Warhead penetration in concrete protective structures

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Licentiate Thesis
Stockholm, October 2011

TRITA-BKN. Bulletin 109, 2011
ISSN 1103-4270
ISRN KTH/BKN/B--109--SE
Akademisk avhandling som med tillstånd av Kungliga Tekniska högskolan framlägges till offentlig granskning för avläggande av teknologi licentiatexamen i byggnetskap fredagen den 9 december 2011 klockan 13.00 i sal B26, Kungliga Tekniska högskolan, Brinellvägen 23, Stockholm.

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Preface

The work presented in this licentiate thesis was carried out at the Swedish Defence Research Agency (FOI), Division for Defence & Security, Systems and Technology, and the Royal Institute of Technology (KTH), Division of Concrete Structures. The main part of the work was financed by the Swedish Armed Forces through research performed at FOI, with additional financial support from “Trygghetsstiftelsen”, “Fortifikationskårens Forskningsfond” and KTH for theoretical studies and the completion of the licentiate thesis.

First of all, I would like to thank my supervisor associated professor Anders Ansell and assistant supervisor Ph.D. Richard Malm for their support, and also for the opportunity to conduct postgraduate studies at KTH.

I would also like to take the opportunity to express my appreciation for my former colleagues at FOI, Division for Defence & Security, Systems and Technology, for their contributions to the research projects, and also for their earlier contribution to this field of research.

Stockholm, October 2011

Håkan Hansson
Abstract

The analysis of penetration of warheads in concrete protective structures is an important part of the study of weapon effects on protective structures. This type of analysis requires that the design load in the form of a warhead is determined, and its characteristic and performance within a protective structure is known. Constitutive equations for concrete subjected to weapon effects have been a major area of interest for a long time, and several material models for concrete behaviour are developed. However, it is not until recent years that it has been possible to use finite element (FE) analyses to simulate the behaviour of concrete targets during projectile penetration with acceptable results. The reason for this is a combination of several factors, e.g. development of suitable material models for concrete, enhancement of numerical methodology and affordable high capacity computer systems. Furthermore, warhead penetration has primarily been of interest for the armed forces and military industry, with a large part of the conducted research being classified during considerable time. The theoretical bases for concrete material behaviour and modelling with respect to FE analyses of projectile penetration are treated in the thesis.

The development of weapons and fortifications are briefly discussed in the thesis. Warheads may be delivered onto a protective structure by several means, e.g. artillery, missiles or aerial bombing, and two typical warhead types were used within the study. These warhead types were artillery shells and unitary penetration bombs for the use against hardened targets, with penetration data for the later warhead type almost non-existing in the literature. The penetration of warheads in concrete protective structures was therefore studied through a combination of experimental work, empirical penetration modelling and FE analyses to enhance the understanding of the penetration phenomenon. The experimental data was used for evaluation of empirical equations for concrete penetration and FE analyses of concrete penetration, and the use of these methods to predict warhead penetration in protective structures are discussed within the thesis.

The use of high performance concrete increased the penetration resistance of concrete targets, and the formation of front and back face craters were prevented with the use of heavily reinforced normal strength concrete (NSC) for the targets. In addition, the penetration depths were reduced in the heavily reinforced NSC. The evaluated existing empirical penetration models did not predict the behaviour of the model scaled hardened buried target penetrators in concrete structures with acceptable accuracy. One of the empirical penetration models was modified to better describe the performance of these penetrators in concrete protective structures. The FE analyses of NSC gave
reasonable results for all simulation cases, with the best results obtained for normal impact conditions of the penetrators.

**Keywords:** Warhead penetration, FE analysis, experiment, material modelling, fortifications.
Sammanfattning

Analyser avseende stridsdelars penetration i skyddskonstruktioner av betong viktigt för studier av vapenverkan mot skyddskonstruktioner. Dessa analyser förutsätter att dimensionerande last i form av stridsdel bestäms, samt att dess karakteristik och verkan mot skyddskonstruktioner är kända. Konstitutiva modeller för betong utsatta för vapenverkan har varit av stort intresse under en lång tid och ett flertal materialmodeller har utvecklats. Det är emellertid först på senare år som det varit möjligt att använda finita element (FE) analyser för att simulera beteendet för betongmål vid projektilpenetration med acceptabla resultat. Anledningen till detta kan tillskrivas kombinationen av ett flertal faktorer, t ex utvecklingen av lämpliga materialmodeller, förbättringar av numerisk metodik och utvecklingen av kostnadseffektiva beräkningsdatorer. Penetration av stridsdelar har dessutom i huvudsak varit av intresse för militären och försvarsindustrin, vilket har resulterat i att en stor del av den bedrivna forskningen har varit hemligstämplad under lång tid. Grunderna avseende betongs materialbeteende och beskrivning av detta med avseende på FE-analyser av projektilpenetration behandlas i denna licentiatuppsats.

Den fortifikatoriska utvecklingen och utvecklingen av vapen diskuteras kortfattad i uppsatsen. Ett flertal olika typer av stridsdelar är av intresse avseende verkan mot skyddskonstruktioner, t ex artillerigranater, missiler eller flygbomber. I denna studie beaktades två typiska stridsdelar, artillerigranater och penetrerande bomber. De senare är specifikt konstruerade för användande mot skyddskonstruktioner och företrädesvis mot betongkonstruktioner. Det visade sig dessutom att data avseende penetration i betong för denna typ av penetrerande stridsdelar i stort sett inte var publicerade. Penetration av stridsdelar i betong studerades därför med en kombination av experimentella metoder, empiriska penetrationsmodeller och FE-analys för att öka förståelsen för problemställningen. De experimentella modellresultaten användes för att utvärdera både de empiriska penetrationsmodellerna och FE-analyserna avseende betongpenetration, med båda metodernas användande diskuterat i uppsatsen.

Användandet av högpresterande betong ökade penetrationsmotståndet för betongmålen i jämförelse med standardbetongmålen. Det var även möjligt att förhindra kraterbildningen på fram- och baksidan av de kraftigt armerande standardbetongmålen, detta medförde även en reducerad penetration för projektilerna i målen. De existerande empiriska penetrationsmodellerna kunde inte förutsäga penetrationen av modellprojektilerna i betongmålen med godtagbara resultat. Istället vidareutvecklades en av dessa modeller för att bättre beskriva denna typ av penetrerande stridsdelar i skyddskonstruktioner av betong. Finita elementanalyserna av standardbetongmålen visade sig ge
ett rimligt beteende för alla analyserade modeller, med de bästa resultaten erhållna för vinkelrätt anslag för de modellprojektilerna av de penetrerande stridsdelarna.

Nyckelord: Stridsdels penetration, FE analyser, experiment, material modellering, fortifikationer.
List of publications

This thesis is based on work contained in the following articles and peer-reviewed conference contributions.


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Notation

Roman letters

$A$ Yield stress at zero plastic strain for the J&C model

$A_i$ Parameter for the polynomial EOS (compression)

$A_2$ Parameter for the polynomial EOS (compression)

$A_4$ Parameter for the polynomial EOS (compression)

$A_{fail}$ Pressure independent parameter for the RHT failure strength surface, this parameter not used in AD

$B$ Hardening constant for the J&C model

$B_0$ Parameter for the polynomial EOS (energy dependency)

$B_1$ Parameter for the polynomial EOS (energy dependency)

$B_{fric}$ Linear parameter for the RHT residual strength surface, this parameter is defined as $B$ in AD

$B_{fail}$ Linear parameter for the RHT failure strength surface, this parameter is defined as $A$ in AD

$BQ$ Brittle to ductile transition parameter

$c_g$ Bulk sound velocity, with $c_g = \sqrt{K/\rho}$

$c_{B,Matrix}$ Bulk sound velocity for the matrix material

$c_{B,Porous}$ Bulk sound velocity for the initial porous state

$c_0$ Bulk sound velocity at zero pressure

$c_s$ Shear sound velocity, with $c_s = \sqrt{G/\rho}$

$C$ Strain rate constant for the J&C model

$C_p$ Specific heat at constant pressure

$C_v$ Specific heat at constant volume

$CRH$ Calibre to radius head

$d$ Projectile diameter

$D$ Accumulated damage

$D_{RHT1}$ Damage parameter $D_1$ for the RHT model

$D_{RHT2}$ Damage parameter $D_2$ for the RHT model

$e$ Internal energy

$e_{0K}$ Internal energy for the 0 K reference state
\( e_{il} \) Internal energy for a Hugoniot reference state
\( E \) Young’s modulus
\( f_c \) Compressive uniaxial strength, static value
\( \hat{f}_c \) Compressive uniaxial strength given in the unit MPa
\( f_{co} \) Compressive uniaxial strength, reference value for strain rate dependence
\( f_{od} \) Compressive uniaxial strength, dynamic value
\( f_{c,el} \) Compressive uniaxial strength, static elastic value
\( f_s \) Shear strength, static value
\( f_t \) Tensile uniaxial strength, static value
\( f_{td} \) Tensile uniaxial strength, dynamic value
\( f_{t,el} \) Tensile uniaxial strength, elastic value
\( f_u \) Ultimate strength for steel, static value
\( f_y \) Yield strength for steel, static value
\( G \) Shear modulus
\( G_{el} \) Shear modulus, elastic
\( G_{pl} \) Shear modulus for strain hardening
\( G_F \) Fracture energy
\( H \) Length of the penetrators nose section
\( HTL' \) Defined according to Eq. (4.32)
\( HTL'^* \) Normalised HTL’, equal to \( HTL'/f_c \)
\( I' \) Impact factor for penetrator
\( k \) Dimensionless crater depth
\( K \) Bulk modulus
\( K_{NDRC} \) Penetrability factor for the NDRC model
\( K_{Matrix} \) Bulk modulus for matrix material
\( l \) Target thickness
\( m \) Thermal softening exponent for the J&C model
\( M \) Mass
\( n \) Hardening exponent for the J&C model
\( n_{P-\alpha} \) Compaction exponent for the P-\( \alpha \) EOS
\( N^* \) Nose factor for penetrator
\( N_{fail} \) Exponent for RHT failure strength surface, this parameter is defined as \( N \) in AD
\( N_{fric} \) Exponent for residual strength surface, this parameter is defined as \( M \) in AD
\( N_{NDRC} \) Nose shape factor for NDRC model
\( N_{Young} \) Nose performance coefficient for the Sandia models
\( p \)  Pressure
\( p^* \)  Normalised pressure, \( p / f_c \)
\( p_{0K} \)  Pressure for 0 K reference state
\( p_{\text{crush}} \)  Initial compaction pressure
\( p_H \)  Pressure for a Hugoniot reference state
\( p_{\text{look}} \)  Solid compaction pressure
\( p_{\text{Matrix}} \)  Pressure in the matrix without pores
\( p_{\text{Porous}} \)  Pressure in the porous material
\( p_{\text{spall}} \)  Spall strength
\( p_{\text{*spall}}^* \)  Normalised spall strength
\( Q_1 \)  Shear to compressive meridian ratio
\( Q_2 \)  Tensile to compressive meridian ratio
\( Q_{2,0} \)  Tensile to compressive meridian ratio, reference value
\( R_{\text{Steel}} \)  Volumetric content of reinforcement steel
\( s \)  Linear shock wave velocity parameter
\( S \)  Empirical target resistance function
\( S_{\text{HBTP}} \)  Modified empirical target resistance function for HBTPs
\( S_{\text{Young}} \)  The Young S-number
\( \text{ShratD} \)  Residual shear modulus fraction
\( \text{SoC} \)  Normalised shear strength, \( f_s / f_c \)
\( T \)  Temperature
\( T_1 \)  Parameter for polynomial EOS (expansion)
\( T_2 \)  Parameter for polynomial EOS (expansion)
\( t_c \)  Cure time for concrete in years
\( T_c \)  Target thickness in projectile diameters
\( T_m \)  Melting temperature
\( T_{\text{Ref}} \)  Reference temperature
\( \text{ToC} \)  Normalised tensile strength, \( f_t / f_c \)
\( U_p \)  Particle velocity
\( U_{pH} \)  Particle velocity for a Hugoniot reference state
\( U_s \)  Shock velocity
\( U_{sH} \)  Shock velocity for a Hugoniot reference state
\( v \)  Volume
\( V \)  Specific volume, \( 1 / \rho \)
\( V_{\text{Impact}} \)  Impact velocity
Exit velocity $V_{Exit}$
Fracture width $w$
Target width $W_c$
Penetration depth $X$

**Greek letters**

- $\alpha$: Compressive strength strain rate exponent for the RHT model
- $\alpha_{P,\alpha}$: Scale parameter for the $P-\alpha$ EOS
- $\alpha_{Porous,0}$: Value of scale parameter $\alpha_{P,\alpha}$ at initial porous state
- $\alpha_{s,CEB}$: Compressive strength strain rate parameter for the CEB-FIP Model Code 90
- $\alpha_{steel, fy}$: Yield strength strain rate exponent for steel (Malvar and Crawford, 1998)
- $\alpha_{steel, fu}$: Ultimate strength strain rate exponent for steel (Malvar and Crawford, 1998)
- $\delta$: Tensile strength strain rate exponent for RHT model
- $\delta_{s,CEB}$: Tensile strength strain rate parameter for the CEB-FIP Model Code 90
- $\delta_{s,MR}$: Tensile strength strain rate parameter for concrete (Malvar and Ross, 1998)
- $\varepsilon$: Strain
- $\varepsilon_{pl}$: Plastic strain
- $\varepsilon_{failure pl}$: Plastic failure strain
- $\varepsilon_{failure pl,min}$: Minimum plastic failure strain
- $\dot{\varepsilon}$: Strain rate
- $\dot{\varepsilon}_0$: Strain rate, reference value
- $\dot{\varepsilon}_{cr,CEB}$: Strain rate, compressive reference value for the CEB-FIP Model Code 90
- $\dot{\varepsilon}_{st,CEB}$: Strain rate, tensile reference value for the CEB-FIP Model Code 90
- $\dot{\varepsilon}_{st,MR}$: Strain rate, tensile reference value for Malvar and Ross (1998)
- $\dot{\varepsilon}_{pl}$: Plastic strain rate
- $\dot{\varepsilon}_{pl,0}$: Plastic strain rate, reference value
- $\Gamma$: Grüneisen gamma
- $\Gamma_0$: Grüneisen gamma at reference density
- $\mu$: Compaction, $\rho/\rho_0 - 1$
- $\theta$: Lode angle
- $\rho$: Density
- $\rho_0$: Reference density
- $\rho_c$: Concrete density
- $\rho_{Matrix}$: Density for matrix material
- $\rho_{Matrix,0}$: Initial density for matrix material
$\rho_{\text{Porous,0}}$ Initial density for porous material

$\sigma$ Stress

$\sigma_1$ Maximum principal stress

$\sigma_2$ Middle principal stress

$\sigma_3$ Minimum principal stress

$\sigma_{\text{eff}}$ Effective stress

$\sigma_{\text{yield}}$ Yield stress

$\psi$ Calibre to radius head, $CRH$

**Abbreviations**

2D Two dimensional

3D Three dimensional

ACE Army Corps of Engineers

AISI American Iron and Steel Institute

Al Aluminium

ALE Arbitrary Lagrange Euler

AD Autodyn

AP Armour piercing

BLU Bomb live unit

BHN Brinell hardness number

B-W Bao and Wierzbicki

Comp. B Composition B, a mixture of 40% TNT and 60% RDX

CP Concrete piercing

EMI Ernst-Mach-Institut

EOS Equation of state

GBU Guided bomb unit

GREAC Gauged reactive confinement

HBTP Hardened buried target penetrator

HE High explosive

HF High fragmentation

HHA High hardness armour steel

HPC High performance concrete

HRC Hardness Rockwell C

HV Hardness Vickers

J&C Johnson and Cook
<table>
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<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tr>
<td>MS</td>
<td>Max shear stress</td>
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<tr>
<td>NDRC</td>
<td>National Defense Research Committee</td>
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<td>NSC</td>
<td>Normal strength concrete</td>
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<tr>
<td>PBXN</td>
<td>Plastic bounded explosive</td>
</tr>
<tr>
<td>RDX</td>
<td>Hexogen (high explosive)</td>
</tr>
<tr>
<td>RHA</td>
<td>Rolled homogenous armour (steel)</td>
</tr>
<tr>
<td>SPH</td>
<td>Smoothed particle hydrodynamics</td>
</tr>
<tr>
<td>TNT</td>
<td>Trinitrotoluene (high explosive)</td>
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Chapter 1

Introduction

The analysis of dynamic events is an important part of studies of weapon effects on protective concrete structures. Two major areas of interest are the penetration of a warhead into a structure and the effects due to detonation of the high explosive within the warhead, with the penetration of warheads considered within this thesis.

Two different types of warhead were used within the study, e.g. artillery shells and hardened buried target penetrators. These warhead types relating to different scenarios. Military camps or field fortifications used for international out of area operations may be subjected to artillery fire, e.g. artillery shells, mortar bombs or artillery rocket. The vulnerability of concrete structures to these warheads was therefore of interest for analyses, with penetration of artillery shells in high performance concrete targets selected for one part of the study. A world-wide interest regarding warheads designed to destroy hardened buried targets exist since the nineties, with the use of penetrating unitary warheads against Iraq’s underground concrete bunkers as an example. New assessments of existing underground concrete protective structures impacted by these improved penetrating warheads were therefore needed. Furthermore, strengthening of existing protective structures and improved designs for future protective structures may also be required for these structures to withstand weapon attacks from improved penetrating warheads. A methodology for the assessment of the vulnerability of concrete protective structures is therefore needed. The penetration in concrete protective structures for hardened buried target penetrators (HBTP) was therefore selected for the main part of this study. The investigation of penetration in concrete protective structures included extensive model scaled penetration tests, the use of empirical penetration models and finite element (FE) analysis. These are complementary methods to aid in the assessment of the penetration performance of warheads.
Chapter 1. Introduction

1.1. Background

The interest of the penetration performance of projectiles and the damage to fortified structures have been of great interest since the construction of catapults in ancient time, through the use of black powder smoothbore cannons, to the current use of modern artillery and penetrating warheads. However, the basic problem is still how to determine if a structure is strong enough to withstand an attack, or if the used warhead is sufficient to destroy a target. Different semi-empirical or empirical equations to analyse projectile penetration in geological material were early of interest for military engineers, as an aid for the design of protective structures.

The invention of modern concrete in the 19th century, and later the introduction of reinforced concrete, resulted in a widespread use of concrete for protective structures. The designs and use of concrete protective structures have changed with the changes of weapon designs and military strategy, but assessing the vulnerability of concrete protective structures to weapon effects and other extreme loading conditions is still subjected to extensive research. Two main areas of interest has been identified, these were the vulnerability of hardened underground (buried) concrete structures to impact of modern types of penetrating warheads and shelters impacted by artillery shells.

Empirical penetration equations were early adopted for assessment of penetration depths in concrete (Corbett et al., 1996), and new empirical or semi-empirical penetration models are still developed. The development of the non-linear finite element (FE) analysis and the introduction of advanced models for the description of materials subjected to extreme loading conditions, e.g. high pressures and loading rate, resulted in a new tool for the analysis of weapon effects on protective structures. However, both the numerical methodology and the used material models need to be thoroughly verified to produce reliable results.

1.2. Aims and scope

The main objective of the presented research is to evaluate the use FE analysis for projectile penetration in concrete protective structures. This study considers unitary warheads, with artillery shells and modern types of penetrating bombs used to defeat hardened buried concrete structures chosen as typical warheads for the study. One objective for the study was to determine the penetration performances of hardened buried target penetrators (HBTP) in different types of concrete targets. This was investigated with the use of model scaled penetration experiments, and these experimental data was used for the evaluation of both empirical penetration models and comparisons with results from FE analyses. One important objective for the study was to identify research areas in need of improvements to further enhance the methodology for FE analysis of penetration in concrete protective structures.
1.3. Limitations

The global structural response of protective structures impacted by warheads, or the effects due to detonation of high explosive warheads, are not considered within the thesis. Furthermore, the penetration of fragments from detonating cased charges, the penetration of shaped charges and the impact of soft missiles (e.g. jet fighters or jet airliners) on concrete structures are not considered within the study.

1.4. Outline of report

The contents of the chapters and appendices are presented below to give an overview of the structure of this thesis.

The historical development of weapons and fortifications are briefly discussed in Chapter 2, with warhead characteristics for artillery and hardened buried target penetrators given in Chapter 3. This part of the thesis was not covered within the appended papers, and it is presented here to give a historical background and perspective for the conducted research.

The static and dynamic behaviour of concrete is briefly described in Chapter 4, with the behaviour of high strength concrete also discussed. The main part of this chapter relates to modelling of concrete with respect to FE analyses of concrete penetration. A thorough description of the used material model for concrete are given in sections 4.2 and 4.3. This part of the thesis is an extension of the brief descriptions presented within Papers I-III and Paper V.

The main contribution to the field of research is presented in Chapters 5 and 6, with studies of warhead penetration in concrete targets. Model scaled penetration tests based on the design of modern types of HBTPs are presented in Chapter 5. Parts of this experimental data were then used for studies of empirical penetration models in Paper IV, with an increased number of empirical models presented in section 6.1. Furthermore, a part of the experimental data obtained for the NSC targets was also used as a base for the presented FE analyses in Paper V and section 6.2. It should be noted that Chapter 5 presents both the experimental data used for Paper IV and V, and additional experimental data not used within these studies. Furthermore, section 5.1 presents additional information regarding both the test set-up and the penetration test evaluation.

Summary and discussions of the experimental program, empirical penetration model approach and the FE analyses are presented in Chapter 7. Conclusions and recommendations for further research regarding material modelling, numerical methodology and penetration studies are finally given in Chapter 8.
Constitutive models for strength modelling of typical metal alloys used for both projectiles and protective materials are given in Appendix A. These material models were used for the FE analyses for modelling of projectiles, reinforcement bars and steel confinement of unreinforced concrete targets.

The penetration of a modified artillery shell in high performance concrete was also studied with FE analysis. The aim for this part of the study was to identify the basic limitations for different target formulations regarding the modelling technique. Furthermore, the limitations of 2D FE analyses compared to 3D simulations were also investigated. These FE analyses were based on benchmark tests presented by Svinsås et al. (2001) and are presented in Appendix B, with additional data given within Papers I and II. The results from the FE analyses were used as an aid in the design of the experimental program. Furthermore, the chosen numerical methodology for the FE analyses of the HBTPs was based on these initial simulations.

1.5. Enclosed papers

**Paper I:**

Penetration of 152 mm artillery shells in high performance concrete (HPC) were studied with 2D rotational symmetry FE analyses in this conference paper. The main objective for this paper was the evaluation of different numerical formulations for the representation of the cylindrical unreinforced target, with Lagrangian, Eulerian and smoothed particle hydrodynamics (SPH) target formulations used. The RHT concrete model was used for all FE analyses, with the compaction of the HPC determined from experimental data.

**Paper II:**

This conference paper is a continuation of Paper I, with 3D FE analyses used for evaluation of Lagrangian and SPH formulations for targets impacted by 152 mm artillery shells. The uses of reinforcement in the HPC targets were also numerically studied within this paper, with the reinforcement bars modelled with beam elements. Furthermore, the influences of oblique impacts and induced yaw angles for the projectiles on the FE analyses results were also investigated.
Paper III:
Air blast loaded reinforced concrete beams were analysed with 3D FE analyses in Paper III. The RHT material model was with different material parameters the different concrete types, with uniaxial compressive strengths between 50 and 175 MPa. The RHT material model was combined with additional tensile failure models, and the influence of the used tensile failure model for the concrete was investigated. Furthermore, the reinforcement bars were modelled with Lagrangian solid elements, and the interaction of the reinforcement bars with the surrounding concrete was calibrated from pull experiments. The author of the thesis contributed with work regarding the numerical simulations of the beams.

Paper IV:
Penetration of model scaled hardened buried target penetrators (HBTP) in unreinforced normal strength (NSC) and high performance concrete (HPC) targets were studied with semi-empirical penetration models. Furthermore, heavily reinforced NSC targets were also considered in the paper. Experimental data for normal impacts of HBTPs with different nose shapes, mass and impact velocities were used for comparison with the semi-empirical penetration models.

Paper V:
Penetration of model scaled HBTPs in NSC targets were studied with 3D FE analyses in Paper V. The study investigated the influences of normal and oblique impacts of the penetrators on heavily reinforced and unreinforced NSC targets. Furthermore, two impact velocities and different target thickness were also studied with the FE analyses. The RHT material model and the chosen numerical methodology proved useful for simulations of both the unreinforced and reinforced targets, with reasonable agreement between simulation results and experimental data.
Chapter 2

Weapon and fortification developments

There has been an on-going battle between the best weapon and fortification designs since the ability to launch projectiles against an opponent first appeared. The designs have continuously evolved, with revolutions in warfare occurring at frequent times in history, with dramatic changes of weapons, protection designs or strategy. These developments have on several occasions made the existing fortification designs obsolete.

2.1. Ancient time to mid-19th century

The catapult was used as a siege weapon for almost two thousand years, from ancient time to the use of introduction of black powder cannons in the 14th century. The introduction of siege cannons made the existing fortifications vulnerable to projectile impacts due to the increased impact velocity and also extended shooting range, with a requirement of new fortification designs. Stone shots were used during the first epoch of ordnance development from year 1313 to 1520, with iron round shots used for the second epoch during 1520-1854 (Johnson, 1991). The iron round ball increased the penetration performance compared to the stone shot, with the later prone to shattering on impact. Typical fortifications for coastal defence from this period used low walled structures with earthwork covering of brick or wood structures. This was considered to present low exposure and cushion the effect from an impacting solid ball. Earthwork used for protection of a coastal defence position is shown in Figure 2.1. It was common that the walls were laid out at angles to each other to allow for defenders to fire on the bases of adjacent walls, forming a typical star shaped fort. An example is Nässkansen shown in Figures 2.1 and 2.2. A weakness of the low walled forts was that a solid shot passing parallel with a wall might destroy several guns in a row, and the gun crews were also vulnerable to shrapnel from exploding shells.
The development of casemates to protect the guns and crew followed, with use of thick masonry walls to withstand the pounding of gun fire. This design also allowed stacked rows of guns in high walls to increase the firepower. A fortification from this era is the Vaxholm castle placed at one of the shipping routes into Stockholm. This coastal artillery fort was modernised in the mid of the 18th century and is shown in Figure 2.3.

![Figure 2.1. The south wall of the Näskansen earth work fortification.](image1)

![Figure 2.2. The star shaped low walled earth work fortification Näskansen was used in the 17th and 18th centuries to block access to Södertälje at the strait Skanssundet.](image2)
2.2. Mid-19\textsuperscript{th} century to modern time

A new leap in gun designs was the introduction of rifled artillery and breach loading in the mid-19\textsuperscript{th} century; this was the start of the third epoch (Johnson, 1991). This allowed for the use of gyroscopic stabilised projectiles with a length larger than its diameter, resulting in increased accuracy and penetration performance. The masonry structures were crushed and penetrated by these new projectiles, e.g. with the modernised Vaxholm castle obsolete a few years after it was finished, and new fortification designs were needed.

Smokeless gun propellants and high explosives for detonating shells developed in the second half of the 19\textsuperscript{th} century increased the fire power for the artillery, and allowed for the use of air burst of fragmenting shells to defeat unprotected personal, e.g. gun crews behind parapets. The need for shielding of gun positions and the development of casemates followed, e.g. for coastal and land based artillery forts. Modern concrete and also iron concrete, i.e. reinforced concrete, were introduced at the same time, resulting in a widespread use of concrete for construction of military installations. Concrete was used for the construction of land based artillery forts at the end of the 19\textsuperscript{th} century and beginning of the 20\textsuperscript{th} century, e.g. the Belgian Brialmont forts constructed between 1859 and 1890 and the French Maginot line at the French-German border constructed in the thirties. The use of howitzers with calibre up to 42 cm showed effective against the unreinforced concrete Brialmont forts in the beginning of World War I. It is worth mentioning that the 42 cm German howitzer used 1 ton shells to destroy Belgian concrete forts (Johnson, 1991). In World War II, the German army initially bypassed the Maginot line by cutting through the neutral states Belgium and the Netherlands, and the Ardenes forest north of the main defence line. The Maginot forts were constructed of reinforced concrete, with an increased resistance to artillery shelling compared to the Belgian Brialmont forts. However, the German forces later broke through the fortification line and also isolated the remaining parts of the fortification line from the rest of
France. Furthermore, the use of aeroplanes and later aerial bombing during World War I introduced a new threat against protective structures. This development of aeroplanes and bombs made the large calibre howitzers used in World War I obsolete in the twenties. The major development of planes, e.g. fighters and bombers, and bombs, in the thirties and during World War II (WW II) resulted in a demand for improved protective structures, with thick reinforced concrete structures used to protect military installations, e.g. coastal artillery positions and submarine docks. An example is the Atlantic Wall ("Der Atlantikwall") built during WW II by Germany along the occupied west coast of the Atlantic (Heber, 2003). The individual fortifications where constructed with the use of standardised designs, with examples shown in Figure 2.4. The most widely used thickness for the external walls and ceilings of bunkers and gun casemates for example in Normandy were 2.0 m, which falls into the category B grade used by the Germans (Zaloga, 2005).

