



KTH Architecture and  
the Built Environment

# **Strength variability in lime-cement columns and its effect on the reliability of embankments**

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## Preface

The work in this thesis was carried out between October 2008 and October 2011 at the Division of Soil and Rock Mechanics, Department of Civil and Architectural Engineering, Royal Institute of Technology (KTH), Stockholm, Sweden. The work was supervised by Professor Stefan Larsson. My personal fund is granted by Kurdistan regional government in Iraq. I would like to take this opportunity to express my acknowledgement to the people who made the work of this thesis to be possible. I must express my appreciation to my supervisor Professor Stefan Larsson, without whom this work would not have been possible, and I look forward to continuing and completing this work with him. Special thanks are due to Professor Håkan Stille for giving me an opportunity to do the present work at the Division of Soil and Rock Mechanics, and for his valuable comments on the work. I would also like to thank my colleagues from the Division of Soil and Rock Mechanics, in particular Lena Wennerlund and Zein-Eddine Merouani. Finally, I would like to express my special gratitude to my family and my parents for their support.

Stockholm, November 2011

Mohammed Al-Naqshabandy

## **Abstract**

Ground improvement by deep mixing (DM) is a generic term used for a number of methods in which a binding agent, often lime and/or cement, is mechanically mixed with the soil to increase its engineering properties. The inherent variability with respect to the engineering properties of the improved soil is high due to the variations in geology and the complex mixing process. High variability introduces uncertainty in estimating improved soil properties and the performance of the structure.

Current design methodology deals with soil properties deterministically and the uncertainties involved are incorporated in a single value represented by a total factor of safety (FS). The chosen FS is highly dependent on the engineer's judgment and past experience, in which both of these factors vary between different geotechnical designers. Therefore, current design methodology used in practice for DM does not deal with uncertainties in a rational way. In order to design a geotechnical system with the desired level of confidence, the uncertainties involved must be integrated in the DM design. This can be achieved by using reliability-based design (RBD) methods.

The research work in this thesis is presented as a collection of three papers. In the first paper, a comprehensive statistical analysis of cone penetration test (CPT) data is described. The objective was to make a contribution to empirical knowledge by evaluating the strength variability of lime-cement columns within the group of tested columns. In the second paper, the effect of the spatial variability and statistical uncertainty with regard to the embankment's reliability was investigated within the framework of RBD. The study in the third paper investigated the strength variability in lime-cement columns based on two test methods, namely CPT and column penetration test (KPS). In this study, the effect of different test methods on the evaluation of the design value was addressed.

The main conclusions from this study can be summarized as follows. First, the probability distribution function (PDF) for the undrained shear strength of lime-cement columns can be modeled in RBD as normal or log-normal distributions. However, the use of log-normal distribution is recommended for RBD analyses. Second, the evaluated scales of fluctuation indicate ranges of 2 to 4 m and 0.2 to 0.8 m in the horizontal and the vertical directions respectively. This means that in order to fulfill the requirements of independent/uncorrelated samples for assessment of the design value, the spacing between samples must exceed the horizontal scale of fluctuation. It is therefore proposed that the spacing between individual samples should be at least 4 meters. Third, the design values evaluated using CPT and KPS were approximately the same. However, the inherent variability evaluated differs due to the larger volume tested with the KPS probe than with the CPT probe. However, this difference was not significant between the two tests. Fourthly, due to the limitation in the deterministic design in terms of dealing with uncertainties, it is recommended that RBD design should be used in parallel with the deterministic design of lime-cement column.

## Sammanfattning

Jordförstärkning genom kemisk djupstabilisering är ett samlingsnamn för ett antal metoder där ett bindemedel, ofta kalk och/eller cement, mekaniskt blandas med jorden för att förbättra dess hållfasthets- och deformationsegenskaper. Variationerna i dessa egenskaper blir ofta stora på grund av variationer i jordens geologi och på grund av den komplexa blandningsprocessen. Vidare ger dessa stora variationer upphov till osäkerheter i bedömningen av konstruktioners beteende.

Gällande dimensioneringsmetodik använder deterministiskt data och osäkerheter representeras av en säkerhetsfaktor. Säkerhetsfaktorn är i sin tur beroende av den enskilda ingenjörens erfarenhet och bedömningar, varför den ofta skiljer sig från fall till fall. Således behandlar inte gällande dimensioneringsmetodik osäkerheter på ett rationellt sätt. För att geotekniska system ska kunna dimensioneras med en önskad säkerhet måste man ta hänsyn till osäkerheter. Detta kan göras genom införandet av sannolikhetsbaserad dimensionering.

Arbetet i denna uppsats presenteras som en sammanläggningsavhandling bestående av tre artiklar. I den första artikeln beskrivs statistiska analyser av CPT data. Målet med uppsatsen var att bidra med empiriska data genom att utvärdera den rumsliga spridningen av skjuvhållfastheten i en grupp med kalk-cementpelare. I den andra artikeln analyseras effekten av rumslig spridning och statistiska osäkerheter på en stabilitetsberäkning, inom ramen för sannolikhetsbaserad dimensionering. I den tredje artikeln utvärderas och jämförs spridningen av skjuvhållfastheten i grupp med kalk-cementpelare, baserat på två olika testmetoder, CPT och KPS. Här studeras effekten av olika testmetoders inverkan på utvärderingen av den dimensionerande skjuvhållfastheten.

Slutsatsen från detta arbete kan summeras enligt följande:

- 1) Fördelningsfunktionen för den odränerade skjuvhållfastheten i kalkcementpelare kan modelleras som normalfördelad och log-normalfördelad. En log-normalfördelning rekommenderas dock.
- 2) Utvärderade fluktuationsavstånd är i storleksordningen 2-4 m i horisontalled och 0,2-0,8 m i vertikalled. Detta betyder att avståndet mellan provtagningspunkter måste vara större än dessa avstånd för att villkoret om okorrelerade prover ska uppfyllas. Det föreslås därför att avståndet mellan provtagningspunkter i horisontalled ska vara minst 4 meter.
- 3) Utvärderat designvärde blir uppskattningsvis lika om det utvärderas med KPS eller CPT. Storleken på den utvärderade inneboende spridningen skiljer dock lite beroende på att en större volym tests med KPS jämfört med CPT.
- 4) På grund av begränsningarna med deterministisk design med avseende på hanteringen av osäkerheter, det rekommenderas att sannolikhetsbaserad dimensionering utförs parallellt med deterministisk design av kalkcementpelare.

## **List of papers**

The following papers are appended:

### **Paper I**

Al-Naqshabandy, M.S., Bergman, N.S. and Larsson, S. (2012). “*Strength variability in lime-cement columns based on CPT data.*” *Ground improvement*, (165) in press.

*The field work was performed by Larsson and Nilsson (2009). Developed analyses were performed by Al-Naqshabandy. Al-Naqshabandy, Bergman and Larsson jointly wrote the paper.*

### **Paper II**

Al-Naqshabandy, M.S., Bergman, N.S. and Larsson, S. “*Effect of spatial variability of the strength properties in lime-cement columns on embankment stability.*” Accepted for publication at the 4th International Conference on Grouting and Deep Mixing. GSP Marriott New Orleans, February 15-18, 2012.

*Al-Naqshabandy performed the RBD analyses and wrote the paper. Larsson and Bergman contributed to the paper with valuable comments and revisions*

### **Paper III**

Bergman, N.S., Al-Naqshabandy, M.S. and Larsson, S. “*Strength variability in lime-cement columns evaluated using CPT and KPS.*” Submitted to *Georisk* 2011.

*Bergman performed the analyses and wrote the paper. Al-Naqshabandy contributed to the paper by supervising the field work and giving valuable comments on the paper. Larsson contribute to the paper with writing, and giving valuable comments and revisions.*

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# 1. Introduction

## 1.1. Background

Ground improvement by deep mixing (DM) is a generic term used for a number of methods in which a binding agent, often lime and/or cement, is mechanically mixed with the soil to increase its engineering properties which are; permeability, deformation and shear strength (Porbaha, 1998; Terashi, 2003; Broms, 2004; Larsson, 1999). In Sweden, where exclusively dry binders are distributed in the soil with compressed air, the method is traditionally called lime-cement columns or dry deep mixing (Fig.1). The engineering properties of the improved soil depend mainly on the amount and characteristics of the binder, the characteristics of the soil, and mixing and curing conditions. Lime-cement columns are mainly used to increase stability and reduce settlement of highway and railway embankments. The inherent variability of engineering properties in the improved soil is high due to variations in geology and the complex mixing process (Larsson et al., 2005). Although there have been significant advances in the equipment and methods used for deep mixing, the inherent variability of the engineering properties is still high. The variability of results evaluated from compression tests on samples taken from in-situ stabilised soil has been quantified in terms of a coefficient of variation (*COV*), and found to be within the range of 14-75% (as summarised by Larsson, 2005a). High variability introduces uncertainties in estimating improved soil properties and the performance of the structure.

