Estimation of reduced electrical distribution losses depending on dispersed small scale energy production

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1. ABSTRACT

The value of dispersed energy production in the power system mainly origin from the produced energy. In addition to this, there is normally a capacity credit. A third factor which is not always considered in power system studies is the impact on the electrical distribution losses. When power production is installed in a distribution system, the electricity to the local consumers is not transported so far and the consequence is that the energy consumed in the transmission lines and cables is reduced. These reduced losses is an extra value for power sources located close to the consumers. The size of this value is very system dependent and is in the range of some percents of total energy production [1,2].

In this paper the loss reduction effect of a local generator is estimated using second order sensitivities.

KEYWORDS : Distribution system, dispersed generation, energy economics, wind power, energy losses, sensitivity analysis.

2. INTRODUCTION

In Sweden small scale power production is defined as units with an installed capacity of less than 1500 kW. There are around 1200 small hydro power stations with a total installed capacity of 400 MW and 160 wind power stations with a total installed capacity of 35 MW. The intention of the new suggested legislation [3] concerning these small stations is that the owner of a small scale power plant, connected to a distribution grid, should get compensation from the owner of the distribution grid. This compensation should include the value of reduced energy losses in the distribution grid depending on the production in the power plant.

The reduced energy losses can be estimated by calculating the power losses for several representative load and production situations. This can be performed with one load flow calculation for each situation. To reduce the calculation time and to obtain a better overview of the loss reduction potential, first and second order sensitivities can be introduced. First order sensitivities are e.g. applied in [6] but here the accuracy is improved using second order sensitivities.

In section 3 basics for load flow and loss calculations are introduced and in section 4 sensitivity analysis is described. In section 5 the method used to estimate the energy losses is found and in section 6 the method is applied to a real 95 bus system.

3. LOAD FLOW AND POWER LOSSES

For the load flow calculation a formulation with only slack and PQ nodes is used, since a distribution system is considered. The dispersed small scale energy source is here modelled as an active and reactive power source without voltage regulation. In the presentation, node 1 is the slack node and nodes 2→N are PQ-nodes. A full Newton-Raphson algorithm is used [5].

For each node in the power system the following equations for active and reactive power injections in the nodes can be derived.

\[ P_i = V_i \sum_{k=1}^{N} V_k (G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}) \]  

\[ Q_i = V_i \sum_{k=1}^{N} V_k (G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik}) \]  

where

\[ V_i \Delta \delta_i = \text{voltage in node } i \]

\[ \delta_{ik} = \delta_i - \delta_k \]

The matrices \( G \) and \( B \) are real and imaginary parts of the node admittance matrix. The demand, \( P_{Di} + jQ_{Di} \), is known at all nodes which implies that the net generation \( P_{Gi} + jQ_{Gi} = -P_{Di} - jQ_{Di} \). In these nodes the active and reactive power mismatches are

\[ \Delta P_i = P_{GiD} - P_i \]

\[ \Delta Q_i = Q_{GiD} - Q_i \]

In this formulation the dispersed generation, \( P_{Gi} + jQ_{Gi} \), is modelled as a negative load.

The state variables for the system are defined as

\[ x = [\delta_2 \, \delta_3 \ldots \delta_N \, V_2 \, V_3 \ldots V_N]^T \]
and since we will study the consequence of a modified load and/or dispersed generation the disturbance variables are net generation of active and reactive power in the PQ-nodes:

\[
u = [P_{G21} P_{G31} \ldots P_{GDN} Q_{G21} Q_{G31} \ldots Q_{GDN}]^T
\]  

(5)

The set of nonlinear equations solved by the Newton-Raphson method can be written

\[
F(x, u) = \begin{bmatrix} \Delta P(x, u) \\ \Delta Q(x, u) \end{bmatrix} = 0
\]

(6)

The system total active power losses can be expressed as the sum of the active power injections in (1) as

\[
P_L(x, u) = \sum_{i=1}^{N} V_i \sum_{k=1}^{N} V_k (G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik})
\]

(7)

4. Sensitivity Analysis of Power Losses

By the use of a second order Taylor approximation of (7), the following expression for the modified power losses as a function of modified disturbance and state variables can be written with \(W(x, u) = P_L(x, u)\) [4]:

\[
\Delta P_L = \Delta W = W_x \Delta x + W_u \Delta u + \frac{1}{2} \Delta x^T W_{xx} \Delta x + \frac{1}{2} \Delta u^T W_{uu} \Delta u + \Delta u^T W_{ux} \Delta x
\]

(8)

This equation will now be further explained.

