Investigation of Vibrations During Internal Turning:
An Experimental and Numerical Study
Utredning av vibrationer vid invändig svarvning:

*Experimentell och numerisk studie*
Investigation of Vibrations During Internal Turning:  
An Experimental and Numerical Study

This thesis is about a manufacturing problem with vibrations during turning operation. The vibrations or so called chatter creates an undulated surface on the work-piece which will not fulfil the specified requirements. The thesis aim is to investigate the problem and reach conclusions about the source of error and give suggestions for solutions or further work. Both experimental and numerical investigations will be carried out. The experimental part consists of a modal analysis of the work-piece – fixture and the tool while the numerical investigation is preformed to derive the eigen-frequencies and frequency response function for the work-piece under ideal clamping conditions.

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Thesis place:
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Thesis period:
2010-10-04 to 2011-02-25

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This thesis deals with vibration problems in a cutting process. The cutting process is internal turning with a relatively stiff tool and the work-piece and clamping sets the stability limit. During the operation, some work-pieces are subjected to chatter. Previous research has derived numerous ways to calculate stability limits and theories about chatter elimination. In this thesis, single degree of freedom models are used to model the system behavior for different modes.

The main goal is to investigate the cause of chatter and give suggestions for improvements to eliminate chatter. Experimental modal analysis of the work-piece – clamping and tool system was done in order to analyze the system behavior and derive SDOF models for stability calculation. Measurement of the sound during operation was also analyzed but without any applicability. Finite element modeling of the work-piece was used to calculate the eigen-frequencies and frequency response function. Measurement of chatter wavelength in finished work-pieces gave useful information about the dominating vibration mode for this problem.

The chatter frequency, from the work-piece, consists with the chatter stability calculation of the work-piece – clamping system's lowest mode. The results from finite element calculation do not show this mode and it strongly imply that this mode is a property of the work-piece clamping. The stability calculation shows that the current operation conditions are close to optimal, but a small change is recommended. The main result is that the chatter is due to insufficient clamping and that the easiest way to eliminate the problem is most likely to change clamping.
Sammanfattning


Acknowledgement

I would like to say thank you to all the friendly people at the department, especially my supervisor; Tech. Lic. Lorenzo Daghini, for discussions and suggestions to improvements. I also appreciate LEAX in Falun for stopping the production when I wanted to make measurements and their manufacturing engineering manager, Erik Nordgren, for fast replies over e-mail.
# Table of content

Nomenclature and abbreviations ............................................................................................. I

1. Introduction .......................................................................................................................... 1
   1.1 Background ..................................................................................................................... 1-1
   1.2 Problem definition .......................................................................................................... 1-2
   1.3 Thesis aim ....................................................................................................................... 1-2
   1.4 Proceedings ................................................................................................................... 1-3
   1.5 Thesis layout .................................................................................................................. 1-3

2. Theory ..................................................................................................................................... 2
   2.1 Basic machine dynamics ............................................................................................... 2-1
   2.2 Chatter and stability ....................................................................................................... 2-3
      2.2.1 Chatter frequency from work-piece ..................................................................... 2-5
      2.2.2 Cutting models ....................................................................................................... 2-5
      2.2.3 Stability criterion chart ....................................................................................... 2-6
      2.2.4 Graphical ............................................................................................................... 2-7
      2.2.5 Analytical ............................................................................................................... 2-8
      2.2.6 Non-linear ............................................................................................................. 2-10
   2.3 Damping of chatter ......................................................................................................... 2-11
   2.4 Surface roughness .......................................................................................................... 2-12

3. Method ................................................................................................................................... 3
   3.1 Specimen ......................................................................................................................... 3-1
   3.2 Current cutting conditions ............................................................................................. 3-2
   3.3 Chatter frequency from work-piece ............................................................................... 3-3
   3.4 Experimental modal analysis ....................................................................................... 3-3
      3.4.1 Modal analysis of the tool ..................................................................................... 3-4
      3.4.2 Modal analysis of the work-piece ......................................................................... 3-5
   3.5 Sound measurement ....................................................................................................... 3-6
   3.6 Numerical calculations ................................................................................................. 3-7
      3.6.1 Mesh ....................................................................................................................... 3-8
      3.6.2 Eigen-frequency calculation .................................................................................. 3-8
      3.6.3 Frequency response calculation ............................................................................ 3-8
   3.7 Identify physical mode properties .................................................................................. 3-9
   3.8 Calculation of stability ................................................................................................. 3-10

4. Results ................................................................................................................................... 4
   4.1 FRF measurements ......................................................................................................... 4-1
   4.2 Comparison between measured and synthesised FRF .................................................. 4-1
   4.3 Synthesised FRF ............................................................................................................ 4-1
   4.4 Sound measurement ....................................................................................................... 4-4
   4.5 Chatter frequency from work-piece ............................................................................. 4-5
   4.6 Single degree of freedom models ................................................................................. 4-5
   4.7 FEM calculations .......................................................................................................... 4-7
      4.7.1 Mesh ....................................................................................................................... 4-7
      4.7.2 Eigen-frequency .................................................................................................... 4-7
      4.7.3 FRF calculation ..................................................................................................... 4-7
   4.8 Stability charts .............................................................................................................. 4-8
5. Discussion and conclusions

5.1 Measurements.................................................................5-1
5.2 SDOF models........................................................................5-1
5.3 FEM..................................................................................5-1
5.4 Stability chart......................................................................5-3
5.5 Chatter free work-pieces......................................................5-4

6. Suggestions for further work.....................................................6

7. References..............................................................................7

8. Appendix.................................................................................8
   A. Measurement results..........................................................8-1
   B. Comparison between measured and synthesised FRF..................8-5
# Nomenclature and abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CP</td>
<td>Cutting Process</td>
<td></td>
</tr>
<tr>
<td>DOF</td>
<td>Degree of Freedom</td>
<td></td>
</tr>
<tr>
<td>EMA</td>
<td>Experimental Modal Analysis</td>
<td></td>
</tr>
<tr>
<td>ES</td>
<td>Elastic Structure</td>
<td></td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
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<tr>
<td>FRF</td>
<td>Frequency Response Function</td>
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<tr>
<td>MRR</td>
<td>Metal Removal Rate</td>
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<tr>
<td>PD</td>
<td>Process Damping</td>
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<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
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<tr>
<td>SDOF</td>
<td>Single Degree of Freedom</td>
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- $c$ – Damping coefficient
- $k$ – Spring stiffness
- $CT$ - Instantaneous chip thickness (Tlusty and Ismail)
- $L$ – Chatter wavelength
- $ds$ – Instantaneous chip thickness (Sweeney and Tobias)
- $m$ – Mass
- $\varepsilon$ – Chatter phase shift
- $N$ – Spindle speed
- $F_{C}$ – Cutting force
- $\omega$ – Angular frequency
- $F_{f}$ – Cutting force in feed direction
- $R_{a}$ – Arithmetical mean surface roughness
- $F_{r}$ – Cutting force in radial direction
- $R_{s}$ – Mean surface roughness
- $F_{t}$ – Cutting force in tangential direction
- $\mu$ – Tool overlap factor between succeeding cuts
- $f_{c}$ – Chatter frequency
- $v_{c}$ – Cutting speed
- $H$, $h_{0}$ – Average chip thickness
- $x_{\omega}$ – Cutting coordinate
- $K$ – Cutting coefficient
1. Introduction

