Evaluation of HMA Fracture Mechanics-Based Thermal Cracking Model

Master Degree Project

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**Abstract:** Low temperature cracking is an important form of asphalt pavement deterioration in cold regions. The cracks develop when thermally induced stresses exceed the fracture resistance of the asphalt pavement. In this study, by incorporating HMA fracture mechanics into thermal cracking model, a new integrated model is introduced to investigate low temperature cracking performance. To evaluate its reliability and accuracy, the predicted thermally induced stress and failure temperature are compared with the fracture stress and fracture temperature obtained from thermal stress restrained specimen test. The findings indicate that this HMA fracture mechanics-based thermal cracking model has a great potential to reliably evaluate the performance of asphalt mixtures subjected to thermally induced damage.

**Key Words:** Hot Mix Asphalt; Low temperature cracking; HMA fracture mechanics; Thermal cracking model; SuperPave IDT; Thermal Stress Restrained Specimen Test
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Also I wish to thank Dr. Michael Behn and other PhD students in the Division of Highway and Railway Engineering who helped me during my thesis project work, for their great advices and supports.

Finally, I owe my sincere gratitude to all my classmates and friends who accompany me during this two-year master program.
<table>
<thead>
<tr>
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<th>Definition</th>
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<tbody>
<tr>
<td>$a_i$</td>
<td>Half of crack length</td>
</tr>
<tr>
<td>$a_g$</td>
<td>Slope of lower asymptote</td>
</tr>
<tr>
<td>$a_l$</td>
<td>Slope of higher asymptote</td>
</tr>
<tr>
<td>$C$</td>
<td>Crack depth</td>
</tr>
<tr>
<td>$C_0$</td>
<td>Current crack length</td>
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<tr>
<td>$C_{\text{comp}}$</td>
<td>Non-dimensional creep compliance factor</td>
</tr>
<tr>
<td>$C_r$</td>
<td>Creep compliance</td>
</tr>
<tr>
<td>$C_{sx}$</td>
<td>Horizontal stress correction factor</td>
</tr>
<tr>
<td>$C_v$</td>
<td>Constant without physical significance</td>
</tr>
<tr>
<td>$d$</td>
<td>Diameter of specimen</td>
</tr>
<tr>
<td>$D$</td>
<td>Thickness of asphalt pavement surface layer</td>
</tr>
<tr>
<td>$D_0$</td>
<td>Creep compliance parameter</td>
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<tr>
<td>$D_1$</td>
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<tr>
<td>$D(\xi)$</td>
<td>Creep compliance at reduced time $\xi$</td>
</tr>
<tr>
<td>$D(\xi - \xi')$</td>
<td>Creep compliance at reduced time $\xi - \xi'$</td>
</tr>
<tr>
<td>$D_{\text{CSE}}$</td>
<td>Dissipated Creep Strain Energy to failure</td>
</tr>
<tr>
<td>$D_{\text{CSE}_{\text{min}}}$</td>
<td>Minimum Dissipated Creep Strain Energy</td>
</tr>
<tr>
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<td>Prony series parameter</td>
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<td>$E(\xi)$</td>
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<td>$GL$</td>
<td>Gauge length</td>
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<tr>
<td>$h$</td>
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<tr>
<td>$L[D(t)]$</td>
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<td>$L[E(t)]$</td>
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<td>$m$</td>
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<tr>
<td>$M_R$</td>
<td>Resilient modulus</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of Kelvin elements</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of loading cycle</td>
</tr>
<tr>
<td>$\text{Pen}_{77}$</td>
<td>Penetration value at 77°F (25°C)</td>
</tr>
<tr>
<td>$r$</td>
<td>Distance from crack tip</td>
</tr>
<tr>
<td>$R$</td>
<td>Parameter representing the curve between asymptotes</td>
</tr>
<tr>
<td>$P$</td>
<td>Load</td>
</tr>
<tr>
<td>$s$</td>
<td>Laplace parameter</td>
</tr>
<tr>
<td>$S$</td>
<td>Cracks spacing of asphalt pavement surface layer</td>
</tr>
<tr>
<td>$S_t$</td>
<td>Tensile strength</td>
</tr>
<tr>
<td>$t_{\text{CSE}}$</td>
<td>Time to 5 mm crack length Due to Dissipated Creep Strain Energy</td>
</tr>
<tr>
<td>$t_{\text{FE}}$</td>
<td>Time to 5 mm crack length Due to Fracture Energy</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
</tr>
<tr>
<td>$T_g$</td>
<td>Glass transition temperature</td>
</tr>
<tr>
<td>$\dot{T}(\xi')$</td>
<td>Temperature change rate</td>
</tr>
<tr>
<td>$S_t$</td>
<td>Tensile strength</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
</tr>
<tr>
<td>$X/Y$</td>
<td>Ratio of horizontal to vertical deformation</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Thermal contraction coefficient</td>
</tr>
<tr>
<td>$\alpha_T$</td>
<td>Time - temperature shift factor</td>
</tr>
<tr>
<td>$\varepsilon_f$</td>
<td>Failure strain</td>
</tr>
<tr>
<td>$\varepsilon(\xi)$</td>
<td>Thermal strain at reduced time $\xi$</td>
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<tr>
<td>$\varepsilon_{cr}(\xi)$</td>
<td>Thermal creep strain at reduced time $\xi$</td>
</tr>
<tr>
<td>$\varepsilon(\xi')$</td>
<td>Thermal strain rate</td>
</tr>
<tr>
<td>$\eta_v$</td>
<td>Binder viscosity</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>Prony Series parameter</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson’s Ratio</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Reduced time</td>
</tr>
<tr>
<td>$\xi'$</td>
<td>Real time integrating variable</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stress</td>
</tr>
<tr>
<td>$\sigma_{AVG}$</td>
<td>Average stress</td>
</tr>
<tr>
<td>$\sigma_{FA}$</td>
<td>Faraway stress from pavement</td>
</tr>
<tr>
<td>$\sigma(\xi)$</td>
<td>Stress at reduced time $\xi$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Prony series parameter</td>
</tr>
<tr>
<td>$\upsilon$</td>
<td>Specific volume change</td>
</tr>
<tr>
<td>$\Delta H$</td>
<td>Horizontal deformation</td>
</tr>
<tr>
<td>$\Delta S/\Delta T$</td>
<td>Slope of stress-temperature curve of Thermal Stress Restrained Specimen Test</td>
</tr>
</tbody>
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**List of Abbreviations**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>AASHTO</td>
<td>American Association of State Highway and Transportation Officials</td>
</tr>
<tr>
<td>ABT</td>
<td>Allmän Teknisk Beskrivning (General Technical Standards)</td>
</tr>
<tr>
<td>CCMC</td>
<td>Creep Compliance Master Curve</td>
</tr>
<tr>
<td>DCSE</td>
<td>Dissipated Creep Strain Energy</td>
</tr>
<tr>
<td>ESALs</td>
<td>Equivalent Single Axle Load</td>
</tr>
<tr>
<td>EE</td>
<td>Elastic Energy</td>
</tr>
<tr>
<td>ER</td>
<td>Energy Ratio</td>
</tr>
<tr>
<td>FE</td>
<td>Fracture Energy</td>
</tr>
<tr>
<td>HMA</td>
<td>Hot Mix Asphalt</td>
</tr>
<tr>
<td>IDT</td>
<td>Indirect Tensile Test</td>
</tr>
<tr>
<td>LVDT</td>
<td>Linear Variable Differential Transducer</td>
</tr>
<tr>
<td>NCHRP</td>
<td>National Cooperative Highway Research Program</td>
</tr>
<tr>
<td>SHRP</td>
<td>Strategic Highway Research Program</td>
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<tr>
<td>TCMODEL</td>
<td>Thermal Cracking Model</td>
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<tr>
<td>TSRST</td>
<td>Thermal Stress Restrained Specimen Test</td>
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1. Introduction

The common distress of asphalt pavement in cold regions is the low temperature cracking, which results from the contraction and expansion of the asphalt pavement under extreme temperature changes (Birgisson et al., 2004). Generally it appears as regularly spaced transverse cracks across pavement surface as shown in Figure 1. Water entering the crack may freeze during wintertime and result in the formation of ice lenses, which in turn may produce frost heave. Pumping of fines through the crack may produce voids under the pavement, which means that the bearing capacity may be reduced. Consequently, the low temperature cracking may cause poor ride quality, reduce service life and increase costs for rehabilitation. Recently, a Hot Mix Asphalt (HMA) fracture mechanics-based thermal cracking model was developed by Das et al. (2011). In this project, the susceptibility of asphalt mixtures to low temperature cracking will be studied by this model which incorporates a fundamental crack growth mechanism associated with damage accumulation and the dissipated energy threshold.

Figure 1. Typical Thermal Cracking in Asphalt Pavement

The principal objective of this project is to evaluate the HMA fracture mechanics-based thermal cracking model. The work comprises the following subtasks:

i. Reviewing the energy-based HMA fracture mechanics and the Thermal Cracking performance Model (TCMODEL).

ii. Implementing the HMA fracture mechanics-based thermal cracking model with the experimental input data.
iii. Validating the predicted thermally induced stress and fracture temperature with the Thermal Stress Restrained Specimen Test (TSRST).

In experimental design, the SuperPave Indirect Tension Tests (SuperPave IDT) are conducted to collect the mechanical properties of three design asphalt mixtures, which serve as the input parameters in the HMA fracture mechanics-based thermal cracking model. Then first the model generates Creep Compliance Master Curve (CCMC) to obtain creep compliance. By converting the creep compliance to relaxation modulus, the model in turn calculates thermal stress and creep strain. Based on energy principle, the total accumulated Dissipated Creep Strain Energy (DCSE) and the total accumulated Fracture Energy (FE) govern the crack development until failure. Finally the model outputs the thermal stress with the changing temperatures and gives an accurate prediction as crack length versus temperature or time.

With different binder grades (Pen77 35/50, 70/100 and 160/220), in total 27 core specimens (150 mm in diameter and 50 mm in thickness) are tested by SuperPave IDT at different temperatures (0 °C, -10 °C and -20 °C), respectively. Also each 4 beam specimens (38 mm × 38 mm × 200 mm) with the same 3 binder grades as SuperPave IDT are tested by TSRST. The predicted thermal stress and fracture temperature are compared with the corresponding values obtained from the TSRST to evaluate the reliability and accuracy of the model.
2. HMA Fracture Mechanics

Cracking mechanism in asphalt mixture may be better understood with the theory of fracture mechanics which combines the mechanics of crack initiation or propagation with the mechanical properties of material. An HMA fracture mechanics model developed by Zhang et al. (2001) at University of Florida provided a fundamental crack growth law for evaluating performance of asphalt mixture. The HMA fracture mechanism primarily consists of two principal theories: theory of linear viscoelasticity and energy-based fracture mechanics. The following sections provide an explanation of the basic principles of HMA fracture mechanics.

2.1 Fracture Thresholds

The fundamental fracture threshold is the core of the HMA fracture mechanics model presented by Zhang et al. (2001). The concept is based on the observation of the discontinuous crack growth in HMA material (Figure 2). Micro-damage (i.e. the damage which does not result in crack initiation or growth) appears to be fully healable, while macro-damage (i.e. the damage which results in crack initiation or growth) does not appear to be healable.

![Figure 2. Crack Propagation in Asphalt Mixture (Birgisson et al., 2004)](image)

The macro-damage development at any time during either crack initiation or propagation is mastered by the lower and upper thresholds: Dissipated Creep Strain Energy to failure ($DSCE_f$) and Fracture Energy ($FE$). Accordingly, Birgisson et al. (2007) suggested two possibilities that the macro-damage develops in asphalt mixture. One is due to creep strain energy, a number of continuously repeated loads with stresses significantly below the tensile strength would cause damage accumulation and lead to fracture when the $DSCE_f$ is reached. The other way is when any large single load applied during the loading cycle exceeds the
FE threshold then fracture would occur.

DSCE has been found to be independent of loading mode or loading history. Therefore, the development of crack in asphalt mixture can be determined by calculating the amount of DSCE induced by any loading condition and comparing it to the DSCE of the asphalt mixture. As illustrated in the stress-strain response (Figure 3), DSCE is determined by the difference between FE and Elastic Energy (EE) for a single load. FE is the area under stress-strain curve to the failure strain.