![Figure 2.4. Examples of cross sections of German WW II gun casemates, with figure (a) showing a position for field artillery and figure (b) for a permanent gun position. The fortifications are category B designs with 2.0 m thick concrete outer walls and ceilings.](image)

The category A was the strongest grade for standard military fortifications, and was used for submarine docks, some heavy gun casemates and some radar bunkers. This category used a concrete thickness equal to 3.5 m for the protective structure. Furthermore, the German WW II category E fortifications had 5.0 m thick exterior walls and ceilings, and was reserved for the Führer bunkers and special facilities, e.g. V-weapons launch bunkers (Zaloga, 2005). Specialised penetrating bombs were developed to destroy these types of thick reinforced concrete structures in during WW II, with further improved penetrating 25 000 lb (11.4×10³ kg) bombs developed after the war (Bentz, 1949). The development of large penetrating bombs for the use against hardened structures ended in the fifties, with the focus for weapon development and protective research shifting towards nuclear weapons and their effects. Furthermore, at the same time the use of coastal artillery fortifications was considered obsolete by most countries, with the use of marine and air forces used for coastal defence instead. Two of the exceptions were Norway and Sweden that continued to develop artillery for coastal defence, with decommission of the coastal artillery in Sweden during the nineties. A late design for a Swedish coastal artillery fortification is shown in
2.2. Mid-19th century to modern time

Figure 2.5, i.e. the ERSTA 12/70 120 mm coastal artillery gun. This system was designed to be placed in hard rock, which provided the main protective structure for installation. However, reinforced concrete and armour steel were also used for parts of the protective structure. Furthermore, the system was designed against the effects of nuclear warheads, except obviously for the case of a close in nuclear blast. However, the development of precision guided conventional warheads provided a new threat to these installations, and Sweden has phased out the system.

![An ERSTA 12/70 coastal artillery 120 mm gun located at Öja. Photo courtesy of Jörgen Carlsson.](image)

Today reinforced concrete structures are still used for underground installations and other types of protective structures, e.g. aeroplane hangars. Furthermore, protection against field artillery warheads, e.g. from mortars, howitzers and rocket artillery, is a threat that needs to be considered for almost all military operations. An example of a hardened protective structure is shown in Figure 2.6, with the use of rock boulders to give enhanced penetration protection and special reinforcement to prevent back face spalling of the concrete. This structure type has been used for protective structures near the ground surface in hard rock. Furthermore, a facility with a considerable cover of hard rock will have good protection against most available warheads types. This requires that vulnerable parts of the facility, e.g. entrances, are well designed and protected.
In areas without hard rock bedrock close to the surface it is necessary to build traditional types of underground bunkers. However, the protection level for this type of facilities may be increased by including burster slabs to prevent penetration into the protected area, and this also increases the distance from the detonation point of a warhead to the main part of the facility. Furthermore, a shock isolation layer may be used in combination with the burster slab to mitigate the ground shock from the blast. These two types of protective structures with burster slabs are shown in Figure 2.7.

The use of layered structures has the ability to increases the protection of any protective structure compared to a single reinforced slab or wall. However, it is necessary to identify design load for the structure, e.g. the warhead type and its properties, to optimise a layered structure. The advantages of layered protective structures were discussed by Eytan (1985) for four types of warhead attacks, with the design types shown in Figure 2.8 and the optimal layered structure configurations given in Table 2.1. However, the more severe warhead threats were not considered, e.g. direct hits of an air delivered bomb or an artillery shell with delayed fuse with the ability to perforate reinforced concrete. Furthermore, a more comprehensive study is needed for each considered combination of warhead type and layered protective structures, with this applying to both above ground and underground structures. The characteristics of artillery warheads and hardened buried target penetrators are discussed in Chapter 3.
2.2. Mid-19th century to modern time

Figure 2.7. Cross sections of earth covered protective structures with burster slabs of reinforced concrete. Structure in figure (a) is without shock isolation layer, with a shock protection layer added to the structure shown in figure (b).

Figure 2.8. Design types for layered protective structures (after Eytan, 1985).
Table 2.1  Optimal layered protective structures for different types of attacks (Eytan, 1985).

<table>
<thead>
<tr>
<th>Structure location</th>
<th>Type of attack</th>
<th>Layered structure types, see Figure 2.8.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Type 1</td>
</tr>
<tr>
<td>Underground structure</td>
<td>Near miss of air bomb</td>
<td>x</td>
</tr>
<tr>
<td>Above ground structure</td>
<td>Direct hit of an artillery shell</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>Near miss of air bomb</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>Direct hit of shaped charge projectile</td>
<td>x</td>
</tr>
</tbody>
</table>

Furthermore, today’s case of out of area missions, with deployment of personal in for example Afghanistan, it is crucial to obtain a protective structure shortly after deployment. This sets constraints on the design methods that can be applied for the protective structures, e.g. the time required construction time may be crucial. However, it is likely that the most severe threats, e.g. air delivered bombs or concrete piercing artillery warheads, does not need to be considered. Furthermore, protective structures that use local material or easily transported material have an advantage over specialised protection solution. Open top gabions filled with geological material provide can provide both protection against fragments and surface burst of artillery warheads, with design examples shown in Figures 2.9 and 2.10a. Furthermore, standard transportation containers may be easily available and can be used as bases for protective structures, with the use of complementary material such as burster slabs and inserted protective wall. Figure 2.10 show protective designs based on these types of containers.

![Field fortifications under construction, with geological material filled open top gabions used for protection. Photo courtesy Anders Carlberg/FOL.](image-url)
The terrorist threat to non-military targets has raised a need to consider weapon effects also for high risk civilian buildings and installations. However, it is likely that the main types of threats that need to be considered in this case are air blast and impact of fragment, and not penetrating warheads. One exception from this is protection against shaped charges, which is outside this study.
Chapter 3

Warhead characteristics

Two types of warheads were selected for the present study, i.e. artillery shells and unitary penetration bombs, and the characteristics of different warheads within these types are discussed here. This overview is not complete, but should give the reader a basic knowledge of typical threats that might need to be considered for different types of protective structures.

3.1. Artillery warheads

Artillery weapons types are normally divided into two main categories, i.e. rocket launchers and artillery with barrels. The later type includes cannons, howitzers and mortars. Generally cannons use longer barrels than howitzers, and thereby higher projectile velocities and extended ranges can be obtained. It should be mentioned that the length of the barrel in this discussion relates to a relative length when compared to the diameter of the bore (calibre), with the length of the barrel often given in calibres.

Artillery shells for howitzers exist in various calibres (Gander and Cutshaw, 2001), e.g. 75/76, 105, 122, 130, 152/155, 180 and 203 mm. However, the number of field artillery guns or howitzers with a calibre larger than 155 mm can be considered very small. The earlier use of large bore howitzers has been replaced on the battlefield by the use of multiple rocket launchers, large calibre mortars or air delivered weapons. The early generations of artillery warheads, e.g. the Swedish 155 mm m/54 and the US designated M107, use relative ductile steels for the body of the shell. The later have been manufactured in a number of countries and have seen a widespread use in western countries. Modern types of artillery instead use high fragmentation (HF) steels optimised to provide an increased number of effective fragments, e.g. Swedish 155 mm m/77 and m/77B shells. Furthermore, these later projectiles are constructed with relative thin body walls to further enhance the fragmentation and also the velocity of the fragments. The artillery shells m/54 and m/77 are shown in Figure 3.1, and Figure 3.2 shows a 105 mm shell for comparison. Furthermore, the fragmentation of an artillery shell is also influenced by the properties of the used high explosive.
(HE) filling, with TNT or Comp. B (TNT/RDX) abundantly used. The HE may also contain additives of aluminium to enhance the blast effects. The fuse for a HE fragmenting shell is located at the nose of the projectile, and this normally also applies to mortar bombs and artillery rockets.

The desired effects can to some extent be varied by the chosen fuse and also the setting of the fuse, e.g. proximity, time, point detonating (PD) or delay after impact. The use of a proximity fuse allows for detonation of a shell at a predetermined distance from a target to optimise the effects from fragments and the blast wave. Furthermore, the effect from the fragments is decreased if detonation occurs at contact with the target, i.e. a surface burst. The use of a delayed detonation setting at typical values of 50 or 60 ms, allow for penetration of a target before initiation of the detonation.

**Figure 3.1.** High explosive artillery shells, (a) photo of the Swedish 155 mm m/54 shown without fuse (Hansson, 2006), (b) cross section of the Swedish m/77 shown without the hollow boat tail skirt and fuse, cross sections of the Russian 152 mm OF-540 HE shell (c) and concrete piercing shell G-530 (d). Figure (b) is after Andersson and Lithén (1987), and figures (c) and (d) are after Morgan and Pittman (1997).
3.1. Artillery warheads

Figure 3.2. Cross section of the US 105 mm M1 HE shell (after Crull, 1998), with measurements given in inches (1” = 25.4 mm).

The passing through a concrete target may damage the fuse, and result in a failure to initiate a warhead. One response to this was the development of specialised fuses designed of high strength steel. The uses of these fuses enhance the performance of a HE fragmenting artillery shells against concrete structures. Furthermore, concrete piercing HE shells with the placement of the fuse in the base of the body are also available. The designs of two Russian 152 mm shell bodies are also shown in Figure 3.1, showing the difference between the HE fragmenting OF-540 projectile and the concrete piercing (CP) G-530 projectile. Data for a few selected artillery shells are given in Table 3.1.

Mortar bombs have a similar characteristic as shells for field artillery, and also use similar fuses. However, the structural strengths and impact velocities for mortar bombs are lower than for artillery shells. The later type of warheads needs to withstand considerable forces during the acceleration of the shell within the howitzer or gun barrel, with the forces acting on a mortar shell during launch is considerably lower. This allows for the use of more brittle material for the casing of mortar shells, e.g. cast iron. Mortar shells are normally not designed to penetrate hardened concrete structures, and have a relative limited penetration performance in concrete. However, large mortar bombs may cause considerable damage due to contact detonation at impact. Mortar bombs varies in size from small 50 to 81/82 mm calibre bombs, through midsized bombs with a calibre of 120 to 160 mm, to large calibre warheads with a calibre of up to 240 mm (Gander and Cutshaw, 2001). Cross sections of two Russian mortar bombs are shown in Figures 3.3 and 3.4, with data for selected mortar bombs given in Table 3.1.
Warheads for multiple rocket launchers are available in several sizes, from small 70 mm rockets, through midsize 122 and 130 mm rockets, and up to large 273 and 333 mm rockets (Gander and Cutshaw, 2001). Furthermore, artillery rockets may be launched with improvised launchers, e.g. mounted on small trucks or assembled at a launch site. Rocket artillery warheads are more fragile than artillery shells, with a relative thin casing and in some cases relative slender warheads. The desired weapon effect is normally a combination of air blast and fragment impacts, with the use of proximity fuses or PD fuses with short delays. However, artillery rockets with diameters 122 mm and larger, may penetrate a considerable thickness of concrete if equipped with long delay fuses. Data for selected artillery rockets are given in Table 3.1.

Accurate penetration studies of mortar shells and rocket artillery warheads impacting hardened concrete structures require that the deformations and breakup of the warheads can be analysed with reliable results. Furthermore, the penetration performances of these warheads are normally of secondary interest and these warhead types are therefore not considered within the study. However, with improved material modelling regarding the strength and fracturing of ductile material discussed in the appendix it may be possible to study also these types of warheads. This also applies to artillery shells impacting protective structures or vehicles constructed of materials with higher strength, e.g. aluminium alloys, steel or ceramics. The requirements for modelling of fracturing projectiles are briefly discussed later.
Table 3.1. Data for selected high explosive artillery warheads compiled from Gander and Cutshaw (2001) and Morgan and Pittman (1997).

<table>
<thead>
<tr>
<th>Warhead</th>
<th>Mass</th>
<th>Alternative high explosive filling</th>
<th>Body length</th>
<th>Total length</th>
<th>Maximum velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Artillery shells</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>105 mm M1</td>
<td>14.97 kg</td>
<td>2.3 kg Comp. B 2.2 kg TNT</td>
<td>0.399 m</td>
<td>0.495 m</td>
<td>494 m/s</td>
</tr>
<tr>
<td>152 mm OF-540</td>
<td>43.51 kg</td>
<td>6.24 kg TNT</td>
<td>0.650 m</td>
<td>0.710 m</td>
<td>655 m/s</td>
</tr>
<tr>
<td>152 mm CP G-530</td>
<td>40 kg</td>
<td>5.10 kg TNT</td>
<td>0.603 m</td>
<td>0.603 m without fuze</td>
<td>---</td>
</tr>
<tr>
<td>152 mm 3OF-25</td>
<td>43.56 kg incl. fuze</td>
<td>6.8 kg RDX/Al/wax</td>
<td>---</td>
<td>0.710 m</td>
<td>655 m/s</td>
</tr>
<tr>
<td>152 mm 3OF-45</td>
<td>43.56 kg incl. fuze</td>
<td>7.7 kg RDX/Al/wax</td>
<td>---</td>
<td>0.864 m</td>
<td>810 m/s</td>
</tr>
<tr>
<td>155 mm Bofors M/77B</td>
<td>41.8 kg incl. fuze</td>
<td>7.9 kg TNT</td>
<td>0.728 m</td>
<td>0.825 m</td>
<td>880 m/s</td>
</tr>
<tr>
<td>155 mm M107</td>
<td>43.88 kg incl. fuze</td>
<td>6.99 kg Comp. B 6.62 kg TNT</td>
<td>0.605 m ----</td>
<td>830 m/s</td>
<td></td>
</tr>
<tr>
<td><strong>Mortar bombs</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120 mm OF-843</td>
<td>16.02 kg incl. fuse</td>
<td>2.68 kg TNT</td>
<td>0.430 m</td>
<td>0.656 m</td>
<td>272 m/s</td>
</tr>
<tr>
<td>160 mm F-853U</td>
<td>41.18 kg incl. fuse</td>
<td>8.99 kg TNT</td>
<td>0.706 m</td>
<td>1.12 m</td>
<td>---</td>
</tr>
<tr>
<td>240 mm F-864</td>
<td>130.84 kg incl. fuse</td>
<td>31.93 kg TNT</td>
<td>0.982 m</td>
<td>1.57 m</td>
<td>---</td>
</tr>
<tr>
<td><strong>Artillery rockets</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>107 mm Norinco Type 63</td>
<td>8.3 kg</td>
<td>1.3 kg TNT</td>
<td>---</td>
<td>0.841 m</td>
<td>375 m/s</td>
</tr>
<tr>
<td>122 mm M-21-OF</td>
<td>18.3 kg incl. fuse</td>
<td>6.4 kg undisclosed HE estimated</td>
<td>0.56 m</td>
<td>Approx. 2.87 m</td>
<td>699 m/s</td>
</tr>
<tr>
<td>130 mm Norinco Type 63</td>
<td>14.7 kg</td>
<td>3.05 kg TNT</td>
<td>---</td>
<td>1.05 m</td>
<td>437 m/s</td>
</tr>
<tr>
<td>220 mm 9M27F FRAG HE</td>
<td>100 kg</td>
<td>51.7 kg undisclosed HE</td>
<td>---</td>
<td>4.83 m</td>
<td>---</td>
</tr>
<tr>
<td>230 mm TOROS 230A</td>
<td>120 kg</td>
<td>---</td>
<td>1.45 m</td>
<td>4.1 m</td>
<td>---</td>
</tr>
<tr>
<td>260 mm TOROS 260A</td>
<td>145 kg</td>
<td>---</td>
<td>1.45 m</td>
<td>4.8 m</td>
<td>---</td>
</tr>
<tr>
<td>273 mm Norinco WM-80</td>
<td>150 kg</td>
<td>34 kg RDX/TNT 40/60</td>
<td>4.58 m</td>
<td>1140 m/s</td>
<td>---</td>
</tr>
</tbody>
</table>
3.2. Hardened buried target penetrators

The requirement for bombs to destroy hardened buried targets was identified during World War II, with the use of thick reinforced concrete structures providing shelter for submarines and other military installations. The perforation capability for the American 1600 lb (730 kg) AN-Mk 1 armour piercing bomb was 2.7 m of concrete with a compressive strength of approximately 35 MPa, if dropped from 30 000 feet (9100 m) (Office of the chief of ordnance, 1945). Other large specialised bombs were for example the English 12 000 lb (5400 kg) Tallboy and 22 000 lb (10 000 kg) Grand slam bombs. The armour piercing bombs manufactured in other countries at that time were likely to have similar performances. The demand for this type of warheads diminished shortly after WW II.

The development of a 2000 lb (900 kg) penetrating bomb was conducted by Lockheed Martin, with initial deliveries of the BLU-109/B warhead in 1985. The 2.4 m long warhead is constructed of 25 mm thick forged gun-barrel steel and was designed as a free-falling bomb, but can be used as a warhead in precision guided bombs such as the GBU-27/B or cruise missiles (Lennox, 2001), and is shown in Figure 3.5. The length to diameter ratio for this warhead is approximately 6.5. This warhead was used extensively during the Gulf War to destroy concrete targets. However, not all the targets could be efficiently attacked by this warhead and there was an urgent demand for an improved penetrating bomb. This resulted in the development of the GBU-28/B guided bomb unit. The bomb casings were manufactured from decommissioned gun barrels with approximately 200 mm calibre, fitted with a hardened nose section and filled with tritonal explosives. The existing guidance systems and tail sections used for the BLU-109/B warhead were modified, with the development completed in 17 days for the initial delivery (Lennox, 2001). The warhead was later designated as the BLU-113, and estimations for the mass and length are 2040 kg and 3.9 m, respectively. The use of this warhead in 1991 created a worldwide interest for both vulnerability modelling of hardened buried protective structures and development of new designs for penetrating warheads. Furthermore, the BLU-113 was replaced by an improved design, i.e. the BLU-122 warhead. A small guided free-falling bomb was also developed with a mass of 250 lb (113 kg), and with the intent to have a similar penetration performance in concrete as the BLU-109/B. The project I-250, or Small Smart Bomb (SSB), started in 1995 and test fires with the penetrator started the same year. This guided bomb was later designated as the Small Diameter Bomb (SDB, GBU-39/B), with an illustration shown in Figure 3.6. The length of the actual warhead is likely to be approximately 1.3 m, and this gives a length to diameter ratio of approximately 8.9 for the penetrator. The increase of length to diameter ratios and improved warhead designs have increased the penetration performance for a given weight of a warhead. Furthermore, similar warheads are already under development, or likely to be developed in the future, by other weapon manufacturers or countries. Data for selected penetrating unitary bombs and warheads are given in Table 3.2.
3.2. Hardened buried target penetrators

Figure 3.5. Illustration of the casing for the penetrating BLU-109/B and BLU-118/B warheads.

Figure 3.6. Illustration of the SDB (GBU-39/B) unitary bomb.

Table 3.2. Data for selected penetrating unitary warheads compiled from Lennox (2001).

<table>
<thead>
<tr>
<th>Penetrating bomb/warhead</th>
<th>Mass (kg)</th>
<th>Alternative HE filling</th>
<th>Diameter (m)</th>
<th>Length (m)</th>
<th>Concrete penetration (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BLU-109/B</td>
<td>874</td>
<td>240 kg TNT/Al</td>
<td>0.370</td>
<td>2.4</td>
<td>1.8 – 2.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>240 kg PBXN-109</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GBU-27/B</td>
<td>984</td>
<td>240 kg TNT/Al</td>
<td>0.370</td>
<td>4.24</td>
<td>1.8 – 2.4</td>
</tr>
<tr>
<td>GBU-28/B</td>
<td>2130</td>
<td>306 kg TNT/Al</td>
<td>0.370</td>
<td>5.84</td>
<td>6</td>
</tr>
<tr>
<td>SDB</td>
<td>113</td>
<td>22.6 kg undisclosed HE</td>
<td>0.152</td>
<td>1.83</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Note: PBXN refer to plastic bounded explosive, with a lower sensitivity than the alternative high explosive tritonal (TNT/Al).

A parallel weapon development is the use of dual charge warheads, normally equipped with a shaped charge warhead to penetrate or damage the concrete and a secondary penetrating warhead delivering the high explosive into the target. Dual charge warheads are likely to obtain the desired penetration depth of more than 6 m of concrete with warheads weighting well below the 2000 kg for the BLU-113 or BLU-122 warheads. However, this type of warheads is not considered within this study.
Chapter 4

Concrete material behaviour

Concrete is a complex composite material with aggregates, varying in size, embedded in a matrix of porous grout. Thus, due to the inherent inhomogeneity it is difficult to describe the mechanical behaviour of concrete. Furthermore, concrete is sensitive to tensile loading and fractures at small deformations, like many other hard and brittle materials. On the other hand, with increasing pressure the strength of concrete also increases, and the flow resistance of crushed concrete under confined compressive states can be significant. The mixture proportions and the used type of aggregate are likely to influence the behaviour of concrete, and easily obtained strength parameters may not be adequate to determine the behaviour of concrete subjected to other loading conditions. Furthermore, it may be very expensive to obtain the necessary experimental data to describe the mechanical behaviour of a specific concrete mixture. Examples of desirable experiments for the used concrete are plate impact tests, triaxial strength tests and dynamic uniaxial tensile test.

For a structural response due to a low velocity impact, or an air-blast loading, it is important to accurately consider the tensile failure of concrete, and also the interaction between reinforcement and concrete may be crucial for the predicted structural response. However, for deep penetration it is likely that the properties for concrete in a confined state, or at least in a partly confined state, are important to consider due to the confinement of the concrete close to the penetrator. Concrete material models may for example include strain rate dependent tensile and compressive strengths, non-linear compaction of the material, pressure dependent failure strength and strength properties of crushed material described by varied accuracy. Furthermore, the residual strength properties of the failed concrete may also be important if more complex loadings need to be considered, e.g. the detonation of a high explosive filled penetrator after penetration into a concrete target.

The static and dynamic uniaxial tensile and compressive failures for concrete are here treated in section 4.1. The main part of the material descriptions relates to the use of the so called P-$\alpha$ equation of state for porous material in section 4.2, and the properties of the RHT material model for concrete in section 4.3. This material is only briefly covered within the appended papers.
Furthermore, the material parameters for the investigated concrete types are also given here, with complimentary parameters given in Papers I-III and Paper V.

### 4.1. Static and dynamic material behaviour

The static and dynamic material behaviour of concrete has been extensively studied through the years, and this brief description provides a limited insight of the complex behaviour at different loading conditions for the material. Furthermore, the requirement for the description of the behaviour varies considerably depending on the type of problem that needs to be analysed. The static tensile failure of concrete was for example studied by Hillerborg (1977), Petterson (1981), Gylltoft (1983), Gopalaratnam and Surendra (1985) and Reinhardt (1985), with the dynamic tensile behaviour for example studied by Weerheijm (1992), Schuler (2004), Brara and Klepaczko (2007), Weerheijm and Van Doormaal (2007), Zhang et al. (2009) and Lu and Li (2011). The uncertainty of the material behaviour increases with increased loading rates, since tensile strength data for strain rates $>300$ s$^{-1}$ is almost non-existing. Furthermore, data for the compressive behaviour of concrete at high strain rates were compiled and evaluated by Bischoff and Perry (1991). The relative dynamic increase for the uniaxial compressive failure strength of concrete is smaller than for the uniaxial tensile failure strength. However, the compressive strength for a concrete sample subjected to a high strain rate compressive loading is considerably higher than the static strength.

The CEB-FIP Model Code 90 (CEB, 1993) gives the strain rate dependent compressive strength for concrete as:

$$
\begin{align*}
DIF_c &= \frac{f_{ct}}{f_c} = \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_{sc,\text{CEB}}}\right)^{1.026 \alpha_{s,\text{CEB}}} & \text{for } 30 \times 10^{-6} \leq |\dot{\epsilon}| \leq 30 \text{ s}^{-1} \quad (4.1a) \\
DIF_c &= \frac{f_{ct}}{f_c} = \gamma_s \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_{sc,\text{CEB}}}\right)^{1/3} & \text{for } 30 \leq |\dot{\epsilon}| \leq 300 \text{ s}^{-1} \quad (4.1b)
\end{align*}
$$

with

$$
\alpha_{s,\text{CEB}} = \frac{1}{5 + 9 f_c / f_{co}} \quad (4.2)
$$

and

$$
\log \gamma_s = 6.156 \alpha_{s,\text{CEB}} - 2 \quad (4.3)
$$
4.1. Static and dynamic material behaviour

where

\[ f_{cd} \] dynamic compressive strength
\[ f_c \] static compressive strength
\[ \dot{\varepsilon} \] strain rate
\[ f_{co} = 1 \times 10^6 \text{ Pa} \]
\[ \dot{\varepsilon}_{s,\text{CEB}} \] strain rate, compressive reference value CEB-FIP, \(-30 \times 10^{-6} \text{ s}^{-1}\)

and the relationships for the tensile uniaxial strength are given as:

\[
DIF = \frac{f_{td}}{f_t} = \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_{s,\text{CEB}}} \right)^{0.016\delta_{s,\text{CEB}}} \quad \text{for} \quad 3 \times 10^{-6} \leq \dot{\varepsilon} \leq 30 \text{ s}^{-1} \tag{4.4a}
\]

\[
DIF = \frac{f_{td}}{f_t} = \beta_{s,\text{CEB}} \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_{s,\text{CEB}}} \right)^{1/3} \quad \text{for} \quad 30 \leq \dot{\varepsilon} \leq 300 \text{ s}^{-1} \tag{4.4b}
\]

with

\[
\delta_{s,\text{CEB}} = \frac{1}{10 + 6 f_t / f_{co}} \tag{4.5}
\]

and

\[
\log \beta_{s,\text{CEB}} = 7.112\delta_{s,\text{CEB}} - 2.33 \tag{4.6}
\]

where

\[ f_{td} \] dynamic tensile strength
\[ f_t \] static tensile strength
\[ \dot{\varepsilon}_{s,\text{CEB}} \] strain rate, tensile reference value CEB-FIP, \(3 \times 10^{-6} \text{ s}^{-1}\)

A modified model for the strain rate dependent tensile strength for concrete subjected to strain rates in the range \(1 \times 10^{-6}\) to \(160 \text{ s}^{-1}\) was reported by Malvar and Ross (1998), with the equations given as:

\[
DIF = \frac{f_{td}}{f_t} = \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_{s,\text{MR}}} \right)^{\delta_{s,\text{MR}}} \quad \text{for} \quad 1.0 \times 10^{-6} \leq \dot{\varepsilon} \leq 1 \text{ s}^{-1} \tag{4.7a}
\]

\[
DIF = \frac{f_{td}}{f_t} = \beta_{s,\text{MR}} \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_{s,\text{MR}}} \right)^{1/3} \quad \text{for} \quad 1 < \dot{\varepsilon} \leq 160 \text{ s}^{-1} \tag{4.7b}
\]
with
\[ \log \beta_{s,MR} = 6\delta_{s,MR} - 2 \] (4.8)
and
\[ \delta_{s,MR} = \frac{1}{1 + \frac{8f_c}{f_{co}}} \] (4.9)

where
- \( f_{td} \) dynamic tensile strength
- \( f_t \) static tensile strength
- \( \dot{\varepsilon} \) strain rate
- \( \dot{\varepsilon}_{st,MR} \) static reference strain rate, \( 1 \times 10^{-6} \text{ s}^{-1} \)
- \( f_{co} \) uniaxial compressive strength

Figure 4.1 shows the tensile dynamic increase factor for concrete with varying compressive strengths for both the modified model by Malvar and Ross and the original model according to the CEB-FIP Model Code 90 (CEB, 1993). The two models give approximately the same values for strain rates less than \( 1 \text{ s}^{-1} \), which is the threshold value for the second branch of the function for the modified model (Malvar and Ross, 1998). However, this later model for the strain rate dependent tensile strength was considered to better describe available experimental data, including data for HPC with approximately 130 MPa compressive strength (Schuler and Hansson, 2006). Comparisons between models and experimental data are shown in Figure 4.2. The strain rate enhancements for different compressive uniaxial strengths according to the CEB-FIP Model Code 90 are given in Figure 4.3, with the dynamic increase factor for the compressive strength being lower than for the tensile strength.
4.1. Static and dynamic material behaviour

Figure 4.1. Dynamic increase factor for the tensile strength of concrete versus strain rate.

Figure 4.2. Dynamic increase factor for the tensile strength of concrete (Schuler and Hansson, 2006).
Chapter 4. Concrete material behaviour

Figure 4.3. Dynamic increase factor for the compressive strength of concrete versus strain rate (CEB, 1993).

The principal deformation properties of the concrete outside and within the fracture zone are shown in Figure 4.4, with the area under the descending part of the stress-displacement curve defined as the fracture energy $G_F$ (Hillerborg, 1977). Several simplified assumptions have been suggested for this relationship between tensile stress and the fracture width $w$, e.g. by Hillerborg (1977), Gylltoft (1983), Reinhardt (1985) and Schuler (2004). The simplest model for consideration of the fracture energy is the linear crack softening model, and this model together with bi-linear models and the model suggested by Schuler (2004) are shown in Figure 4.5. Linear and bi-linear fracture models were both investigated for the dynamic tensile failure of concrete at low strain rates within Paper III.

Figure 4.4. Deformation properties of the material outside the fracture zone (a), and the deformation properties of the fracture zone (b) (Peterson, 1981).
4.1. Static and dynamic material behaviour

![Figure 4.5](image1.png)

**Figure 4.5.** Linear and bi-linear crack softening models shown for concrete with $G_f=120$ Nm/m², with the crack softening model suggested by Schuler (2004) also shown.

The investigation of tensile fracturing of concrete by Weerheijm (1992) did not show any influence on the fracture energy for a crack opening velocity lower than approximately 0.3 m/s. However, Schuler (2004) later experimentally investigated the tensile fracture behaviour for concrete at higher strain rates, showing increased tensile fracture energy for crack opening velocities of approximately 10 to 30 m/s. Furthermore, an increase for the fracture energy for notched high strength concrete beams with a compressive strength of 127 MPa subjected to drop weight impact tests was shown by Zhang et al. (2009).