Internal turning is often associated with problems due to vibrations. The turning is often done in pre-drilled holes or cast cavities which require boring bars to be long and slender in order to avoid unwanted contact with the work piece. This shape of the tool has a big disadvantage since it is weak and therefore easy to excite. Therefore knowledge about the tool - tool holder eigen-frequencies is an important property to know in order to avoid resonant behaviour. Also the properties of geometry and clamping condition of the work-piece can be of great importance. A long and thin-walled work-piece has a low dynamic stiffness and can give the dominating vibration contribution.

In general the structure's first (lowest frequency) eigen-modes gives the highest contribution of vibration. Knowledge about the system behaviour is indispensable in order to avoid unnecessary high energy input in system modes. Work-pieces with low dynamic stiffness are more likely to suffer from bad clamping, which may be the limiting factor and should be chosen correctly if one are to limit problems with chatter.

There are two ways in which chatter can be generated: regenerative and forced chatter. Regenerative chatter is when the machined surface has an undulated surface which during the next revolution creates an fluctuating cutting force. This force fluctuation then gives a new undulated surface. This process is self generating since the cutting force will always fluctuate due to the wavy surface and the relative motion between tool and work-piece will continue and the finished machined surface will be undulated. Forced chatter is when an external force excites the structure motion. Regenerative chatter is often eliminated by changing cutting parameters, such as spindle speed, depth of cut or feed rate, while forced chatter is cancelled by removing or damp the external force.

A combination of experimental modal analysis and cutting tests are the most common way to create an estimated stability chart. One major drawback with this modelling is the absence of an complete model with all properties (mostly non-linear properties are excluded) and the time consuming cutting tests.

Metal Removal Rate (MRR)
Shops striving to shorten the machine time for cutting machines try to have as high Metal Removal Rate (MRR) as possible. The MRR can be changed in a number of ways, for instance deeper cut, faster feed rate or higher cutting speed. All these changes gives different output when it comes to vibration level. It is therefore possible to get optimized cutting parameters for high MRR, no chatter and long tool life.

1.1 Background

Machining has gone trough a lot of changes during the history. Manufactures today strive to have a high MRR in order to reduce the time for machining and get a higher profit. This sets high requirements on the machine since high acceleration of the tool is a must. High acceleration is easier to have with an light tool – tool holder than with a heavy one. The drawback, with vibrations in mind, of light structures is that they are far more easily excited and vibrations is a big problem.
Turning with chatter vibrations create an surface with worse surface roughness than chatter free turning. It is therefore important to have knowledge about when chatter occurs to have high MRR and fine surface roughness.

Internal turning has yet one more disadvantage in tool choice due to geometrical reasons. A boring bar is often used in order to get full reach and fit in the cavity. The drawback of a boring bar is that is it long and slender (length compared to cross-section dimensions). This shape of tool has a low dynamic stiffness and it is therefore often subjected to self-excited chatter. The boring bar can successfully be modelled as a clamped-free cantilever beam using Euler-Bernoulli or Timoshenko beam theory, Andrén et al. (2004 b), in order to simulate the dynamic properties of the tool. Weak boring bars can also give a loss in dimensional accuracy of the finished work-piece.

The clamping of the work-piece has to be considered since a rigid clamping gives a system with mostly the properties of the work-piece (other influences are the spindle, bearings and housing). A real clamping devise have its own system behaviour and can therefore add an additional resonance.

1.2 Problem definition

During internal turning of a hub reduction gear housing severe vibrations are experienced, close to the open free edge. Only one of the cutting operations will be investigated. The operation of interest is internal turning in the outer part of the work-piece. The vibrations cause an undulated surface which does not meet the required surface roughness. Chatter is not always present and possible causes to the problem are to be discussed.

Important parts are:
- Experimental modal analysis of the tool – tool holder and work-piece – fixture to extract structure modes and make a modal model.
- FEM model of the work-piece to calculate eigen-frequencies and make frequency response function calculations to draw conclusions about the clamping.
- Measurement during machining to get live operation information.

Preferred conclusions are: Structure modes interaction (both tool and work-piece), if the spindle speed can be changed for stable cutting, if work-piece natural frequencies are close to the tool's natural frequency or the spindle speed, if it is regenerative chatter.

1.3 Thesis aim

The thesis main goal is to investigate the problem to find what is causing the chatter and hopefully reach conclusions to solve the problem or give suggestions for further work. Secondary benefits, increased tool life and a less noisy environment, can also be achieved if the vibrations are successfully reduced.

(1 - 2)
1.4 Proceedings

Experimental modal analysis will be carried out on both the tool – tool holder and the work-piece - fixture in order to investigate the structure modes. An FEM model will be made to analyse the work-piece eigen-frequencies and frequency response function with ideal clamping. The thesis start with a literature survey to get knowledge about machining and vibrations during turning. The survey is also an important part to get information in which way others have attacked similar problems in order to avoid pitfalls and spending time on already solved problems. It is equally important in order to get a deeper insight and useful “tools”. A proposal to literature survey is basic machining, vibrations in turning operations, chatter – cause and solutions, vibrations in boring bar. There are two ways to solve chatter related problems, with passive or active control, the focus in the literature survey will be on passive solutions. Results which has to be drawn are: excitation frequency/-ies, work-piece eigen-frequencies, system modes and damping, stability charts.

1.5 Thesis layout

This thesis is divide in chapters.