![Graphical Illustration of DCSE](image)

**Figure 3. Graphical Illustration of DCSE** (Birgisson et al., 2004)

### 2.2 Crack Growth Law

Based on the concept of energy threshold, Zhang et al. (2001) developed the HMA fracture mechanics model. It was assumed to have a 10 mm initial crack size based on the typical aggregate size of asphalt mixtures. According to Birgisson et al. (2007), the continuous cyclic loading will increase the accumulative DCSE in the initiation zone until it reaches the DCSE threshold as showed in Figure 4a. Then the stress at the crack tip will draw a high rate of DCSE accumulation in the process zone next to the crack tip (Figure 4b). The length of the process zone \( \gamma \) is defined by Equation [1].

\[
\gamma_i = \frac{1}{2} \cdot \left( \frac{\sigma_{FA}}{S_t} \right)^2 \cdot a_i \quad (i > 1)
\]

where \( S_t \) is tensile strength, \( \sigma_{FA} \) is faraway stress and \( a_i \) is current crack length. In Figure 4c, the DCSE accumulation process continues in the new process zone and the crack grows at a faster rate in the new process zone.
2.3 Energy Ratio

Roque et al. (2004) introduced Energy Ratio (ER) into the HMA fracture mechanics model to further evaluate the fracture performance of asphalt mixture. ER is defined as $\text{DCSE}_f$ of asphalt mixture over $\text{DCSE}_{\text{min}}$ which is the minimum $\text{DCSE}$ required for a good cracking performance that serves as a single criterion for cracking performance by considering both asphalt mixture properties and pavement characteristics. ER is a measure of fracture resistance of asphalt mixture and can be calculated as follow:

$$\text{ER} = \frac{\text{DCSE}_f}{\text{DCSE}_{\text{min}}} = \frac{\text{DCSE}_f [0.0299\sigma^{-3.1}(6.36-S_t)+246\cdot10^8]}{m^{2.98} \cdot D_1}$$

where $\text{DCSE}_f$ is dissipated creep strain energy to failure, $\text{DCSE}_{\text{min}}$ is minimum dissipated creep strain energy for adequate cracking performance and $S_t$ is tensile strength. $\text{DCSE}_{\text{min}}$ is a function of creep compliance power law parameters ($m$-value and $D_1$) as illustrated in Figure 5. For a good field performance of the mixture, the ER should be greater than 1.0.

The basic principles of HMA fracture mechanics are shown in Figure 6 if two asphalt mixtures with different properties are compared. For either asphalt mixture, the DCSE increases with number of load applications (ESALs). The asphalt mixture with higher creep
compliance power law (m-value and $D_1$) exhibits a higher rate of dissipated creep strain energy accumulation. However, this does not necessarily imply that cracks will initiate or propagate more quickly in this asphalt mixture, as it depends on the energy threshold, which varies significantly between asphalt mixtures and is not necessarily related to its creep characteristics.

Figure 5. Creep Compliance Power Law Parameters (Roque et al., 2004)

Figure 6. Basic Principles of HMA Fracture Mechanics Model (Roque et al., 2004)

It is evident that no single asphalt mixture property can be used to predict asphalt mixture cracking performance reliably. In order to use the HMA fracture mechanics-based thermal cracking model, resilient modulus ($M_R$), creep compliance power law parameters (m-value and $D_1$), tensile strength ($S_t$), dissipated creep strain energy to failure (DCSE$_f$) and Fracture Energy (FE) are required. More details about how to obtain all these parameters from the SuperPave IDT were specified by Du (2010).
3. HMA Fracture Mechanics-Based Thermal Cracking Model

Low temperature cracking is a thermally induced distress caused by the extremely temperature change in cold climate. The existence of transverse cracks eventually causes thermal contraction and failure, resulting in different kinds of degradation to asphalt pavement. Several different thermal cracking models have been developed by applying empirical and/or analytical methods. Few of them, however, attempt to incorporate a fundamental crack growth mechanism associated with damage accumulation and the dissipated energy threshold in asphalt pavement. Therefore, a mechanics-based thermal cracking performance model, which is able to incorporate the energy-based HMA fracture mechanics, may provide a proper framework to evaluate the thermal cracking performance of asphalt pavement.

3.1 Thermal Cracking Performance Model

As part of the Strategic Highway Research Program (SHRP), a mechanics-based Thermal Cracking performance Model (TCMODEL, Hiltunen and Roque, 1994) was developed based on the theory of linear viscoelasticity. TCMODEL predicts the amount of thermal cracking that will develop in an asphalt pavement as a function of time, which provides the basis for a true performance-based mixture specification for thermal cracking.

3.1.1 Thermal Cracking Mechanism

The thermal contraction strains induced by pavement cooling always lead to thermal tensile stress development in the restrained surface layer. On the other hand, the tensile strength of asphalt pavement increases only to a maximum and then decreases. Thermal tensile stress development is mostly in the longitudinal direction of pavement since this direction is restrained. Also thermal tensile stress is greatest at the surface of pavement because of the lower temperature and the higher temperature change (Witczak et al. 2000). In Figure 7, an asphalt pavement surface layer of thickness $D$ is shown to be subjected to a tensile stress distribution with depth and the potential crack sites are uniformly spaced at a distance $S$.

There are two different types of thermal cracking. In case of very severe cooling cycles, e.g. extremely low temperature and/or fast cooling rates, low-temperature cracking may occur when the thermal tensile stress in asphalt mixture exceeds its tensile strength. If the tensile stress is below the tensile strength, the pavement will not crack under a single daily temperature cycle but could still crack after a long time tensile stress accumulation. This is usually referred to as thermal fatigue cracking. Essentially the failure mechanisms are same for these two types of cracking and the only difference is the rates at which cracks occur.
3.1.2 Crack Propagation Model

Temperature-induced thermal cracking is a “top-down” initiation and propagation. During the cooling process, stresses develop due to the contraction of asphalt pavement. The stresses are not constant with depth because of a thermal gradient in the pavement and the temperatures vary with the depth. Within the surface layer there are potential crack zones uniformly spaced at a regular distance (Figure 7). At each of these crack zones the thermally induced stresses can cause a crack to propagate through the surface layer (Figure 8), where $\Delta C$ is the crack growth due to the cooling cycle and $C_0$ is the initial crack length for the next cooling cycle. Due to spatial variation of the relevant material properties within the surface layer, each of these cracks can propagate at a different rate (Witczak et al. 2000).

3.2 Thermal Cracking Prediction

The physical behavior of asphalt mixture can be approximated by theory of linear viscoelasticity. It exhibits viscosity even at low temperature which will affect the thermal cracking performance of an asphalt pavement.
Thermal cracking prediction is conducted by the following steps:

i. Gathering input data and introducing them into the TCMODEL.

ii. Developing Creep Compliance Master Curve (CCMC).

iii. Converting creep compliance to relaxation modulus.

iv. Predicting thermal stress using viscoelastic transformation theory.

3.2.1 Thermal Contraction Coefficient

Thermal contraction coefficient ($\alpha$) is one of the thermo-mechanical properties required in the prediction of thermal cracking of asphalt pavements because thermally induced stresses and strains are directly proportional to it. The $\alpha$ value of asphalt mixture is dependent on the thermal properties of selected binder and mineral aggregate.

Although the thermal contraction coefficient of asphalt mixture has non-linear behavior, a simplified $\alpha$ value (constant, linear or bi-linear) has been using in many thermal cracking models. The reliability of thermal cracking prediction is mostly depends upon the accuracy of this value. In this study, the constant thermal contraction coefficients obtained from stress-temperature curve of TSRST are applied. Alternatively, the non-linearity of thermal contraction coefficient is taken into account while calculating thermally induced stresses and strains by using the following Equations developed by Nam and Bahia (2004):

\[
\alpha = \frac{du}{dT}
\]

\[
u = c_v + a_g (T - T_g) + R (a_l - a_g) \ln \left(1 + \exp \left[\frac{(T - T_g)}{R}\right]\right)
\]

where $\nu$ is specific volume change in ml/g; $c_v$ is a constant without physical significance; $T_g$ is glass transition temperature; $a_g$ and $a_l$ is slope of lower and higher asymptote respectively and $R$ is a parameter representing the curvature between asymptotes. More information about how to apply the no-linear thermal contraction coefficient on the same three design asphalt mixtures was specified by Das et al. (2011).

3.2.2 Creep Compliance Master Curve

Since the viscoelastic properties of asphalt mixture are temperature dependent, the model requires creep compliance parameters ($D_0$, $D_1$ and m-value) at three different temperatures (0 °C, -10 °C and -20 °C). These parameters can be determined from the SuperPave IDT
static creep test. Whereas, creep compliance is simply the time independent stain divided by the constant stress used in the creep test. Once the parameters are known, Creep Compliance Master Curve (CCMC) can be generated by using time-temperature superposition principle (Hiltunen and Roque 1994; Buttlar et al. 1998). The lowest temperature (-20 °C) is set as reference temperature. Then as showed in Figure 9, the CCMC is fitted with power model in log-log scale and mathematically it can be expressed as follow (Witczak et al. 2000):

\[ D(\xi) = D_0 + D_1 \xi^m \]  \[5\]

where \( D(\xi) \) is creep compliance at reduced time \( \xi \) and \( D_0, D_1 \) and \( m \) are the creep compliance parameters. In chapter 5 Figures (17 - 22), the different creep compliance data and the CCMC fitted by power law for each of the three design asphalt mixtures are illustrated, respectively.

![Figure 9. Power Model of Creep Compliance (Witczak et al., 2000)](image)

### 3.2.3 Conversion of Creep Compliance to Relaxation Modulus

Since creep and stress relaxation phenomena are two aspects of the same viscoelastic behavior of material, they are obviously related. The relationship between creep compliance and relaxation modulus can be presented by a convolution integral as follows:

\[ \int_0^\infty D(t - \xi) \frac{dE(\xi)}{d\xi} d\xi = 1 \]  \[6\]

Applying the Laplace transformation to Equation [6] then yield Equation [7],

\[ L[D(t)] \cdot L[E(t)] = \frac{1}{s^2} \]  \[7\]

where \( L[D(t)] \) is Laplace transformation of the creep compliance, \( L[E(t)] \) is Laplace transformation of the relaxation modulus, \( s \) is the Laplace parameter and \( t \) is time (for the master curve, the reduced time \( \xi \) is used).
Once the relaxation modulus is known, it can be presented by a generalized Maxwell model to represent the viscoelastic properties. Mathematically, the relaxation modulus for a generalized Maxwell model can be expressed according to the following Prony Series.

$$E(\xi) = \sum_{i=1}^{N+1} E_i e^{-\xi/\lambda_i}$$  \[8\]

where $E(\xi)$ is relaxation modulus at reduced time, $E_i$ and $\lambda_i$ are the Prony series parameters for relaxation modulus master curve. In chapter 5 Figures (23 - 25), the relaxation modulus fitted by Prony Series for each of the three design asphalt mixtures are illustrated, respectively.

### 3.2.4 Calculation of Thermal Stress

For transient temperature conditions where the temperature varies with the time, thermal stress is generally involved and developed due to the thermal contraction. For linear viscoelastic materials, the thermal stress can be predicted by using Boltzmann’s Superposition Principle.

$$\sigma(\xi) = \int_0^\xi E(\xi - \xi') \frac{d\varepsilon(\xi')}{d\xi'} d\xi'$$  \[9\]

where $\sigma(\xi)$ is the stress at reduced time $\xi$, $E(\xi - \xi')$ is relaxation modulus at reduced time and $\xi'$ is the real time integrating variable. Morland and Lee (1960) introduced the following reduced time, which is able to take into account both effects of temperature gradients and time variations coincidently.

$$\xi(t) = \int_0^t \frac{1}{\alpha_T(T(t'))} dt'$$  \[10\]

where $\alpha_T$ is the time-temperature shift factor which can be determined from Arrhenious Equation or Williams-Landel-Ferry (WLF) Equation.

The other parameter thermal strain rate $\dot{\varepsilon}(\xi')$ \[i.e. \frac{d\varepsilon(\xi')}{d\xi'}\] which is directly related to the thermal contraction coefficient ($\alpha$).

$$\dot{\varepsilon}(T, \xi') = \alpha(T) \times \dot{T}(\xi')$$  \[11\]

Where the rate of change in temperature $\dot{T}(\xi') = dT(\xi')/d\xi'$. 
3.3 General Concept of HMA Fracture Mechanics-Based Thermal Cracking Model

The general concept used to calculate the amount of crack development of the HMA fracture mechanics-based thermal cracking model consists of 5 steps:

i. Defining the process zones.

ii. Predicting thermal stress.

iii. Calculating average stresses within each process zone.

iv. Calculating and assigning DCSE within each individual process zone.

v. Calculating and assigning FE within each individual process zone.

