described in Section 2.5.1.

Aircraft weights At least one aircraft weight needs to be defined, which is taken as a reference for design. This weight is typically the maximum take-off weight (MTOW) which is used to evaluate critical maneuver loads. For the evaluation of critical gust states two additional weights need to be provided, the maximum landing weight (MLW) and the maximum zero fuel weight (MZFW).

3.2.2 Structural Input Information

In addition, wingbox-specific structural information has to be provided.

Structural settings The wingbox structural settings include the structural boundary conditions, the front and rear spar position for each wing partition as well as the fueled span of the wing.
3.3 General Approach

**External weights and forces**  The position and mass of external weights and loads, other than the aerodynamic loads, have to be defined if applicable. This includes wing mounted engines and gears. Furthermore, the weight of the wing secondary structure needs to be set. This value accounts for the inertial relief of all structural wing components that are not part of the wingbox, for example control surfaces or leading and trailing edge components. The fraction of the wing secondary mass is typically between 20% and 40% of the total wing mass.

**Material properties**  The properties for the wingbox material need to be defined. This includes allowable tensile strength, young’s modulus, shear modulus and material density.

3.3 General Approach

Based on the input data described before, a wingbox layout can be performed. The procedure starts by calculating aerodynamic forces on the wing for a given flight state. The resulting loads are passed to the structural model also including additional external loads such as engine and gear forces. Now the required wingbox skin thickness distribution is calculated, which results in a certain wingbox stiffness and weight. The eigenmass of the resulting wingbox also contributes to the wing loading, therefore the sizing process has to be reiterated until a converged solution is obtained. This first sizing allows the computation of deflections which in turn requires the recalculation of the aerodynamic forces. The sizing of the structure is then reiterated until a quasi-static equilibrium between the structural and the aerodynamic forces is obtained. The described layout process is performed for every critical design state and an optimal wingbox structure for every state is computed. These structures are merged into one optimal structure able to withstand all critical loading conditions. A refinement loop can be performed with the obtained merged wingbox structure to improve the accuracy of the result.

With respect to the software architecture, a systematic and structured approach is essential to handle the complexity of the problem. An object oriented implementation facilitates the structuring of the code since functions and variables belonging together can be encapsulated in classes. These classes can easily be extended, exchanged and subclasses can be derived by inheritance. The platform used for implementation is MATLAB (MATLAB, 2010). The tool is separated into sev-
eral distinct modules with clearly defined interaction ports. The two fundamental modules are the structures module and the aerodynamics module. These form the basis for the other modules such as the aeroelastics module and the critical design module.

### 3.4 Module Description

This section details the previously introduced modules of the tool *dAEDalus* where the main focus is put on the structures module.

#### 3.4.1 Structures Module

The structures module implements the linear and nonlinear finite element models as described in the theoretical part of this thesis. It is capable of modeling single beams, as well as coupled systems of beams, which enables the modeling of complex wing systems. The module is shown in Figure 20 with its input and output ports.

At first, the module needs to be initialized with the wing geometry, material properties and boundary conditions. The height and width of the wingbox at an arbitrary cross-section is determined by the wing planform, the wing profile and the user-defined front and rear spar positions. These outer dimensions are set automatically during the initialization process. The module is then able to perform a structural self design based on the input loads. Once the structure has been sized, the internal forces, as well as deflections and stresses, can be computed.

![Figure 20: Input and output ports of the structures module](image)

The structures module has two main operational modes, i.e. the self-design mode and the flight state calculation mode. In the first mode an automatic layout of the structure based on the input loads is performed using the sizing rules defined in
Section 2.1. The flight state calculation mode is a standard finite element solution where deflections, stresses and internal forces are computed based on external forces. Instead of using the automatic layout functionality the beam model can as well be initialized manually by providing the element-wise stiffness and material properties.

**Self design mode** The self design process requires an internal loop as shown in Figure 21 since the initially unknown mass of the structure also contributes to the loading conditions. At first all known external forces are collected in a nodal load vector. This comprises aerodynamic loads, engine forces, gear forces or forces due to wing fueling and wing secondary structure. The compiled load vector along with the properties of the material is forwarded to the finite element beam model. Based on this input the internal forces are computed. Knowing the internal forces the cross-section is dimensioned element-wise using the sizing rules defined in Section 2.1, on page 11. Once the thickness of all elements has been computed the mass is calculated, and this multiplied with the gravitational acceleration vector provides the load contribution due to structural eigenmass. This load is added to the external forces in the following iteration step and the structural layout is repeated until convergence is obtained.

![Diagram](structures_module_load_based_self_design_mode.png)

**Figure 21**: Structures module running in self-design mode

**Flight state mode** Once the structure is sized and all required parameters are provided the structures module can be run in the flight state calculation mode, see
3.4 Module Description

Figure 22. In this mode the deflections, internal forces and stresses are calculated based on the provided input loading condition. This enables the calculation of the wing’s displacements at an arbitrary flight state.

![Structures module: Flight state calculation mode](image)

Figure 22: Structures module running in flight state calculation mode

**Software Architecture**  The object oriented architecture allows the collection of attributes and operations in classes. Figure 23 on page 43 shows all classes and their most important attributes (prefix -) and operations (prefix +) as well as their relations to each other in an UML class diagram (Ambler, 2005). Note that this figure does not contain the full set of attributes and operations but rather serves as an overview of the functionality. In the following, the most important classes and their purpose is explained shortly.