- |   |   |  |
|---|---|--|
| <b>1) Penetration of mixing tool to desired depth</b> | <b>2) Dispersion of binder agent in the soil.</b><br>Sub-phases:<br>a) incorporation and spreading of binder agent;<br>b) wetting of dry binder particles;<br>c) breakdown of agglomerates;<br>d) distribution (and stabilizing of dispersion). | <b>3) Manufacturing completed.<br/>Mixing continues by molecular diffusion</b> |
|---|---|--|

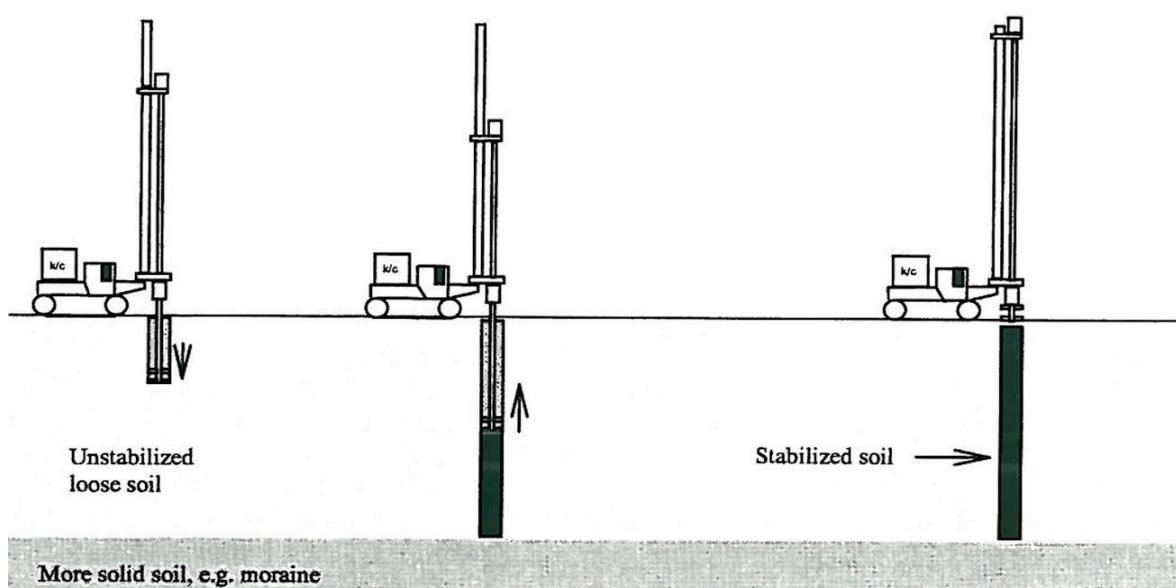


Fig.1. The mixing process of dry deep mixing (Larsson, 2003)

In the ultimate limit state design, geotechnical engineers are traditionally more accustomed to deterministic design for evaluating the safety of the geotechnical structures by means of a total factor of safety (FS). In this study the term *deterministic design* refers to any design methodology that merely considers the characteristic values (i.e. normally the mean values ( $\mu$ )) of the acting forces (i.e. loads (L)) and the resisting forces (i.e. resistance (R)) parameters in the design of embankments, i.e. no randomness is involved. However, estimation of the  $\mu$  values of these parameters is always associated with inaccuracy (i.e. uncertainty). In the deterministic designs, the uncertainties are incorporated in a single value of FS, which is defined as a ratio of the R and the L parameters evaluated along the potential slip failure surface. The chosen value of FS is highly dependent on the engineer's judgment and past experience, in which both of these factors vary between different geotechnical engineers. The safety requirement for embankments improved with lime-cement columns should normally fulfil  $FS>1.5$  for an undrained analysis (Broms, 1999). Current design methods used to assess stability of embankments improved with lime-cement columns thus do not deal with the uncertainties involved rationally. In order to design geotechnical systems with the desired level of confidence, the uncertainties involved must be taken into consideration in a rational way. This can be achieved by using reliability-based design (RBD) approaches (Alonso, 1976; Grivas and Chowdhury, 1983; Li and Lumb, 1987; Christian et al., 1994; Alén, 1998; Phoon et al., 2003; El-Ramly et al., 2002, 2003, Möller and Hansson, 2008 among many others).

Many years of research and great effort have been devoted to evaluating the reliability of geotechnical systems within the framework of RBD. RBD is currently used extensively as a tool for reliability assessment and risk analysis in many geotechnical projects such as the stability of earth slopes and embankments and the bearing capacity of strip footings, laterally loaded piles and earth-retaining structures (recently by Liang et al., 1999; Honjo and Suzuki, 2000; Griffiths et al., 2002; Loukidis et al., 2003; Dasaka et al., 2005; Foye et al., 2006; Cho, 2007; Haldar and Babu, 2008a, 2008b, 2009; Griffiths et al., 2009; Goh et al., 2009; Basha and Babu, 2010).

Despite the fact that the variability in DM has been addressed previously, very little has been published on the application of RBD related to DM design (Filz and Navin, 2006; Kasama et al., 2009; Tokunaga et al. 2009). This is probably due to several factors such as; the presence of high variability in the improved soil properties, very complex interaction between treated and untreated soil which increases uncertainty about the failure mechanism, and also because the application of DM methods is relatively young.

Previous researchers have been specific as regards quantifying the inherent variability and the spatial variability. However, they did not consider other sources of uncertainties (e.g. statistical, measurement errors and model transformation uncertainties).

## *1.2. Objectives*

The objectives of the study are summarized as follows:

1. Make a contribution to empirical knowledge by investigating the spatial variability with respect to the undrained shear strength of the improved soil.
2. Incorporate the variability into the design of DM by using reliability-based design methodologies.
3. Investigate the effect of different sources of uncertainty on the embankment reliability with respect to spatial variability and sampling associated with evaluation of the strength properties of improved soil.
4. Investigate the effect of different test methods (i.e. cone penetration (CPT) and column penetration (KPS) tests), used to evaluated the strength variability of lime-cement columns, on the evaluation of the design value.

## *1.3. Limitations*

This study focuses mainly on the effect of inherent spatial variability on the reliability of the embankment at ultimate limit state. Other uncertainties such as measurement and transformation errors are only considered briefly. The analyses consider only the shear failure mode. Other failure modes such as tilting, overturning and bending of lime-cement columns are not considered.

Stability of embankments is a complex problem in geotechnical engineering. In order to simplify the problem, a number of assumptions were made. Only resistance parameters of the improved and the surrounding soils were considered to be random variables (i.e.  $c_{u,col}$  and  $c_{u,soil}$ ). In the reliability analyses, the random variables were treated as uncorrelated. This is due to the present size of the failure domain where the mixing process has a major influence on variability with respect to  $c_{u,col}$ . No traffic load is considered. In addition, the study focused on Swedish conditions, where only low-strength columns are used.

The analyses were performed on the field measurements based on CPT data a short time, i.e. three weeks, after the installation of the columns. However, strength properties normally increase with time and thus affect reliability but this particular issue is beyond the scope of this study.

#### *1.4. Thesis layout*

This thesis is a summary of three appended papers. It also contains a number of chapters that introduce a description of the area of research and current design methodology used in the field of ground improvement with lime-cement columns, as follows:

##### **Chapter 2 Literature review of RBD related to DM**

A brief overview of previous research that has used and/or addressed the use of probabilistic methods for the design of DM is presented.

##### **Chapter 3 Deterministic design of DM**

Different deterministic design methods used in the design of DM are presented. Some characteristics of the limit equilibrium methods and the difference between them are also presented. The limitation of these methods as a single design method for lime-cement columns is highlighted.

##### **Chapter 4 Uncertainties**

The sources of uncertainties associated with geotechnical soil properties are described. The limitations of the deterministic design methods of DM and their approach in dealing with these uncertainties are demonstrated. The need for rational quantification of uncertainties and their incorporation in the design of DM is also addressed.

##### **Chapter 5 Reliability-based design**

Different levels of reliability-based design and their components are presented. A brief description of each component is given. The need for reliability-based design in DM is motivated.

##### **Chapter 6 Summary of appended papers**

A summary of each attended paper is presented in a chronological order.

##### **Chapter 7 Conclusions and further research**

This section contains general conclusions from the research and a proposal for further research is included.

## 2. Literature review of RBD related to DM

The main aim of this section is to provide a brief overview of previous research that has used and/or addressed the use of probabilistic methods for the design of DM. However, the published work related to DM design is limited. Below is presented a summary of some works that have been published in the international journals, at conferences and in theses.

Honjo (1982) has utilized statistical methods to evaluate the shear strength and its variability of heterogeneous soil improved by the deep mixing method. The scale of fluctuation ( $\delta$ ) was evaluated.  $\delta$  is the distance within which soil properties show strong correlation, beyond this distance the properties are uncorrelated. It was found that  $\delta$  was influenced by the in-situ soil properties, cement content and mixing process. In his study, Honjo proposed a new failure mode for the improved material, a so-called *bundle* failure model, which is a combination of the *weakest link* failure mode for brittle materials (e.g. concrete and rock) with the *average* failure mode for ductile materials (e.g. cohesive soils). The bundle failure mode is a reasonable failure mode for the improved materials, but has not been included in design due to its complexity and the fact that improved materials are highly variable with space.

Larsson (2005b) used statistical methods to assess the mixture quality and the uniformity of lime-cement columns at two tested sites. In this study the uniformity of the lime-cement columns over the cross-sectional area was evaluated by means of the variability. In another study, by Larsson et al. (2005), the effect of the variability of a property with respect to the analysis of the mechanical system of the column-supported embankments was discussed. It was emphasized that the variability and the concept of probability of failure can be considered in design of lime-cement columns by using partial factors of safety to evaluate the design value of the improved soil properties. A parametrical study based on the simple design procedure for evaluation of the design value showed that the design value is affected by the variability and the sensitivity of the improved soil property.