A first order approximation of the function \(F\) calculated around the solution of (6) (i.e. \(F = 0\)) is

\[
\Delta F = F_x \Delta x + F_u \Delta u
\]

(9)

where \(F_x\) and \(F_u\) are Jacobian matrices. By introducing (5) in (3), \(F_u\) can be estimated as

\[
F_u = I
\]

(10)

where \(I\) is the identity matrix. With a modification of the disturbance variables, the load flow equations should still be fulfilled, i.e. \(\Delta F = 0\). This implies that (9) can be rewritten as

\[
\Delta x = -F_x^{-1} \Delta u
\]

(11)

The loss equation gradient vectors \(W_x\) and \(W_u\) can be estimated as

\[
W_x = \begin{bmatrix} \frac{\partial^2 P_L}{\partial \delta^2} \\ \frac{\partial^2 P_L}{\partial V^2} \end{bmatrix}
\]

(12)

\[
W_u = \begin{bmatrix} \frac{\partial^2 P_L}{\partial P_{GD}} \\ \frac{\partial^2 P_L}{\partial Q_{GD}} \end{bmatrix}
\]

(13)

where

\[
\frac{\partial P_L}{\partial \delta_i} = -2V_i \sum_{k=1}^{N} V_k G_{ik} \sin \delta_{ik}
\]

(14)

\[
\frac{\partial P_L}{\partial V_i} = 2 \sum_{k=1}^{N} V_k G_{ik} \cos \delta_{ik}
\]

(15)

\[
\frac{\partial P_L}{\partial P_{GD}} = 0
\]

(16)

\[
\frac{\partial P_L}{\partial Q_{GD}} = 0
\]

(17)

The loss equation Hessian matrices \(W_{xx}\), \(W_{uu}\) and \(W_{ux}\) are defined as

\[
W_{xx} = \begin{bmatrix} \frac{\partial^2 P_L}{\partial \delta \partial \delta} \\ \frac{\partial^2 P_L}{\partial V \partial \delta} \end{bmatrix}
\]

(18)

\[
W_{uu} = \begin{bmatrix} \frac{\partial^2 P_L}{\partial V \partial V} \\ \frac{\partial^2 P_L}{\partial P_{GD} \partial Q_{GD}} \end{bmatrix}
\]

(19)

\[
W_{ux} = \begin{bmatrix} \frac{\partial^2 P_L}{\partial \delta \partial P_{GD}} \\ \frac{\partial^2 P_L}{\partial \delta \partial Q_{GD}} \end{bmatrix}
\]

(20)

where

\[
\frac{\partial^2 P_L}{\partial \delta^2} = -2V_i \sum_{k=1}^{N} V_k G_{ik} \cos \delta_{ik}
\]

(21)

\[
\frac{\partial^2 P_L}{\partial \delta \partial \delta} = 2V_i V_k G_{ik} \cos \delta_{ik}
\]

(22)

\[
\frac{\partial^2 P_L}{\partial V \partial V} = -2 \sum_{k=1}^{N} V_k G_{ik} \sin \delta_{ik}
\]

(23)

\[
\frac{\partial^2 P_L}{\partial \delta \partial P_{GD}} = -2V_i V_k G_{ik} \sin \delta_{ik}
\]

(24)

\[
\frac{\partial^2 P_L}{\partial \delta \partial Q_{GD}} = 2G_{ii}
\]

(25)

\[
\frac{\partial^2 P_L}{\partial V \partial V} = 2G_{ik} \cos \delta_{ik}
\]

(26)

\[
W_{uu} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
\]

(27)

\[
W_{ux} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
\]

(28)

Equation (8) can now be written as

\[
\Delta P_L = A \Delta u + \frac{1}{2} \Delta u^T D \Delta u
\]

(29)

where

\[
A = -W_x F_x^{-1}
\]

