Towards a Uniform Fracture Mechanics-Based Framework for Flexible Pavement Design

Master Thesis

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Abstract: Cracking is an important potential failure mechanism for pavement structures. By combining a strain energy-based fracture criterion with conventional fracture mechanics based on the Energy Ratio (ER) concept, crack growth in asphalt can be investigated, and a low temperature Thermal Cracking model (TCMODEL) can be introduced. This thesis presents the implementation of the Florida cracking model into a Mechanistic-Empirical (ME) flexible pavement design framework. An improved analysis procedure for better converting raw data from the Superpave Indirect Tensile Test (IDT) into fundamental viscoelastic properties of the asphalt mixture allows for calibration of the TCMODEL. This thesis involves a detailed review of Florida cracking model and TCMODEL. Finally, a MATLAB tool is prepared for the thermal cracking model to investigate the cause and effect of the problems.

KEY WORDS: Crack growth; Low temperature cracking; Fracture mechanics; Dissipated creep strain energy; Asphalt pavements; Energy Ratio
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List of Symbols

\(a_T\)       Temperature shift factor
\(A\)       Intercept of binder Viscosity-Temperature relationship
\(A_f\)      Field aging parameter
\(B_f\)      Field aging parameter
\(C\)       Crack depth
\(C_0\)      Current crack length
\(C_f\)      Field aging parameter
\(C_{tc}\)   Observed amount of thermal cracking
\(c_f\)      Function of binder viscosity
\(D_0\)      Creep compliance parameter
\(D_1\)      Creep compliance parameter
\(D_f\)      Field aging parameter
\(D(\xi)\)   Creep compliance at reduced time \(\xi\)
\(DCSE_f\)   Dissipated Creep Strain Energy to failure
\(DCSE_{\text{min}}\) Minimum Dissipated Creep Strain Energy
\(E\)       Mixture stiffness
\(E_1\)      Dynamic modulus in compression
\(E^*\)      Dynamic modulus from test
\(E(\xi - \xi')\) Relaxation modulus at reduced time \(\xi-\xi'\)
\(E(\xi)\)    Relaxation modulus at reduced time \(\xi\)
\(ER_{\text{OPT}}\) Optimum energy ratio
\(f\)       Loading frequency
\(F_{AV}\)  Field correlative constant
\(F_f\)      Reduction factor
\(h_{ac}\)   Thickness of asphalt layer
\(K\)       Stress intensity factor
\(L[D(t)]\) Laplace transformation of the creep compliance
\(L[E(t)]\) Laplace transformation of the relaxation modulus
\(m\)       Creep compliance parameter
$M_R$  Resilient modulus  
$n$  Fracture parameter  
$N ()$  Standard normal distribution evaluated at ()  
$p_{3/4}$  Percent weight retained on 3/4 inch (19mm) sieve  
$p_{3/8}$  Percent weight retained on 3/8 inch (9.5mm) sieve  
$p_4$  Percent weight retained on No. 4 (4.75-mm) sieve  
$p_{200}$  Percent weight passing through No. 200 (0.75-mm) sieve  
$Pen_{77}$  Penetration value at 77°F  
$S$  Laplace parameter  
$S_f$  Tensile stiffness  
$S_t$  Tensile strength  
$STD$  Standard deviation of the log of the depth of cracks  
$t$  Time  
$t_{red}$  Reduced time  
$t_y$  Time in years  
$T_R$  Temperature in Rankine  
$V_a$  Percent air void content  
$V_{be}$  Effective asphalt content  
$VFA$  Void Filled with Asphalt  
$VTS$  Slope of binder Viscosity-Temperature relationship  
$\alpha$  Curve fitting parameter  
$\beta$  Curve fitting parameter  
$\beta_1$  Regression coefficient determined through field calibration  
$\beta_c$  Calibration parameter  
$\gamma$  Traffic factor  
$\Delta C$  Change in the crack depth due to a cooling cycle  
$\Delta K$  Change in the stress intensity factor due to a cooling cycle  
$\lambda_1$  Creep compliance parameter  
$\lambda_r$  Tensile stiffness reduction factor  
$\nu$  Poisson’s ratio  
$\sigma_m$  Undamaged mixture tensile strength
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{AVE}$</td>
<td>Average stress</td>
</tr>
<tr>
<td>$\sigma_{\text{max}}$</td>
<td>Maximum tensile stress</td>
</tr>
<tr>
<td>$\sigma_f$</td>
<td>Far-field stress from pavement</td>
</tr>
<tr>
<td>$\sigma_{FA}$</td>
<td>Faraway stress from pavement</td>
</tr>
<tr>
<td>$\sigma(\xi)$</td>
<td>Stress at reduced time $\xi$</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Structural resistant factor</td>
</tr>
<tr>
<td>$\varepsilon_{\text{mix}}$</td>
<td>Strain rate of the asphalt mixture</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Binder viscosity</td>
</tr>
<tr>
<td>$\eta_{\text{aged}}$</td>
<td>Aged binder viscosity</td>
</tr>
<tr>
<td>$\eta'_{\text{aged}}$</td>
<td>Corrected aged binder viscosity</td>
</tr>
<tr>
<td>$\eta_r$</td>
<td>Binder viscosity at the reference temperature</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Reduced time</td>
</tr>
</tbody>
</table>
List of Abbreviations

AASHTO - American Association of State Highway and Transportation Officials
AC - Asphalt Concrete
CCMC - Creep Compliance Master Curve
DCSE - Dissipated Creep Strain Energy
DDM - Displacement Discontinuity Method
EE - Elastic Energy
ER - Energy Ratio
ESALs - Equivalent Single Axle Load
FE - Fracture Energy
FEM - Finite Element Method
FHWA - Federal Highway Administration
HMA - Hot Mix Asphalt
IDT - Indirect Tensile Test
MAAT - Mean Annual Air Temperature (in Fahrenheit)
MCCC - Master Creep Compliance Curve
ME - Mechanistic Empirical
NCHRP - National Cooperative Highway Research Program
PG - Performance Grade
RTFO - Rolling Thin Film Oven
SHRP - Strategic Highway Research Program
TCMODEL - Thermal Cracking Model
VFA - Voids Filled with Asphalt
VMA - Voids in Mineral Asphalt
WAPA - Washington Asphalt Pavement Association
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1. Introduction

The main distress in asphalt pavements built in northern countries is the low temperature cracking resulting from the contraction and expansion of the asphalt pavement under extreme temperature changes (Birgisson et al., 2004). Low temperature cracking is manifested as a set of parallel surface-initiated transverse cracks of various lengths and widths. The cracks are generally perpendicular to the center line of roadway shown in Figure 1. The existence of transverse cracks leads to different types of degradation of the pavement structure. Water enters the pavement through these cracks and weakens the pavement base and sub-base. Under moving loads water and fine materials may pump out and leading to a progressive deterioration of the asphalt layer. In winter the presence of water may leads to differential frost heave of the pavement and causes distresses. Due to the diversity in pavement designs and construction procedures, as well as depending on the corresponding loading conditions and boundary conditions, thermal cracking may develop within asphalt pavement. Top-down cracking and thermal cracking problems in asphalt pavements can be predicted by implementing fracture mechanics in the design procedure.

Figure 1. Typical Thermal Cracking in Asphalt Pavement
This work involves a detailed review of the existing fracture mechanics-based Thermal Cracking Performance Model (TCMODEL). TCMODEL predicts the amount of thermal cracking that will develop in a pavement as a function of time. Several of the pavement materials fundamental properties are used as the inputs for the model, for example the master creep compliance curve, and the failure limits as a function of temperature. Both of these are obtained from the Superpave IDT test (AASHTO, 1996), the pavement geometry and site-specific weather data. Ultimately, this system provides the basis for the development of a true performance-based mixture specification for thermal cracking.

The objectives of this paper are i) to introduce Hot Mix Asphalt (HMA) fracture mechanics into the design procedure and implement it to determine the minimum thickness of the HMA layer; ii) to present an overview of HMA top-down cracking depending on energy ratio concept and iii) to review and implement of the thermal cracking model.
2. Background Overview

Hot Mix Asphalt (HMA) pavements are typically a layered system. One or more asphalt layers are placed on top of a granular aggregate base layer, which in turn is placed on compacted soil layers. Thermal cracking can form across the width of the pavement if an asphalt pavement is subjected to a thermal loading due to temperature change (Roque et al., 1995). It is one of the most destructive distresses that can occur in asphalt pavements in cold climates. Over the time, various empirical and “mechanistic-empirical” models (Fromm and Phang, 1972; Roque et al., 1995) have been proposed for predicting this distress. To analyze the elasticity properties of pavements, the local stress and strain can calculate by using the finite element method (Waldhoff et al., 2000). It is not straightforward to extend these results to general cases since the quality of numerical simulations depends on the quality of the meshing. Thus, analytical solutions are needed as an important tool for model verification and to gain a better insight into mechanical responses.