- **interface_structure** defines the interfaces any structures module must have in order to be compliant with the rest of the code. These interfaces are the ports of the module as also shown in Figure 20. Depending on the scenario, either a beam, a wing or a beam collection can act as structures module since all classes have the same interfaces.

- The **class_beam** owns a list of beam elements belonging to the same beam and a list of boundary conditions. The most important attributes are the global stiffness matrix, $K$, the global force vector, $f$, and the structural boundary conditions such as the clamping of the wing root. The class is able to assemble the global stiffness matrix and force vector from the element matrices of its elements. In addition it contains the solvers for the linear and nonlinear system for a simple
beam and implements the interface for structures. 

`class_wing` is a subclass of `class_beam` and contains wing specific extensions in addition to the general beam model. These extensions include information about wing mounted engines and gears and the excentricity for the aerodynamic interaction.

Sometimes a wing cannot be represented by a single beam model. For box-wings, joint wings, C-wings or other complex wing systems, it can be beneficial to represent each lifting surface as a separate beam. The `class_beam_collection` contains all functionalities that enable solving beam systems. It collects several classes of `class_beam` or its subclasses (e.g. `class_wing`) and has the capability of coupling beams together with certain coupling conditions. This class can solve the coupled system equations and updates the contained beams with respective information.

The `class_beam_element` contains all information required for a finite beam element. This includes the orientation of the element relative to the global coordinate system, the resulting transformation matrix, $T$, the element length, $l_e$, the element stiffness matrix, $K_e$, and the element force vector, $f_e$, along with all formulas required for their computation. Each beam element owns an object of cross-section as shown in the diagram.

The `class_crosssection_wingbox` encapsulates all cross-sectional information, including geometry, materials and the sizing rules for the cross-section. If a different cross-sectional model is required, this class merely has to be exchanged or modified.

### 3.4.2 Aerodynamics Module

The aerodynamics module provides an interface to both aerodynamic solvers described in Section 2.3 and is shown in Figure 24. The module can calculate the aerodynamic forces and coefficients based on a given aircraft state once it has been initialized with a wing geometry. In case the wing geometry is already deformed, the deflections can be provided as an additional input. Both aerodynamic tools have been extended with mesh morphing functionalities in order to enable the computation of the aerodynamic forces on a deformed mesh.

A UML class diagram for the aerodynamics module is shown in Figure 25. `interface_aero` defines all functions the aerodynamics module must have in-
3.4 Module Description

Figure 23: Class dependencies for structures module

Figure 24: Input and output ports of the aerodynamics module
dependent of the applied aerodynamic solver. The format of the input and output data for these functions is standardized. These unified ports make all other implementations independent of the used solver and a different aerodynamic method can be added with a minimum of implementation effort.

![Figure 25: Interface class for the aerodynamics module](image)

### 3.4.3 Aeroelastics Module

The aeroelastics module implements the fluid-structure interaction (FSI) solver required for aeroelastic computations. The FSI solver couples the structures and the aerodynamics module by forwarding the aerodynamic loads to the structures module. Then the deflections output of the structures module is connected to the aerodynamics module to deform the aerodynamic mesh according to the structural deformations. This creates a loop, as depicted in Figure 26, which is called the aeroelastic loop. One step of an aeroelastic computation is done by running the aerodynamic solver and then performing the structural computation. After each step, the convergence criterion is checked. If the convergence criterion is met, or a maximum number of iterations is reached, the calculation loop is stopped. The implemented solver requires that the change of the wing tip deflection is less than $1\%$ from one step to the next for a converged solution. Potentially occurring convergence problems may result from an instability condition such as divergence.

Two different FSI solvers are available: one for a simple flight state calculation as shown in Figure 26 and a second one for the structural design process, see the structural sizing loop in Figure 27. The wingbox sizing process is performed
on the aeroelastic model, since an elastic wing subjected to aerodynamic loading will influence the pressure distribution and hence the thicknesses and mass of the load-carrying structure.

![Diagram of aeroelastic loop]

**Figure 26: Standard aeroelastic loop**

### 3.4.4 Critical Design Module

The critical design module provides the functionality to perform the critical design of a wingbox. The two major tasks of the module are the assistance in the definition of critical design conditions and the execution of a critical wingbox self design. The most important critical design states according to the FAR, are predefined and implemented in the module. This includes the symmetric pull-up maneuver, the maximum aileron maneuver and critical gust states. In addition to these design states, the user is free to define other design criteria.

Figure 27 shows the process of a critical wingbox design. At first, a set of critical design cases is collected in a list of aircraft states as introduced in Section 3.2.1, page 37. An outer loop runs the structural sizing loop for each of these critical states. Thus, the optimal structure for each single loading condition is obtained. The different structures are merged into one combined optimal structure by taking the maximum thickness of each segment resulting from any of the single load cases. This is then the final result of the critical design process.
3.4 Module Description

![Flow chart of the critical design process](image)

Figure 27: Flow chart of the critical design process
4 Tool Validation and Evaluation of Results

In this section the developed software is validated by simulating different types of aircraft. At first, the structural model is validated separately to ensure its functionality in the aeroelastic loop. The aerodynamic solvers already have been validated in (Melin, 2000) and (Steiner, 2010). Afterwards the fully coupled aeroelastic model is validated in different steps, starting with a detailed analysis of an Airbus A320-200. For this aircraft the torsional and bending deformations as well as the wingbox mass are compared to reference data. Thereafter the wingbox masses for a range of aircraft of different size and type are computed and compared to available data. Finally, the wingbox mass for two innovative concepts, the C-wing and the box-wing, is predicted.