The number of process zones contributes as an important factor that affects the computation time. In this model, a 10 mm initial crack length followed by 5 mm of processing zone is assumed, which generally corresponds to half the nominal maximum aggregate size. Also a 100 mm crack limit is set, so the program is automatically stopped when the total length of crack reaches 100 mm.

The procedure of thermal stress prediction has been discussed in section 3.2.4. In each process zone, an average value of the thermally induced tensile stress is calculated at small time increments. These average values are subsequently used to calculate the DCSE and FE over the process zone. The details of DCSE and FE calculations for each process zone are presented in the following sections. An overall flow chart for crack development calculation of HMA fracture mechanics-based thermal cracking model is illustrated as Figure 10.
No

No

No

No

No

No

No

No

No

Figure 10. General Steps of HMA Fracture Mechanics-Based Thermal Cracking Model (Das et al., 2011)
3.3.1 Calculation of Dissipated Creep Strain Energy

Viscoelastic materials subjected to a step constant stress experience a time-dependent increase in strain. This phenomenon is known as viscoelastic creep and the time-dependent total strain produced by applied stress is known as creep strain.

Although the power model has been successfully used as a fitting function of the creep behavior for linear viscoelastic materials, its mathematical deficiency does not allow predicting the thermal stress of viscoelastic materials under multiple temperature ranges. In SHRP A-005 study, the thermal creep strain was predicted by viscosity $\eta_v$ obtained from the Prony series (Generalized Voight-Kelvin model) (Buttlar et al. 2009).

$$D(\xi) = D_0 + \sum_{i=1}^{n} \left(1 - e^{-\frac{\xi}{\tau_i}}\right) + \frac{\xi}{\eta_v}$$  \[12\]

where $D(\xi)$ is the creep compliance at reduced time $\xi$; $D_0$, $D_1$, $\tau_i$ are Prony series parameters and $n$ is the number of Kelvin elements. The presence of viscous flow at long loading times can be presented by $\eta_v$ which is the viscosity as $\xi \to \infty$. Considering only the viscous component representing the rate of damage of viscoelastic media, Equation [12] can be simplify as

$$D(\xi) = \frac{\xi}{\eta_v}$$  \[13\]

The thermal creep strain $\varepsilon_{cr}$ can be predicted from time-temperature constitutive strain Equation [15] combining with irreversible creep component $\eta_v$.

$$\varepsilon(\xi) = \int_0^\xi D(\xi - \xi') \frac{d\sigma(\xi')}{d\xi'} d\xi'$$  \[14\]

$$\varepsilon_{cr}(\xi) = \int_0^\xi \frac{1}{\eta_v} (\xi - \xi') \frac{d\sigma(\xi')}{d\xi'} d\xi'$$  \[15\]

where $\varepsilon(\xi)$ is the stain at reduced time $\xi$, $D(\xi - \xi)$ is creep compliance at reduced time and $\xi'$ is the real time integrating variable.

The thermal strain resulting from the tensile stress is irreversible. The dissipated creep strain energy (DCSE) is an irreversible parameter representing fundamental energy loss in viscoelastic materials, which generally can be determined from the thermal stress and creep strain relationship. Therefore, DSCE can be obtained at each time increment if the thermal stress and thermal creep strain are known. It may be more convenient to detect the failure if the DCSE$_f$ are same at any temperature. However in reality, DCSE$_f$ is constant at fixed temperature but it may vary with changing temperature. Based on energy principle, DCSE at
small time increment ($\Delta t$) can be determined by transferring DCSE$_f$ at the reference temperature using the following equation (Kim et al. 2008):

$$DCSE(\Delta t) = \left[ \frac{\sigma(t) - \sigma(t - \Delta t)}{2} \epsilon_{cr}(t) - \epsilon_{cr}(t - \Delta t) \right] \frac{DCSE_f \text{ at Reference Temperature}}{DCSE_f \text{ at Given Temperature}}$$ \[16\]

So total accumulated DCSE can be obtained from the summation of DCSE at each time increment.

$$DCSE(t) = \sum DCSE(\Delta t)$$ \[17\]

### 3.3.2 Calculation of Fracture Energy

Fracture Energy ($FE$) is another threshold which develops due to the temperature change in the pavement. The area under the thermally induced stress-strain curve is known as fracture energy. The calculated average stresses over the process zones calculated by the thermal stress at small time increments are used to get the FE over the process zone. As FE limits are changing with the temperature change so FE limits at the given temperature should be transferred to a correspondent FE limit at a reference temperature as follow (Kim et al. 2008):

$$FE(t) = \int_0^t \sigma_{AVG}(t) \epsilon(t) \, dt \left( \frac{FE \text{ at Reference Temperature}}{FE \text{ at Given Temperature}} \right)$$ \[18\]

### 3.3.3 Calculation of Crack Length

Once the DCSE and FE are obtained then assigned to each process zone. The process zone near the crack tip is failed once the total accumulated DCSE reaches DCSE$_f$ or the total accumulated FE reaches FE limit. At the same time as the crack length is increasing so the stress distribution along the process zone is also changing. Then in the next step the stress redistributes along the processing zone. This iteration process continues until the crack length reaches total 100 mm.
4 Experimental Design

A series of the SuperPave InDirect Tension Test (SuperPave IDT) and the Thermal Stress Restrained Specimen Test (TSRST) are conducted in this experimental design. The details of raw materials, sample preparations and test introductions are presented in the following sections. All the results from each type of tests then are followed in Chapter 5 for the data analytical studies.

4.1 Sample Preparation

In these tests, three dense graded asphalt mixtures with different binders (Pen\textsubscript{77} 35/50, 70/100 and 160/220) are prepared according to ATB 11 of the Swedish Road Standards (ABT VÄG 2004). The aggregates consisted of a crushed granite from Skäalunda Quarry in Sweden with the maximum size of 11 mm are selected. The aggregate graduation used to prepare the design asphalt mixtures is shown in Table 1 and Figure 11. The 3 binders with different penetration grades are used in these tests. The softening point (EN 1427) and penetration at 25 °C (EN 1426) were measured and given in Table 2.

Table 1. Aggregate Gradation Design

<table>
<thead>
<tr>
<th>Sieve Size (mm)</th>
<th>Specification Limits %</th>
<th>Targeted Gradation %</th>
<th>Selected Gradation %</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.4</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>16</td>
<td>98</td>
<td>99</td>
<td>100</td>
</tr>
<tr>
<td>11.2</td>
<td>85</td>
<td>92</td>
<td>91.2</td>
</tr>
<tr>
<td>8</td>
<td>70</td>
<td>79</td>
<td>80.9</td>
</tr>
<tr>
<td>5.6</td>
<td>58</td>
<td>66.5</td>
<td>66.9</td>
</tr>
<tr>
<td>4</td>
<td>48</td>
<td>57</td>
<td>59.0</td>
</tr>
<tr>
<td>2</td>
<td>33</td>
<td>42.5</td>
<td>43.5</td>
</tr>
<tr>
<td>1</td>
<td>23</td>
<td>32.5</td>
<td>31.3</td>
</tr>
<tr>
<td>0.5</td>
<td>16</td>
<td>23.5</td>
<td>22.6</td>
</tr>
<tr>
<td>0.25</td>
<td>11</td>
<td>16.5</td>
<td>16.2</td>
</tr>
<tr>
<td>0.125</td>
<td>8</td>
<td>11.5</td>
<td>11.5</td>
</tr>
<tr>
<td>0.063</td>
<td>6</td>
<td>7.5</td>
<td>6.3</td>
</tr>
</tbody>
</table>
The aggregates and binders to be mixed are preheated at mixing temperature 155 °C for 3 hours and 1.5 hours, respectively. The compaction temperature is 135 °C. The normal binder content is 6.2% by mass and the air void is 2 ± 0.5% by volume. The slabs are produced by using gyratory compactor (Model ICRT-150R/RB). The compacted slabs denoted as Mix-35-50, Mix-70-100 and Mix-160-220 are extruded from moulds and cool at room temperature for 24 hours.

For each design asphalt mixture, 9 cylindrical specimens (150 mm in diameter and 50 mm in thickness) are extracted for SuperPave IDT and 4 beam specimens with 38mm × 38mm × 200mm are sawed for TSRST, respectively. When preparing IDT specimens, it is significantly important to enable a smooth surface for mounting gage points.
4.2 SuperPave IDT

As a part of the Strategic Highway Research Program (SHRP) (NCHRP, 2004), the SuperPave IDT was developed to determine resilient modulus ($M_R$), creep compliance ($C_r$), m-value, $D_1$, tensile strength ($S_t$), failure strain ($\sigma$), fracture energy (FE) and dissipated creep stain energy to failure ($DCSE_f$). It consists of resilient modulus, creep compliance and tensile strength tests.

In this study, these tests were performed at 0 °C, -10 °C and -20 °C for establishing the Creep Compliance Master Curve (CCMC). For each design asphalt mixture at each temperature, 3 cylindrical specimens with 150 mm in diameter and 50 mm in thickness are prepared, so a total of 27 specimens are tested. As showed in Figure 12, two strain gauges with a length of 38.1 mm were placed at the center of the specimen to measure vertical and horizontal deformations during loading.

![Figure 12. Layout of Strain Gauges for SuperPave IDT Specimen (Birgisson et al., 2007)](image)

4.2.1 Resilient Modulus Test

The resilient modulus is a measure of a material’s elastic stiffness. The ratio of the applied stress to recoverable strain under applied repeated loads is known as resilient modulus. The test is conducted in load control model by applying a repeated haversine waveform load to specimen for a 0.1 second followed by 0.9 seconds rest period resulting in horizontal strain within the range of 200 to 300 micro-strains. The resilient modulus is calculated using the following equation:

$$M_R = \frac{P \times GL}{\Delta H \times h \times d \times C_{comp}} \quad [19]$$
\[ C_{\text{comp}} = 0.6345 \left( \frac{X}{Y} \right) - 1 - 0.332 \]  

where \( M_R \) is resilient modulus, \( P \) is maximum load, \( GL \) is gauge length, \( \Delta H \) is horizontal deformation, \( h \) is thickness of specimen, \( d \) is diameter of specimen and \( C_{\text{comp}} \) is non-dimensional creep compliance factor, \( \left( \frac{X}{Y} \right) \) is ratio of horizontal to vertical deformation.

### 4.2.2 Creep Compliance Test

As creep compliance is a function of time-dependent strain over stress so the time-dependent behavior of asphalt mixture can be represented by the creep compliance master curve. Thus, it can be used to evaluate the rate of damage accumulation of asphalt mixture. The creep compliance test is carried out by applying a constant load for 1000 seconds resulting in horizontal strain within the range of 200 to 750 micro-strains. If the horizontal deformation is greater than 180 micro-inches at 100 seconds, the load is immediately removed from the specimen and then the specimen is allowed to recover for a minimum 3 minutes before reloading at a lower level.

Three parameters (\( D_0 \), \( D_1 \) and m-value) will be obtained from creep compliance test. \( D_0 \) describes the instantaneous elastic response, \( D_1 \) describes the initial portion of the creep compliance curve, while m-value expresses the long-term portion of the curve. An asphalt mixture with a low m-value exhibits a low rate of damage accumulation.

### 4.2.3 Tensile Strength Test

The tensile strength test is conducted to determine the strength and strain of the specimen in a displacement control model by applying a constant rate of 50.8 mm/min until the specimen fails. With the stress strain response, the dissipated creep strength energy (DCSE\(_F\)) is determined by deducting from fracture energy (FE) to elastic energy (EE). Fracture energy is the area under the stress strain curve to the failure strain. The tensile strength is calculated as:

\[ S_t = \frac{2PC_{sx}}{\pi dh} \]  

where \( S_t \) is indirect tensile strength, \( P \) is load of specimen, \( d \) is diameter of specimen, \( h \) is thickness of specimen and \( C_{sx} \) is horizontal stress correction factor.

\[ C_{sx} = 0.948 - 0.0114 \left( \frac{h}{d} \right) - 0.2693\nu + 1.436 \left( \frac{h}{d} \right) \nu \]  

[22]
where Poisson’s ratio $\nu = 0.1 + 1.480 \left( \frac{X}{Y} \right)^2 - 0.778 \left( \frac{h}{d} \right)^2 \left( \frac{X}{Y} \right)^2$, $\left( \frac{X}{Y} \right)$ is ratio of horizontal to vertical deformation.