In a PhD thesis, Navin (2005) assessed the stability of embankments constructed on improved soil with the DM method. The safety of the embankment was evaluated by using both a limit equilibrium method<sup>1</sup> (LEM) and a numerical analysis<sup>2</sup> method (NM). Additionally, a section of his study was committed for using reliability analyses for the analysed embankments. However, his study mainly focused on demonstrating the limitations of using LEM methods in the design of DM. With regard to the reliability analysis, Navin showed that reliability analyses are necessary to address the impact of the variability of the improved soil properties and the variability of other materials included. However, the significance of variability as regards reliability was not shown, and it does not account for the uncertainties that are usually associated with the evaluation of the improved soil properties.

A number of conference papers have been published recently on the use of RBD related to DM. Kasama et al. (2009) used probabilistic methods to assess the reliability of the bearing

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<sup>1</sup> Limit equilibrium method in this study refers to the stability methods used to investigate the equilibrium of the structure tending to slide along a potential shear surface.

<sup>2</sup> Numerical analyses used to provide an approximate solution for the very complex problems which cannot be solved by limit equilibrium methods.

capacity of cement treated ground considering spatial variability. In this study, the spatial variability parameters (i.e. coefficient of variation and scale of fluctuation) were chosen based on the literature review and assumed that the cement-treated ground has the same unit weight as the unimproved ground. The main conclusion from this study was that the bearing capacity of the cement-treated ground decreases with increasing variability.

Tokunaga et al. (2009) performed parametric studies on RBD applied in the deep mixing method. In this study, external, internal and overall stability were examined. The study was performed on five case histories. The patterns of improved soil were varied between block and wall type improvements. The main conclusion from this study was that the RBD can be established for any failure mechanisms (i.e. sliding, overturning, bearing capacity, internal and extrusion failures). It was also found that the reliability index increases with increasing factor of safety.

Filz and Navin (2010) proposed a statistical analyses for evaluating the design value of the undrained shear strength of columns ( $c_{u,col}$ ) that support embankments. The assessed design value was mainly based on the assumption that the probability that the actual  $c_{u,col}$  will exceed the shear stresses in DM ground is equal to the probability that the actual  $c_{u,soil}$  will exceed the shear stress in the untreated soil along the potential failure surface. The proposed method account for the variability associated with  $c_{u,col}$  and  $c_{u,soil}$  by introducing a factor called *variability factor*, which is the ratio of the design strength of the DM ground to the specified strength of the DM ground.

According to this review some important notes can be highlighted as follows. Firstly, the performed studies were instructive in terms of identifying the problems that are associated with the variability of DM. They also succeed in quantifying the variability parameters of the improved soil properties, in particular the scale of fluctuation that can be used by others at similar sites. Secondly, it can be seen that there is a trend toward applying RBD in parallel with the deterministic design of DM. Despite the fact that previous researchers have considered the effect of the inherent variability on the mechanical system, they have not considered the effect of other sources of uncertainties, e.g. statistical, measurement errors, and model transformation uncertainties. Furthermore, the spatial variability along the failure surface has not been dealt with in DM design.

### **3. Stability of embankments on DM columns**

The safety of geotechnical structures, such as embankments, founded on DM foundations can be assessed by considering different failure patterns. The most common failure patterns considered in DM design are; internal stability due to instability in DM columns, external stability due to sliding or overturning of DM columns and global stability (Broms, 1999; Terashi and Kitazume, 2011; Filz et al., 2011). However, these failure patterns are normally assessed on basis of the deterministic design by using LEM methods. Soil properties are treated based on a number of samples taken to evaluate their characteristic values, normally the mean values ( $\mu$ ). However, estimation of  $\mu$  of the soil properties is always associated with uncertainties due to sampling, measurement errors, inherent variability, and model transformation. Deterministic design incorporates uncertainties into a single value represented by the total FS.

In the design of DM, the internal stability is normally the predominant factor. This is due to the high uncertainty regarding material properties and the behaviour of DM columns. This study focuses on the assessment of the internal stability of lime-cement columns in the ultimate limit state due to the embankment load. The study was conducted within the framework of deterministic and reliability-based designs. Other failure patterns, i.e. external and global stability are not considered here.

The following section will focus on describing the design of lime-cement columns in general, types of limit equilibrium method of slices, and the behaviour of lime-cement columns.

#### *3.1. Design of lime-cement columns in general*

Broms (2004) has pointed out that the design of lime-cement columns should consider different limit states. First, the ultimate bearing capacity for the columns and the surrounding soil should be adequate. Second, the total and the differential settlement and the lateral deformations should not be excessive under the working load. Third, the design should guarantee the safety of nearby buildings and other facilities during and after column installation. However, this study will only focus on the assessment of the stability of embankments due to the ultimate limit state in lime-cement columns. Other limit states are beyond the scope of the current study.

The engineering properties of the columns depend on many factors such as the type and geological formation of the original soil, initial water content, the amount of binder and the type of DM, and the mixing process. The behaviour of the improved soil due to the applied loads is uncertain owing to the high variability in its properties and the complex interaction between the improved and the unimproved soils, as shown in Fig. 2 (Kivelö, 1996).

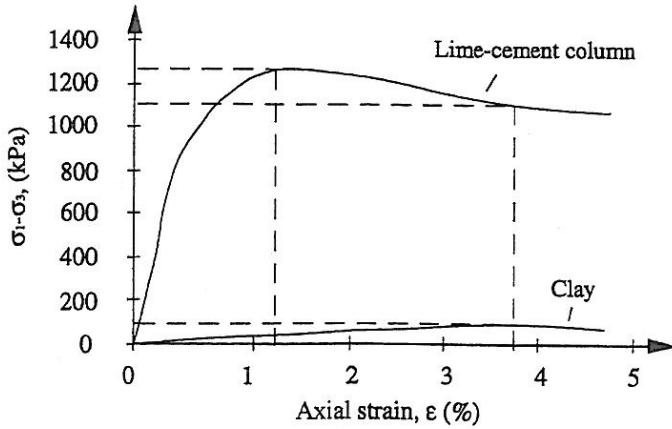


Fig. 2. Stress-strain curves of lime-cement columns and of soft clay as evaluated from undrained tri-axial tests (Kivelö, 1996)

Some assumptions have been adopted to overcome this complexity: complete interaction between the columns and the surrounding soil is normally assumed; the shear failure in the columns and in the surrounding soil is assumed to occur in parallel along a circular slip surface; stability analysis is assessed based on the undrained condition for both the columns and the soil; the method of slices with a circular slip surface is normally used; and the undrained shear strength of the total improved area is evaluated based on the weighted average method, as in Eq.(1). Many of these assumptions are currently used in many countries in the design of DM column-supported embankments (Broms, 2004; Porbaha, 1998; Kivelö, 1996; Kitazume et al., 1996; Terashi and Kitazume, 2011).

$$c_{u,comp} = \bar{a}c_{u,col} + (1 - \bar{a})c_{u,soil} \quad (1)$$

where  $c_{u,comp}$  is the undrained shear strength of the composite material (i.e. the columns and the surrounding soil),  $\bar{a}$  is the area replacement ratio defined as the ratio of the area of the columns and the total area, and  $c_{u,col}$  and  $c_{u,soil}$  are respectively the undrained shear strength of the columns and the soil.

According to Swedish practice (Larsson, 2006), Eq. (1) is normally adopted and  $c_{u,col}$  is normally evaluated from column penetration tests or occasionally from unconfined compression tests performed on samples taken from the site.  $c_{u,col}$  is normally assumed to be equal to a half of the unconfined compressive strength and the maximum value of  $c_{u,col}$  that is utilized in design should be within the range of 100-150 kPa.

In this study the undrained shear strength of the composite material was calculated from Eq.(1). The properties  $c_{u,col}$  and  $c_{u,soil}$  were evaluated from in situ CPT tests and fall cone tests respectively. The stability analyses were performed according to Swedish practice by using the method of slices. Shear failure was assumed for the columns and the surrounding soil to have occurred along the circular slip surface. Fellenius's method of slices was used.

However, due to the complex interaction between the improved and unimproved soils the assumptions made may not all be satisfied in some cases. For example, the mobilized shear resistance of lime-cement columns at failure can be very low when the axial load applied on columns is low. Therefore, lime-cement columns may fail before the peak shear strength of the soft soil is mobilized due to progressive failure (Broms, 1999; Larsson and Broms, 2000). In this case the LEM may overestimate the internal stability of the DM foundations. Since this subject was investigated by some researchers specialised in the field of DM, this issue will therefore be further underlined in the following sections.

### 3.2. Limit equilibrium methods of slices

Generally, stability of embankments can be evaluated deterministically in terms of the total FS by using either numerical methods (NM), e.g. Finite element methods or limit equilibrium methods (LEM). In this study, LEM refers to the method of slices (Duncan, 1996). In the design of DM, the methods most applied for checking internal stability of lime-cement columns are LEM methods. This is because they are considered to be the most accustomed methods for geotechnical designers, since they are simpler than the NM, very handy for stability calculations, and are based on simple assumptions. In addition, extensive experience has been built up over the years and these methods are widely used around the world, see Fig.3 and Eq.(2).

Stability of embankments is regarded as a complex system, in terms of applying forces caused by loads and the resisting forces from the strength of the material properties. Due to this complexity, stability analyses are statically indeterminate (Duncan, 1996). Consequently, LEM methods employ assumptions (e.g. the soil behaves as an elastic perfectly plastic material and the failure surface takes a certain shape (often cylindrical), FS is constant along the slip surface, and the strength parameters are not dependent on the developed shear strains) to fulfil the requirement of statically determinant analyses. As a result of the differences in these assumptions, various methods have been developed and are summarized in Table 1.

