Modeling and Optimization of a Fuel Cell Hybrid System

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Abstract
The purpose of this project was the modeling, optimization and prediction of a hybrid system composed of a fuel cell, a dc-dc converter and a supercapacitor in series. Lab tests were performed for each device to understand their behavior, and then each one was modeled using software (Simulink). The validation of the model was done by comparing its results with measured data; finally the model was used for the optimization and the prediction of the hybrid system.
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0. Project idea and objectives

A fuel cell hybrid vehicle is a vehicle which is powered by more than one energy supply, and one of them is a fuel cell. Depending on the configuration of the devices, the vehicle can be powered by a parallel hybrid system or by a series hybrid system. This project deals with the study of a vehicle powered by a fuel cell hybrid system with configuration in series. In particular it is composed of a fuel cells stack, a DC-DC converter and a supercapacitor. According to those devices features, it is useful to define a strategy so that the system can work with the optimal performance. The best strategy can be found with an empirical method: performing many tests in different condition, the optimal way of working of the system can be defined. However, this method takes a lot of time. Another method is modeling: a model which describes the system can be constructed and implemented in a software. Afterward, the optimization of the strategy can be reached by performing simulations. This method saves time and economic sources. The modeling process can be divided in three main phases:

• Construction of the model
• Validation of the model
• Using the model for simulations

In this project the chosen software is Simulink, a programming environment based on Matlab. The model of the three main devices of the system is supposed to be defined, and validated singularly. Later on, these models shall be joined together to form the model of the complete system. As for each device, also the complete model has to be validated. Finally the model is meant to be used for the two goals of the project: optimization and prediction. In particular optimization concerns the strategy of collaboration between fuel cell and supercapacitor, while the prediction concerns the influence of changes of the characteristics of the vehicle on its performance. However, according to the predictions, the preferable changes to the system are meant to be presented, and an optimization for the future system will be done.
1.1 Shell Eco-marathon

Shell Eco-marathon is a competition taking place every year. High schools and universities coming from all over Europe challenge each other in a race. The winner is the team whose car is able run a certain distance with the least amount of energy. This year the race took place at Eurospeedway Lausitz in Germany. In the first place the vehicles have to pass an accurate control to ensure that they fulfill all the rules. Afterward the teams have five attempts to run their car for six laps (about 19 km) within 45 minutes; the best result of these five tries is the final result. There are two main categories: Prototype category and Urban concept category; each one is divided in other subcategories depending on which type of energy the vehicles are powered by (solar, internal combustion engine, fuel cell etc.) [1].

KTH Royal Institute lined up in the Urban concept category Spiros IV, a four wheels vehicle, able to carry one person with an average speed of 25-30 Km/h:

- Weight: 135 (Kg)
- Width: 125 (cm)
- Length: 220 (cm)
- Height: 105 (cm)
- Fuel: hydrogen (H₂)
The team was composed of students from different programs [2]:

- 3 Mechatronics students.
- 6 Chemistry students.
- 1 Electrical student.
- 2 Machine Design students.
- 9 PhD students.

Spiros placed in fifth position in the Urban concept hydrogen class out of 17 participants with a result of 60 km/kWh, corresponding to about 530 km with the equivalent of one liter of fuel [3].

1.2 The system and the car

Spiros IV is powered by hybrid system composed of a fuel cell, a DC-DC converter and a supercapacitor in series.
By the oxidation of hydrogen, the fuel cell converts chemical energy to electric energy at low voltage and high current. The DC-DC converter takes this input power and transforms it in a high voltage (the same of the supercapacitor) low current power. Finally the supercapacitor has the task of feeding both motor and additional devices (water pump, air pump, cooling fan, recirculation pump and control boxes).

1.3 The fuel cell

The fuel cell has the task of transforming the chemical energy of the fuel, in this case hydrogen, into an energy form which is more suitable to supply other devices, the electrical energy. Due to their high efficiency, low pollution and high flexibility, fuel cells are getting more and more interesting for replacing combustion engines powered by fossil fuels.

1.3.1 Basic principle

There are different kinds of fuel cells, but the general principle is always the same; there are four fundamental parts which are common to every device: the anode, the cathode, the
electrolyte and an external circuit connecting anode and cathode. The production of electric energy is based on two reactions:

- On the anode side the fuel is oxidized, reacting into an electron, which has a negative charge, and an ion, which has a positive charge. In case the fuel is hydrogen, we have this reaction:

\[
2H_2 \rightarrow 4H^+ + 4e^- \tag{1.1}
\]

- The products move from the anode to the cathode passing by different ways: the ions pass through the electrolyte that divides the anode from the cathode, and the electrons pass through the external circuit, giving electrical current.
- Ions and electrons meet together in the cathode, and reacting with oxygen they produce water [4]:

\[
O_2 + 4e^- + 4H^+ \rightarrow 2H_2O \tag{1.2}
\]

Fig. 1.5: Diagram of a fuel cell [5]

1.3.2 Open circuit voltage

The reversible open circuit voltage is the theoretical maximum voltage that a fuel cell can deliver. To calculate this parameter some chemical considerations have to be done: for every reaction the difference between the Gibbs free energy of the products and the reactants is a measure of the ‘external work’ which the reaction needs or delivers.

\[
\Delta G_f = G_{prod} - G_{react} \tag{1.3}
\]
Inside a fuel cell, this external work is used to move electrons in the circuit which connects anode and cathode; 2N electrons pass inside the circuit for each mole of hydrogen oxidized, where N is the Avogadro’s number.

\[-2Ne = -2F\]  \[\text{[1.4]}\]

Where:
e: charge of one electron (C)
F: Faraday’s number (C)

If all the Gibbs free energy is used to move electrons, the reaction has no losses \[\text{[4]}\]:

\[
\Delta g_f = -2F \cdot E \quad \rightarrow \quad E = \frac{-\Delta g_f}{2F}
\]  \[\text{[1.5]}\]

Where:
\(\Delta g_f\): Gibbs free energy released by one mole of hydrogen (KJ/mol)
E: reversible open circuit voltage (V)

1.3.3 Fuel cell efficiency

To be able to calculate the maximal voltage that it is possible to obtain from a fuel cell, the enthalpy of formation has to be used in place of the Gibbs free energy in the equation 1.5:

\[
E = -\frac{\Delta h_f}{2F}
\]  \[\text{[1.6]}\]

\(\Delta h_f\) can assume two different values, depending on the state of aggregation of the water produced:

\[
H_2 + \frac{1}{2} O_2 \rightarrow H_2O \quad \text{steam}
\]  \[\text{[1.7]}\]

\[
\Delta h_f = -241.83 \frac{KJ}{mol} = LHV
\]

\[
H_2 + \frac{1}{2} O_2 \rightarrow H_2O \quad \text{liquid}
\]  \[\text{[1.8]}\]

\[
\Delta h_f = -285.84 \frac{KJ}{mol} = HHV
\]

Putting both the values of the enthalpy of formation inside equation (6), two different values of the reversible open circuit voltage can be found.
These are the voltages which the fuel cell would deliver if its efficiency was 100%. The voltage of the cell drops due to the losses, so its efficiency can be considered almost proportional to its voltage. Considering that not all the hydrogen reacts inside the fuel cell, the efficiency can be expressed as [4]:

\[
\eta = \mu_{H_2} \cdot \frac{V_c}{E} \cdot 100\% \quad [1.9]
\]

Where:
- \( \mu_{H_2} \): utilization coefficient
- \( V_c \): actual voltage (V)
- \( \mu_{H_2} \) will be defined in the paragraph 1.3.6.

**1.3.4 Fuel cell irreversibilities**

The voltage drop results from four major irreversibilities:

- **Activation losses**: in the transfer of electrons from or to the electrode a part of the energy is lost.
- **Fuel crossover and internal currents**: part of the fuel and of the electrons pass through the electrolyte, without giving useful energy.
- **Ohmic losses**: the electrodes and the interconnections have their own resistance to the passage of electrons. As a result a part of energy is lost in heat.
- **Mass transport or concentration losses**: the concentration of the reactants at the surface of the electrode decreases with the increasing of the output current [4].

![Fig. 1.6: Example of polarization curve [6]](image)
1.3.5 Fuel cell design

A fuel cell alone can only deliver a very low voltage, so usually they have to be connected in series: such a connection is known as a ‘stack’. Thus the quality of the interconnection between the different cells is important due to the ohmic losses it can cause. As a consequence lots of solutions were developed to avoid this problem, so that a higher number of cells can be connected. The output current depends on the area of the electrodes. As an example a single fuel cell with a certain constructive solution has a certain current density: if a series of cells is used to compose a stack, its total output current is proportional to the total area of the cells. Fuel cell stacks can be designed as the application requires them, deciding power, voltage and current. For this reason this device is flexible and suitable for lots of applications [4].

1.3.6 Fuel cell consumption

The consumption of a fuel cell can be calculated just knowing its output current. Each mole of hydrogen oxidized releases a charge of \(2F\), where \(F\) is the Faraday constant. As a consequence the consumption rate can be calculated with the following equation:

\[
CR = \frac{I}{2F}
\]  

[1.10]

Where:
- \(CR\): consumption rate (mol/s)
- \(I\): output current (A)

This is just a theoretical equation. As said in paragraph 1.3.3, not all the hydrogen contributes to the production of current, but a part of it passes through the cells without oxidizing. As a result a coefficient of utilization can be defined:

\[
\mu_{H_2} = \frac{\text{mass of fuel input to the cells}}{\text{mass of hydrogen reacted in the cells}}
\]  

[1.11]

Finally the equation for the consumption rate becomes [4]:

\[
CR = \mu_{H_2} \cdot \frac{I}{2F}
\]  

[1.12]

Using the ideal gas law and integrating the result, the total normal cubic meters of hydrogen consumed can be found:
\[ V = \int \frac{CR \cdot R \cdot T}{P} \]  

[1.13]

Where:
R: ideal gas constant = 8.314 (m³Pa)/(K mol)
T: thermodynamic temperature = 273 (K)
P: pressure = 101325 (Pa)

1.3.7 Spiros fuel cell

Spiros is equipped with an eight cells PEMFC stack with an area of 170 cm² each one. In the PEM (proton exchange membrane) fuel cell the electrolyte is an polymer which, if hydrated with water, can conduct ions; this kind of membrane has good features for the application in this field:

- It has a chemical high resistance.
- It has a mechanical high resistance.
- High absorption capacity.
- High proton conduction [4].

