Degree project

Approximation of Antenna Patterns With Gaussian Beams in Wave Propagation Models

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Abstract

The topic of antenna pattern synthesis, in the context of beam shaping, is considered. One approach to this problem is to use the method of point matching. This method can be used to approximate antenna patterns with a set of uniformly spaced sources with suitable directivities. One specifies a desired antenna pattern and approximates it with a combination of beams. This approach results in a linear system of equations that can be solved for a set of beam coefficients. With suitable shifts between the matching points and between the source points, a good agreement between the assumed and the reproduced antenna patterns can be obtained along an observation line. This antenna modelling could be used in the program NERO to compute the field at the receiver antenna for a realistic 2D communication link. It is verified that the final result is not affected by the details of the antenna modelling.

Keywords: Gaussian beam, Pattern synthesis, Point matching, Wave propagation models
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1. Introduction

Wave propagation is important in the context of telecommunications where one typically has a transmitting antenna that sends a signal through a radio channel with properties that are determined by the terrain between the transmitting and the receiving antenna. Approximate high frequency methods have been used to estimate the strength of the received signal [1]. In order to verify the accuracy of these methods, full wave simulations were done [2, 3].

For high frequencies and long distances the solution of integral equations requires a large number of basis functions [4]. This difficulty is handled by means of the FMM method that dramatically reduces the memory and computing time [5, 6]. The iterative solution of linear systems is dependent on the conditioning of the system [4, 7].

The modelling of the terrain profile can be done in various ways [8, 9]. A problem is to reduce the three-dimensional problem to a two-dimensional problem by modelling the field from the transmitting antenna properly [10].

In section 2 of this report an overview of the program NERO is given. Section 3 gives a comparison of the results produced by this program and the approximate high frequency method UTD, in the case of diffraction from a thin half-screen. Section 4 discusses the parameters of the gaussian beam. In Section 5, the use of gaussian beams in a procedure to approximate the radiation pattern of an antenna is verified. In section 6 the stability of the procedure for a realistic communication link was investigated by means of the program NERO.

2. Overview of the program NERO

One can describe a full wave electromagnetic simulator in terms of pre-processing, processing and post-processing [4]. In the pre-processing part one specifies the geometry, the excitation, the operational frequency and the output. In NERO this part is known as the wxGBTool or graphics toolbox [11]. In this toolbox one can choose geometries with predefined shapes such as circles or polygons. There is also a possibility to define geometries by introducing a finite number of points in 2D Cartesian coordinates in a counterclockwise fashion. One can specify perfectly conducting bodies or dielectric bodies with complex permittivity and permeability.

The excitation could be a point source with a complex current, or a gaussian beam with variable waist, position and direction. Plane waves with a direction relative to the horizontal line are also included. For plane waves one can define the polarization as transverse magnetic TM, or transverse electric TE. The operating frequency and the complex permittivity and permeability of the surrounding medium are also given.

In the wxGBtool, one can define how to observe the fields in the vicinity of the scatterer. A straight line, a circle surrounding the scatterer or a bitmap that shows a picture of the incident and scattered field throughout the geometry can be used.

The processing part, known as the NERO solver, includes an MoM-based multilevel FMM code, known as MLFMA, together with an iterative solver which allows for a time and memory complexity of \( O(N \log N) \) per iteration [11]. \( N \) denotes the number of basis functions on the scatterer boundary.
Generally outputs from the processing part are not directly interpretable for the user and one should convert output data files to a suitable format. In the post-processing part we use a FORTRAN code con.f [12], in order to convert these outputs to a form suitable for Mathematica scripts for Cartesian or polar plots.

By analysing the plots one can view the surface current generated on the boundary of the geometry or calculate the strength of the scattered field and compare the results with the exact analytical solutions [12].

3. Diffraction by a thin absorbing half-screen

As a verification we can see how the NERO results compare with the approximate high frequency solution, UTD, for the field behind a thin absorbing half-screen illuminated by a TM plane wave.

The diffraction has been much studied with the Geometrical Theory of Diffraction (GTD) [13], with the objective of defining a diffraction coefficient. This coefficient is used to calculate the strength of the diffracted field, in a direction away from the diffracting object, by multiplying it into the incident field. The calculated diffracted field must be added to the incident field in order to obtain the total field behind the half-screen.

The problem with this method is that the diffraction coefficient diverges at the shadow boundary. In reality the field behind the screen must be bounded and continuous at the shadow boundary and equal to one-half of the incident field [13].