To predict tensile stress distributions Shen and Kirkner, (1999) and Timm et al. (2003) have developed a one-dimensional (1-D) pavement models. However the 1-D model cannot solve the shear stress distribution in the overlay, as it has limitations in the prediction field temperature along the thickness. Thus for describing the thermal stress distribution along the thickness a two-dimensional (2-D) model is needed. At first, Beuth (1992) presented solutions for fully and partially cracked film problems for elastic films. After that Hong et al. (1997) came out with a model to predict the crack spacing and crack depth in highway pavements. The fracture mechanics based cracking model can deals with all the viscoelastic properties of the asphalt that can correlate with the crack initiation and as well as propagation of the crack.
3. HMA Fracture Mechanics

Fracture mechanics combines the mechanics of crack initiation or growth with the mechanical properties of that material. Mostly, fracture mechanics deals with the fracture phenomena i.e., the crack initiation and propagation. In an HMA pavement fracture simulator the HMA fracture mechanics can be introduced as the viscoelastic displacement discontinuity method (DDM). This displacement discontinuity method is employed to obtain the viscoelastic solution to the problem under consideration and also calculation of the Dissipated Creep Strain Energy (DCSE) in the process zone in front of the crack. After that, the HMA fracture mechanics crack growth rule is used for determining when and where the crack starts and propagates. According to Sangpetngam et al. (2003b), a natural length for the process zone is assumed to be associated with the aggregate size of the asphalt mixture.

3.1 Fracture Threshold in Hot Mix Asphalt

Zhang et al. (2001a) suggested the existence of a fracture threshold by observing that discontinuous (stepwise) crack growth in HMA materials (Figure 2).

![Figure 2. Illustration of Crack Propagation in Asphalt Mixtures (Birgisson et al., 2007)](image-url)
Below a certain threshold damage is limited to micro-crack which is not related to crack initiation or crack growth. After a rest period the micro-cracks are fully healable. On the other hand, non-healable macro-crack that is associated with crack initiation or growth occurs when the threshold is reached or exceeded. Zhang (2000) found that the DCSE limit is independent of the mode of loading (strength mode or cyclic mode) and can be used as the fundamental threshold for crack propagation.

Birgisson et al. (2007) discussed two possible ways that fracture can develop in asphalt mixtures. The first is due to creep strain energy a number of continuously repeated loads with stresses significantly below the tensile strength can cause damage accumulation and lead to fracture when the dissipated creep strain energy (DCSE) threshold is reached. The second way to initiate fracture is when any large single load during the loading cycle exceeds the Fracture Energy (FE) threshold then fracture can occur. The FE is generally higher than the DCSE threshold. Elastic energy (EE) is the difference between the FE and DCSE for a single load (Figure 3).

**Figure 3.** Graphical Illustration of the DCSE (Birgisson et al., 2004)
3.2 HMA Fracture Model for IDT Test

On the basis of the concept on energy threshold, Zhang et al. (2001a) developed a model for the cyclic IDT test. In that model, a small circular hole is located in the center of the sample where the crack initiates. The crack initiation length was assumed 10 mm (Figure 4a) based on the typical aggregate size for asphalt mixtures. Zhang et al. (2001a) proposed a simple Equation for the DCSE at each cycle.

$$\text{DCSE / cycle} = \left( \frac{\lambda_1}{20} \right) \cdot m \cdot (100)^{m-1} \cdot \sigma_{\text{AVE}}^2$$

[1]

where $\lambda_1$ and $m$ are creep compliance parameters, $\sigma_{\text{AVE}}$ is the average stress in the process zone with the length of 10 mm in the initiation phase.

![Diagram of crack growth process in IDT Test](image)

**Figure 4.** Crack Growth Process in IDT Test (Birgisson et al., 2007)
According to Birgisson et al. (2007), the continuous cyclic loading will increase the accumulative DCSE in the initiation zone until it reaches the DCSE threshold as shown in Figure 4a. After initiation, the stress at the crack tip will draw a high rate of DCSE buildup in the process zone next to the crack tip (Figure 4b). The length of the process zone $r_i$ is defined in Equation 2.

$$r_i = \frac{1}{2} \cdot \left( \frac{\sigma_{FA}}{S_t} \right)^2 \cdot a_i, \quad (i > 1)$$

where $S_t$ is the tensile strength, $\sigma_{FA}$ is the faraway stress, and $a_i$ is the current crack length. In Figure 4c, the DCSE accumulation process continues in the new process zone. Since the newer zones are always weaker than the earlier zones due to the prolonged DCSE accumulated from the beginning, the crack grows at a faster rate (i.e., fewer number of load cycles ($N$) to fail in the new process zone). That shows, the applied loads not only damage the initiation zone or the zone next to crack tip but they also cause smaller damage throughout the crack growth path.

### 3.3 HMA Fracture Simulator Framework

An HMA fracture simulator is developed (Birgisson et al., 2007) based on the numerical solution obtained from the viscoelastic displacement discontinuity method by using the dissipated creep strain energy (DCSE) (or permanent damage) threshold concept. Figure 5 shows a flowchart of the HMA fracture simulator.

The problem is modeled by placing displacement discontinuity (DD) elements on the boundaries; with the possible crack initiation specified locations then define the process zone in front of the critical location(s). Next the displacement discontinuity method (DDM) is used to calculate the tensile-mode dissipated creep strain energy (DCSE) step by step according to a specified loading spectrum. Finally, the accumulated DCSE is used to determine whether the crack will grow or not. If the accumulated DCSE exceeds or reaches the damage threshold (i.e., DCSE limit) then a macro-crack forms in the critical zone and causes the crack to grow by length of the zone in the direction of maximum DCSE.
The fracture simulator continues to calculate and accumulate DCSE in the current critical zone through the remaining loading spectrum. Throughout the process of crack growth, the history of load applications at each step of crack length is recorded for illustrating the rate of crack growth (Birgisson et al., 2007). The implementation details for the HMA fracture simulator and its key features are described in the following discussion.

The length of the equal-sized process zone indicates the location where stress exceeds the limit, as illustrated in Figure 6. The process zone is subdivided into two segments $P_1$ and $P_2$. In zone $P_1$ the active stress is equal to...
the tensile stress limit and in $P_2$ the active stress is less than the tensile stress limit. The total DCSE in the process zone is determined by adding the parts associated with $P_1$ and $P_2$.

![Figure 6. Stress in the Process Zone (Birgisson et al., 2007)](image1)

After the DCSE at several critical locations is computed, the average DCSE in the process zone can be obtained by numerical integration. Sangpetngam (2003) found that the average DSCE in the process zone for the Superpave IDT test can be approximated from two trapezoidal areas under the DCSE curve as shown in Figure 7. The DCSE are required at three locations (Figure 7), which are denoted by (1), (2) and (3) i.e., the crack tip, the dividing point of $P_1$ and $P_2$; and the front of the process zone respectively.

![Figure 7. Approximate DCSE in the Process Zone (Birgisson et al., 2007)](image2)
3.4 Crack Growth in Superpave IDT Test

Superpave is a term which comes from the results of the asphalt research portion of the 1987 - 1993 Strategic Highway Research Program (SHRP). The final product of this research program is a new system referred to as "Superpave", which stands for SUperior PERforming Asphalt PAVEments. Superpave consists of three basic components: an asphalt binder specification, a design and analysis system based on the volumetric properties of the asphalt mix, and finally, mix analysis tests and performance prediction models (WAPA, 2002). The Superpave Indirect Tensile Test (IDT) is used to determine the creep compliances and indirect tensile strengths of asphalt mixtures at low and intermediate pavement temperatures. These measurements can use in performance prediction models, such as Superpave, to predict the low-temperature thermal cracking potential and intermediate-temperature fatigue cracking potential of asphalt pavements (FHWA, 2006).

An asphalt disk specimen with a small central hole is subjected to cyclic haversine loads, with 0.1 second loading and 0.9 second rest period in each loading cycle. Two Superpave mixtures were previously produced (a coarse-graded and a fine-graded mix) and tested by Honeycutt (2000) and Zhang (2000). The experiment setup is shown in Figure 8a. The diameter and thickness of the specimen are 150 mm and 25 mm respectively, whereas the hole diameter is 8 mm.

Figure 8. Superpave IDT Test with a Vertical Crack and Its Representative DD Model (Birgisson et al., 2007)
4. Application to Top-Down Cracking

Load-related top-down fatigue cracking (cracking that initiates at the surface and propagates downward) commonly occurs in asphalt pavements. This phenomenon has been reported to occur in many parts of the United States, as well as in Europe and China. The typical top-down cracking observed from a field core is shown in Figure 9. Among different types of distresses occurred in asphalt pavements (such as the bottom-up and top-down fatigue cracking, thermal cracking, reflective cracking), the top-down cracking seems to be the most problematic. This failure mode cannot be explained by traditional fatigue approach that used to explain load-associated fatigue cracking which generally initiates at the bottom of the pavement.