(30)

\[
D = (F_x^{-1})^T W_{xx} F_x^{-1}
\]

(31)

5. ESTIMATION OF ENERGY LOSSES

How to estimate the system energy losses will here be illustrated with a simple power system where a
transmission line feeds a load. There is also a local generator in the load bus. The transmission line, with line impedance \( R + jX \) is fed in one end with a constant voltage \( V_1 \angle \delta_1 = 0^\circ \). This power system

\[
V_1 \angle \delta_1 = 0^\circ \quad R + jX \quad V_2 \angle \delta_2 \quad P_{G2} + jQ_{G2} \quad P_{D2} + jQ_{D2}
\]

Figure 1: Power transmission in a one-line system

now consists of two nodes where node 1 is the slack bus and node 2 is a PQ-bus. For each load and production situation the load flow problem can be formulated according to (3) and by solving (6) the resulting values of \( V_2 \) and \( \delta_2 \) are obtained. From this solution the system power losses are estimated with (7).

Assume that \( V_1 = 10 \text{ kV} \) and \( R + jX = 3.5 + j3.5 \Omega/\text{phase} \). With a constant load power factor \( \cos \phi = 0.95 \), lagging, the system losses as a function of net load level can be estimated. In figure 2 the losses as a function of net generation (= negative net load) is shown. Assume then a simple case where we

![Graph](image)

Figure 2: System losses as a function of net load level

have a constant load, \( P_{D2} = 0.9 \text{ MW} \) and a generator which during 50% of a year produces \( P_{G2} = 0.6 \text{ MW} \) at power factor 0.95, lagging. During the other half of the year the unit is not in operation. This implies that there are two net generation situations: \( P_{GD2-A} = -0.9 \text{ MW} \) when the unit is not operating and \( P_{GD2-B} = 0.6 - 0.9 = -0.3 \text{ MW} \) when the unit is in operation. The power losses for these two situations are shown in figure 2.

The yearly generated energy in node 2 is

\[
W_{G2} = 8760 \cdot 0.50 \cdot P_{G2} = 2628 \text{ MWh} \tag{32}
\]

and the yearly mean losses can be estimated as

\[
P_{L-AB} = 0.5 P_{L-A} + 0.5 P_{L-B} =
\]

\[
= 0.5 \cdot 0.0344 + 0.5 \cdot 0.0036 =
\]

\[
= 0.0190 \text{ MW}
\]

The yearly energy losses without local generation is

\[
W_{L1} = 8760 \cdot P_{L-A} = 301 \text{ MWh}
\tag{34}
\]

and with generation it becomes

\[
W_{L2} = 8760 \cdot P_{L-AB} = 166 \text{ MWh}
\tag{35}
\]

This implies that a local generation of 2628 MWh reduces the losses with 301-166 = 135 MWh i.e. it reduces the delivered energy in node 1 with 2628+135 = 2763 MWh which corresponds to \( W_{G2} + 5.1 \% \). This effect is here denoted \( C = \text{loss reduction effect} \).

\[
C = 5.1 \% \tag{36}
\]

An approximative way of estimating the loss reduction caused by a local generation source is to only perform one load flow calculation for a mean load situation, and compare this value with the original losses. Applying this method on the here used system implies that the load flow is performed at \( P_{GD2-AB} = 0.5 P_{GD2-A} + 0.5 P_{GD2-B} = -0.6 \text{ MW} \). The losses at this level are

\[
P_{L-appr} = P_{L}(-0.6) = 0.0148 \text{ MW}
\tag{37}
\]

so the approximative energy losses can be estimated as

\[
W_{L-appr} = 8760 \cdot P_{L-appr} = 130 \text{ MWh}
\tag{38}
\]

and the loss reduction effect becomes

\[
C_{app} = 100 \cdot \frac{301 - 130}{W_{G2}} = 6.5 \%
\tag{39}
\]

i.e. an overestimation of the loss reduction caused by the local generator.

In figure 3 it is shown that the approximative method underestimates the mean losses i.e. it overestimates the loss reduction caused by the local generator. This depends on the nonlinear system loss function.