Instead one can use the Uniform Theory of Diffraction, UTD. In this method a transition function removes the singularity of the solution at the shadow boundary. Another approach to remove this singularity is to use the Fresnel integrals [13].

![Figure 1: The Diffracted plane wave over a thin absorbing half-screen at 900 MHz.](image)

Fig. 1 shows the diffracted field over an absorbing thin half-screen in bitmap form.
The total field produced by the UTD method is given by:

\[
E = A_0 \frac{e^{-ikx}}{2} + A_0 \frac{e^{i\pi/4}}{\sqrt{2}} e^{-ikx} \left[ C\left(\frac{k}{\pi x}\right) - iS\left(\frac{k}{\pi x}\right) \right].
\]  \hspace{1cm} (1)

From Eq. 1, one can use a Mathematica script in order to visualize the solution for a thin half-screen, illuminated by a TM plane wave in free space (see Appendix A). \(x\) is the horizontal distance between the tip of the screen and the vertical observation line. Using this script we can compare the UTD and the NERO results.

![Figure 2](image1.png)  
**Figure 2:** The field at 0.9 GHz just behind a half-screen as a function of height along a vertical line 0.3 meters after the screen. The UTD result is given in solid line and the NERO result in dashed line.

In Fig. 2, the fields computed by means of UTD and NERO are shown. Since the field is computed relatively close to the screen, the UTD result is not very accurate due to the approximations and the edge diffraction at the tip of the screen.

![Figure 3](image2.png)  
**Figure 3:** The field at 0.9 MHz behind the half-screen as a function of height along a vertical line 1 meter after the screen. The UTD result is given in solid line and the NERO result in dashed line.
Fig. 3 shows a better agreement since the approximations in UTD have less effect further from the screen.

![Graph](image1)

**Figure 4:** The field at 0.9 MHz behind the half-screen as a function of height along a vertical line 7 meters after the screen. The UTD result is given in solid line and the NERO result in dashed line.

![Graph](image2)

**Figure 5:** The field at 0.9 MHz behind a half-screen as a function of height along a vertical line 15 meters after the screen. The UTD result is given in solid line and the NERO result in dashed line.

Figs. 4 and 5 show that the agreement improves as the observation line moves further from the tip of the screen. The accuracy of the NERO results are verified by varying the number of basis functions per wavelength (SEG_LENGTH) [11].
4. The modelling of antenna patterns using gaussian beams

Gaussian beams are the simplest and most regular type of beams used in the study of electromagnetic waves [14]. By using this type of beam one can obtain a point matching procedure in an efficient form.

Although the actual wave propagation is three-dimensional, it is necessary to describe the properties of this beam in two dimensions since the NERO simulation is two-dimensional, with a transversal dimension \( y \) and a longitudinal dimension \( x \).

A 2D gaussian beam with a complex electric field can be written in the form:

\[
E(x, y) = E_0 \sqrt{\frac{2}{\pi W_0 q(x)}} e^{-\frac{i}{k} \left(\frac{y^2}{2q(x)} + x\right)},
\]

with an amplitude \( E_0 \) and the width at the waist \( W_0 \). \( q(x) \) denotes the complex radius of curvature that describes both the curvature of the wave front and the transversal size of the propagated beam:

\[
q(x) = x + i x_R.
\]

![Figure 6: The waist of a gaussian beam.](image)

As shown in Fig. 6, at the beam waist (\( x = 0 \)) the radius of curvature is infinite. This means that the wave front at this point is plane and corresponds to \( q_0 \). The beam has the minimum radius of curvature at the distance \( \pm x_R \) from the beam waist, and the width of the beam at this position is equal to \( \sqrt{2} W_0 \) [14]. \( x_R \) is known as the Rayleigh range and relates to the behaviour of the beam along the propagation axis:

\[
x_R = \frac{\pi W_0^2}{\lambda}.
\]

\( \lambda \) is the wavelength in the material where the beam is propagating.
One can focus the beam by optimizing the width at the waist of the beam for different operational frequencies and distances between the source and observation points.