The stack is a prototype designed by Power Cell Sweden AB in collaboration with KTH. The stack was manufactured with structural plates in stainless steel that weighed 11 kg per head, thus a total weight of 22 kg. This weight was considered too great to be overlooked. As a consequence new lightweight plates were designed by two Machine Design students with a weight of 1.5 Kg each [7].

![Fig. 1.8: Spiros stack](image)
The fuel cell behavior can be seen from the following graphs.

![Polarization curve of Spiros stack](image1.png)

*Fig. 1.9: Polarization curve of Spiros stack*

![Efficiency of Spiros stack](image2.png)

*Fig 1.10: Efficiency of Spiros stack*

Due to the growth of the losses the output voltage drops with the increasing of the current. The polarization curve changes if the current is dropping or growing: this is because at the downward step the stack already has wet cells so that their membranes have a better
proton transport. Another important reason is a better reaction kinetics at the cathode, because after working at low potentials the catalyst surface is less covered with oxides. The efficiency is calculated taking as reference the LHV which is the most common used in literature [7].

1.4 The DC-DC converter

The DC-DC converter is an electronic device which is able to transform the voltage from a value to another.

1.4.1 Spiros DC-DC converter

The Spiros system is equipped with a switch-mode DC-DC converter: the operation of this device is based on the storage and release of the input energy with a certain frequency using a switch; thus, adjusting the time of storing and of releasing (duty cycle), the level of the output voltage can be changed [8].

![DC-DC converter of Spiros](image)

The fuel cell can deliver a power at high current (between 0 and about 70 A) and low voltage (less than 8V). Therefore it is not possible to supply the supercapacitor which works at higher voltage (more than 28V). The DC-DC converter has to step up the output voltage of the fuel cell to the voltage of the supercapacitor, with a consequent current drop. All the process takes place with a certain efficiency, which depends on the level of power which is converted.
\[ P_{fc} = V_{fc} \cdot I_{fc} = \eta \cdot V_{dc-dc} \cdot I_{dc-dc} = P_{dc-dc} \]  

Where:

- \( P_{fc} \) = output power of the fuel cell (W)
- \( V_{fc} \) = output voltage of the fuel cell (V)
- \( I_{fc} \) = output current of the fuel cell (A)
- \( \eta \) = efficiency of the DC-DC converter
- \( V_{dc-dc} \) = output voltage of the DC-DC converter (V)
- \( I_{dc-dc} \) = output current of the DC-DC converter (A)
- \( P_{dc-dc} \) = output power of the DC-DC converter (W)

The following graph represents the efficiency of the DC-DC converter depending on the power.

![Graph showing efficiency of DC-DC converter vs. power](image)

**Fig. 1.12: Efficiency of the DC-DC converter**

In counter trend respect to the fuel cell, the DC-DC converter has an efficiency which grows with increased input power, and over about 180W is quite constant.

### 1.5 The supercapacitor

A supercapacitor is an electrochemical energy storage device characterized by a higher power density than the conventional batteries and a higher energy density than conventional capacitors. Due to their features, supercapacitors are able to support fast changes in the stored energy level. As a consequence they are suitable for the application in hybrid electric vehicles, especially combined with fuel cells. Fuel cells are characterized by a high energy
density and low power density; in addition they are not able to storage energy. Therefore the 
supercapacitor is suitable to complement the limits of the fuel cell. With this implementation, 
the fuel cell can increases its efficiency, working most of the time at moderate power, and, due 
to the recuperation of braking energy, the vehicle can save fuel [9].

1.5.1 Structure

The basic general structure of the double-layer capacitor consists of a pair of polarisable 
electrodes suspended in an electrolytic solution and of a separator between the electrodes. 
Two collectors enable the charging of the electrodes [11].

The electrodes are composed of porous material in order to have a very high specific surface 
area. The electrolyte is a solution containing charged ions. Finally the separator is a 
membrane which enables the passage of the ions [12].
1.5.2 Basic principle

When a voltage is applied, the positive ions are collected on the surface of the electrode with negative charge, and vice versa. Thus two layers form at the interface between the solid and the liquid: an external layer mainly composed of ions surrounded by solvent, and an internal layer mainly composed of solvent. As a consequence the internal layer works as dielectric separator between the electrode and the charged ions.

As shown in figure 1.15, the distance between the positive and the negative charges is small. Consequently the capacitance of the device is high, according to the following equation [13]:

$$C = \frac{\varepsilon \cdot A}{d}$$

Where:
- $C$: capacitance (F)
- $\varepsilon$: dielectric constant (F/m)
- $d$: distance between charge (m)
- $A$: interface area (m$^2$)

1.5.4 How supercapacitors work

The charged ions are free to move inside the solution. Once the electrodes are charged, the ions are attracted on their surfaces: the higher the difference of potential between the
electrodes, the higher is the number of ions collected on the surface. As a consequence the capacitance of the supercapacitor is not constant, but it increases with its voltage.

Fig. 1.16: Ions collections on the electrodes surfaces [12]

The value of the upper limit of the voltage is defined by the properties of the electrolyte: once this value is reached the electrolyte starts to react and produces gases, with the consequent breaking of the device [12].

1.6 The motor

The motor is the main user of the power produced by the fuel cell. The motor is an electric DC brushless motor with the following characteristics:

- Model: 160ZWX02
- N. of phases: 3
- Rated voltage: 36 (V)
- Rated speed: 175±20 (rpm)
- Rated torque: 26 (Nm)
- Rotor inertia: 6350 (Kgmm²)
- Weight: 4.75 (Kg)
- Length: 85 (mm)

Brushless motors are the least generation of DC motors; they differ from the classic brushed motors because the position of the permanent magnet and the phases are inverted: the phases are on the case and the permanent magnet is on the rotor. The position of the rotor is followed by an encoder: the signal out from the encoder is read by a controller which synchronizes the rotation of the phases supplied by current with the rotation of the magnetic field. This evolution from brushed to brushless enables to remove the brushes from the motor, which are mainly responsible for losses; on the other hand the brushless motor presents a more complicated and expensive control compared to the brushed one [14].
The motor has two main aims:

- Transforming the electrical energy in mechanical energy for moving the wheels, and consequently the car;
- Regenerating energy during the breaking;

The second point is interesting: in conventional cars (powered by an internal combustion engines) the energy gained during the acceleration is then lost during the braking in the form of heat due to the friction; the hybrid electric cars enable to regenerate a part of this energy just changing the way of working of the motor, from actuator to generator. The change can be done just setting a commanded speed lower than the effective one [15]. This leads to energy and consequently fuel saving, which increases the efficiency of the system. In the graph below the motor efficiency depending on the speed and the resistive torque is shown:

![Motor Efficiency Graph](image)

*Fig. 1.18 Motor efficiency [16]*

The data cover just a small range of speed, but it is enough to understand which is the average value of the efficiency, and its drop with the growing of the resistive torque: this will be useful to understand the high peak of current during the accelerations.
1.7 The additional devices

There are some devices that are not directly involved in the main functions of the system, but are fundamental to ensure that it works properly:

- Water pump
- Air pump
- Cooling fan
- Recirculation pump
- Control boxes

The water pump has the task of pumping water inside the fuel cell, to ensure cooling of the device; it needs more or less 7 W at 12 V voltage, and this power is quite constant over the range of function of the stack.

The air pump has the task of filling the cathode with air to ensure enough oxygen for the reactions; it needs a voltage at 12 V that can vary between 15 and 25 W depending on the fuel cell operating status.

\[ P_{fe, grows} \rightarrow \dot{H}_2 \text{ grows} \rightarrow \dot{O}_2 \text{ grows} \rightarrow P_{ap, grows} \]

Where:

- \( P_{fe} \): fuel cell output power (W)
- \( \dot{H}_2 \): input hydrogen flow rate of the fuel cell (mol/s)
- \( \dot{O}_2 \): input oxygen flow rate of the fuel cell (mol/s)
- \( P_{ap} \): air pump input power (W)

The cooling fan has the task of cooling the stack when its temperature is too high; it needs a power that can vary between 2 and 3 W at 12V voltage.

The recirculation pump to take advantage of the unreacted hydrogen from the stack and enable lower fuel consumption, a reflux system with an active pump was installed for leading back unreacted hydrogen to the fresh influx of hydrogen to the stack [7]. This device needs a constant power of 3 W at 12V voltage.

The control boxes are all the boxes that contribute to the control of all the system; they take in input information coming from the sensors and they take decisions about what the system has to do. Measurements about the power absorbed by these devices are not available, so a constant power of 20W at 12 V voltage is assumed.
1.8 The forces, the power and the torque

The output power from the motor is defined by the resistive forces that the car has to overcome. Therefore the power load of the vehicle can be calculated just knowing its speed. There are four main types of forces:

- Inertial force
- Aerodynamic resistive force
- Rolling resistive force
- Force due to inclination of the track (gravity)

_The inertial force_ is the force that opposes every change of state of motion of an object; its definition comes directly from Newton’s second law of classical mechanics. It is possible to distinguish two components: one due to the linear acceleration and one due to the rotational acceleration:

\[
F_i = m \cdot a \quad \quad M_i = J \cdot \frac{d\omega}{dt} \quad [1.15]
\]

Where:

- \( F_i \): inertial force (N)
- \( m \): weight (Kg)
- \( a \): acceleration (m/s\(^2\))
- \( M_i \): angular momentum variation (Nm)
- \( J \): moment of inertia (m\(^2\)Kg)
- \( \omega \): angular speed (1/s\(^2\))

In Spiros, two rotating parts are considered: the motor, and the wheels. The moment of inertia of the motor is given (\( J_m \)), while the one of the wheels has to be calculated. Some simplifications have to be assumed: the wheel can be considered as a cylinder (the rim) and an annulus (the tire) with their own weight and bound together.