### 4.3 Thermal Stress Restrained Specimen Test

Thermal Stress Restrained Specimen Test (TSRST) was developed under SHRP A-400 contract by Jung and Vinson (1994), which can be used to evaluate the low-temperature cracking susceptibility of asphalt mixture. As shown in Figure 13 and 14, the equipment comprises of three subsystems: a cooling system, a load/displacement system, and a test control/data acquisition system. In TSRST, the specimen is subjected to thermal stress due to the decrease of temperature. During the cooling process, the length of the specimen is held constant so the specimen is restrained from shrinkage. As the specimen contracts, two linear variable differential transducers (LVDTs) sense the movement and a signal is sent to the computer, which in turn causes the screw jack to stretch the specimen to its original length. As the temperature continues to decrease, the thermal stress increases until the specimen breaks. The temperature at which the specimen fails is called fracture temperature and the corresponding thermally induced stress is fracture stress.

![Figure 13. Specimen Alignment Stand of TSRST](image)

In Figure 15, a typical TSRST result with the obtained test parameters is shown. At the beginning of the test, a relatively slow increase in thermal stress is observed due to relaxation of asphalt mixture. However, below the transition temperature, the thermally induced stress is
linearly related to temperature. At the transition temperature, the sample changes from a viscoelastic to elastic state. The transition temperature and the slope of the stress-temperature curve, $\Delta S/\Delta T$, below the transition temperature, may play an important role in characterizing the rheological behavior of the asphalt mixture at cold temperature.

Figure 14. Schematic of TSRST Apparatus (OEM Inc.)

Figure 15. Typical Stress versus Temperature Curve of TSRST (OEM Inc.)

For each design asphalt mixture, 4 beam specimens (38 mm × 38 mm × 200 mm) are prepared. In total, 12 specimens are tested in this project. The tests are performed according to AASTHO TP-10-93 specification, the specimen is pre-cooled at 2°C for h before the test began, the initial temperature is 2 °C and the cooling rate is 10 °C/h.
5 Data Analysis

To be a reliable and accurate thermal cracking model, it should be able to provide clearly expected results. For example, a higher thermal contraction coefficient should accelerate crack development of asphalt mixture. In this chapter, the output results of the HMA fracture mechanics-based thermal cracking model, such as the predicted thermally induced stress and fracture temperature, will be compared with the fracture stress and fracture temperature obtained from TSRST, to finally evaluate the low temperature cracking prediction of the model.

5.1 SuperPave IDT and TSRST Data

As described in Chapter 4, the SuperPave IDT and TSRST were designed in this project and the test results are presented in this section. Table 3 shows the different parameters of the three design asphalt mixtures collected from SuperPave IDT at different temperatures (0 °C, -10 °C and -20 °C), which are served as the input data of the model. More details about the Matlab software for obtaining these parameters were specified by Du (2010). Figure 16 shows the thermally induced stresses of the three design asphalt mixtures develop with the changing cooling temperatures, which basically match the typical TSRST result illustrated as Figure 15.

Table 3. Model Input Parameters Obtained from SuperPave IDT

<table>
<thead>
<tr>
<th>Temp 0°C</th>
<th>Mixture ID</th>
<th>D₀ (1/GPa)</th>
<th>D₁ (1/GPa)</th>
<th>m</th>
<th>St (MPa)</th>
<th>FE (KJ/m³)</th>
<th>DCSEf (KJ/m³)</th>
<th>ER</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Mix-35-50</td>
<td>0.054</td>
<td>0.031</td>
<td>0.546</td>
<td>3.832</td>
<td>2.450</td>
<td>2.049</td>
<td>2.956</td>
</tr>
<tr>
<td>-10</td>
<td>Mix-35-50</td>
<td>0.038</td>
<td>0.010</td>
<td>0.477</td>
<td>4.112</td>
<td>1.172</td>
<td>0.837</td>
<td>5.192</td>
</tr>
<tr>
<td>-20</td>
<td>Mix-35-50</td>
<td>0.032</td>
<td>0.004</td>
<td>0.425</td>
<td>4.483</td>
<td>0.779</td>
<td>0.450</td>
<td>10.281</td>
</tr>
<tr>
<td>0</td>
<td>Mix-70-100</td>
<td>0.057</td>
<td>0.070</td>
<td>0.653</td>
<td>3.344</td>
<td>4.045</td>
<td>3.707</td>
<td>1.537</td>
</tr>
<tr>
<td>-10</td>
<td>Mix-70-100</td>
<td>0.041</td>
<td>0.014</td>
<td>0.547</td>
<td>3.856</td>
<td>0.820</td>
<td>0.510</td>
<td>1.663</td>
</tr>
<tr>
<td>-20</td>
<td>Mix-70-100</td>
<td>0.033</td>
<td>0.005</td>
<td>0.440</td>
<td>3.394</td>
<td>0.581</td>
<td>0.391</td>
<td>7.505</td>
</tr>
<tr>
<td>0</td>
<td>Mix-160-220</td>
<td>0.071</td>
<td>0.086</td>
<td>0.669</td>
<td>3.030</td>
<td>3.246</td>
<td>2.912</td>
<td>0.979</td>
</tr>
<tr>
<td>-10</td>
<td>Mix-160-220</td>
<td>0.052</td>
<td>0.026</td>
<td>0.644</td>
<td>3.451</td>
<td>1.372</td>
<td>1.050</td>
<td>1.195</td>
</tr>
<tr>
<td>-20</td>
<td>Mix-160-220</td>
<td>0.040</td>
<td>0.010</td>
<td>0.497</td>
<td>3.767</td>
<td>1.059</td>
<td>0.769</td>
<td>4.582</td>
</tr>
</tbody>
</table>
Figure 16. Stress versus Temperature Curve of TSRST

5.2 Creep Compliance Master Curve and Relaxation Modulus

Figures (17 – 22) show the different creep compliance data and the Creep Compliance Master Curves (CCMC) fitted by power law for each of the three design asphalt mixtures, respectively. The model horizontally shifts the data obtained at various temperatures to establish a smooth, continuous curve which is the CMCC at the reference temperature (-20 °C), then the CCMC is fitted with power model (cf. Equation [5]). Since the mathematical deficiency of power law under multiple temperature ranges, the different deviations are observed between the CCMCs and the power law fitted curves.
Figure 17. Different Creep Compliance Data at Various Temperatures for Mix-35-50

Figure 18. Creep Compliance Master Curve Fitted by Power Law for Mix-35-50
Figure 19. Different Creep Compliance Data at Various Temperatures for Mix-70-100

Figure 20. Creep Compliance Master Curve Fitted by Power Law for Mix-70-100
Figure 21. Different Creep Compliance Data at Various Temperatures for Mix-160-220

Figure 22. Creep Compliance Master Curve Fitted by Power Law for Mix-160-220
Figure 23. Relaxation Modulus Fitted by Prony Series for Mix-35-50

Figure 24. Relaxation Modulus Fitted by Prony Series for Mix-70-100
Figures (23 – 25) show the relaxation modulus fitted by Prony Series for each of the three design asphalt mixtures, respectively. Governing by the convolution integral (cf. Equation [6]), the creep compliance and the relaxation modulus are clearly related by the following curves. Also the relaxation modulus master curves for the generalized Maxwell model are perfectly expressed by the Prony series (cf. Equation [8]).

5.3 Thermally Induced Stress with Constant Thermal Contraction Coefficient

With constant thermal contraction coefficients, Figures (26 – 37) show the comparisons of the thermally induced stresses predicted by the HMA fracture mechanics-based thermal cracking model (cf. Equation [9]) and the stresses collected from TSRST. It can be easily seen that the predicted stresses were quite close to the observed stresses from TSRST for all samples, which indicate that the HMA fracture mechanics-based thermal cracking model can provide reasonable and reliable predictions for the thermal stress development during the cracking process of HMA.
Figure 26. Comparison of Thermal Stress Using Two Methods for Mix-35-50-1

Figure 27. Comparison of Thermal Stress Using Two Methods for Mix-35-50-2
Figure 28. Comparison of Thermal Stress Using Two Methods for Mix-35-50-3

Figure 29. Comparison of Thermal Stress Using Two Methods for Mix-35-50-4
**Figure 30.** Comparison of Thermal Stress Using Two Methods for Mix-70-100-1

**Figure 31.** Comparison of Thermal Stress Using Two Methods for Mix-70-100-2
Figure 32. Comparison of Thermal Stress Using Two Methods for Mix-70-100-3

Figure 33. Comparison of Thermal Stress Using Two Methods for Mix-70-100-4
Figure 34. Comparison of Thermal Stress Using Two Methods for Mix-160-220-1

Figure 35. Comparison of Thermal Stress Using Two Methods for Mix-160-220-2
Figure 36. Comparison of Thermal Stress Using Two Methods for Mix-160-220-3

Figure 37. Comparison of Thermal Stress Using Two Methods for Mix-160-220-4
5.4 Fracture Temperature

To accurately evaluate the HMA fracture mechanics-based thermal cracking model, every sample’s predicted fracture temperature is compared with the corresponding value collected from the TSRST. According to Table 4, with constant thermal contraction coefficients, the fracture temperatures predicted by the model have evident divergences with the TSRST results. As a comparison, the average fracture temperature predictions of the same three design asphalt mixtures using the non-linear thermal contraction coefficient are showed in Table 5.

It can be clearly seen that the variances of all three design asphalt mixtures dramatically decrease in Table 5, which indicate that the non-linear thermal contraction coefficient is more suitable than the constant $\alpha$ value on predicting fracture temperature. The comparison also demonstrates that the accuracy of the fracture temperature prediction is greatly depends upon the thermal contraction coefficient. More details about using no-linear thermal contraction coefficient for fracture temperature prediction were specified by Das et al. (2011).

Table 4. Comparisons of Fracture Temperatures with constant contraction coefficient

<table>
<thead>
<tr>
<th>Sample ID</th>
<th>Predicted $^\circ$C</th>
<th>TSRST $^\circ$C</th>
<th>Variance %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mix-35-50-1</td>
<td>-20,05</td>
<td>-22,20</td>
<td>9,70</td>
</tr>
<tr>
<td>Mix-35-50-2</td>
<td>-20,00</td>
<td>-23,23</td>
<td>13,89</td>
</tr>
<tr>
<td>Mix-35-50-3</td>
<td>-23,54</td>
<td>-23,98</td>
<td>1,80</td>
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<td>Mix-35-50-4</td>
<td>-23,23</td>
<td>-23,53</td>
<td>1,24</td>
</tr>
<tr>
<td>Avg. Mix-35-50</td>
<td>-21,71</td>
<td>-23,23</td>
<td>6,56</td>
</tr>
<tr>
<td>Mix-70-100-1</td>
<td>-19,27</td>
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<tr>
<td>Avg. Mix-70-100</td>
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<td>-26,47</td>
<td>23,87</td>
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<tr>
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<td>-36,35</td>
<td>19,43</td>
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<td>-34,93</td>
<td>25,81</td>
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<td>Mix-160-220-3</td>
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<td>-30,85</td>
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<td>Mix-160-220-4</td>
<td>-25,20</td>
<td>-33,78</td>
<td>25,39</td>
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<tr>
<td>Avg. Mix-160-220</td>
<td>-26,06</td>
<td>-33,98</td>
<td>23,30</td>
</tr>
</tbody>
</table>
Table 5. Comparisons of Fracture Temperatures using non-linear contraction coefficient (Das et al., 2011)

<table>
<thead>
<tr>
<th>Mixture ID</th>
<th>Predicted $^\circ$C</th>
<th>TSRST $^\circ$C</th>
<th>Variance %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Mix-35-50</td>
<td>-22.80</td>
<td>-22.00</td>
<td>-3.64</td>
</tr>
<tr>
<td>Avg. Mix-70-100</td>
<td>-23.60</td>
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<td>10.94</td>
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<tr>
<td>Avg. Mix-160-220</td>
<td>-27.70</td>
<td>-32.00</td>
<td>13.44</td>
</tr>
</tbody>
</table>

5.5 Effect of Thermal Contraction Coefficient

In this section the fracture temperatures and fracture time predicted by the HMA fracture mechanics-based thermal cracking model using the different constant thermal contraction coefficients are analyzed. For the Mix-70-100 and Mix-160-220, Table 6 and Figures (39 – 40) clearly show that the higher thermal contraction coefficient was applied, the higher fracture temperature and the less fracture time the sample fails at. For the Mix-35-50, there are a few exceptions of either fracture temperature or fracture time, which may be due to the faults of data collection during the TSRST where the thermal contraction coefficient originally deducted from. However, the predicted fracture temperature and fracture time do not vary too much from the expected ranges, so these data are mostly in accordance with the expected results.