![Figure 9. Typical Top-Down Cracking Observed from a Field Core (Birgisson et al., 2004)](image)

4.1 Top-Down Cracking Model

The top-down cracking model is also known as Florida cracking model. The top-down cracking mostly initiates at the top of the asphalt pavement layer in a direction along the wheel path and grows into the pavement layer (Roque et al., 2000; Uhlmeyer et al., 2000). Conventional pavement analyses models are incapable of explaining the initiation and propagation of top-down longitudinal cracks. Most of these models generally predict the bending stresses in a layered pavement system by considering the instantaneous elastic
response but ignore the delayed elastic and creep behavior of the asphalt layer. The key features of this method, called the Florida cracking model, are summarized below:

- The accumulated dissipated creep strain energy (DCSE) is equal to the damage in asphalt mixture
- A damage threshold or limit exists in asphalt mixtures and is independent of loading mode or loading history
- Damage below the cracking threshold is fully healable
- A macro-crack will initiate when the damage (accumulated DCSE) exceeds the damage threshold (DCSE limit) or propagate the crack which is already present
- Macro-cracks are not healable.

According to the cracking model, for any loading condition the initiation and propagation of cracks can be determined by calculating the amount of accumulated DCSE and finally, comparing that DCSE with the DCSE threshold of the mixture. The value of DCSE depends on the structural properties (used to determine the tensile stress) and the creep compliance parameter $D_1$ and slope of the curve m-value, which are parameters in the creep compliance function, as shown in Figure 10.

![Graphic Illustration of the Creep Compliance Curve and the DCSE\textsubscript{min}](Birgisson et al., 2004)
4.1.1 Energy Ratio Concept

The basic principles of energy ratio can be shown if two mixtures with different properties can be compared, as illustrated in Figure 11. The DCSE increases with number of load applications in terms of Equivalent Single Axle Load (ESALs). Higher creep compliance power law parameters (m-value and $D_1$) will lead to a higher rate of DCSE accumulation for the mixture.

Roque et al. (2004) introduced the energy ratio into the HMA Fracture Mechanics Model. The energy ratio with the DCSE limit can be used to distinguish between pavements that exhibited top-down cracking and those that did not. The Energy Ratio (ER) is a dimensionless number that can be defined as the Dissipated Creep Strain Energy threshold (DCSE$_f$) of the mixture divided by the minimum Dissipated Creep Strain Energy (DCSE$_{min}$).

\[ ER = \frac{DCSE_f}{DCSE_{min}} \]

where DCSE$_f$ is dissipated creep strain energy threshold, and DCSE$_{min}$ is defined as the minimum dissipated creep strain energy. Roque et al. (2004) expressed the relation between DCSE$_{min}$ and the creep parameters $D_1$ and $m$-value in a single function, as shown in Figure 10:

**Figure 11. Basic Principles of HMA Fracture Mechanics Model** (Birgisson et al., 2004)
\[ \text{DCSE}_{\text{min}} = m^{2.98} \cdot D_1 / f (S_t, \sigma_{\text{max}}) \]  

[4]

where \( m \) and \( D_1 \) are the creep compliance parameters and the function \( f (S_t, \sigma_{\text{max}}) \) can be expressed as

\[ f (S_t, \sigma_{\text{max}}) = 0.0299 \cdot \sigma_{\text{max}}^{-3.10} \cdot (6.36 - S_t) + 2.46 \cdot 10^8 \]  

[5]

where \( S_t \) is the tensile strength (in MPa), and \( \sigma_{\text{max}} \) is the maximum tensile stress (in psi).

### 4.1.2 Traffic and Reliability Factors

From the above discussion it follows that for a pavement section, the optimum energy ratio (ER\text{OPT}) can be used to determine the top-down cracking performance (Birgisson et al., 2004). A mixture with higher minimum ER is needed for a pavement with more load applications and higher reliability. The ER can be used as a standard value for evaluating the reliability of a pavement system, with ER = 1 as the reference point. ER lower than 1 indicates a weak asphalt mixture which cracks easily, whereas ER greater than 1 leads to a good asphalt mixture which resists cracking. For this purpose Birgisson et al. (2004) established a factor which counts in reliability of a pavement and another factor which is related to traffic level. This can provide a rational basis to adjust the minimum ER criterion for pavements with different traffic and reliability levels.

![Figure 12. The Traffic Factor \( \gamma \) as a Function of the Number of ESALs (Birgisson et al., 2004)](image)
As ER criterion is 1.0, and then the minimum or optimum Energy Ratio could be determined as a function of traffic level and reliability.

\[
\text{ER}_{\text{OPT}} = \frac{\gamma}{\varphi}
\]

[6]

where \(\gamma\) is the traffic factor and \(\varphi\) is structural resistant factor (or reliability factor). The equation was obtained based on the calibration of lowest ER value for an uncracked pavement section. In the equation the required minimum \(\text{ER}_{\text{OPT}}\) is expressed in terms of traffic factor and reliability factor, which are the functions of design number of ESALs and the reliability level. Once the reliability and traffic information are obtained, the minimum required ER can be uniquely defined from \(\text{ER}_{\text{OPT}} = \gamma / \varphi\) where \(\gamma\) can be determined from Figure 12 and \(\varphi\) can be determined from the Figure 13.

![Figure 13. The Reliability Factor Expressed in Terms of the Reliability for Different Traffic Levels (Birgisson et al., 2004)]
4.1.3 Design Framework

For Level 3 top-down cracking design the ER criterion is used which accounts for the structure and mixture for “averaged” environmental conditions. The design scenario is to determine the asphalt layer thickness for ER ≈ ER optimum at the pavement design life (Birgisson et al., 2004). The Level 3 M-E design flowchart for top-down cracking is shown in Figure 14.

![Level 3 M-E Design Flowchart for Top-Down Cracking (Birgisson et al., 2004)](image_url)

**Figure 14.** Level 3 M-E Design Flowchart for Top-Down Cracking (Birgisson et al., 2004)

An initial thickness for the asphalt layer is assumed and the material properties for the asphalt mixture are obtained from volumetric relations developed based on the master curve. Next the structural information for the base layer is applied as input and performs linear elastic analysis to obtain the maximum tensile stress in the AC layer. Then the ER could be found using the tensile stress and the IDT fracture parameters of the asphalt mixture at the end of the pavement life. From the traffic information and reliability level, the minimum required ER is calculated. Finally, it needs to check the calculated ER is equal to the ER_{opt}. If this ER is close to or equal to ER_{opt} within a certain specified tolerance, the design is optimized. If not, then the AC thickness is adjusted and the above steps are repeated until a final design is achieved.
4.2 Top-Down Cracking in Level 3 M-E Design Tool

Level 3 Mechanistic-Empirical (M-E) design deals with a series of semi-empirical models which were developed for estimation of time dependent material properties (Jianlin et al., 2007). With the help of this material properties model, the design tool can perform pavement thickness design as well as pavement life prediction for top-down cracking based on the Florida design model. The thickness design is an automated process in this level 3 M-E design. The designed thickness is optimized for different traffic levels, mixture types and binder selections.

4.2.1 Material Property Model

The Level 3 analysis is introduced in 2002 AASTHO design guide where a brief description about it can be found. In Level 3 design to evaluate the top-down cracking performance, the material properties need to be determined without performing laboratory testing. In the cracking model, the mixture properties necessary for the top-down cracking design procedure are binder viscosity and elastic properties of the mixtures for stress calculation, such as the dynamic modulus $E^*$ and the Poisson’s ratio ($\nu$). The Superpave IDT fracture parameters are needed for calculation of the ER, which depends on the tensile strength $S_t$, the creep parameters such as $D_1$ and $m$-value and the dissipated creep strain energy limit to failure (DCSE$_f$).

4.2.1.1 Dynamic Modulus

In analyzing the response of pavement systems dynamic modulus $E_1$ of asphalt concrete is an important property. One of the most available comprehensive mixture dynamic modulus models is the predictive equation developed by Witczak and Fonseca (1996). The dynamic modulus $E_1$ is represented by a sigmoid function in Witczak’s model as given below:

$$\log |E_1| = \delta + \alpha /[1 + \exp (\beta + \gamma \log t_{\text{red}})]$$  \hspace{1cm} [7]

where,

- $E_1$ is the dynamic modulus in compression (in psi)
- $t_{\text{red}}$ is reduced time of loading (in seconds) at the reference temperature ($t_{\text{red}} = 1/f$, and $f$ is loading frequency)
\( \delta \) and \( \alpha \) are curve fitting parameters for a given set of data, \( \delta \) represents the minimum value \( E^* \), and \( \delta + \alpha \) represents the maximum value of \( E^* \); \( \delta \) are \( \alpha \) are dependent on aggregate gradation, binder content, and air void content.

\( \beta \) and \( \gamma \) are curve fitting parameters describing the shape of the sigmoid function (dependent on the viscosity of asphalt binder).