4.1 Validation of the Structural Model

The structural model is validated separately in order to ensure the proper functionality and correctness of the implemented structural finite element method. The validation is done by comparing the results to the established finite element code Nastran.

As sketched in Figure 28 a beam with a point load serves as validation object. Two different test cases are computed with both solvers, dAEDalus and Nastran. In the first case the beam is clamped at one end and a point load is applied on the other end. For the second case both ends are clamped and the load is applied in the middle.

![Validation cases](image)

Figure 28: Test scenarios for the structures model validation

The required details for the computations are given by the beam length, $l = 100$ in, the force of, $F = 6000$ lb, the young’s modulus of, $E = 30 \cdot 10^6$ lb/in$^2$, the Possion
ratio of, $\theta = 0.3$, and the dimensions of the solid cross-section profile with the height of, $h = 2 \text{ in}$, and the width of, $w = 1 \text{ in}$. For both cases different discretization levels are applied, dividing the beam into 8 or 16 equidistant elements. The resulting maximum deflections are presented in Table 1 and Table 2. The analytical result serves as reference for the linear solution. The relative difference between $dAEDalus$ and $Nastran$ is shown in the last row, where $Nastran$ is used as a reference.

Table 1: Maximum deflections for test case 1, in $[\text{in}]$

<table>
<thead>
<tr>
<th>Discretization</th>
<th>8 elements</th>
<th>16 elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>linear</td>
<td>nonlinear</td>
</tr>
<tr>
<td>Analytical</td>
<td>100</td>
<td>-</td>
</tr>
<tr>
<td>Nastran</td>
<td>98.71</td>
<td>56.65</td>
</tr>
<tr>
<td>$dAEDalus$</td>
<td>99.99</td>
<td>60.41</td>
</tr>
<tr>
<td>Rel. difference $[%]$</td>
<td>+1.29</td>
<td>+6.22</td>
</tr>
</tbody>
</table>

Table 2: Maximum deflections for test case 2, in $[\text{in}]$

<table>
<thead>
<tr>
<th>Discretization</th>
<th>8 elements</th>
<th>16 elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>linear</td>
<td>nonlinear</td>
</tr>
<tr>
<td>Analytical</td>
<td>1.56</td>
<td>-</td>
</tr>
<tr>
<td>Nastran</td>
<td>1.44</td>
<td>1.18</td>
</tr>
<tr>
<td>$dAEDalus$</td>
<td>1.56</td>
<td>1.25</td>
</tr>
<tr>
<td>Rel. difference $[%]$</td>
<td>+7.69</td>
<td>+5.60</td>
</tr>
</tbody>
</table>

The results obtained from $Nastran$ and $dAEDalus$ match well for the better refined discretization level using 16 elements whereas they differ more using only 8 elements. Since the differences for the higher amount of elements are very small it can be concluded that a sufficient $h$-convergence has been reached. $h$-convergence denotes the convergence with respect to discretization (Baier, 2010). Note that $Nastran$ uses a nonlinear Updated Lagrangian formulation whereas $dAEDalus$ incorporates a Total Lagrangian formulation. Finally it can be concluded that both FEM-models, the linear and the nonlinear, have been implemented correctly.
4.2 Tool Validation for Conventional Aircraft Types

Figure 29 shows the linear and nonlinear deformations resulting for both test cases using 16 elements. It can be seen that for large deflections the difference between linear and nonlinear theory is significant. The deflection for the nonlinear model is lower compared to the linear model due to large deflection and geometric stiffening effects. Large deflection effects result from the small angle approximations made in the linear theory. The geometric stiffening effect results from the normal stress induced by the deformation which additionally stiffens the structure.

Figure 29: Length-wise deflections computed by dAEDalus using 16 elements

4.2 Tool Validation for Conventional Aircraft Types

This section presents validation results for a group of conventional types of aircraft.

4.2.1 Validation for the Airbus A320-200

Because a large amount of data is accessible for the A320-200, it serves as the main validation case. The critical design state can be approximated using data found in (Airbus, 2002). The maximum operating limit speed is \( v_{MO} = 350 \text{ KIAS} \) and the maximum operating Mach number is \( M_{MO} = 0.82 \). The required design speeds can be obtained using the relations posed in Section 2.5.1, i.e. \( v_{C,EAS} = 181 \text{ m/s} \) and \( v_{D,EAS} = 208.15 \text{ m/s} \). The calculation mesh used for the simulations is shown in Figure 15 on page 28. Four different critical design cases are considered for the wingbox layout, these are:
4.2 Tool Validation for Conventional Aircraft Types

- 2.5g pull up maneuver at MTOW
- \( \frac{2}{3} \times 2.5g \) rolling maneuver at MTOW
- Gust load case with a discrete gust from above at MZFW
- Gust load case with a discrete gust from below at MZFW

Lateral gusts are not considered for the layout since the wing does not feature any winglets. In order to size the wingbox, a critical layout is performed as described in Section 3.4.4 considering the implemented critical load cases. The assumed tensile yield strength for the aluminum alloy is \( \sigma_{alw,lim} = 250 \, MPa \). Figure 30 shows the resulting critical design case for each discrete segment of the structure and Figure 31 shows the corresponding equivalent thickness distribution. Results for both aerodynamic methods are presented in order to point out associated differences, sensitivities and uncertainties.