Figures (38-40) also clearly show the predicted 100-mm cracking development versus time in the three design asphalt mixtures. In the most fracture times, the damages increase at a relatively slow rate. The crack lengths increase when the total accumulated DCSE reaches DCSE$_f$ or the total accumulated FE reaches FE limit in the every process zone. Since the newer process zones are always weaker than the earlier zones due to the prolonged DCSE accumulated from the beginning, the cracks grow at increasingly fast rates (i.e. fewer number of loading cycles cause failure in the new process zone) until the samples fail.
Table 6. Fracture Temperature and Fracture Time versus Thermal Contraction Coefficient

<table>
<thead>
<tr>
<th>Sample ID</th>
<th>Fracture Temperature °C</th>
<th>Fracture Time Sec</th>
<th>Thermal Contraction Coefficient e⁻⁵/°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mix-35-50-1</td>
<td>-20.05</td>
<td>8900</td>
<td>3.13</td>
</tr>
<tr>
<td>Mix-35-50-2</td>
<td>-20.00</td>
<td>9000</td>
<td>3.06</td>
</tr>
<tr>
<td>Mix-35-50-3</td>
<td>-23.54</td>
<td>10600</td>
<td>2.00</td>
</tr>
<tr>
<td>Mix-35-50-4</td>
<td>-23.23</td>
<td>10200</td>
<td>1.84</td>
</tr>
<tr>
<td>Mix-70-100-1</td>
<td>-19.27</td>
<td>7800</td>
<td>5.88</td>
</tr>
<tr>
<td>Mix-70-100-2</td>
<td>-19.62</td>
<td>8000</td>
<td>5.45</td>
</tr>
<tr>
<td>Mix-70-100-3</td>
<td>-20.58</td>
<td>8300</td>
<td>4.99</td>
</tr>
<tr>
<td>Mix-70-100-4</td>
<td>-21.13</td>
<td>8400</td>
<td>4.92</td>
</tr>
<tr>
<td>Mix-160-220-1</td>
<td>-29.29</td>
<td>13100</td>
<td>4.46</td>
</tr>
<tr>
<td>Mix-160-220-2</td>
<td>-25.91</td>
<td>11200</td>
<td>6.39</td>
</tr>
<tr>
<td>Mix-160-220-3</td>
<td>-23.83</td>
<td>10200</td>
<td>8.01</td>
</tr>
<tr>
<td>Mix-160-220-4</td>
<td>-25.20</td>
<td>10800</td>
<td>6.96</td>
</tr>
</tbody>
</table>

Figure 38. Effect of Thermal Contraction Coefficient for Mix-35-50
Figure 39. Effect of Thermal Contraction Coefficient for Mix-70-100

Figure 40. Effect of Thermal Contraction Coefficient for Mix-160-220
6 Summary and Conclusion

The HMA fracture mechanics model is based on the theory of viscoelasticity and energy-based fracture mechanics, which deals with fracture associated with a fundamental dissipated creep strain energy loss in viscoelastic materials. The TCMODEL developed based on the theory of linear viscoelasticity is a proper mechanistic-empirical design tool to predict thermal cracking in asphalt pavement. In this project, the HMA fracture mechanics-based thermal cracking model developed by Das et al. (2011) was studied. Linear viscoelasticity from the TCMODEL, thermally induced dissipated creep strain energy threshold and fracture energy threshold were used to investigate low temperature cracking performance. SuperPave IDT test results were used to compute thermally induced stress and fracture temperature.

In experimental design, 27 SuperPave IDT specimens were tested at different temperatures (0 °C, -10 °C and -20 °C) and then the different parameters were input in the HMA fracture mechanics-based thermal cracking model to step by step calculate the thermally induced stresses, dissipated creep strain energy, fracture energy and crack length. The data analysis of the thermal stresses and fracture temperatures gained from the model and the TSRST respectively were conducted and led to the following findings:

- The model-predicted thermally induced stresses match well with observed TSRST stress, which means the basic physics of viscoelastic stress generated with temperature change is properly followed.

- Thermal contraction coefficient is the key parameter for thermal cracking performance prediction. By comprising the predicted results which used the constant thermal contraction coefficient and the no-linear thermal contraction coefficient, the model is observed to be able to predict more accurate fracture temperatures with the no-linear thermal contraction coefficient.

- The data analysis of the model-predicted crack length versus fracture time validates that the higher thermal contraction coefficient fastens the low temperature cracking development in asphalt mixture.

- Since the new process zones are always weaker than the last zones due to the prolonged DCSE accumulation, the cracks grow at increasingly fast rates until the samples fail.

These conclusions indicated that this HMA fracture mechanics-based thermal cracking model has the potential to reliably evaluate the performance of asphalt mixtures subjected to thermally induced damage. It is able to detect thermal stress with the changing temperature and give an accurate output of crack development versus temperature or time.
**Future research area:**

The findings of this study indicate that the HMA fracture mechanics-based thermal cracking model has the potential to reliably evaluate low temperature cracking resistance of asphalt mixtures. However, this research needs to be continued to address the following issues:

- Further study the performance of the model by testing the asphalt mixture with polymer-modified binder.

- Applying different thermal contraction coefficients, e.g. linear and bi-linear $\alpha$ values to evaluate the reliability and accuracy of the model.
References


Appendix

MATLAB Codes for HMA Fracture Mechanics-Based Thermal Cracking Model

Code of Final.m

%% Prabir Das
% version: 03-05-2010
% Mixture ID: 35-50-1
clear
clc
close all

%% Input Data
% alpha=50e-6; %Thermal coefficient
coolingrate=10; %rate of cooling C/hr
testdata=load('stress-35-50-1.txt'); % data from TSRST (Temp. vs Stress)
data=load('35-50.txt'); % Data from IDT (Temp.,Do, D1, m)
%% Loading Data from Input files
temp=data(1:end,1);
D0=data(1:end,2); % in 1/GPa
D1=data(1:end,3); % in 1/GPa
m=data(1:end,4);
FE_test=data(1:end,5); % FE from IDT test in KJ/m3
ST=data(1:end,6); % Tensile strength from IDT test in MPa
DCSE_test=data(1:end,7); %DCSEf from IDT test in KJ/m3
time=[1;5;10;20;50;100;200;500;1000];
nn=7;
mm=7;

%%%% Analysis for Master curve, Relaxation curve
[reducedtime,CCredtim,aT2,aT3]=shifting(D0,D1,m,time,nn,nm);
redtime=10.^reducedtime;
fCC=(10.^CCredtim);
[jk,AA,ljk,crpcom,AAA,mvalue]=master(redtime,fCC);
[ljk,Et]=relaxation(ljk,crpcom,mvalue);

%%
t=ljk;
E=log(Et);
figure,
plot(t,Et,'mo')

%% Relaxation modulus fitted by Prony Series
[cf_,R,C,Coe]=relaxmodelAG2MW(t,Et);
E1 = C(1);
E2 = C(2);
E3 = C(3);
E4 = C(4);
E5 = C(5);
i = [E1; E2; E3; E4; E5];
t1 = -1/C(6);
t2 = -1/C(7);
t3 = -1/C(8);
t4 = -1/C(9);
t5 = -1/C(10);

lambda = [t1; t2; t3; t4; t5];
[ljk, Et] = relaxation(ljk, crpcom, mvalue);
hold on
EE = E1*exp(-t./t1) + E2*exp(-t./t2) + E3*exp(-t./t3) + E4*exp(-t./t4) + E5*exp(-t./t5);
loglog(t, EE, 'g+');
legend('D(t)', 'E(t) By Laplace Transformation', 'Fitted by Prony Series', 4)
%
figure
plot(t, Et, 'mo')
hold on
plot(t, EE, 'g+');

eta = 1000 / (D1(3) * m(3) * (1000)^m(3) - 1);

[Alpha] = alphatopu(aT, T, TSRST_stress, TSRST_T, E1, E2, E3, E4, E5, t1, t2, t3, t4, t5, coolingrate);
alpha = Alpha;
deltatime = 100;
% tim=(0:deltim:15000)';
tim=(0:deltim:time_T)';
[stress,time,ttr]=stress_calculation_f(alpha,aT,T,tim,E1,E2,E3,E4,E5,
t1,t2,t3,t4,t5,coolingrate);

TT=TSRST_Temp(1)-time.*(coolingrate/3600); %correponding Temp
figure;plot(TT,stress,'r*')
hold on
plot(TSRST_Temp,TSRST_stress)
title ('Mix - 35-50-1')
xlabel('Temperature[^0C]')
ylabel('Stress \sigma [MPa]')
legend('Predicted Stress','TSRST Stress',1)
titleStr = sprintf('Thermal Coefficient = %d /^0C', alpha);
gtext(titleStr);

figure;plot(time,stress,'r*')
title ('Mix - 35-50-1')
xlabel('Time [sec]')
ylabel('Stress \sigma [MPa]')

%% Crack growth symulator
% close all
% clc
figure
a=10; %initial crack length
z=5; %zone size
n=19; %number of zone
St=ST(3); % tensile srength from IDT test data
[fracture_time,crack_length,Fracture_Temp]=
crackgrowth(n,a,z,alpha,stress,ttr,eta,DCSE_test,temp,St,TT,time,FE_t
est)

**Code of shifting.m**

function [reducedtime,CCcredtim,aT2,aT3]=shifting(D0,D1,m,time,nn,nm)
for n=1:3
    if n==1
        CC0=D0(n)+D1(n).*(time.^m(n)); %Creep Compliance D(t)at 0C
    end
    if n==2
        CC_10=D0(n)+D1(n).*(time.^m(n)); %Creep Compliance D(t) at -10C
    end
end
if n==3
    CC_20=D0(n)+D1(n).* (time.^m(n));  %Creep Compliance D(t) at -20C
end

figure
plot(time,CC0,'-.r*')
hold on
plot(time,CC_10,'--bs')
hold on
plot(time,CC_20,':g+')
title('Mix - 35-50-1')
xlabel('Time,t')
ylabel('D(t)')
legend('0C','-10C','-20C',2)
hold off

%% Curve fitting
CC0=log10(CC0);
CC_10=log10(CC_10);
CC_20=log10(CC_20);
logtime=log10(time);
figure
plot(logtime,CC0,'-.r*')
hold on
plot(logtime,CC_10,'--bs')
hold on
plot(logtime,CC_20,':g+')
title('Mix - 35-50-1')
xlabel('Time, log(t)')
ylabel('log D(t)')
legend('0C','-10C','-20C',2)
hold off

figure
X=logtime;
Y=CC0;
[cf_,R,C0,Coe]=linearfit(X,Y);
ncc0=C0(1)*logtime+C0(2);
Y=CC_10;
[cf_,R,C10,Coe]=linearfit(X,Y);
ncc10=C10(1)*logtime+C10(2);
Y=CC_20;
[cf_,R,C20,Coe]=linearfit(X,Y);
ncc20=C20(1)*logtime+C20(2);
\% figure
\% plot(logtime,ncc0,'r*')
\% hold on
\% plot(logtime,ncc10,'bs')
\% hold on
\% plot(logtime,ncc20,'g+')
\% xlabel('time,t')
\% ylabel('D(t)')
hold off

\%
y_20=C20(1)*logtime(nn)+C20(2);
x_20=(y_20-C20(2))/C20(1);
x_10=(y_20-C10(2))/C10(1);
aT2=x_10-x_20; \% Shift factor for -10C ref.-20
redtime2=logtime-aT2;
plot(redtime2,CC_10,'bs',logtime,CC_20,'g+')

y_10=C10(1)*logtime(nm)+C10(2);
x_10=(y_10-C10(2))/C10(1);
x_0=(y_0-C0(2))/C0(1);
aT31=x_0-x_10; \% Shift factor for -10C ref.-20
aT3=aT2+aT31;
redtime3=logtime-aT3;
figure
plot(redtime3,CC0,'r*',redtime2,CC_10,'bs',logtime,CC_20,'g+')
xlabel('Reduced time,log(tr)')
ylabel('log D(t)')
legend('0C','-10C','-20C',2)
hold off
reducedtime=[logtime;redtime2;redtime3];
CCredtim=[CC_20;CC_10;CC0];
figure
plot(10.^reducedtime,10.^CCredtim,'mo')