*Fig. 1.19: Spiros wheel and its simplification*
\[ J_r = \frac{1}{2} \cdot m_r \cdot r_r^2 \quad J_i = \frac{1}{2} \cdot m_i \cdot \left( r_i^2 + r_r^2 \right) \quad \rightarrow J_w = J_r + J_i \quad [1.17-1.18-1.19] \]

Where:
- \( r_r \): rim radius (m)
- \( r_i \): tire radius (m)
- \( m_r \): rim weight (Kg)
- \( m_i \): tire weight (Kg)
- \( J_r \): rim moment of inertia (m²Kg)
- \( J_i \): tire moment of inertia (m²Kg)
- \( J_w \): wheel moment of inertia (m²Kg)

The rotational inertia can be transformed in a force applied to the periphery of the rotating objects (wheels and motor):

\[ F_{ir} = \frac{M_i}{r} = \frac{J}{r} \cdot \frac{d\omega}{dt} \quad \frac{d\omega}{dt} = \frac{a}{r} \quad \rightarrow \quad F_{ir} = \frac{J}{r^2} \cdot a \quad [1.20] \]

Where:
- \( \omega \): angular speed (1/s)
- \( a \): acceleration of the car (m/s²)

So the total inertial force can be expressed:

\[ F_i = \left( m_{tot} + \frac{4 \cdot J_w + J_m \cdot \tau}{r_w^2} \right) \cdot a \quad [1.21] \]

Where:
- \( m_{tot} \): total weight of the car and the driver (Kg)
- \( \tau \): transfer ratio

The aerodynamic resistive force is the resistance which an object, moving inside a fluid. The aerodynamic resistive force can be calculated according to the following equation:

\[ F_a = \frac{1}{2} \cdot \rho \cdot A \cdot C_d \cdot v^2 \quad [1.22] \]

Where:
- \( \rho \): air density (Kg/m³)
- \( C_d \): air drag coefficient
- \( A \): car cross section area (m²)
- \( v \): speed (m/s)
As shown by the equation, when a vehicle is designed, a particular attention has to be paid to its front area and its shape: a large area increases the resistance, and the shape influences the drag coefficient. Many studies about the aerodynamic of the shape have been done: because of the growing of the drag depending on the square of the speed, the aerodynamic resistance is one of the biggest forces that the power train of a vehicle has to overcome. On the graph below the behavior of the power dissipated by the aerodynamic resistance depending on the speed is shown:

![Graph showing the power aerodynamic resistance of Spiros](image)

*Fig. 1.20: Power aerodynamic resistance of Spiros*

The parameters assumed for Spiros are:

\[
\rho = 1.184 \text{ (Kg/m}^3\text{)}
\]

\[
C_d = 0.2
\]

\[
A = 0.8 \text{ (m}^2\text{)}
\]

While the air density is measured at a temperature of 25°C and it comes from the literature, the air drag coefficient and the cross section area come from measurements performed on Spiros by a PhD student, Daniel Wanner.

The rolling resistive force is the force that opposes the rolling of a round object on a surface. In the case of a wheel, it is mainly caused by the deformation of the tire: when the vehicle moves on a street, the rubber of the tire is subjected to cycles of deformation and recovery in which a production of heat occurs. This heat represents an energy loss which makes the car slow. The problem can be seen also from another point of view: due to the difference between the deformation and recovery energy, the reaction of the surface is not completely vertical, but displaced. A component opposes the motion:
Where:

- $M_r$: rolling resistive momentum (Nm)

This force can be considered as an horizontal force applied to the periphery of the tire:

$$M_r = F_{py} \cdot d \quad [1.23]$$

Where:

- $C_r$: rolling resistance coefficient
- $N$: normal reaction of the surface (N)
- $r_w$: wheel radius (m)

The normal reaction varies depending on the inclination of the track:

$$F_r = C_r \cdot N = \frac{M_r}{r_w} \quad [1.24]$$
The force due to the inclination of the track: Depending on the inclination of the track the car can be subjected to a force that opposes or favors the motion; this is due to the gravity:

\[ F_t = m_{tot} \cdot g \cdot \sin(\vartheta) \]  \[1.26\]

If the inclination of the track is positive, the force will oppose the motion, if it is negative it will favor it.

Once all the forces are calculated, the output power of the motor can be calculated:

\[ P = (F_i + F_t + F_a + F_r) \cdot \nu \]  \[1.27\]

Knowing which are the forces and their points of application, the resistive torque can be also calculated: the actual positions of the gravity center and of the point of application of the aerodynamic resistance are not known, so an approximation has to be done. They are considered to be at half of the height of the car; here is the equation for its calculation:

\[ T = (F_a + F_i + F_r) \cdot \left(\frac{h}{2} - r\right) + F_r \cdot r \]  \[1.28\]
2. The model

Modeling is the process of generating conceptual, graphical or mathematical description of the empirical objects, phenomena and physical processes; the aim of the modeling is the prediction of the outcomes of a process beginning from certain conditions. Humans have always trying to model the world surrounding them, researching the general laws at the base of to the singular phenomena. With the passing of the time the tools for modeling became more and more powerful, especially with the advent of computers: their high speed revolutionized the research, enabling to solve complex and long calculations in shorter time. Modeling software was implemented, with a simpler user interface, making possible to use the calculation power of the computer without knowing low level programming language.

2.1 Simulink

Simulink is a programming environment based on Matlab, which is particularly suitable for designing dynamic and embedded systems. With its graphical environment and its vast libraries, Simulink can be used for designing, simulating, implementing and testing of time-varying systems in all their aspects: controls, video processing, signal processing, communications and image processing [18].

2.2 Hybrid system model

A model for the entire hybrid system was developed, joining together the models of the single devices:

![Diagram of the complete hybrid system](image)

All the components of the system are represented by boxes of different color and connected by lines which carry the signals from a subsystem to another; the range and the step of the
time can be chosen according to the duration of the simulation and the accuracy requested. The input signal of the fuel cell block is the output power of the stack. The block gives as output the output current of the fuel cell, its efficiency and its output power, which is the same of the input. Afterward the current and the power output, together with the voltage of the supercapacitor, get inside the DC-DC converter block, which gives as output the current that supplies the supercapacitor. While the motor power output block takes in input the speed of the car, and calculates the output power of the motor, the resistive torque and the rotational speed, the additional devices FC block, beginning from the output power of the fuel cell, defines the power absorbed by the additional devices. In conclusion the signal coming from the DC-DC converter block together with the signals coming from the additional devices FC block and the power of the control boxes get inside the supercapacitor block, where the voltage of the supercapacitor is calculated. The clock generates a series of numbers representing the time, which are displayed and at the same time saved in a vector of name ‘tsimu’. This vector is useful for elaborations and plotting operations in Matlab environment. The output box block takes in input the efficiency of the fuel cell, the power output of the motor, the power absorbed by the additional devices and by the control boxes, the power output of the fuel cell and the time, and calculates some parameters for checking the performance of the system. Finally the control block takes in input the voltage of the supercapacitor, and gives in output the output power of the fuel cell. However this block is used only in auto mode, as explained in the paragraph 2.2.3.

2.2.1 Signals declaration

All the input signals must be declared in Matlab environment: constant inputs are declared as scalar numbers; dynamic inputs are matrices composed of two columns : the first one represents the time, and the second one is the value assumed by the signal at that particular time, so every row is a corner of the signal. Here is an example:

```
signal = [ 0 1
           10 1
           10 2
           20 3
           30 3
           40 4 ];
```

![Fig.2.2: Example of input signal to the model](image)

2.2.2 Lookup tables

Inside the program many lookup tables are used. Usually they are built up using data coming measurements, so they are not defined all over the range in which the signal varies, but only...
in a few points. For covering all the range of interest, a linear interpolation is made in Matlab environment, using the function ‘fit’: as result function representing the interpolation of the measured points is calculated. Here is an example:

```matlab
fuel_cell_curr_out = [0;10;25;30;40;50;60;70];
fuel_cell_volt = [7.6400;6.5200;6.0400;5.8100;5.8000;5.6400;5.5600;5.4800];
pol_curve = fit(fuel_cell_curr_out,fuel_cell_volt,'linearinterp');
```

![Polarization curve](image)

**Fig.2.3: Example of construction of a lookup table**

### 2.2.3 Manual and auto mode

With the commutation of the ‘mode switch’, the model can work in two modes:

- Manual mode
- Auto mode

With manual mode the system needs two inputs: the speed of the car and the output power from the fuel cell; this is a good way of working for the validation of the model: giving in input the real output power from the fuel cell and the real speed of the car during the race, we can compare the behavior of the model with the real behavior of the system; the manual mode was also useful during the construction of the vehicle for the prediction of a possible power cycle for the fuel cell during the race. The results will be discussed in the following chapters. With Auto mode the system needs just one input, the speed, while the output power from the fuel cell is decided step by step by the control box; how it works will be explained more in detail.

### 2.3 Simulink model: fuel cell

The fuel cell is a complex device: there are many parameters which influence its behavior, such as the temperature and the pressure of hydrogen and oxygen, the temperature of the stack, the hydration of the membranes etc. Because of this, it is also complex to model. There are different ways of modeling a fuel cell depending on the aim of the study. There are two different approaches: the static one and the dynamic one. The static behavior is totally defined by the polarization curve, which represents the trend of the voltage depending on the output current; the dynamic behavior is the answer of the system in the time domain to the changing
of the load. Both behaviors can be faced in two different ways: a mathematical way and an empirical way. The mathematical way is based on utilization of general theoretical equations for describing a real phenomena: it has the advantage to be general and more suitable for applying at different study cases, but it has low accuracy due to the approximations of general laws; the empirical way is based on the utilization of equation or lookup table directly coming from measurements, sometimes without having a theoretical correspondence: this way of working provides very high precision for the studied case, but with a loss in generality of the model.