![Figure 4.6](image2.png)

**Figure 4.6.** Dynamic increase factor for the fracture energy ($DIF_G$) of concrete (Schuler and Hansson, 2006).
4.2. Equation of states for solids

The concept of an equation of state for a material requires that only certain sets of the variables pressure \( (p) \), internal energy \( (e) \) and specific volume \( (V) \) can exist. Furthermore, there exists one fundamental difference between equation of states (EOS) for non-porous solids and porous materials, i.e. porous materials are subjected to a considerably volumetric compaction at a relatively low pressure. Most non-porous solids, e.g. construction steel, other metals and hard rock such as granite, have an increased shock wave velocity for an increase of the particle velocity. The shock wave velocity defines the propagation speed of a shock wave through a material subjected to a high pressure loading, and it can be given as a function of pressure or particle velocity at the shock front. However, the shock wave velocity may be considerably reduced due to the pore collapse within a porous material as concrete. Furthermore, the bulk sound velocity at zero pressure \( (c_0) \) relates to the elastic state, and is obtained for zero particle velocity. The fundamental concepts and relationships for the modelling of these two types of materials with respect to penetration applications are given here.

4.2.1. Shock equation of state for non-porous material

The shock EOS, where pressure as a function of density and specific internal energy is described by a Mie-Grüneisen form of EOS is briefly described here, with a more thorough description given by for example Meyers (1994). The changes in shock velocity and bulk modulus can be neglected at low particle velocities, resulting in a linear EOS with respect to the density of the material. The shock EOS can thereby be approximated by a linear EOS with constant bulk modules for low velocity impacts of solid materials without any significant changes for simulation results. Both the bulk modulus and parameters for a higher order polynomial EOS can be determined from the shock EOS, which will be shown later for the use with equation of states for porous materials.

The shock EOS relates a state \( (p, e, V) \) to the pressure and internal energy at 0 K temperature according to:

\[
p = p_{0K} + \Gamma \rho (e - e_{0K})
\]  

(4.10)

Furthermore, another reference state can also be used for this EOS, e.g. a point on the Hugoniot curve according to:

\[
p = p_{H} + \Gamma \rho (e - e_{H})
\]  

(4.11)
The Hugoniot curve is given by a locus of points representing uniaxial strain plate impact experiments on the surface in the $p$, $e$, and $V$ space. This curve can be given on the form $p = f(V)$, although each point on the Hugoniot curve has a value of the internal energy $e$ associated with it (Zukas, 1990).

Thus, the pressure and internal energy of the material are related to the pressure and internal energy for a point on the Hugoniot curve with the same volume. Furthermore, the pressure varies linearly with internal energy at constant volume as seen in the Mie-Grüneisen form. It is assumed that $\Gamma \rho = \Gamma_0 \rho_0 = \text{constant}$, with the Grüneisen gamma defined as:

$$\Gamma = \frac{1}{\rho} \left( \frac{dp}{de} \right)_V$$  \hspace{1cm} (4.12)

The following relationships apply for the pressure along the solid Hugoniot line in the thermodynamic space (Meyers, 1994)

$$p_H = \rho_0 U_{siH} U_{pH}$$  \hspace{1cm} (4.13)

$$e_H = \frac{1}{2} \frac{p_H}{\rho_0} \left( \frac{\mu}{1 + \mu} \right)$$  \hspace{1cm} (4.14)

Equation (4.13) can be expressed according to Eq. (4.16) with the assumption that particle velocity $U_p$ is linearly related to the shock velocity $U_s$ according to Eq. (4.15), with $c_0$ being the bulk sound velocity at zero pressure and $s$ the linear shock wave velocity parameter. This gives:

$$U_s = c_0 + s U_p$$  \hspace{1cm} (4.15)

$$p_H = \frac{\rho_0 c_0^2 \mu (1 + \mu)}{[1 - (s-1)\mu]^2}$$  \hspace{1cm} (4.16)

The input data for the Mie-Grüneisen EOS is usually given as a linear relationship between the shock velocity and the particle velocity, but other relationships may also be used. Furthermore, an approximation for the Grüneisen gamma at the reference density can be applied to unknown materials according to Meyers (1994) as:

$$\Gamma_0 = 2s - 1$$  \hspace{1cm} (4.17)
Table 4.1 presents material data for the Mie-Grüneisen shock equation of state for selected metals, with Figure 4.7 showing a typical linear $U_p - U_s$ relationship. Furthermore, it is likely that the material parameters from different publications are based on the same, or partly the same, experimental results. To estimate the uncertainties for the evaluated parameters it would be necessary to have access to the original experimental data.

**Table 4.1. Mie-Grüneisen shock data parameters for selected materials.**

<table>
<thead>
<tr>
<th>Material</th>
<th>Density $(\text{kg/m}^3)$</th>
<th>$c_0$ $(\text{m/s})$</th>
<th>$s$</th>
<th>$\Gamma_0$</th>
<th>$C_p$ $(\text{J/kgK})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stainless steel 304 $^1$</td>
<td>7.890×10³</td>
<td>4.578×10³</td>
<td>1.488</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Stainless steel 304 $^2$</td>
<td>7.90×10³</td>
<td>4.57×10³</td>
<td>1.49</td>
<td>1.93</td>
<td>--</td>
</tr>
<tr>
<td>Stainless steel 304 $^3$</td>
<td>7.90×10³</td>
<td>4.57×10³</td>
<td>1.49</td>
<td>2.2</td>
<td>4.4×10³</td>
</tr>
<tr>
<td>Steel 4340 HRC 38 $^2$</td>
<td>7.81×10³</td>
<td>4.578×10³</td>
<td>1.33</td>
<td>1.67</td>
<td>--</td>
</tr>
<tr>
<td>Al 6061-T6 $^2$</td>
<td>2.703×10³</td>
<td>5.24×10³</td>
<td>1.40</td>
<td>1.97</td>
<td>--</td>
</tr>
<tr>
<td>Al 7075-T6 $^2$</td>
<td>2.804×10³</td>
<td>5.20×10³</td>
<td>1.36</td>
<td>2.20</td>
<td>--</td>
</tr>
<tr>
<td>Al 7039 $^4$</td>
<td>2.77×10³</td>
<td>5.328×10³</td>
<td>1.338</td>
<td>2.00</td>
<td>--</td>
</tr>
</tbody>
</table>


**Figure 4.7.** Relationship between shock wave velocity and particle velocity for stainless steel 304 (Marsh, 1980).
4.2. Equation of states for solids

4.2.2. Equation of states for porous materials

Concrete has non-linear compression behaviour due to the inhomogeneity and the porosity of the material. The so called P-α model (Herrman, 1969) is a general equation of state that accounts for the porosity of materials, and this EOS used as an important part of the concrete material model discussed later. The description of the P-α model reported here is extended to include the application of this EOS to concrete behaviour (Riedel, 2000). Furthermore, the original references are recommended for more detailed descriptions of the models.

The equations below describe the most essential parts of the P-α porous equation of state. Generally the behaviour of the matrix material is described by a solid EOS, e.g. a polynomial or Mie-Grüneisen shock EOS, while the pressure for the porous material is scaled using the parameter $\alpha_{p-a}$ with respect to the porosity of the material according to Eqs. (4.18) and (4.19). Compaction and unloading paths for a porous material are shown in Figure 4.8. Furthermore, it is stated that the pressure in the porous material is approximate $1/\alpha_{p-a}$ times the pressure in the matrix material. This later assumption is an extension of the original model presented by Herrman (1969).

$$\alpha_{p-a} = \frac{\rho_{Matrix}}{\rho}$$  \hspace{1cm} (4.18)

$$p_{\text{Matrix}} = f(\rho \alpha_{p-a}, e)$$  

$$p_{\text{Porous}} = \frac{1}{\alpha_{p-a}} f(\rho \alpha_{p-a}, e)$$  \hspace{1cm} (4.19a)

$$p_{\text{Porous}} = \frac{1}{\alpha_{p-a}} f(\rho \alpha_{p-a}, e)$$  \hspace{1cm} (4.19b)

Figure 4.8. Compaction and unloading of porous material with non-linear EOS for the matrix material, modified from Riedel (2000).
The porous sound speed $c_{B_{\text{Porous}}}$ relates to the initial elastic state of the concrete according to the Eq. (4.20), with $\alpha_{p,a}$ equal to its maximal value $\alpha_{\text{Porous},0}$ according to Eq. (4.21).

$$\frac{dp}{d\rho} = c_{B_{\text{Porous}}}^2 \quad (4.20)$$

$$\alpha_{\text{Porous},0} = \rho_{\text{Matrix},0}/\rho_{\text{Porous},0} \quad (4.21)$$

At the pressure $p_{\text{lock}}$ the material is fully compacted and $\alpha_{p,a}$ is reduced to unity. The pressure is then calculated using only the solid equation of state, and for an assumption of a linear elastic behaviour of the matrix the following equation is used:

$$p = c_{B_{\text{Matrix}}}^2 (\rho - \rho_{\text{Matrix},0}) = K_{\text{Matrix}} \mu \quad \text{with} \quad \mu = \frac{p}{\rho_0} - 1 \quad (4.22)$$

However, a polynomial EOS is normally used in combination with the RHT material model for the compression and the expansion states according to:

$$\left\{ \begin{array}{ll} p = A_1 \mu + A_2 \mu^2 + A_3 \mu^3 + (B_0 + B_1 \mu) \rho_0 e & \text{for} \quad \mu \geq 0 \quad (4.23) \\ p = T_1 \mu + T_2 \mu^2 + B_0 \rho_0 e & \text{for} \quad \mu < 0 \quad (4.24) \end{array} \right.$$ 

An approximate method to determine the parameters $A_1$, $A_2$ and $A_3$ for a polynomial EOS for the fully compacted material is to use the following set of equations (Riedel, 2000):

$$A_1 = \rho_0 c_0^2 \quad (4.25)$$

$$A_2 = \rho_0 c_0^2 \left[ 1 + s(s - 1) \right] \quad (4.26)$$

$$A_3 = \rho_0 c_0^2 \left[ 2(s - 1) + 3(s - 1)^2 \right] \quad (4.27)$$

The pressure is scaled with respect to the scale parameter $\alpha_{p,a}$ for pressures between $p_{\text{crush}}$ and $p_{\text{lock}}$ according:

$$\alpha_{p,a} = 1 + (\alpha_{\text{Porous},0} - 1) \left[ \frac{p_{\text{lock}} - p}{p_{\text{lock}} - p_{\text{crush}}} \right]^\gamma_{p,a} \quad (4.28)$$
Furthermore, the unloading modulus is determined by a linear interpolation between values corresponding to the initial porous state and the fully compacted state:

\[
e_B = e_{B,\text{Porous}} + (e_{B,\text{Matrix}} - e_{B,\text{Porous}}) \left[ \frac{\alpha_{P\alpha} - \alpha_{\text{Porous,0}}}{1 - \alpha_{\text{Porous,0}}} \right]
\]  

(4.29)

An initial compaction pressure for the concrete needs to be assumed for the considered loading path, with a recommended value of \(2f_c/3\) for penetration studies (Riedel, 2000). This is necessary since the \(P-\alpha\) EOS is not coupled to the failure surfaces within the RHT strength model for the concrete. Furthermore, the initial temperature and the heat capacity \(C_v\) for the material are also input for the EOS and the temperature calculations. The \(P-\alpha\) EOS input parameters for a normal strength concrete with 35 MPa compressive strength are given in Table 4.2, together with a modified data set for the 48 MPa NSC. Parameters for the HPC with 92 MPa compressive strength was determined by calibration of data from gauged reactive confinement (GREAC) tests, with the parameter set also given in Table 4.2. The value of 80.0 MPa used for \(p_{\text{crush}}\) within this parameter set for the HPC relates to the calibration of the parameters against data obtained with confined compressive tests. Furthermore, an alternative parameter set for this concrete was presented by Svinsås et al. (2001).
### Table 4.2. \( P-\alpha \) equation of state parameters for NSC with a uniaxial compressive strength of 35 MPa, and the modified data set used for the 48 MPa NSC and 92 MPa HPC.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NSC (Riedel, 2000)</th>
<th>NSC, 48 MPa (Paper V)</th>
<th>HPC, 92 MPa (Hansson, 2001, Paper I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_{\text{Porous},0} ) (kg/m³)</td>
<td>2.314\times10³</td>
<td>2.314\times10³</td>
<td>2.390\times10³</td>
</tr>
<tr>
<td>( c_{\text{B, Porous}} ) (m/s)</td>
<td>3000</td>
<td>3000</td>
<td>3000</td>
</tr>
<tr>
<td>( P_{\text{crash}} ) (MPa)</td>
<td>23.3</td>
<td>35.0</td>
<td>80.0</td>
</tr>
<tr>
<td>( P_{\text{lock}} ) (MPa)</td>
<td>6000</td>
<td>6000</td>
<td>1800</td>
</tr>
<tr>
<td>( n_{P-\alpha} ) (kg/m³)</td>
<td>3</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>( \rho_{\text{Matrix},0} ) (kg/m³)</td>
<td>2.75\times10³</td>
<td>2.75\times10³</td>
<td>2.54\times10³</td>
</tr>
<tr>
<td>( A_1 ) (GPa)</td>
<td>35.27</td>
<td>35.27</td>
<td>40.00</td>
</tr>
<tr>
<td>( A_2 ) (GPa)</td>
<td>39.58</td>
<td>39.58</td>
<td>0.00</td>
</tr>
<tr>
<td>( A_3 ) (GPa)</td>
<td>9.04</td>
<td>9.04</td>
<td>0.00</td>
</tr>
<tr>
<td>( B_0 )</td>
<td>1.22</td>
<td>1.22</td>
<td>1.22</td>
</tr>
<tr>
<td>( B_1 )</td>
<td>1.22</td>
<td>1.22</td>
<td>1.22</td>
</tr>
<tr>
<td>( T_1 ) (GPa)</td>
<td>35.27</td>
<td>35.27</td>
<td>40.00</td>
</tr>
<tr>
<td>( T_2 ) (GPa)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>( T_{\text{Ref}} ) (K)</td>
<td>300</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>( C_v ) (J/kgK)</td>
<td>654</td>
<td>654</td>
<td>654</td>
</tr>
</tbody>
</table>

### 4.3. RHT concrete model

Material models that describe the mechanical response of the materials with an acceptable accuracy need to be included in FE codes to obtain accurate simulation results. Constitutive equations for concrete exposed to weapons effects have been a major area of interest for a long time, and several material models for concrete behaviour have been developed. Furthermore, it is not until recent years that it has been possible to simulate the behaviour of concrete targets during projectile penetration with acceptable results. One of the limitations for FE analyses of penetration in concrete structures has been the lack of validated material models for concrete subjected to the extreme loading conditions occurring during warhead impacts. The different tensile and compressive behaviour of the concrete under high strain rate deformation, and with the need to consider the residual strength of the material under compression, require that a complex strength model is needed for the concrete during these loading conditions. One material model for FE codes which include these phenomena is the RHT concrete model. This material model was used for the all the FE analyses presented in Papers I, II, III and V. The RHT model is described here with notations closely following the original documentation and implementation, with a thorough description of the model available in Riedel (2000). Furthermore, material parameters for normal
4.3. RHT concrete model

Strength concrete with a uniaxial compressive strength of 35 MPa were also given in the original documentation. The yield surfaces of the concrete for the material model are scaled with reference to the uniaxial compressive strength $f_c$ for the concrete to obtain consequent data set for different concrete strengths. However, a new parameter set based on extensive material testing needs to be established if major changes of the behaviour of the concrete are anticipated. Furthermore, minor changes were introduced for the version of RHT material model implemented in the Autodyn version 4.2 and higher (Century Dynamics, 2005b), and these are noted in this description of the RHT material model. The background of the RHT model and its area of application are discussed by Riedel (2009).

The description of the stress state in the material model relates to three pressure dependent yield surfaces for the definition of the elastic limit surface, failure surface and remaining residual strength surface for the crushed material, with the three yield surfaces described below. The failure surface can be seen as a function of the strength along the compression meridian $Y_{TXC}(p)$ multiplied by the factors $F_{Rate}(\dot{\varepsilon})$ and $R_3(\theta,Q_2)$ as:

$$Y_{fail}(p,\theta,\dot{\varepsilon}) = Y_{TXC}(p)F_{Rate}(\dot{\varepsilon})R_3(\theta,Q_2)$$

(4.30)

The strength along the meridian is given by Eq. (4.31a), where $Y_{TXC}^*(p)$ define the pressure dependent static compressive meridian normalised to the unconfined compression strength $f_c$. The parameter $A_{fail}$ is not included in the RHT version implemented in Autodyn (Century Dynamics, 2005b), with the equation according to the Autodyn notation given in Eq. (4.31b).

$$Y_{TXC}^*(p) = \frac{Y_{TXC}(p)}{f_c} = A_{fail} + B_{fail} \left[ p^* - HTL'^* \right]^{N_{sol}}$$

for $p^* \geq \frac{1}{3}$ (Riedel, 2000)

(4.31a)

$$Y_{TXC}^*(p) = \frac{Y_{TXC}(p)}{f_c} = A \left[ p^* - HTL'^* \right]^N$$

for $p^* \geq \frac{1}{3}$ (Century Dynamics, 2005b)

(4.31b)

with $A_{fail}$, $B_{fail}$ and $N_{fail}$ are taken as material constants characteristic for the specific concrete. Furthermore, the intersection HTL$'^*$ normalised to $f_c$ is defined by:

$$HTL'^* = \frac{1}{3} - \frac{1}{N_{sol}} \sqrt{\frac{1}{B_{fail}}}$$

with

$$HTL' = f_c \times HTL'^*$$

(4.32)
Eqs. (4.31) and (4.32) above describes the high pressure compressive meridian, and are not used for pressure below $f_c/3$. Instead a piece-wise linear approximation of the compressive meridian is used between stress states representing the tensile strength ($f_t/Q_1$), the shear strength ($f_s/Q_1$) and the compressive strength according to Figure 4.9. The correction factors $Q_1$ and $Q_2$ are applied since the shear and uniaxial tensile stress states are located on the tensile and shear meridian, respectively. The reference points for the failure surface are shown in Figure 4.10, with a graphical representation of the correction factors shown later in Figure 4.12a.

![Figure 4.9. The linear approximation of the compressive meridian at low pressures, modified from Riedel (2000).](image-url)
The factor $F_{\text{RATE}}(\dot{\varepsilon})$ takes the strain rate enhancement into account according to:

$$
F_{\text{RATE}}(\dot{\varepsilon}) = \begin{cases} 
\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^\alpha & \text{for } p \geq f_c / 3, \text{ with } \dot{\varepsilon}_0 = 3 \times 10^{-6} \text{ s}^{-1} \\
\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^\delta & \text{for } p \leq -f_c / 3, \text{ with } \dot{\varepsilon}_0 = 3 \times 10^{-6} \text{ s}^{-1}
\end{cases}
$$

(4.33a) (4.33b)

Different strain rate enhancements are used for compressive and tensile loading conditions, with $\alpha$ and $\delta$ being material constants. Furthermore, the strain rate enhancement factor is linearly interpolated between the pressures $-f_c / 3$ and $f_c / 3$. Testing of concrete at increased strain rates by using a split Hopkinson pressure bar (SHPB) have resulted in an increased dynamic compressive strength, e.g. the CEB-FIP model code 1990 (CEB, 1993) is suggesting a bilinear approximation of the strain rate enhancement factor both in tension and compression. This results in a rapid increase of the dynamic compressive strength according to the CEB-FIP model code at a strain rate greater
than 30 s\(^{-1}\). However, it seems that the major part of the dynamic increase of the compressive strain rate is related to the pressure dependent yield strength of concrete. The simulation of SHPB tests has shown that a pressure dependent constitutive model is likely to account for the major part of the dynamic increase of the compressive strength for concrete subjected to strain rates \(>30\) s\(^{-1}\) (Li and Meng, 2003).

![Figure 4.11. The dynamic increase of the compressive meridian for \(p \geq f_c/3\).](image)

The reduced failure strength for states off the compression meridian on the failure surface is introduced and given by the factor \(R_3(\theta, Q_2)\). This factor scales the strength from the highest value at the compression meridian, and is given by:

\[
R_3(\theta, Q_2) = \frac{2(1 - Q_2^2) \cos \theta + (2Q_2 - 1)[4(1 - Q_2^2) \cos^2 \theta + 5Q_2^2 - 4Q_2]]}{4(1 - Q_2^2) \cos^2 \theta + (1 - 2Q_2)^2} \tag{4.34}
\]

Thus with \(\theta\) rotating around the hydrostatic axis the entire failure surface can be calculated, see Figure 4.12a. The parameter \(Q_2\) gives the distance from the hydrostatic axis to the tensile meridian divided by the distance between the hydrostatic axis to the compressive meridian as:

\[
0.5 < Q_2 = Q_{2,0} + BQ \times p^* \leq 1 \tag{4.35}
\]

The extreme case of \(Q_2\) equal to 0.5 is found at low pressures and gives a triangular failure surface in the deviatoric plane. At the other extreme \(Q_2\) is equal to 1, and this gives a circular cross section of the failure surface according to Figure 4.12b. Thus at large confining pressures the surface
approaches the circular form, with the pressure dependence of $Q_2$ according to Eq. (4.35) above. This method to account for the reduced concrete strength of the compressive meridian was first used by William and Warnke (1975). Furthermore, the parameter $Q_1$ used for definition of the failure surface at low pressures relates the distance from the shear meridian to the hydrostatic axis to distance from the compressive meridian to the hydrostatic axis. The elastic limiting surface $Y_{el}(p,\theta,\dot{\varepsilon})$ shown in Eq. (4.36) and Figure 4.13 is scaled from the failure surface by the scaling factor $YoF$ along the loading paths. This scaling factor varies linearly from $f_{c,el}/f_c$ to $f_{t,el}/f_t$ between the pressures $-f_{t,el}/3$ and $f_{c,el}/3$ according to Figure 4.14.

![Diagram](image1)

**Figure 4.12.** Cross section through the failure surface at a deviatoric plane, modified from Riedel (2000).

![Diagram](image2)

**Figure 4.13.** A schematic figure of the elastic limit, modified from Riedel (2000).
Chapter 4. Concrete material behaviour

Figure 4.14. The pressure dependent scale function for the ratio between elastic and failure strength surfaces, modified from Riedel (2000).

Linear strain hardening is used between the elastic and failure surfaces, and hardening modulus is given by the input ratio $G_{el}/(G_{el} - G_p)$ (Century Dynamics, 2005b), which defines the ratio between the original shear modulus and the hardening modulus. The elastic strength surface is then defined as:

$$Y_{el}(p, \theta, \varepsilon) = Y_{fail}(p, \theta, \varepsilon) Y_{oF} F_{Cap}(p)$$

Furthermore, the elastic part of the deformation decreases at high pressures and the option to use a cap on the elastic surface ensures that the elastic surface closes at high pressures. The elastic strength surface within the RHT material model can be forced to close at high pressures by activation of dimensionless cap function $F_{Cap}(p)$, which goes smoothly from unity to zero. The cap function is unity up to the pressure $p_u$ where the uniaxial compression path intercepts with the elastic surface. At higher pressures $F_{Cap}(p)$ decreases and reaches zero at the pressure $p_0$, which is obtained as $p_{crash}$ from the P-\(\alpha\) EOS input data. The mathematical expression for $F_{Cap}(p)$ is given as:

$$F_{Cap}(p) = \begin{cases} 
1 & \text{for } p \leq p_u \\
1 - \left( \frac{p - p_u}{p_0 - p_u} \right)^2 & \text{for } p_u < p < p_0 \\
0 & \text{for } p_0 \leq p
\end{cases}$$

(4.37)
4.3. RHT concrete model

The damage in the material grows after the stress point passes the failure surface according to:

$$D = \sum \frac{\Delta \varepsilon_{pl}}{\varepsilon_{pl}^\text{failure}}$$  \hspace{1cm} (4.38)

$$\varepsilon_{pl}^\text{failure} = D_{\text{RHT1}} \left( p^* - p^*_\text{spall} \right)^{D_{\text{RHT2}}} \geq \varepsilon_{pl,\text{min}}$$  \hspace{1cm} (4.39)

with $D_{\text{RHT1}}$ and $D_{\text{RHT2}}$ taken as material specific parameters.

At low pressures, a lower limit of the failure strain is set by introducing a minimum failure strain $\varepsilon_{\text{min}}$, with the damage evolution calibrated for cyclic uniaxial compressive stress conditions according to Holmquist et al. (1993). The residual strength $Y^*_\text{fric}$ (normalised to the unconfined compression strength) of the fully damaged concrete is calculated from Eq. 4.40a, with the Autodyn notation given in Eq. 4.40b (Century Dynamics, 2005b).

$$Y^*_\text{fric} = B_{\text{fric}} \times \left( p^* \right)^N$$  \hspace{1cm} (Riedel, 2002)  \hspace{1cm} (4.40a)

$$Y^*_\text{fric} = B \times \left( p^* \right)^M$$  \hspace{1cm} (Century Dynamics, 2005b)  \hspace{1cm} (4.40b)

The strength is interpolated from the strength values for the undamaged material ($D = 0$) at the failure surface and the completely damaged material ($D = 1$) according to:

$$Y^*_\text{fractured} (D) = (1 - D) Y^*_\text{fail} + D \times Y^*_\text{fric}$$  \hspace{1cm} (4.41)

Furthermore, the shear modulus is reduced with respect to the damage parameter according to:

$$G_{\text{fractured}} (D) = G_{D=0} (1 - D) + G_{D=1} \times D \quad \text{with} \quad G_{D=1} = \text{Shrat} D \times G_{D=0}$$  \hspace{1cm} (4.42)

In Figure 4.15 characteristics of the RHT model are schematically shown. The input parameters for the RHT material model for NSC are given in Table 4.3 (Riedel, 2000), with modifications for the parameter set for the 48 MPa NSC shown. Furthermore, the base parameter set for the HPC with 92 MPa concrete strength is also shown. Data from confined GREAC tests of this HPC were used for calibration of this parameter set (Hansson, 2001, Paper I), with an alternative parameter set presented by Svinsås et al. (2001).
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Figure 4.15. Figure (a) shows the failure surface (outer) and the elastic limit surface (inner), with figure (b) showing the failure surface (outer) and the residual strength surface (inner) (From Paper V).

Table 4.3. RHT strength model parameters for 35 MPa normal strength concrete according to Riedel (2000), and modified base parameter set used for concrete types with 48 and 92 MPa strengths.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NSC, 35 MPa (Riedel, 2000)</th>
<th>NSC, 48 MPa (Paper V)</th>
<th>HPC, 92 MPa (Hansson, 2001, Paper I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{el}$ (GPa)</td>
<td>16.7</td>
<td>16.7</td>
<td>18.0</td>
</tr>
<tr>
<td>$f_c$ (MPa)</td>
<td>35.0</td>
<td>48.0</td>
<td>92.0</td>
</tr>
<tr>
<td>$f_t/f_c$</td>
<td>0.10</td>
<td>0.083</td>
<td>0.057</td>
</tr>
<tr>
<td>$f_r/f_c$</td>
<td>0.18</td>
<td>0.18</td>
<td>0.30</td>
</tr>
<tr>
<td>$A_{fail}$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$B_{fail}$</td>
<td>1.60</td>
<td>1.60</td>
<td>1.90</td>
</tr>
<tr>
<td>$N_{fail}$</td>
<td>0.61</td>
<td>0.61</td>
<td>0.60</td>
</tr>
<tr>
<td>$Q_{2,9}$</td>
<td>0.6805</td>
<td>0.6805</td>
<td>0.6805</td>
</tr>
<tr>
<td>$BQ$</td>
<td>0.0105</td>
<td>0.0105</td>
<td>0.0105</td>
</tr>
<tr>
<td>$G_{el}/(G_{el}-G_{pl})$</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>$f_{r,el}/f_t$</td>
<td>0.70</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>$f_{r,c}/f_c$</td>
<td>0.53</td>
<td>0.53</td>
<td>0.75</td>
</tr>
<tr>
<td>Cap option</td>
<td>Active</td>
<td>Active</td>
<td>Active</td>
</tr>
<tr>
<td>$B_{frac}$</td>
<td>1.6</td>
<td>1.6</td>
<td>1.6</td>
</tr>
<tr>
<td>$N_{frac}$</td>
<td>0.61</td>
<td>0.61</td>
<td>0.61</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.032</td>
<td>0.032</td>
<td>0.010</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.036</td>
<td>0.036</td>
<td>0.013</td>
</tr>
<tr>
<td>$D_{RHT1}$</td>
<td>0.04</td>
<td>0.04</td>
<td>0.08</td>
</tr>
<tr>
<td>$D_{RHT2}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\varepsilon_{failure}$</td>
<td>0.01</td>
<td>0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>ShratD</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
</tr>
</tbody>
</table>
Chapter 5

Experimental program for warhead penetration

The study of penetration of HBTPs in concrete was aided by an experimental program to obtain the necessary penetration data, both for analytical modelling with empirical penetration models and for comparisons with FE analyses of concrete penetration. This chapter describes the experimental program for the performance of HBTPs in different types of concrete targets, with an initial test report earlier published (Hansson, 2005a). These model scale tests with ogive nosed penetrators with a length to diameter ratio of approximately nine against concrete targets were performed at FOI (Swedish Defence Research Agency), with the designs for the penetrators representing a generic hardened buried target penetrator. The performed tests used targets constructed of unreinforced NSC and HPC, and also heavily reinforced NSC. The experimental study considered both penetration tests with measurement of the penetration depths and perforations of targets with measurements of exit velocities for the penetrators. Furthermore, the tests were performed at two nominal impact velocities, i.e. 420 and 460 m/s, and two impact angles for the penetrators, i.e. 90.0° and 59.5°. The experimental set up and material properties for the tests are described in section 5.1, with the test results given in section 5.2. However, a part of the test results for experiments with normal impact conditions is also briefly given in Paper IV, with additional test results for oblique impacts of NSC given in Paper V. Furthermore, this chapter contains additional information regarding the test set-up and the evaluation of the penetration experiments compared to Papers IV and V, with post-test photos presented for all the targets.