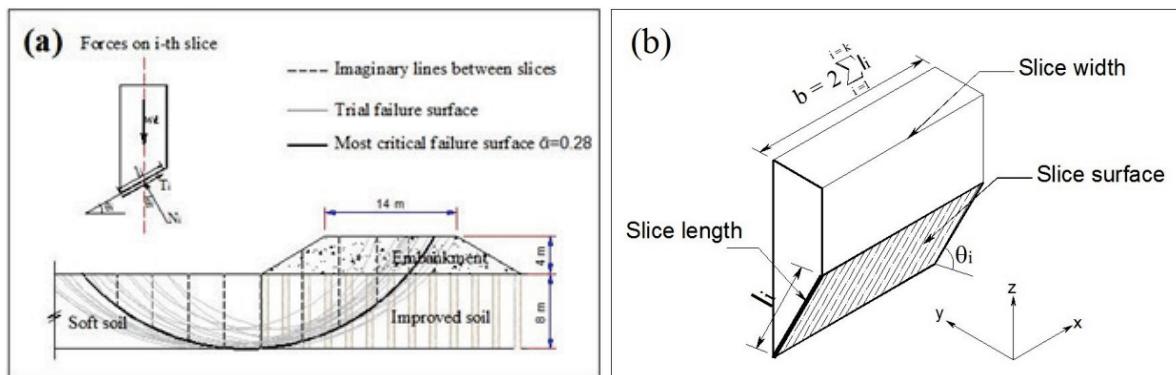


Fig. 3. a) Cross-section of the typical embankment considered in the analyses, and b) Illustration of geometrical parameters for a typical slice.

Table 1: *Characteristics of Equilibrium Methods of Slope Stability Analysis (Duncan, 1996)*

Method (1)	Characteristics (2)
Janbu's Slope Stability Charts	Accurate enough for many purposes; Faster than detailed computer analyses
Ordinary Method of Slices (Fellenius's method)	Only for circular slip surfaces; Satisfies moment equilibrium; Does not satisfy horizontal or vertical force equilibrium
Bishop's Method of Slices	Only for circular slip surfaces; Satisfies moment equilibrium; Satisfies vertical force equilibrium; Does not satisfy horizontal force equilibrium
Force Equilibrium Methods	Any shape of slip surfaces; Do not satisfy moment equilibrium; Satisfies both vertical and horizontal force equilibrium
Janbu's Generalized Procedure of Slices	Any shape of slip surfaces; Satisfies all conditions of equilibrium; Permits side force locations to be varied; More frequent numerical problems than some other methods
Morgenstern and Price's Method	Any shape of slip surfaces; Satisfies all conditions of equilibrium; Permits side force locations to be varied
Spencer's Method	Any shape of slip surfaces; Satisfies all conditions of equilibrium; Side forces are assumed to be parallel

As can be seen from the table, there are differences between some of the characteristics. However, these methods share some common features and limitations: for example they all employ the same definition of the FS; Mohr-Columb is the most dominant failure criterion; and the stability system is a statically indeterminate structure. As a consequence of the common features, many studies were performed to compare and assess the accuracy of these methods for evaluating the safety of the embankment (e.g. Garber and Baker, 1979 and Duncan and Wright, 1980, among many others). All these studies have reached broadly the same conclusions: first, no significant differences were found between FSs evaluated by different LEM methods, especially when the undrained case is considered; second, the circular critical slip surface was the dominant shape in the analyses where the slip surface was allowed to take any shape (Duncan, 1996). Regarding the design of DM in natural soils, the same assumptions are normally adopted, as mentioned earlier.

Despite the simplicity of LEM methods, they may overestimate the safety of the embankment constructed on DM foundations. This is probably due to the adopted assumptions and the possibility of more than one failure mode occurring in the columns at failure. This issue has been investigated recently by Han et al. (2005), Filz and Navin (2006) and Adams et al. (2009). They have evaluated the safety of embankments founded on DM columns by using both NM and LEM methods. The main conclusions from their studies were that the NM method in general gives a conservative FS compared to the LEM method. In addition, different failure modes were observed to occur in the columns, e.g. bending and tilting rather than shearing. Moreover, they showed that the difference between both methods increases significantly with increasing strength of the columns, as shown in Fig. 4.

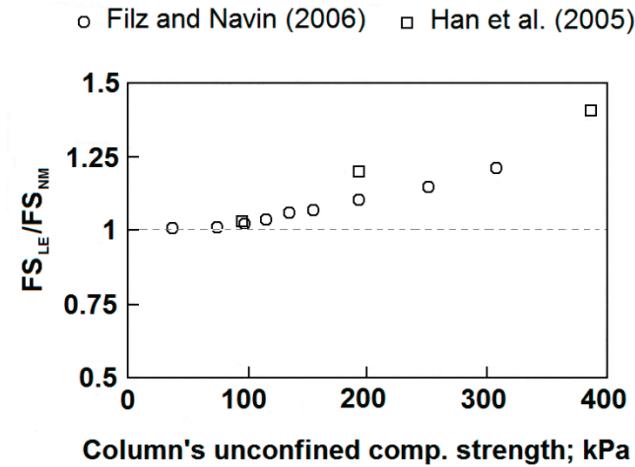


Fig. 4. Comparison of limit equilibrium and numerical analyses of slope stability as a function of unconfined compressive strength.

### 3.2.1. Fellenius's method of slices

Fellenius's method is normally considered to be the simplest method of slices of all methods for assessing the stability of slopes and embankments. It is very handy for stability calculations, provides conservative deterministic FS compared to other sophisticated methods (e.g. Bishop's or Spencer's methods), and it does not require any iteration calculation to evaluate FS. Due to its simplicity Fellenius's method of slices was used to evaluate the safety of the analysed embankment in Fig.3 as follows:

$$FS = \frac{R}{L} = \sum_{i=1}^{i=k} \left( l_i c_{u,i} + (w_i \cos \theta_i - u l_i) \tan \phi_i \right) \Bigg/ \sum_{i=1}^{i=k} w_i \sin \theta_i \quad (2)$$

where  $i$  is the slice number ( $i = 1, 2, 3, \dots, k$ ),  $k$  is the total number of slices,  $c_{u,i}$ ,  $\phi_i$ ,  $w_i$  and  $u_i$  are the cohesion, internal friction angle, weight and pore water pressure on the base of slice  $i$ , respectively. In the present study, only undrained condition was considered in the soil and the improved soil and thus  $u_i$  is set to zero and the shear strength was defined by the undrained shear strength.

### 3.3. Behaviour of lime-cement columns

The possible behaviour of lime-cement columns has been investigated by Kivelö (1998). Kivelö pointed out that individual lime-cement columns may fail at different patterns along the potential slip surface (Fig. 5). Failure modes a, b and c in Fig. 5 represent column bending, d represents flow of soil around the column, and e, f, g and h represent column tilting, column translation, shearing through the column and compression failure in the column, respectively. Kivelö has also derived expressions for evaluating the shear resistance

of the columns along assumed slip surfaces for each failure mode. It has been shown that the failure of relatively high-strength columns can occur by exceeding the moment capacity of the columns (i.e. failure modes a, b and c) or the lateral resistance of the soft soil around the columns (i.e. failure modes d, e and f). Failure of low-strength columns can be caused by shear failure along a slip surface (i.e. failure mode g) or when the compressive strength of the columns is exceeded (i.e. failure mode h). The low-strength column is defined as the columns that possesses shear strength equal to or lower than 150 kPa (Broms, 2004). It should be noted that the proposed failure modes has not been verified by load tests and has therefore not yet been applied in practice.

In stability analyses, shear strength of lime-cement columns should be evaluated along the most critical failure surface. The most critical failure surface is the surface with the lowest FS among the possible failure surfaces. Determining the location of the most critical failure surface is thus a prerequisite in stability analyses. However, searching for the most critical slip surface by the method proposed by Kivelö requires information with respect to strength and its variability with depth for every single column in order to identify its failure mode, which at present is not practicable.

Laboratory studies have been carried out to investigate the failure mechanism of the columns installed with different patterns. For example, Larsson (2008) has conducted 23 shear tests on the lime-cement columns installed with various patterns. In this study, bending failure mode was found to be the predominant mode (failure modes a, b and c in Fig. 5) in individual columns and in columns installed in rows. Bending failure has also been observed by Kitazume et al. (2009) from centrifuge tests carried out to determine the failure modes. However, as mentioned earlier, the proposed failure modes proposed by Kivelö (1998) are not yet used in practice in the design of lime-cement columns, nor have they been validated by field tests. Shear failure is therefore currently the most adopted mode in the design of lime-cement column.