The Spiros stack is modeled with a static empiric model; the best would have been to make a dynamic model, because Spiros’ behavior is studied in the time domain, but it is very difficult to make a mathematical model which gives good results; so the only one choice would have been the empirical one, which means a lot of experiments and accurate measurements. Due to the time lack this was not possible. Anyway the model would have become heavier and complex, contradicting the purpose of simplicity that was agreed in the beginning of the project. Instead, the mathematical way would have been possible for the static approach: using the Nernst equations, which give the reversible cell voltage depending on the temperature and the pressure, and the equations for each kind of irreversibility, the polarization curve can be found. The biggest problem is that inside these equations there are some empiric parameters, such the constants used in the definition of the irreversibilities of the stack, which request measures to be defined [4]; approximate values of these parameters can be found also in literature, but that would involve low precision. The following figure represents the implementation in Simulink of the model:

![Simulink model of the fuel cell](image)

The output power of the stack is the input signal to the subsystem; then the signal passes through two different kind of lookup table: one giving the output current and one the efficiency; because of the real behavior of the stack, the lookup tables are defined both for dropping and increasing current, beginning from the polarization curves presented in the paragraph 1.3.6.
At each iteration the output power from the fuel cell is compared with the value of the last iteration: depending on it is growing or dropping, a switch lets pass only the signal coming from a branch or from the other. While the power and the current are plotted on a scope and saved in a matrix in Matlab environment, the efficiency is only plotted on another scope. The utilization of a model based on lookup tables has its advantages and disadvantages: it is very light, so suitable to be inserted inside a bigger system, and it is very simple to implement without running into bugs that are difficult to find and solve; on the other hand it can only be applied to one type of stack: this is not a very big problem, because the measurement and the insertion to the program of the polarization curve does not takes a long time.

### 2.4 Simulink model: DC-DC converter

The DC-DC converter is a quite complex electrical device, so it would be difficult to model it with an equivalent circuit which describes exactly its way of working. Therefore, focusing more on the model of the supercapacitor and of the fuel cell, a very simple model of this device was made.
There are three input signals to the subsystem: the supercapacitor voltage, the fuel cell output current and the fuel cell output power; on one branch the fuel cell voltage is calculated by dividing the power by the current; on the other branch the power passes through a lookup table which gives the efficiency of the DC-DC converter as output, according to the measurements. The current and the output voltage from the fuel cell and the efficiency are multiplied and then divided by the voltage of the supercapacitor: this operation gives the current that is going to supply the supercapacitor. The block that defines if this subsystem can work well is the lookup table of the efficiency: this means that good measurements result in a good model for the DC-DC converter.

### 2.5 Simulink model: resistive forces

The definition of the load that the hybrid system has to overcome to move the vehicle is of great importance for the model. For this purpose it is important to choose a signal which is simple to measure and to understand. Based on these considerations, the speed of the car was chosen as signal for the calculation of the load: the idea was to define the output power of the motor calculating, based on the speed, all the resistive forces that the hybrid system has to overcome. This part of the model is based on a similar model in Matlab environment by the PhD student Daniel Wanner.

As shown in the graph above, the input signal to this subsystem is the speed expressed in Km/h, that is converted to m/s, the SI unit of measurement. There are four boxes, each one has the task of calculating each type of force considered. Then the forces are summed up and multiplied by the speed, to get the output power from the motor. In the following diagrams the content of the each block is shown:
All the forces were calculated as described in the introduction chapter. Particular attention has to be paid to the lookup table representing the inclination of the track: information about the altitude in each point of the track could not be found. However, comparing the logged data of the input current to the motor and of the speed, it was possible to see that in some points of
the track there was an acceleration of the car without an increasing of the current requested by the motor and vice versa. These points were appeared in each lap of the race, and also in different races. The differences between the measured current and the calculated current were attributed to the inclination of the track. Thus a diagram of the inclination of the track depending on the distance was built, and a force was defined, as explained in paragraph 1.8. The corners of the curve were changed many times, till the calculated current fitted with the measured one. Inside the program the inclination of the track is implemented by a lookup table depending on the distance. For sure this is not an accurate way of proceeding, but, with the available information, it is the only one.

![Inclination of the track depending on the distance](image)

**Fig. 2.13: Inclination of the track depending on the distance**

There are two other important tasks executed by this subsystem: the calculation of the resistive torque and of the rotating speed of the wheels.

![Calculation of the resistive torque](image)

**Fig. 2.14: Calculation of the resistive torque**

The rotation speed is calculated just multiplying the linear speed for a constant value which comes out from the following equation:
The rotation speed and the resistive torque will be the inputs in the motor efficiency lookup tables.

### 2.6 Simulink model: additional devices and control boxes

Even if these devices do not participate directly to the main functions of the system, they have a high influence on its behavior. Developing a good model for each one would take a long time. Here it is shown how they have been implemented in Simulink environment:

![Simulink model of the additional devices](image)

The input signal to the subsystem is the fuel cell output power. While the water pump and the hydrogen recirculation pump need a constant power, the operation of the air compressor and of the air fan is strictly connected with the operating status of the fuel cell. There are two lookup tables describing the behavior of these two devices:

![Air fan and air compressor lookup tables](image)

The figure 2.17 shows that the power consumed by the air compressor never reaches to zero, but there is a lower limit at more or less 18.5 W: this is because during the race even if the output current of the fuel cell goes under 20 A, the compressor goes on giving always the

$$\text{rpm} = \frac{v}{2\pi r} \cdot 60 \rightarrow c = \frac{60}{2\pi r} \quad [2.1]$$
same amount of air, consuming the same amount of energy. This lower limit can be changed easily in Matlab environment. As said in paragraph 1.7, these devices all work at a voltage of 12 V. Therefore a converter has the task of transforming the input current from the voltage of the supercapacitor to the voltage of the devices. We assume that this conversion takes place with an efficiency of 0.85. The control boxes are electrical devices that are always on during all the race independently of the operating status of the system. As a consequence they are modeled with a constant power of 20 W.

2.7 Simulink model: supercapacitor

2.7.1 Model choice

In paragraph 1.5 the structure and the operation of a supercapacitor were explained. Based on its features, the super capacitor cannot be described just by a capacitance, but an equivalent circuit composed of resistances and non linear capacitances has to be constructed [11].

The one in figure 2.18 is a very complex circuit which is very difficult to model and to apply to a real device. Consequently simpler equivalent circuits were investigated. At first a RC circuit was considered: it is a one branch circuit composed of a resistance and of a capacitance in series. The resistance (ESR, equivalent series resistance) describes the ohmic losses of the device, while the capacitance describes the behavior of the supercapacitor during charging and discharging [19].

![Fig. 2.18: Ideal equivalent circuit of a supercapacitor](image)

![Fig. 2.19: RC equivalent circuit](image)
A RC circuit was implemented in Simulink environment. Taking in input the current absorbed or provided by supercapacitor, the model is able to calculate the voltage of the device.

![Simulink model of an RC circuit](image)

**Fig. 2.20: Simulink model of an RC circuit**

The operation of the model can be explained showing the transfer function of the circuit:

\[
V(s) = ESR \cdot I(s) + \frac{I(s)}{C \cdot s}
\]  

[2.2]

The two branches of the model represent the two components of equation 2.2, which together give the voltage of the capacitor. Finally a constant provides the initial value of the voltage. A tuning of the model was done just performing a sequence of simulation. Beginning from the nominal value of the capacitance of the supercapacitor (33F), after each simulation the results were compared with data measured and the values of the parameters were changed. This operation was repeated until the best results of the model were gotten (C=35F, ESR= 0.0193). The performance of the model is shown in figure 2.21:

![Comparison between the results of the RC model and the measurements](image)

**Fig. 2.21: Comparison between the results of the RC model and the measurements**

A cycle of charges at 2 A and discharges at 10, 7, 5 and 2 A was performed. Figure 2.21 shows that the simulated voltage diverges more and more with the passing of the time. This is because the self-discharge of the supercapacitor is not described by the RC model. In addition
the RC model is not able to follow the non-linear behavior of the device. Thus, even if this model is simple to implement and tune, it is not accurate enough.

The second analyzed model is the RC parallel branch model. This model is characterized, as the name suggests, by a certain number of RC branches connected in parallel. Each branch has a different time constant in order to completely describe the behavior of the device [20].

![Ideal RC parallel branch model](image)

An infinite number of branches cannot be used, so two RC branches are considered sufficient. In particular the branch with the smaller time constant describes the behavior of the device in the short period (charge and discharge), while the other one describes the behavior of the supercapacitor in the medium and long period [11].

A particular equivalent circuit has been chosen: the capacitance of the branch with the smaller time constant depends on the voltage with a linear relation.

![Two RC branches circuit](image)

Figure 2.23 shows that the capacitance of the first branch is composed of a constant part (C₀) and of a part which depends linearly on the supercapacitor voltage; the EPR (Equivalent parallel resistance) describes the self-discharge of the device. The total capacitance of the first branch results from the following equation:
Where:

\[ C_1 = C_0 + k_v \cdot V \]  

- \( C_1 \) = total capacitance of the first branch (F)
- \( C_0 \) = constant component (F)
- \( k_v \) = constant of proportionality \((F^2/C)\)
- \( V \) = voltage of the supercapacitor (V)

Also this circuit was implemented in Simulink environment:

The modeling and the implementation were done as explained in [21]. As a result the following values were derived for the parameters of the circuit:

\[ R_0 = 0.0577(\Omega) \]
\[ C_0 = 7.8272(F) \]
\[ k_v = 0.7302(F/V) \]
\[ R_2 = 868.8132(\Omega) \]
\[ C_2 = 0.2072(F) \]
As for the previous model, also for the two branches model a comparison with a real cycle of charges and discharges was performed in order to test its quality. In figure 2.25 the results are shown:

The plot shows that the simulated voltage fits well with the measurements. This is because, contrary to the RC model, the two branches model takes into account the self-discharge of the device. In addition, due to its variable capacitance, it can follow the non-linear behavior of the supercapacitor. However it has two main limits: the tuning is very time-consuming and difficult, and the resolution its slow because of its complexity.