![Figure 7: The spreading of a Gaussian beam centred at 20 m:](a) 1 GHz with \( w_0 = 7 \) m (solid line), \( w_0 = 9 \) m (sparse), \( w_0 = 5 \) m (dense), (b) 2 GHz with \( w_0 = 5 \) m (solid line), \( w_0 = 7 \) m (sparse), \( w_0 = 3 \) m (dense).](image)

In Fig. 7, for a fixed value of \( E_0 \) and \( x \), the optimum value of \( W_0 \), in solid line, decreases from 7 to 5 m as we increase the frequency. Increasing the distance, between the source and observation point, increases the optimum value of \( W_0 \).

Applying Eq. 4 in Eq. 3, one has:

\[
q(x) = x + i\pi \frac{W_0^2}{\lambda},
\]

and \( q_0 = i\pi \frac{W_0^2}{\lambda} \). By considering the function:

\[
f = \frac{q_0}{W_0q(x)} = \frac{i\pi W_0}{x\lambda + i\pi W_0^2},
\]

one may let the derivative with respect to \( W_0 \), of the function \( f f^* \), equal to zero and find the value:

\[
W_1 = \sqrt{\frac{x\lambda}{i\pi}},
\]

which yields the narrowest beam for a specific distance and frequency.
5. Point Matching

To obtain a combination of gaussian beams that approximates a given antenna pattern, one may use the method of point matching. In this method one assumes an antenna with a desirable radiation pattern. The coefficients for the gaussian beams are obtained by equating the field of the assumed antenna pattern and the combination of beams at a number of matching points along an observation line. The number of beams is equal to the number of matching points. By solving the resulting linear system of equations with direct methods, one obtains the complex magnitude of each of the beams that reproduce the assumed antenna pattern.

The convergence of the point matching method depends on the selection of points. By proper adjustment of the shift between source points and the shift between matching points, in combination with the optimum value \( W_1 \) for each beam, convergence is achieved. Using a FORTRAN code nu.f (see Appendix B) we can solve the relevant system for a suitable number of beams.

The linear system of equations associated with the above procedure can be expressed in matrix form as:

\[
AG = P. \quad (8)
\]

\( G = [g_1, g_2, \ldots, g_N]^T \) is the vector of beam coefficients. \( N \) denotes the number of matching points and \( T \) denotes the transpose. \( A \) is the beam matrix given in the form:

\[
A_{mn} = \frac{2}{\pi} \frac{q_0}{q(x_0)W_1} e^{-ik \left( \frac{y_m^2}{2q(x_0)+x_0} \right)}. \quad (9)
\]

In this equation \( q_0 \) and \( q(x) \) are the gaussian beam parameters from Eq. 3 and \( W_1 \) is obtained from the Eq. 7. \( x_0 \) is the longitudinal coordinate for the matching points and \( Y_{mn} \) is given:

\[
Y_{mn} = y_m - \left( y_s + s \left( n - \frac{N}{2} \right) \right). \quad (10)
\]

\( y_s \) is the position of the source midpoint along the y-axis and \( s \) gives the shift between source points. \( y_m \) gives the position of the matching points:

\[
y_m = y_0 + l \left( m - \frac{N}{2} \right). \quad (11)
\]

In this expression \( y_0 \) is the position of the matching midpoint along the y-axis and \( l \) denotes the shift between matching points.

In Eq. 8, \( P \) represents the vector of values for the assumed antenna pattern. The pattern for \( P \) is defined in the form:
\[ P = \frac{e^{-ikx}}{x} \left(1 - \left(\frac{y_m - y_s}{W_L}\right)^2\right), \quad |y_m - y_s| < W_L \]  

(12)

where \( W_L \) gives the width of the main lobe.

As a first example one can solve this system for an array of 10 shifted gaussian beams matched at 10 equidistant points along the observation line. The frequency is set to 500 MHz with a shift between the source and matching points equal to 2 m and \( W_1 = 6 \) m for each beam. The distance between the line of source points and the line of matching points is set to 200 m.

| Figure 8: Matching result with 10 gaussian beams centred at 50 m for a distance of 200 m at 500 MHz. The assumed antenna pattern is given in solid line and the reproduced pattern in dashed line. |

In Fig. 8, the solid line represents the pattern produced by the function \( P \) from Eq. 12 and the dashed line shows the reproduced pattern with 10 gaussian beams.

Because of the discontinuity in the derivative of the assumed antenna pattern (at 36 and 64) the computed antenna pattern is not accurate for a small number of beams. By increasing the number of beams to 20, as shown in Fig. 9, one can get a better matching between the assumed and the reproduced pattern.
Figure 9: Matching result with 20 gaussian beams centred at 50 m for a distance of 200 m at 500 MHz. The assumed antenna pattern is given in solid line and the reproduced pattern in dashed line.