The detailed expressions for \( \delta, \alpha, \beta, \gamma \) are based on the gradation and volumetric properties of the mixture. Based on these expressions for Florida mixtures a new set of regression constants found by Birgisson et al. (2004) from extensive complex modulus test data, the fitting parameters \( \delta, \alpha, \beta, \gamma \) can be expressed as:

\[
\delta = 2.718879 + 0.079524 \cdot p_{200} - 0.007294 \cdot (p_{200})^2 + 0.002085 \cdot p_4 - 0.01293 \cdot V_a + 0.08541 \cdot V_{be} / (V_{be} + V_a) \quad [8]
\]

\[
\alpha = 3.559267 - 0.005451 \cdot p_4 + 0.020711 \cdot p_{3/8} - 0.000351 \cdot (p_{3/8})^2 + 0.00532 \cdot p_{3/4} \quad [9]
\]

\[
\beta = -0.513574 - 0.355353 \cdot \log(\eta_r) \quad [10]
\]

\[
\gamma = 0.37217 \quad [11]
\]

where,

- \( V_a \) is percent air void content by volume;
- \( V_{be} \) is effective asphalt content, percent by volume;
- \( p_{3/4} \) is percent weight retained on the 3/4 inch (19.05 mm) sieve;
- \( p_{3/8} \) is percent weight retained on the 3/8 inch (9.51 mm) sieve;
- \( p_4 \) is percent weight retained on No 4 (4.76 mm) sieve;
- \( p_{200} \) is percent weight passing No 200 (0.74 mm) sieve;
- \( \eta_r \) is binder viscosity at the reference temperature (°F) in \( 10^6 \) poise.
4.2.1.2 Binder Viscosity and Global Aging Model

Typical A-VTS values used in Level 3 design which is provided in the Design Guide software based on the binder performance grade (PG) to estimate the binder viscosity at mix condition (Birgisson et al., 2004) is given below:

\[
\log \log (\eta) = A + VTS \cdot \log (T_R) \tag{12}
\]

where \( \eta \) is the binder viscosity in centipoises \((10^{-2} \text{ poise})\), \( T_R \) is the temperature in Rankine, and \( A \) and \( VTS \) are the regression constants.

According to Mirza and Witczak (1995), the viscosity of the asphalt binder for aged conditions \((\eta_{aged})\), at near the pavement surface (depth \( z = 0.25 \) in or 6.25 mm) could be estimated from the following in-service surface aging model:

\[
\log \log (\eta_{aged}) = F_{AV} \cdot \left[ \log \log (\eta_{t=0}) + A_f \cdot t \right] / (1 + B_f \cdot t) \tag{13}
\]

where \( t \) is the time in months and \( F_{AV} \) is the field correlative constant. \( A_f \) and \( B_f \) are field aging parameters given as:

\[
A_f = -0.004166 + 1.41213 \cdot C_f + C_f \cdot \log (\text{MAAT}) + D_f \cdot \log (\eta_{t=0}) \tag{14}
\]

\[
B_f = 0.197725 + 0.068384 \cdot \log C_f \tag{15}
\]

the parameters \( C_f \) and \( D_f \) are given by,

\[
C_f = 10 \cdot \exp(274.4946 - 193.831 \cdot \log T_R + 33.9366 \cdot \log^2 T_R) \tag{16}
\]

\[
D_f = -14.5521 + 10.4762 \cdot \log T_R - 1.88161 \cdot \log^2 T_R \tag{17}
\]

where MAAT is the Mean Annual Air Temperature in Fahrenheit (°F) and \( \eta_{t=0} \) is the unaged binder viscosity. Birgisson et al. (2004) proposed a simple empirical correction on the current aging model by introducing a reduction factor \( F_r \):
\[
\log \log (\eta_{\text{aged}}') = F_r \cdot \log \log (\eta_{\text{aged}})
\]  \hspace{1cm} [18]

and

\[
F_r = 1 - (c_r/\Pi) \cdot \arctan (t_y)
\]  \hspace{1cm} [19]

where \( \eta_{\text{aged}}' \) is the corrected aged viscosity, \( t_y \) is the time in years, and \( c_r \) is a constant between 0.06 and 0.1 (in MATLAB model \( c_r = 0.08 \) is used).

4.2.1.3 Tensile Strength

The tensile strength is an important factor that needs to estimate in the evaluation for the cracking performance of asphalt mixture. Deme and Young (1987) discovered that the tensile strength of mix is well correlated with the mixture stiffness at a loading time \( t = 30 \) minutes. In their evaluation work on the low temperature cracking performance, they used the temperature range of \(-40\) to \(25^\circ\)C. These data is digitized and plotted in Figure 15.

\[\text{Figure 15. Relation between Mix Stiffness and Tensile Strength (Birgisson et al., 2004)}\]

The research team in Florida (Birgisson et al., 2004) proposed the following relation between the mix stiffness and the tensile strength using nonlinear regression:
\[ S_t = \sum_{n=0}^{5} a_n (\log S_f)^n \]  \quad [20a]

\[ S_f = \lambda_r \cdot E_1 \]  \quad [20b]

where \( S_t \) is the tensile strength (in MPa), and the tensile stiffness \( S_f \) (in psi) is obtained from the dynamic modulus by introducing a reduction factor (\( \lambda_r \)).

The constants \( a_n \) in the Equation 20a, were given as follows:

\[
\begin{align*}
a_0 &= 284.01, & a_1 &= -330.02, & a_2 &= 151.02, & a_3 &= -34.03, \\
a_4 &= 3.7786, & a_5 &= -0.1652
\end{align*}
\]

4.2.2 Creep Compliance Parameters and DCSE Limit

There is no existing model to predict damage, fracture properties and the changes in these properties induced by aging. The development, calibration, and validation of a mixture model are necessary to predict damage, healing, and fracture properties. These also involve the use of correlations from rheological properties and mixture characteristics to predict damage and fracture properties.

4.2.2.1 Creep Compliance Parameters

For viscoelastic materials creep compliance is the property that describes the relation between the time dependent strain and applied stress. As seen earlier, the ER value strongly depends on the creep compliance parameters \( D_1 \) and \( m \). The graphical relation between the creep compliance function \( D(t) \) and the corresponding creep parameters is shown in Figure 16.
It is important to estimate the creep parameters $D_0$ and $m$. The $D_0$ and $D_1$ can be obtained from the following equations:

\[
\log(D_0) = -\delta - \alpha - \log \lambda_r \quad [21a]
\]

\[
\log(D_0 + D_1) = -\delta - \left[ \frac{\alpha}{1 + e^\beta} \right] - \log \lambda_r \quad [21b]
\]

where $\lambda_r$ is the tensile stiffness reduction factor as introduced in Equation 20. The parameters $\delta$, $\alpha$, and $\beta$ obtained from the mixture volumetric and binder viscosity as mentioned in section 4.2.2.1. The $m$-value obtained by taking the derivative of the master curve Equation 4 with suitable modification. The research team in Florida (Birgisson et al., 2004) used the slope of $\log t$ vs. $\log |E^*|$ curve at $t = 1000$ s (denoted as $m_0$) as a base value:

\[
m_0 = \alpha \gamma \cdot \frac{\exp(\beta + 3\gamma)}{[1 + \exp(\beta + 3\gamma)]^2} \quad [22]
\]

After that, the viscosity change due to aging effects takes into account and the final predictive equation of $m$-value can be given by the Equation 23, where $k$ is a constant ($k = 0.408$) and $\eta$ is the binder viscosity in Mega Poise.
\[ m = m_0 + \frac{k}{\log \log \eta} \]  \[23\]

\[4.2.2.2 \textbf{Dissipated Creep Strain Energy Limit}\]

The dissipated creep strain energy limit (or DCSE to failure) has been connected to the resistance to top-down cracking in field. It is extremely difficult to estimate the DCSE\textsubscript{f} as its change induced by aging. It is found that the DCSE\textsubscript{f} is directly related to the strain rate of the asphalt mixture (\(\varepsilon_{\text{mix}}\)). It is inversely proportional to the mix viscosity, the tensile strength (\(S_t\)) and the resilient modulus (\(M_R\)). It is believed that it was reasonable to express the DCSE\textsubscript{f} as a function of \(\varepsilon_{\text{mix}}\) at \(t = 1000s\), the tensile strength, and the creep parameters.

\[
\text{DCSE}_f = c_f \cdot S_t \left( \frac{m \cdot D_1}{10^3(1-m)} \right) \]  \[24\]

where \(c_f\) is a function of binder viscosity. For simplicity, take \(c_f = 6.9 \times 10^7\) design (Birgisson et al., 2004).
5. Review of Thermal Crack Model

Thermal cracking is a severe problem for asphalt pavements. Generally it is regularly spaced transverse cracks across the complete pavement surface. Thermal crack is environmentally induced problem which is caused by the change of pavement temperature. Finally, it causes extreme thermal contraction and fracture of the asphalt surface. It permits water infiltration into the underlying pavement layer that can cause structural failure of the pavement. Thermal cracking also contributes to the loss of smoothness.

5.1 The Thermal Cracking Mechanism

The primary mechanism leading to thermal cracking is shown in Figure 17. In the restrained surface layer, thermal contraction induces strain by cooling which lead to thermal tensile stress development. Thermal stress development is mostly in the longitudinal direction of the pavement as this direction is more restraint. Thermal stresses are greatest at the surface of the pavement because of the lower pavement temperature (Witczak et al., 2000). Transverse cracks may develop at different points along the length of the pavement depending upon the magnitude of these stresses and the asphalt mixture's resistance to fracture (crack propagation).