The third analyzed model is the ESR-EPR model; it is composed of a resistance, called equivalent series resistance (ESR), in series with a capacitance and a resistance in parallel, called equivalent parallel resistance (EPR).
This circuit differs from the first one for the EPR. This parallel resistance was already used in the two branches model for describing the self-discharge. However in this model it is in parallel only with the capacitance and not with all the RC branch as in the previous model. In the diagram below the implementation in Simulink environment is shown:

![Simulink model of ESR-EPR circuit](image)

**Fig. 2.27: Simulink model of ESR-EPR circuit**

The operation of this model is almost the same as the RC branch model. The difference is that not all the current passes through the capacitance, but a part crosses the parallel resistance. For this reason an additional backwards branch has to be inserted. The following diagram explains better what it happens:

![Explanation of the operation of the model](image)

**Fig. 2.28 Explanation of the operation of the model**

An interesting feature of this model is the possibility of tuning it with the System identification toolbox: this is a Matlab toolbox specifically designed for estimating linear and nonlinear mathematical models of dynamic systems from measured data. So in this case, giving as input the measured current and voltage of the supercapacitor and the shape of the transfer function between these two signals, it is easy to get a good tuning of the model. Because of its simple equivalent circuit, the transfer function of the ESR-EPR model can be found without problems.

\[
G(s) = \frac{V(s)}{I(s)} = \frac{ESR + EPR + C \cdot ESR \cdot EPR \cdot s}{1 + EPR \cdot C \cdot s}
\]  

[2.4]
The transfer function proposed by System Identification toolbox is:

\[
T(s) = K_p \cdot \frac{(1 + T_z \cdot s)}{(1 + T_p \cdot s)}
\] \hspace{1cm} \text{[2.5]}

So comparing the two equations the following relation can be extrapolated:

\[
K_p = ESR + EPR \quad T_z = \frac{C \cdot ESR \cdot EPR}{ESR + EPR} \quad T_p = EPR \cdot C
\] \hspace{1cm} \text{[2.6-2.7-2.8]}

With these relations it is possible to find the value of all three parameters of the circuit in a very short time, and with a good accuracy. In practice the values resulting from the tuning are:

\[
C = 31.8(F) \\
ESR = 0.0193(\Omega) \\
EPR = 43.761(\Omega)
\]

Finally the quality of the model was tested, comparing it with the measured data:

![Fig.2.29: Comparison between the results of the ESR-EPR model and the measurements](image_url)

It can be seen that the ESR-EPR model has the advantages of both previous models: it is simple to implement and to tune, it is fast, and it has a very high accuracy. It cannot describe the nonlinear behavior of the real supercapacitor, but this does not involve a great precision loss. Therefore this is the chosen model for the implementation of the model of the system.
2.7.2 Insertion into the system

Once a model is chosen, it has to be inserted into the hybrid system; as for the other devices, a subsystem has been constructed for the supercapacitor:

This block has six input signals: the output current from the DC-DC converter, the output power of the motor, the resistive torque, the rotational speed of the wheels, the power consumed by the additional devices and the control boxes. All these signals, except of the current from the DC-DC converter, pass through another subsystem in which, beginning from the power values, all the currents that the supercapacitor has to supply are calculated. Here its contents are shown:

The resistive torque and the rotational speed are the two inputs for the lookup table of the motor efficiency. This lookup table is different from the others used in the rest of the model,
because it has two inputs instead of one; so the efficiency of the motor is not described by a
curve, but by a surface:

The measurements of the motor efficiency are incomplete: data is available only in a small
operation range of the motor. The measurements were performed in a range between 200
and 250 rpm, and between 4 and 10 Nm, while during the race the wheels rotation speed goes
from 0 up to 330 rpm, and the torque 0 up to 20 Nm. Based on the trend of the data and the
trend found in literature, the efficiency for the missing ranges was defined.
The output power of the motor passes by two branches. The first one calculates the output
current of the supercapacitor to the motor when it works as an actuator. The second one,
using a constant efficiency of 0.2, calculates the input current of the supercapacitor from the
motor when it works as a generator. A switch, depending on the value of the reference speed,
allows the signal coming from one branch or from the other to pass. The reference speed is a
signal which represents at each moment the speed commanded by the control boxes. When
the reference speed becomes very high, at least over 100 Km/h, it means that the motor is
working as a generator. The power absorbed by the additional devices and the control boxes
is divided by the supercapacitor voltage, giving the output currents. Finally all the currents
are summed up and they are given as output from the subsystem.

Going back to the supercapacitor block, the currents calculated are subtracted to the current
coming from the DC-DC converter, giving the total input-output current of the supercapacitor.
Afterwards this signal goes inside the model previously defined and yields the voltage of the
supercapacitor. Two parameters can be set: the initial value of the voltage and its upper limit.
Finally the voltage calculated in this series of passages is the output of the supercapacitor
subsystem.
2.8 Simulink model: control

The control subsystem performs tasks which in the real system are performed by the control boxes. In particular this part of the model reads the voltage of the supercapacitor at each iteration and decides which has to be the output power of the fuel cell. Without the control block, at each simulation, the user has to insert manually the output power in the way explained in the paragraph 2.2.1, with a waste of time. However with the control block there is the opportunity of working in auto mode, so that the only thing which the user has to insert is the logic of control. Here it is shown in more detail how the control block works:

Fig. 2.33: Simulink model of the control

The input signals are the supercapacitor voltage, and the speed of the car: the supercapacitor voltage passes through two different lookup table. One for the first lap, and one for the remaining laps. In the first lap the fuel cell does not work properly yet: the temperature is still low, and the membranes are not completely hydrated. Therefore a different control logic can be chosen for the first lap. Integrating the speed, the distance is calculated: this is the signal that commands the commutation of the switch from the first lap lookup table to the other one. In addition another switch commands an output power of the fuel cell of 0 W when the speed is about 0 Km/h. Here is an example of control curve:

Fig. 2.34: Control curve
The control curve has to be modified every time there is a change inside the system. In the plot below the comparison between the real voltage of the supercapacitor during a full race (the 5th performed in Lausitz) and the one calculated by the model working in auto mode with the previous control curve is shown.

![Comparison between the actual voltage and the simulated one with auto mode](image)

The calculated cycle fits quite well with the real one: this means that the logic used by the control block of the model is similar to the one of the real car. Anyway the control curve can be modified by the user with the aim of finding a better strategy than the actual one.

### 2.9 Simulink model: output box

All the measurement made in the system were collected together in one subsystem, the output box: here four main parameters are calculated: average output power from the fuel cell, fuel cell consumption, average partial efficiency, average total efficiency. The consumption is measured in Km/KWh: this is the unit of measurement used for the results of the race. In paragraph 1.3.6 it was figured out how to calculate the total consumption expressed in normal cubic meters; the following equation explains how to pass from this value to the value expressed in Km/KWh:

\[
\frac{Km}{KWh} = \frac{l \cdot 6}{V \cdot 3.0858}
\]

Where:
- \(l\): length of the track (m)
- \(V\): normal cubic meters of hydrogen (V)
The value 3.0858 is the energy content in KWh of one normal cubic meter of hydrogen considering its lower heating value:

\[
EC_{KWh} = \frac{EC_{MJ}}{3.6} = 3.0858
\]  

[2.10]

Where:

\( UEC_{MJ} \): energy content of a normal cubic meter considering the LHV = 11.109 MJ

\( EC_{KWh} \): energy content of a normal cubic meter expressed in KWh

The difference between the partial efficiency and the total efficiency has to be explained. The partial efficiency takes into account only the losses of the main devices of the system, the fuel cell, the DC-DC converter, the supercapacitor and the motor. The total efficiency considers also the losses due to the additional devices and the control boxes; here are their equations:

\[
\eta_{par} = \frac{E_w + E_{ad}}{E_H} \\
\eta_{tot} = \frac{E_w}{E_H}
\]  

[2.11-2.12]

Where:

\( E_w \): energy to the wheels (J)

\( E_H \): energy content of the hydrogen (J)

\( E_{ad} \): energy to the additional devices and control boxes (J)

All the equations defined before are implemented inside the output box:

Fig. 2.36: Output box in Simulink
3. Model validation

The validation is a very important part in the development of a model. In this step of the project the model results are compared with the experimental data to see how much they differ.

3.1 Data for the validation

Before the Shell-Eco marathon took place, there was no data available with which the results of the complete model could be compared. Lab tests on each component of the system were performed, providing good data. However, these were not enough for having a plausible validation of the model of the complete system, but only for the model of the singular devices, like the supercapacitor. During the race, Spiros was provided with a wireless acquisition system. The trend of all the signals measured could be checked on a big screen, and at the same time they were logged. Consequently, a lot of useful information coming from all the system could be collected.

Five races were performed: data was logged for the last four. However, due to problems with the reception, data for the second and the third race are not complete. For this reason only the data of the forth and the fifth race have been used for the validation. In particular there were five signals that were useful for the validation of the model: the fuel cell voltage, the input current of the supercapacitor, the output current of the supercapacitor, the voltage of supercapacitor and the speed of the car.

3.2 Validation steps

The validation process can be described by these few steps:

1. Calculating the actual output power cycle of the fuel cell
2. Turning the model to the manual modality
3. As input to the model putting the actual speed and the actual output power cycle of the fuel cell
4. Compare the calculated voltage of supercapacitor with the actual one

The first step can be carried out in two way. On the one hand the output voltage of the fuel cell can be used, and on the other the input current and the voltage of the supercapacitor can be used. In the first case, using the polarization curve, the output current and consequently the output power can be calculated. However this method shows some problems: the data is not accurate, and the conversion between voltage and current takes place with a high sensitivity (see figure 3.2). The combination of these two factors result in large errors.

![Inverse polarization curve](image1)

*Fig. 3.2: Inverse polarization curve*

In the current range between 30 to 68 A the voltage drops from 5.8 to 5.5 V. Therefore this lookup table is not the best way to find the power cycle of the fuel cell. In the second case the input current of the supercapacitor is multiplied by its voltage yielding the output power of the DC-DC converter. Then, it is divided by the efficiency of the DC-DC converter getting the output power of the fuel cell. The following graph represents the comparison between the calculated output current of the DC-DC convert and the real one:

![DC-DC output current comparison](image2)

*Fig. 3.3: Comparison between the calculated output current of the converter and the real one*
The two curves fit very well. For this reason this method is choosen for the calculation of the output power of the fuel cell.