5.1. Experimental set-up

The designs of penetrators and targets are described, together with the material properties for the used materials and the used measurement techniques.

5.1.1. Penetrators

Penetrator designs developed by FOI were used for the tests, with the different designs shown in Figure 5.1 and the properties for the penetrators given in Table 5.1. Three nose designs were used
within the test series, corresponding to Calibre Radius Head (CRH) values of 3.0, 8.0 and 12.0. The CRH value gives the ratio of the ogive radius and the diameter of the penetrator, see Figure 5.1b. The penetrators were fabricated from forged standard steel 34CrNiMo6 (Swedish standard SS 14 2541). The hardness of 450 to 520 HV (Hardness Vickers) for the penetrators used for test series no. 2004 corresponds to an estimated yield strength for the steel of approximately 1.6 GPa. Furthermore, the stress-strain relationships for the 34CrNiMo6 steel are shown for steel hardness HV 300, 450 and 600 at a nominal strain rate of approximately 400 s⁻¹ in Figure 5.2. Furthermore, the AISI 4340 steel has similar properties as the 34CrNiMo6 steel. An inert ballast material consisting of cement based mortar with a density of 2.4×10³ kg/m³ was used to obtain the desired mass of the penetrator for test series no. 2002-10. A dentist mould plaster with density of 1.8×10³ kg/m³ was used as ballast for the improved penetrators used for test series no. 2004. The increased case thickness in combination with a reduced density of the filling material results in a penetrator with an increased performance compared with test series no. 2002.

Table 5.1. The penetrator properties for the test series no. 2002 and 2004.

<table>
<thead>
<tr>
<th>Test series</th>
<th>Series no 2002</th>
<th>Series no. 2004</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penetrator design</td>
<td>CRH 8.0</td>
<td>CRH 3.0 CRH 8.0 CRH 12.0</td>
</tr>
<tr>
<td>Ogive radius (mm)</td>
<td>400</td>
<td>150 400 600</td>
</tr>
<tr>
<td>Length (mm)</td>
<td>450</td>
<td>450 450 470</td>
</tr>
<tr>
<td>Solid nose length (mm)</td>
<td>Approx. 83</td>
<td>Approx. 85 Approx. 85 Approx. 105</td>
</tr>
<tr>
<td>Body diameter (mm)</td>
<td>50.0</td>
<td>50.0</td>
</tr>
<tr>
<td>Total mass (kg)</td>
<td>3.65 ±0.02</td>
<td>4.50 ±0.02</td>
</tr>
<tr>
<td>Casing thickness (mm)</td>
<td>5.0</td>
<td>10.0</td>
</tr>
<tr>
<td>Hardness</td>
<td>HV 560 – 620</td>
<td>HV 450 – 520</td>
</tr>
</tbody>
</table>

Figure 5.1. The penetrator designs with CRH 3.0, 8.0 and 12.0 for the ogive nose are shown from the top (a) (Paper IV). Figure (b) shows the definition of the ogive radius for the penetrator with CRH 3.0.
5.1. Experimental set-up

![Stress-strain relationship graph]

Figure 5.2. Typical stress-strain relationships at a nominal strain rate of approximately 400 s⁻¹ for 34CrNiMo6 steel with HV 300, 450 and 600. The data in the figure are not valid for strains obtained after the onset of necking of the samples, and these parts of the curves only shown to give an estimation of the ductility of the material (after Hansson, 2005a).

5.1.2. Concrete types

Normal strength concrete and two HPC types were used for the first test series no. 2002, with only NSC and one type of HPC were used for test series no. 2004. The different HPC types and batches are designated by the approximate uniaxial cylinder strength in MPa at the time of testing, e.g. HPC 97. The mix proportions for NSC and HPC 133/146 are given in Table 5.2 below. The mix proportions for the HPC 97 that was used for the first test series are not available. However, the cement content and amount of mixing water were 440 kg/m³ and 145 kg/m³, respectively. Furthermore, the largest aggregate size for this concrete was 20 mm. Standard strength tests were performed on both the NSC and HPC concrete types (Hansson, 2005a), with the results given in Tables 5.3 and 5.4. The HPC batch used for test series no. 2004 (HPC 133) showed a slightly lower compressive strength that the batch of the same type of concrete used for test series no. 2002 (HPC 146).
Table 5.2. Mix proportions of two of the concrete types.

<table>
<thead>
<tr>
<th>Material</th>
<th>NSC Amount (kg/m³)</th>
<th>HPC 133 Amount (kg/m³)</th>
<th>HPC 146 Amount (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement</td>
<td>330</td>
<td>578</td>
<td></td>
</tr>
<tr>
<td>Aggregate 0 – 4 mm</td>
<td>990</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>Aggregate 4 – 8 mm</td>
<td>825</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>Aggregate 0 – 2 mm</td>
<td>--</td>
<td>548</td>
<td></td>
</tr>
<tr>
<td>Aggregate 2 – 5 mm</td>
<td>--</td>
<td>231</td>
<td></td>
</tr>
<tr>
<td>Aggregate 5 – 8 mm</td>
<td>--</td>
<td>485</td>
<td></td>
</tr>
<tr>
<td>Aggregate 8 – 11 mm</td>
<td>--</td>
<td>312</td>
<td></td>
</tr>
<tr>
<td>Water</td>
<td>215</td>
<td>166</td>
<td></td>
</tr>
<tr>
<td>Superplastizers</td>
<td>--</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td><strong>Total:</strong></td>
<td><strong>2360</strong></td>
<td><strong>2471</strong></td>
<td></td>
</tr>
</tbody>
</table>
Table 5.3. Properties for NSC and HPC (Ø100×200 mm cylinders).

<table>
<thead>
<tr>
<th>Concrete type</th>
<th>Series no.</th>
<th>Approx. age (days)</th>
<th>Compressive strength, $f_c$ Average (MPa)</th>
<th>Compressive strength, $f_c$ Std. dev. (MPa)</th>
<th>Tensile splitting strength Average (MPa)</th>
<th>Tensile splitting strength Std. dev. (MPa)</th>
<th>Young’s modulus Average (GPa)</th>
<th>Young’s modulus Std. dev. (GPa)</th>
<th>Density Average (kg/m$^3$)</th>
<th>No of samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSC 2002</td>
<td>2002</td>
<td>28</td>
<td>45.5</td>
<td>0.9</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>4</td>
</tr>
<tr>
<td>NSC 2002</td>
<td>2002</td>
<td>91</td>
<td>48.2</td>
<td>1.7</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>5</td>
</tr>
<tr>
<td>NSC 2004</td>
<td>2004</td>
<td>131</td>
<td>42.5</td>
<td>0.3</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>4</td>
</tr>
<tr>
<td>NSC 2004</td>
<td>2004</td>
<td>291</td>
<td>54.8</td>
<td>1.2</td>
<td>--</td>
<td>31.3</td>
<td>1.0</td>
<td>2.31×10$^3$</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>HPC 97 2002</td>
<td>2002</td>
<td>46</td>
<td>83.2</td>
<td>2.0</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>2.42×10$^3$</td>
<td>3</td>
</tr>
<tr>
<td>HPC 97 2002</td>
<td>2002</td>
<td>270</td>
<td>97.0</td>
<td>2.0</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>2.42×10$^3$</td>
<td>3</td>
</tr>
<tr>
<td>HPC 133</td>
<td>2004</td>
<td>307</td>
<td>132.9</td>
<td>1.2</td>
<td>--</td>
<td>45.0</td>
<td>0.7</td>
<td>2.51×10$^3$</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>HPC 146</td>
<td>2002</td>
<td>28</td>
<td>128.9</td>
<td>1.9</td>
<td>6.2</td>
<td>0.5</td>
<td>--</td>
<td>--</td>
<td>2.50×10$^3$</td>
<td>3</td>
</tr>
<tr>
<td>HPC 146</td>
<td>2002</td>
<td>210</td>
<td>145.7</td>
<td>0.9</td>
<td>7.3</td>
<td>0.8</td>
<td>--</td>
<td>--</td>
<td>2.49×10$^3$</td>
<td>3</td>
</tr>
</tbody>
</table>

Note 1: Cured in water the first four days and then stored dry at 20°C. 2: Cured with the targets. 3: Cored cylinders. 4: Cured in water the first four days and then stored with the targets.

Table 5.4. Fracture energy for HPC 133 measured on beams cured in water until testing.

<table>
<thead>
<tr>
<th>Concrete type</th>
<th>Series no.</th>
<th>Approx. age (days)</th>
<th>Sample geometry (mm × mm × mm)</th>
<th>Fracture energy, $G_f$ Average (N/m)</th>
<th>Fracture energy, $G_f$ Std. dev. (N/m)</th>
<th>Density Average (kg/m$^3$)</th>
<th>No of samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPC 133</td>
<td>2004</td>
<td>307</td>
<td>840 × 100 × 100</td>
<td>158</td>
<td>7</td>
<td>2.55×10$^3$</td>
<td>4</td>
</tr>
</tbody>
</table>
5.1.3. Concrete targets

The NSC target for the test no. 2002-10 was cast in a steel pipe with 1.25 m diameter, with the diameter of the thin walled steel cylinders reduced to 1.20 m for the targets used for the main test series. The dimensions of the thin walled steel cylinders used for the unreinforced targets are given in Table 5.5. Steel reinforcement bars were welded inside the thin walled steel cylinders to obtain a mechanical connection between the concrete and the steel cylinder. The locations of these bars were chosen to minimise the influence on the penetration path. An additional unreinforced NSC target was cast for test no. 2004-21 without the use of a thin walled steel cylinder. The dimensions of this target were 1.20×1.20×1.20 m³. The reinforcement design for a reinforced NSC target is shown in Figure 5.3. The reinforcement layers consist of 19 bars in each direction with a centre to centre distance equal to 60 mm for the 1.20×1.20×0.60 m³ target, and with the properties of the reinforced targets given in Table 5.6. The cover of concrete for the first and last reinforcement layers were approximately 30 mm, and layer two to four were then distributed equally along the length of the target. The individual reinforcement layers were welded to the longitudinal reinforcement bars, which had a centre to centre distance equal to 180 mm. The second and fourth layers of the reinforcement were shifted 28 mm in both horizontal and vertical directions.

Table 5.5. Dimensions of thin walled steel cylinders used for unreinforced targets.

<table>
<thead>
<tr>
<th>Concrete type</th>
<th>NSC</th>
<th>HPC 97</th>
<th>HPC 146</th>
<th>NSC</th>
<th>HPC 133</th>
<th>NSC</th>
<th>HPC 133</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impact angle (°)</td>
<td>90.0</td>
<td>90.0</td>
<td>90.0</td>
<td>90.0</td>
<td>90.0</td>
<td>59.5</td>
<td>59.5</td>
</tr>
<tr>
<td>Target diameter (m)</td>
<td>1.25</td>
<td>1.20</td>
<td>1.20</td>
<td>1.20</td>
<td>1.20</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>Steel thickness (mm)</td>
<td>8.0</td>
<td>5.0</td>
<td>5.0</td>
<td>8.0</td>
<td>8.0</td>
<td>8.0</td>
<td>8.0</td>
</tr>
</tbody>
</table>
5.1 Experimental set-up

![Figure 5.3](image)

*Figure 5.3. The reinforcement design for the 1.20×1.20×0.60 m³ targets, with the side view (a), the front view (b) and a perspective view (c) shown.*

Table 5.6. Specification for the reinforced concrete targets.

<table>
<thead>
<tr>
<th>Target type</th>
<th>Reinforced NSC 0.60 m targets</th>
<th>Reinforced NSC 0.54 m targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penetrator impact angle (°)</td>
<td>90.0</td>
<td>59.5</td>
</tr>
<tr>
<td>Target thickness (m)</td>
<td>0.60</td>
<td>0.54</td>
</tr>
<tr>
<td>Target width (m)</td>
<td>1.20</td>
<td>1.20</td>
</tr>
<tr>
<td>Target height (m)</td>
<td>1.20</td>
<td>1.50</td>
</tr>
<tr>
<td>Reinforcement type</td>
<td>B500 BT type 1</td>
<td>B500 BT type 1</td>
</tr>
<tr>
<td>Bar diameter (mm)</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>No. of reinforcement layers</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Centre to centre distance (mm)</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Amount of reinforcement steel (vol-%)</td>
<td>4.71</td>
<td>5.19</td>
</tr>
<tr>
<td>(kg/m³)</td>
<td>291</td>
<td>325</td>
</tr>
</tbody>
</table>

5.1.4. Shooting and measurement techniques

A 61 mm smooth bore gun was used to launch the penetrators, with each penetrator centred in the barrel with a guidance ring and a pusher plate, i.e. cup sabot, during the launch as shown in Figure 5.4. The masses for the guidance ring and cup sabot were approximately 130 and 350 g, respectively. Several different types of sabots are developed for projectiles, with all designed to separate from the projectile shortly after the projectile has left the barrel of the gun. The cup sabot
for the rear end of the penetrator was used since it easy to manufacture and provided easy handling of the penetrators before launching. Furthermore, the guidance ring was only attached to the penetrator with three small screws. These screws were assumed to be sheared off after the guidance ring made contact with the concrete target. The targets were placed so the sight-line through the gun barrel passed through the targets at approximately mid-height. The horizontal and vertical views of the penetrator were both recorded with two 70 mm Lexander high-speed cameras before impact and after perforation of the targets, with the high-speed cameras running at frame rates between 900 and 960 frames/s during the tests. The films from these cameras were used to determine the yaw and pitch angles for the penetrators, and exit velocities for the penetrators. High-speed film frames showing a penetrator after perforation of an unreinforced NSC target are shown in Figure 5.5. The set-up of the high-speed filming was improved after the initial test series no. 2002, with increased accuracy for determination of the yaw and pitch angles for test series no. 2004. The placements of targets for normal impact and oblique impact conditions are shown in Figure 5.6, with the placements of short circuit screens used to determine the impact velocities for the penetrators also shown in this figure. Furthermore, the impact velocities were also estimated from the high-speed film with an estimated error of ±10 m/s.

![Figure 5.4](image_url)  
*Figure 5.4. The penetrator design with CRH 8.0 shown with guidance ring and cup sabot of aluminium (pusher plate).*

![Figure 5.5](image_url)  
*Figure 5.5. Frames from the high-speed film from test no. 2004-6 with a frame rate of 916 frames/s.*
The placements of unreinforced concrete targets for tests with approximately 90.0° (a) and 59.5° impact angles (b), and with the placement of reinforced NSC targets for tests with oblique penetrator impacts shown in figure (c). The positions of the short circuit screens for velocity measurement are shown in figures (a) and (c), with the third short circuit screen mounted on the surface of the target.
5.2. Penetration experiments

The results given here were obtained for penetration experiments conducted with two nominal impact velocities, two impact angles, three nose shapes and two masses for the penetrator, with each of the penetration tests briefly described.

5.2.1. Test series no. 2002

All penetrators within test series no. 2002 impacted the target at approximately a normal impact angle, with estimated pitch and yaw angles according to Table 5.7. The penetration results from the test series no. 2002 are given in Tables 5.8 and 5.9, together with the data obtained for test series no. 2004. Only one NSC test was performed within series no. 2002, with a post-test photo of the target and the frames from the high-speed film used to determine the yaw and pitch angles for the penetrators shown in Figure 5.7.

Table 5.7. The estimated pitch and yaw angles for the penetrators for test series no. 2002.

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Pitch (°)</th>
<th>Yaw (°)</th>
<th>Test no.</th>
<th>Pitch (°)</th>
<th>Yaw (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002-10</td>
<td>0.8</td>
<td>1.0</td>
<td>2002-4</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>2002-1</td>
<td>0.6</td>
<td>&lt; 0.2</td>
<td>2002-7</td>
<td>1.5</td>
<td>1.0</td>
</tr>
<tr>
<td>2002-2</td>
<td>1.3</td>
<td>0.7</td>
<td>2002-11</td>
<td>0.6</td>
<td>0.2</td>
</tr>
<tr>
<td>2002-3</td>
<td>0.5</td>
<td>2.2</td>
<td>2002-12</td>
<td>1.1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Seven HPC tests were conducted with the HPC 97 and HPC 146 concretes, see Table 5.9. Post-test photos of HPC 97 targets impacted at the velocities 411 and 417 m/s are shown in Figure 5.8. Furthermore, the test no. 2002-3 was performed in the same concrete type, but with an increased impact velocity of 462 m/s. This test was not used for any further evaluation due to the fracturing of the penetrator since this may have influenced the penetration depth for this test. Furthermore, this experiment was not presented within Papers IV or V. Post-test photos of this target, including a close up of the penetrator, are shown in Figure 5.10. Four tests were conducted with the HPC 146 concrete, see Table 5.9. Penetration depths of approximate 0.40 m were obtained for the two targets with a length of 1.20 m for an impact velocity of approximately 410 m/s. Perforations of the targets occurred for the two 0.45 m thick targets impacted at approximate 420 m/s, with exit velocities of the penetrators equal to 129 and 69 m/s, respectively. Post-test photos of these tests are shown in Figures 5.10 to 5.12. The recovered penetrators are shown in Figure 5.13 after perforation of targets no. 2002-11 and 2002-12, with the penetrator from test no. 2002-7 shown for comparison.
5.2. Penetration experiments

Figure 5.7. Post-test photo of NSC target no. 2002-10 (a), and high-speed film frames showing vertical and horizontal projections of the penetrator before impact for the test (b).

Figure 5.8. Post-test photos of the HPC 97 targets from tests no. 2002-1 (a) and 2002-2 (b).

Figure 5.9. Post-test photo of the HPC 97 target from test no. 2002-3 with an increased impact velocity of 462 m/s (a), and a close up of the fractured penetrator from the test (b).
Chapter 5. Experimental program for warhead penetration

Figure 5.10. Post-test photos of the HPC 146 targets from tests no. 2002-4 (a) and 2002-7 (b).

Figure 5.11. The perforated target from test no. 2002-11 with HPC 146 concrete, with the front face (a) and the back face (b) of the target shown.

Figure 5.12. The perforated target from test no. 2002-12 with HPC 146 concrete, with the front face (a) and the back face (b) of the target shown.
5.2. Penetration experiments

Figure 5.13. Recovered penetrators after perforation of HPC targets for tests no. 2002-11 (a) and 2002-12 (b). Figure (c) shows the penetrator from HPC test no. 2002-7 without perforation of the target for comparison.

5.2.2. Test series no. 2004

The main test series were performed with the improved penetrator designs using an increased casing thickness and mass for the penetrators. Furthermore, tests with heavily reinforced normal strength concrete targets were also included in the test series and two impact angles were used for the tests. These were normal impact, i.e. 90.0°, and oblique impact at 59.5°. The performance of the penetrators was also investigated used for the two nominal impact velocities 420 and 460 m/s. The test results are compiled in Tables 5.8 and 5.9, together with the data obtained for test series no, 2002. The estimated error of the impact angle for the penetrator due to placement of the target was estimated to be ±0.5°, with error for the yaw and pitch angles estimated to equal or less than 0.20°. The penetration and crater depths were measured perpendicular to the front face of the targets, and were estimated in 5 mm diversions with an estimated error of ±5 mm. Furthermore, the diameters of front and back face craters were estimated in 50 mm diversions. The given impact velocities were calculated from the registered arrival times of the penetrators at the locations of three short circuit screens. The measured impact velocities, yaw angles and pitch angles for the penetrators are given in Figures 5.14 and 5.15. The impact velocities for the penetrators measured with short circuit screens were in good agreement with velocities obtained from the high-speed films, with the velocities estimated from the high speed films within ±1.9% of the velocities determined by the short circuit screens. Furthermore, the measured velocities from the short circuit screen registrations were within ±2.7% of the nominal impact velocities. The average values for the pitch and yaw angles for the penetrators for these tests were approximately 1.0° and 0.5°, respectively. The penetration depths for the individual tests with normal impact conditions are compiled in Figure 5.16.
### Table 5.8 Target configuration and experimental results for NSC tests (Hansson, 2005a).

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Target type</th>
<th>Approx. age of concrete (days)</th>
<th>Target length (m)</th>
<th>Penetrator mass (kg)</th>
<th>Nose shape (CRH)</th>
<th>Impact angle (°)</th>
<th>Impact velocity (m/s)</th>
<th>Exit velocity (m/s)</th>
<th>Penetration depth (m)</th>
<th>Front face crater Diameter (m)</th>
<th>Front face crater Depth (m)</th>
<th>Back face crater Diameter (m)</th>
<th>Back face crater Depth (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002-10</td>
<td>NSC</td>
<td>240</td>
<td>1.50</td>
<td>3.65</td>
<td>8.0</td>
<td>90.0</td>
<td>420</td>
<td>--</td>
<td>0.49</td>
<td>0.55</td>
<td>0.12</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>2004-1</td>
<td>NSC</td>
<td>134</td>
<td>0.90</td>
<td>4.50</td>
<td>3.0</td>
<td>90.0</td>
<td>415</td>
<td>--</td>
<td>0.545</td>
<td>0.55</td>
<td>0.100</td>
<td>Fracture</td>
<td>--</td>
</tr>
<tr>
<td>2004-2</td>
<td>NSC</td>
<td>145</td>
<td>0.90</td>
<td>4.50</td>
<td>3.0</td>
<td>90.0</td>
<td>419</td>
<td>--</td>
<td>0.570</td>
<td>0.80</td>
<td>0.100</td>
<td>Fracture</td>
<td>--</td>
</tr>
<tr>
<td>2004-3</td>
<td>NSC</td>
<td>146</td>
<td>0.90</td>
<td>4.50</td>
<td>8.0</td>
<td>90.0</td>
<td>409</td>
<td>--</td>
<td>0.620</td>
<td>0.60</td>
<td>0.125</td>
<td>Fracture</td>
<td>--</td>
</tr>
<tr>
<td>2004-4</td>
<td>NSC</td>
<td>152</td>
<td>1.20</td>
<td>4.50</td>
<td>8.0</td>
<td>90.0</td>
<td>463</td>
<td>--</td>
<td>0.690</td>
<td>0.80</td>
<td>0.150</td>
<td>Fracture</td>
<td>--</td>
</tr>
<tr>
<td>2004-5</td>
<td>NSC</td>
<td>153</td>
<td>0.90</td>
<td>4.50</td>
<td>12.0</td>
<td>90.0</td>
<td>422</td>
<td>--</td>
<td>0.640</td>
<td>0.90</td>
<td>0.155</td>
<td>Fracture</td>
<td>--</td>
</tr>
<tr>
<td>2004-6</td>
<td>NSC</td>
<td>154</td>
<td>0.60</td>
<td>4.50</td>
<td>8.0</td>
<td>90.0</td>
<td>425</td>
<td>139</td>
<td>--</td>
<td>0.65</td>
<td>0.110</td>
<td>0.90</td>
<td>0.155</td>
</tr>
<tr>
<td>2004-25</td>
<td>NSC</td>
<td>248</td>
<td>0.54</td>
<td>4.50</td>
<td>8.0</td>
<td>59.5</td>
<td>424</td>
<td>16</td>
<td>--</td>
<td>0.90</td>
<td>0.240</td>
<td>1.05</td>
<td>0.220</td>
</tr>
<tr>
<td>2004-26</td>
<td>NSC</td>
<td>249</td>
<td>0.54</td>
<td>4.50</td>
<td>8.0</td>
<td>59.5</td>
<td>422</td>
<td>--</td>
<td>0.500</td>
<td>0.65</td>
<td>0.230</td>
<td>1.05</td>
<td>0.180</td>
</tr>
<tr>
<td>2004-19</td>
<td>NSC, RC</td>
<td>187</td>
<td>0.60</td>
<td>4.50</td>
<td>3.0</td>
<td>90.0</td>
<td>422</td>
<td>--</td>
<td>0.485</td>
<td>0.60</td>
<td>0.045</td>
<td>0.70</td>
<td>0.050</td>
</tr>
<tr>
<td>2004-20</td>
<td>NSC, RC</td>
<td>188</td>
<td>0.60</td>
<td>4.50</td>
<td>8.0</td>
<td>90.0</td>
<td>424</td>
<td>--</td>
<td>0.530</td>
<td>0.60</td>
<td>0.045</td>
<td>0.80</td>
<td>0.050</td>
</tr>
<tr>
<td>2004-23</td>
<td>NSC, RC</td>
<td>193</td>
<td>0.54</td>
<td>4.50</td>
<td>3.0</td>
<td>59.5</td>
<td>423</td>
<td>--</td>
<td>0.340</td>
<td>0.60</td>
<td>0.030</td>
<td>Fracture</td>
<td>--</td>
</tr>
<tr>
<td>2004-23b</td>
<td>NSC, RC</td>
<td>194</td>
<td>0.54</td>
<td>4.50</td>
<td>8.0</td>
<td>59.5</td>
<td>424</td>
<td>--</td>
<td>0.350</td>
<td>0.90</td>
<td>0.105</td>
<td>N/A</td>
<td>--</td>
</tr>
<tr>
<td>2004-24</td>
<td>NSC, RC</td>
<td>195</td>
<td>0.54</td>
<td>4.50</td>
<td>8.0</td>
<td>59.5</td>
<td>421</td>
<td>--</td>
<td>0.390</td>
<td>0.60</td>
<td>0.080</td>
<td>Fracture</td>
<td>--</td>
</tr>
<tr>
<td>2004-24b</td>
<td>NSC, RC</td>
<td>195</td>
<td>0.54</td>
<td>4.50</td>
<td>8.0</td>
<td>59.5</td>
<td>420</td>
<td>--</td>
<td>0.345</td>
<td>1.00</td>
<td>0.085</td>
<td>Scabbing</td>
<td>--</td>
</tr>
<tr>
<td>2004-21</td>
<td>NSC</td>
<td>155</td>
<td>1.20</td>
<td>4.50</td>
<td>8.0</td>
<td>90.0</td>
<td>424</td>
<td>N/A</td>
<td>&gt;1.20</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Note:  
1: Impact velocities were determined with short circuit screens.  
2: Secondary penetrator impact of targets.  
3: Test performed in an unreinforced NSC target without steel confinement.  
4: Penetrator recovered behind the target.  
N/A: Not applicable.
Table 5.9. Target configuration and experimental results for HPC tests (Hansson, 2005a).