- Chapter 1: Introduction and basic information about the thesis
- Chapter 2: Theory of basic machine dynamics, cutting force, tool and work-piece motion chatter and stability
- Chapter 3: Methods used to get results, experimental and numerical
- Chapter 4: Results from measurements and calculations
- Chapter 5: Discussion about used methods and results, conclusions and suggestions to changes
- Chapter 6: A few suggestions for further work
- Chapter 7: Used references
- Chapter 8: Appendix: Measurement results and comparison between measured and synthesised response
2. Theory

2.1 Basic machine dynamics

A lathe structure dynamic properties are very complicated due to the complex built up structure. All joints and splices add damping and parts have different dynamic behaviour. Many researchers (Tobias and Tlusty among others) have divided the machine, in this case a lathe, in two sub structures: the spindle – chuck – work-piece and tool – tool holder. The natural frequencies are then calculated or experimentally derived to find important characteristics. Andrén et al. (2004a) reports that chatter frequency is often close, and slightly higher, to a system mode and that is why natural frequencies are of interest when trying to eliminate chatter.

There are lots of parameters to change when dealing with chatter. The most common are spindle speed, depth of cut and feed rate. But it is also possible to change the tool geometry in numerous ways. Lindström (2003) summed some parameters and their influence on chatter, see figure 2-1 for tool geometry. The Side Cutting Edge Angle has a big influence on the radial component of the cutting force and can therefore be set to minimise it, since in general the dynamic stiffness is the lowest in this direction. Side Cutting Edge Angles other then 90º has an negative influence on chatter. The nose radius can also be used to reduce the radial cutting force component. The Back Rake Angle can be used to reduce the dynamic forces by choosing a positive angle.

![Tool geometry](image)

The reason for chatter is fluctuating cutting forces. Low cutting forces keeps a small deflection and the system have less energy in it. The cutting force can be decomposed in components corresponding to tangential, radial and axial directions, see figure 2-2, in order to evaluate changes.
In figure 2-2, $F_C$ is the cutting force, $F_t$ is the tangential component (or the component in cutting speed direction), $F_r$ is the radial component, $F_f$ is the axial or feed component. There are five basic tool oscillation components, tool relative to work-piece, which are shown in figure 2-3. One can also imagine torsional oscillation. In real life cutting, a sum of all motion types is the true oscillation.

**Figure 2-2.** Cutting force and its components.

**Figure 2-3.** Different components of tool oscillation. a - axial oscillation, b – radial or Type A chatter, c – tangential or Type B chatter, d – axial sweep, e – tangential rotation.
Tobias (1961) among others reports a reduced tool life when chatter is present during operation. This is due to high impact loading of the cutting edge. The article also reports that in some cases an increased tool life was gained by chatter. Vast numbers of research has been made on tool failure and various ways to detect it, for example by acoustic emission or analysis of radiated sound.

2.2 Chatter and stability

Tobias (1961) reports two types of chatter motions, Type A and Type B chatter. Type A is when the cutting structure is relatively stiff in all directions except for in the normal direction of the work-piece, see figure 2-3. Type B chatter is when the cutting structure is stiff in all directions except in the tangential, see figure 2-3.

In figure 2-3 it can easily be seen that the relative displacement between the cutting tooth and work-piece is of great importance for optimal cutting. With a tool with low dynamic stiffness, large deflection and chatter of Type B is easily excited.

Chatter excitation can be divided in two forms of classes, external and regenerative excitation. External excitation is when an external force excites the structure and creates chatter. The external force can for instance be a hard spot in the work-piece or change in feed speed or cutting angle. Regenerative chatter is self-excited and occurs when the chip thickness changes due to the previous and current cut. Sweeney and Tobias (1961) described the chip thickness as a function of previous cut, see equation (2-a).

\[ ds = x(t) - \mu x(t-T) \]  

(2-a)

where \( ds \) is the chip thickness variation, \( x(t) \) is the current relative motion between tool and work-piece, \( x(t-T) \) is the relative motion \( T \) seconds earlier and \( \mu \) is an overlap factor, \( 0 \leq \mu \leq 1 \), between successive cuts (\( \mu = 0 \) for threading and \( \mu = 1 \) for parting). If \( \mu = 0 \) regenerative chatter is not possible. Tlusty and Ismail (1981) described the chip thickness as result of waviness, see equation (2-b).

\[ CT = Z_{\min} - Z(N); \quad Z_{\min} = \min(Z_{N-L} + H, Z_{N-2L} + 2H, Z_{N-3L} + 3H, ...) \]  

(2-b)

where \( CT \) is the current cut chip thickness, \( Z_{(i)} \) is the tool coordinate, \( H \) is the average chip thickness and \( L \) is a step equal to one revolution. The chip thickness is dependent on chatter frequency and corresponding wavelength on the work-piece. On the circumference of the work-piece a full number of chatter wavelengths and an phase shift factor, \( \epsilon \), will be cut, see figure 2-4. For different phase shift, \( \epsilon \), different chip thickness variations is a fact. Figure 2-5 shows simulated cutting, over a short part of the work-piece, with \( \mu = 1 \); the work-piece is assumed to be straight and not round (which is an convenient approximation if the wavelength is small compared to the work-piece radius).
Figure 2-4. Undulated work-piece surface with wavelength $L$ and phase shift $\epsilon$

Figure 2-5. Simulated cutting motion with two different phase shifts, $\epsilon$
The chatter amplitude and average chip thickness will set different chip forms. With high amplitudes and a small average chip thickness the chip will be broken. For opposite properties the chip will be continuous with an undulated surface. This chip form is also dependent on \( \varepsilon \), as seen in figure 2-4. In theory a continuous chip can be the case with a small average chip thickness, high chatter amplitude and a small phase shift, see figure 2-6.

![Figure 2-6](image)

**Figure 2-6**, Different \( \varepsilon \) gives different chip form. In a \( \varepsilon = 0^\circ \) and in b \( \varepsilon = 180^\circ \) in a the chip thickness is continuous and in b it has its maximum change

2.2.1 Chatter frequency from work-piece

The undulated surface left on the work-piece by chatter vibrations gives information about the wavelength. The chatter frequency can be calculated by measuring the distance between two wave-crest, see equation 2-c, if the cutting speed is known.

\[
f_c = \frac{v_c}{L} \tag{2-c}
\]

where \( L \) is the length between two wave-crests.