### 3.3 Validation results

Once the two signals are defined, the validation can be done. In particular a simulation of a race is performed, and then the simulated voltage curve of the supercapacitor is compared with the real one. Before showing the results, it is interesting to show how sensible the supercapacitor is. Here is the validation cycle of the supercapacitor already shown in the paragraph 2.7.1.

![Figure 2.29](image1)

Afterward, the cycle of the current is increased with 0.1 A, in order to have a constant error all over it. The following plot represents the answer of the model:

![Figure 3.4](image2)

*Fig.3.4: Comparison between the results of the ESR-EPR model with an error and the measurements*
Figure 3.4 shows that at small error in the current input corresponds to a large error in the voltage output. According to this it is obvious that trend of the simulated voltage has not exactly the same behavior as the real one. The main errors that can influence the behavior of the voltage of the supercapacitor:

- Acquisition errors
- Errors in the calculation of the output power of the motor
- Accumulation of errors

The output power of the motor is calculated beginning from the forces. It is difficult to predict all the resistive forces due to their complexity. The main lack is the wind speed: it can cause a rather big change in the aerodynamic resistance, but it is very difficult to determine. In addition the errors increase with the time due to the integral behavior of the supercapacitor. However the validation was successful. Two comparisons were performed: one using data from the 4th race and one using data from the 5th race.

Fig.3.5 : Comparison between the simulated voltage and the real one during the 4° race

Fig 3.6 : Error depending on the time for the 4° race
The root mean square error and the root mean square percentage error have been also calculated:

\[
RMSE = \sqrt{\frac{err_1^2 + err_2^2 + \ldots + err_n^2}{n}} = 1.8071 \text{(V)}
\]  

[3.1]

\[
RMSPE = \sqrt{\frac{\left(\frac{err_1}{U_{r1}}\right)^2 + \left(\frac{err_2}{U_{r2}}\right)^2 + \ldots + \left(\frac{err_n}{U_{rn}}\right)^2}{n}} = 5.2443\%
\]  

[3.2]

Where:
- RMSE: root mean square error (V)
- \( err \): error at each point of acquisition (V)
- \( n \): number of points of acquisition
- RMSPE: root mean square percentage error
- \( U_r \): value of the measured voltage at each point of acquisition (V)

**Fig. 3.7:** Comparison between the simulated voltage and the real one during the 5° race

**Fig. 3.8:** Error depending on the time for the 5° race
\[ RMSE = 1.5175(V) \]
\[ RMSPE = 4.2373\% \]

The simulated voltage fits quite well with the actual one in some parts of the race. Even in some parts where there are large errors, the trend is almost always equal to the real one: when the actual voltage grows the simulated voltage grows and vice versa. Finally the root mean square of the error is lower than 2 V for both race, with percent value lower than 5 \%. Another important validation concerns the consumption. Actually this parameter is the most interesting during simulation. No official result has been obtained for the 4\textsuperscript{th} race, so it is only possible to validate the result of the 5\textsuperscript{th} race. The results are presented in the following table.

<table>
<thead>
<tr>
<th>Race</th>
<th>Official result (Km/KWh)</th>
<th>Calculated result (Km/KWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>not available</td>
<td>60.17</td>
</tr>
<tr>
<td>5</td>
<td>59.8</td>
<td>59.53</td>
</tr>
</tbody>
</table>

*Tab. 3.1: Comparison between real and calculated consumption*

**4. Results and discussion**

After the validation, the model is ready to be used for the final aim of the project: prediction and optimization of the fuel cell hybrid system. Therefore, it is interesting to see what happens, in terms of the behavior of the system, if some characteristics of the car or the devices are changed. These results are supposed to assist with decisions about the future changes of the vehicle.

**4.1 Simulating with auto mode**

The automatic modality is useful in the phase of simulation. Every time a modification is made to the system, its behavior changes. Consequently, a unique power cycle of the fuel cell cannot be used in different simulations. For this reason a specific power cycle for each simulation would have to be inserted by the user, requesting a considerable time. This step can be avoided thanks to the automatic modality. Thus, only the control curve has to be changed in every simulation (paragraph 2.8): it can be modified in Matlab environment saving a lot of time.

**4.2 Planning of the simulations**

Before beginning with the simulations, it is important to define a plan concerning the method that is going to be used, and which type of simulations have to be performed. In the first place the method used has to guaranty that the results of the different simulations can be compared. For example some simulations can be performed changing the supercapacitor: the system has a different behavior because one of the device is changed. Consequently the level of the
voltage of the supercapacitor differs from one simulation to the other in each point of the race. However a parameter that can be calculated in all the simulations is the average value of the voltage: the control curve has to be fixed every time so that this value is approximately equal to the real one.

In the second place, sets of simulation have to be designed depending on which characteristics or devices seem to be interesting. The sets of simulations performed are focused on three particular aspects of the car: the fuel stack, the supercapacitor and the weight of the car. The motivations of this choices will be explained in the following paragraph.

4.3 Controlled parameters

During the different simulations some parameters have to be checked in order to understand the performance of the system: they have to provide a complete characterization of the vehicle. The following four parameters were chosen:

- Average output power from the fuel cell (W)
- Fuel consumption (Km/KWh)
- Average partial efficiency
- Average total efficiency

All these parameters have been presented and explained in paragraph 2.9.

4.4 Weight influence

The first set of simulations performed concerns the influence of the weight on the performance of the car. This characteristic was chosen due to the high weight of Spiros IV: the car which won the race weighted 70 Kg, while Spiros weighed 135 Kg. For this reason, it would be interesting to know which behavior Spiros would have had with the weight of the first ranked. Therefore, beginning from the actual characteristic of Spiros, simulations were performed decreasing the weight with steps of 10 Kg down to 70 Kg.

![Average power and Consumption depending on the weight of the car](image)
As expected, the average output power of the fuel cell increases with increasing weight: a higher weight means higher resistive forces and consequently more power delivered by the stack. As a result, the consumption increases and the Km/KWh drop. More interesting is the plot of the curves of the efficiencies: while the average partial efficiency drops with increasing weight, the total one increases. An explanation can be found from the equation of the total efficiency. Beginning from equation 2.12:

\[ \eta_{tot} = \frac{E_w}{E_{H_2}} \]

It’s possible to express the energy to the wheels with the following equation:

\[ E_w = E_{H_2} - \sum |L_{all}| \]  \[4.1\]

Where:
\[ \sum |L_{all}| = \text{The sum of the absolute value of all the losses in the car (J)} \]

Entering the [4.1] inside the 2.12

\[ \eta_{tot} = \frac{E_{H_2} - \sum |L_{all}|}{E_{H_2}} \Rightarrow \eta_{tot} = 1 - \frac{\sum |L_{all}|}{E_{H_2}} \]  \[4.2\]

Equation [4.2] can be expressed also as following:

\[ \eta_{tot} = 1 - \left( \frac{|L_{fc}| + |L_{con}| + |L_{sc}| + |L_{mot}| + |L_{ad}|}{E_{H_2}} \right) \]
And also:

$$\eta_{tot} = 1 - \left[ \frac{L_{fc} + L_{con} + L_{sc} + L_{mot}}{E_{H_2}} \right] + \frac{L_{ad}}{E_{H_2}}$$

In this particular set of simulations the following trend can be observed:

$$\eta_{tot} = 1 - \left[ \frac{L_{fc} + L_{con} + L_{sc} + L_{mot}}{E_{H_2}} \right] + \frac{L_{ad}}{E_{H_2}}$$

The first component represents the losses in the main devices of the system: the fuel cell, the DC-DC converter, the supercapacitor and the motor. In particular the first component represents the losses that are involved in the definition of the partial efficiency. According to the graph, this part increases with the weight. The second component describes the losses due to the additional devices and control boxes. While the numerator is almost constant when the output power of the fuel cell is changed, the denominator increases. As a result, the ratio drops. In this particular case the drop of the second component is bigger than the growth of the first one, so that the total efficiency increases with the growing of the weight.

**4.5 8 Cells stack and supercapacitor influence**

Another set of simulations concerns the influence of the supercapacitor on the performance of the vehicle. The actual supercapacitor is quite small, so it is interesting to see what would happen if its capacitance was increased. In particular three different sizes were compared: the actual one (31.8 F), a capacitance twice as large as the actual one (64.6 F) and a capacitance three times as large as the actual one (95.4 F). Moreover, the weight of the system was increased due to the growth of the supercapacitor weight: 7 Kg for the actual one, 12.36 Kg for the 64.6 F one, and 17.72 Kg for the 95.4 F one. The results are shown in the following graphs:

*Fig. 4.3: Average power and Consumption of a 8 cells stack depending on the capacitance*
The graphs show that the trends are quite similar to the previous set of simulation. This is mainly because a growth in the capacitance means a growth in the weight of the system. The average output power of the fuel cell increases with the growth of the capacitance, while the Km/KWh drops. While the average partial efficiency drops with the increase of the capacitance, the average total efficiency is almost constant: this means that the growth of the losses due to the main devices of the system is more or less equal to the drop of the losses in the additional devices. According to the behavior of the controlled parameters, it seems that it is not convenient to replace the actual capacitance with a bigger one. However, it is interesting to watch the power cycle of the fuel cell with a 95.4 F capacitance:
The output power of the fuel cell is constant in big intervals of the race, with good consequences for the dynamic behavior of the stack. However, the model of the fuel cell is a static model, so it cannot predict such benefits.