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Target type</th>
<th>Approx. age of concrete (days)</th>
<th>Target length (m)</th>
<th>Penetrator mass (kg)</th>
<th>Nose shape (CRH)</th>
<th>Impact angle (°)</th>
<th>Impact velocity 3 (m/s)</th>
<th>Exit velocity (m/s)</th>
<th>Penetration depth (m)</th>
<th>Front face crater Diameter (m)</th>
<th>Front face crater Depth (m)</th>
<th>Back face crater Diameter (m)</th>
<th>Back face crater Depth (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002-1</td>
<td>HPC 97</td>
<td>240</td>
<td>1.50</td>
<td>3.65</td>
<td>8.0</td>
<td>90.0</td>
<td>411</td>
<td>--</td>
<td>0.45</td>
<td>0.75</td>
<td>0.21</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>2002-2</td>
<td>HPC 97</td>
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<td>1.50</td>
<td>3.65</td>
<td>8.0</td>
<td>90.0</td>
<td>417</td>
<td>--</td>
<td>0.47</td>
<td>0.65</td>
<td>0.19</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>2002-3</td>
<td>HPC 97</td>
<td>240</td>
<td>1.50</td>
<td>3.65</td>
<td>8.0</td>
<td>90.0</td>
<td>462</td>
<td>--</td>
<td>0.46</td>
<td>0.75</td>
<td>0.20</td>
<td>--</td>
<td>--</td>
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<tr>
<td>2002-4</td>
<td>HPC 146</td>
<td>210</td>
<td>1.20</td>
<td>3.65</td>
<td>8.0</td>
<td>90.0</td>
<td>407</td>
<td>--</td>
<td>0.38</td>
<td>0.65</td>
<td>0.25</td>
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<tr>
<td>2002-7</td>
<td>HPC 146</td>
<td>210</td>
<td>1.20</td>
<td>3.65</td>
<td>8.0</td>
<td>90.0</td>
<td>410</td>
<td>--</td>
<td>0.41</td>
<td>0.65</td>
<td>0.20</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>2002-11</td>
<td>HPC 146</td>
<td>210</td>
<td>0.45</td>
<td>3.65</td>
<td>8.0</td>
<td>90.0</td>
<td>424</td>
<td>129</td>
<td>--</td>
<td>0.70</td>
<td>0.21</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>2002-12</td>
<td>HPC 146</td>
<td>210</td>
<td>0.45</td>
<td>3.65</td>
<td>8.0</td>
<td>90.0</td>
<td>417</td>
<td>64</td>
<td>--</td>
<td>0.75</td>
<td>0.20</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>2004-8</td>
<td>HPC 133</td>
<td>174</td>
<td>0.51</td>
<td>4.50</td>
<td>8.0</td>
<td>90.0</td>
<td>421</td>
<td>136</td>
<td>--</td>
<td>0.70</td>
<td>0.150</td>
<td>N/A</td>
<td>0.210</td>
</tr>
<tr>
<td>2004-10</td>
<td>HPC 133</td>
<td>177</td>
<td>0.75</td>
<td>4.50</td>
<td>8.0</td>
<td>90.0</td>
<td>422</td>
<td>--</td>
<td>0.480</td>
<td>0.70</td>
<td>0.210</td>
<td>--</td>
<td>Fracture</td>
</tr>
<tr>
<td>2004-12</td>
<td>HPC 133</td>
<td>188</td>
<td>0.65</td>
<td>4.50</td>
<td>8.0</td>
<td>90.0</td>
<td>426</td>
<td>--</td>
<td>0.500</td>
<td>1.05</td>
<td>0.250</td>
<td>Scabbing</td>
<td>0.225</td>
</tr>
<tr>
<td>2004-13</td>
<td>HPC 133</td>
<td>189</td>
<td>0.90</td>
<td>4.50</td>
<td>3.0</td>
<td>90.0</td>
<td>422</td>
<td>--</td>
<td>0.410</td>
<td>0.65</td>
<td>0.210</td>
<td>Fracture</td>
<td>--</td>
</tr>
<tr>
<td>2004-14</td>
<td>HPC 133</td>
<td>190</td>
<td>0.90</td>
<td>4.50</td>
<td>12.0</td>
<td>90.0</td>
<td>417</td>
<td>--</td>
<td>0.495</td>
<td>0.60</td>
<td>0.165</td>
<td>Fracture</td>
<td>--</td>
</tr>
<tr>
<td>2004-15</td>
<td>HPC 133</td>
<td>191</td>
<td>0.90</td>
<td>4.50</td>
<td>8.0</td>
<td>90.0</td>
<td>457</td>
<td>--</td>
<td>0.535</td>
<td>0.85</td>
<td>0.160</td>
<td>Fracture</td>
<td>--</td>
</tr>
<tr>
<td>2004-16</td>
<td>HPC 133</td>
<td>192</td>
<td>0.90</td>
<td>4.50</td>
<td>12.0</td>
<td>90.0</td>
<td>460</td>
<td>--</td>
<td>0.560</td>
<td>0.80</td>
<td>0.190</td>
<td>Fracture</td>
<td>--</td>
</tr>
<tr>
<td>2004-36</td>
<td>HPC 133</td>
<td>196</td>
<td>0.90</td>
<td>4.50</td>
<td>3.0</td>
<td>90.0</td>
<td>456</td>
<td>--</td>
<td>0.475</td>
<td>0.80</td>
<td>0.155</td>
<td>Fracture</td>
<td>--</td>
</tr>
<tr>
<td>2004-17</td>
<td>HPC 133</td>
<td>197</td>
<td>0.66</td>
<td>4.50</td>
<td>8.0</td>
<td>90.0</td>
<td>456</td>
<td>--</td>
<td>0.540</td>
<td>0.95</td>
<td>0.165</td>
<td>Fracture</td>
<td>--</td>
</tr>
<tr>
<td>2004-18</td>
<td>HPC 133</td>
<td>218</td>
<td>0.66</td>
<td>4.50</td>
<td>12.0</td>
<td>90.0</td>
<td>455</td>
<td>--</td>
<td>0.550</td>
<td>1.20</td>
<td>0.250</td>
<td>Fracture</td>
<td>--</td>
</tr>
<tr>
<td>2004-27</td>
<td>HPC 133</td>
<td>330</td>
<td>0.45</td>
<td>4.50</td>
<td>8.0</td>
<td>59.5</td>
<td>421</td>
<td>--</td>
<td>N/A</td>
<td>N/A</td>
<td>0.25</td>
<td>Fracture</td>
<td>--</td>
</tr>
<tr>
<td>2004-28</td>
<td>HPC 133</td>
<td>331</td>
<td>0.45</td>
<td>4.50</td>
<td>8.0</td>
<td>59.5</td>
<td>456</td>
<td>--</td>
<td>N/A</td>
<td>N/A</td>
<td>0.25</td>
<td>Fracture</td>
<td>--</td>
</tr>
</tbody>
</table>

Note: 1: Back face of target from test no. 2002-1 was used for the test. 2: The penetrator fractured. 3: Impact velocities determined with short circuit screens. N/A: Not applicable.
Figure 5.14. Impact velocities for the penetrators measured with short circuit screens and determined from high-speed films.

Figure 5.15. Measured yaw and pitch angles for the penetrators.
5.2. Penetration experiments

Experiments with normal impact conditions

The normal impact experiments, with an impact angle close to 90.0°, were performed with both unreinforced and heavily reinforced NSC targets, and also unreinforced HPC 133 targets.

Unreinforced NSC targets

The tests of unreinforced NSC targets were performed with the concrete confined by a thin walled steel cylinder. Two tests were performed with CRH 3.0 penetrators, and impact velocities 415 m/s and 419 m/s, respectively. This resulted in penetration depths of approximately 0.545 m and 0.570 m for tests for test 2004-1 and 2004-2, respectively. Post-test photos of these targets are shown in Figures 5.17 and 5.18. An increased penetration depth of approximately 0.620 m was obtained for a CRH 8.0 penetrator at an impact velocity of 409 m/s for test no. 2004-3. Post-test photos of this target are shown in Figure 5.19. The impact velocity for test 2004-4 increased to 463 m/s, resulting in an increased penetration depth to approximately 0.690 m in NSC for the CRH 8.0 penetrator. The post-test condition of this target is shown in Figure 5.20. The penetrator design with a very sharp CRH 12.0 was used for penetration test no. 2004-5, resulting in a penetration depth of approximately 0.640 m at the impact velocity of 422 m/s. The post-test condition for this target is shown in Figure 5.21.

A test was performed against a 0.60 m thick target resulted in perforation of the target with an exit velocity of approximately 139 m/s obtained for an impact velocity of 425 m/s and the penetrator design with CRH 8.0. The target from this test, i.e. 2004-6, is shown in Figure 5.22.
Chapter 5. Experimental program for warhead penetration

Figure 5.17. Front and back face views of a NSC target after test no. 2004-1 are shown.

Figure 5.18. Front and back face views of a NSC target after test no. 2004-2 are shown.

Figure 5.19. Front and back face views of a NSC target after test no. 2004-3 are shown.
5.2. Penetration experiments

Figure 5.20. Front and back face views of NSC target after test no. 2004-4 are shown.

Figure 5.21. Front and back face views of NSC target after test no. 2004-5 are shown.

Figure 5.22. Perforated NSC target after test no. 2004-6, with front and back face views shown.
Unreinforced NSC target with free boundaries

The influence from the boundary effect was studied by comparing the earlier shown targets with steel confinements, with an unreinforced NSC target without confinement. The dimensions of this target were 1.20 x 1.20 x 1.20 m³. The target was completely destroyed, with the \textit{CRH} 8.0 penetrator recovered behind the target. The impact velocity for this test, i.e. 2004-21, was 424 m/s. Photos of the target before and after impact are shown in Figure 5.23, with the recovered penetrator shown in Figure 5.24. Note that this test was not included in Papers IV or V.

![Figure 5.23. The unreinforced NSC target without steel confinement is shown before and after test no. 2004-21.](image1)

![Figure 5.24. The recovered penetrator from test no. 2004-21.](image2)

Reinforced NSC targets

Two tests were performed in reinforced concrete with approximately a normal impact angle for the penetrators. The targets were 1.20×1.20 m², with a thickness of 0.60 m. The tests no. 2004-19 and 2004-20 were performed using the nose designs with \textit{CRH} 3.0 and 8.0, respectively. The penetrator with \textit{CRH} 3.0 impacted the target at a velocity of 422 m/s and penetrated to a depth of approximately 0.485 m. The penetrator with \textit{CRH} 8.0 impacted the target at a velocity of 424 m/s and penetrated to an increased depth of approximately 0.530 m. Back face scabbing with concrete removed to a depth of approximately 50 mm occurred for both tests. Post-test photos of the reinforced NSC targets are shown in Figure 5.25.
5.2. Penetration experiments

Unreinforced HPC targets

Perforation occurred for test no. 2004-8 with a 0.51 m thick HPC target and an impact velocity for the penetrator of 421 m/s, with an exit velocity of approximately 136 m/s for the CRH 8.0 penetrator. The target from this test is shown in Figure 5.26. The target thickness was increased to estimate a ballistic thickness for the HPC 133 and the penetrator design with CRH 8.0 at an impact velocity of 420 m/s. The tests no. 2004-10 and 2004-12 used 0.75 and 0.65 m thick targets, respectively. The impact velocity for test no. 2004-10 was 422 m/s, resulting in a penetration depth of approximately 0.480 m. The penetration depth for test no. 2004-12 was approximately 0.500 m for an impact velocity of 426 m/s, with the penetrator visible in the back face crater of the target. These targets are shown in Figures 5.27 and 5.28.
Figure 5.26.  Post-test photos of a HPC 133 target after test no. 2004-8, with front and back faces views shown.

Figure 5.27.  Post-test photos of a HPC 133 target after test no. 2004-10, with front and back faces views shown.

Figure 5.28.  Post-test photos of a HPC 133 target after test no. 2004-12, with front and back faces views shown.
The influence of the ogive radius of the penetrator on the penetration in HPC 133 was also studied. A penetrator with CRH 3.0 was used for test no. 2004-13, which impacted the target at 422 m/s and penetrated to approximately 0.410 m. Test no. 2004-14 used a penetrator with CRH 12.0 that impacted the target at 417 m/s, resulting in a penetration depth of approximately 0.495 m. The post-test conditions of these are shown in Figures 5.29 and 5.30, respectively. Furthermore, tests were also performed at an increased impact velocity. The penetration depth for the penetrator with CRH 3.0 increased to 0.475 m at an impact velocity of 456 m/s for test no. 2004-36. Post-test photos of this target are shown in Figure 5.31. An increase of the impact velocity from 422 to 457 m/s, for the penetrator design with CRH 8.0, increased the penetration depth from approximately 0.480 m for test no. 2004-10 to approximately 0.535 m for test no. 2004-15. Furthermore, an increase of the impact velocity to 460 m/s for the penetrator design with CRH 12.0 increased the penetration depth to approximately 0.560 m for test no. 2004-16. Post-test photos of these are shown in Figures 5.32 and 5.33, respectively.

The influence of target length on the penetration depth was studied by reducing the target thickness from 0.90 m to 0.66 m for two tests at the increased nominal impact velocity of 460 m/s. The penetration depth was approximately 0.540 m for test no. 2004-17 with a CRH 8.0 penetrator and an impact velocity of 456 m/s. Test no. 2004-18 used the penetrator design with CRH 12.0 and an impact velocity of 455 m/s, resulting in a penetration depth of approximately 0.550 m. However, the influence on the penetration depths was not significantly changed with the reduced target thickness compared to the results from tests no. 2004-15 and 2004-16. The post-test conditions of targets no. 2004-17 and 2004-18 are shown in Figures 5.34 and 5.35, respectively.
Figure 5.30. Post-test photos of a HPC 133 target after test no. 2004-14, front and back face views.

Figure 5.31. Post-test photos of a HPC 133 target after test no. 2004-36, front and back face views.

Figure 5.32. Post-test photos of a HPC 133 target after test no. 2004-15, front and back face views.
5.2. Penetration experiments

Figure 5.33. Post-test photos of a HPC 133 target after test no. 2004-16, front and back face views.

Figure 5.34. Post-test photos of a HPC 133 target after test no. 2004-17, front and back face views.

Figure 5.35. Post-test photos of a HPC 133 target after test no. 2004-18, front and back face views.
Tests with oblique impact angle

Penetration tests were performed with an oblique impact angle for the penetrator, with an angle equal to 59.5° ±0.5° between the target’s front face and the velocity vector for the penetrator. The tests with oblique impacts were performed with targets of unreinforced NSC, unreinforced HPC 133 and heavily reinforced NSC.

Unreinforced NSC targets

The unreinforced NSC targets impacted at an oblique angle by CRH 8.0 penetrators had an increased diameter of 1.50 m, with a thickness of 0.54 m. The impact velocity for test no. 2004-25 was 424 m/s, this resulted in perforation of the target and an exit velocity of approximately 16 m/s for the penetrator. The penetrator was stopped near the back face of the target for test no. 2004-26, with an almost identical impact velocity of 422 m/s. The angle between the front face of target and the stopped penetrator was approximate 40° for this test. These targets are shown in Figure 5.36, with the recovered penetrator from test no. 2004-25 shown in Figure 5.37.

Figure 5.36. The unreinforced NSC targets shown after tests no. 2004-25 (a) and 2004-26 (b) with oblique impacts of the penetrators. Front views of the targets are shown to the left, with back faces shown to the right.
Figure 5.37. The recovered penetrator from test no. 2004-25 with oblique impact conditions.

Reinforced NSC targets

The influence of reinforcement on the penetration performance was also studied for the case of the oblique impact condition, with the same thickness of 0.54 m used for both the reinforced and unreinforced NSC targets. The heights of the reinforced NSC targets were 1.50 m, with widths equal to 1.20 m. Two tests were performed in each of the two reinforced NSC targets, with the impact points for the penetrators moved 0.25 ±0.01 m sideways before the second test.

The CRH 3.0 penetrator design was used for two tests in target no. 2004-23. The first test was designated as test no. 2004-23, with the second test designated as 2004-23b. These two tests were not presented in Papers IV or V. The impact velocity for the first test was 423 m/s, which resulted in a penetration depth of approximately 0.340 m/s. The angle between the front face of the target and the stopped penetrator was approximately 43° for this test. Furthermore, the impact velocity for the second test was 424 m/s, which resulted in a penetration depth of approximately 0.350 m. The angle between the front face of the target and the second stopped penetrator was approximately 39° for this test. The penetration depths were measured perpendicular to the front face of the target. A major fracture occurred in the penetrator used for test no. 2004-23, and the penetrator from second test no. 2004-23b was broken into two pieces approximately 280 mm from the nose. Post-test photos of the targets are shown in Figures 5.38 and 5.39, with the recovered penetrators shown in Figure 5.40.

Target no. 2004-24 was used for two tests with CRH 8.0 penetrators. The impact velocity for the first test no. 2004-24 was 421 m/s, resulting in a penetration depth of approximately 0.390 m. The angle between the front face of the target and the stopped penetrator was approximately 46° for this test. The impact velocity was 420 m/s for the second test, i.e. test no. 2004-24b, which resulted in a penetration depth of approximately 0.345 m. The angle between the front face of the target and the second stopped penetrator was approximately 29°. This secondary test in the target was not presented in Papers IV or V. The penetration depths were measured perpendicular to the front face of the target as for target no. 2004-23. The penetrators from both these tests were broken into two pieces, with the penetrator from test no. 2004-24 fractured approximately 100 mm from the nose and the penetrator from test no. 2004-24b fractured approximately 320 mm from the nose. Post-test photos of the target are shown in Figures 5.41 and 5.42, with the recovered penetrators shown in Figure 5.43.
Figure 5.38. The reinforced NSC target after test no. 2004-23 with an oblique impact of the penetrator, front and back face views are shown.

Figure 5.39. The reinforced NSC target after test no. 2004-23b with an oblique impact of the penetrator, front and back face views are shown.

Figure 5.40. The recovered penetrators from tests no. 2004-23 (top) and 2004-23b (bottom).
5.2. Penetration experiments

Figure 5.41. The reinforced NSC target after test no. 2004-24 with an oblique impact of the penetrator, front and back face views are shown.

Figure 5.42. The reinforced NSC target after test no. 2004-24b with an oblique impact of the penetrator, front and back face views are shown.

Figure 5.43. The recovered penetrators from tests no. 2004-24 (top) and 2004-24b (bottom).
Unreinforced HPC targets

Two tests of oblique impacts of HPC 133 targets were performed at impact velocities 421 and 456 m/s with similar results for the tests. The penetrator was recovered in front of the target for test no. 2003-27 performed at the lower velocity, and the penetrator was recovered inside the front face crater for the second test performed with the increased impact velocity. Post-test photos of the HPC targets from these tests are shown in Figures 5.44 and 5.45, with the recovered penetrators shown in Figure 5.46. These tests were not presented in the appended Papers IV or V.

Figure 5.44. The unreinforced HPC 133 target after test no. 2004-27 with an oblique impact of the penetrator, front and back face views are shown.

Figure 5.45. The unreinforced HPC 133 target after test no. 2004-28 with an oblique impact of the penetrator, front and back face views are shown.
Figure 5.46. The recovered penetrators from tests no. 2004-27 (top) and 2004-28 (bottom).
Chapter 6

Modelling of warhead penetration

Penetration of HBTPs in concrete targets were analysed with both empirical penetration models and FE analyses. Furthermore, FE analyses of a modified artillery shell impacting an HPC target are presented separately in Appendix B. This earlier work includes complementary evaluations of basic numerical concepts for 2D and 3D FE analyses of artillery warhead penetration in concrete targets. This part of the project was necessary to determine suitable modelling techniques for the FE analyses of the HBTPs.

6.1. Empirical penetration models

This section on empirical penetration models is an extension of the evaluation of empirical penetration models presented in Paper IV, with an increased number of empirical penetration models presented and evaluated. As discussed in Paper IV, empirical or semi-empirical penetration models have been used to study penetration of different types of projectiles in various materials for a long time. The semi-empirical models are normally based on a derived target resistance to the projectile penetration, e.g. by cavity expansion theory. This methodology allows for the incorporation of non-linear material properties of the target material, e.g. the influence of the compaction for the concrete. A pure empirical model may not include any physical behaviour for the target, and can be created by a statistical analysis of selected penetration data. These two types of penetration models are sometimes not easily distinguished from each other and the boundary between the two types is not well defined since the models may be more or less physically based. The empirical, or semi-empirical, penetration models are normally calibrated versus an easily measured strength parameter, e.g. the uniaxial compressive strength. These two types of models are therefore both referred to as empirical penetration models herein. Furthermore, this is only a brief overview of empirical penetration models and not a comprehensive evaluation of the existing penetration models. Empirical penetration equations for concrete penetration have been evaluated in several studies, e.g. by NDRC (1946), Amirikian (1950), Kennedy (1976), Sliter (1980), Adeli and Amin (1985), Corbett et al. (1996), Li and Chen (2003), Li and Tong (2003) and Chen et al. (2008).
Penetration in concrete protective structures is a complex problem to analyse due to the non-linear mechanical properties of concrete subjected to large deformations, high pressures and high strain rate deformations. The inhomogeneity of concrete also makes it difficult to determine representative macroscopic material properties. Furthermore, the tensile stresses within a concrete structure are transferred to the adjacent reinforcement bars used within concrete structures to reduce tensile cracking. The projectiles impacting a protective structure can be of quite different types, e.g. fragments from detonating warheads, artillery shells or air-delivered bombs, with varied geometrical characteristics and impact conditions. Furthermore, industrial accidents may also generate various types of missiles that may impact concrete structures. It is therefore not surprisingly that a large number of empirical penetration formulas exist for concrete. However, this methodology should be used with care since the results from different penetration models may be substantially different (Kennedy, 1976, Sliter, 1980, Li and Chen, 2003, Li et al., 2005). Empirical penetration models normally consider the projectile as a rigid body with only one translation degree of freedom, i.e. along its longitudinal axis. This set constrains on the various types of penetration cases that are possible to study, e.g. deforming projectiles and non-normal projectile impacts further add to the complexity of the penetration phenomena.

The major part of the existing empirical penetration models was calibrated versus the uniaxial compressive of the used concrete, which may cause a problem since different age of the concrete and different test sample geometries will give different compressive strengths. However, a compressive concrete strength at the time of testing is often used for the penetration models. Furthermore, the concrete strength is normally determined on concrete cylinders with a height to diameter ratio of 2.0 and with a diameter of 150 to 200 mm. This may cause a problem for testing of HPC due to the great compressive force needed to obtain the failure stress, and a smaller diameter for the test samples may be required to perform uniaxial compressive testing. However, it is not obvious that the uniaxial compressive strength of concrete is a good measure of the penetration resistance for a concrete structure. The penetration phenomenon in concrete structures may be divided into three major parts, e.g. penetration during the formation of the front crater, deep penetration in concrete without any major influence from the free surfaces and with the last stage creating an exit crater at the back face of the target. The major deformation state of the concrete cannot be considered to be represented by a uniaxial compressive stress state, instead the formation of the front and back face craters are likely to be strongly influenced by the tensile strength and shear strength at low confinement pressure, and with the behaviour of the confined concrete relating to the penetration resistance in between. However, these data are seldom available for the used concrete and it will not be practical to use these types of data for calibration of penetration models, hence the need to calibrate the models versus the static uniaxial compressive strength for the concrete. Furthermore, the penetration resistance during crater formation is relative low due to
the limited amount of energy required to create the craters, compared to the energy needed to crush concrete during deep penetration in a concrete structure.

Data from the experimental program described in Chapter 5 was used for an evaluation of empirical penetration models. The studied penetration cases were limited to normal impact conditions of the penetrator and tests without perforation of the targets. However, the data included NSC and the two types of HPC. The geometry of hardened buried target penetrators falls outside the validated range for the major part of the existing empirical penetration models, and therefore these models are not likely to predict the behaviour of these penetrators. The experimental program showed that the crater formation could be reduced by a careful design of a heavily reinforced target. Furthermore, it is likely that the influence on the penetration depth or exit velocity for a projectile from the use of reinforcement may be neglected in many cases, and to incorporate the influence of reinforcement on the penetration performance of projectile into concrete structures would further stretch the complicity of empirical penetration models. Furthermore, it should be noted that empirical penetration models often are dimensionally incorrect, and several of the existing models were developed for imperial units. Table 6.1 gives the conversation factors to SI units for commonly used Imperial units.

<table>
<thead>
<tr>
<th>Property</th>
<th>Imperial unit</th>
<th>SI unit conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>1 inch (in)</td>
<td>$25.4 \times 10^{-3}$ m</td>
</tr>
<tr>
<td>Mass</td>
<td>1 pound (lb)</td>
<td>0.453 kg</td>
</tr>
<tr>
<td>Velocity</td>
<td>1 ft/s</td>
<td>0.3048 m/s</td>
</tr>
<tr>
<td>Force</td>
<td>1 lbf</td>
<td>4.448 N</td>
</tr>
<tr>
<td>Pressure</td>
<td>1 psi</td>
<td>$6.895 \times 10^{3}$ Pa</td>
</tr>
</tbody>
</table>

Several empirical penetration models were based on work performed during World War II, e.g. the National Defense Research Committee (NDRC, 1946) proposed a model for penetration of non-deforming projectiles in massive concrete structures. However, this model included an undefined penetrability factor $K_{NDRC}$. This penetrability factor was a function of the concrete strength, and the original model had a limited ability to predict penetration in concrete. Typical experimental data sets used to determine the penetrability factor $K_{NDRC}$ resulted in values between 2.0 and 5.0. The penetration equations for this NDRC model are given as:
with the nose shape factor $N_{NDRC}$ estimated from the following:

$$N_{NDRC} = 0.72 + 0.25\sqrt{CRH-0.25}$$

(6.3)

An expression for the penetrability function $K_{NDRC}$ was obtained by Kennedy (1966, 1976) by fitting:

$$K_{NDRC} = \frac{180}{\sqrt{f_c}}$$

(Imperial units) \hspace{1cm} (6.4)

$$K_{NDRC} = 14.9 \times 10^3 \sqrt{f_c}$$

(converted to (SI units)) \hspace{1cm} (6.5)

to available penetration data, with the improved penetration model referred to as the “Modified NDRC formula”. The values for the nose shape factor $N$ according to Kennedy (1976) are given in Table 6.2. This model has been used extensively for different penetration cases.

**Table 6.2. Nose shape factor $N_{NDRC}$ for the modified NDRC penetration model (Kennedy, 1976).**

<table>
<thead>
<tr>
<th>Penetrator nose geometry</th>
<th>Nose shape factor, $N_{NDRC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat</td>
<td>0.72</td>
</tr>
<tr>
<td>Hemispherical</td>
<td>0.84</td>
</tr>
<tr>
<td>Blunt</td>
<td>1.00</td>
</tr>
<tr>
<td>Very sharp nose</td>
<td>1.14</td>
</tr>
</tbody>
</table>

Another early empirical penetration model was developed by the Army Corps of Engineers (ACE) in 1946 (Kennedy, 1976). The penetration depth is independent of nose shape and is given as:
6.1 Empirical penetration models

\[ \frac{X}{d} = \frac{282 M}{d^{2.785} \sqrt{f_c}} \left( \frac{V_{\text{impact}}}{1000} \right)^{1.5} + 0.5 \]  
(Imperial units) \hspace{1cm} (6.6)

\[ \frac{X}{d} = \frac{11.1 \cdot M}{d^{2.785} \sqrt{f_c}} \left( \frac{V_{\text{impact}}}{1000} \right)^{1.5} + 0.5 \]  
(converted to SI units) \hspace{1cm} (6.7)

These formulas have their heritage from work performed during World War II and are still in use today. However, it may be difficult to determine their ability to predict penetration for general penetration cases. The predicted penetration depths for the NSC tests calculated by the NDRC and ACE penetration models are given in Figures 6.1 and 6.2, respectively.

Figure 6.1. The ratio between predicted and measured penetration depths according to the modified NDRC formula, with a nose shape factor equal to 1.14 used for all the tests, resulting in an average value for the tests equal to 57%. 

\( X \) and \( d \) are the penetration depth and the diameter of the penetrator, respectively. \( m \) is the mass of the penetrator, and \( f_c \) is the characteristic fracture toughness of the target material. The impact velocity \( V_{\text{impact}} \) is the velocity of the penetrator at the time of impact.

The graph shows the ratio of predicted to measured penetration depths for various test conditions, with different symbols representing different mass and target combinations. The average ratio is 57%, indicating the accuracy of the formula in predicting penetration.
Chapter 6. Modelling of warhead penetration

The empirical penetration models used at Sandia National Laboratories were compiled by Young (1997). The influences on the penetration depth from the amount of reinforcement in the target, cure time for the concrete and the ratio of target to projectile diameter are for example considered within the Sandia penetration model. However, it was noted that 11 vol-% of reinforcement would result in a zero penetration depth, and it is not likely that the model is intended for use with reinforcement content of a few volumetric percentages although no validated range is given. Young’s S-number for concrete is defined as:

\[ S_{\text{Young}} = 0.085 K_e (0.11 - R_{\text{steel}})(t_c T_e)^{-0.06} (35 \times 10^5 / f_c)^{0.3} \]  

(6.8)

where \( R_{\text{steel}} \) is the volumetric content of reinforcement steel, \( t_c \) is the cure time for the concrete in years and \( T_e \) is the thickness of the target given in projectile diameters. The parameter \( t_c \) should be given equal to 1.0 for targets older than one year, and \( T_e \) is equal to 6.0 for target a thickness greater than 6.0 projectile diameters. Furthermore, the equation may be inadequate for \( T_e < 0.5 \). A default value for Young’s S-number equal to 0.9 is recommended for targets were the data is insufficient to permit the calculation of the value for this parameter. There are only limited validations performed for Eq. (6.8) for very high strength concrete with a uniaxial compressive concrete strength greater than 124 MPa. The function \( K_e \) increases the penetration depth for small targets, and for reinforced targets it is given as:
6.1. Empirical penetration models

\[
K_e = \begin{cases} 
(20d/W_e)^{0.3} & \text{for } W_e/d < 20 \\
1 & \text{for } W_e/d \geq 20
\end{cases} 
\]  
\hspace{2cm} (6.9a)
\[
K_e = \begin{cases} 
(30d/W_e)^{0.3} & \text{for } W_e/d < 30 \\
1 & \text{for } W_e/d \geq 30
\end{cases} 
\]  
\hspace{2cm} (6.9b)

with \( W_e \) equal to the target width and \( d \) the diameter of the projectile. The function \( K_e \) for an unreinforced target is given as:

\[
K_e = \begin{cases} 
(30d/W_e)^{0.3} & \text{for } W_e/d < 30 \\
1 & \text{for } W_e/d \geq 30
\end{cases} 
\]  
\hspace{2cm} (6.10a)
\[
K_e = \begin{cases} 
1 & \text{for } W_e/d < 20 \\
(20d/W_e)^{0.3} & \text{for } W_e/d \geq 20
\end{cases} 
\]  
\hspace{2cm} (6.10b)

The nose performance coefficient for the Sandia penetration model is given as:

\[ N_{Young} = 0.18(CRH - 0.25)^{0.5} + 0.56 \]  
\hspace{2cm} (6.11)

The penetration depth in concrete is then given as:

\[ X_{Young} = 18 \times 10^{-6} S_{Young} N_{Young} \left( \frac{4M}{\pi d^2} \right)^{0.7} (V_{impact} - 30.5) \quad \text{for } V_{impact} \geq 61 \text{ m/s} \]  
\hspace{2cm} (6.12)

with parameter \( M \) being the mass of the projectile, \( V_{impact} \) the impact velocity, and \( d \) the diameter of the projectile. Furthermore, the penetration depths for lightweight penetrators with a mass less than 182 kg are obtained by multiplying the calculated penetration depth \( X_{Young} \) with the correction factor \( K_h \) given as:

\[ K_h = 0.46M^{0.15} \quad \text{for } M < 182 \text{ kg} \]  
\hspace{2cm} (6.13)

The predicted penetration depths for the model scaled penetrators were considerably reduced by this factor, with the predicted penetrations for the test shown in Figure 6.3.
Chapter 6. Modelling of warhead penetration

Figure 6.3. The ratio between predicted and measured penetration depths according to Young (1997), with an average value for the tests equal to 121%.