![Figure 30: Resulting critical load cases for each wing segment](image)

**Validation of the wingbox mass** The actual A320-200 wing mass can be found in (LTH, 2006) and is given by \( W_{wing} = 8811 \, kg \). The wingbox mass is presented in (Ajaj et al., 2011), which sets the mass of the secondary wing structure to \( W_{wing,secondary} = 2644 \, kg \) or 30% of the total wing mass. This mass is assumed to be distributed uniformly across the wing and contributes to the inertial relief during the sizing process. Table 3 lists the results for both aerodynamic solvers and each load case, as well as the resulting mass for the combination of all critical design load cases.
4.2 Tool Validation for Conventional Aircraft Types

Figure 31: Equivalent skin thickness for different wing segments resulting from the critical wingbox sizing

Table 3: Calculated masses for the A320-200, in [kg]

<table>
<thead>
<tr>
<th>Design case</th>
<th>Wingbox</th>
<th>Wingbox</th>
<th>Wing</th>
<th>Wing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
<td>Tornado</td>
<td>PAWAT</td>
<td>Actual</td>
</tr>
<tr>
<td>2.5g</td>
<td>-</td>
<td>4844.4</td>
<td>4568.0</td>
<td>-</td>
</tr>
<tr>
<td>Max aileron</td>
<td>-</td>
<td>4887.4</td>
<td>4678.8</td>
<td>-</td>
</tr>
<tr>
<td>Gust</td>
<td>-</td>
<td>5079.4</td>
<td>4987.4</td>
<td>-</td>
</tr>
<tr>
<td>Combined</td>
<td>6167.0</td>
<td>5687.8</td>
<td>5510.6</td>
<td>8811.0</td>
</tr>
<tr>
<td>Deviation [%]</td>
<td>0.00</td>
<td>-7.77</td>
<td>-10.64</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Compared to the reference value from the literature the predicted wing mass is about 5 – 10 % lighter. First, the applied aerodynamic model has a significant influence on the results. Further reasons for the deviation have been identified and are listed in the following:

- **Manufacturability:** The wingbox sizing performed does not consider any manufacturing restrictions. Each element has the minimum thickness required to bear the load. In practice it is unrealistic to have a varying sheet thickness across the entire wing span. This could be solved by either introducing a general manufacturing penalty factor accounting for undersizing or a minimum skin thickness applied for entire wing partitions.
• **Allowable tensile strength:** Additional studies revealed that the allowable tensile yield strength has a high impact on the wingbox mass, hence an incorrect estimate of that value can lead to significant deviations.

• **Secondary wing masses:** An inappropriate estimation of the secondary wing mass results in an erroneous contribution to inertial relief which influences the wingbox mass.

• **Design point:** Since the design point defines the maximum dynamic pressure and the maximum aerodynamic forces that the aircraft is subjected to, it has a significant influence on the wingbox sizing.

In the following, the influence of the factors mentioned above are investigated in more detail.

**Influence of the tensile strength** A sensitivity study has been made to investigate the influence of the allowable tensile strength, $\sigma_{alw}$, on the resulting wingbox mass. A value of 250 MPa has been used as a reference and is varied in steps of 10 MPa which correlates to a change of 4%. The results are presented in Table 4 and demonstrate that already small changes of $\sigma_{alw}$ have a significant impact on the wingbox mass prediction. A variation of 10 MPa in the tensile strength changes the wingbox mass by approximately 4%. Furthermore, it can be seen that the sensitivities are approximately equal for both aerodynamic solvers.

<table>
<thead>
<tr>
<th>Tensile strength [MPa]</th>
<th>230</th>
<th>240</th>
<th>250</th>
<th>260</th>
</tr>
</thead>
<tbody>
<tr>
<td>TORNADO</td>
<td>6149.8</td>
<td>5909.8</td>
<td>5687.8</td>
<td>5482.0</td>
</tr>
<tr>
<td>Relative difference [%]</td>
<td>+8.12</td>
<td>+3.90</td>
<td>0</td>
<td>-3.62</td>
</tr>
<tr>
<td>PAWAT</td>
<td>5965.6</td>
<td>5729.4</td>
<td>5510.6</td>
<td>5307.6</td>
</tr>
<tr>
<td>Relative difference [%]</td>
<td>+8.26</td>
<td>+3.97</td>
<td>0</td>
<td>-3.68</td>
</tr>
</tbody>
</table>

**Influence of the secondary wing mass** Now, the sensitivity of the wingbox mass towards a change of the secondary wing mass is investigated. A fraction of 30% of the total wing mass is taken as reference and varied by ±10%. Table 5 shows the resulting wingbox masses for a secondary wing mass varying from 20% to 40% of the total wing mass, which is in a realistic range for most
4.2 Tool Validation for Conventional Aircraft Types

Aircraft. It was observed that a 10% change of the reference value influences the wingbox mass by approximately 1.5%, where the sensitivities again are almost independent of the aerodynamic solver.