4.6 12 cells stack and supercapacitor influence

This set of simulations concerns the influence of the supercapacitor on a system powered by a bigger stack than the actual one, a 12 cells stack. First of all, the polarization curve of the fuel cell has to be changed, but data for such a stack is not available. Thus, a polarization curve is constructed using the characteristic of the single cell of the actual stack. Here are the results:

Moreover, the weight of the stack has to be changed: the actual weight of the 12 cells stack is unknown, so 5 Kg are summed to the weight of the car. Another parameter which has to be increased is the limit under which the power absorbed by the air compressor is constant: it is changed from 123 W to 190W which corresponds to an output current of almost 20 A. For more explanations see the paragraph 2.6. The trend of the results is almost the same as for the previous set of simulations:

![Polarization curves and efficiency curves of the 12 cells stack](image)

![Average power and Consumption of a 12 cells stack depending on the capacitance](image)
The reason of this behavior is the same as before: a bigger supercapacitor means a higher weight. Anyway also in this set of simulations a bigger supercapacitor admits to have a more constant power cycle for the fuel cell, so a better dynamic behavior. The differences due to the different size of stack will be shown in paragraph 4.8.

4.7 5 cells stack and supercapacitor influence

The last set of simulations concerns the influence of the supercapacitor on a system powered by a smaller stack, a 5 cells stack. In this case, data for the polarization curve is available, but not for both growth and drop of current. This stack was used in the Spiros project in the beginning of the year, before it has been changed to the actual one. Therefore, the characteristics of the single cells are the same.

Also in this case the weight of the stack is unknown, so 5 Kg were subtracted from the total weight of the car due to the smaller size of the fuel cell. In addition, the limit under which the
power absorbed by the air compressor is constant is decreased: it is changed from 123W for the actual stack to 73W, which corresponds to an output current of about 18.3 A. While the average output power of the fuel cell, the consumption and the average total efficiency have the same trend as the previous set of simulations, the average partial efficiency has a different behavior:

![Graph showing the relationship between capacitance and power or consumption.](image)

**Fig. 4.12: Average power and Consumption of a 5 cells stack depending on the capacitance**

![Graph showing the relationship between capacitance and efficiency.](image)

**Fig. 4.13: Total and partial average efficiency of an 5 cells stack depending on the capacitance**

As shown in the graph, the average partial efficiency grows with the increase of the capacitance. An explanation can be found looking at the trend of the efficiency of the stack and of the DC-DC converter depending on the output power of the fuel cell:
The two curves are in countertrend. While in the previous sets of simulation the drop of efficiency of the stack was bigger than the growth of the efficiency of the DC-DC converter, here it is just the opposite. As a consequence both components of the equation 4.2 drop:

$$
\eta_{tot} = 1 - \left[ \left( \frac{|L_{fc}| + |L_{con}| + |L_{sc}| + |L_{mot}|}{E_{H_2}} \right) + \frac{|L_{ad}|}{E_{H_2}} \right] \quad [4.2]
$$

4.8 Stack influence

Finally the performance of the system depending on the size of the stack is compared. In order to investigate the influence of the number of cells of the stack, the capacitance of the supercapacitor is kept constant, and equal to the actual one. As before, the weight of the system and the limit under which the power absorbed by the air compressor is constant are changed depending on the size of the fuel cell: the values are the same as on the previous sets, thus they are not repeated. The results are presented in the graphs below:

Fig. 4.15: Average power and consumption depending on the size of the stack
Even if the average output power of the fuel cell is increasing with the growing of the number of cells, the consumption decreases: this is because both partial and total average efficiency grow. The explanation of the trend of the efficiency of the system can be found comparing the curve of the efficiencies of the different stack:

Over 200 W the difference of efficiency between the three stacks is big. Therefore, even if the weight increases, it seems to be convenient to use a bigger stack, because it can work at a higher efficiency.
4.9 Summary of the results

Each set of simulations has the goal to highlight the influence of a particular device or a particular characteristic on the behavior of the vehicle. After that it is possible to identify the trend of the performance in the cases considered during the simulations. In particular three main trends are evidenced:

1. If the weight grows, the performance of the car decreases
2. If the capacitance grows, the performance of the car decreases
3. If the number of cells grows, the performance of the car increases

The trend number 2 is not totally sure: the performance decreases with the growth of the capacitance because the weight of the system increases, but, due to the static model of the fuel cell, it is not possible to take into account the benefits for the dynamic behavior of the stack. Anyway, according to tendencies evidenced, it is possible to identify the best system between all the simulations performed. Its parameters and results are shown in the following table:

<table>
<thead>
<tr>
<th>System</th>
<th>Weight (Kg)</th>
<th>Capacitance(F)</th>
<th>N. of cells</th>
<th>Av. power (W)</th>
<th>Cons. (Km/KWh)</th>
<th>Total eff.</th>
<th>Partial eff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual</td>
<td>135</td>
<td>31.8</td>
<td>8</td>
<td>228.7</td>
<td>61.51</td>
<td>0.262</td>
<td>0.4136</td>
</tr>
<tr>
<td>Best</td>
<td>75</td>
<td>31.8</td>
<td>12</td>
<td>187.9</td>
<td>82.28</td>
<td>0.2729</td>
<td>0.4733</td>
</tr>
</tbody>
</table>

Tab. 4.1: Comparison between the best and the actual system
5. Conclusions

Not all the objectives that were stated in the beginning of the project were accomplished. The main missed goal is the optimization of the strategy. Before the race there has been the idea of running Spiros in auto mode: some corners should have been defined in specific points of the track in which speed and voltage of the supercapacitor were set. The values of these parameters would have been chosen in order to have the best performance of the car. However, before Shell-Eco marathon the model was not validated yet due to a lack of data. Therefore, it would have been dangerous to run Spiros with a strategy coming from a not verified model. In addition, the car was not working properly prior to the beginning of the competitions, thus more basic problems have had to be solved before an auto mode could be used.

Another consideration can be done about the fuel cell model: as said in the previous chapters, this is a static model. On the contrary the final goal of the project was the description of a dynamic system. Therefore the model of the stack is not the best that can be used: the dynamic behavior should be taken into account. However it is quite difficult to construct a dynamic model of a fuel cell: plenty data has to be collected, and consequently many tests have to be performed. In addition a badly constructed dynamic model, can result in larger errors in the simulation than a static one. Finally, the model based on lookup tables was preferred to a dynamic one.

On the other hand, there are some objectives which have been reached successfully, and sometimes with better results than expected. First of all the model has overcome the validation: this is the most difficult phase in the modeling process. Furthermore, some parts that have not been defined since the beginning of the project have been inserted into the model: the calculation of the forces beginning from the speed and the motor. As a result, the load which the system has to overcome can be set in terms of speed of the vehicle, a parameter much more simple to measure and understand by the user than the power output from the supercapacitor.

Finally, simulations concerning the prediction of the consequences due to changes in the system were performed: these showed interesting results how Spiros can be modified in the future.
6. Outlook

This project deals with a first attempt of modeling the hybrid system of Spiros building each device from zero. As a consequence, lots of improvement can be performed. First of all the fuel cell model is a static model which is part of a dynamic system. As a result, advantages and disadvantages due to its dynamic behavior cannot be recognized. In addition, there is no model of the motor available. Even if this device heavily influences the behavior of the system, it is modeled just with a lookup table. In addition the measurements of the efficiency are not complete. Another improvement concerns the additional devices and the control boxes: due to the lack of trustworthy data, they are modeled with approximate values. Finally a map of the inclination of the track should be provided, in order to have a more accurate calculation of the output power of the motor.
In the following two tables the value of the parameters and of the results of all the sets of simulations are summed up.

### Appendix A - Simulation summary

#### Tab 7.1: Parameters values for the simulations

<table>
<thead>
<tr>
<th>Set of simulations</th>
<th>N. simulation</th>
<th>N. of cells</th>
<th>Capacitance (F)</th>
<th>Weight (Kg)</th>
<th>Low limit (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight influence</td>
<td>1</td>
<td>8</td>
<td>31.8</td>
<td>135</td>
<td>123</td>
</tr>
<tr>
<td>Weight influence</td>
<td>2</td>
<td>8</td>
<td>31.8</td>
<td>125</td>
<td>123</td>
</tr>
<tr>
<td>Weight influence</td>
<td>3</td>
<td>8</td>
<td>31.8</td>
<td>115</td>
<td>123</td>
</tr>
<tr>
<td>Weight influence</td>
<td>4</td>
<td>8</td>
<td>31.8</td>
<td>105</td>
<td>123</td>
</tr>
<tr>
<td>Weight influence</td>
<td>5</td>
<td>8</td>
<td>31.8</td>
<td>95</td>
<td>123</td>
</tr>
<tr>
<td>Weight influence</td>
<td>6</td>
<td>8</td>
<td>31.8</td>
<td>85</td>
<td>123</td>
</tr>
<tr>
<td>Weight influence</td>
<td>7</td>
<td>8</td>
<td>31.8</td>
<td>70</td>
<td>123</td>
</tr>
<tr>
<td>Weight influence 8 cells-supercapacitor</td>
<td>1</td>
<td>8</td>
<td>31.8</td>
<td>135</td>
<td>123</td>
</tr>
<tr>
<td>Weight influence 8 cells-supercapacitor</td>
<td>2</td>
<td>8</td>
<td>63.6</td>
<td>140.36</td>
<td>123</td>
</tr>
<tr>
<td>Weight influence 8 cells-supercapacitor</td>
<td>3</td>
<td>8</td>
<td>95.4</td>
<td>145.72</td>
<td>123</td>
</tr>
<tr>
<td>Weight influence 12 cells-supercapacitor</td>
<td>1</td>
<td>12</td>
<td>31.8</td>
<td>140</td>
<td>190</td>
</tr>
<tr>
<td>Weight influence 12 cells-supercapacitor</td>
<td>2</td>
<td>12</td>
<td>63.6</td>
<td>145.36</td>
<td>190</td>
</tr>
<tr>
<td>Weight influence 12 cells-supercapacitor</td>
<td>3</td>
<td>12</td>
<td>95.4</td>
<td>150.72</td>
<td>190</td>
</tr>
<tr>
<td>Weight influence 5 cells-supercapacitor</td>
<td>1</td>
<td>5</td>
<td>31.8</td>
<td>130</td>
<td>73</td>
</tr>
<tr>
<td>Weight influence 5 cells-supercapacitor</td>
<td>2</td>
<td>5</td>
<td>63.6</td>
<td>135.36</td>
<td>73</td>
</tr>
<tr>
<td>Weight influence 5 cells-supercapacitor</td>
<td>3</td>
<td>5</td>
<td>95.4</td>
<td>140.72</td>
<td>73</td>
</tr>
<tr>
<td>Weight influence Stack influence</td>
<td>1</td>
<td>5</td>
<td>31.8</td>
<td>130</td>
<td>73</td>
</tr>
<tr>
<td>Weight influence Stack influence</td>
<td>2</td>
<td>8</td>
<td>31.8</td>
<td>135</td>
<td>123</td>
</tr>
<tr>
<td>Weight influence Stack influence</td>
<td>3</td>
<td>12</td>
<td>31.8</td>
<td>140</td>
<td>190</td>
</tr>
</tbody>
</table>