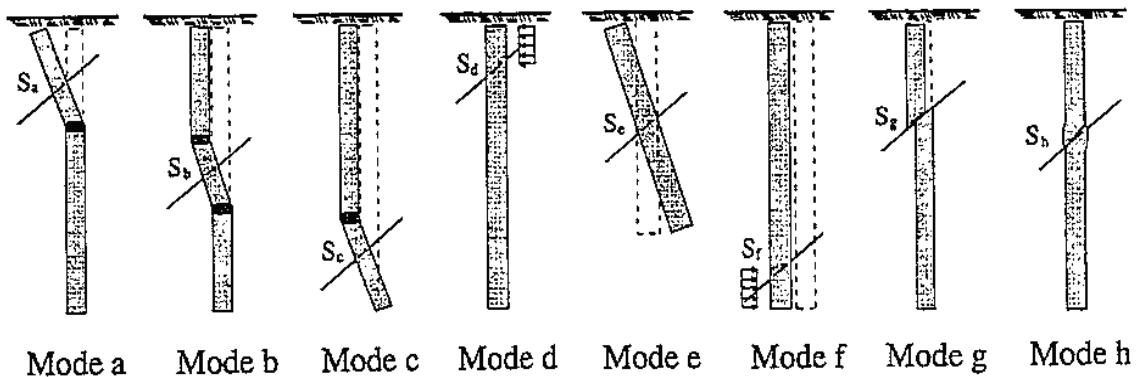


Fig.5. Possible failure modes in DM columns (after Kivelö, 1998)

## 4. Uncertainty

### 4.1. General

Soil properties are normally evaluated either by laboratory, penetration or in situ tests. The measurement will always be associated with uncertainties that arise mainly from two sources, *data scatter* and *systematic error* (Christian et al., 1994), as shown in Fig.6. Data scatter is also called *aleatory* uncertainty which is the combination of the uncertainty in the “inherent” randomness of natural process manifesting as variability over space at different locations (i.e. spatial variability) and the uncertainty from measurement errors (i.e. equipment or testing procedure). In the design of any geotechnical system, inherent variability is usually considered to be the major and unavoidable source of uncertainty, as a consequence of the complex geological process involved with the deposition and formation of soil and rock (Orchant et al., 1988; Christian et al., 1994; Phoon and Kulhawy, 1999). Inherent variability can be evaluated by finding two parameters: the mean ( $\mu$ ) and the standard deviation ( $\sigma$ ). In order to better understand how a property varies at different locations, there is a need to evaluate a third parameter, called the scale of fluctuation ( $\delta$ ) (Vanmarcke, 1977).

Systematic error on the other hand is also called *epistemic* uncertainty and covers all sources of uncertainties that are related to knowledge uncertainty due to lack of data and lack of information and understanding of the physical behaviour of the material’s properties in reality. Systematic error can be divided into two sub divisions which are; statistical uncertainty (i.e. related to the amount of data), and bias in the measurement process (i.e. related to an idealized model of reality). As Stille et al. (2003) have pointed out, when discussing uncertainty it is important to distinguish between different sources of error. These concepts are illustrated in Fig.7.

Due to the existence of such uncertainties, the life cycle and the estimated performance of the embankments will be uncertain. However, the effect of these sources of uncertainty on the design of DM cannot be achieved by using deterministic design as they are not incorporated in the design procedure. In order to clearly identify the effect of the uncertainties on system safety, the concepts of probability and statistic can be incorporated in the design of geotechnical systems (Baecher and Christian, 2003; Ang and Tang, 2007).

As mentioned earlier, the current design method used in DM is based on the deterministic design by choosing conservative values of the uncertain quantities and/or FS as an approach for dealing with uncertainties. The chosen value of FS is entirely dependent on two factors: engineering judgement and past experience. Both of those factors vary between geotechnical engineers. To overcome these problems, there is a trend towards using RBD approaches which are considered to be more rigorous design methods in terms of dealing with uncertainties compared to deterministic methods. Furthermore, as pointed out by Ang and Tang (2007), irrespective of the type of uncertainty, probability and statistics provide proper tools for modelling material variability and design and/or analysis of the geotechnical structures. The need of using RBD in DM design will be further emphasized in the following section.

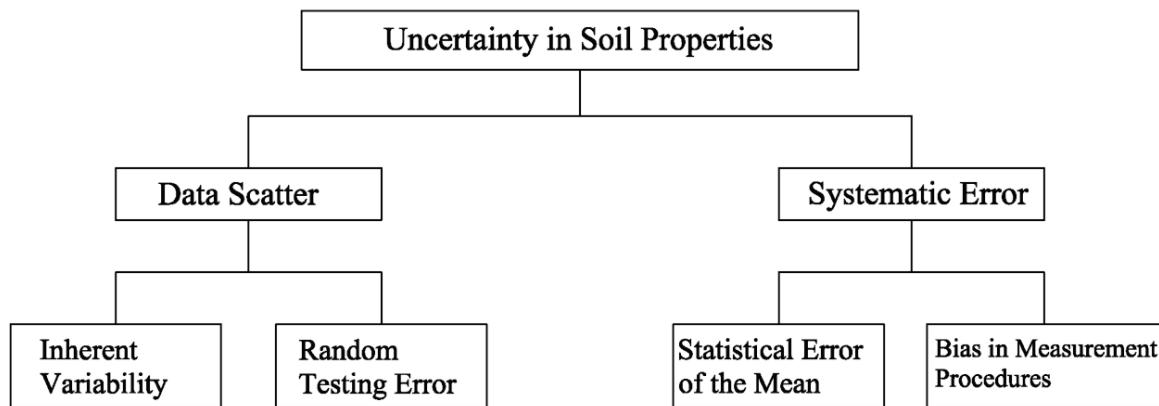


Fig. 6. *Categories of uncertainty in soil properties (after Christian et al. 1994)*

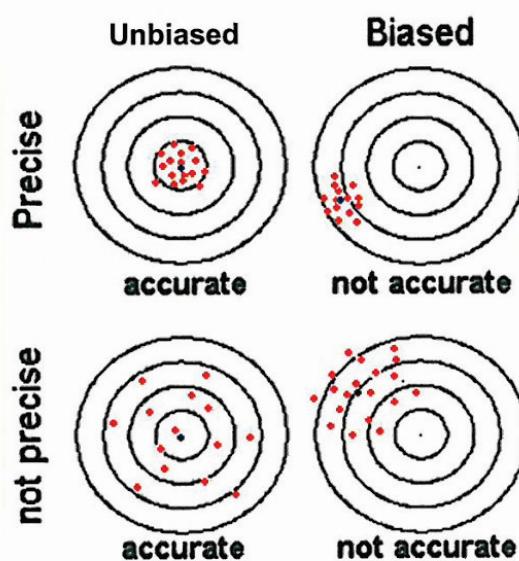


Fig. 7. *Sources of error; Accuracy, precision and bias (after Stille et al. 2003)*

#### 4.2. Uncertainty in improved soil properties

The main sources of uncertainty related to the improved soils are the same as those related to the natural soils. However, uncertainty due to inherent variability in Fig.6 can be sub-divided into two sources of uncertainties, i.e. inherent variability due to original soil and the mixing process. The inherent variability due to the improved soil properties is the source of uncertainty that arises from the natural variability in the original soil itself, while the mixing process is the predominant source of uncertainty associated with all factors that are related to the complex mixing process. However, according to the currently available knowledge and methods it is very difficult to evaluate and quantify each source of uncertainty involved in

data scatter separately. The term inherent variability is therefore widely used to represent the data scatter in both natural and improved soils. Fig. 6 can therefore be modified to represent the sources of uncertainties associated with the improved soil properties, as shown in Fig.8.

Despite the complexity of separating the sources of uncertainty, Christian et al. (1994), Baecher and Ladd (1997) and Jaksa et al. (1999) have shown in their studies that the total uncertainty can be accounted for as the sum of the individual sources of uncertainty as follows:

$$COV_{total} = \sqrt{COV_{spt}^2 + COV_{err}^2 + COV_{stat}^2 + COV_{trs}^2} \quad (3)$$

where  $COV$  is the coefficient of variation, which is the ratio of standard deviation to the mean value of the soil property, and the subscripts  $spt, stat, err$  and  $trs$  respectively denote spatial variability, statistical, measurement errors and transformation uncertainties.

Previous research has shown that the inherent variability of the engineering properties in the improved soils is very high, and is an unavoidable source of uncertainty because it is related to the natural inherent variability and the complex mixing process. Although there have been significant advances in the equipment and methods used for deep mixing, a considerable amount of the variability in improved soil properties still remains, e.g. in the undrained shear strength ( $c_{u,col}$ ). Inherent variability in DM under different conditions has been quantified by means of  $COV$ , and found to be within the range of 14-99% (Larsson, 2005a; Kasama and Zen, 2009; Navin and Filz, 2005). High variability increases uncertainty in estimating soil properties and could have a negative impact on stability. It is worth noting that uncertainty not only exists in the material property but also in the loads and the geometry of engineering structures such as embankments.

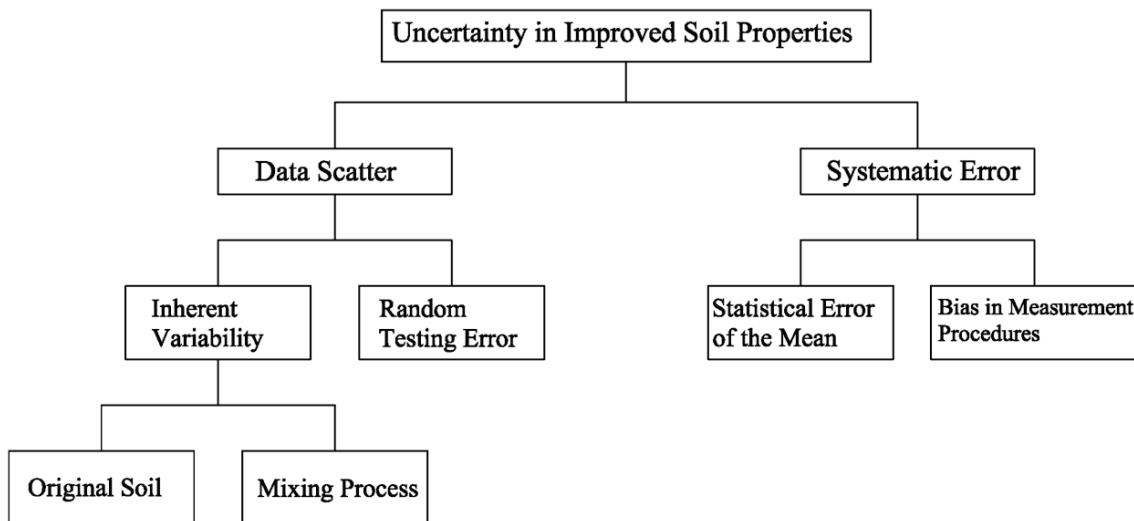


Fig. 8. Sources of uncertainty in DM associated with the evaluation of the improved soil properties.