![Figure 17. Schematic of Physical Model of Pavement Section (Witczak et al., 2000)](image)

In case of very severe cooling cycles (very low temperatures and/or very fast cooling rates) transverse thermal cracks may develop at the surface layer of pavement. This is usually referred to as low temperature cracking. As the
pavement is exposed to subsequent cooling cycles additional cracks can develop at different locations. For example, cracks may advance and develop at a slower rate for milder cooling conditions, so that it may take several cooling cycles to propagate cracks completely through the surface layer. This phenomenon is well known as thermal fatigue cracking.

Low temperature Cracks will develop faster at some locations within the pavement than at others. It is important to notice that the mechanism of failure is same for low temperature cracking and as well as thermal fatigue cracking. The only difference is in the rate at which cracking occurs.

5.2 Crack Propagation Fracture Model

Figure 17 shows an asphaltic surface layer is subjected to a tensile stress distribution along the depth (D). During the cooling process, stresses develop due to the contraction of the asphalt material. The stresses are not constant with depth because of a thermal gradient in the pavement temperatures vary with depth. Within the surface layer there are potential crack zones uniformly spaced at an assumed distance. At each of these crack zones the induced thermal stresses can cause a crack to propagate through the surface layer as shown in Figure 18, where \( C \) is the initial crack, \( \Delta C \) is the crack growth due to the cooling cycle and \( C_0 \) is the initial crack length for the next cooling cycle. Due to spatial variation of the relevant material properties within the surface layer each of these cracks can propagate at different rates (Witczak et al., 2000).

![Figure 18. Schematic of Crack Depth Fracture Model (Witczak et al., 2000)](image)
As per NCHRP 9-19 (National Cooperative Highway Research Program), the thermal cracking model consists of two main assumptions: The first is a mechanics-based model that calculates the downward progression of a vertical crack, at a single site, having average material properties. The second is a probabilistic model that calculates the global amount of thermal cracking visible on the pavement surface from the current average crack depth and the assumed distribution of crack depths within the surface layer.

5.3 Use of Superpave IDT in TCMODEL

Thermal stress development and crack propagation of thermal cracking shown in Figure 17. It is viscoelastic properties that control thermal stress development and the fracture properties, and this control the rate of crack development. These properties that can be measured and controlled by the Superpave Indirect Tensile Test (IDT), is shown in Figure 19. A description of the material models on IDT test data and a description of the IDT transformation model are necessary to introduce it in TCMODEL.

Figure 19. Materials Characterization with the IDT (Witczak et al., 2000)

5.4 Viscoelastic Properties

The level of stress development during cooling is mostly control by the viscoelastic properties of the asphalt concrete mixture. The time and
temperature dependent relaxation modulus is needed to calculate thermal stresses in the pavement as shown in Equation 25.

\[
\sigma(\xi) = \int_0^\xi E\left(\xi - \xi'\right) \frac{d\epsilon}{d\xi} d\xi'
\]

where \(\sigma(\xi)\) is the stress at reduced time \(\xi\), \(E(\xi - \xi')\) is relaxation modulus at reduced time and \(\xi'\) is the variable of integration.

The relaxation modulus for a generalized Maxwell model can be expressed mathematically according to the following Prony series.

\[
E(\xi) = \sum_{i=1}^{N+1} E_i e^{-\xi/\lambda_i}
\]

where \(E(\xi)\) is relaxation modulus at reduced time, \(E_i\) and \(\lambda_i\) are the Prony series parameters for master relaxation modulus curve.

Figure 20. Superpave Indirect Tensile Strength Device (Witczak et al., 2000)
5.5 Creep Compliance Curve and m-value

The creep compliance curve is used to calculate thermally-induced stresses in asphalt pavements. The m-value from the master curve depends on the fracture resistance of asphalt mixtures at low temperatures. The viscoelastic response of asphaltic materials as a function of time and temperature can be described by using master creep compliance curve. This viscoelastic relationship can be found from Superpave IDT test which is described in section 3.4.

For linear viscoelastic materials, the time-temperature superposition principle can be illustrated in the Figure 21. There is a correspondence between loading time and temperature, as defined by the relationship between shift factor (\(a_T\)) and temperature. According to Witczak et al. (2000), “The relationship can be obtained from creep compliance curves obtained at multiple test temperatures by shifting the compliances horizontally on a log compliance-log time plot to form one smooth continuous curve at a single temperature”.

![Figure 21. Development of Master Creep Compliance Curve (MCCC) (Witczak et al., 2000)](image)

This temperature is known as the reference temperature and the resulting curve is the creep compliance master curve. Creep compliance over a wide range of loading times and temperature can be obtained from this curve and the shift factor-temperature relationship. The real time is replaced by reduced time using the shift factor, \(a_T\).
where $\xi$ is reduced time, $t$ is the real time and $a_T$ is known as temperature shift factor.

To describe mathematically the compliance curve for asphalt mixture four Kelvin elements were generally suitable. So, four Kelvin elements can be presented by $N=4$ in the Prony series.

$$D(\xi) = D_o + \sum_{i=1}^{N} D_i \left(1 - e^{-\frac{\xi}{\tau_i}}\right) + \frac{\xi}{\eta_v}$$  \[28\]

where $D(\xi)$ is the creep compliance at reduced time, $D_o$, $D_i$, $\tau_i$, $\eta_v$ are proney series parameters and $N$ is number of Kelvin elements.

The exponents $\tau_1$ through $\tau_4$ in the Generalized Voight-Kelvin model suggest nonlinear regression is needed for better fitting of the model to the master curve. The best results can be obtained when the assumed $\tau$’s were evenly distributed across the range of reduced time covered by the master curve. Generally, the log of the longest reduced loading time was equal to log $(1/a_{T3})+3$ for 1000 seconds data and log $(1/a_{T3})+2$ for 100 seconds data. Based on this observation, the following scheme to generate $\tau$’s was developed (Witczak et al., 2000).

$$\log(\tau_1) = 0.33 \left[\log \left(1 \frac{1}{a_{T3}}\right)\right] + N$$  \[29\]

$$\log(\tau_2) = 0.58 \left[\log \left(1 \frac{1}{a_{T3}}\right)\right] + N$$  \[30\]

$$\log(\tau_3) = 0.75 \left[\log \left(1 \frac{1}{a_{T3}}\right)\right] + N$$  \[31\]

$$\log(\tau_4) = 1.00 \left[\log \left(1 \frac{1}{a_{T3}}\right)\right] + N$$  \[32\]

where $N=2$ for 1000 seconds test data and $N=1$ for 100 second test data.
The slope of the linear portion of the master curve plotted on a log-log plot, is necessary to fit the power model with the shifted fitted creep compliance-time data (Figure 22) and mathematically it can be expressed as follows (Witczak et al., 2000).

\[
D(\xi) = D_0 + D_1 \xi^m
\]  \[33\]

where \(D(\xi)\) is creep compliance at reduced time, \(\xi\) is reduced time, \(D_0, D_1\) and \(m\) are the Power model parameters.

To perform calculations of crack depth in the thermal cracking predictions TCMODEL uses the parameter \(m\).

![Power Model for Master Creep compliance Curve](Witczak et al., 2000)

**Figure 22. Power Model for Master Creep compliance Curve** (Witczak et al., 2000)

### 5.6 Master Relaxation Modulus Curve

It is accepted that creep tests on viscoelastic materials are typically easier to conduct and the results are more reliable than relaxation test results. For that reason, an indirect tensile creep test was developed for measuring the viscoelastic properties which is known as the creep compliance. The creep compliance is the time dependent strain divided by the constant stress and also the relaxation modulus can be approximated as simply the inverse of the creep compliance. However, the inverse of the creep compliance is the creep modulus (or creep stiffness) is not the relaxation modulus. Although with hard materials at low temperatures and short loading times, the two moduli are approximately equal (Witczak et al., 2000).
If a generalized Voight-Kelvin model is used to represent the master creep compliance curve the calculations become particularly easy. For a viscoelastic material, the relationship between creep compliance and relaxation modulus is given by the hereditary integral.

\[ \int_{0}^{\infty} D(t - \tau) \frac{dE(\tau)}{d\tau} d\tau = 1 \]  \hspace{1cm} [34]

Taking the Laplace Transformation of each side,

\[ L[D(t)] \cdot L[E(t)] = \frac{1}{s^2} \]  \hspace{1cm} [35]

where \( L[D(t)] \) is Laplace transformation of the creep compliance, \( L[E(t)] \) is Laplace transformation of the relaxation modulus, \( S \) is the Laplace parameter and \( t \) is time (or reduced time, \( \eta \)).