2.2.2 Cutting models

Various models of dynamic cutting and stability have been made during the past years. The simplest ones are one degree of freedom systems (1-DOF), for instance Tobias and Fishwick (1958) and Deshpande and Fofana (2001). The 1-DOF system was considered to simplified and did not explain the non-linearities which was assumed to have an significant role in the cutting process. Tlusty and Ismail (1981) assumed a 2-DOF model to simulate the cutting motion with tool jumps, the model can be seen in figure 2-7.
2.2.3 Stability criterion chart

There are basically two methods to determine stability, one analytical and one graphical. Both have been used in different ways and with different parameters in mind. A stability chart can be seen in figure 2-8. The operation is stable if the cutting conditions gives a value under the stability line and unstable if it is above.
### 2.2.4 Graphical

A graphical method was developed by Gurney and Tobias (1961) where the harmonic response locus is used for describing the machine structure characteristics. The model assumes a change in cutting forces due to a change in chip thickness, also other parameters are claimed to be considered in the model. This force variation and the equation of motion of the system makes it possible to draw the harmonic response locus. Since the limit of stability creates a sinusoidal vibration with stable amplitude, the same amplitude for previous and current cut. The excitation force can then be represented as a vector in the harmonic response locus. Using geometrical properties and making the calculation for a frequency interval the critical conditions can be found. The force oscillation due to change in chip thickness is modelled as

\[ dP(t) = k_1 [x(t) - \mu x(t-T)] \]  

(2-d)

where \( k_1 \) is the chip thickness coefficient. The equation of motion is then

\[ f(x, \dot{x}, \ddot{x}) = -dP(t) \]  

(2-e)

Stability requires \( x(t) \) to be sinusoidal and therefore the harmonic response locus is a pertinent tool. The excitation force, \( -dP(t) \), is represented by the vector \( OP \) and \( dP(t) \) is represented by \( OP' \), that is \( OP'=-OP \), see figure 2-9. Since stability requires sinusoidal motion with constant amplitude, neither increasing nor decreasing, the vectors \( k_1x(t) \) and \( k_1x(t-T) \) are of equal magnitude. These vectors meets on the perpendicular bisector of \( OP' \) so a triangle can be drawn with \( OR'=k_1x(t) \) and \( P'R'=\mu k_1x(t-T) \). This gives \( P'R'=\mu OR' \) and the normalization of \( OR=\lambda s x(t) \) leads to

\[ \frac{k_1}{\lambda_s} = \frac{OR'}{OR} \]  

(2-f)

where \( \lambda_s \) is the statical stiffness between the tool and work-piece. By repeating this for a series of frequencies \( k_1 \) can be found as a function of the frequency, \( \omega \). Harmonic functions gives the phase angle between \( k_1x(t) \) and \( k_1x(t-T) \) to \( \omega T \), denotes as \( \theta \) in figure 2-9. \( T \) has multiple values of \( \omega \) since the angle can be \( \theta + 2\pi n \), where \( n \) is any integer.
2.2.5 Analytical

Various models have been used, Tlysty and Ismail (1981); Sweeney and Tobias (1996); Altintas and Weck (2004) for example, to calculate the limit of stability. Basically they take the structure motion into account to derive a stability chart, depth/width of cut to spindle speed for instance. Altintas and Weck (2004) wrote the following equations:

$$F_t = K_t a h_0$$
$$F_r = K_r a h_0$$

(2-g)

where $F$ is the force, $t$ denotes the tangential and $r$ the radial direction, $a$ is the width of cut, $h_0$ is the statical chip thickness and $K_t$ is a cutting coefficient, which may be dependent on tool geometry, chip thickness, cutting speed, lubrication. The change in chip thickness is written as

$$h_{(t)} = h_0 - \mu [y(t) - y(t-T)]$$

(2-h)

where $T$ is the spindle rotation period and $y(t)$ and $y(t-T)$ is the vibration amplitude for current and previous cut, in radial direction. The dynamic orthogonal cutting was then expressed as a 1-DOF system with a mass, $m$, stiffness, $k$, and damping, $c$ at the cutting point. This gives a delayed differential equation which was described by an closed loop system, see figure 2-10, simulating the following equation

(2 - 8)
\[ m \ddot{y} + c \dot{y} + k y = F, \]  
the transfer function was then given as
\[ \frac{h_{(s)}}{h_{0(s)}} = \frac{1}{1 + (1 - e^{-sT})K_r \alpha \phi_{(s)}} \]  

(2-i)

(2-j)

The characteristics equation of equation (2-j) determines the chatter stability. The frequency response of the given system was given as:
\[ \phi_{(s)} = m s^2 + c s + k s, \quad \rightarrow s = i \omega \]
\[ \phi_{(\omega)} = G_{(\omega)} + i H_{(\omega)} \]

(2-k)

\( s \) is the Laplace operator and \( \omega \) is the frequency. An absolute chatter stability law, equation (2-j), was used to determine stability as a function of with of cut, \( a \).

\[ a_{\text{lim}} = - \frac{1}{2K_r \Re(\phi_{(\omega)})} = - \frac{1}{2K_r G_{(\omega)}} \]

(2-l)

In figure 2-10 \( R \) is the control signal, \( F \) is the cutting force, \( X' \) is the cutting motion with respect of the elastic structure deflection, \( V \) is an external disturbance, \( X \) is the real cutting motion and \( P \) is the force modulation due to CP.

Nigm (1981) summed the frequency response functions of the machine tool, chip thickness modulation and the metal cutting process in one model to calculate the chatter stability. The model considered cutting in one direction but could be extended to more directions, with oriented transfer function. All transfer functions represents a subsystem in the cutting motion. The transfer functions corresponding frequency response functions are expressed in a complex polar form, in frequency domain. The machine tool was represented as in equation (2-m).

\[ G_{(\omega)} = M e^{i\phi} \]

(2-m)

where \( \omega \) is the angular frequency, \( i = \sqrt{-1} \), \( M \) is the gain and \( \Phi \) is the phase angle.

(2 - 9)
The chip thickness modulation is expressed as by Sweeney and Tobias, equation (2-a). This gives the frequency response function in equation (2-n).

\[ \phi_1(i) = C e^{i\psi} \]  
where \( C = \sqrt{1+\mu^2-2\mu \cdot \cos(\beta)} \), \( \beta = \omega T = \frac{60\omega}{N} \), \( \psi = \arctan\left( \frac{\mu \sin(\beta)}{1-\mu \cos(\beta)} \right) \) and \( N \) is the number of revolutions per minute.

The metal cutting process subsystem gets its input from the chip thickness modulation and the output is the dynamic cutting force. The frequency response function is then:

\[ \phi_2(i) = K e^{i\theta} \]  
where \( K = W k \), \( \theta = \arctan\left( \frac{\lambda \omega}{v_c} \right) \). \( W \) is the width of cut, \( k \) is the dynamic cutting coefficient and since \( v_c = \frac{\pi DN}{60} \), \( \theta \) can be rewritten as: \( \theta = \arctan\left( \frac{\lambda \beta}{\pi D} \right) \). \( D \) is the work-piece diameter and \( \lambda \) is a parameter which is dependent of the undeformed chip thickness.