When increasing the frequency for a fixed distance one should reduce the shift between source points and the shift between matching points. If these shifts become smaller or larger than a specific value the point matching produces near identical equations. This results in poor conditioning of the linear system.

It is impractical to set all the parameters manually in the point matching procedure. By automatically setting the shift between matching points, the shift between source points and the width of the assumed and reproduced antenna patterns one can simplify the handling.

The shift between source points can be computed from:

\[ s = x \frac{FNBW}{2N} \]  

(13)

and similarly for \( l \), the shift between matching points.

One can specify the widths of the main lobe by means of the First Null Beam Width (FNBW) [15], and from that obtain the linear width of the pattern. By defining the width of the assumed antenna pattern as a function of distance we can match the main lobe of the assumed and reproduced antenna pattern in a better way in the code:

\[ W_L = x \frac{FNBW}{2} \]  

(14)

One now needs to manually adjust the number of beams, the frequency and the range between the line of source points and the line of matching points.
Fig. 10 shows that the range can be increased to 5000 m with good results. Here 20 beams and a frequency of 1 GHz are used. The shifts between source and matching points and the value of $W_1$ are set automatically to 9 and 21 m, respectively.

6. Antenna models applied to 2D wave propagation in terrain

A description of a communication link must include the geometrical features as well as the electromagnetic properties of the ground. In NERO one can introduce a terrain profile and investigate the stability of the point matching for a realistic wave propagation problem. To produce such a terrain profile, via the NERO graphic toolbox, one can define a polygon by means of a large number of points.

A FORTRAN code pol.f (see Appendix C) can generate these points in a counterclockwise fashion. One defines the number of points, the number of hills $N$ and the position and height of each hill in the geometry. When plugging the produced points, from the data file poly.dat, into the NERO graphic toolbox one should also specify the values of the complex permittivity and permeability of the profile.
Fig. 11 shows a 2D terrain profile, 130 m long, with the relative permittivity and permeability of 2.5 and 1, respectively. Hilltops are located at 20, 50 and 85 m away from the origin.

As shown in this figure, one can place the receiver point \((x_o, y_o)\) some distance before the end point of the geometry in order to remove the effect of the scattered field produced at the corner of the end point.

In this profile we define a receiver point at 100 m away from the origin and the source point above the origin. The highest hill (36 m) is located at 85 m on the x-axis. The matching points along the observation line range from deep inside the shadow region behind the highest hilltop and well into the illuminated region. The output from NERO is a line output along the vertical line of matching points.

By adjusting the corresponding parameters in the program nu.f we compute the complex coefficients together with the position and the width of the waist of each beam for later use in the NERO graphic toolbox. These values are readable from the data file Amp.dat (see Appendix B).

One should first check the quality of the produced matching results from the program nu.f and then plug the computed values into the NERO graphic toolbox in order to get complete results from NERO.

Fig. 12 shows the matching result from the program nu.f for 8 gaussian beams at a distance of 100 m and a frequency of 100 MHz. The matching points and source points are centred on 70 m altitude along the y-axis. The shift between matching points and the shift between source points is set to 5.85 m and the width \(W_1\) to 9.7 m. The conditioning of the linear system is good, so one can rely on the calculated coefficients from the program nu.f.

If one changes the shift between matching points and the shift between source points within a considerable range, while keeping distance, number of beams and frequency fixed, the method is stable if the agreement with the assumed pattern is unaffected.
As shown in Fig. 13, a change in the shift between matching points and the shift between source points from 5.85 to 6.52 m, yields nearly identical results. By increasing the matching shift and the beam shift up to 7.5 m we still get similar result, due to fair conditioning.

With these matching results one can include the predefined geometry, within the program NERO, and evaluate the overall stability of the results.

Fig. 14 illustrates the total fields from NERO using the two available point matching results in the presence of the predefined terrain. Here both the matching midpoint and the source midpoint are placed at 70 m. Good agreement is obtained. In this figure the dashed line corresponds to a 6.5 m shift between both the matching points and the source points and the solid line corresponds to a 5.9 m shift.
Figure 15: The total field produced by 8 beams centred at \( y_0 = 35 \) m for a distance of 100 m at 100 MHz. The solid line corresponds to shifts of 5.94 m and the dashed line to shifts of 6.52 m.