**Code of linearfit.m**

```matlab
function [cf_,R,C,Coe]=linearfit(X,Y)
%LINEARFIT    Create plot of datasets and fits
% LINEARFIT(X,Y)
% Creates a plot, similar to the plot in the main curve fitting
% window, using the data that you provide as input. You can
% apply this function to the same data you used with cftool
% or with different data. You may want to edit the function to
```
% customize the code and this help message.
% Number of datasets: 1
% Number of fits: 1

% Data from dataset "Y vs. X":
% X = X:
% Y = Y:
% Unweighted

% Set up figure to receive datasets and fits
f_ = clf;
figure(f_);
%set(f_,'Units','Pixels','Position',[658 330 688 486]);
legh_ = [];
legt_ = {};
% handles and text for legend
xlim_ = [Inf -Inf];
% limits of x axis
ax_ = axes;
set(ax_,'Units','normalized','OuterPosition',[0 0 1 1]);
set(ax_,'Box','on');
axes(ax_); hold on;

% --- Plot data originally in dataset "Y vs. X"
X = X(:);
Y = Y(:);
h_ = line(X,Y,'Parent',ax_,'Color',[0.333333 0 0.666667],... 'LineStyle','none', 'LineWidth',1,... 'Marker','.', 'MarkerSize',12);
xlim_(1) = min(xlim_(1),min(X));
xlim_(2) = max(xlim_(2),max(X));
legh_(end+1) = h_;
legt_(end+1) = 'Y vs. X';

% Nudge axis limits beyond data limits
if all(isfinite(xlim_))
    xlim_ = xlim_ + [-1 1] * 0.01 * diff(xlim_);
    set(ax_,'XLim',xlim_)
else
    set(ax_,'XLim',[-10.1, 0.10000000000000001]);
end

% --- Create fit "fit 1"
ok_ = isfinite(X) & isfinite(Y);
if ~all( ok_ )
    warning( 'GenerateMFile:IgnoringNansAndInfs', ...
Ignoring NaNs and Infs in data
end
ft_ = fittype('poly1');

% Fit this model using new data
[cf_,R]= fit(X(ok_),Y(ok_),ft_);
C = coeffvalues(cf_);
Coe=coeffnames(cf_);
% Or use coefficients from the original fit:
if 0
    cv_ = {0.0084970826000000006, 0.0919708260000000006};
    cf_ = cfit(ft_,cv_(:));
end

% Plot this fit
h_ = plot(cf_,'fit',0.95);
legend off; % turn off legend from plot method call
set(h_(1),'Color',[1 0 0],
    'LineStyle','-', 'LineWidth',2,
    'Marker','none', 'MarkerSize',6);
legh_(end+1) = h_(1);
legt_(end+1) = 'fit 1';

% Done plotting data and fits. Now finish up loose ends.
hold off;
leginfo_ = {'Orientation', 'vertical', 'Location', 'NorthEast'};
h_ = legend(ax_,legh_,legt_,leginfo_(:)); % create legend
set(h_,'Interpreter','none');
xlabel(ax_,''); % remove x label
ylabel(ax_,''); % remove y label

Codes of master.m
function [jk,AA,ljk,crpcom,AAA,mvalue]=master(redtime,fCC)
A=log10(redtime);
B=log10(fCC);
hold off
[cf,a]=curvefitting(A,B);
jk=[A(1):.1:A(end)]';%Reduce time in a constant interval of 0.4 in log scale
sqtim=jk.*jk;
AA=a(1)*sqtim+a(2)*jk+a(3);%corresponding D(t)
AAA=10.^AA;%D(t) in normal scale
ljk=10.^jk;%reduced time in normal scale
figure
loglog(ljk,AAA,'r*')

title('Mix - 35-50-1')
xlabel('Reduced time, tr')
ylabel('D(t)')
hold off
figure

%% Calculating m-value from fitted curve by D(t)=D_0+D_1 t^m
[cfinfo,xy,yy]=powerlaw(ljk,AAA);
mvalue=xy(2);
crpcom=xy(3)+xy(1)*(ljk.^xy(2));
logcrpcom=log10(crpcom);
figure
loglog(ljk,crpcom,'g+')
hold on
loglog(ljk,AAA,'r*')

title('Mix - 35-50-1')
xlabel('Reduced time, tr')
ylabel('D(t)')
legend('Curve fitted by Power law','Creep-Compliance Master Curve',2)
hold off

Code of curvefitting.m

function [cf_,aa,bb]=curvefitting(A,B)
%CURVEFITTING  Create plot of datasets and fits
% CURVEFITTING(A,B)
% Creates a plot, similar to the plot in the main curve fitting
% window, using the data that you provide as input. You can
% apply this function to the same data you used with cftool
% or with different data. You may want to edit the function to
% customize the code and this help message.
%
% Number of datasets:  2
% Number of fits:  1

% Data from dataset "B vs. A":
%  X = A;
%  Y = B;
%  Unweighted

% Data from dataset "B vs. A (smooth)":
%  X = A;
%  Y = B;
Unweighted
Set up figure to receive datasets and fits

```matlab
f_ = clf;
figure(f_);
set(f_, 'Units', 'Pixels', 'Position', [658 330 688 486]);
legh_ = [];
legt_ = {};
xlim_ = [Inf -Inf];
ax_ = axes;
set(ax_, 'Units', 'normalized', 'OuterPosition', [0 0 1 1]);
set(ax_, 'Box', 'on');
axes(ax_); hold on;
```

Plot data originally in dataset "B vs. A"

```matlab
A = A(:);
B = B(:);
% This dataset does not appear on the plot
% Add it to the plot by removing the if/end statements that follow
% and by selecting the desired color and marker
if 0
    h_ = line(A, B, 'Color', 'r', 'Marker', '.', 'LineStyle', 'none');
    xlim_(1) = min(xlim_(1), min(A));
    xlim_(2) = max(xlim_(2), max(A));
    legh_(end+1) = h_
    legit_(end+1) = 'B vs. A';
end
```

Plot data originally in dataset "B vs. A (smooth)"

```matlab
sm_.y2 = smooth(A, B, 0.25, 'lowess', 0);
```

```matlab
h_ = line(A, sm_.y2, 'Parent', ax_, 'Color', [0.333333 0.666667 0], ...
    'LineStyle', 'none', 'LineWidth', 1, ...
    'Marker', '.', 'MarkerSize', 12);
```

```matlab
xlim_(1) = min(xlim_(1), min(A));
xlim_(2) = max(xlim_(2), max(A));
legh_(end+1) = h_
legg_(end+1) = 'B vs. A (smooth)';
```

Nudge axis limits beyond data limits

```matlab
if all(isfinite(xlim_))
    xlim_ = xlim_ + [-1 1] * 0.01 * diff(xlim_);
    set(ax_, 'XLim', xlim_)
else
    set(ax_, 'XLim', [-0.056010086722827597, 5.6570187590055871]);
end
```
% --- Create fit "fit 2"
ok_ = isfinite(A) & isfinite(sm_.y2);
if ~all(ok_)
    warning( 'GenerateMFile:IgnoringNansAndInfs', ... 
        'Ignoring NaNs and Infs in data' );
end
ft_ = fittype('poly2');

% Fit this model using new data
cf_ = fit(A(ok_),sm_.y2(ok_),ft_);
aa = coeffvalues(cf_);
bb=coeffnames(cf_);

% Or use coefficients from the original fit:
if 0
    cv_ = { 0.080415734549208001, -0.054481304357972923, 
        -1.2738041899509212};
    cf_ = cfit(ft_,cv_{:});
end

% Plot this fit
h_ = plot(cf_,'fit',0.95);
legend off;  % turn off legend from plot method call
set(h_,'Color',[1 0 0],...
    'LineStyle','-','LineWidth',2,...
    'Marker','none', 'MarkerSize',6);
legh_{end+1} = h_{1};
legt_{end+1} = 'fit 2';

% Done plotting data and fits. Now finish up loose ends.
hold off;
leginfo_ = {'Orientation', 'vertical'};
h_ = legend(ax_,legh_,legt_,leginfo_{:}); % create and reposition legend
set(h_,'Units','normalized');
t_ = get(h_,'Position');
t_(1:2) = [0.276647,0.776406];
set(h_,'Interpreter','none','Position',t_);
xlabel(ax_,'');               % remove x label
ylabel(ax_,'');               % remove y label

Code of powerlaw.m

function [cf_,xy,yy]=powerlaw(ljk,AAA)
%POWERLAW  Create plot of datasets and fits
POWERLAW(LJK,AAA)

Creates a plot, similar to the plot in the main curve fitting window, using the data that you provide as input. You can apply this function to the same data you used with cftool or with different data. You may want to edit the function to customize the code and this help message.

Number of datasets: 1
Number of fits: 1

Data from dataset "AAA vs. ljk":
X = ljk:
Y = AAA:
Unweighted

This function was automatically generated on 14-Oct-2009 17:37:39

Set up figure to receive datasets and fits
f_ = clf;
figure(f_);
set(f_, 'Units', 'Pixels', 'Position', [658 330 688 486]);
legh_ = [] ; legt_ = {} ; % handles and text for legend
xlim_ = [Inf -Inf]; % limits of x axis
ax_ = axes;
set(ax_, 'Units', 'normalized', 'OuterPosition', [0 0 1 1]);
set(ax_, 'Box', 'on');
axes(ax_); hold on;

--- Plot data originally in dataset "AAA vs. ljk"
ljk = ljk(:,);
AAA = AAA(:,);

h_ = line(ljk,AAA,'Parent',ax_, 'Color',[0.333333 0 0.666667],...
    'LineStyle','none', 'LineWidth',1,...
    'Marker', '.', 'MarkerSize',12);
xlim_(1) = min(xlim_(1),min(ljk));
xlim_(2) = max(xlim_(2),max(ljk));
legh_(end+1) = h_;
legt_(end+1) = 'AAA vs. ljk';

Nudge axis limits beyond data limits
if all(isfinite(xlim_))
    xlim_ = xlim_ + [-1 1] * 0.01 * diff(xlim_);
    set(ax_, 'XLim', xlim_)
else
% --- Create fit "fit 1"
ok_ = isnan(ljk) & isnan(AAA);
if ~all( ok_ )
    warning( 'GenerateMFile:IgnoringNansAndInfs', ... 
        'Ignoring NaNs and Infs in data' );
end
st_ = [0.14965436610687932 0.18676589928965109 0.55008973389125626 ];
ft_ = fittype('power2');

% Fit this model using new data
cf_ = fit(ljk(ok_),AAA(ok_),ft_, 'Startpoint', st_);
xy= coeffvalues(cf_);
yy=coeffnames(cf_);
% Or use coefficients from the original fit:
if 0
    cv_ = { 0.00029532868001074121, 0.79757663209753638, 
        0.091005693352234004};
    cf_ = cfit(ft_,cv_{:});
end

% Plot this fit
h_ = plot(cf_, 'fit', 0.95);
legend off; % turn off legend from plot method call
set(h_(1),'Color',[1 0 0],...  
    'LineStyle','-','LineWidth',2,...
    'Marker','none','MarkerSize',6);
legh_(end+1) = h_(1);
legt_{end+1} = 'fit 1';

% Done plotting data and fits. Now finish up loose ends.
hold off;
leginfo_ = {'Orientation', 'vertical'};

h_ = legend(ax_,legh_,legt_,leginfo_{:}); % create and reposition legend
set(h_,'Units','normalized');
t_ = get(h_,'Position');
t_(1:2) = [0.265746,0.776063];
set(h_,'Interpreter','none','Position',t_);
xlabel(ax_,'');               % remove x label
ylabel(ax_,'');               % remove y label
Code of relaxation.m

```matlab
function [ljk,Et]=relaxation(ljk,crpcom,mvalue)
figure
loglog(ljk,crpcom,'r*')
hold on
Ds=gamma(1+mvalue)*crpcom;
Es=1./Ds;
Et=Es./gamma(1-mvalue);
loglog(ljk,Et,'bs')
hold on
ff=1./crpcom;
%loglog(ljk,ff,'*r')
xlabel('Reduced time, tr')
ylabel('D(t)                                               E(t)')
legend('D(t)','E(t) By Laplace Transformation',4)
title('Mix - 35-50-1')
```