Since the FRFs are derived with linear theory, superposition is valid. This gives the total frequency response function:

\[ \phi(i) = G(i) H_1(i) H_2(i) \]  
expressed in complex polar form equation (2-p) turns to

\[ \phi(i) = A e^{i\alpha} \]  
where \( A = MCK \) and \( \alpha = \phi + \psi + \theta \). The stability criterion is then given as: When \( \alpha = \pi \) the system is stable if \( A < 1 \), unstable if \( A > 1 \) and on the border to stability if \( A = 1 \). The equations are then reduced to

\[ \phi = \pi - \psi - \theta \]  
and

\[ K = \frac{1}{MC} \]

to calculate the limit of stability. If one knows the dynamic cutting coefficient, \( k_1 \), it is possible to plot the stability as a function of width of cut, \( W \), dependent on the spindle speed, \( N \).

2.2.6 Non-linear

One major idea of non-linear models is to take the tool jump in consideration. The fact that vibration amplitude does not grow to infinity but stabilizes at a finite value was one reason to model non-linear cutting dynamics. One other reason was that the stability predicted with linear theory is not the same as in measurements Ahmadi and Ismail (2010). The same researchers showed that the transition from stable to unstable cutting conditions changed gradually and not
abruptly. Cutting operation in turning with strong chatter is a non-linear phenomena since the cutting force is not fluctuating in a harmonic way. This is due to the motion of the tool which sometimes leaves the work-piece, or do a so called tool jump, see figure 2-5. The cutting force which is proportional to the chip thickness during continuous cutting becomes zero when the tool leaves the work-piece (no cutting of material). This non-linearity was first described by Tlusty and Ismail (1981). It was also shown that this non-linearity sets the vibration amplitude limit to a stable value, when linear theory would suggest that it would grow to infinity. Damping of the motion is generated by the tool – work-piece interference, see figure 2-11. When the tool is in contact with the work-piece the cutting force acts contrary the tool motion and therefore acts as a damper. Different simulations and experimental set-ups were preformed by Ahmadi and Ismail (2010) to evaluate linear and non-linear models. Tlusty (1993) reports the effect of spindle speed and stability for high speed milling in terms of Process Damping (PD). It was also reported that process damping is high for low spindle speed.

![Figure 2-11. Process damping tool contact](image)

### 2.3 Damping of chatter

Chatter reduction can be done by changing cutting parameters (and most likely the MRR) or with damping, either passive or active. Both passive and active damping have been successfully used and they have different advantages and drawbacks.

Passive damping is based upon a damped tool/cutting tool support structure or work-piece clamping/spindle. The basic theory is that a viscous elastic material is added in the structure to damp the critical motion for the limiting frequencies. The damping can be added in the tool, tool – holder, the cutting tool support structure, work-piece clamping or spindle bearing. Different clamping conditions give different results since the structure properties can be changed a lot. Passive damping is limited to the designed purpose since only the accounted frequencies can be damped. One big advantage with passive damping is that very small changes in machine operation has to be done.

Active damping uses online data recorded during the operation and tunes the tool to create anti resonance. Active damping can be used for a number of different frequencies, within the tuning range. The drawback of active damping is that it requires expensive equipment and changes in the machine set-up.
2.4 Surface roughness

There are two types of surface roughness specified for the surface of interest, the arithmetical mean roughness ($R_a$) and the mean roughness ($R_z$). In this case $R_z$ is the limiting requirement. $R_a$ is specified as:

$$R_a = \frac{1}{r} \int_{0}^{r} |f(x)| \, dx$$

(2-t)

where $r$ is the test length and $y=f(x)$, see figure 2-12. $R_z$ is defined with the mean line between peaks and valleys, see equation (2-u).

$$R_z = \frac{\left|y_{p1} + y_{p2} + y_{p3} + y_{p4} + y_{p5}\right| + \left|y_{v1} + y_{v2} + y_{v3} + y_{v4} + y_{v5}\right|}{5}$$

(2-u)

where $y_{pi}$ is the peak value and $y_{vi}$ is the valley value, see figure 2-12.

![Surface roughness description](image)

**Figure 2-12, Surface roughness description, m is the mean line**
3. Method

3.1 Specimen

The specimen showed in figure 3-1 in this thesis is a hub reduction gear housing for trucks. Its main geometry can be seen in figure 3-2. It is cast and the operation of interest has previously been roughed. The operation to be studied is finish internal turning at the open end of the housing, see figure 3-1.

![Figure 3-1](image1.png)  
**Figure 3-1**, *Hub reduction gear housing. The specimen of the thesis*

![Figure 3-2](image2.png)  
**Figure 3-2**, *Main geometry properties*

The work-piece is made of cast iron, GJ 0720, due to lack of material properties a similar material data have been used for calculations; see table 3-1 for material properties, Sundström (1998).

<table>
<thead>
<tr>
<th>Material</th>
<th>Young's modulus</th>
<th>Density</th>
<th>Poisson's ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS0727-02</td>
<td>167 GPa</td>
<td>7200 kg/m³</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table 3-1, Material properties
A vertical lathe is used for the turning operation, see figure 3-3. The work-piece stands on a ring and it is clamped with an expanding mandrel in the centre hole.

![Figure 3-3](image)

**Figure 3-3, Vertical lathe in which the turning is performed.**

### 3.2 Current operation conditions

Cutting data used during experiment was the same as during manufacturing of the work-piece, see table 3-2 for data.

<table>
<thead>
<tr>
<th>Tool insert</th>
<th>CNMG 120408-WMX 3210 Sandvik Coromant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side Cutting Edge Angle</td>
<td>95°</td>
</tr>
<tr>
<td>End Cutting Edge Angle</td>
<td>5°</td>
</tr>
<tr>
<td>Depth of cut</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Feed</td>
<td>0.11 mm/rev</td>
</tr>
<tr>
<td>Cutting speed</td>
<td>170-180 m/min</td>
</tr>
</tbody>
</table>

The spindle speed is given by

\[
N = \frac{v_{cut}}{\pi D}
\]  

(3-a)

which results in \( 203 \leq N \leq 215 \) rev/min.

The overlap factor, \( \mu \), was calculated as in equation (3-b), the tool geometry and cutting parameters gives a theoretical area of contact, see figure 3-4, and an overlapping area between cuts.
This gives an overlap factor of $\mu = 0.76$.