In Fig. 15, the height of the midpoint for both matching and source points is reduced from 70 to 35 m, which is near to the shadow boundary relating to the highest hill (36 m). Still the results agree.

By increasing the range from 100 m to 500 m and repeating the procedure for different shifts between the source and the matching points one can further investigate the stability. In Fig. 16, the geometry for this range has four hills at 80, 180, 250 and 400 m, with heights of 37, 55, 44 and 71 m, respectively. The relative permittivity and permeability of the profile are set to 2.5 and 1 respectively, and the frequency is set to 80 MHz.

Figure 16: Terrain profile for a distance \( x_0 = 500 \) m.

By setting the shift between matching points and the shift between source points to 12 m there is a good agreement for the assumed and reproduced antenna pattern from the program nu.f. When increasing the shifts to 15 m, the reproduction of the radiation pattern is still good.
Figure 17: The antenna pattern produced by 12 beams centred at $y_0 = 90$ m for a distance of $x_o = 500$ m at 80 MHz. The solid line corresponds to shifts of 12 m and the dashed line to shifts of 15 m.

Fig. 17 shows the radiation pattern produced by two different matching results from NERO in the absence of terrain. The plot in solid line corresponds to the result with matching shifts and beam shifts of 12 m and the plot in dashed line corresponds to 15 m. The purpose of this simulation is to show that since one has a good conditioning of the system in the program nu.f, the same results are achievable from NERO.

Figure 18: The total field produced by 12 beams centred at $y_0 = 90$ m for a distance of 500 m at 80 MHz. The solid line corresponds to shifts of 12 m and the dashed line to shifts of 15 m.

Fig. 18 shows the effect of terrain for the two above point matching results, with the source and matching midpoints at 90 m. Good agreement is obtained.
Figure 19: The total field produced by 12 beams centred at $y_0 = 65$ m for a distance of 500 m at 80 MHz. The solid line corresponds to shifts of 12 m and the dashed line to shifts of 15 m.

By reducing the height of the source and matching midpoints from 90 to 65 m, one observes the diffracted field deep inside the shadow region related to the highest hilltop at 71 m. Fig. 19 shows that there is a good agreement between the two matching results when the shifts are changed from 12 to 15 m. After increasing the operating range we turn to the stability of the point matching procedure for higher frequencies.

Figure 20: The total field produced by 10 beams centred at $y_0 = 65$ m for a distance of 500 m at 1 GHz. The solid line corresponds to shifts of 4 m and the dashed line to shifts of 6 m.
Results at 1.0 and 1.8 GHz frequency, for a distance of 500 m, are shown in Figs. 20 and 21, respectively. By including the 2D terrain profile, shown in Fig. 16, one can verify that this range of frequencies still yields nearly identical results for two different point matching solutions. The shift between matching points and the shift between source points was changed from 4 to 6 m, which is sufficient to verify the stability.

The stability of the method for a complete communication link was verified in this section. The possibility of including matching points at the hilltops in the point matching procedure was also tested briefly, but the numerical stability and the conditioning of the system seems to be more problematic in that case.

7. Conclusion

A linear combination of two-dimensional gaussian beams that approximates the antenna pattern is obtained by means of point matching along the observation line. The 3D problem can be reduced to 2D since a realistic incident field can be modeled. Thus, by considering the 2D terrain profile of a communication link, the stability of the point matching method was verified using the program NERO. By changing the shift between source points and the shift between matching points, within a considerable range, one can get nearly identical field patterns along the observation line. This verification was done for various distances and frequencies and shows that modifications of these parameters do not affect the wave propagation model.
References

URL http://urn.kb.se/resolve?urn=urn:nbn:se:vxu:diva-6127
URL http://urn.kb.se/resolve?urn=urn:nbn:se:lnu:diva-8767
Appendix A: UTD Approximation (Mathematica)

\[ f = 900000000 \]
Print["f = ", f]
\[ \lambda = \frac{300000000}{f} \]
\[ k = \frac{2\pi}{\lambda} \]
plim = 2
ppts = 500
xobs = 0.3 (* endpoints: (xobs,-ylim), (xobs,ylim) *)
\[ \text{ylim} = 3 \]
Print["xobs = ", xobs]
Print["ylim = ", ylim]
Print["plim = ", plim]
Print["ppts = ", ppts]