Table 5: Influence of the secondary wing mass, in [kg]

<table>
<thead>
<tr>
<th>Secondary wing mass [%]</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>TORNADO</td>
<td>5770.2</td>
<td>5687.8</td>
<td>5606.2</td>
</tr>
<tr>
<td>Relative difference [%]</td>
<td>1.45</td>
<td>0</td>
<td>1.43</td>
</tr>
<tr>
<td>PAWAT</td>
<td>5595.0</td>
<td>5510.6</td>
<td>5426.4</td>
</tr>
<tr>
<td>Relative difference [%]</td>
<td>1.53</td>
<td>0</td>
<td>1.53</td>
</tr>
</tbody>
</table>

Influence of the design point Finally, the influence of the design point is investigated, which is represented by the dynamic pressure $\eta_{\text{design}}$. The used reference value is 26537 Pa, which results from the equivalent design dive speed of 208.15 m/s. This reference value is then varied by ±10%. The resulting wingbox masses are listed in Table 6, which shows that the variations increase or decrease the wingbox mass by approximately 2.5% independent of the aerodynamic solver. Using PAWAT, no results could be obtained for the reduced dynamic pressure, since the maximum lift coefficient is extended for the critical layout process, which means that stall occurs before the critical loading condition is reached.

Table 6: Influence of the maximum dynamic pressure on the wingbox mass, in [kg]

<table>
<thead>
<tr>
<th>Max. dyn. pr.</th>
<th>$\eta_{\text{design,ref}} - 10%$</th>
<th>$\eta_{\text{design,ref}}$</th>
<th>$\eta_{\text{design,ref}} + 10%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TORNADO</td>
<td>5557.8</td>
<td>5687.8</td>
<td>5838.0</td>
</tr>
<tr>
<td>Rel. diff. [%]</td>
<td>2.29</td>
<td>0</td>
<td>2.64</td>
</tr>
<tr>
<td>PAWAT</td>
<td>-</td>
<td>5510.6</td>
<td>5634.4</td>
</tr>
<tr>
<td>Rel. diff. [%]</td>
<td>-</td>
<td>0</td>
<td>2.64</td>
</tr>
</tbody>
</table>

From the sensitivity studies, it can be concluded that the allowable yield strength has the highest influence on the wingbox mass followed by the design dynamic pressure, whereas a change of estimated secondary wing structure has less impact. For example, a change of the allowable tensile strength by approximately
4.2 Tool Validation for Conventional Aircraft Types

4% increases or decreases the wingbox mass by 4%. In contrast, a change of the secondary wing mass by 10% makes the wingbox heavier or lighter by only 1.4%.

Comparison of wing displacements Next, the wing displacements resulting from the aeroelastic model are compared to the displacements of the actual aircraft. For this purpose, the jig shape is modeled and the flight shape is computed after performing a critical wingbox design. Figure 32 compares torsional and bending deformation resulting from the model to the data of the original aircraft.

![Deformations of the A320-200 wing in flight, $Ma = 0.78$, $C_L = 0.475$ and $h = 35000 \text{ ft}$ (Bauhaus Luftfahrt, 2011)](image)

It can be seen that there is a match between the original and the model. The deflections are slightly lower compared to the original values which can be a result from the idealized formulas for the cross-sectional stiffness parameters.
4.2.2 Further Validation on Existing Aircraft

The code has also been tested for a number of different aircraft. The investigated aircraft types include the Dornier Do-728, the Boeing 747-100 and the Bombardier Global Express. The input data is mainly found in (Jackson, 2000) and is listed in the Appendix. All critical layouts have been performed for the critical maneuver and gust load cases. Plots from the simulations are shown in the Appendix. Table 7 summarizes the obtained results and compares them to reference data found in (Ajaj et al., 2011). The predicted values all turn out slightly lighter compared to the reference values. Possible reasons for this have already been discussed in the previous subsection.

Table 7: Validation of wingbox mass prediction (TORNADO), in $[\text{kg}]$

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Do-728</td>
<td>2693.0</td>
<td>2371.4</td>
<td>-11.94 %</td>
<td>4356.0</td>
<td>4034.4</td>
<td>-7.38 %</td>
</tr>
<tr>
<td>A320-200</td>
<td>6167.0</td>
<td>5687.8</td>
<td>-7.77 %</td>
<td>8811.0</td>
<td>8331.8</td>
<td>-5.44 %</td>
</tr>
<tr>
<td>B747-100</td>
<td>21464.0</td>
<td>19528.2</td>
<td>-9.00 %</td>
<td>39184.4</td>
<td>37248.2</td>
<td>-4.94 %</td>
</tr>
<tr>
<td>Global Ex.</td>
<td>3387.0</td>
<td>3030.0</td>
<td>-10.54 %</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

4.3 Tool Application for Unconventional Wing Configurations

The model has been validated on a wide range of aircraft featuring different wing geometries and a varying number of wing mounted engines. Considering the model simplifications, all results were in an acceptable range, providing wingbox masses that are approximately 7.5 % to 12 % lighter compared to the reference values.

The implementation of nonlinear aeroelasticity enables the software to also predict the wingbox mass for wings with large deflections. Furthermore, coupled wing systems and multiple lifting surfaces can be modeled, which allows the simulation of unconventional wing configurations. Since the model does not contain any restrictions to a conventional wing, it can be concluded that the predicted mass for any other configuration is in the same range of accuracy as for the performed validation runs.