However, current design methods used for the assessment of internal stability of DM columns do not account for the sources of uncertainty involved in the DM process individually. This is because current design methods are based on the deterministic design where the uncertainties are incorporated in a single value of FS. This concept can be visualized more in Eq.(2). The FS evaluated from Eq.(2) is the deterministic FS because it is evaluated from the characteristic value of the soil properties ( $c_{u,i}$ ,  $\phi_i$ ,  $w_i$  and  $u_i$ ). The safety requirement for embankments improved with lime-cement columns should fulfil  $FS>1.5$  for an undrained analysis (Broms, 1999). The characteristic value is normally represented by  $\mu$  of the soil properties. However, estimation of  $\mu$  is always associated with uncertainties. Current design methods used to assess stability of embankments improved with lime-cement columns do not therefore deal with the uncertainties associated with the complex mixing process, the measurements, and the inherent variability rationally. In order to design geotechnical systems with desired confidence, the uncertainties involved must be taken into consideration in a rational way. This can be achieved by using RBD approaches (Li and Lumb, 1987; Alèn, 1998; Phoon et al., 2003; Möller and Hansson, 2008 among others).

However, RBD methods are not well established for DM because of the difficulties in predicting the strength and deformation properties in improved soil in relation to the mechanical system (Larsson et al., 2005). Furthermore, the application of RBD is based on several assumptions that are rarely all satisfied in geotechnical systems, for example the assumptions were made based on the normal distribution of the random variables, and this is seldom the case with geotechnical properties (Lo and Li, 2007). Despite the rationality of RBD it is still rarely implemented in geotechnical systems in practice. El-Ramly et al. (2002) have attributed this to the lack of publications which illustrate the execution and benefits of RBD approaches, and also to the lack of clarity in identifying the acceptable levels of performance and hence the acceptable risk in design. Accordingly, when using RBD in geotechnical systems it is important that it be provided in its simplest form, not only because of its rational treatment of uncertainties but also to be an applicable design tool for geotechnical engineers. One approach for achieving this objective is to use partial factor design as proposed by (Eurocode 7). In the partial factor design, the design value of the soil properties can be evaluated from its characteristic value and the partial factor. The design value can be used directly in the deterministic analysis (e.g. in Eq.2) and the evaluated FS should be greater or equal to one. The partial factors should be evaluated so that they account for all sources of uncertainties involved and it should also be calibrated with the target safety level and validated with more sophisticated RBD methods. The present study focuses on demonstrating the effect of the inherent variability and other sources of uncertainty on the reliability of embankments. The analyses were carried out within the framework of deterministic and RBD designs. The following chapter (chapter 5) will focus on describing the levels of reliability and the components that involve in RBD procedure.

#### 4.3. Uncertainties considered in the analyses

In order to find the effect of the improved soil properties on the reliability of embankment and due to the information and data in hand, this study only considered the uncertainties with respect to inherent variability and statistical error for the improved soil properties (i.e.  $c_{u,col}$  and  $c_{u,soil}$ ), the variability in the embankment parameters (i.e. unit weight and friction angle) was not considered. In this study the spatial variability of lime-cement columns were quantified by using the CPT test. The measurements obtained from CPT are the cone tip resistance ( $q_c$ ) which is related to the  $c_{u,col}$  as proposed by Robertson and Campanella (1983) as follows:

$$c_{u,col} = \frac{(q_c - \sigma_{v0})}{N_k} \quad (4)$$

where  $\sigma_{v0}$  is the total overburden pressure and  $N_k$  the bearing capacity factor. The lack of reference methods makes the choice of a proper bearing capacity factor difficult. Porbaha et al. (2001) proposed  $N_k = 22 - 23$  related to direct shear tests and  $N_k = 18$  related to unconfined compression tests for cement-treated ground. The latter was chosen as a value of  $N_k$  and adopted in this study. Due to the wide range of  $N_k$  values, high model transformation uncertainty is therefore expected to be associated with Eq. (4). However, transformation uncertainty has not been quantified for CPT in lime-cement columns and choosing a proper value for it is therefore not a trivial task. The study of the embankment focused mainly on investigating the effect of the spatial variability on its reliability. The effect of other sources of uncertainty on the analysed embankment have therefore not been shown but rather considered to show their impact on the evaluated design values from CPT and KPS tests.



## 5. Reliability-based design

Nowadays, the trend in many engineering disciplines is towards providing designs at specified levels of safety. Often, this objective requires predicting the performance of a system with the presence or lack of information or knowledge about the properties involved in the system (Ang and Tang, 2007). Reliability analysis deals with the relation between the loads ( $L$ ) a system must carry and its ability to carry those loads ( $R$ ). Both  $L$  and  $R$  can be uncertain the resulting performance of their interaction is therefore also uncertain (i.e. FS). The following section will focus on describing the levels of reliability methods and the components involved in the RBD procedure, giving a brief description of each.

### 5.1. Levels of reliability

According to Thoft-Christensen and Baker (1982) the method of structural reliability can be divided into three levels of safety checking which are defined as follows:

Level 1 method: Design methods (i.e. partial factor design) in which appropriate degrees of structural reliability are provided on a structural element basis by the use of a number of partial safety factors, or partial coefficients, related to pre-defined characteristic or nominal values of the major structure and loading variables.

Level 2 method: Methods (e.g. first order reliability (FORM), first order second moment reliability (FOSM) and point estimation method) involving certain approximate iterative calculation procedures to obtain an approximation to the failure probability of a structure or structural system.

Level 3 method: Methods (e.g. Monte-Carlo simulation and direct integration) in which simulations and/or calculations are made to determine the probability of failure for a system or system component, making use of a full probabilistic description of the system and taking into account the true nature of the failure domain.

However, the three levels of safety checking are actually connected to each other, where level 2 methods are an approximation to level 3 methods and level 1 methods are a discretization of level 2 methods. In order to use reliability analyses in practice it is necessary to have a method of reliability analysis which is computationally fast and efficient, so that the expected performance level can be estimated with the desired degree of accuracy. Level 2 methods are widely used that fulfill the requirements (Baecher and Christian, 2003; Thoft-Christensen and Baker, 1982). In this study, level 2 methods are used by means of FOSM methods to analyse the safety of the embankment and also to find the effect of the spatial variability and statistical uncertainty on the reliability of the embankment in Fig.3. However, for the sake of simplicity, level 1 was used in a parametrical study to determine the effect of the other sources of uncertainty on the design value.

## 5.2. Performance function

In reliability analyses the performance of the geotechnical systems is normally described by the margin of safety ( $M$ ), which is the difference between  $R$  and  $L$  (i.e.  $M = R - L$ ). The performance function can also be described by means of  $FS$  as a ratio of  $R$  and  $L$  (i.e.  $FS = R/L$ ). However, in reliability analyses both  $FS$  and  $M$  are called performance functions. Failure will occur, theoretically, when  $M < 0$  or  $FS < 1$ , in which both correspond to the probability of failure  $p_f > 0.5$ . Since geotechnical engineers are more accustomed to deterministic design represented by traditional  $FS$ , it is therefore useful to provide the performance function as a function of the  $FS$  according to (Baecher and Christian, 2003) as follows:

$$M = FS - 1 \quad (5)$$

Most geotechnical systems consist of multiple random variables from both  $L$  and  $R$  and it is therefore reasonable to describe  $M$  as a function of the random variables  $g(X)$ :

$$M = g(X_i) = g(x_1, x_2, x_3, \dots, x_n) \quad (6)$$

where  $X$  is an  $n$ -dimensional vector of random variables. The performance function of the analysed embankment based on Fellenius's method of slices will then be

$$M = \left[ \sum_{i=1}^{i=k} (l_i c_{u,i} + (w_i \cos \theta_i - u l_i) \tan \phi_i) \right] / \sum_{i=1}^{i=k} w_i \sin \theta_i - 1 \quad (7)$$

## 5.3. Limit states

Limit states are those conditions in which the system or its components cease to fulfil the intended function. In other words, limit states are identified when  $M=0$ . Limit state can be described as a surface, in multivariate space of the two random variables  $R$  and  $L$ , in the margin between the safe and the unsafe regions (Fig. 9).

The two limit states of interest for most foundations are *ultimate limit state* (ULS) and *serviceability limit state* (SLS). The former deals with states concerning the stability, i.e. ultimate bearing failure, which include exceeding the load carrying capacity. The latter relate to conditions concerning deformation, i.e. performance requirements are exceeded under normal service loads, e.g. settlement (Fenton and Griffiths, 2008). Basically, the design of any geotechnical system must satisfy both failure criteria. With regard to the design of lime-cement columns, Broms (2004) pointed out that in addition to ULS and SLS the design of lime-cement columns should also consider the environmental impact of the columns, and also the safety of nearby buildings or buried services should be guaranteed during and after the installation of the columns. However, since this study deals with assessment of the stability of embankments, the reliability analyses were carried out in the ultimate limit state.