### 3.3 Chatter frequency from work-piece

The wavelength was measured in four work-pieces. In each work-piece five measurements were performed. The arithmetic mean value was then calculated.

### 3.4 Experimental modal analysis

To extract the mode shapes and natural frequencies experimentally a modal analysis was performed. LMS Test.Lab Rev 8b. was used to record and analyse data. The synthesised modal model was derived with LMS PolyMAX method. The structure was excited with an impact hammer and the response was measured with one accelerometer. The accelerometer was moved between all points and the hammer strike was always on the same point. All double impacts or obvious miss strikes were excluded. Sufficient number of valid impacts, for averaging, was selected to five. The upper frequency limit of interest was set to 4096 Hz, sampling frequency $f_{\text{sample}} = 8192$ Hz and an anti-aliasing filter was used in order to avoid aliasing/folding. An exponential window was used on the force signal. The frequency response functions were calculated as displacement over force, compliance function, since the relative displacement between tool and work-piece is of interest.

**Equipment**

**Impact hammer**

- **Brand:** Ziegler
- **Model:** Ixys H2
- **SN:** 9117
- **Sensitivity:** 2.24 mV/N
- **Attached weight:** 77g
- **Tip:** Steel

(3 - 3)
Accelerometer
Brand: Dytran
Model: 3225F
SN: 6283
Sensitivity: 10.3 mV/g
Weight: 0.6g, 6g with cable

When adding the accelerometer to the work-piece the structure properties are changed. If the added mass is too high the response will not be accurately measured. The added mass tends to lower the resonance frequency, see equation (3-c).

\[ f_m = f_s \sqrt{\frac{M}{M + m_{acc}}} \]  

(3-c)

where \( f_m \) is the measured resonance frequency, \( f_s \) is the structure's resonance frequency, \( M \) is the structure's effective moving mass and \( m_{acc} \) is the accelerometer mass.

From equation (3-c) a minimum effective moving mass for the structure can be calculated as a function of allowed frequency change, see equation (3-d).

\[ M_{\text{min}} = \frac{m_{a} \left( \frac{f_m}{f_s} \right)^2}{1 - \left( \frac{f_m}{f_s} \right)^2} \]  

(3-d)

If a resonance frequency change of 1% is allowed, the minimum effective moving mass can be calculated. From equation (3-d) and accelerometer weight, page 3-4, follows that \( M_{\text{min}} = 29.6 \) g, which is equivalent to 4.1 cm\(^3\) of SS 0227-02 cast iron. The accelerometer weight is therefore considered to be negligible.

Microphone
Brand: Brüel & Kjær
Model: 4191
SN: 1832989
Sensitivity: 12.5 mV/Pa

Pre-amplifier, microphone
Brüel & Kjær, Model: 2669, SN: 183442 pre-amplifier
Brüel & Kjær, Microphone Power Supply Type 2804

Data acquisition system
LMS Test.Lab Rev. 8b on personal computer with SCADAS Mobile front end.

3.4.1 Modal analysis of the tool

Five points were used in the direction of the cutting speed, see figure 3-5a. Point 2 to 5 was placed on the tool axis and Point 1 next to the insert. For measurement in radial direction four points were used, see figure 3-5b. Point 2 was the striking point for both measurements.
3.4.2 Modal analysis of work-piece

Two EMAs were performed on the work-piece. One on the side of the work-piece x-direction, and one on top of it, y-direction, see figure 3-6a and b, note that the sketches are very simplified. Five points on the side and four points on the top. The work-piece was clamped as during standard operation when the modal analysis was performed.
3.5 Sound measurement

The sound was recorded during the operation. The microphone was placed inside the lathe enclosure on the back wall, see figure 3-7. The microphone membrane was placed perpendicular to the line between the approximated cutting and microphone.
The autocorrelation of the sound was calculated with LMS Test.Lab for different time steps and the average power estimate, power spectral density – PSD, was performed with Welch's method in MatLab (command pwelch) with 50% overlap and an hanning window, to avoid leakage. The data from the sound measurements is only for additional information about the cutting process. Therefore it was not calibrated and the levels are presented in Volt, not sound power level. The recordings contain both the wanted sound from the cutting as well as background noise, such as the coolant jet and other sources from the work shop.

3.6 Numerical calculations

The finite element method was used for calculating the natural frequencies and response function for the work-piece. The model was created in SolidEdge CAD software and the FEM analysis where executed in Comsol Multiphysics 4.0a. The FEM model has a reduced geometrical complexity (holes are filled, slits removed and edges made round) in order to avoid sharp corners which in linear elastic material models may give infinite strain, see figure 3-8 and 3-9. A linear-elastic material model was used with quadratic Lagrange shape functions. The Multifrontal Massively Parallel sparse direct Solver or MUMPS was used for calculations. The material data given in table 3-1 was used. No structural damping was added to save calculation time.

Figure 3-8. Simplified geometry for FEM model (with cross section)
3.6.1 Mesh

In order to know if the solution converged the solution change from current mesh to the next refined mesh had to be 1% or less. Comsol meshing and mesh refinement satisfying the convergence gave the meshes presented in table 3-3.

<table>
<thead>
<tr>
<th>Mesh 1</th>
<th>Mesh 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tetrahedal elements</td>
<td>56230</td>
</tr>
<tr>
<td>Triangular elements</td>
<td>20034</td>
</tr>
<tr>
<td>Edge elements</td>
<td>4459</td>
</tr>
<tr>
<td>Vertex elements</td>
<td>322</td>
</tr>
<tr>
<td>Number of elements:</td>
<td>56230</td>
</tr>
<tr>
<td>Mesh volume</td>
<td>2405000.0 mm$^3$</td>
</tr>
<tr>
<td>Tetrahedal elements</td>
<td>224644</td>
</tr>
<tr>
<td>Triangular elements</td>
<td>40610</td>
</tr>
<tr>
<td>Edge elements</td>
<td>6127</td>
</tr>
<tr>
<td>Vertex elements</td>
<td>322</td>
</tr>
<tr>
<td>Number of elements:</td>
<td>224644</td>
</tr>
<tr>
<td>Mesh volume</td>
<td>2409000.0 mm$^3$</td>
</tr>
</tbody>
</table>

3.6.2 Eigen-frequency calculation

The first six eigen-frequencies where calculated.