Physically based empirical penetration models also exists as noted earlier, e.g. with the use of cavity expansion theory and its ability to include physical material behaviour to describe a target during the penetration of a projectile. Cavity expansion theory was for example used by Forrestal et al. (1994) to develop an empirical penetration model. This model was later modified by Frew et al. (1998) to include the influence of concrete strength, with further modifications proposed by Li and Chen (2003). Furthermore, the interface friction between the projectile and target is neglected for this model. The version of the model according to last reference is given below, and this model was used for a preliminary evaluation of the penetration tests with model scaled HBTPs. The nose factor $N^*$ for a penetrator with ogive nose is defined as a function of the calibre to radius head $\psi$ as:

$$N^* = \frac{1}{3\psi} - \frac{1}{24\psi^2}$$

with $0 < N^* \leq \frac{1}{2}$  

(6.12)

The impact factor $I^*$ for the penetrator is defined as:

$$I^* = \frac{MW_{\text{Impact}}^2}{d^3f_c}$$

(6.13)

where $M$ is the penetrator’s mass, $V_{\text{Impact}}$ is the impact velocity of the penetrator, $d$ is the penetrator’s diameter and $f_c$ is the compressive strength of the concrete. The geometry function $N$ is defined as:
6.1. Empirical penetration models

\[ N = \frac{M}{\rho_c d^3 N^*} \]  \hspace{1cm} (6.14)

where \( \rho_c \) is the density of the concrete. The empirical function \( S \) versus the uniaxial concrete strength \( \hat{f}_c \) given in MPa was estimated from penetration tests performed on grout and concrete with compressive strengths from 14 to 97 MPa (Frew et al., 1998):

\[ S = 82.6 \hat{f}_c^{-0.544} \]  \hspace{1cm} (6.15)

The impact function \( I \) is then defined as:

\[ I = \frac{I^*}{S} \]  \hspace{1cm} (6.16)

The dimensionless crater depth \( k \) is a function of the length of the nose section \( (H) \) and the diameter of the penetrator (Li and Chen, 2003), as:

\[ k = \left(0.707 + \frac{H}{d}\right) \]  \hspace{1cm} (6.17)

Specially, \( k = 2.37 \) for an ogive nose with \( CRH 3.0 \), \( k = 3.49 \) for an ogive nose with \( CRH 8.0 \) and \( k = 4.13 \) for an ogive nose with \( CRH 12.0 \). The dimensionless penetration depth is finally given as:

\[ \frac{X}{d} = \begin{cases} \sqrt{\frac{4kI/\pi}{(1+I/N)}} & \text{for } \frac{X}{d} \leq k \\ \frac{2}{\pi} N \ln \left[ \frac{1+I/N}{1+k \pi / 4N} \right] + k & \text{for } \frac{X}{d} > k \end{cases} \]  \hspace{1cm} (6.18a/b)

The original penetration model for assumes a constant crater depth of \( 2d \) for penetration depths greater than the crater depth (Forrestal et al., 1994), and is obtained by substituting the variable \( k \) with a constant value of 2.0 in Eq. (6.18). As a result, the two equation sets predict the same penetration depths for this nose shape for an ogive nose with \( CRH 2.0 \). An alternative optimisation for the empirical resistance function \( S \) versus concrete compressive strength was proposed in Paper IV as:

\[ S_{HBTP} = 130.3 \hat{f}_c^{-0.733} \]  \hspace{1cm} (6.19)

The empirical resistance function \( S \) calculated from the experimental data sets and according to Eqs. (6.15) and (6.19) are shown in Figure 6.4.
Chapter 6. Modelling of warhead penetration

The dimensionless empirical constant $S$ versus the unconfined compressive concrete strength (Paper IV).

The ratios of calculated and experimental determined penetration depths are shown in Figures 6.5 and 6.6 for the original $S$ value and $S_{HTBP}$, respectively. Improved estimations of the penetration depths were obtained with the modified values for the empirical function according to the Eq. (6.19), with the calculated penetration depths for the tests given in Figure 6.7.

Figure 6.4. The dimensionless empirical constant $S$ versus the unconfined compressive concrete strength (Paper IV).

Figure 6.5. The ratio between predicted penetration depths according to Eq. (6.18b) and experimental penetration depths, with the empirical parameter $S$ according to Eq. (6.15) (Paper IV), with an average value for the tests equal to 77%. A value of $k$ equal to 1.0 was used for the calculation of the penetration depths in the reinforced concrete targets.
6.1. Empirical penetration models

Figure 6.6. The ratio between calculated penetration depths according to Eq. (6.18b) and experimental penetration depths, with the empirical parameter $S_{HBTP}$ according to Eq. (6.19) (Paper IV), with an average value for the tests equal to 104%. A value for $k$ equal to 1.0 was used for the calculation of the penetration depths in the reinforced concrete targets.

Figure 6.7. Calculated penetration depths according to Eq. (6.18b) and experimental penetration depths, with the modified empirical parameter $S_{HBTP}$ according to Eq. (6.19). A value for $k$ equal to 1.0 was used for the calculation of the penetration depths in the reinforced concrete targets.
Chapter 6. Modelling of warhead penetration

6.2. Numerical penetration analyses

Investigations of highly dynamic events like projectile penetration, or loading by blast waves, are often supported by numerical simulations. These calculations are usually performed with explicit FE analysis. The aim for this part of the study was to evaluate the possibilities of using numerical simulations for future predictions of penetration of warheads in concrete structures. Furthermore, preliminary results from FE analyses of the HBTP experiments has been presented earlier (Hansson, 2005b).

6.2.1. Modelling of reinforcement bars

This section briefly discuss the modelling techniques for the inclusion of reinforcement into the NSC targets, with the rebars modelled at beam elements fully coupled to the concrete material. A piece-wise strength model was used for the description of the reinforcement bars, see Figure 6.8. A modified version of the Johnson and Cook (1983) strength model with a piece-wise strain hardening relationship instead of the original strain hardening function was used for the reinforcement (Paper V). Furthermore, the parameters for the strain rate dependency and thermal softening for the reinforcement steel were taken as data for AISI 4340 steel (Johnson and Cook, 1983). The Johnson and Cook (J&C) material model and variations of this constitutive model are further described in Appendix A.

Figure 6.8. Typical measured stress-strain relationship for 16 mm B500BT type 1 reinforcement bar shown with extended data for an assumed failure strain equal to 0.5 (Paper V).
Furthermore, the original implementation of the J&C strength model uses a lower cut off equal to 1.0 s\(^{-1}\) for the influence of the strain rate on the material behaviour. However, a modified strain rate dependency should be used for FE analysis of structural response, e.g. for reinforcement bars. This methodology was applied in Paper III, with solid elements for the rebars and a reduced coupling to the concrete for the reinforcement.

The dynamic properties of reinforcement steels were thoroughly discussed by Malvar and Crawford (1998), with the following equation given to estimate the dynamic increase factors (\(DIF_{\text{steel}}\)) for yield and ultimate strength for reinforcement steel:

\[
DIF_{\text{steel}} = \left( \frac{\dot{\varepsilon}}{1 \times 10^{-4}} \right)^{\alpha_{\text{steel}}} 
\]  

(6.20)

where for the yield stress;

\[
\alpha_{\text{steel},f_y} = 0.074 - 0.04 \frac{f_y}{414 \times 10^6} 
\]  

(6.21a)

and for the ultimate yield stress;

\[
\alpha_{\text{steel},f_u} = 0.019 - 0.009 \frac{f_u}{414 \times 10^6} 
\]  

(6.21b)

The formulation is valid for strain rates between 10\(^{-4}\) and 225 s\(^{-1}\) and for rebars with static yield strengths (\(f_y\)) between 290 and 710 MPa.

An engineering failure strain of 0.50 was assumed for the reinforcement bars. This corresponds to a measured elongation at failure for a gauge length of approximately 1.2 times the rebar diameter according to Figure 6.9. An example of a failed reinforcement bar with developed necking is shown in Figure 6.10. It is likely that only the beam elements close to a projectile path are subjected to large deformations during penetration of a projectile, with a small influence on the global response of the failure conditions for the reinforcement steel. Furthermore, only a few beam elements are likely to be subjected to high strain rate deformations due to the rapid decrease of the strain rate with increasing distance from the projectile path.
Figure 6.9. The measured engineering failure strain vs. the initial measurement length given in rebar diameters for a B500BT type 1 reinforcement bar. The diameter of the tested rebar was 16 mm.

Figure 6.10. A failed 16 mm B500BT type 1 grade rebar after tensile testing.

6.2.2. FE analyses of HBTP experiments

The FE analyses of the HBTP experiments in NSC were performed with the nominal impact velocities of the 50 mm model scaled penetrators, i.e. at 420 and 460 m/s. The FE analyses in Paper V all refer to test series no. 2004, with FE analyses of the additional test no. 2002-10 also presented here. The nose design with CRH 8.0 closets resembles the nose of a modern HBTP, and this design was therefore chosen for all the FE analyses. However, FE analyses were performed for both 5.0 and 10.0 mm casing thickness for the penetrators, with the 5.0 mm case thickness used for test no. 2002-10. The analyses were performed with Autodyn 3D (Century Dynamics, 2005a), with the use of 8-node single integration point Lagrange elements for both the targets and the penetrators. The use of this element formulation for the concrete requires that heavily distorted elements needs to be removed during penetration simulations, and an erosion strain equal to 2.0 was used to avoid numerical problems associated with degenerated elements. The RHT material model was used to
model the NSC (Riedel, 2000), with the material parameters modified to better describe the used concrete batch according to sections 4.2 and 4.3.

The yaw and pitch angles for the penetrators were neglected for the FE analyses, which allowed for the use $\frac{1}{4}$ and $\frac{1}{2}$ symmetries for the unreinforced targets to reduce the number of elements. However, the use of symmetry conditions set restrictions on the displacements along the symmetry planes, but this was assumed to not significantly influence the global response for simulations of NSC. The meshes and material locations for the penetrators are shown in Figure 6.11, with approximately 900 elements used for the $\frac{1}{2}$ symmetry model. Furthermore, the material parameters for the J&C material model for the steel casing are taken as AISI S-7 tool steel which is of similar hardness and strength. The data for AISI S-7 steel according to Johnson and Cook (1983) are given in Appendix A.1. Furthermore, a shear modulus equal to 81.8 GPa and a reference temperature of 300 K were also used for the penetrator casing. However, at the initial contact between the penetrator and the target only one node in the penetrator was in contact with the target surface. This results in severe deceleration of this node and a risk of a progressive failure of the elements in the penetrator. The yield strength parameter $A$ for the J&C strength model was therefore increased from the original value of 1.59 to 2.5 GPa for the first two element rows in the penetrator. Note that the simulation of the penetrator with the lower mass and thinner casing corresponding to test no. 2002-10 was not included in Paper IV.

![Figure 6.11](image.png)

**Figure 6.11.** Mesh and material location for projectiles with 5.0 and 10.0 mm casing thickness and mass of 3.64 and 4.53 kg, respectively. The element rows in the nose with increased yield strength for the steel are shown in a darker shade of grey in the cross sections.

The interaction between the concrete target and the penetrator is one of the most difficult parts of a penetration model to define, and not much work has been performed within this area. In many cases the friction is neglected, and this might be justified for projectiles with small length to diameter ratios or at high impact velocities, see for example Appendix B. A friction coefficient equal to 0.05 was used for the major part of the simulations, with a frictionless model also analysed for
comparison and to verify that extensive erosion of the targets was not introduced due to shear forces at the interface between target and penetrator.

**Penetration in unreinforced NSC**

A mesh size equal to 5.0 mm in the centre of the targets was considered adequate for the FE analyses. Furthermore, simulations with a coarser target mesh using 10 mm elements were also performed. However, an increased mesh density with the use of substantial smaller element size than 5.0 mm was not considered practical for this initial study. The geometries for the unreinforced targets are given in Table 6.3. It is difficult to obtain a full coupling between the concrete and the steel cylinder for the experiments, e.g. due to the risk for shrinkage of the concrete during curing. The simulation models therefore used a reduced steel thickness equal to 5.0 mm for the confining thin steel cylinders to consider this reduced stiffness.

Table 6.3. Geometries and properties for unreinforced NSC target FE analyses.

<table>
<thead>
<tr>
<th>Simulation case</th>
<th>FEA1</th>
<th>FEA2</th>
<th>FEA0, FEA3, FEA4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impact angle (°)</td>
<td>90.0</td>
<td>90.0</td>
<td>60.0</td>
</tr>
<tr>
<td>Symmetry condition</td>
<td>½</td>
<td>½</td>
<td>¼</td>
</tr>
<tr>
<td>Target length (m)</td>
<td>1.20</td>
<td>0.60</td>
<td>0.54</td>
</tr>
<tr>
<td>Target diameter (m)</td>
<td>1.20</td>
<td>1.20</td>
<td>1.50</td>
</tr>
<tr>
<td>No. of brick elements</td>
<td>1 094 400</td>
<td>1 648 510</td>
<td>820 800</td>
</tr>
<tr>
<td>Average time step (µs)</td>
<td>35×10⁻³</td>
<td>35×10⁻³</td>
<td>35×10⁻³</td>
</tr>
</tbody>
</table>

Normal impact of unreinforced NSC

Simulations of normal impacts of unreinforced NSC were performed both for 0.60 and 1.20 m thick targets, with the results from the FE analyses compiled in Tables 6.4 and 6.5, respectively. The influence of the used tensile strength for the concrete was studied by increasing the tensile strength from 4.0 to 4.8 MPa. This increase of the tensile strength resulted in a minor decrease equal to 4% of the penetration depths in the 1.20 m long target for an impact velocity of 420 m/s for simulation cases FEA0 and FEA1, with 3.64 and 4.53 kg penetrators, respectively. Furthermore, the calculated penetration performance was approximately 15% less for the penetrator with the lower mass for both the used concrete tensile strengths. A damage and deformation plot for model no. FEA0-01 with the 3.64 kg penetrator and a penetration depth of 0.450 m is shown in Figure 6.12. The influence on the projectile deceleration of projectile mass and impact velocity for FE analyses of a 1.20 m NSC target are shown in Figure 6.13. Furthermore, the penetration depth was increased from 0.527 m for the base parameter model FEA1-01 to 0.609 m for simulation no. FEA1-03 without friction between the target and the penetrator.
Table 6.4.  **FE analyses of penetration in unreinforced NSC targets with a length of 1.200 m, with a 90.0° impact angle and ¼ symmetry used for all models.**

<table>
<thead>
<tr>
<th>Simulation identity</th>
<th>FEA0-01</th>
<th>FEA0-02</th>
<th>FEA3-01</th>
<th>FEA03-02</th>
<th>FEA3-03</th>
<th>FEA04-01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element size (mm)</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Projectile mass (kg)</td>
<td>3.64</td>
<td>3.64</td>
<td>4.53</td>
<td>4.53</td>
<td>4.53</td>
<td>4.53</td>
</tr>
<tr>
<td>Impact velocity (m/s)</td>
<td>420</td>
<td>420</td>
<td>420</td>
<td>420</td>
<td>420</td>
<td>460</td>
</tr>
<tr>
<td>Reference density $\rho_0$ (kg/m³)</td>
<td>2314</td>
<td>2314</td>
<td>2314</td>
<td>2314</td>
<td>2314</td>
<td>2314</td>
</tr>
<tr>
<td>Tensile concrete strength (MPa)</td>
<td>4.0</td>
<td>4.8</td>
<td>4.0</td>
<td>4.8</td>
<td>4.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.0</td>
<td>0.05</td>
</tr>
<tr>
<td>Penetration depth (m)</td>
<td>0.450</td>
<td>0.432</td>
<td>0.527</td>
<td>0.505</td>
<td>0.609</td>
<td>0.587</td>
</tr>
<tr>
<td>Prediction error (%)</td>
<td>-8.2</td>
<td>-11.8</td>
<td>-15.0</td>
<td>-18.5</td>
<td>-1.8</td>
<td>-14.9</td>
</tr>
</tbody>
</table>

Table 6.5.  **FE analyses of perforation of 0.600 m unreinforced NSC targets, with a 90.0° impact angle and ½ symmetry used for all models.**

<table>
<thead>
<tr>
<th>Simulation identity</th>
<th>FEA1-01</th>
<th>FEA1-02</th>
<th>FEA1-03</th>
<th>FEA1-04</th>
<th>FEA1-05</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element size (mm)</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td>Projectile mass (kg)</td>
<td>4.53</td>
<td>4.53</td>
<td>4.53</td>
<td>4.53</td>
<td>4.53</td>
</tr>
<tr>
<td>Impact velocity (m/s)</td>
<td>420</td>
<td>420</td>
<td>420</td>
<td>420</td>
<td>420</td>
</tr>
<tr>
<td>Reference density $\rho_0$ (kg/m³)</td>
<td>2314</td>
<td>2330</td>
<td>2330</td>
<td>2314</td>
<td>2314</td>
</tr>
<tr>
<td>Tensile concrete strength (MPa)</td>
<td>4.0</td>
<td>4.0</td>
<td>4.8</td>
<td>4.0</td>
<td>4.8</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Exit velocity (m/s)</td>
<td>143</td>
<td>155</td>
<td>98</td>
<td>64</td>
<td>60</td>
</tr>
</tbody>
</table>
Figure 6.12. Damage and deformations for model FEA0-01 with the 3.64 kg penetrator and 4.0 MPa tensile strength for the concrete.

Figure 6.13. The influence of projectile mass and impact velocity on penetration depth for assumed semi-infinite targets.

The simulations of the 0.60 m thick target for simulation case FEA1 showed a decrease for the exit velocity from approximately 150 to 100 m/s for an increase of the tensile strength for the concrete from 4.0 to 4.8 MPa, corresponding to an approximately 9% decrease for the kinetic energy for the penetrator. Damage and deformation plots for models with different tensile concrete strength are shown in Figure 6.14. Furthermore, it was noted that there was no correlation between the increase of tensile strength for the concrete and the exit velocity of the penetrator for an increased element size to 10.0 mm, indicating that the use of the coarse grid causes numerical problems and was not
likely to produce numerical reliable results. The influence of a change of the initial density for the concrete from 2314 to 2330 kg/m$^3$ was also analysed, with only approximately 11 m/s difference of the exit velocity for simulations FEA1-01 and FEA-02. Figure 6.15 shows the velocities versus the penetrator displacements for the FE analyses of the perforation case.

Figure 6.14. Damage and deformations for models FEA1-01 (a) and FEA1-03 (b) with 4.0 and 4.8 MPa tensile strength, respectively.

Figure 6.14. Damage and deformations for models FEA1-01 (a) and FEA1-03 (b) with 4.0 and 4.8 MPa tensile strength, respectively.
Chapter 6. Modelling of warhead penetration

The angle between the front face of the target and the projectile was 60° for the FE analyses of oblique impacts of unreinforced NSC targets. The tests earlier described in Chapter 5 showed that a 4.53 kg penetrator impacting the target at a nominal velocity of 420 m/s was close to the ballistic limit for this target. An attempt was therefore made to determine the required thickness to prevent the perforation of the target with FE analyses. The simulations were performed with an element size of 5.0 mm in the central part of the 1.500 m diameters target, with a graded element size to reduce the number of elements. Furthermore, the FE mesh also used one symmetry plane to reduce the number of elements and the required simulation time.

The simulation results for oblique impacts of the unreinforced concrete targets are compiled in Table 6.6, with target thickness varying from 0.540 to 0.600 m. The target with 0.540 m thickness was of the same thickness as used for the earlier described tests, and the thickness was then increased until the projectile was stopped in the target. The damage and deformations for the models with 0.540 and 0.600 m target thickness are shown in Figure 6.16. The velocities versus projectile displacements for models with varying target thickness are shown in Figure 6.17.

Figure 6.15. Influence of element size, reference density and tensile strength for concrete.

Oblique impact of unreinforced NSC

The angle between the front face of the target and the projectile was 60° for the FE analyses of oblique impacts of unreinforced NSC targets. The tests earlier described in Chapter 5 showed that a 4.53 kg penetrator impacting the target at a nominal velocity of 420 m/s was close to the ballistic limit for this target. An attempt was therefore made to determine the required thickness to prevent the perforation of the target with FE analyses. The simulations were performed with an element size of 5.0 mm in the central part of the 1.500 m diameters target, with a graded element size to reduce the number of elements. Furthermore, the FE mesh also used one symmetry plane to reduce the number of elements and the required simulation time.

The simulation results for oblique impacts of the unreinforced concrete targets are compiled in Table 6.6, with target thickness varying from 0.540 to 0.600 m. The target with 0.540 m thickness was of the same thickness as used for the earlier described tests, and the thickness was then increased until the projectile was stopped in the target. The damage and deformations for the models with 0.540 and 0.600 m target thickness are shown in Figure 6.16. The velocities versus projectile displacements for models with varying target thickness are shown in Figure 6.17.
6.2. Numerical penetration analyses

Table 6.6. Performed FE analyses of oblique impacts of unreinforced NSC targets, with a 60.0° impact angle and $\frac{1}{2}$ symmetry used for all models.

<table>
<thead>
<tr>
<th>Simulation identity</th>
<th>FEA2-01</th>
<th>FEA2-02</th>
<th>FEA2-03</th>
<th>FEA2-04</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target length</td>
<td>(m)</td>
<td>0.540</td>
<td>0.567</td>
<td>0.589</td>
</tr>
<tr>
<td>Projectile mass</td>
<td>(kg)</td>
<td>4.53</td>
<td>4.53</td>
<td>4.53</td>
</tr>
<tr>
<td>Impact velocity</td>
<td>(m/s)</td>
<td>420</td>
<td>420</td>
<td>420</td>
</tr>
<tr>
<td>Reference density $\rho_0$ (kg/m$^3$)</td>
<td>2314</td>
<td>2314</td>
<td>2314</td>
<td>2314</td>
</tr>
<tr>
<td>Tensile concrete strength (MPa)</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td></td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Penetration depth</td>
<td>(m)</td>
<td>---</td>
<td>---</td>
<td>&gt; 0.479</td>
</tr>
<tr>
<td>Exit velocity</td>
<td>(m/s)</td>
<td>117</td>
<td>$\approx$ 57</td>
<td>&lt; 8.0</td>
</tr>
<tr>
<td>Corresponding tests</td>
<td></td>
<td>2004-25</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2004-26</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

Figure 6.16. Damage and deformations for model FEA2-01 of a 0.54 m target (a) and model FEA2-04 of a 0.60 m target (b). The models FEA2-01 and FEA2-04 are shown 4.0 and 3.9 ms after impact, respectively.
Figure 6.17. The influence of target thickness for oblique impact of unreinforced NSC targets (PaperV).

Penetration in reinforced NSC

The FE analyses of reinforced NSC were performed both with normal and oblique impact conditions for the penetrators. The reinforced concrete targets were simulated with the use of beam elements representing the reinforcement and solid Lagrange elements representing the concrete. The layout of the reinforcement was earlier described in section 5.1.3, with a simplified mesh used for the FE analyses shown in Figure 6.18a. The element size for the solid elements representing concrete was $7.5 \times 7.5 \times 3.75$ mm$^3$, with the shortest measurement in the direction of the target thickness. The beam elements representing the reinforcement bars were 15.0 and 7.5 mm, with the shortest elements used through the thickness of the target. An additional reinforcement layout with a reduced amount of reinforcement was used to study the influence of the connecting reinforcement bars between the main reinforcement layers, see Figure 6.18b. The geometries of the targets and some model properties for these FE analyses are given in Table 6.7.
Figure 6.18. The reinforcement layouts for meshes with the full reinforcement (a) and with the reduced amount of reinforcement (b) (Paper V).

Table 6.7. The target geometries and properties for FE analyses of reinforced NSC targets.

<table>
<thead>
<tr>
<th>Simulation case</th>
<th>FEA5</th>
<th>FEA6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model symmetry</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Target thickness (m)</td>
<td>0.600</td>
<td>0.540</td>
</tr>
<tr>
<td>Width and height of target (m)</td>
<td>1.20</td>
<td>1.20</td>
</tr>
<tr>
<td>No. of brick elements</td>
<td>1 792 000</td>
<td>1 612 800</td>
</tr>
<tr>
<td>No. of reinforcement bars</td>
<td>239</td>
<td>239</td>
</tr>
<tr>
<td>No. of beam elements</td>
<td>17 968</td>
<td>17 772</td>
</tr>
<tr>
<td>Projectile mass (kg)</td>
<td>4.53</td>
<td>4.53</td>
</tr>
<tr>
<td>Impact velocity (m/s)</td>
<td>420</td>
<td>420</td>
</tr>
<tr>
<td>Reference density $\rho_0$ (kg/m³)</td>
<td>2314</td>
<td>2314</td>
</tr>
<tr>
<td>Tensile concrete strength (MPa)</td>
<td>4.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Average time step (μs)</td>
<td>$35 \times 10^{-3}$</td>
<td>$35 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Normal impact of reinforced NSC.

Normal impact of the reinforced NSC target resulted in a penetration depth of 0.510 m for the FE analysis FEA5-01 with the full amount of reinforcement, and an increase of the penetration depth to 0.535 m for the model FEA5-02 with the reduced amount of reinforcement according to Figure 6.18b. Furthermore, the removal of the reinforcement entirely resulted in a perforation of the target with an exit velocity of approximately 170 m/s for the penetrator. The properties of the models are...
given in Table 6.8, with Figures 6.19 and 6.20 showing the damage and deformations for the models. The unreinforced target for model FEA5-03 shows an enlarged damage zone compared to the two reinforced targets, i.e. models FEA5-01 and FEA5-02. The velocities of the projectiles versus the penetration depth for the models with the different reinforcement amount are shown in Figure 6.21.

Table 6.8. FE analyses of the influence of reinforcement on penetration of a 0.600 m NSC target, with a 90.0° impact angle for all models.

<table>
<thead>
<tr>
<th>Simulation identity</th>
<th>FEA5-01</th>
<th>FEA5-02</th>
<th>FEA5-03</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exit velocity (m/s)</td>
<td>---</td>
<td>---</td>
<td>174</td>
</tr>
<tr>
<td>Penetration depth (m)</td>
<td>0.510</td>
<td>0.535</td>
<td>---</td>
</tr>
<tr>
<td>Corresponding test</td>
<td>2004-20</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Prediction error (%)</td>
<td>-3.8</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

Figure 6.19. Damage and deformation plots for the model FEA5-01 with the full amount of reinforcement, shown approximately 3.9 ms after impact. The figures show a cut through the model (a), the front face of the target (b), and the back face of the target (c).
6.2. Numerical penetration analyses

Figure 6.20. Damage and deformation plots for the model FEA5-02 with the reduced amount of reinforcement (a) and FEA5-03 without reinforcement (b). The models FEA5-02 and FEA5-03 are shown approximately 3.6 and 3.2 ms after impact, respectively.

Figure 6.21. The influence of the amount of reinforcement for normal impacts of NSC targets (Paper V).
Oblique impact of reinforced NSC

The calculated penetration depth for the analysis FEA6-01 of an oblique impact of the reinforced 0.54 m thick NSC target was 0.391 m, with perforation of the target obtained for analysis FEA6-02 of the NSC target with removed reinforcement. The properties for FE the analyses of oblique impacts of reinforced NSC are given in Table 6.9, with Figures 6.22 and 6.23 showing the damage and deformations for the models. The unreinforced target shows an enlarged damage zone also for this case with the oblique impact of the target, compared to the reinforced target for model FEA6-01. The velocity decreases for the penetrators within the targets are shown for the reinforced and unreinforced targets in Figure 6.24.

Table 6.9. FE analyses of the influence of reinforcement on oblique penetration of a 0.540 m NSC target, with a 60.0° impact angle for all models.

<table>
<thead>
<tr>
<th>Simulation identity</th>
<th>FEA6-01</th>
<th>FEA6-02</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reinforcement</td>
<td>Full reinforcement</td>
<td>No reinforcement</td>
</tr>
<tr>
<td>Exit velocity (m/s)</td>
<td>---</td>
<td>128</td>
</tr>
<tr>
<td>Penetration depth (m)</td>
<td>0.391</td>
<td>---</td>
</tr>
<tr>
<td>Corresponding test no.</td>
<td>2004-24</td>
<td>---</td>
</tr>
<tr>
<td>Prediction error (%)</td>
<td>0.0</td>
<td>---</td>
</tr>
</tbody>
</table>

Figure 6.22. Damage and deformation plots of the model FEA6-01 with the full amount of reinforcement. Figures (a) and (b) are shown 1.2 and 4.4 ms after impact, respectively. The same damage scale is used for the NSC as in Figure 6.20.
6.2. Numerical penetration analyses

Figure 6.23. Damage and deformation plots of model FEA6-02 with removed reinforcement. Figures (a) and (b) are shown 1.2 and 3.6 ms after impact, respectively. The same damage scale is used for the NSC as in Figure 6.20.

Figure 6.24. The influence of the amount of reinforcement for an oblique penetrator impact of 0.540 m thick NSC targets.
Chapter 7

Discussion

The discussion is divided into three parts, with the parts relating to the experimental program, empirical penetration models and FE penetration analyses.

7.1. Experimental program

7.1.1. Performance of hardened buried target penetrators

The tests in NSC and different types of HPC show that the penetration depths for this type of model scaled HBTP are approximately 0.8 to 1.6 times the length of the penetrator, depending on the concrete type and impact conditions for the penetrator. The penetration depths for the penetrators were considerably increased when the $CRH$ value increased from 3.0 to 8.0. However, a further increase of the $CRH$ to 12.0 for the sharpest nose design of penetrators only resulted in a minor influence of the penetration depths, see Figure 5.16. Furthermore, an average increase of the penetration depth by 12% was obtained for an increase of the impact velocity from the nominal velocity 420 m/s to the nominal velocity 460 m/s by approximately 10% for both unreinforced NSC and HPC 133. However, the increase in kinetic energy corresponds to approximately 20%.