In order to compare the weight of an unconventional wing configuration to a con-
4.3 Tool Application for Unconventional Wing Configurations

A conventional wing, two ratios are introduced: the wing weight divided by the maximum takeoff weight

\[ E_1 = \frac{W_{wingbox}}{W_{MTOW}} \]  \hspace{1cm} (91)

and the wingbox mass per planform area,

\[ E_2 = \frac{W_{wingbox}}{S} \]  \hspace{1cm} (92)

### 4.3.1 Analysis of a Box-Wing

The first application scenario is the Bauhaus Luftfahrt Claire-Liner featuring a box-wing design. The critical design speeds are assumed with \( v_{C,EAS} = 180 \) m/s and \( v_{D,EAS} = 200 \) m/s, the maximum takeoff weight is given by \( W_{MTOW} = 167 \) t. Figure 33 shows the lift distribution for the undeflected wing shape which is close to the butterfly lift distribution of a best wing system as suggested by Prandtl (Prandtl, 1924). The internal forces of the jig shape at \( n = 1g \) are visualized in Figure 34. The moment and force distributions are standardized by the maximum appearing value in order to point out the forces and moments that are most crucial for the wing sizing. The shear forces and the wing bending moment caused by the aerodynamic lift force outweigh the other forces and moments in their magnitude. Their distribution differs from a conventional wing since the box-wing is double clamped and statically overdetermined. It can be seen that these forces are carried partly by the front and the rear wing in contrast to conventional wings or the C-wing configuration where the entire shear force and bending moment is carried by the wing root.

![Box-wing lift distribution for the undeflected shape](image-url)
4.3 Tool Application for Unconventional Wing Configurations

Figure 34: Box-wing internal forces and moments in local coordinates at $1g$ flight

- $Q_z_{\text{max}} = 409.9\text{kN}$
- $M_x_{\text{max}} = 3.78\text{MNm}$
- $Q_y_{\text{max}} = 12.0\text{kN}$
- $M_t_{\text{max}} = 327.5\text{kNm}$
- $Q_x_{\text{max}} = 11.1\text{kN}$
- $M_z_{\text{max}} = 148.9\text{kNm}$
Critical design  For the mass prediction, a structural self-design is performed for a 2.5\(g\) load case. Figure 35 displays the lift distribution for a deflected wing at a 2.5\(g\) maneuver. It can be seen that the lift distribution is strongly shifted towards the rear wing. The reason for this is the bending torsion coupling of the rear wing showing a nose-up airfoil rotation for increased lift which increases the lift even further. For the front wing this effect is the opposite, thus the local angle of attack is reduced for higher wing loading. The coupling of the wing tips with the tip fin can not fully prevent the unfavorable effect from the forward swept rear wing. The aerodynamic loads across the tip region of the front wing appear to be lower compared to the 1\(g\) load case, which implies that the 1\(g\) loading condition is critical for a small part of the wing, cp. Figure 36. Figure 37 shows the resulting skin thickness distribution. For the rear wing the required skin thickness is much higher than for the front wing because the wing loading for the 2.5\(g\) maneuver is clearly shifted backwards. From a dynamic point of view, it is questionable if the configuration is capable of performing a 2.5\(g\) maneuver because of the strong increase of the overall nose-down pitching moment. Finally, Figure 38 shows the original and bent box-wing including the aerodynamic and structural mesh.

Mass prediction  The overall wing mass considering the 1\(g\) and the 2.5\(g\) load cases results with \(W_{\text{wing}} = 27674\) \(kg\). In order to compare the box-wing weight to conventional designs, the parameters \(E_1\) and \(E_2\) are calculated and shown in Table 8. Both weight efficiency ratios are considerably higher compared to conventional configurations, hence the structural mass efficiency is lower for a box-wing. This means that the aerodynamic benefit for a box-wing may be diminished by higher structural weight. On the other hand, weight savings can be obtained because of the omission of the horizontal stabilizer. Which effect is predominant for a certain mission remains to be investigated.

Linear and nonlinear results  Next, the influence of the structural nonlinearities is presented. Figure 39 shows the wing deformations for a 2.5\(g\) maneuver for both the linear and the nonlinear model. It is obvious the beam becomes elongated for the linear model because of the approximation \(\cos\Theta = 1\), cp. Section 2.2.4. For the nonlinear model the beam length stays constant also in deflected state resulting in a slight span reduction of \(\Delta y\). 
4.3 Tool Application for Unconventional Wing Configurations

Figure 35: Box-wing lift distribution at a 2.5g maneuver

Figure 36: Critical load cases for the box-wing configuration (gusts not considered)
4.3 Tool Application for Unconventional Wing Configurations

Figure 37: Equivalent skin thickness distribution for the box-wing configuration

Figure 38: Box-wing deformations for a $2.5g$ maneuver
4.3 Tool Application for Unconventional Wing Configurations

4.3.2 Analysis of a C-Wing

The C-wing is another unconventional configuration that significantly reduces induced drag compared to a conventional wing and is therefore used as a second test scenario. Figure 40 shows the modeled C-wing configuration including the structural and aerodynamic mesh. The maximum takeoff weight is set to $W_{MTOW} = 100\, t$ and the design airspeeds are chosen equal to the ones used for the box-wing scenario. The internal forces resulting for the $1g$ state are displayed in Figure 41, again forces and moments are standardized by the largest appearing values. It can be seen that the down force produced by the rear wing slightly unloads the front wing tip compared to a conventional wing, which reduces the wing root shear force and bending moment. Secondly, the down force of the rear wing induces a significant torsional moment at the wing tip of the front wing due to the lever arm of the tip fin.