![Graph](image)

Figure 3: Approximative loss error

Now we will apply first order sensitivities to this example. With the method described in section 4 we can estimate \( A \) in (29) for the mean net generation point to

\[
A(P_{GD2} = -0.6) = [-0.0457 - 0.0158] \tag{40}
\]

If we neglect the second order terms the losses at different net generation levels can be estimated as

\[
P_{L-1} = P_{L}(-0.6) + A \cdot \frac{\Delta P_{GD2}}{\Delta Q_{GD2}}
\tag{41}
\]

where

\[
\Delta P_{GD2} = P_{GD2}(-0.6)
\tag{42}
\]
\[ \Delta Q_{GD2} = \frac{Q_{GD2} - Q_{GD2}(P_{GD2} = -0.6)}{\sin \phi / \cos \phi} \Delta P_{GD2} = 0.3287 \cdot \Delta P_{GD2} \]

since we assume a constant generation and load power factor. Now (41) can be rewritten as

\[ P_{L-1} = P_L(-0.6) + \lambda_{PGD2}[P_{GD2} - (-0.6)] \quad (44) \]

where \( \lambda_{PGD2} \) is the loss function slope in the point \( P_{GD2} = -0.6 \). \( \lambda_{PGD2} \) can be estimated as

\[ \lambda_{PGD2} = \frac{1}{\sin \phi / \cos \phi} \begin{bmatrix} 0.0457 \\ -0.0158 \end{bmatrix} = -0.051 \quad (45) \]

As shown in figure 4, using first order sensitivities

![Figure 4: First order loss sensitivity](image)

The losses for both operation states. The loss reduction effect, can now be estimated as

\[ C_1 = 100 \cdot \frac{0.5P_{L-1}(-0.9) - 0.5P_{L-1}(-0.3)}{0.5 \cdot 0.6} = \]

\[ = 100 \cdot \lambda_{PGD2} = 5.1 \% \quad (46) \]

i.e. the loss function sensitivity calculated from the first order sensitivities gives an approximative estimation of the loss reduction effect of a dispersed generator.

It can be noted that in (46) the losses at no production is also estimated using the piece-wise line in figure 4. The loss reduction effect in (46) is therefore close to the one in (36) since the piece-wise lines in figures 3 and 4 have about the same slopes.

Now the second order sensitivities will be used in the simple examples. The \( D \)-matrix in (31) can be estimated as

\[ D = \begin{bmatrix} 0.0766 & 0.0034 \\ 0.0034 & 0.0743 \end{bmatrix} \quad (47) \]

Equation (29) can now be applied by introducing (43) as

\[ \Delta P_{L-2} = A \Delta u + \frac{1}{2} \Delta u^T D \Delta u = \]

\[ = -0.051 \cdot \Delta P_{GD2} + 0.0869 \cdot \Delta P_{GD2}^2 \]

i.e. the losses at different generation levels can be estimated with \( \Delta P_{GD2} = P_{GD2} - (-0.6) \) as

\[ P_{L-2}(P_{GD2}) = P_L(-0.6) - 0.051 \cdot \Delta P_{GD2} + \]

\[ + 0.0869 \cdot \Delta P_{GD2}^2 \quad (49) \]

This function is shown with a piece-wise line in figure 5 together with the exact line. As shown in the figure

![Figure 5: Second order loss sensitivity function](image)

the correspondence between the exact and the second order approximation is good. The mean losses based on second order sensitivities calculated in the mean generation point can now be estimated as

\[ P_{L-2-AB} = 0.5P_{L-2-A} + 0.5P_{L-2-D} = \]

\[ = 0.5 \cdot 0.0340 + 0.5 \cdot 0.0035 = \]

\[ = 0.0187 \text{ MW} \quad (50) \]

and the loss reduction effect can be calculated as

\[ C_2 = 100 \cdot \frac{8760(P_{L-2-A} - P_{L-2-AB})}{W_{G2}} = 5.2 \% \quad (51) \]

It can be noted that the yearly losses without local generation is estimated to 301 MWh in (34). Using the second order sensitivity function this value becomes 8760 \(-0.0340 = 293 \text{ MWh}, \) i.e. 1 \% too low. Using this method to estimate the reference losses, only one load flow calculation has to be performed.