3.6.3 Frequency response calculation

Two different boundary conditions where used for FRF calculation. Boundary condition 1 with the bottom hole fixed and boundary condition 2 with the bottom and upper holes fixed. The FRF response from 100 to 1000 Hz was calculated. The reason lower frequencies where excluded was the low interest of the response for this band. The upper limit was set due to the time consuming calculations.
3.7 Identify physical mode properties

In order to use proposed analytical methods the measured frequency response has to be converted into physical parameters in a single degree of freedom system (SDOF), see figure 3-10. In a system with well separated modes these can be extracted by data from response plot and some simple calculations, see figure 3-11 and equation (3-e) to (3-i), Bodén et al. (1999). The damping ratio is calculated with Half-energy bandwidth method.

From the graphs the physical parameters can be extracted by the following equations:

- Natural angular frequency, $\omega = 2\pi f_n$  \hspace{1cm} (3-e)
- Viscous damping ratio: $\zeta = \frac{f_2 - f_1}{2f_n}$  \hspace{1cm} (3-f)
- Stiffness: $k = \frac{-1}{2\zeta \cdot 3(H_{\omega})}$  \hspace{1cm} (3-g)
- Mass: $m = \frac{k}{\omega_n^2}$  \hspace{1cm} (3-h)
- Damping: $c = 2\zeta \cdot \sqrt{k \cdot m}$  \hspace{1cm} (3-i)

(3 - 9)
3.8 Calculation of stability

The stability chart was calculated in the way Nigm (1981) proposed. All three models were used to calculate the stability limit. Since there is an unknown parameter dependent on the undeformed chip thickness, $\lambda$, a range of values was used, $0.01 \leq \lambda \leq 1.5$ [mm].
4. Results

4.1 FRF measurements

The results from modal testing are presented in appendix A. The FRFs are given as accelerance function, acceleration over force.

4.2 Comparison between measured and synthesised FRF

Comparisons between the measured and synthesised accelerance FRFs are presented in appendix B.

4.3 Synthesised FRF

The synthesised compliance FRF, see figure 4-1 to 4-5, is the base for the single degree of freedom models, in section 4.6.

Figure 4-1, Synthesised FRF, Compliance and phase for the work-piece in x-direction
Figure 4-2. Synthesised FRF; Compliance and phase for work-piece in x-direction, zoomed

Figure 4-3. Synthesised FRF; Compliance and phase for work-piece, y-direction
Figure 4-4, Synthesised FRF, Compliance and phase for tool, x-direction

Figure 4-5, Synthesised FRF, Compliance and phase for tool, tangential direction
4.4 Sound measurement

Autocorrelation of the operation sound signal for three measurements can be seen in figure 4-6. The PSD for the same measurements are presented in figure 4-7.

Figure 4-6. Autocorrelation for cutting operation

Figure 4-7. PSD of measurement signals
4.5 Chatter frequency from work-piece

Measurement of the wavelength in work-pieces is shown in figure 4-8. The mean value was calculated, from four different work-pieces on five locations, to \( L = 12.28 \) mm. This gives an chatter frequency of about \( f_c = 220 \) Hz.

![Figure 4-8. Measurement of wavelength in work-piece](image)

4.6 Single degree of freedom models

Three models, extracted from the synthesised frequency response, section 4.4, where created for stability calculation. Two models of the work-piece and one of the tool. The SDOF models of the work-piece model the behaviour for the mode at 204 and 856 Hz, in radial direction at point 2. The model for the tool was extracted from point 1 in radial direction. Properties for all systems are presented in table 4-1. Model 1 represents the work-piece at 204 Hz, model 2 represents the work-piece at 856 Hz and model 3 represents the tool.

<table>
<thead>
<tr>
<th>Table 4-1, SDOF model properties</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model 1</strong></td>
</tr>
<tr>
<td>Mass [kg]</td>
</tr>
<tr>
<td>Damping [Ns/m]:</td>
</tr>
<tr>
<td>Stiffness [N/m]:</td>
</tr>
</tbody>
</table>

The response of all models where plotted with the synthesised FRF in order to compare them, see figure 4-9 and 4-10.
Figure 4-9, Comparison between model 1, 2 and FRF

Figure 4-10, Comparison between model 3 and FRF
4.7 FEM calculations

4.7.1 Mesh

The final mesh quality is plotted in figure 4-11.

![Mesh quality, by Comsole (dark red = good, deep blue = bad)](image)

**Figure 4-11**, *Mesh quality, by Comsole (dark red = good, deep blue = bad)*

4.7.2 Eigen-frequencies

The first six eigen-frequencies are presented in table 4-2.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency</th>
<th>Mode</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>450 Hz</td>
<td>4</td>
<td>1470 Hz</td>
</tr>
<tr>
<td>2</td>
<td>832 Hz</td>
<td>5</td>
<td>1556 Hz</td>
</tr>
<tr>
<td>3</td>
<td>1305 Hz</td>
<td>6</td>
<td>1742 Hz</td>
</tr>
</tbody>
</table>

**Table 4-2**, *FEM calculated eigen-frequencies*

4.7.3 FRF calculation

Figure 4-12 shows the resulting response in point 2, note that the model is made without damping, for the two boundary conditions, given in 3.6.
4.8 Stability charts

Stability charts for all models are presented in figure 4-13 to 4-15.
Figure 4-14. Stability chart for model 3, mode at 856 Hz

Figure 4-15. Stability chart for model 3, Tool
5. Discussion and conclusions

5.1 Measurements

The FRF measurements shows, as common, bad coherence in low frequency and at anti-resonances. At resonance and over all the coherence is good and the measurements are considered to be accurate. The synthesised responses correlates well with the measured response. The synthesised tool response in x-direction only takes the dominating mode into account, since it is the most important. The tool compliance can be considered low and non-contributing to the stability problem.

Looking at the response in fig 4-2 it is clear that the displacement for the mode at 856 Hz will be smaller the closer to the clamping base. By looking at the FEM results this is obvious since the work-piece wall acts like a pendulum, the closer to the hinge the smaller the displacement. The mode at 204 Hz does not indicate the same behaviour, the compliance does decline but not as much as at 856 Hz. This this indicates that the 204 Hz mode is not a property of the work-piece but of the clamping or spindle.

The sound measurements can be excluded from the work since almost no energy was carried in 220 Hz. The low frequency content is probably generated by the machine and surrounding environment. The peak at 850 Hz is probably generated by the work-piece first mode, see FEM discussion in 5.3, since it is the frequency where the whole work-piece acts as a acoustic radiator. The contribution at 2500 Hz is most likely to come from the tool and work-piece.

Wavelength measurements gave the chatter frequency to about 220 Hz, this coincides with the lowest mode for the work-piece – fixture. This result with measurements excludes the tool from the list of possible candidates to chatter exciters.