Critical Design  Again the $1g$ and $2.5g$ load case are considered for the critical layout, the deflections for the $2.5g$ maneuver are visualized in Figure 42. Figure 43 shows the critical loading condition for the different wingbox segments. A similar phenomenon is observed as for the box-wing. Certain parts of the wing are loaded higher in the $1g$ level flight case than for the $2.5g$ pull-up maneuver, this includes the tip area of the front and rear wing, as well as parts of the tip.
4.3 Tool Application for Unconventional Wing Configurations

Figure 40: C-wing deformations for a $2.5g$ maneuver

fin. The reason why the local wing loading can decrease for parts of the wing, even if the global load factor is increasing, is the torsional coupling between the front and the rear wing. This torsional coupling is caused because the increased lift, as occurring for a $2.5g$ maneuver, generates a nose-down torsion of the front wing, which also reduces the incidence angle of the rear wing due to the structural connection by the tip fin. This, in turn, increases the down force of the rear wing, which generates a torsional moment, increasing the front wing tip’s incidence angle.

Figure 44 shows the resulting skin thickness distribution for a C-wing. The webs of the front wing appear to be significantly thicker compared to conventional wings or box-wings since the torsional moment induced from the rear wing on the front wing has to be carried.

**Mass Estimation** The resulting total wing weight is $W_{\text{wing, total}} = 9404 \text{ kg}$. Table 8 shows the weight efficiency parameters of a C-wing. The computed parameters are lower compared to reference values of conventional wings.
4.3 Tool Application for Unconventional Wing Configurations

Figure 41: C-wing internal forces and moments in local coordinates at $1g$ flight

- $Q_z$ with $Q_{z_{\text{max}}} = 498.9 \text{kN}$
- $Q_y$ with $Q_{y_{\text{max}}} = 43.6 \text{kN}$
- $Q_x$ with $Q_{x_{\text{max}}} = 28.6 \text{kN}$
- $M_x$ with $M_{x_{\text{max}}} = 2.98 \text{MNm}$
- $M_t$ with $M_{t_{\text{max}}} = 459.6 \text{kNm}$
- $M_z$ with $M_{z_{\text{max}}} = 237.7 \text{kNm}$
4.3 Tool Application for Unconventional Wing Configurations

Figure 42: C-wing in deflected and non deflected state for a 2.5g maneuver (front view)

Figure 43: Critical load cases for the C-wing
4.3 Tool Application for Unconventional Wing Configurations

Figure 44: Equivalent skin thickness distribution for the C-wing

Table 8: Weight efficiency parameters of C-wing vs. conventional wings

<table>
<thead>
<tr>
<th>Parameter</th>
<th>box-wing</th>
<th>C-wing</th>
<th>A320-200</th>
<th>Do728</th>
<th>Boeing 747-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1 [%]</td>
<td>16.57</td>
<td>9.40</td>
<td>11.98</td>
<td>11.47</td>
<td>12.17</td>
</tr>
<tr>
<td>E2 [kg/m²]</td>
<td>93.72</td>
<td>43.38</td>
<td>71.98</td>
<td>58.08</td>
<td>76.68</td>
</tr>
</tbody>
</table>

The lower weight of a C-wing results from the down force produced by the rear wing which reduces the root bending moment for the front wing. However, the torsional moment on the front wing is increased due to the big lever arm of the rear wing.

The C-wing seems to combine structural and aerodynamic benefits, however the system of a C-wing is very complex to handle since the twist angles of the front and rear wing are coupled directly. More details remain to be investigated but outside the scope of this work.
5 Summary and Outlook

In the following, the performed work and the outcome of this thesis are summarized and suggestions for further work are provided.

5.1 Summary

With this thesis, an aeroelastic design tool including nonlinear structural and aerodynamic methods has been developed, implemented, validated and tested. At first, appropriate structural and aerodynamic models were selected. Then a finite element code for beam structures including a self-design functionality for wing-box cross-sections was developed, incorporating both linear and nonlinear theory. Available aerodynamic solvers were extended with mesh morphing functionalities to enable the aeroelastic coupling. Next, the fluid-structure interaction solver for the structural sizing of an elastic wing, and for regular flight state calculations was realized. Finally, additional functionalities allowing a critical wingbox design were implemented according to FAR design requirements.

The resulting tool enables the preliminary structural design of arbitrary wings and wing systems with a small amount of user input data. The main features of the preliminary design tool include:

- Modeling of very complex wings and wing systems
- Element-wise wingbox self design considering critical load cases
- Easy implementation of critical load cases based on FAR design requirements or user expertise
- Consideration of discrete points loads like engine and gear forces
- Wing and wingbox mass prediction
- Calculation of deflections, rotations and internal forces for quasi-static flight states

The turn-around time of the tool is relatively short, an experienced user is able to model an aircraft within less than four hours. The computation itself takes about five minutes for an aircraft such as the A320-200 considering four different critical load cases. Wing design parameters, such as twist, can be modified easily to obtain desired properties.
5.2 Outlook

The tool has been validated on a number of aircraft of different size and configuration. Reasonable results were obtained for all modeled cases. The predicted wingbox masses are generally slightly lighter compared to the reference values, possible reasons have been named and investigated. A strong dependency of the predicted mass on the allowable tensile yield strength and the design point has been observed. Since these input parameters are undefined during the early design phase, they have to be estimated. Further it has been demonstrated that the method is capable of analyzing very complex wing configurations such as the box-wing and the C-wing. The analyses have shown that the classical design criteria applied for conventional wings are no longer sufficient for unconventional configurations, thus additional critical design states have to be established and implemented.