As part of this thesis a MATLAB code is developed to solve the Equation 34 for the master relaxation modulus of a given master creep compliance. For solving the equation the Laplace transformation and also inverse Laplace transformation is necessary. The following steps are necessary to follow to solve that equation. First the Laplace transformation of the master creep compliance curve, \( L[D(t)] \) is calculated; where \( D(\xi) \) is defined by the Prony series. Next the result is Multiplied by \( s^2 \), and the reciprocal of \( s^2 \cdot L[D(\xi)] \), which is \( L[E(\xi)] \). Finally, computes \( E(\xi) \), which is the inverse Laplace transformation of \( L[E(\xi)] \). This allows for the implementation of the master creep compliance curve into a MATLAB based tool.
6. Implementation of Thermal Crack Model

The expected amount of transverse cracking in the pavement system can be predicted by relating the crack depth to an amount of cracking (Witczak et al., 2000). The total amount of crack propagation for a given thermal cooling cycle can be predicted by using the Paris law of crack propagation.

\[ \Delta C = A \cdot \Delta K^n \]  \[36\]

where \( \Delta C \) is the change in the crack depth due to a cooling cycle, \( \Delta K \) is the change in the stress intensity factor due to a cooling cycle, A and n are the fracture parameters for the asphalt mixture.

The master creep compliance curve can be expressed by the power function shown in Equation 34. The m value can be derived from the compliance curve. The fracture parameter, n can be computed through the following equation:

\[ n = 0.8 \left( 1 + \frac{1}{m} \right) \]  \[37\]

If the n value is known, the fracture parameter A can be computed from the following equation:

\[ A = 10 \cdot \exp \left[ \beta_c \cdot (4.389 - 2.52 \cdot \log (E \cdot \sigma_m \cdot n)) \right] \]  \[38\]

where E is for mixture stiffness, \( \sigma_m \) is the undamaged mixture tensile strength and \( \beta_c \) is the calibration parameter.

6.1 Structural Response Modeling for Thermal Cracking

Many different factors that can affect the magnitude of the thermal cracking prediction in the asphalt layer can be listed as: temperature-depth profile within the asphalt layer, creep compliance, creep compliance test temperature, tensile strength, mixture void in mineral asphalt (VMA), aggregate coefficient of thermal contraction, mix coefficient of thermal contraction, asphalt layer thickness, air voids, voids filled with asphalt (VFA), intercept of binder viscosity-temperature relationship at RTFO (Rolling Thin Film Oven) condition and Penetration value at 77°F Fahrenheit.
6.2 Thermal Cracking Prediction Procedure

The step by step procedure for determining the amount of thermal cracking can be listed as:

i. Gather input data and summarize all inputs needed for predicting thermal cracking.

ii. Development of the master creep compliance curve.

iii. Prediction thermal stress using viscoelastic transformation theory, the compliance can be related to the relaxation modulus of the asphalt mix.

iv. Compute growth of the thermal crack length where Paris Law is used to compute the growth of the thermal crack length within the asphalt layer and

v. Compute length of thermal cracks.

6.2.1 Gathering Input Data

The characterization of the asphalt mixes in Indirect Tensile (IDT) mode is required for the developed thermal cracking approach. The following Table 1 contains information on seven different binders that were used in a thermal fracture analysis, along with the corresponding mix characteristics. It should be clarified that on performing this analysis, which is based on linear viscoelasticity, the creep compliance and the indirect tensile strength were measured, since these are the key visco-elastic properties. In fact, the first one was measured using indirect tensile tests at one or three temperatures (0, -10 and -20°C or 32, 14 and -4°F) depending on the level of the analysis, whereas the latter was evaluated only at one temperature and that is -10°C Celsius.
Table 1. *Binder and Mix Characteristics* (Witczak et al., 2000)

<table>
<thead>
<tr>
<th>Mix</th>
<th>Binder PG Grade</th>
<th>% Retained</th>
<th>% Pass. #200 sieve</th>
<th>Eff. Binder Content (%)</th>
<th>Va (%)</th>
<th>VMA (%)</th>
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<tbody>
<tr>
<td></td>
<td>3/4'' sieve</td>
<td>3/8'' sieve</td>
<td>#4 sieve</td>
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<tr>
<td>0</td>
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<td>52.8</td>
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<td>7.0</td>
</tr>
</tbody>
</table>

1 in. = 2.54 cm

Witczak et al. (2000) estimated A and VTS parameters based on the regression of RTFO viscosity results found in the Design Guide program database. The default parameters are presented in Table 2.

Table 2. *A-VTS Parameters (after RTFO)* (Witczak et al., 2000)

<table>
<thead>
<tr>
<th>Binder Type</th>
<th>A</th>
<th>VTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>82-10</td>
<td>9.514</td>
<td>-3.128</td>
</tr>
<tr>
<td>76-16</td>
<td>10.015</td>
<td>-3.315</td>
</tr>
<tr>
<td>70-22</td>
<td>10.299</td>
<td>-3.426</td>
</tr>
<tr>
<td>64-28</td>
<td>10.312</td>
<td>-3.440</td>
</tr>
<tr>
<td>58-34</td>
<td>10.035</td>
<td>-3.350</td>
</tr>
<tr>
<td>52-40</td>
<td>9.496</td>
<td>-3.164</td>
</tr>
<tr>
<td>46-46</td>
<td>8.755</td>
<td>-2.905</td>
</tr>
</tbody>
</table>
The creep compliance response at time $t$ is given in Equation 39.

$$D(t) = D_1 \ t^m \tag{39}$$

where $D_1$ and $m$ are the fracture coefficients obtained from the creep compliance and strength of the mixture; and the loading time ($t$) in seconds. The $D_1$ and $m$ parameters can be found at each temperature available: -20, -10, and 0°C.

On performing non-linear regression analysis, firstly the parameters $D_1$ and $m$ need to be established (or identified, or evaluated) for each one of the selected mixes (Table 2). Upon completion of this task, the investigation (or the analysis) proceeded by correlating these parameters against different volumetric and mixture properties. The correlation for the $D_1$ fracture parameter is given below (Witczak et al., 2000):

$$\log (D_1) = -8.5241 + 0.01306 \ T + 0.7957 \ \log (V_a) + 2.0103 \ \log (\text{VFA})$$

$$-1.923 \ \log (\text{A}_\text{RTFO}) \tag{40}$$

where,

- $T$ is the test temperature (°C) (i.e., 0, -10, and -20 °C)
- $V_a$ is the Air voids expressed in percent (%)
- VFA means Void Filled with Asphalt (%) $= \frac{V_{\text{beff}}}{V_{\text{beff}} + V_a} \cdot 100$
- $V_{\text{beff}}$ means Effective binder content (%)
- $\text{A}_\text{RTFO}$ means intercept of binders Viscosity-Temperature relationship for the RTFO test.

For the parameter $m$ can be calculated using,

$$m = 1.1628 - 0.00185 \ T - 0.04596 \ V_a + 0.01126 \ \text{VFA}$$

$$+ 0.00247 \ \ T \cdot (\text{Pen}_{77})^{0.4605} \tag{41}$$

where $T$ is the test temperature (°C) (i.e., 0, -10, and -20°C), $V_a$ is Air voids (%), VFA means Void Filled with Asphalt (%), and $\text{Pen}_{77}$ is Penetration value at 77°F Fahrenheit.
\[ \text{Pen}_{77} = 10 \exp(290.5013 - \sqrt{P}) \]  
\[ P = 81177.288 + 257.0694 \cdot 10^A + 257.0694 \cdot VTS \]

where \( A \) is the intercept of binder Viscosity-Temperature relationship and \( VTS \) is the slope of binder Viscosity-Temperature relationship. The outcome of the \( m \) value was set to a lower limit of 0.01 (Witczak et al., 2000).

According to Witczak et al. (2000) the tensile strength at –10°C is also correlated with the design mix properties such as: the air voids, the void filled with asphalt content, the Penetration at 77°F and the \( A \) intercept of the binder temperature-viscosity relationship for the RTFO condition.

\[ S_t = 7416.712 - 114.016 V_a - 0.304 V_a^2 - 122.592 VFA + 0.704 VFA^2 + 405.71 (\text{Pen}_{77}) - 2039.296 \log (A_{RTFO}) \]

where the tensile strength \( (S_t) \) is in psi. However Witczak et al. (2000) suggests a lower limit to the tensile strength of 100 psi.

As per ASHTO 2002 design guide, the default values of creep compliance and tensile strength for these seven binders can be calculated by using the above equations. The volumetric properties of the asphalt mix can be used as input value.

To perform this analysis MATLAB tool was developed as part of this thesis. The program is presented in the appendix. Table 3 summarize the calculated creep compliance parameters calculated using this tool.
<table>
<thead>
<tr>
<th>PG Grade</th>
<th>Time (sec)</th>
<th>Creep Compliance (1/psi) x 10^4</th>
<th>Tensile strength (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-20°C</td>
<td>-10°C</td>
<td>0°C</td>
</tr>
<tr>
<td>82-10</td>
<td>1</td>
<td>0.38</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.40</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.44</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.48</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.51</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.56</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.60</td>
<td>1.03</td>
</tr>
<tr>
<td>76-16</td>
<td>1</td>
<td>0.33</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.36</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.41</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.45</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.49</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.55</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.60</td>
<td>1.06</td>
</tr>
<tr>
<td>70-22</td>
<td>1</td>
<td>0.31</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.34</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.39</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.42</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.47</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.52</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.57</td>
<td>1.09</td>
</tr>
<tr>
<td>64-28</td>
<td>1</td>
<td>0.34</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.37</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.41</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.45</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.49</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.55</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.60</td>
<td>1.28</td>
</tr>
</tbody>
</table>
The change of creep compliance with the time at different three temperature (-20, -10 and 0°C) is calculated. Then these three curves are shifted by using the shift factor for a master creep compliance curve (MCCC) with respect to reduced time. For each of the seven different mixtures, the change of creep compliance value at various temperatures, and the corresponding master curves, is found by using the MATLAB tool. Figures 23 to 36 show the creep compliance data and the MCCC for each of the seven design mixes, respectively.