The penetration resistance for the NSC targets impacted at an oblique angle of 30° from the normal of the target face for the penetrator were only slightly improved compared to the NSC targets impacted at approximately normal impact angles, see Tables 5.8 and 5.9. However, the model scaled penetrators were deflected by the targets for the oblique impact tests performed with HPC 133, see Figures 5.44 to 5.46.

The penetration depths in different types of HPC for normal impact conditions for the model scaled penetrators were reduced compared with the penetration in unreinforced NSC, see Figure 5.16. However, the decrease of the penetration depth for the unreinforced HPC 97 within series no. 2002 was only approximately 6% when compared to the NSC. The HPC 146 instead showed a reduction
of approximately 19% for the penetration depth compared to the NSC for series no. 2002. This corresponds well with the reduction in penetration of 23% obtained for the HPC 133 compared to NSC for series no. 2004. Furthermore, the actual penetration channels in the concrete were approximately 40% shorter for the HPC 133 than for NSC for test series 2004, see Tables 5.8 and 5.9.

The introduction of heavily reinforcement for the NSC targets reduces the penetration depths for the CRH 8.0 penetrator with approximately 14%, and prevented the perforation of the NSC targets both for normal and oblique impact conditions. These targets were shown in Figures 5.25b and 5.41, with the corresponding unreinforced targets shown in Figures 5.22 and 5.36. However, the location of the reinforcement bars within a concrete target in relation to the impact point of the penetrator may influence the behaviour of the warhead within the target. Furthermore, it is likely that extensive testing is required with different target configurations and impact conditions for the penetrator to establish this influence. Furthermore, the reinforcement for the NSC targets limited the craters at both the front and the back faces of the targets, and the major part of the increased penetration resistance was probably due to this prevented crater formation at the front of the target. However, the confinement of the concrete created by the reinforcement steel was also likely to contribute to the reduced penetration depth of the penetrators.

Oblique impact conditions for a penetrator are likely to increase the bending forces acting on the penetrator during penetration of a concrete target, with an increased risk for fracturing the penetrator. The penetrators that impacted the HPC 133 targets at an oblique impact angle showed considerably deformations, see Figure 5.46. However, it seems that the casing thickness of approximately 20% of the diameter for the penetrators used for test series no. 2004 was enough to avoid fracturing of the penetrator for impacts of unreinforced NSC and HPC 133. Furthermore, all penetrators that impacted the reinforced NSC targets at an oblique impact angle were subjected to large bending forces, and this resulted in fracturing of the casing with three out of four penetrators completely broken into two pieces. These penetrators are shown in Figures 5.40 and 5.43. Furthermore, the impact of a second penetrator in a damaged reinforced target resulted in similar results regarding the penetration depths as was obtained for the first projectile, see Table 5.8. However, the estimated lengths for the penetration paths of the projectiles inside the targets were increased for the second tests.

7.1.2. Experimental limitations and uncertainties

The experimental data contains results obtained for several types of concrete, different target configurations, different penetrator designs and impact conditions for the penetrators. As a result, it was necessary to reduce the number of test for each test setup, with only a few test setups used for
more than a single test. The experimental variations for the tests are therefore unknown. However, a few test setups were used for more than one experiment, with similar results for these tests.

The diameters of the concrete targets used for the model scale tests were 1.20 and 1.50 m depending on the impact conditions, with a width of 1.20 m used for the reinforced targets. This gives a relationship between target dimension and penetrator diameter of 24 and 30, which should be sufficient to avoid any significant boundary influence. Only small influences of the target size on the penetration resistance for confined 23 MPa NSC targets were obtained by Frew et al. (2006) for target to projectile diameter ratios of 12, 18 and 24. However, for an unconfined concrete target there may be considerable effects even for targets with large diameters in relation to the diameter of the penetrator. Perforation by the model scaled HBTP of a 1.2 m thick unreinforced and unconfined target was obtained for test no. 2004-21, for an impact velocity of 424 m/s for the penetrator. The relation of the target width to the diameter of the penetrator was 24 for this test, with the target shown before and after the test in Figure 5.23. The penetration depth for the corresponding confined NSC target used for test no. 2004-3 was approximately 0.62 m at an impact velocity of 409 m/s, with the target shown in Figure 5.19.

The relative part of the energy required for cratering of the target is likely to change if the size of the penetration tests changed, e.g. for a test with a full sized warhead. Furthermore, the surface areas of the fractures are scaled by a quadratic scaling law, with the energy required for plastic work or crushing of the material during penetration by a cubic scaling law. However, the energy consumption to obtain a tensile failure due to cratering is only a small part of the energy needed to crush the material during projectile penetration, and for projectiles penetrating deep within a target only a minor part of the energy of the warhead is lost during crater formation at the front and back face of the targets. Furthermore, the energy lost due to frictional work at the interface between the target and penetrator also relates to a surface effect and has a quadratic scaling law. A more crucial effect for a direct evaluation of the experimental data is the influence of the strain rate dependency of the concrete. The downsized model scaled experiments results in an increased strain rate, and thereby the experiments are likely to overestimate the penetration resistance in comparison with a full sized target. However, with the use of 50 mm penetrator diameters this phenomenon is reduced compared to small scale tests.

The mass of the cup sabot shown in Figure 5.4 was approximately 8% of the mass of the penetrator used for test series no. 2004, and because of this the penetration depths were likely to be increased if kinetic energy from the pusher plates were transferred to the penetrator. This would cause uncertainties for the measured penetration depths. However, in most cases the actual transferred kinetic energy from the pusher plate can probably be neglected. The use of expanding sabots, instead of cup sabots and attached guidance rings to the penetrators, for the guidance of the penetrators inside the gun bore eliminates this uncertainty. However, expanding sabots are more
complicate to manufacture and, and there is a higher risk of disturbing the penetrator at during the separation of sabot and penetrator. This may increase the yaw and pitch angles for the penetrators in flight, and at the moments of impacts of the targets.

### 7.2. Empirical penetration models

In general, empirical penetration equations consider concrete strength, impact velocity, projectile mass, nose design and projectile cross section. Minor influences from other factors are also considered for some formulas, e.g. the amount of reinforcement, aggregate type and ratio between projectile length and diameter. Empirical equations for penetration in concrete are suitable for a limited number of penetration cases, e.g. normal impact of semi-infinite unreinforced concrete targets. These equations normally neglect yaw and pitch of projectiles, and it may be difficult to consider oblique impact angles. Furthermore, failure and deformation of the penetrators are normally not considered for this type of empirical penetration equations, and therefore only the performance of penetrators subjected to small deformations should be studied. Furthermore, the earlier discussion regarding the strain rate effects and scaling problem related to surface effects for the experimental evaluation also applies to empirical penetration models, and this effect also needs to be considered for empirical penetration models.

A rough estimate of the penetration depth for a normal impact of this type of penetrators can be obtained by using an empirical equation. However, the penetration depths vary considerably depending on the chosen empirical penetration model, and it is necessary to verify the used penetration model against relevant experimental data. Furthermore, it was shown that the penetration depths could not be accurately predicted by a number of existing empirical penetration models. These evaluations were shown in Figures 6.1, 6.2, 6.3 and 6.5. The averaged value of the penetration predictions for these empirical models varied from 48% to 121% of the measured penetration depths. This was probably due to that the combinations of the geometries for the penetrators, impact conditions, target types and concrete types were outside the intended, or validated, parameter range for the models. Furthermore, the most promising of the evaluated empirical penetration models was modified to better describe the penetration performance of the model scaled hardened target penetrators, see Figure 6.6 and Paper IV. This resulted in an increase of the calculated average penetration depths from 77% to 104% of the experimentally obtained penetration depths for this model. However, the modified empirical model only considers normal impact conditions, and it may not give acceptable results for full size penetrators. As discussed earlier, it is likely that the relative penetration depths given in penetrator diameters increase with an increase of the penetrators scale.
7.3. FE penetration analyses

The use of FE analyses as a tool to estimate the penetration performance for warheads requires a thorough understanding of the behaviour of the material subjected to high pressures, high strain rates and large deformations. Furthermore, the chosen numerical formulations for the FE analyses are likely to influence the simulation results obtained from FE analyses. One limitation of the FE analyses was that only the NSC targets impacted by the penetrators with CRH 8.0 were analysed. This still allowed for the analyses of several different impact conditions and target designs, i.e. normal and oblique impact of both reinforced and unreinforced NSC targets. However, for most cases only one experiment was performed and the experimental error therefore is unknown.

The FE analyses of the HBTP experiments gave reasonable results for the different simulation cases, with the best results for normal impact conditions (Paper IV). As discussed in sections 4.2 and 4.3, the RHT material model contains a large number of parameters, and the values for the constants were determined partly from experimental data and partly from knowledge of the behaviour of similar concrete types. It is well known that the RHT damage evolution model is calibrated for a uniaxial compressive stress state, with a limited ability to predict the damage evolution for the concrete subjected to other stress states. The result is that the fracturing of the concrete due to of the front and back face cratering is not accurately described. Furthermore, the damage evolution near the centre of the targets is overestimated since the confinement of the concrete to some extent should delay the damage evolution due to crushing of the concrete. The FE analyses therefore generally showed unrealistic large volumes of the targets with an accumulated damage level close to unity after the penetration events, see for example Figures 6.12 and 6.14. Furthermore, the parameters for the pressure dependent yield strength of the concrete were determined for a NSC with a compressive strength of 35 MPa, and thereby it is likely that the strength of the material during the penetration was slightly overestimated. The use of a numerical erosion algorithm for removal of heavily deformed elements reduces the rotational force on the projectiles for non-normal impacts. This may have contributed to the increased error for the FE analyses of oblique impacts. Considering this, the simulation results seem quite reasonable, since it would require extensive testing to characterise a specific concrete type.

The result for simulation FEA1-01 with normal impact conditions for the model scaled HBTP showed good agreement with the experimental results from test no. 2004-6, with kinetic energy losses of the projectile given as 89% for the test and 88% for the simulation. However, for simulation FEA2-01, of an oblique impact of the target, there was only at the most a fair agreement (Paper V). The reason for this was probably due to the use of the numerical erosion algorithm for removal of heavily distorted elements. The distorted elements are located equally around the penetrator for the FE analyse FEA1-01, with a normal impact of the target. On the other hand, the
heavily distorted elements were located mainly on one side of the penetrator for the FE analyses of an oblique impact, and the rotating moment acting on a penetrator was thereby likely to be reduced due to the removal of these elements by the erosion algorithm. However, the calculated thickness for an unreinforced NSC target required to prevent the perforation of the model scaled penetrator was probably determined within 10% of the experimental results, see Figures 6.16 and 6.17.

The result for FE analyses FEA0-01, FEA3-01 and FEA4-01 of model scaled HBTPs were considered to have a fair agreement with the test results, with an average underestimation for the calculated penetration depth by 13% (Table 6.4). Furthermore, typical values for the deceleration of the model scaled penetrators for these tests during deep penetration were approximately $2.0 \times 10^5$ m/s$^2$. The FE analyses of the reinforced targets, i.e. simulations no. FEA5-01 and FEA6-01, showed good agreement with the test results and the penetration depths were estimated within 4% of the test results (Paper V), see Tables 6.8 and 6.9. Furthermore, the FE analyses of the reinforced NSC showed an increase of the penetration resistance that corresponded well to the observed experimental results, with an increased penetration resistance for the reinforced NSC compared to the unreinforced NSC targets. The FE analyses results were shown in Figures 6.19 and 6.22, with the corresponding NSC targets from the experiments shown in Figures 5.25b and 5.38. The main contribution to the increased penetration resistance of the reinforced NSC targets due to the reinforcement was likely the prevention of tensile failure in the concrete and the increased confinement of the concrete, with the direct contact between the projectile and reinforcement likely to be of secondary interest. Furthermore, the FE analyses showed that oblique impacts of penetrators resulted in increased lateral forces acting on a warhead, see for example Figure 6.22.

An erosion strain equal to 2.0 was used as the criteria to remove heavily deformed Lagrangian elements in the targets during the FE analyses. However, increasing this value may not increase the penetration resistance of a concrete target, since heavily distorted Lagrange elements may be retained and severe numerical errors may develop that effect the numerical solution. Furthermore, the removal of heavily deformed elements by the erosion algorithm may also reduce the frictional forces acting on the projectile due to a reduced pressure at the penetrator-target interface. This effect may also be more pronounced for an oblique impact of a penetrator, with the erosion of heavily deformed elements likely to reduce the lateral forces acting on the penetrator. The smooth particle hydrodynamics (SPH) or Eulerian formulations are alternative numerical methods that do not require the numerical erosion of the target material, but they have other limitations. The influences on penetration analyses of the numerical formulations for concrete targets are further addressed in Appendix B, with respect to FE analyses of artillery shell penetration in HPC (Paper I and II).
Another uncertain parameter for the FE analyses of the HBTP experiments was the friction coefficient for the target to penetrator interaction. The performed 3D FE analyses of artillery shells impacting concrete targets showed a minor influence of the friction coefficient on the simulation results (Tables B.2 and B.3), and as a result the friction forces for artillery shells impacting concrete targets could be neglected. This might be an acceptable approach for projectiles with small length to diameter ratios, or for high speed impacts. However, during the deep penetration of a HBTP in a concrete target it is likely that the shear forces at the target to penetrator interface can not be neglected, and an interaction without friction forces is therefore unsuitable. A reasonable value for the friction coefficient was assumed to be 0.05 for the simulations of the model scaled penetrators. However, a problem arises with the use of Lagrange solid elements for the targets since these elements may be subjected to increased shear deformations due to the friction forces at the interface between target and penetrator. This increased shear deformation increases the risk of heavily deformed elements, which are removed by the erosion algorithm. As a result the penetration depth increase with an increase of the friction forces acting on the penetrator. Furthermore, the contact surface between the concrete and the penetrators are not continuous for the FE analyses. This is caused by the rapid radial expansion of the target ahead of the penetrator, and also by the removal of heavily deformed elements in the central part of the target. It is likely that the earlier discussed erosion algorithm also may contribute to this effect. It should be noted that the ability to use advanced descriptions of the interfaces between targets and penetrators are almost non-existing within commercial codes, and the friction coefficient is not likely to be a linear function of the contact pressure.
Chapter 8

Conclusions and further research

8.1. Conclusions

8.1.1. Experimental program

The high performance concrete types showed an increased penetration resistance compared to the NSC. However, the HPC with a 97 MPa uniaxial compressive strength only had a slightly better protective performance than the NSC, see for example Figure 5.16 and Tables 5.8 and 5.9. To further decrease the penetration in a HPC target it is necessary to suppress the formation of an impact crater in the target. The impact craters in the HPC 133 targets were relatively deep when compared with the total penetration depth. Furthermore, an oblique impact at an impact angle of 60° in a HPC target with a compressive strength of 133 MPa of an HBTP resulted in a deflection of the penetrator. The use of this type of HPC for protective structures may therefore require impact conditions for a projectile close to a normal impact angle to obtain a considerable penetration depth for the warhead.

The formations of front and back face craters were prevented for the heavily reinforced NSC targets, with only the concrete cover for the reinforcement removed during the penetration of the HBTP. This reduced the penetration and the perforation thickness for the reinforced NSC compared to the unreinforced NSC considerably. Furthermore, as a HE warhead detonates it is likely that a heavily reinforced concrete protective construction will be subjected to reduced damage levels for both the impacted concrete slab and the surrounding structure.

8.1.2. Empirical penetration models

The predictions of penetration depths in concrete targets are likely to vary for different empirical penetration models (section 6.1), with improved results for penetration cases that are similar to the penetration cases that were used to derive the equation. It is therefore important that the predicted
penetration depths obtained for an empirical penetration model are verified by comparison against experimental data for similar projectiles. The empirical penetration model presented by Li and Chen (2003), and based on the earlier work by Forrestal et al. (1994) and Frew et al. (1998), was modified to better describe the data obtained for the experimental program regarding HBTPs (section 6.1). This resulted in an average value for the calculated penetration depths equal to 104% of the penetration depths for the experiments, with a spread of the data from 87% to 114%. The calculated penetration depths are given in Figure 6.7. However, the complexity of empirical penetration models increases with the number of projectile types, impact conditions and target configurations that needs to be considered. This methodology is very cumbersome for evaluation of non-normal impact conditions of projectiles, different types of reinforced targets or special types of HPC. Furthermore, the predictive performance of empirical penetration models is considered to be very limited if experimental data for comparison is not available.

8.1.3. FE penetration analyses

The FE analyses of penetration of HBTPs resulted in realistic estimations of the performance of warheads in concrete targets in comparison to the experimental results. Furthermore, the FE analyses allow for estimations of the stress states both for the steel casing and HE within a warhead, with a possibility of estimating deformations for a warhead casing. However, the accuracy of the predictions of the penetration depth is reduced if the dominating failure mode of the concrete is due to tensile cracking, e.g. caused by the radial expansion of an unreinforced target. Furthermore, the calculated exit velocities after perforation and ballistic limits of concrete targets are likely to be strongly influenced by the tensile failure description of the concrete. However, the RHT material model may be useful for cases were crater formation and radial cracking are suppressed, i.e. for heavily reinforced concrete structures. However, to be able to predict penetration depths in concrete or perforation thickness it is important that correct material properties are determined. It is therefore recommended that the material behaviour for concrete types used for penetration studies are thoroughly tested, e.g. regarding shock properties, tri-axial strength properties and dynamic tensile behaviour. This requires extensive laboratory testing at high pressures and deformation rates of the concrete, e.g. the FE analyses resulted in local pressures and strain rates of approximately 1.0 GPa and $5 \times 10^3 \text{s}^{-1}$ in the concrete, respectively. Furthermore, enhanced descriptions of the damage evolution for both compressive and tensile failures of concrete for use with the RHT model are likely to enhance the description of the concrete during warhead penetration by the FE analyses. However, the use of the Lagrangian element formulation for the target makes it difficult to distinguish between errors caused by the material models and errors caused by numerical problems, e.g. distorted elements and numerical erosion.
For 3D FE analyses it is recommended that a minimum of ten elements within a target is used for a length equal to the diameter of the penetrator, with equal or smaller elements used in the penetrator. The use of larger element sizes may cause numerical problems and lock up of the elements, which may result in an increase of the penetration resistance of a target.

The use of symmetry planes within a 3D FE model constraints the node displacements along the symmetry planes. This may result in numerical instabilities at these boundaries and influence the local response for a FE analysis close to these planes for brittle material. Symmetry planes should therefore be avoided for 3D FE analyses of penetration in quasi-brittle or brittle material, e.g. NSC and HPC. However, the influence on the global response for the models were small for the NSC targets, with an increased influence of the constraints along the symmetry planes for FE analyses of HPC targets perforated by artillery shells (Appendix B, e.g. Figure B.6). Furthermore, with incorporation of improved failure models for concrete there might be an increased risk of inducing unphysical tensile failure conditions close to the symmetry planes due to numerical instabilities.

It was concluded that solid elements representing concrete within the target and beam elements representing reinforcement should have the same length, in order to reduce hourglass deformations of the solid elements. Hourglass deformations, or other zero energy modes, will induce damage propagation in the solid elements, and thereby the strength of the target is likely to be reduced. This may be even more important if structural response is considered, e.g. the failure of an air blast loaded reinforced concrete beam. Therefore, it is preferable that FE analyses are performed with solid elements for both concrete and reinforcement, and with a ratio of 1:1 for the element lengths for concrete and steel. Furthermore, this methodology was used for the FE analyses presented in Paper III.

**8.2. Further research**

Further research is needed within several application areas to enhance the understanding and predicting capabilities for the protective performance of concrete structures, e.g. research regarding material modelling and numerical methodology. Furthermore, this theoretical research needs to be verified against experimental data regarding the penetration of different warhead types in a variety of protective structures.

**8.2.1. FE analysis methodology and material modelling**

The material behaviour and modelling of concrete were discussed earlier in the thesis, with two major areas of future improvements identified. These are the modelling of the tensile failure of concrete close to a target’s surfaces, and the damage to the confined concrete during deep penetration in concrete. A model with a bi-linear crack softening relationship for concrete was
implemented in Autodyn by Leppänen (2004) and used for Paper III. This tensile failure model used a fracture energy that is independent of the strain rate based on Weerheijm (1992), and it may be acceptable for concrete structures subjected to low strain rate deformations. However, later investigations have shown an increase of the fracture energy for concrete at increased loading rates (section 4.1). A crack softening failure model that considers the dependence of the crack opening velocity on both tensile failure strength and fracture energy $G_F$ may therefore improve the modelling of concrete subjected to high strain rate deformations. A concrete damage model that considers this type of tensile behaviour, and also with an improved damage evolution for confined deformations of concrete, was developed by Schuler (2004). The use of improved failure models enhances the modelling capabilities regarding FE analyses of concrete penetration (Schuler, 2004). Further analyses are therefore recommended to evaluate improved damage models for the concrete, and with material properties determined for the specific concrete. However, with strain rates of up to approximately $5 \times 10^3$ s$^{-1}$ obtained for the model scaled HBTP FE analyses it may not be appropriate to extend existing strength and failure models for concrete to the deformation rates obtained for warhead penetration.

Severe deformations of the penetrators were identified for the cases with oblique impacts of the HPC 133 and the reinforced NSC, and as a result its casing may fracture. Furthermore, large deformations or fracturing of the casing of a HBTP are likely to decrease the performance of a warhead, with improved modelling techniques and material models needed for future studies of the casing fracture phenomenon.

An eight node Lagrange element formulation with a single integration point was used for both the target and penetrators for the FE analyses. This has historically been the dominating element formulation for 3D penetration simulations. However, it may be an advantage to use solid elements of a higher order, e.g. cubic element formulation to obtain a better approximation of the stress, e.g. close to the nose of the penetrator. The use of higher order elements for explicit FE analyses may therefore enhance the numerical resolution for FE analyses of projectile penetration. Furthermore, the use of higher order solid elements also for the reinforcement bars would allow for an acceptable geometric resolution without decreasing the time step for explicit FE analyses further, with the possibility of improving the modelling of the local behaviour of a reinforced concrete target. Incorporation of a bond model for the interaction of the concrete and the reinforcement may further improve the modelling of reinforced targets.

It is recommended that an improved numerical methodology should replace the used algorithm for deleting heavily deformed Lagrange elements in future numerical studies. An erosion algorithm with the distorted elements replaced by SPH nodes are for example used within the FE code EPIC (Johnson and Stryk, 2003), with both mass and strength of the target material retained after heavily deformed elements are removed. This type of methodology improves the modelling of the target...
and penetrator interaction for Lagrange element formulations, especially for cases of oblique penetrator impacts. Furthermore, it is recommended that this methodology is combined with a contact formulation using a pressure dependent friction coefficient for the interaction between targets and penetrators. In general, an improved modelling of the interface between concrete targets and projectiles is needed to enhance future studies using both empirical penetration models and FE analyses of penetration in concrete.

8.2.2. Experimental and numerical studies of protective structures

Several research areas regarding protective design against HBTPs are of interest for future experimental studies and for evaluation by FE analyses, with examples given below. Furthermore, the performed FE analyses presented in section 6.2 does not cover the entire experimental program and complementary FE analyses are therefore recommended of the existing HPBT experiments.

Evaluations of typical nose designs for HBTPs (section 3.2), e.g. conical noses with different angles and number of sections, are of great interest for future studies. Furthermore, these nose designs for penetrators may be more effective than the used ogive nose designs in this study, and experimental data for optimised nose design are of great interest for studies of the interaction of the penetrator and target. The use of an optimized nose design for a penetrator may decrease the frictional forces acting on the penetrator, and thereby increase the penetration performance. There is also a need to perform further experimental studies using both inert and live full size penetrators. However, it is recommended that full size live tests with HE filled warheads is preceded by model scaled experiments using live warheads.

Penetration of improved concrete structure configurations are of interest for future experimental and numerical studies, e.g. studies of layered structures and heavily reinforced HPC targets. The use of heavily reinforced HPC 133 targets, i.e. with approximately 5 vol-% reinforcement, may decrease the total penetration depth considerably compared to the unreinforced HPC targets. Furthermore, the reinforcement increases the confinement of the HPC, and thereby it is possible to take advantage of the high compressive strength for this concrete type. However, an earlier study of 75 mm artillery shells penetrating HPC with a compressive strength of approximately 150 MPa did not result in any significantly increased protection level with the introduction of reinforcement for the targets (Unosson and Nilsson, 2006), see also Appendix B.3. It should be noted that these penetration results may have been strongly influenced by the use of artificial aggregate consisting of crushed slag for the concrete, and the data should not be taken as representative for either reinforced or unreinforced HPC.

Several techniques exist to disturb a HBTP, e.g. yaw inducing deflection grids constructed of reinforced concrete (Underwood, 1995) and rock rubble overlays. The used size of the blocks in a
rock rubble overlay is about three times the diameter of the penetrator (Bulson, 1997). These techniques are intended to increase the yaw of an advanced penetrator, and thereby increase the bending forces acting on the penetrator during penetration, with schematic design of a reinforced concrete deflection grid and a rock rubble overlay shown in Figure 8.1. The influence of these types of complementary protective structures needs further evaluation in the future for the protection of hardened underground facilities.

Penetrating weapons with dual-charge warheads are likely to be less sensitive to both the impact angle and yaw/pitch of the penetrator. These warheads use a shaped charge jet that first penetrates the target, and then a penetrating warhead with an explosive charge follows in the damaged concrete structure. Future experimental studies and FE analyses of the performance of dual-charge warheads in different types of concrete targets are therefore recommended.
References


Riedel, W., Beton unter dynamischen lasten, Meso- und makromechanische modelle und ihre parameter, EMI-Bericht 6/00, Freiburg, July 2000.


Steinberg, D.J., Equation of state and strength properties of selected materials, Lawrence Livermore National Laboratory, report no. UCRL-MA-106439 Change 1, 1996.


Appendix A

Plasticity models for ductile materials

Fundamental basics for modelling of ductile materials with respect to strength modelling are given in this appendix. The Johnson and Cook (J&C) (1983) material model describes the constitutive behaviour of metals and is often used for penetration studies. Even if a projectile behaves almost as a rigid body, it is recommended that a deformable projectile is used for FE analyses. The reason is to obtain feedback on the projectile behaviour within a target, and limit the stresses for both the projectile and the target material. Furthermore, several other strength models exist for ductile materials, e.g. a dislocation based strength model presented by Zerilli and Armstrong (1987). However, this model may be more suitable for high velocity impact studies. The J&C strength model may on the other hand not be applicable to high velocity impacts, and the high strain rates associated with this phenomenon.

The thin walled steel confinements for the unreinforced concrete targets were only subjected to elastic stress according to the FE analyses, and an elastic constitutive model would have given identical results. The influence of strain rate on the yield strength of high strength steel are considered to be of less influence then for low strength steels. Furthermore, normal strength concrete is considered to be a relative soft target material in comparison to a forged steel projectile, with the penetrator only subjected to minor deformations. The FE analyses of the penetration cases with normal impact of the unreinforced NSC targets did not show any plastic deformations of the hardened buried target penetrators, and therefore the plastic strain rate was zero during the analyses. Furthermore, oblique impacts of the unreinforced NSC targets resulted in local plastic strain rates of approximately $1 \times 10^2 \text{ s}^{-1}$ for the penetrators. The maximum plastic strain rate for the penetrators was increased to approximately $2 \times 10^3 \text{ s}^{-1}$ for the FE analyses of the reinforced targets. However, this strain rate value only occurs for elements in direct contact with the reinforcement bars. In retrospect, the use of an elastic-plastic constitutive model without strain rate effects and thermal softening for the penetrator for the FE analyses should not have any significant influence on the FE analyses results.
It is also likely that the used strain rate dependency and thermal softening modelling of the reinforcement bars have a negligible effect on the global behaviour of the models, since only reinforcement bars located in the direct path of the penetrator are subjected to high deformation rates. The maximum plastic strain rate for these reinforcement bars was approximately \(5 \times 10^3 \text{ s}^{-1}\), with a considerably increase of the yield strength for these rebar elements. However, the resulting increase of plastic work due to strain rate effects for these elements is likely to be negligible compared to the total energy transferred to the target.