Code of relaxmodel.m

```matlab
function [cf_,R,C,Coe]=relaxmodel (t,Et)
%RELAXMODEL    Create plot of datasets and fits
% RELAXMODEL (T,ET)
% Creates a plot, similar to the plot in the main curve fitting
% window, using the data that you provide as input. You can
% apply this function to the same data you used with cftool
% or with different data. You may want to edit the function to
% customize the code and this help message.
% Number of datasets:  1
% Number of fits:  1
% Data from dataset "Et vs. t":
%   X = t:
%   Y = Et:
% Unweighted

% Set up figure to receive datasets and fits
f_ = clf;
figure(f_);
set(f_,'Units','Pixels','Position',[1111 230 688 486]);
xlim_ = [Inf -Inf];       % limits of x axis
ax_ = axes;
set(ax_,'Units','normalized','OuterPosition',[0 0 1 1]);
set(ax_,'Box','on');
axes(ax_); hold on;
```
% --- Plot data originally in dataset "Et vs. t"
t = t(:);
Et = Et(:);
h_ = line(t,Et,'Parent',ax_, 'Color',[0.333333 0 0.666667],...
    'LineStyle','none', 'LineWidth',1,...
    'Marker','.', 'MarkerSize',12);
xlim_(1) = min(xlim_(1),min(t));
xlim_(2) = max(xlim_(2),max(t));

% Nudge axis limits beyond data limits
if all(isfinite(xlim_))
xlim_ = xlim_ + [-1 1] * 0.01 * diff(xlim_);
set(ax_, 'XLim',xlim_)
else
    set(ax_, 'XLim',[-15847.921924611142, 1600742.1143857252]);
end

% --- Create fit "fit 6"
fo_ = fitoptions('method','NonlinearLeastSquares','Lower',[0 0 0 0 0 -Inf -Inf -Inf -Inf -Inf -Inf -Inf -Inf -Inf -Inf],
    'Upper',[Inf Inf Inf Inf Inf 0 0 0 0 0 0 0 0 0 0 0]);
ok_ = isfinite(t) & isfinite(Et);
if ~all( ok_ )
    warning( 'GenerateMFile:IgnoringNansAndInfs', ... 
        'Ignoring NaNs and Infs in data' );
end
st_ = [0.45800000000000002 0.5 0.29999999999999999 1 0.20000000000000000 
    0 0 -Inf -Inf -Inf -Inf -Inf,'Upper',[Inf Inf Inf Inf Inf 0 0 0 0 0 0 0 0 0 0 0]);
set(fo_, 'Startpoint',st_);
ft_ = fitttype('a1*exp(b1*x)+a2*exp(b2*x)+a3*exp(b3*x)+a4*exp(b4*x)+a5*exp(b5*x)',...
    'dependent',{'y'},'independent',{'x'},...
    'coefficients',{'a1', 'a2', 'a3', 'a4', 'a5', 'b1', 'b2', 'b3', 'b4',
    'b5'});

% Fit this model using new data
[cf_,R]= fit(t(ok_),Et(ok_),ft_,fo_);
C = coeffvalues(cf_);
Coe=coeffnames(cf_);
% Or use coefficients from the original fit:
if 0
cv_ = { 1.2976064027986283, 1.5570838218130905, 2.1114148277151776, 0.47761927846475249, 0.48075772600737371, -1.7684893241346126e-005, -0.0010046448909533216, -0.00014042402404471793, -1.0452388545944326e-006, -0.012440058586048458};

cf_ = cfit(ft_,cv_{:});
end

% Plot this fit
h_ = plot(cf_,'fit',0.95);
legend off; % turn off legend from plot method call
set(h_(1),'Color',[1 0 0],...
     'LineStyle','-', 'LineWidth',2,...
     'Marker','none', 'MarkerSize',6);

% Done plotting data and fits. Now finish up loose ends.
hold off;

Code of alphatopu.m

function [Alpha, stress, time, ttr]=alphatopu(aT,T,TSRST_stress,TSRST_T,E1,E2,E3,E4,E5,t1,t2,t3,t4,t5,coolingrate)
start=[30e-6];
Alpha=fminsearch(@alphacal,start);

function [error]=alphacal(parameter)
close
% parameter=30e-6; %parameter;
alpha=parameter;
X=T; Y=aT*2.303;

[cf_,R,C,Coe]=linearfit(X,Y);
close
A=C(1); B=C(2); % log aT = AT+B

si=size(TSRST_T);
a=A*coolingrate/3600;
reducedtime=exp(-B)*(1/a)*(exp(a.*TSRST_T)-1); % by integration
strain_rate=alpha*coolingrate/3600;
ttr=reducedtime(end)-reducedtime;
t=ttr;

E_tr=E1*exp(-t./t1)+E2*exp(-t./t2)+E3*exp(-t./t3)+E4*exp(-t./t4)+E5*exp(-t./t5);
I=E_tr*strain_rate*1000; % 1000 for GPa to MPa
for n=2:1:(si(1))
    time =TSRST_T(1:n);
    II =I(1:n);
    y(n)=trapz(time,II)
end,
stress=y';

error=sum((TSRST_stress(50:end)-stress(50:end)).^2);
error=norm(abs(TSRST_stress)-abs(stress));
end
end

Code of stress_calculation_f.m

function [stress,time,ttr]=stress_calculation_f(alpha,aT,T,tim,E1,E2,E3,E4,E5,t1,t2,t3,t4,t5,coolingrate)
X=T; Y=aT*2.303;
figure;
[cf_,R,C,Coe]=linearfit(X,Y);
close
A=C(1); B=C(2); % log aT = AT+B

si=size(tim);
a=A*coolingrate/3600;
reducedtime=exp(-B)*(1/a)*(exp(a.*tim)-1); %by integration

figure,plot(tim,reducedtime,'--r')
xlabel('Time [sec]')
ylabel('Reduced time [sec]')
strain_rate=alpha*coolingrate/3600;
ttr=reducedtime(end)-reducedtime;
t=ttr;
E_tr=E1*exp(-t./t1)+E2*exp(-t./t2)+E3*exp(-t./t3)+E4*exp(-t./t4)+E5*exp(-t./t5);
figure;loglog(t,E_tr,'r*')
xlabel('reduced time [sec]')
ylabel('Relaxation Modulus [GPa]')
I=E_tr*strain_rate*1000; % 1000 for GPa to MPa %y_final=trapz(tim(1:end),I);
figure,plot(tim,I,'mo')
for n=2:1:(si(1))
    time =tim(1:n);
    II =I(1:n);

    y(n)=trapz(time,II);
end
stress=y;

Codes of crackgrowth.m

function [fracture_time,crack_length,Fracture_Temp]=
    crackgrowth(n,a,z,alpha,stress,ttr,eta,DCSE_test,temp,St,TT,time,FE_test)
    [DCSEnor,kk,dcselimt]=crackgrowthDCSE_1(n,a,z,stress,ttr,eta,DCSE_test,temp,St,TT,time);
    a1=kk;
    [FEnor,kk,felimt]=crackgrowthFE_1(n,a,z,alpha,stress,FE_test,temp,St,TT,time);
    a2=kk;
    kk=[];
    check=[];
    if a1<a2
        kk=a1;
        check='D';
    else
        kk=a2;
        check='F';
    end
    t1=kk;
    DCSE_final=DCSEnor(1:(kk+1),:); %taking value upto DCSE_limit
    DCSE_next=DCSE_final(end,2:end);

    FE_final=FEnor(1:(kk+1),:); %taking value upto FE_limit
    FE_next=FE_final(end,2:end); %DCSE for adding up to the next crack
    % n-th crack

    frc_tim=[];
    Mode=[];
    crack_l=[];
    topu=[];
    Az=[];
    for ii=1:1:(n-1)
aaa=a+z*ii; %initial crack length
nnn=n-ii; %number of zone
topu=[topu;kk];
Mode=[Mode,check];
Azz=cumsum(topu);
Az=Azz(end);
[DCSEnorm,kk]=crackgrowthDCSE_n(nnn,aaa,z,stress,ttr,eta,DCSE_test,dc
selimt,St,TT,time,DCSE_next,Az);
a1=kk;
[FEnorm,kk]=crackgrowthFE_n(nnn,aaa,z,stress,alpha,FE_test,felimt,St,
TT,time,FE_next,Az);
a2=kk;
if a1<a2
    kk=a1;
    check='D';
else
    kk=a2;
    check='F';
end
DCSE_final=DCSEnorm(1:(kk+1),:); %taking value upto DCSE_limit
DCSE_next=DCSE_final(end,2:end); %DCSE for adding up to the next crack
FE_final=FEnorm(1:(kk+1),:); %taking value upto FE_limit
FE_next=FE_final(end,2:end); %FE for adding up to the next crack
%frc_tim=[frc_tim;frac_time];
crack_l=[crack_l;aaa];
end
Failure_mode=Mode';
tt=[topu;kk];
f_time=cumsum(tt);
fracture_time=time(f_time);
crack_length=[a;crack_l];
Fracture_Temp=TT(Az);
figure;
plot(fracture_time,crack_length,'-*')
title('Mix - 35-50-1')
xlabel('Time [sec]')
ylabel('Crack length [mm]')
end

Code of crackgrowthDCSE_1.m

function[DCSEnor,kk,dcselimt]=crackgrowthDCSE_1(n,a,z,stress,ttr,eta,DCSE_test,temp,St,TT,time)
% Crack growth for the 1st crack
ss = size(stress);
si = ss(2);

% Det. of length (r) from the crack tip
r = [];
for m = 1:1:n
    r1 = z*m;
    r = [r, r1]; % r = length from the crack tip
end

% Calculation of stress in each zone
sig = [];
for i = 1:1:si
    sigmaFA = stress(i);
sig1 = sigmaFA.*(r+a)./(sqrt(r.*(r+2*a))); % corresponding stress
sigma2 = [St, sig1]; % adding the St as the first value
sig = [sig; sigma2]; % finally the stress distribution
end

% NB:: Need to fix the lower and upper boundary
% -----------------------------------------------
% calculating the AVERAGE Stress
sigmaAVE = [];
for ii = 1:n
    sigmaAVE1 = 0.5*(sig(:,ii)+sig(:,ii+1));
    sigmaAVE = [sigmaAVE, sigmaAVE1];
end

% -----------------------------------------------
% Calculation of creep strain corresponding to the stress:
zx = ttr./eta;
b = size(zx);
crepst = [];

for ii = 1:n
    stres = sigmaAVE(:,ii);
    sttt = [];
    for m = 2:1:b(1)
        xx = stres(1:m);
        yy = zx(1:m);
        sttl = trapz(xx, yy); % creep strain
        sttt = [sttt, sttl];
    end
crpst=[crpst, stt']; % creep strain matrix for diff. zone
end

% ssss=trapz(sigmaAVE(:,3),zx) % for cross-checking
%--------------------------------------------------------------------
% Calculation of DCSE in diff zone:
dcse=[];
for ii=1:n
    stres=sigmaAVE(:,ii); % picking up the average stress column
    crpstrain=[0; crpst(:,ii)]; % adding 0 as the starting value
    DCSE=[];
    for nn=2:1:b(1)
        SS=stres(1:nn);
        PP=crpstrain(1:nn);
        DCSE1=trapz(PP, SS)*1000; % for MPa to KJ/m^3
        DCSE=[DCSE, DCSE1];
    end
    DCSE1=[0, DCSE]; % adding 0 as the starting value
    dcse=[dcse, DCSE1];
end
DE=dcse;
%DCSE_f=trapz([0; crpst(:,2)], sigmaAVE(:,2))*1000 % for cross-check
%--------------------------------------------------------------------
% DCSE normalization:
% fitting curve between 0C to -10C
DE1=DCSE_test(1:2);
T1=temp(1:2);
X=T1;
Y=DE1;
[cf_, R, C, Coe] = linearfit(X, Y);
close
D=C;

% fitting curve between -10C to -20C
figure;
DE2=DCSE_test(2:3);
T2=temp(2:3);
X=T2;
Y=DE2;
[cf_, R, C, Coe] = linearfit(X, Y);
E=C;
close
% fitting curve between -20C to -40C

DE3=[DCSE_test(3);0.0000000000000000001];
T3=[-20;-60];
X=T3;
Y=DE3;
[cf_,R,C,Coe]=linearfit(X,Y);
close

% getting data from the fitted curve (above and below -10C nad -20C)
ww=size(TT);
nm=ww(1);
R=[];
M=[];
W=[];
for j=1:1:nm
  tem=TT(j);
  if tem>=-10
  de1=D(1)*tem+D(2);
  R=[R,de1];
  end
  if tem<-10 && tem>=-20
  de2=E(1)*tem+E(2);
  M=[M,de2];
  end
  if tem<-20 && tem>-2000
  de3=C(1)*tem+C(2);
  W=[W,de3];
  end
end
dcselimt=[R,M,W]; %DCSE limit at diff temp,
dcseref=DCSE_test(3); %DCSE limit at ref. Temp (-20C).
factor=dcseref./dcselimt;
topp=DE(:,ii);