6. APPLICATION TO A REAL SYSTEM

In June 1992 a Vestas V27-225 kW wind power station was erected at Utö, which is an island in the archipelago of Stockholm. The Utö wind power station is expected to produce 490 MWh/year which is about 10-15 \% of the yearly energy consumption on the island. The island is fed by a 6 km submarin cable from the mainland. Figure 6 shows the 10 kV grid on Utö island where Arbottna is the feeding station on the main land. The total distance from Arbottna to the most distant substation, T218, is 16.2 km, and the distance from Arbottna to the site of the wind power station, VKV, is 11.1 km.

In this section we will estimate the reduced distribution losses caused by the wind power station. The method is based on one load flow calculation and second order sensitivities. The result will be compared with the "true" result using several load flow runs. All the loads are located on the 400 V side of all transformers denoted T** in figure 6.

There are 21 load transformers and during the year July–92 – June–93 the total energy consumption was 4217 MW. The load power factor can be estimated to be 0.95. The mean wind power production during this period was 55.94 kW. The induction generator
The loss function sensitivities can now be estimated and in table 1 the result for the 400 V side of some transformers is shown. As shown in table 1, the highest loss reduction effect is with a local generator connected to the transformer T206.

The loss reduction effect of the wind power station will now be estimated using second order sensitivities. Here we will limit the number of load situations to 6 and wind production situations to 3 which gives us 18 different net load situations.

The wind power production is divided into three summer production levels and three winter production levels according to table 2. The data are from the year July-92 – June-93. Three load situations are considered for each half of the year, c.f. table 3. The

<table>
<thead>
<tr>
<th>Transformer</th>
<th>$\lambda_{P_{GD1}}$</th>
<th>Transformer</th>
<th>$\lambda_{P_{GD1}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T191</td>
<td>-0.014</td>
<td>T204</td>
<td>-0.046</td>
</tr>
<tr>
<td>T192</td>
<td>-0.023</td>
<td>T206</td>
<td>-0.064</td>
</tr>
<tr>
<td>VKV</td>
<td>-0.017</td>
<td>T208</td>
<td>-0.050</td>
</tr>
<tr>
<td>T199</td>
<td>-0.041</td>
<td>T218</td>
<td>-0.051</td>
</tr>
</tbody>
</table>

Table 1: Loss function sensitivities at Utø

<table>
<thead>
<tr>
<th>Winter</th>
<th>Summer</th>
</tr>
</thead>
<tbody>
<tr>
<td>kW</td>
<td>% of time</td>
</tr>
<tr>
<td>0</td>
<td>31</td>
</tr>
<tr>
<td>68</td>
<td>55</td>
</tr>
<tr>
<td>225</td>
<td>14</td>
</tr>
</tbody>
</table>

Table 2: Wind power production levels

<table>
<thead>
<tr>
<th>Winter</th>
<th>Summer</th>
</tr>
</thead>
<tbody>
<tr>
<td>load factor</td>
<td>% of time</td>
</tr>
<tr>
<td>1.26</td>
<td>20</td>
</tr>
<tr>
<td>1.16</td>
<td>60</td>
</tr>
<tr>
<td>1.06</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 3: Load levels

The exact losses are calculated with (7) based on one load flow solution per situation. The first order values are estimated with

$$P_{L-1} = P_L(\text{base case}) + A \Delta u$$

where $\Delta u$ is a vector with net generation modifications.

The second order values are obtained with

$$P_{L-2} = P_L(\text{base case}) + A \Delta u + \frac{1}{2} A \Delta u^T D A \Delta u$$
The sensitivity analysis is based on two base cases. One with mean wind power production and one with zero wind power where the reactive compensation is disconnected. Table 4 shows that there is a good example. Calculations using first order sensitivities overestimates the value. Using second order sensitivities requires only 2 load flow runs instead of 18 and the result is close to the exact one.

7. CONCLUSIONS

In this paper the reduced electrical distribution losses depending on dispersed small scale energy production has been evaluated. Instead of running several load flows for each load and production situation, the loss reduction effect can be estimated from only a few load flow runs and sensitivity analysis. In the paper it is shown that first order sensitivities gives an approximative estimation of the loss reduction potential of a local generator. To obtain a more exact value second order sensitivities are needed.

8 REFERENCES