#### Tab 7.2: Results of the simulations

<table>
<thead>
<tr>
<th>Set of simulations</th>
<th>N. Simulation</th>
<th>Av. Power (W)</th>
<th>Total eff.</th>
<th>Partial eff.</th>
<th>Consumption (Km/KWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight influence</td>
<td>1</td>
<td>228.7</td>
<td>0.262</td>
<td>0.4136</td>
<td>61.51</td>
</tr>
<tr>
<td>Weight influence</td>
<td>2</td>
<td>221.5</td>
<td>0.261</td>
<td>0.4172</td>
<td>63.74</td>
</tr>
<tr>
<td>Weight influence</td>
<td>3</td>
<td>214.6</td>
<td>0.2665</td>
<td>0.4109</td>
<td>66.04</td>
</tr>
<tr>
<td>Weight influence</td>
<td>4</td>
<td>207.7</td>
<td>0.2683</td>
<td>0.4136</td>
<td>68.54</td>
</tr>
<tr>
<td>Weight influence</td>
<td>5</td>
<td>200.9</td>
<td>0.2765</td>
<td>0.4181</td>
<td>71.18</td>
</tr>
<tr>
<td>Weight influence</td>
<td>6</td>
<td>194.1</td>
<td>0.2944</td>
<td>0.4171</td>
<td>74.01</td>
</tr>
<tr>
<td>Weight influence</td>
<td>7</td>
<td>184</td>
<td>0.2501</td>
<td>0.4032</td>
<td>78.32</td>
</tr>
<tr>
<td>Weight influence 8 cells-supercapacitor</td>
<td>1</td>
<td>228.7</td>
<td>0.262</td>
<td>0.4136</td>
<td>61.51</td>
</tr>
<tr>
<td>Weight influence 8 cells-supercapacitor</td>
<td>2</td>
<td>231.7</td>
<td>0.2615</td>
<td>0.4102</td>
<td>60.75</td>
</tr>
<tr>
<td>Weight influence 8 cells-supercapacitor</td>
<td>3</td>
<td>234</td>
<td>0.2635</td>
<td>0.4107</td>
<td>60.12</td>
</tr>
<tr>
<td>Weight influence 12 cells-supercapacitor</td>
<td>10</td>
<td>232.8</td>
<td>0.2861</td>
<td>0.4503</td>
<td>65.16</td>
</tr>
<tr>
<td>Weight influence 12 cells-supercapacitor</td>
<td>11</td>
<td>235.6</td>
<td>0.2858</td>
<td>0.4468</td>
<td>64.41</td>
</tr>
<tr>
<td>Weight influence 12 cells-supercapacitor</td>
<td>12</td>
<td>238.8</td>
<td>0.2879</td>
<td>0.4467</td>
<td>63.64</td>
</tr>
<tr>
<td>Weight influence 5 cells-supercapacitor</td>
<td>13</td>
<td>224.8</td>
<td>0.2423</td>
<td>0.3844</td>
<td>58.75</td>
</tr>
<tr>
<td>Weight influence 5 cells-supercapacitor</td>
<td>14</td>
<td>228.1</td>
<td>0.2452</td>
<td>0.3867</td>
<td>58.32</td>
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<tr>
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<td>231.4</td>
<td>0.2484</td>
<td>0.3891</td>
<td>57.85</td>
</tr>
<tr>
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<td>228.7</td>
<td>0.262</td>
<td>0.4136</td>
<td>61.51</td>
</tr>
<tr>
<td>Weight influence Stack influence</td>
<td>10</td>
<td>232.8</td>
<td>0.2861</td>
<td>0.4503</td>
<td>65.16</td>
</tr>
</tbody>
</table>
close all
clear

%Fuel cell
n_cells = 8; %number of cells

%Fuel cell lookup table for growing current
fuel_cell_curr_out_grow = [0;10;25;30;40;50;60;70];
fuel_cell_vol_grow = ([0.95*8;0.81*8;0.75*8;5.77;0.72*8;0.7*8;0.69*8;0.68*8]+0.04); fuel_cell_power_out_grow = fuel_cell_curr_out_grow.*fuel_cell_vol_grow;
%calculation of the power
fuel_cell_curr_volt_int_grow = fit(fuel_cell_curr_out_grow,fuel_cell_vol_grow,'linearinterp'); %fuel cell polarization curve with growing current
fuel_cell_power_curr_int_grow = fit(fuel_cell_power_out_grow,fuel_cell_curr_out_grow,'linearinterp'); %fuel cell current output depending on the power
fuel_cell_eff_grow = fuel_cell_vol_grow/1.01/1.23/n_cells; %fuel cell efficiency using LHV
fuel_cell_power_eff_grow = fit(fuel_cell_power_out_grow,fuel_cell_eff_grow,'linearinterp'); %fuel cell efficiency depending on the power
fuel_cell_curr_eff_grow = fit(fuel_cell_curr_out_grow,fuel_cell_eff_grow,'linearinterp'); %fuel cell efficiency depending on current

%Fuel cell lookup table for dropping current
fuel_cell_curr_out_drop = (0:10:70);
fuel_cell_vol_drop = ([0.96*8;0.83*8;0.8*8;0.76*8;0.72*8;0.7*8;0.68*8]+0.04);
fuel_cell_power_out_drop = fuel_cell_curr_out_drop.*fuel_cell_vol_drop;
%calculation of the power
fuel_cell_curr_volt_int_drop = fit(fuel_cell_curr_out_drop,fuel_cell_vol_drop,'linearinterp'); %fuel cell polarization curve with dropping current
fuel_cell_power_curr_int_drop = fit(fuel_cell_power_out_drop,fuel_cell_curr_out_drop,'linearinterp'); %fuel cell current output depending on the power
fuel_cell_eff_drop = fuel_cell_vol_drop/1.01/1.23/n_cells; %fuel cell efficiency using LHV
fuel_cell_power_eff_drop = fit(fuel_cell_power_out_drop,fuel_cell_eff_drop,'linearinterp'); %fuel cell efficiency depending on the power
fuel_cell_curr_eff_drop = fit(fuel_cell_curr_out_drop,fuel_cell_eff_drop,'linearinterp'); %fuel cell efficiency depending on current

%DC-DC converter lookup table
dc_dc_con_eff = [0.75;0.759;0.8156;0.8684;0.870;0.8711;0.8603];
dc_dc_con_power_input = [0;66.4;128;182.4;240;288;336;380.8];
dc_dc_con_power_eff_int = fit(dc_dc_con_power_input,dc_dc_con_eff,'linearinterp'); %DC-DC converter efficiency depending on the power input
%Supercapacitor

EPR=43.7617; %equivalent parallel resistance
ESR=0.0193; %equivalent series resistance
C=31.8; %equivalent capacitance

%motor
%vectors of the efficiency depending on the rpm
motr_eff_0_Nm=[0.7571,0.7571,0.81,0.81,0.78];
motr_eff_4_Nm=[0.7571,0.7571,0.81,0.81,0.78];
motr_eff_8_Nm=[0.7373,0.7373,0.7624,0.7624,0.73];
motr_eff_10_Nm=[0.721,0.721,0.7563,0.7563,0.72];
motr_eff_20_Nm=[0.721,0.721,0.7563,0.7563,0.72]-0.1;

%Control system

%control curve for the first lap
control_voltage_step_first = [31.4;31.9;32.00001;36.6;37;37.5;38.1];
control_power_step_first = [265;265;240;240;190;142.5;0.0001];
control_voltage_power_step_first_int=fit(control_voltage_step_first,control_power_step_first,'linearinterp');

%control curve for the other laps
control_voltage_step = [31.4;31.9;32.00001;36.37;37.5;38.1];
control_power_step = [325;325;295;295;250;212.5;0.0001];
control_voltage_power_step_int = fit(control_voltage_step,control_power_step,'linearinterp');

%oxygen flow rate lower limit
flow_rate_lower_limit = 123; %power lower limit for the air compressor
curr_lower_limit = fuel_cell_power_curr_int_grow(flow_rate_lower_limit);
hydr_pow_lower_limit = flow_rate_lower_limit/fuel_cell_power_eff_grow(flow_rate_lower_limit);

%Air pump lookup table
ter_pump_pow_int = fit([0;350],[15;25],'linearinterp');
air_pump_inp = [0;flow_rate_lower_limit;350;400];
air_pump_power = ter_pump_pow_int(flow_rate_lower_limit);ter_pump_pow_int(flow_rate_lower_limit);
air_pump_inp_pow_int = fit(air_pump_inp,air_pump_power,'linearinterp');

%Air fan lookup table
power_fuel_cell_fan = [0;249.99;250;350;400];
power_air_fan = [0;0;2;3;3];
power_fan_power_fuel_cell_fit = fit(power_fuel_cell_fan,power_air_fan,'linearinterp');

%declaration of the inputs (speed and output power of the fuel cell)

%speed declaration
load time.txt %vector of the time
load act_speed.txt %vector of the speed
speed_smoothed=smooth(act_speed,5);
Speed=[time,speed_smoothed]; %input signal of the speed

%lookup table for the inclination of the track
load angolo.txt %vector of the sinus of the inclination angle of the track
load laps.txt %distance
Incl_angle=fit(laps,angolo,'linearinterp'); %input signal of the inclination

%reference speed declaration
load ref_speed.txt
ref_speed=[time,ref_speed];

%output power of the fuel cell declaration
load power_output_fuel_cell.txt
power_out=[time,smooth(power_output_fuel_cell,10)];
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