<table>
<thead>
<tr>
<th>PG Grade</th>
<th>Time (sec)</th>
<th>Creep Compliance (1/psi) x 10⁴</th>
<th>Tensile strength (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-20°C</td>
<td>-10°C</td>
<td>0°C</td>
</tr>
<tr>
<td>58-34</td>
<td>1</td>
<td>0.38</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.43</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.50</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.56</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.63</td>
<td>1.27</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.73</td>
<td>1.68</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.82</td>
<td>2.08</td>
</tr>
<tr>
<td>52-40</td>
<td>1</td>
<td>0.45</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.55</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.71</td>
<td>1.28</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.86</td>
<td>1.78</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>1.05</td>
<td>2.46</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>1.37</td>
<td>3.79</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>1.67</td>
<td>5.24</td>
</tr>
<tr>
<td>46-46</td>
<td>1</td>
<td>0.55</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.81</td>
<td>1.29</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>1.34</td>
<td>2.69</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>1.98</td>
<td>4.67</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>2.91</td>
<td>8.12</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>4.84</td>
<td>16.80</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>7.11</td>
<td>29.30</td>
</tr>
</tbody>
</table>

1 psi = 6.89 kPa
Figure 23. Different Creep Compliance Data at Various Temperatures for Mix-0

Figure 24. Creep Compliance Master Curve for Mix-0
Figure 25. Different Creep Compliance Data at Various Temperatures for Mix-1

Figure 26. Creep Compliance Master Curve for Mix-1
Figure 27. Different Creep Compliance Data at Various Temperatures for Mix-2

Figure 28. Creep Compliance Master Curve for Mix-2
Figure 29. Different Creep Compliance Data at Various Temperatures for Mix-3

Figure 30. Creep Compliance Master Curve for Mix-3
Figure 31. Different Creep Compliance Data at Various Temperatures for Mix-4

Figure 32. Creep Compliance Master Curve for Mix-4
Figure 33. Different Creep Compliance Data at Various Temperatures for Mix-5

Figure 34. Creep Compliance Master Curve for Mix-5
Figure 35. Different Creep Compliance Data at Various Temperatures for Mix-6

Figure 36. Creep Compliance Master Curve for Mix-6
6.2.2 Development of the Master Creep Compliance Curve

The results of the creep compliance master curve (CCMC) analysis can be fitted to a Prony series defined by the following equation (Witczak et al., 2000).

\[
D(\xi) = D(0) + \sum_{i=1}^{N} D_i \left( 1 - e^{-\xi/\eta_i} \right) + \frac{\xi}{\eta_v} \tag{44}
\]

where,
\(\xi\) or \(t_{\text{red}}\) is reduced time \(= t/a_T\)
\(t\) is real time
\(a_T\) is temperature shift factor
\(N\) is the Kelvin parameter
\(D(\xi)\) is creep compliance at reduced time \(\xi\)
\(D(0), D_i, \tau_i,\) and \(\eta_v\) are the proney series parameters.

The results of the master creep compliance curve can also be fitted to a power model. The fitted curve is shown in Figure 37.

![Figure 37: Creep Compliance Master Curve Fitted by Prony Series](image-url)
6.2.3 Prediction of Thermal Stresses

The relaxation modulus represented by a generalized Maxwell model and expressed by a Prony series relationship in Equation 26. The relaxation modulus can be found by using inverse lapus transformation of the creep compliance master curve (Witczak et al., 2000). By using the MATLAB code, the changing behavior of relaxation modulus with the reduced time at a reference temperature -20°C is illustrated in the figure below:

![Stress Relaxation Master Curve](image)

Figure 38. Stress Relaxation Master Curve

6.2.4 Growth of the Thermal Crack Length Computation

Fracture mechanics is a tool that can be used for computing the growth of the thermal crack length within the asphalt layer. The input parameters such as the stress intensity factor (K), as well as the A and n fracture parameters can be obtained from the creep compliance and strength of the mixture. The stress intensity parameter (K), has been formulated by Witczak et al. (2000) who developed a simplified equation based upon theoretical FEM studies and from that analysis.
where $K$ is the stress intensity factor, $\sigma_f$ is the far-field stress from pavement response model and $C_0$ is current crack length in feet. The crack propagation model can be used in the thermal fracture model, is illustrated in Equation 37. The crack increasing in terms of depth with the cooling cycle is shown in Figure 39.

\[ K = \sigma_f (0.45 + 1.99 C_0^{0.56}) \]  

**Figure 39. Growth of Crack Depth with the Cooling Cycle**

### 6.2.5 Length of Thermal Cracks Computation

The degree of cracking is generally expressed as the length of thermal-transverse cracks occurring in a pavement length of 500 ft. It is predicted from the relationship between the probability distribution of the log of the crack depth to HMA layer thickness ratio and the percent of cracking (Witczak et al., 2000). The relation of the computed crack depth to an amount of cracking can be presented by the following Equation 46.
\[ C_{tc} = \beta_1 \cdot P_r (\log C > \log h_{ac}) \quad \text{or,} \]

\[ C_{tc} = \beta_1 \cdot N \left( \frac{\log C}{\frac{h_{ac}}{\text{STD}}} \right) \]

where,
- \( C_{tc} \) is the observed amount of thermal cracking in ft
- \( \beta_1 \) is regression coefficient determined through field calibration
- \( N(\cdot) \) is the standard normal distribution evaluated at (·)
- \( \text{STD} \) is standard deviation of the log of the depth of cracks in the pavement
- \( C \) is crack depth and
- \( h_{ac} \) is the thickness of asphalt layer.

### 6.3 Comparison of Tensile Strength

Tensile strength is an important material property for prediction crack formation and crack growth. In general, pavements with high tensile strength are less susceptible to cracking. It is therefore very important to be able to predict the tensile strength accurately. A comparison of tensile strength at -10°C is done by using the AASHTO method and the formula developed by Birgisson et al. (2004) is shown in Figure 40.

The tensile strength got by using the empirical formula (AASHTO, 2002) where the results come only for the -10°C. The temperature was not used as an input parameter variable in this empirical formula. As asphalt is a viscoelastic material and most of the properties of viscoelastic material change with the temperature changes. Asphalt shows high stiffness in the low temperature whereas at high temperature it becomes soft (i.e., shows low stiffness). So it makes clear that there is a relation between temperature and the tensile strength. The tensile strength is dependent on the various properties of the asphalt mix and also related with the environment. For the limitation of the AASHTO empirical formula, Birgisson et al. (2004) developed the correlation where the tensile strength can be found as a function of binder type, binder content, mix proportion and temperature.
Figure 40. Comparison of Tensile Strength Using Two Methods at \(-10^\circ C\)

In this case the input values for the seven binder used from the Table 1 and 2. Among all of these mixes mostly the performance grade of the binder and the binder content changes. For mix 0 the binder content is 10.5%, for mix 1 and 2 it is 10% and for rest of the mixes binder content increases gradually (Table 1). If the binder content increase in a mix then the tensile strength generally decreases. As per AASHTO formula the tensile strength increases rapidly with the increasing percent of binder content (Figure 40). For the Birgisson et al. (2004) method the tensile strength is higher for mix 1 and 2 because of lower binder content and it decreases gradually for mix 3 to 6 as the binder content increases.
7. Summary and Conclusion

It is known from the HMA fracture mechanics that non-healable crack initiation or crack growth occurs when the cumulative DCSE reaches the cracking threshold. For modeling the crack growth as an accumulation of micro-damages described by DCSE, the referenced HMA fracture mechanics with a fundamental crack growth threshold is being employed. In order to implement the DCSE threshold concept in the numerical analysis, a critical (process) zone needs to be defined in front of the crack tip, which is where the maximum limit of the tensile stress occurs, when the DCSE in a zone exceeds the energy threshold, i.e. when it exceeds the DCSE limit, the crack shall propagate by the length of the critical zone, as per the HMA fracture mechanics suggestion, and the direction of the crack growth shall be governed by the maximum dissipated creep strain energy direction.

For modeling of the TCMODEL, the approach recently developed in University of Florida has been reviewed. It is found that the using of fracture mechanics is a useful tool in evaluating crack initiation and propagation technique. In particularly it is good for the investigation of top-down down cracking and as well as thermal cracking. Based on the research performed in this study, the following important conclusions can be drawn:

- The minimum thickness for asphalt layer against top-down cracking, based on the minimum energy ratio concept, can be found by using fracture mechanics.

- Taking consideration of the minimum asphalt layer, binder properties and environmental condition thickness we can get a better prediction of the thermal cracking response.