The object oriented, module-based software can be further extended in many aspects to increase the level of detail of the analysis.

5.2 Outlook

Some ideas for potential code extensions will now be listed in a prioritized order.

- **Refinement of cross-section model** The cross-section model used in this code is relatively simple and is only intended as a first approximation. Section 4.2.1 shows that the resulting wingbox weight is relatively sensible with regard to the allowable tensile stress. Since the elements of a wingbox are usually made of different materials, better results are expected by implementing a multi-material model. In addition, the simplified rectangular cross-section should be extended to a more realistic shape like a polygon.

- **Elastic fuselage and tail surfaces** The aeroelastic model can also be transferred to a fully aeroelastic aircraft model. The coupling of beams and their computation is already implemented for the purpose of handling complex wing geometries. A possible approach would be to implement a further derivative from the class beam, for the fuselage and the fin with representative cross-sections. The visualization functionality then also has to be extended. A flexible beam could be important for heavily staggered box-wing configurations since bending of the fuselage will influence the wing root incidence of the front and rear wing.
• **Preliminary sizing for fuselage and tail surfaces** Once the fuselage and tail surfaces are structurally modeled, critical design cases and sizing rules for the self-design can be defined. This would enable the preliminary layout of an entire aircraft within a couple of minutes based on the most critical design cases. A mass breakdown of the aircraft will then be available shortly after the first design sketch has been made. Since there is no restriction in the number of critical design cases the layout can be refined by adding more potentially critical cases to the layout process.

• **Optimization and parameter studies** Optimization and parameter studies are very important in preliminary aircraft design. An additional set of functions would be a useful extension of the code and could be incorporated as an additional module.

• **Refinement of critical loading conditions** So far, relatively conservative critical loading conditions are assumed since the fuel distribution along the wing is unknown. An extension of the class wing would offer the consideration of all possible fueling and weight conditions for all maneuvers as soon as the respective information is available.

• **Analytical wing rib model** The main purpose of wing ribs is to prevent buckling. In the scope of this thesis, the structural analysis of ribs is neglected and therefore skin buckling cannot be considered. The mass of the ribs is, however, taken into account using a Torenbeek estimation formula. In order to enable a more accurate prediction of the rib mass, an analytical model could be implemented considering buckling criteria.

• **Dynamic aeroelasticity** A further step could be the extension of the static equations to dynamic equations, which then will permit the analysis of flutter phenomena. This can be done without a huge implementation effort, the required mass matrix can be computed with the available mass information obtained by the critical design. An instationary aerodynamic model would have to be implemented or linked to the software. Another possibility would the approximation of the instationary aerodynamic forces by a Theodorsen function.

• **Extended nonlinearities** The usage of the nonlinear method may cause convergence problems because the Newton-Raphson method applied in the
5.2 Outlook

code is not able to follow the equilibrium path for all possible cases. To improve convergence, extended nonlinear solution procedures such as load stepping and the arc-length method could be implemented. Then, nonlinear effects such as snapping can also be investigated.
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A.1 Simulation Input Data

Appendix: Simulation Input Data and Results

A.1 Simulation Input Data

A.1.1 Dornier Do-728

\[
\begin{align*}
ALT &= 26000 \text{ ft} \\
M_C &= 0.82 \\
\sigma_{alw} &= 250 \text{ MPa} \\
\end{align*}
\]

MTOW=37990 kg
MZFW=31450 kg
MLW=35250 kg

Figure 45: Structural and aerodynamic mesh for the Dornier 728

A.1.2 Boeing 747-100

\[
\begin{align*}
ALT &= 30000 \text{ ft} \\
M_C &= 0.90 \\
\sigma_{alw} &= 250 \text{ MPa} \\
\end{align*}
\]

MTOW=322050 kg
MZFW=238815 kg
MLW=265350 kg
A.1 Simulation Input Data

Figure 46: Structural and aerodynamic mesh for the Boeing 747-100

A.1.3 Bombardier Global Express

\[ ALT = 26000 \text{ ft} \]  \quad \text{MTOW}=44452 \text{ kg}
\[ M_{C}=0.82 \quad \text{MZFW}=25401 \text{ kg} \]
\[ \sigma_{\text{alw}}=250 \text{ MPa} \quad \text{MLW}=35652 \text{ kg} \]
A.2 Simulation Results

A.2.1 Dornier Do-728

Figure 47: Structural and aerodynamic mesh for the Global Express

Figure 48: Dornier 728 wing deformation during maximum aileron maneuver
A.2 Simulation Results

Figure 49: Critical design cases for the Do728

Figure 50: Equivalent skin thickness distribution for the Dornier 728
A.2 Simulation Results

A.2.2 Boeing 747-100

Figure 51: Equivalent skin thickness distribution for the Boeing 747-100

A.2.3 Bombardier Global Express

Figure 52: Equivalent skin thickness distribution for the Global Express