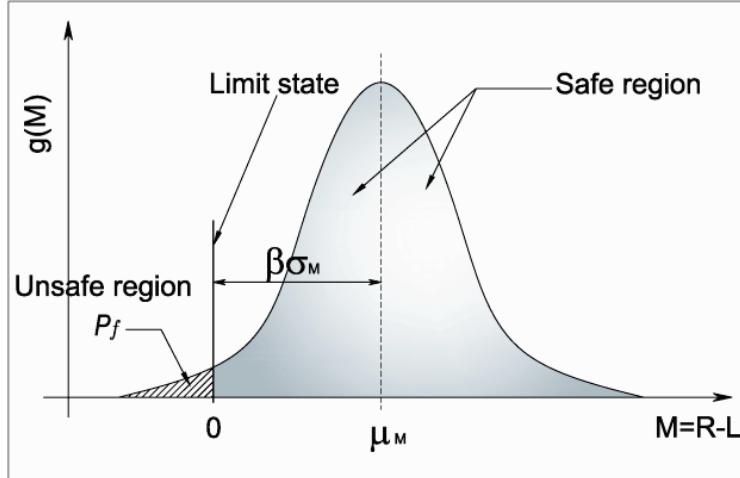


Fig. 9. Probability distribution function (PDF) of the performance function,  $M$ .

The failure criteria or the ultimate limit state function and the failure domain for the multiple random variables ( $X_i$ ) for the embankment can be expressed as follows

$$M = FS - 1 = 0 \quad (8)$$

$$M = g(X_i) = \left[ \sum_{i=1}^{i=k} \left( l_i c_{u,i} + (w_i \cos \theta_i - u l_i) \tan \phi_i \right) \middle/ \sum_{i=1}^{i=k} w_i \sin \theta_i \right] - 1 < 0 \quad (9)$$

#### 5.4. Reliability index

The reliability index ( $\beta$ ) can be defined geometrically as a minimum distance in a unit standard deviation between the mean margin of safety and the limit state  $M=0$  (Fig.9). In general,  $\beta$  can also be defined mathematically as a ratio of the mean margin of safety and its standard deviation ( $\beta = \mu_M / \sigma_M$ ). In the present study,  $\beta$  is considered as a relative measure of the degree of safety. The significance of  $\beta$  is that it identifies the coordinates of the failure point in the multivariate space for each random variable when its important factor is known. Level 2 methods of reliability are normally used to evaluate  $\beta$ , e.g. FOSM, FORM and point estimation methods. In this study,  $\beta$  was evaluated by using FOSM as follows:

$$\beta = \frac{E[FS] - 1}{\sigma_{FS}} \quad (10)$$

$$E[FS] = \mu_{FS} \approx FS(X_1, X_2, \dots, X_n) \quad (11)$$

$$\sigma_{FS} \approx \left( \sum_{i=1}^n \sum_{j=1}^n \frac{\partial FS}{\partial x_i} \frac{\partial FS}{\partial x_j} \rho_{X_i X_j} \sigma_{X_i} \sigma_{X_j} \right)^{1/2} \quad (12)$$

where  $E[FS]$  and  $\sigma_{FS}$  are the expected value and the standard deviation of the factor of safety,  $X_1, X_2, \dots, X_n$  are the  $n$  number of random variables,  $x_i$  denotes for the values of the random variables,  $\sigma_{X_i}$  is the standard deviation of the random variable  $i$ ,  $\rho_{X_i X_j}$  is the correlation coefficient between random variables  $i$  and  $j$ , and  $\partial$  stands for the partial derivative notation. If the random variables are uncorrelated, Eq. 11 can be rewritten as follows:

$$\sigma_{FS} \approx \left( \sum_{i=1}^n \left( \frac{\partial FS}{\partial x_i} \right)^2 \sigma_{X_i}^2 \right)^{1/2} \quad (13)$$

It is clear from Eqs. (10, 12 and 13) how reliability analyses deal with the variability of soil properties in design. It has been shown that the gap between deterministic  $FS$  and RBD can be bridged according to Eqs. (5-9). It is therefore recommended that RBD be conducted alongside deterministic design.

## 6. Summary of appended papers

The following section is a summary of the three appended papers. The papers presented in chronological order. The summary of each paper is presented separately. However, the reader is advised to read it continuously. The main focus in the summaries is to describe the aims of the study, followed by the methodology used and the important findings from the study, and finally the major conclusions. At the end of each summary the connection to the next paper is described.

### 6.1. Paper I

#### **Strength variability in lime-cement columns based on CPT data**

*Mohammed Salim Al-Naqshabandy, Niclas Bergman and Stefan Larsson*

*Ground improvement, in press (2012)*

In this paper the statistical evaluation of the cone penetration tests (CPT) data was comprehensively described. The objective was to make a contribution to the empirical knowledge with regard to strength variability within the volume of lime-cement columns. The methodologies used were motivated on the basis of the literature review. This study was based on the field test, in which 30 CPT soundings were performed in lime-cement columns. The test site was in Lidatorp on Road 73, which is located 50 km south of Stockholm. The tested area was 15 m x 15 m, in this area a total of 312 lime-cement columns were manufactured in a grid pattern with an area ratio of 55%. The spatial variability of the cone tip resistance ( $q_c$ ) measured in lime-cement columns was evaluated within the group of lime cement columns.

In order to fully define the inherent variability of soil properties three statistical parameters should be evaluated (in this study the term *variability parameters* is used). These are the mean ( $\mu$ ), the variance ( $\sigma^2$ ) and the scale of fluctuations ( $\delta$ ), which is the distance within which soil properties show strong correlation. Beyond this distance, soil properties are uncorrelated. The mean, variance and probability distribution function PDF of  $q_c$  data were evaluated using simple statistical approaches. The variability was assessed by means of the coefficient of variation (COV). However,  $\delta$  cannot be easily evaluated by applying the same statistical approaches. Random field theory as described by Vanmarcke (1977) was used to evaluate  $\delta$  in the X, Y and Z directions, in this method by fitting a theoretical autocorrelation function (ACF) to the sample ACF. In order to detect the existence of correlation, two conditions must be satisfied: the number of samples should be sufficient and the samples should be very close to each other in order to detect the correlation within the tested area. Single exponential ACF found to be the best fit for vertical  $\delta$ , while due to the limited number of data in the horizontal direction, linear function was the best fit for the horizontal  $\delta$ . In this study, two sources of uncertainty, i.e. inherent variability and statistical uncertainties, were accounted for to find their effect on the evaluation of the design value. For this purpose a parametrical study was conducted. Furthermore, an equation has been proposed for calculating normalized

design values for log-normally distributed soil properties, which implicitly accounts for the spatial variability and statistical uncertainties.

The results showed that the *COV* of the tested columns with respect to  $q_c$  range from 22-67%. This wide range in the *COV* indicates the high variability. The variability is highly volume dependent and the high *COV* associated with CPT data is probably because the size of the CPT probe is small relative to the column segment, and it is also too small to represent the strength of the whole column cross section. The distribution of PDF for  $q_c$  followed log normal distribution. This means that in the reliability analysis, the strength property of lime-cement columns based on  $q_c$  of CPT data can be modelled as log normally distributed random variable. The evaluated  $\delta$  ranged from 20-70 cm and 2-4 m in the vertical and horizontal directions respectively. The variance reduction ( $\Gamma^2$ ) within the volume tested was evaluated. A parametric study conducted to find the effect of different parameters (i.e. spatial variability, statistical uncertainty, type of PDF and sensitivity factor) on the evaluation of the design value. When the variability is high (e.g. 60%) the effect of the type of PDF and statistical uncertainty were crucial for the assessed design value. However, when the spatial variability is taken into account the variance reduces to (e.g. 25%) the effect of the type of PDF was negligible and the effect of statistical uncertainty decreased significantly. Consequently, when the variability is very high in DM ground, the spatial variability and the type of PDF should be considered in design.

Some significant conclusions can be drawn from the study. The *COV* of the  $q_c$  was very high, which is probably a result of the small volume tested by CPT. This hypothesis can be validated by testing with a larger probe (e.g. column penetration test). In the reliability analysis, the PDF of  $q_c$  should be detected. The scales of fluctuation of 2-3 m in the horizontal direction suggest that the spacing between test points should be larger than 3 m to ensure independence between samples. A simple design consideration shows that the spatial variability and the statistical uncertainty have a major influence on the assessment of the design value. As the variability is high in DM ground, the spatial variability should be considered in the design and the reliability analysis should be executed.

In order to better understand the inherent variability of the improved soil, there is a need for further research to validate the hypothesis of the high variability due to the small volume tested associated with the CPT probe. In order to find the effect of the spatial variability and statistical uncertainty on the geotechnical systems (e.g. embankments), reliability-based design should be conducted for stability assessment in DM design. Since inherent variability and statistical uncertainty were found to have a significant influence on the design value, other sources of uncertainty such as transformation uncertainty and random testing errors may also have an influence on the assessed design value. These sources of uncertainty need to be evaluated for DM design and their effect on the reliability of embankments should be further investigated.

## 6.2. Paper II

### Effect of spatial variability of the strength properties in lime-cement columns on embankment stability

Mohammed Salim Al-Naqshabandy, Niclas Bergman and Stefan Larsson

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The main objective of this paper was to investigate the effect of the inherent spatial variability of the improved soil property on the stability of embankments. The study was based on the field tests, in which 30 CPT soundings were performed in a random manner in lime-cement columns to evaluate column undrained shear strength ( $c_{u,col}$ ). The undrained shear strength of the soil ( $c_{u,soil}$ ) was also evaluated based on one CPT sounding. The test site was located in Kista and was part of a major ground improvement project for construction of a new highway 10 km north of Stockholm. The tested area was 16 m x 16 m, in this area a total of 236 lime-cement columns were manufactured individually with an area ratio of 28%.