### A.1. J&C strength model

The Johnson and Cook (1983) material model describes the relation between the flow stress \(\sigma\) of a metal and the plastic strain \(\varepsilon_{pl}\), plastic strain rate \(\dot{\varepsilon}_{pl}\) and temperature \(T\) as:

\[
\sigma_{yield} = \left( A + B \varepsilon_{pl}^n \right) \left( 1 + C \ln \frac{\dot{\varepsilon}_{pl}}{\dot{\varepsilon}_{pl,0}} \right) \left( 1 - \left( \frac{T - T_{Ref}}{T_m - T_{Ref}} \right)^m \right)
\]  

(A.1)

where

- \(T_{Ref}\) \: reference temperature
- \(\dot{\varepsilon}_{pl,0}\) \: reference value for plastic strain rate
- \(A\) \: yield stress at zero plastic strain
- \(B\) \: hardening constant
- \(n\) \: hardening exponent
- \(C\) \: strain rate constant
- \(m\) \: thermal softening exponent
- \(T_m\) \: melting temperature

The reference strain rate is normally \(1 \text{ s}^{-1}\), with the reference temperature equal to 293 K or 300 K. However, other values may be specified for a parameter set for a material. Furthermore, the parameter values given in different sources for the same material may differ both due to the selected experimental data for the material and the assumptions made during the analyses of the experimental data. Parameters for this strength model are given in Table A.1 for a few selected materials. The abbreviation HHA refers to high hardness armour steel with a Brinell hardness number (BHN) of approximately 500, while RHA (rolled homogenous armour) normally refers to older armour steels with approximately BHN 300.
## Table A.1. Johnson and Cook strength model parameters for selected materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>Steel AISI 4340(^1)</th>
<th>Steel AISI 1006(^2)</th>
<th>Steel AISI S-7(^2)</th>
<th>Al. 7039 (^2)</th>
<th>Al. 7039(^2)</th>
<th>HHA (^3)</th>
<th>Armox 500T(^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardness</td>
<td>HRC 30</td>
<td>HRF 94</td>
<td>HRC 50</td>
<td>HRB 76</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Density (kg/m(^3))</td>
<td>7830</td>
<td>7890</td>
<td>7750</td>
<td>2770</td>
<td>--</td>
<td>--</td>
<td>7850</td>
</tr>
<tr>
<td>(C_p) (J/kgK)</td>
<td>477</td>
<td>452</td>
<td>477</td>
<td>875</td>
<td>--</td>
<td>--</td>
<td>1450</td>
</tr>
<tr>
<td>(T_m) (K)</td>
<td>1793</td>
<td>1811</td>
<td>1763</td>
<td>877</td>
<td>933</td>
<td>1783</td>
<td>1800</td>
</tr>
<tr>
<td>(A) (MPa)</td>
<td>792</td>
<td>350</td>
<td>1539</td>
<td>337</td>
<td>220</td>
<td>1504</td>
<td>1470</td>
</tr>
<tr>
<td>(B) (MPa)</td>
<td>510</td>
<td>275</td>
<td>477</td>
<td>343</td>
<td>500</td>
<td>569</td>
<td>702</td>
</tr>
<tr>
<td>(n)</td>
<td>0.26</td>
<td>0.36</td>
<td>0.18</td>
<td>0.41</td>
<td>0.22</td>
<td>0.22</td>
<td>0.199</td>
</tr>
<tr>
<td>(C)</td>
<td>0.014</td>
<td>0.022</td>
<td>0.012</td>
<td>0.010</td>
<td>0.016</td>
<td>0.003</td>
<td>0.00549</td>
</tr>
<tr>
<td>(m)</td>
<td>1.03</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.905</td>
<td>0.90</td>
<td>0.811</td>
</tr>
<tr>
<td>(\dot{\varepsilon}_{pl,0}) (s(^{-1}))</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>(T_{Ref}) (K)</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>298</td>
<td>--</td>
<td>293</td>
</tr>
<tr>
<td>(E) (GPa)</td>
<td>207</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>207</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.29</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

Note: 
\(^1\) Ref. Johnson and Cook (1985).  
\(^3\) Ref. Gray et al. (1994).  
\(^4\) Ref. Nilsson (2003), Brinell hardness measured with a tungsten indenter (HBW) is 480 to 540 according to the manufacturer SSAB (www.ssabox.com).

The stress-strain relationships for AISI 1006 and AISI 4340 steels are shown in Figures A.1 and A.2, respectively. The stress-strain relationships for AISI S-7 tool steel and Armox 500T armour steel are shown in Figures A.3 and A.4, respectively.

![Figure A.1. The yield strength of AISI 1006 steel (Johnson and Cook, 1983).](image-url)
Appendix A: Plasticity models for ductile materials

Figure A.2. The yield strength of AISI 4340 steel (Johnson and Cook, 1985).

Figure A.3. The yield strength of AISI S-7 tool steel (Johnson and Cook, 1983).
A.2. Modifications to the J&C strength model

The J&C strength model uses a power relationship to describe the strain hardening of the material, see Eq. (A.1). This approximation is suitable for a large number of materials. However, low strength mild steel grades, and also several reinforcement steel grades, have a yield point elongation associated with large plastic deformations before the onset of the strain hardening. A piece-wise strain hardening model is therefore sometimes used to better describe the strain hardening of these materials instead of the original power law for the J&C strength model.

Furthermore, the original formulation of the J&C strength model uses a reference strain rate equal to 1.0 s\(^{-1}\), with the yield strength defined at this strain rate. For structural behaviour it is of interest to consider strain rates relating from quasi-static loading to approximately 10 s\(^{-1}\). Therefore, the reference strain rate \(\dot{\varepsilon}_{pl,0}\) is an input parameter in several computer codes. A modified strain rate enhancement factor was proposed by Børvik et al. (2001) to avoid numerical problems related to strain rates below the reference strain rate, giving the following equation:

\[
\sigma_{yield} = \left( A + B \varepsilon_{pl}^n \right) \left( 1 + \frac{\dot{\varepsilon}_{pl}}{\dot{\varepsilon}_{pl,0}} \right)^C \left( 1 - \frac{T - T_{Ref}}{T_m - T_{Ref}} \right)^m
\]  \(\text{(A.2)}\)

Note that the values for the material parameters are not equal to the parameters determined for the original J&C model. A modified strain rate dependency of the J&C strength model was also proposed by Rule and Jones (1998), with the incorporation of three new material parameters to obtain a better approximation of the strength of materials at high strain rates.
According to Rohr et al. (2005) it was not possible to obtain a good representation of data for the strain hardening of high strength steel, with specifications according to the standard 35NiCrMoV109, at elevated temperatures. An improved strength model with different temperature dependencies for the yield strength parameter $A$ and the strain hardening was instead proposed, with a good fit to experimental data up to 823 K. Furthermore, the strain rate dependency of the J&C model was also modified with the use of changed parameter values for $A$ and $C$ for strain rates exceeding 2000 s$^{-1}$. However, the use of an enhanced strength model for steel would require extensive testing to obtain the necessary experimental data to determine the specified material parameters.
Appendix B

FE analyses of artillery shell penetration in HPC

Finite element analyses of penetration of a modified 152 mm semi-armour-piercing shell in unreinforced HPC shells were analysed in Paper I and II. The HPC used for these targets was of the same type as the HPC 97 used for the HBTP experiments, but with this batch only reaching a uniaxial compressive strength of 92 MPa. One important aim for the benchmark tests was to obtain reliable data for numerical simulations, with the benchmark tests and initial FE analyses were published by Svinsås et al. (2001). Furthermore, typical warheads for different types of artillery were given in section 3.1.

B.1. Benchmark experiments

The artillery shell and target set-up for the benchmark experiments are shown in Figure B.1. The mass and length for the modified artillery shell were 46.2 kg and 552 mm, respectively. The artillery shells only showed minor surface erosion after perforation of the targets. Furthermore, the ratio between the diameters for the target and shell was approximately 16, which in combination with the weak confinement resulted in severe radial cracking out to boundary of the target. Post-test photos of the targets are shown in Figure B.2. The shells impacted the 0.75 m thick HPC target with a velocity of approximately 460 m/s and exited the target at approximately 190 m/s, see Table B.1.

Table B.1. Test results for 152 mm modified artillery shell (Paper I).

<table>
<thead>
<tr>
<th>Test no.</th>
<th>1999-23</th>
<th>1999-24</th>
<th>1999-25</th>
<th>Mean value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{\text{Impact}}$ (m/s)</td>
<td>460.0 ±0.5</td>
<td>455.5 ±0.2</td>
<td>458.8 ±0.2</td>
<td>458.1 ±0.2</td>
</tr>
<tr>
<td>$V_{\text{Exit}}$ (m/s)</td>
<td>183 ±6</td>
<td>204 ±4</td>
<td>181 ±4</td>
<td>190 ±14</td>
</tr>
</tbody>
</table>
Appendix B: FE analyses of artillery shell penetration in HPC

Figure B.1. A modified 152 mm artillery shell shown before and after test, and set-up of the HPC target with 2.4 m diameter used for the benchmark tests.

Figure B.2. Post-test photos of HPC targets used for the benchmark tests with 152 mm shells. The upper row shows the front faces of the targets, with the lower row showing the back faces (Hansson, 2001).
B.2. FE analyses of artillery shell penetration in HPC

Only very limited material data that was available for the HPC, and the uncertain boundary conditions of the targets, made predictive FE analysis impossible to perform. It should be noted that the steel culverts with helical corrugation used as moulds for the concrete targets have a low ability to confine the target during the penetration of projectiles. The steel culvert was therefore omitted from the FE analyses. The aims for the simulations were instead to identify important parameters that were needed to be considered for future penetration experiments and FE analyses. The FE analyses were performed with the RHT model using data for the 92 MPa HPC given in sections 4.2 and 4.3. The FE analyses were performed with both 2D and 3D models, and with different formulations used for the target, i.e. Lagrange, Eulerian, and smoothed particle formulation (SPH) formulations. Furthermore, the influences of the angle of attack and impact angle for the shells on the penetration were also studied with FE analyses. It should be noted that the velocity vector and the symmetry axis of a projectile do not coincide for a spin stabilised projectile, e.g. artillery shells or small arms bullets. This deviation between the velocity vector and the axis of the projectile also varies with the shooting range and between tests.

Lagrangian meshes were used for the shell for both the 2D and the 3D FE analyses. The length and diameter of the cylindrical part of the shell were 324 and 152 mm, respectively. The ogive radius for the shell was 380 mm, resulting in CRH 2.5. The used element size for the projectile was approximately 7.4 and 21.5 mm in radial and axial directions, respectively. The inner part of the shells was instrumented with accelerometers for the benchmark tests. The FE models instead used a simplified mesh with a homogeneous material for the inner parts of the shell to obtain the correct projectile mass. Furthermore, the J&C strength model with parameters for AISI S-7 tool steel (Johnson and Cook, 1983) was used for the shell casing, see also Appendix A.

The 2D FE analyses were performed with a rotation symmetry model and a constant element length of 10.0 mm, or a graded element size with 5.0 mm element length in the central part of the model. The 3D Lagrange models used a graded element mesh, with element lengths of 5.0 and 10.0 mm used for the central part of the target. This reduced the computational time considerably with a negligible effect on the penetration path for the shell. The 2D model geometry with 5.0 mm element size consisted of $27 \times 10^3$ elements, while the corresponding 3D $\frac{1}{4}$ symmetry models consisted of $1.2 \times 10^6$ elements. The FE analyses with 3D $\frac{1}{2}$ symmetry models of oblique impacts and yaw/pitch for the projectiles were conducted with the 10.0 mm element length to reduce the number of elements. Furthermore, it is recommended that SPH models should be used with a uniform particle size to avoid numerical problems, and the 2D rotational symmetry SPH models used a constant node size of 5.0 mm for comparison with the Lagrangian and Eulerian FE analyses.
Appendix B: FE analyses of artillery shell penetration in HPC

(Paper I). An erosion algorithm to remove heavily distorted elements was used for the Lagrangian elements, with the erosion strain equal to 1.5.

The main part of the FE analyses was performed without friction to reduce the distortion of the elements close to the projectile path, and to obtain comparable models for the Lagrange, Eulerian and SPH formulations. The influence of friction for the target and projectile interaction was discussed in section 6.2.2 for studies of the penetration of HBTPs, and this subject was also studied for the penetration of shells in HPC.

Reinforcement was also introduced for the 3D SPH simulations within the FE analysis study, with an unconfined concrete target with a rectangular 2.0×2.0 m² cross section used for the simulations. The reinforcement was modelled by a piece-wise linear strain hardening material model. The failure strain for the reinforcement bars was approximately 12% engineering strain for the main part of the simulations (Paper II), with additional FE analyses of the influence on the exit velocity of an increased failure strain to 30% engineering strain also investigated (Hansson, 2003).

B.2.1. 2D penetration simulations

The 2D rotational symmetry FE analyses of the benchmark tests were performed with Autodyn 2D using Lagrange, Eulerian and SPH formulations for the concrete targets. The SPH formulation eliminates the numerical problems associated with heavily deformed elements in the concrete target since no elements are used. Therefore, there is no need to use a numerical erosion algorithm for the FE analysis. Figure B.3 show the principal differences between a deformed 2D element mesh and a meshless SPH model.

![Figure B.3. Typical figures of distorted targets during penetration, with Lagrange mesh (a) and SPH formulation (b) from Hansson and Skoglund (2002).](image)
For the 2D FE simulations with the same material parameters and impact conditions, it was shown that the exit velocity depended on the chosen formulation for the target (Paper I). The exit velocity was highest for the Eulerian target and decreased for the target with Lagrangian elements, with the lowest exit velocity obtained for the SPH analysis. Data for FE analyses with 5.0 mm element lengths and SPH node sizes are given in Table B.2, with plots of damage for targets with different target formulations shown in Figure B.4. Furthermore, an increase in element length to 10 mm reduced the exit velocity for the Lagrangian and Eulerian targets to 189 and 246 m/s, respectively. These data were for the FE analyses cases without friction between target and projectile. It should be noted that Autodyn did not allow frictional forces between the target and projectile. It should be noted that Autodyn did not allow frictional forces between the target and projectile.

### Table B.2. 2D FE analyses of 152 mm modified artillery shell penetrating HPC (Paper I).

<table>
<thead>
<tr>
<th>Model identity</th>
<th>B99056</th>
<th>B99057</th>
<th>B99E55</th>
<th>B99S00</th>
<th>B99S01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target formulation</td>
<td>Lagrange</td>
<td>Lagrange</td>
<td>Euler</td>
<td>SPH</td>
<td>SPH</td>
</tr>
<tr>
<td>Element length (mm)</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Particle size (mm)</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>0.0</td>
<td>0.05</td>
<td>0.0</td>
<td>0.0</td>
<td>0.05</td>
</tr>
<tr>
<td>Exit velocity (m/s)</td>
<td>237</td>
<td>207</td>
<td>255</td>
<td>214</td>
<td>186</td>
</tr>
</tbody>
</table>

**Figure B.4.** Damage plots for Lagrange element model (a), Eulerian element model (b) and SPH model (c). All models used 5.0 mm elements or particles and were without friction between target and projectile (Paper I). The scale shows the scaling for the concrete damage used for all contour plots in this appendix.
B.2.2. 3D penetration simulations

Penetration simulations using 3D models with the use of Lagrange and SPH formulations for the targets were published in Paper II. The 2D simulations required that a rotational symmetry axis was used for the problems. This assumption is not normally valid for the geometry of a penetration case, e.g. due to a non-zero angle of attack or a non-normal trajectory angle for the projectile. Furthermore, the rotation symmetry for a circular target ceases to exist at the onset of damage within the concrete.

B.2.2.1. 3D Lagrange penetration simulations

The 3D Lagrange FE analyses showed less influence from variations of element sizes and friction coefficient than the earlier shown 2D analyses, see Table B.3. Furthermore, the element length of 5.0 mm resulted in an increased exit velocity for the shell 3D FE analyses. The studies of other than normal impact conditions require the use of ½ symmetry models, or models without symmetry conditions. Table B.4 show the simulation results for different impact cases for ½ symmetry FE analyses. The analyses for the case of a normal impact condition resulted in a lower exit velocity for the ½ symmetry model compared to the ¼ symmetry model. This decreased exit velocity was probably related to the allowed rotation of the projectile in one direction for the ½ symmetry model. Furthermore, a small 1.0° deviation between the projectile axis and the velocity vector for the projectile resulted in a 25 m/s decrease of the exit velocity to 186 m/s compared to the case with a normal impact condition. This initial 1.0° yaw for the projectile resulted in a wider penetration channel and also a curved path for the projectile through the target. A consequence of this is that more energy was dissipated into the concrete and the exit velocity decreased. The penetration paths for these models are shown in Figure B.5.

Table B.3. Data for ¼ symmetry 3D Lagrangian FE analyses of a 152 mm modified artillery shell penetrating HPC (Paper II).

<table>
<thead>
<tr>
<th>Model identity</th>
<th>B99311</th>
<th>B99312</th>
<th>B99307</th>
<th>B99309</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target formulation</td>
<td>Lagrange</td>
<td>Lagrange</td>
<td>Lagrange</td>
<td>Lagrange</td>
</tr>
<tr>
<td>Element size (mm)</td>
<td>10.0</td>
<td>10.0</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Symmetry condition</td>
<td>¼</td>
<td>¼</td>
<td>¼</td>
<td>¼</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>0.0</td>
<td>0.05</td>
<td>0.0</td>
<td>0.05</td>
</tr>
<tr>
<td>Exit velocity (m/s)</td>
<td>217</td>
<td>205</td>
<td>225</td>
<td>213</td>
</tr>
</tbody>
</table>
Table B.4. Data for ½ symmetry 3D Lagrangian FE analyses of a 152 mm modified artillery shell penetrating HPC (Paper II).

<table>
<thead>
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<th>B99320</th>
<th>B99321</th>
<th>B99323</th>
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<td>Target formulation</td>
<td>Lagrange</td>
<td>Lagrange</td>
<td>Lagrange</td>
</tr>
<tr>
<td>Element size (mm)</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td>Symmetry condition</td>
<td>½</td>
<td>½</td>
<td>½</td>
</tr>
<tr>
<td>Impact conditions</td>
<td>Normal impact at 90°</td>
<td>1.0° yaw</td>
<td>Impact angle 60°</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Exit velocity (m/s)</td>
<td>211</td>
<td>186</td>
<td>111</td>
</tr>
</tbody>
</table>

Figure B.5. Damage plots and penetration paths for models B99320 with normal impact conditions (a) and B99321 with 1° yaw (b) (Hansson, 2003).

An oblique impact angle of 60° causes different pressures on the upper and lower side of the projectile, and as a consequence an increasing curvature of the trajectory. This resulted in a decreased exit velocity of 111 m/s for model B99323, with Figure B.6 showing the projectile trajectory for this model. Furthermore, a radial crack developed along the symmetry line in the middle of the target in this FE analysis. The prescribed nodal displacements along a symmetry plane may introduce numerical instabilities at this plane, with tensile stresses exceeding the tensile strength of the concrete. This phenomenon is likely to be more pronounced for a material with high compressive strength in relation to its tensile strength. Furthermore, this may influence the global behaviour for the projectile, especially for non-normal impact conditions. The use of model geometries without symmetry planes would eliminate this numerical problem.
B.2.2.2. 3D SPH penetration simulations

The main focus for the 3D SPH analyses was to evaluate the use the SPH formulation for concrete targets subjected to projectile impacts. These analyses were performed with ½ and ¼ symmetry models, different node size, different boundary conditions, and different impact angles for the projectile. Furthermore, FE analyses were conducted with and without reinforcement of the HPC targets for the SPH models. Selected simulation results are given in Tables B.5 and B.6, with further descriptions of the FE models and results given in Paper I and II. The increase of target thickness to 0.76 m for the SPH models with 20 mm particle size was assumed to have no significant influence on the calculated exit velocity of the projectile.

Damage plots for ¼ and ½ symmetry SPH models with normal impact conditions are shown in Figure B.7. Furthermore, these two models resulted in similar exit velocities. However, there is a risk of an unacceptable influence on the results for an analysis if symmetry planes are used. This is due to that the unsymmetrical damage propagation in the target is likely to induce rotations for a shell. An increase of the target size to 3.0 m for model SPH206 resulted in a minor decrease of the exit velocity with 20 m/s to 185 m/s. This model used normal impact conditions and 20 mm SPH particles.

The rotation of the target to obtain a 60° impact angle for model SPH200 decreased the exit velocity to 164 m/s, compared to 204 m/s for the normal impact case. Furthermore, a decrease of

Figure B.6. Damage plots and penetration path for an unreinforced HPC target impacted at a trajectory angle of 60° for the projectile, with the figures shown 2.0 and 3.8 ms after impact, respectively.
the particle size to 12.5 mm for this impact condition resulted in a reduction of the exit velocity with 20 m/s to 144 m/s for model shown in Figure B.9. Furthermore, the 3D SPH model resulted in a higher calculated exit velocity and less projectile deflection than the earlier shown Lagrangian element models for the case of an oblique impact. The 3D SPH models also showed similar damage evolution along the symmetry plane in the models that was observed for the Lagrangian analysis of the target.

**Table B.5.** Data for unreinforced 3D SPH FE analyses of a 152 mm modified artillery shell penetrating HPC (Paper II).

<table>
<thead>
<tr>
<th>Model identity</th>
<th>SPH000</th>
<th>SPH204</th>
<th>SPH200</th>
<th>SPH203</th>
<th>SPH206</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target formulation</td>
<td>SPH</td>
<td>SPH</td>
<td>SPH</td>
<td>SPH</td>
<td>SPH</td>
</tr>
<tr>
<td>Symmetry condition</td>
<td>¼</td>
<td>½</td>
<td>½</td>
<td>½</td>
<td>½</td>
</tr>
<tr>
<td>Target diameter (m)</td>
<td>2.4</td>
<td>2.4</td>
<td>2.4</td>
<td>2.4</td>
<td>3.0</td>
</tr>
<tr>
<td>Target length (m)</td>
<td>0.760</td>
<td>0.760</td>
<td>0.760</td>
<td>0.750</td>
<td>0.760</td>
</tr>
<tr>
<td>Particle size (mm)</td>
<td>20.0</td>
<td>20.0</td>
<td>20.0</td>
<td>12.5</td>
<td>20.0</td>
</tr>
<tr>
<td>Impact angle (°)</td>
<td>90.0</td>
<td>90.0</td>
<td>60.0</td>
<td>60.0</td>
<td>90.0</td>
</tr>
<tr>
<td>Boundary condition</td>
<td>Free surface</td>
<td>Free surface</td>
<td>Free surface</td>
<td>Free surface</td>
<td>Free surface</td>
</tr>
<tr>
<td>Exit velocity (m/s)</td>
<td>199</td>
<td>203</td>
<td>164</td>
<td>144</td>
<td>185</td>
</tr>
</tbody>
</table>

**Table B.6.** Data for reinforced 3D SPH FE analyses of a 152 mm modified artillery shell penetrating HPC (Paper II).

<table>
<thead>
<tr>
<th>Model identity</th>
<th>SPH220</th>
<th>SPH221</th>
<th>SPH222</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target formulation</td>
<td>SPH</td>
<td>SPH</td>
<td>SPH</td>
</tr>
<tr>
<td>Symmetry condition</td>
<td>½</td>
<td>½</td>
<td>½</td>
</tr>
<tr>
<td>Target cross section (m²)</td>
<td>2.0×2.0</td>
<td>2.0×2.0</td>
<td>2.0×2.0</td>
</tr>
<tr>
<td>Target length (m)</td>
<td>0.760</td>
<td>0.760</td>
<td>0.760</td>
</tr>
<tr>
<td>Particle size (mm)</td>
<td>20.0</td>
<td>20.0</td>
<td>20.0</td>
</tr>
<tr>
<td>Impact angle (°)</td>
<td>90.0</td>
<td>60.0</td>
<td>90.0</td>
</tr>
<tr>
<td>Boundary condition</td>
<td>Free surface</td>
<td>Free surface</td>
<td>Constrained target</td>
</tr>
<tr>
<td>Exit velocity (m/s)</td>
<td>168</td>
<td>149</td>
<td>88</td>
</tr>
</tbody>
</table>
Figure B.7. Damage plots for the unreinforced ¼ symmetry model SPH000 (a) shown 1.9 ms after impact, and the unreinforced ½ symmetry model SPH204 (b) shown 2.2 m/s after impact. Normal projectile impact conditions and 20 mm SPH particles were used for both models.

Figure B.8. Damage plots for a HPC target during penetration for model SPH200 with an oblique impact and 20 mm SPH particle size, shown 2.0 and 4.4 ms after impact.
B.2. FE analyses of artillery shell penetration in HPC

Figure B.9. Damage in a HPC target during penetration for model SPH203 with an oblique impact and decreased SPH particle size to 12.5 mm, shown 2.0 and 4.4 ms after impact.

Reinforced HPC targets were also included in the FE analysis study. The reinforced targets for the FE analyses had a cross section of 2.0×2.0 m², with one symmetry plane used for the models. Furthermore, the thickness for the targets was 0.76 m for models with 20 mm SPH node size, i.e. the same as for the unreinforced targets. The front and back faces reinforcement for the targets consisted of 25 mm diameter bars in the vertical and horizontal direction, with a centre to centre distance of 100 mm. The bars at the front and back faces were joined with longitudinal bars with 10 mm diameter and equally spaced at distances of 200 mm, resulting in a three dimensional grid. The steel reinforcement contents in the targets for the FE analyses were approximately 2.7 vol-\%. The reinforcement was modelled with beam elements for the 25 mm diameter bars and with truss elements for the longitudinal 10 mm bars, with both types of element having 50 mm length. The model included totally 1812 truss and beam elements, with the nodes for the beam and truss elements joined to coincident SPH particles to obtain a coupling to the concrete. The reinforcement elements were eliminated at an erosion strain slightly greater than the failure strain used for reinforcement. The calculated damage of the target and deformations of the reinforcement for the base data model SPH220 with a normal impact of the projectile are shown in Figure B.10. Introducing the reinforcement decreased the exit velocity from 203 m/s for the unreinforced model SPH204 to 168 m/s for the reinforced model SPH220. Furthermore, the exit velocity decreased from 164 m/s for the unreinforced model SPH200, to 149 m/s for the reinforced model SPH221, and for a 60° oblique impact of the targets. A modified stress-strain relationship that considered necking and engineering strains up to 30% engineering strain for the reinforcement bar elements before failure was used to study the influence of the failure strain on the analyses. This value for the
Appendix B: FE analyses of artillery shell penetration in HPC

failure strain was chosen due to the fact that the length to diameter ratios for the reinforcement elements were two or five depending on the diameter of the used rebars, i.e. 25 mm rebars for front and back face reinforcement and 10 mm rebars for the interconnection reinforcement. However, the increased failure strain for the reinforcement did not result in any significant change of the exit velocity for the projectile (Hansson, 2003). This was probably due to the fact that only a few reinforcement bars close to the projectile path obtain strains greater than 10%.

A constraint of a reinforced SPH target was introduced to study the influence of the boundary condition. This boundary condition was applied at the outer surfaces of the model that were parallel to the projectile path, i.e. not to the front and back faces of the targets. The boundary condition limited the movement at the surface of the target in the normal direction to within $\pm 1.0$ mm. This substantially reduced the exit velocity to 88 m/s for the reinforced model SPH222, from 168 m/s for the model SPH220 with the normal free surface boundary condition.

Figure B.10 Concrete damage and deformed reinforcement for a normal impact model SPH220 with 20 mm node size. The figures are shown 3.1 ms after impact.

The SPH model with 20 mm particles was modified to study different impact angles, with Figure B.11 showing the model SPH221 with a 60° oblique impact for the artillery shell (Paper II). This simulation resulted in an exit velocity of 149 m/s, which is approximately 20 m/s lower exit velocity than obtained for the normal impact of the reinforced target and the 60° oblique impact of the unreinforced target with 20 mm particles. However, the reinforced model also shows a slightly increased rotation for the projectile within the target.
B.3. Discussion and conclusions

It should be considered normal that a projectile with a small angle of attack (yaw or pitch angles) begins to rotate during the penetration phase. This phenomenon may be caused by small initial instabilities of the projectile that are enhanced after the impact of the target. A yaw of one degree is not unlikely for ballistic testing, and the FE analyses show that this small angle has an influence on the penetration path of artillery shells due to their low moment of inertia. Furthermore, the target is initially inhomogeneous and the damage evolution within the target is also likely to induce rotations of the projectile due to an uneven pressure distribution on the shell.

A decrease of the erosion strain to 1.0, or an increase to 2.0, only resulted in minor changes of the exit velocities for both 2D or 3D FE analyses. However, removing elements from the mesh reduces the mass and confinement of the concrete around the projectile. The problem with distorted elements can be eliminated with the use of meshless techniques, e.g. SPH, for concrete targets. However, meshless techniques and Eulerian formulations have their own limitations and advantages that need to be considered. The material transport of the target material through a fixed Eulerian mesh, or ALE mesh, introduces errors for the field variables and makes it difficult to track the deformation history for the material. This phenomenon is due to the coupling of these material variables to elements and not to the moving material, resulting in increasing errors with increased
deformations. This may introduce significant errors for FE analyses including material models with damage or failure models.

The introduction of friction and a friction coefficient of 0.05 for the target-shell interaction resulted in a minor decrease of the exit velocity for the shell for the major part of the FE analyses, with typically a 10 to 20 m/s decrease of the exit velocities for the 3D Lagrange models.

The influence on the exit velocity for an increase of the diameter to 3.0 m for the unreinforced HPC target was approximately the same as introducing reinforcement in the FE analyses. It is likely that the combination of targets with 2.4 m diameter and the weak confinement of the concrete from the chosen corrugation steel culverts results in an undefined boundary condition. Therefore, the targets have been considered as unconfined since the helical (spiral wound) corrugated steel culverts expanded during the penetrating event. The uses of reinforced targets or welded steel pipes as confinement for the targets are likely to decrease the influence of boundary effects on projectile penetration in concrete targets.

The penetration resistance for a projectile in reinforced concrete targets may be influenced by the locations of the rebars relative to the point of impact, with the deceleration of the projectile depending on the location of the impact point as well as the position of the reinforcement in the target. However, the FE analyses show only a minor influence from the reinforcement on the protective performance of HPC impacted by artillery. Furthermore, FE analyses of an unreinforced HPC target and a HPC target with 6.7 vol-% reinforcement impacted by 75 mm artillery shells were performed by Unosson and Nilsson (2006), for a HPC with 153 MPa uniaxial compressive strength. These simulations showed no significant decrease in the exit velocity for the reinforced HPC targets, with the experimental results indicating a similar result. However, it should be noted that artificial aggregate consisting of crushed slag was used in this HPC. This type of aggregate may be unsuitable for concrete used for protective structures, and extensive tests are needed to describe the properties of this HPC for different loading conditions if reliable FE analyses of the tests are to be performed.
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