5.2 SDOF models

One can argue about mode separability in this case but as shown in figure 4-9 and 4-10, all models gives a reasonable correlation between model and synthesised response. Since only a small part of the model response (close to resonance where the model response and synthesised FRF have high correlation) is used for stability calculation, the models can be considered valid.

5.3 FEM

The results from the FEM calculations gives lower resonance frequencies then the measured response. The reason for this is probably the assumption of material data and the simplified model might also change the response. The result gave no indication what so ever that there should exist any mode at 204 Hz. This is a strong result since it eliminates the work-piece and the clamping – spindle is the weak link in this structure. The mode at 450 Hz in FRF calculations was not derived.
for the eigen-frequency calculations for a free work-piece. The mode is not seen in the experimental results either. By looking at the shape for the solution at 450 Hz, figure 5-1, it can be seen that it is a result of inaccurate modelling of the real clamping condition. In reality the work-piece is standing on a support and bending at the base is not possible. This theory is supported by the two different clamping conditions for FRF calculations. By clamping both the upper and lower hole the compliance is lowered significantly and by adding a stiff support in the base this mode will probably be cancelled out.

Figure 5-1, FEM solution at 450 Hz, bending at the base of the work-piece

The lowest eigen-frequency result shows the same shape as the first vibrating mode of a finite cylinder, see figure 5-2, this is a good acoustic source, which can be seen in the sound measurements. It also gives a reason for the high compliance and why the stability is lower than the 204 Hz mode. Therefore this mode cannot be excluded for stability calculations.

Figure 5-2, FEM solution at 822 Hz
5.4 Stability chart

The cutting stiffness coefficient has been left unknown since it requires extensive measurements and the stability chart works as a guidance in choosing spindle speed for stable cutting without it. When choosing parameters for stable cutting all significant modes should be considered for optimum result. Figure 5-3 gives a stability chart with both work-piece modes, the tool model has been excluded since its stability limit is far over the others. The assumption of no vibrations in tangential direction can be considered valid due to the low compliance of the tool.

Figure 5-3 shows that the optimal choice of spindle speed is 214 rpm for model 1 but not for model 2. Although model 1 has a higher border of stability it is important to consider the fact that these stability limits have been calculated with linear stability theory and some simplifications. In reality the operation has more parameters to be included, such as process damping, variation in spindle speed, external disturbances etc. Since the measured chatter frequency in the work-piece gives almost the same frequency as model 1's chatter frequency, better operation condition for this mode cannot be selected (without a big increase). Model 2 have a higher limit of stability at 225 Hz but since this mode is not seen on the chatter marks in the work-piece a change might not improve the result. One suggestion of change is that the spindle speed should not be set to as low as 203 rpm, due to the fact that the stability border is about 1.5 times higher at 215 rpm. A constant spindle speed at 214 gives, according to the calculations, the best result.
5.5 Chatter free work-pieces

There are some different explanations to the fact that chatter does only appear on some work-pieces. One is that the external disturbance, see figure 2-10, is small for the larger part of work-pieces but on some the roughing may have left a bad surface or the tool is close to worn out. Other possibilities can be that the automatic placement of the work-piece gets misaligned and the clamping force is distributed unevenly or that the mandrel sleeve is pulled oblique over the arbor.

After the operation studied in this thesis the tool continues downwards, y-direction, with an increased feed rate. During this operation chatter is not present, regardless is it occurred or not in the previous step. One explanation for this can be that the overlap factor, \( \mu \), is lower which gives a higher stability limit or it may be that the cutting force is changed, in magnitude, direction or both.
6. Suggestions for further work

If one are to continue the study the fixture properties has to be derived. A experimental modal analysis of the spindle will probably rule out the influence of bearings and rotor structure dynamics and the clamping has to be tested. A experimental modal analysis of the mandrel is not recommended since it is a highly non-linear complex built-up structure and different loadings will give different results. Instead it is recommended to study the clamping process from the robot placing the work-piece to the sleeve expansion. This is probably an enormous task since not all work-pieces are subjected to chatter and the experimental set has to be large.

One other suggestion is to find a better clamping device where the work-piece is clamped in radial and axial direction, in order to reduce the degrees of freedom.
7. References


Sundström, B., (1998), Handbok och formelsamling i Hållfasthetslära, Sixth printing, Stockholm, E-PRINT AB, Copy right: Institutionen för hållfasthetslära KTH.


Tlusty, J., (1993), High-Speed Machining, CIRP Annals – Manufacturing Technology, Volume 42, Issue 2, pp 733-738, ISSN 0007-8506


8. Appendix

A. Measurement results

The measured accelerance FRF and coherence is presented in figure A-1 to A-5.

Figure A-1, Measured FRF and coherence for work-piece x-direction
Figure A-2, Zoomed FRF and coherence for work-piece x-direction

Figure A-3, Measured FRF and coherence for work-piece, y-direction
Figure A-4, Measured FRF and coherence for the tool in x-direction

Figure A-5, Measured FRF and coherence for the tool in tangential direction
B. Comparison between measured and synthesised FRF

A comparison between measured and synthesised accelerance FRF is given in figure B-1 to B-18.

Figure B-1, Comparison of FRF for Work-piece x-direction in point 1

Figure B-2, Comparison of FRF for Work-piece x-direction in point 2
Figure B-3. Comparison of FRF for Work-piece x-direction in point 3

Figure B-4. Comparison of FRF for Work-piece x-direction in point 4
Figure B-5. Comparison of FRF for Work-piece x-direction in point 5

Figure B-6. Comparison of FRF for Work-piece y-direction in point 1

(8 - 7)
Figure B-7, Comparison of FRF for Work-piece y-direction in point 2

Figure B-8, Comparison of FRF for Work-piece y-direction in point 3
Figure B-9. Comparison of FRF for Work-piece y-direction in point 4

Figure B-10. Comparison of FRF for Tool x-direction in point 1
Figure B-11, Comparison of FRF for Tool x-direction in point 2

Figure B-12, Comparison of FRF for Tool x-direction in point 3

(8 - 10)
Figure B-13. Comparison of FRF for Tool x-direction in point 4

Figure B-14. Comparison of FRF for Tool tangential direction in point 1
Figure B-15, Comparison of FRF for Tool tangential direction in point 2

Figure B-16, Comparison of FRF for Tool tangential direction in point 3
Figure B-17. Comparison of FRF for Tool tangential direction in point 4

Figure B-18. Comparison of FRF for Tool tangential direction in point 5