The variability parameters of  $c_{u,col}$  were evaluated within the volume of the improved area. The strength property of the improved area was evaluated based on the weighted average method. Reliability-based design (RBD) was performed to find the effect of the spatial variability and statistical uncertainty of the improved soil property on the reliability of the embankment. The probability distribution functions (PDF) were detected and the scale of fluctuations ( $\delta$ ) evaluated for the columns and the soil. The statistical analyses for the CPT data described in the previous paper were used to evaluate the variability parameters. Knowledge about the variability along the failure surface is required in order to perform RBD, hence the variance reduction factor ( $\Gamma^2$ ) was evaluated along the failure surface. The stability of the embankment was assessed using Fellenius's method of slices. Reliability analyses were performed on the most critical failure surface, i.e. the one with the minimum factor of safety (FS) found among 24 trial failure surfaces. In the calculations the soil and the columns were assumed to be entirely interacting and only shear failure mode of the columns was examined. Since the objective of this study was to determine the effect of the variability of the strength properties from the improved soil on the embankment stability, only resistance parameters from the columns and from the soil (i.e.  $c_{u,col}$  and  $c_{u,soil}$ ) were treated as random variables. Due to the lack of information and for the sake of simplicity the correlation between the random variable was ignored.

Results from statistical analyses indicate that the evaluated mean values of  $c_{u,col}$  and  $c_{u,soil}$  were 110 kPa and 10 kPa. The COV were 10% and 27% for the soil and columns respectively. The PDF distribution of both  $c_{u,col}$  and  $c_{u,soil}$  was found to be normally distributed. However, CPT data were also statistically tested to determine whether the data fit log normal distribution. The results showed that the CPT data can be modelled in this particular tested area as both normal or log normal distribution. These results are consistent with the relatively low variability (i.e. COV=27%) compared to the Lidatorp site for example, where the COV was 67%. The evaluated  $\delta$  for the columns for the regional average data were 4 m and 0.4 cm

for the horizontal and vertical direction respectively. The scale of fluctuation for  $c_{u,soil}$  was 0.2 cm in the vertical direction. Due to the lack of data the  $\delta$  for  $c_{u,soil}$  was not detected in the horizontal direction. However,  $\delta$  in the horizontal direction was assumed in the reliability analysis to be greater than the size of the failure domain. The evaluated FS for the tested embankment was 1.63. The corresponding RBD analyses were carried out in two cases; first when the effect of spatial variability of the random variables is not considered (i.e.  $\Gamma_{i,surf}^2 = 1$ ), and second when the  $\Gamma_{i,surf}^2$  is evaluated along the failure surface and considered in the analyses. The evaluated  $\beta$  according to FOSM were 2.18 and 5.7 for the two cases respectively. This simple example shows that the utilization of variance reduction has a significant influence on the evaluated  $\beta$ . The deterministic design practice cannot address the impact of the statistical uncertainty and spatial variability. Since the variability with respect to  $c_{u,col}$  is high in DM, RBD may lead to a great underestimation of  $\beta$  if the spatial variability is not considered.

The effect of the spatial variability on the reliability of an embankment was addressed in this study. Statistical analyses of CPT data show that PDF of  $c_{u,col}$  and  $c_{u,soil}$  could be normal or log-normal distributions. However, log-normal distribution is recommended for RBD analyses. RBD was found to be a powerful design tool for integrating variability into the design of DM. Therefore, it has been recommended that RBD should be conducted alongside with the deterministic analyses. The analyses presented illustrate the significant influence of the spatial variability that should be considered in the design of DM. The analyses also illustrate the significant influence of the number of tests on the reliability. This study only considered the resistance parameters from the improved area as uncorrelated random variables. However, treating other parameters (i.e. from the loads) as random variables and considering the correlation between them may influence the results of reliability analyses. There is therefore a need for further research related to this subject to address the issues in question. Furthermore, there is also a need for further investigation to determine the effect of other sources of uncertainties on the embankment's stability.

### *6.3. Paper III*

## **Strength variability in lime-cement columns evaluated using CPT and KPS**

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The aim of the paper was to investigate the strength variability in lime-cement columns based on two test methods: i.e. the cone penetration test (CPT) and the column penetration test (KPS). The study also addressed their effect on the evaluation of the design value. Two different sites were tested. The first site was located in Kista, 10 km north of Stockholm, where 60 columns were tested randomly. The second site was located in Lidingö, slightly east of Stockholm, where 12 columns were tested in groups of four. A parametric study was conducted to study the effect of different sources of uncertainty, i.e. transformation uncertainty, spatial variability, measurement error and statistical uncertainty, on the design value.

Results show that the *COV* from CPT data ranged from 18-59% with an average of 29%, while *COV* from KPS ranged from 19-47% with an average of 22%. The vertical scale of fluctuations ( $\delta$ ) were 40 cm and 60 cm for CPT and KPS respectively. The results suggested that less variability can be achieved by using a larger probe. Accordingly, the KPS test could be more representative test method than the CPT test for evaluating the average strength property and its variability in lime-cement columns, due to the increase in the local average that results from using a larger probe in the KPS test. Results of scale of fluctuation from both tests are also consistent with *COV* results, because the larger scale of fluctuation from KPS indicates smoother spatial variability than the smaller scale of fluctuation from CPT which indicates rapid spatial variability. Results from the assessment of the design value show no significant difference in the design values assessed using both test methods. Parametrical studies from simple design procedure show, after taking spatial variability into account, that transformation uncertainty and statistical uncertainty both have a significant influence on the assessed design values. These matters need to be further investigated and the question of the optimum number of tests required for a reliable design also needs to be studied further.



## 7. Conclusions and further research

### 7.1. Conclusions

In this study, the statistical evaluation of the undrained shear strength of lime-cement column based on CPT and KPS data have been presented. The tests were conducted at three different test sites, the first in Lidatorp and the second and third in Kista and Lidingö respectively. The study identified the sources of uncertainty associated with the improved soil properties. The effect of different sources of uncertainty on the assessment of the design value and the embankment's reliability were demonstrated. A reliability-based design method was used to evaluate the safety of the embankment.

The spatial variability parameters of individual columns,  $COV$  and  $\delta$ , with respect to CPT data, in the vertical and horizontal direction at the test site in Lidatorp ranged from 25-67%, 20-70 cm and 2-3 m respectively. The  $COV$  and  $\delta$  in the vertical direction from CPT measurements for the test site in Kista were 18-59% and 40cm respectively. The  $COV$  and  $\delta$  in the vertical direction from KPS measurements at the test site in Kista were 19-47% and 60 cm respectively. The range of the evaluated  $COV$  and the vertical  $\delta$  was large, indicating the high variability in lime-cement columns. Horizontal  $\delta$  between individual columns is probably due to the inherent variability from the original soil rather than the installation process. In order to fulfil the requirements of no correlation and independency between samples, the spacing between them should be greater than the scales of fluctuation. It is recommended that RBD analyses should consider spatial variability, and the spacing between samples taken for design should be at least 4 m.

Results from a simple design procedure for assessing the design value based on two tests (i.e. CPT and KPS) indicated no significant difference in the evaluated design values. This means that reliability analysis is insensitive to the type of penetration test used. It was found that KPS is a more representative test method for evaluating the variability and the strength of lime-cement columns than CPT. This is due to the large volume tested using KPS, where the variability is highly volume dependent. However, measurements from both test methods are associated with high transformation uncertainty, due to the indirect estimation of the real property,  $c_{u,col}$ . Transformation uncertainty is highly recommended to be considered in the RBD design of lime-cement columns.

Deterministic design cannot capture the uncertainty associated with lime-cement columns. The reliability of the analysed embankment affected by the uncertainty and the spatial variability were found to have a significant influence on the embankment reliability. In order to take the uncertainty into account in design, it is recommended that RBD and deterministic method be conducted alongside for DM design. The sources of uncertainty (i.e. spatial variability, statistical, measurement errors and transformation uncertainties) should therefore be considered in the design of lime-cement columns.

## *7.2. Further research*

The conclusions drawn from the present study were based on some assumptions and limitations and the results can therefore not be generalized for many other cases. As a consequence, there is a need for further research of this nature to address the following points:

- 1- The variability parameters with respect to the strength properties of the improved soil should be evaluated under different conditions. This can be achieved by manipulating many factors such as type and amount of binder(s) used in the improvement, size of the examined field test and type of test used. Better understanding of the spatial variability at different conditions may reduce the uncertainty and increase the reliability and performance of the mechanical system.
- 2- Data recorded from CPT and KPS tests provide indirect measurements of the soil strength. Empirical equations are normally used to transform data from CPT and KPS tests to represent the soil strength. Thus, a high degree of transformation uncertainty is expected to coincide with this transformation. However, model transformation uncertainty associated with CPT and KPS is not yet quantified for the improved soil with DM methods.
- 3- Limit equilibrium methods may overestimate the safety of embankments constructed on DM foundations. Uncertainty in relation to the stability method used therefore needs to be investigated for DM methods.
- 4- Geotechnical engineers are accustomed to traditional deterministic design. In order to use RBD in practice, it should be presented in its simplest form. One approach for achieving this is to conduct benchmark calculations to set values for partial factors that include property variability and the associated uncertainties in the design of DM.
- 5- The random variables in this study were considered to be uncorrelated. Since the correlation between the parameters may have an influence on the design value, there is therefore a need for further research of this nature that address the correlation between random variables in DM design.
- 6- The strength of DM columns increases with time and their variability could therefore also be increased. This temporal variation in the soil properties will increase uncertainty in the improved soil properties. However, the effect of temporal variation on an embankment's reliability has not yet been investigated. This source of uncertainty can be handled in design by observational methods or Bayesian updating.

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