DCSEnor=[];
for ii=1:n
  DCSEnor1=DE(:,ii).*factor'; %Multiplying with factor in each column of dcse
DCSEnor=[DCSEnor,DCSEnor1];
end
%----------------------------------------------------------------------------------------------------------
% Checking DCSE>= DCSE_limit::
% info---> Here...DCSE=DCSEnor & DCSE_limit=dcseref
jk=DCSEnor(:,1);
h=size(jk);
DE_f=[];

% info--> DCSE nor-e je sokol value DCSE_f-er cheye boro, sei gula-k alada
% korar 1ta loop
for ii=1:1:h(1)
dcse_i=jk(ii);
if dcse_i<=dcseref
    tt=dcse_i;
    DE_f=[DE_f;tt];
else
    tm=dcse_i;
end
end
k=size(DE_f);
kk=k(1);

Code of crackgrowthFE_1.m

function [FEnor,kk,felimt]=crackgrowthFE_1(n,a,z,alpha,stress,FE_test,temp,St,TT,time)
%Crack growth for the 1st crack
ss=size(stress);
si=ss(2);

% Det. of length (r) from the crack tip
r=[];
for m=1:1:n
    rl=z*m;
    r=[r,rl]; %r= length from the crack tip
end
%----------------------------------------------------------------------------------------------------------
% Calculation of stress in each zone
sig=[];
for i=1:1:si
    sigmaFA=stress(i);
sigmal=sigmaFA.*(r+a)./(sqrt(r.*(r+2*a))); %corresponding stress
    sigma2=[St,sigmal]; %adding the St as the first value
sig=[sig;sigma2]; % finally the stress distribution
end

% NB:: Need to fix the lower and upper boundary
%-------------------------------------------------------------
% calculating the AVERAGE Stress
sigmaAVE=[];
for ii=1:n
    sigmaAVE1=0.5*(sig(:,ii)+sig(:,ii+1));
    sigmaAVE=[sigmaAVE,sigmaAVE1];
end

%Ddd=sigmaAVE(:,1);
% figure(1);plot(time,Ddd,'*')
% for ii=1:si
%    D=sigmaAVE(ii,:);
%    plot(r,D,'*-')
%    hold on
% end
%-------------------------------------------------------------
% Calculation of <thermal strain corresponding to the stress:
strain_th=alpha*(TT(1)-TT);
%-------------------------------------------------------------
% Calculation of FE in diff zone:
FE=[];
for ii=1:n
    stres=sigmaAVE(:,ii);
    fe=[];
    for i=2:1:si
        sts=stres(1:i);
        PPP=strain_th(1:i);
        FE1=trapz(PPP,sts)*1000; % for MPa to KJ/m3
        fe=[fe;FE1];
    end
    FE1=[0;fe]; % adding 0 as the starting value
    FE=[FE,FE1];
end
% FE_f=trapz(strain_th,sigmaAVE(:,3))*1000 % for cross-check

% FE normalization:
% fitting curve between 0C to -10C
FE1=FE_test(1:2);
T1=temp(1:2);
X=T1;
Y=FE1;
[cf_,R,C,Coe]=linearfit(X,Y);
close
D=C;

% fitting curve between -10C to -20C
figure;
FE2=FE_test(2:3);
T2=temp(2:3);
X=T2;
Y=FE2;
[cf_,R,C,Coe]=linearfit(X,Y);
E=C;
close

% fitting curve between -20C to -40C
figure;
FE3=[FE_test(3);0.0000000000000000001];
T3=[-20;-60];
X=T3;
Y=FE3;
[cf_,R,C,Coe]=linearfit(X,Y);
close

% getting data from the fitted curve (above and below -10C nad -20C)
ww=size(TT);
nm=ww(1);
R=[];
M=[];
W=[];
for j=1:1:nm
    tem=TT(j);
    if tem>=-10
        de1=D(1)*tem+D(2);
        R=[R,de1]
    end
    if tem<-10 && tem>=-20
        de2=E(1)*tem+E(2);
        M=[M,de2]
    end
end
if tem<-20 && tem>-2000
    de3=C(1)*tem+C(2);
    W=[W,de3];
end
end
felimit=[R,M,W]; % DCSE limit at diff temp,

feref=FE_test(3); % DCSE limit at ref. Temp (-20C).
factor=feref./felimit;

FEnor=[];
for ii=1:n
    FEnor1=FE(:,ii).*factor'; % Multiplying with factor in each column of dcse
    FEnor=[FEnor,FEnor1];
end

% Checking FE>= FE_limit::
% info--> Here...FE=FEnor & FE_limit=feref
jk=FEnor(:,1);
h=size(jk);
FE_f=[];
% info--> FE_nor-e je sokol value FE_f-er cheye boro, sei gula-k alada
% korar lta loop
for ii=1:h(1)
    fe_i=jk(ii);
    if fe_i<=feref
        tt=fe_i;
        FE_f=[FE_f;tt];
    else
        tm=fe_i;
    end
end

k=size(FE_f);
kk=k(1);

Code of crackgrowthDCSE_n.m

function [DCSEnorm,kk]=crackgrowthDCSE_n(nnn,aaa,z,stress,ttr,eta,DCSE_test,dcselimt,St,TT,time,DCSE_next,Az)
% Crack growth for the n-th crack
n=nnn;
a=aaa;
ss=size(stress);
si=ss(2);

% Det. of length (r) from the crack tip
r=[];
for m=1:1:n
  r1=z*m;
  r=[r,r1]; %r= length from the crack tip
end

% Calculation of stress in each zone
sig=[];
for i=Az:1:si
  sigmaFA=stress(i);
  sigma1=sigmaFA.*(r+a)./(sqrt(r.*(r+2*a))); %corresponding stress
  sigma2=[St,sigma1]; %adding the St as the first value
  sig=[sig;sigma2]; % finally the stress distribution
end
% NB:: Need to fix the lower and upper boundary

%calculating the AVERAGE Stress
sigmaAVE=[];
for ii=1:n
  sigmaAVE1=0.5*(sig(:,ii)+sig(:,ii+1));
  sigmaAVE=[sigmaAVE,sigmaAVE1];
end

% Calculation of creep strain corresponding to the stress:
zx=ttr(Az:end)./eta;
b=size(zx);
crpst=[];
for ii=1:n
  stres=sigmaAVE(:,ii);
  stt=[];
  for m=2:1:b(1)
    xx =stres(1:m);
    yy =zx(1:m);
    stt1=trapz(xx,yy); %creep strain
    stt=[stt,stt1];
  end
  crpst=[crpst,stt']; %creep strain matix for diff. zone
end
% ssss=trapz(sigmaAVE(:,3),zx) %for cross-checking

% Calculation of DCSE in diff zone:
dcse=[];
for ii=1:n
    stres=sigmaAVE(:,ii);  % picking up the average stress column
    crpstrain=[0;crpst(:,ii)];  % adding 0 as the starting value
    DCSE=[];
    for nn=2:1:b(1)
        SS=stres(1:nn);
        PP=crpstrain(1:nn);
        DCSE1=trapz(PP,SS)*1000;  % for MPa to KJ/m3
        DCSE=[DCSE,DCSE1];
    end
    dcse=[dcse,DCSE'];
end
DE=dcse;

% DCSE_f=trapz([0;crpst(:,2)],sigmaAVE(:,2))*1000  % for cross-check
%------------------------------------------------------------------------
% DCSE normalization:
% getting data from the fitted curve (above and below -10C nad -20C)
dcseref=DCSE_test(3);  % DCSE limit at ref. Temp (-20C).
factor=dcseref./dcselimt;

DCSEnor=[];
for ii=1:n
    DCSEnor1=DE(:,ii).*factor(Az+1:end);  % Multiplying with factor in each
    column of dcse
    DCSEnor=[DCSEnor,DCSEnor1];
end

% Adding up the DCSE from the crack before
DCSEnorm=[];
for ii=1:n
    DCSEnorm1=DCSEnor(:,ii)+DCSE_next(:,ii);
    DCSEnorm=[DCSEnorm,DCSEnorm1];
end
%------------------------------------------------------------------------
% Checking DCSE>= DCSE_limit::
% info--> Here...DCSE=DCSEnor & DCSE_limit=dcseref
jk=DCSEnorm(:,1);
h=size(jk);
DE_f=[];
% info--> DCSE_nor-e je sokol value DCSE_f-er cheye boro, sei gula-k alada
% korar 1ta loop
for ii=1:1:h(1)
    dcse_i=jk(ii);

if dcse_i<=dcseref
    tt=dcse_i;
    DE_f=[DE_f;tt];
else
    tm=dcse_i;
end
end
k=size(DE_f);
kkt=k(1);
kkk=kkt(1)+1;
if h(1)<kkk
    k2=h(1);
else
    k2=kkk;
end

Code of crackgrowthFE_n.m

function[FEnorm,kk]=crackgrowthFE_n(nnn,aaa,z,stress,alpha,FE_test,fe
limt,St,TT,time,FE_next,Az)

% Crack growth for the n-th crack
n=nnn;
a=aaa;
ss=size(stress);
si=ss(2);

% Det. of length (r) from the crack tip
r=[];
for m=1:1:n
    r1=z^m;
    r=[r,r1]; %r= length from the crack tip
end
% Calculation of stress in each zone
sig=[];
for i=Az:1:si
    sigmaFA=stress(i);
sigmal=sigmaFA.*(r+a)./(sqrt(r.*(r+2*a)))); %corresponding stress
    sigma2=[St,sigmal]; %adding the St as the first value
sig=[sig;sigma2]; % finally the stress distribution
end
% NB:: Need to fix the lower and upper boundary
%--------------------------------------------------------------------
% calculating the AVERAGE Stress
sigmaAVE=[];
for ii=1:n
    sigmaAVE1=0.5*(sig(:,ii)+sig(:,ii+1));
    sigmaAVE=[sigmaAVE,sigmaAVE1];
end
%--------------------------------------------------------------------
% Calculation of THERMAL strain corresponding to the stress:
strain_th=alpha*(TT(1)-TT);
%--------------------------------------------------------------------
% Calculation of FE in diff zone:
zx=time(Az:end);
b=size(zx);
FE=[];
for ii=1:n
    stres=sigmaAVE(:,ii); % picking up the average stress column
    strain=strain_th(Az-1:end); %adding 0 as the starting value
    fe=[];
    for nn=2:1:b(1)
        SS=stres(1:nn);
        PP=strain(1:nn);
        FE1=trapz(PP,SS)*1000; %for MPa to KJ/m3
        fe=[fe,FE1];
    end
    FE=[FE,fe'];
end
%DCSE_f=trapz([0;crpst(:,2)],sigmaAVE(:,2))*1000 % for cross-check
%--------------------------------------------------------------------
% DCSE normalization:
% getting data from the fitted curve (above and below -10C nad -20C)

feref=FE_test(3); %DCSE limit at ref. Temp (-20C).
factor=feref./felimt;

FEnor=[];
for ii=1:n
    FEnor1=FE(:,ii).*factor(Az+1:end)'; %Multiplying with factor in each
column of dcse
    FEnor=[FEnor,FEnor1];
end
% Adding up the DCSE from the crack before
FEnorm=[];
for ii=1:1:n
FEnorm1=FEnor(:,ii)+FE_next(:,ii);
FEnorm=[FEnorm,FEnorm1];
end
%--------------------------------------------------------------------
% Checking FE>= FE_limit::
% info--> Here...FE=FEnor & FE_limit=feref
jk=FEnorm(:,1);
h=size(jk);
FE_f=[];
% info--> FE_nor-e je sokol value FE_f-er cheye boro, sei gula-k alada
% korar 1ta loop
for ii=1:1:h(1)
    fe_i=jk(ii);
    if fe_i<=feref
        tt=fe_i;
        FE_f=[FE_f;tt];
    else
        tm=fe_i;
    end
end
k=size(FE_f);
kk=k(1);
% kkk=kk(1)+1;
% FE_final=FEnorm(1:kkk,:); %taking value upto DCSE_limit
% %frac_time=time(kkk); %Corresponding time to grow the crack
% FE_next=FE_final(end,2:end); %DCSE for adding up to the next crack