- A complete MATLAB tool is developed for calculating the thermal crack growth with the cooling cycle, which follows the TCMODEL.

- Two methods were compared for predicted tensile stress. If the new formula by Birgisson et al. (2004) is used to calculate the tensile strength from the raw data then the change of tensile strength is too low but on the other hand if we use AASTHO
formula then it changes rapidly. The new formula it is possible to apply the different temperature but for AASTHO it is not possible. For AASTHO the formula directly gives the tensile stress at a reference temperature of \(-10^0\text{C}\) i.e., temperature was not used as a variable.

**Future research area:**

The findings of this study represent a start in developing an improved asphalt binder and asphalt mixture specifications, as well as improving the low temperature cracking model. However, this research effort needs to be continued to address the following key issues:

- The critical condition for thermal cracking
- Link the critical condition with the final design stage.
Appendix

MATLAB code for thermal cracking model:

```matlab
%% Input Data
clc
T=[0 -10 -20];  % Temperature
Va=7;    % Air voids (%)
Vbc=14;  % Effective bitumen content (Vb eff)
VFA=(Vbc/(Vbc+Va))*100;      % Void filled with Asphalt (%)

PG=46-46;  % Bindre type
    if PG==82-10; A=9.514; VTS=-3.128;
    elseif PG==76-16; A=10.015; VTS=-3.315;
    elseif PG==70-22; A=10.299; VTS=-3.426;
    elseif PG==67-22; A=10.6316; VTS=-3.548;
    elseif PG==64-28; A=10.312; VTS=-3.440;
    elseif PG==58-34; A=10.035; VTS=-3.350;
    elseif PG==52-40; A=9.496; VTS=-3.164;
    elseif PG==46-46; A=8.755; VTS=-2.905;
end

logD1=-8.5241+.01306.*T+0.7957*log10(Va)+2.0103*log10(VFA)-
1.923*log10(A);
Pen77=10^(290.5013-sqrt(81177.288+257.0694*10^(A+2.72973*VTS)));
mm=1.1628-0.00185.*T-.04596*Va-.01126*VFA+0.00247*Pen77+.001683*T*(Pen77^0.4605);
if mm<=0.01; mm=.01;end
m=mm;

TS=7416.712-114.016*Va-0.304*(Va^2)-
122.592*VFA+.704*(VFA^2)+405.71*log10(Pen77)-2039.296*log10(A)
if TS<=100; TS=100;end
St=TS;  % Undamaged tensile strength
```

%%%
```matlab
% Time points
t=[2 5 10 20 50 100 200 500 1000];
logt=log10(t);

% Constants
logD0=logD1(1)+m(1).*logt;
logD10=logD1(2)+m(2).*logt;
logD20=logD1(3)+m(3).*logt;

D0=10.^[logD0];
D10=10.^[logD10];
D20=10.^[logD20];

% Plotting
plot(logt,logD0,'--')
hold on
plot(logt,logD10,'r')
hold on
plot(logt,logD20,'g')
hold on
xlabel('Time, log(t)')
ylabel('logD(t)')
legend('Temp= 0C','Temp= -10C','Temp= -20C',2)
hold off

%% Shift factor calculation
a1=1;
loga1=log10(a1);

logtr2=(logD0(1)-logD1(2))/m(2); % Reference time for 0C to -10C
loga2=log(1)-logtr2; % Shift factor for 0C to -10C

bbb=logt-loga2; % Reference time for 0C to -10C
tr=[log t bbb];
log=[logD10 logD0];

logtr3=(log(1)-logD1(3))/m(3); % Reference time for -10C to -20C
loga3=log(1)-logtr3; % Shift factor for -10C to -20C

cc=logt-loga3;
```
ccc=bbb-loga3;
trr=[logt cc ccc];
logDtr=[logD20 log];
Dtm=10.^logDtr;

%% Develop of Creep Compliance Curve
P=polyfit(trr,logDtr,1)    %fitted curve by linear regression
f=polyval(P,trr);
figure,plot(trr,f,'b')
legend('Creep Compliance Curve',2)
hold on
plot(trr,logDtr,'P')
xlabel('reduced time,log(tr)')
ylabel('log D(tr)')
hold off
m=P(1); %   m-value of creep compliance curve

%figure.plot(trr,Dtm,'o')
% xlabel('reduced time,log(tr)')
% ylabel('logaD(tr)')

%% Shift factor-Temperature Curve
loga22=loga2+loga3; %Shift factor for 0C for shifting to -20C
at3=10^(loga22);
loga=[loga22 loga3 loga1];

figure,plot(T,loga,('--',T,loga, '*')) %plotting of shift factor.
xlabel('temparature')
ylabel('log a(T)')

%% Prony series Parameter Calculation:

N=2;
logt1=.33*(log10(1/at3)+N);
t1=10^(@logt1);
logt2=.58*(log10(1/at3)+N);
t2=10^(@logt2);
logt3=.75*(log10(1/at3)+N);
t3=10^(@logt3);
\[
\log t4 = 1.00^* (\log 10(1/at3)+N);
\]
\[
t4 = 10^{(\log t4)};
\]
\[
Sp = (\text{trr}(27)-\text{trr}(1))/5;
\]
\[
T_tr = [\text{trr}(1) + \text{Sp} \cdot \text{trr}(1) + \text{Sp}^2 \cdot \text{trr}(1) + \text{Sp}^3 \cdot \text{trr}(1) + \text{Sp}^4 \cdot \text{trr}(1) + \text{Sp}^5];
\]
\[
DTr = P(1).^* T_tr + P(2);
\]
\%
\%
---------------------
hold off;
Tr = Tr(:);
DTr = DTr(:);
ww = \{t1 t2 t3 t4\};
ok_ = isfinite(Tr) & isfinite(DTr);
if ~all(ok_)
    warning( 'GenerateMFile:IgnoringNansAndInfs', ... 
        'Ignoring NaNs and Infs in data');
end
st_ = [0.00000000000001 0.000000000000025 0.000000000000022 
    0.000000000000029 0.000000000000011];
ft_ = fittype('D0+D1*(1-exp(-x/t1))+D2*(1-exp(-x/t2))+D3*(1-exp(-x/t3))+D4*(1-exp(-x/t4))+D5*x',... 
    'dependent',\{'y'\},'independent',\{'x'\},'problem',\{'t1', 't2', 't3', 't4'\},... 
    'coefficients',\{'D0', 'D1', 'D2', 'D3', 'D4', 'D5'\});
%
Fit this model using new data
cf_ = fit(Tr(ok_),DTr(ok_),ft_,\{'Startpoint',st_,'problem',ww\})
hold on;
plot(Tr, DTr, 'o');
xlabel('reduced time, \log(\text{tr})')
ylabel('\log D(\text{tr})')
legend('Fitted by prony series','Estimated point',2)
%
---------------------
Laplace transformation
P(1)=0.1678;
P(2)=-6.5328;
syms a s t w x
\[ f = (P(1) \cdot t + P(2)) \]
\[ L = \text{laplace}(f) \]
\[ d = L \cdot s^2 \]
\[ D = 1/d \]
\[ l = \text{ilaplace}(D) \]

% Plotting relaxation modulus curve:
\[ tt = 10:100:100000; \]
\[ t = \log10(tt); \]
\[ n = 10.^(\text{-625}/4083*\exp(839/32664*t)); \]
\[ \text{figure, loglog(tt, n.*1e6, 'b', tt, n.*1e6, 'P')} \]
\[ \text{ylim([1.38e5 1.05e7])} \]
\[ \text{xlabel('Reduced Time (Sec)')} \]
\[ \text{ylabel('Relaxation modulus (Psi)')} \]
\[ \text{legend('Tref = -20C')} \]

%% Calculating of thermal crack depth with the cooling Cycle

\[ M = P(1); \quad \%M=m= \text{Slope of the creep compliance master curve.} \]
\[ N = 0.8 \cdot (1+1/M); \]
\[ \beta = 1; \quad \% \text{Calibration parameter} \]
\[ k = 10000; \]
\[ \log A = \beta \cdot (4.389 - 2.52 \cdot \log10(k \cdot St \cdot N)); \]
\[ Bmix = 3.45e-4; \quad \% Bmix= \text{Coefficient of thermal contraction.} \]
\[ \sigma = Bmix \cdot 10000 \cdot 20; \]
\[ C(1) = 0; \]
\[ C(2) = 0.6; \quad \% \text{C0=Current crack length in inch} \]
\[ \text{for } n = 3:9 \]
\[ K(n) = \sigma \cdot (0.45 + 1.99 \cdot C(n-1)^{0.56}); \quad \% K= \text{Stress intensity factor.} \]
\[ C(n) = (10^\log A) \cdot (K(n)^N + C(n-1)) \quad \% C= \text{Change in crack depth.} \]
\[ \text{end} \]
\[ \text{figure, plot(C, t)} \]
\[ x = [0 1 2 3 4 5 6 7 8]; \]
\[ \text{close all} \]
\[ \text{plot(x,C, '--', x,C,'O')} \]
\[ \text{xlabel('Cooling Cycle')} \]
\[ \text{ylabel('Depth (inch)')} \]
Bibliography