\[ Dc[\theta_] := -\frac{1}{\sqrt{2\pi k}}(1 + \cos(\theta))/\sin(\theta) \]
\[ CS[z_] := \text{FresnelC}[z] - I*\text{FresnelS}[z] \]
\[ u[\theta_] := (1 + \theta/\text{Abs}[\theta])*0.5 \]
\[ \text{thf}[y_] := \text{ArcTan}[y/xobs] \]
\[ \text{rhot}[\theta_] := xobs/\cos(\theta) \]
\[ Edf[\theta_] := 0.5*\exp[-I*k*xobs] + \exp[I*Pi/4]/\sqrt{2}*\exp[-I*k*xobs]*\text{CS}[\sqrt{k/(\pi*xobs)}*y] \]
a = Plot[Abs[Edf[thf[y]]],{y,-ylim,ylim},
Frame->True,
FrameLabel->
{Text[StyleForm[SubscriptBox["y","o"], DisplayForm, FontSize->14], "|E|[v/m]","",""],
PlotRange->{0,plim},
PlotPoints->ppts,
DisplayFunction->Identity}

Export["UTNe.pdf",Show[a,DisplayFunction->$DisplayFunction],"PDF"]
Appendix B: The FORTRAN program nu

program nu
integer ndim,Nq,j,i,Nq1,Nq0
real*8 k,Pi,thp,Js,Rcond,xs,ys,x,y,x0,y0,gau_shift,ma_shift
real*8 c,f
parameter(ndim= 3000)
real*8 xma(ndim),yma(ndim),pl_shift,FNBW,w0,lambda
complex*16 Ci,Q(ndim,ndim),B(ndim),Uin(ndim),V1(ndim)
complex*16 gaussian,ant_pat,beam
integer IPVT(ndim)
parameter(Ci= (0.d0,1.d0), Pi=3.141592653589793D0)
external gaussian,ant_pat
open(73, file='Amp.dat')
open(74, file='Ant.dat')
open(75, file='Bea.dat')
xma(1)= x0 !match. points
yma(1)= y0 + ma_shift*(i-Nq/2.d0)
enddo
write(6,*)'x0=',sngl(x0),'y0=',sngl(y0)
c= 2.99792458d8 !speed of light
f= 1.d9 ! f < 1000 GHz
write(6,*)'f=',sngl(f)
k= 2*pi*f/c !x= k*a= 2*pi/lambda= 2*pi*f/c
lambda= 2*pi/k
ma_shift= 8*x0*FNBW/(2.d0*Nq)
write(6,*)'ma_shift=',sngl(ma_shift)
do i=1,Nq
xma(i)= x0 !line matching
yma(i)= y0 + ma_shift*(i-Nq/2.d0)
enddo
w0= sqrt(x0*lambda/Pi)
write(6,*)'w0=',w0
gau_shift= 8*x0*FNBW/(2.d0*Nq)
write(6,*)'gau_shift=',sngl(gau_shift)
do i=1,Nq
x=xma(i);y=yma(i) !match. points
B(i)= ant_pat(k,xs,ys,x,y,FNBW)
do j=1,Nq

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Q(i,j)= gaussian(k, xs, ys, x, y, j, Nq, gau_shift, w0)
enddo
enddo
call zGECO(Q, ndim, Nq, IPVT, RCOND, V1) !factorize
write(6,*)'
write(6,*)'Gaussian zGECO; RCOND=', sngl(RCOND) !1/cond.num.
if(RCOND.LT.1.d-8) then
write(6,*)'Bad conditioning'
endif
CALL zGESL(Q, ndim, Nq, IPVT, B, 0) !solve

do j = 1, Nq
!beam #, Amp, j, pos
write(73,*) B(j), j, ys + gau_shift*(j-Nq/2.d0)
endo

Nq1 = 10*Nq
pl_shift = 10*x0*FNBW/(2.d0*Nq1)
do i = 1, Nq1
!position #
x = x0
y = y0 + pl_shift*(i-Nq1/2.d0)
write(74,*) sngl(y), sngl(abs(ant_pat(k, xs, ys, x, y, FNBW)))
beam = (0.d0, 0.d0)
do j = 1, Nq
beam = beam + B(j)*gaussian(k, xs, ys, x, y, j, Nq, gau_shift, w0)
endo
write(75,*) sngl(y), sngl(abs(beam))
endo

write(6,*)' '
write(6,*)'PMATCH/pla'
stop
end

c---------------------------------

function gaussian(k, xs, ys, x, y, j, Nq, gau_shift, w0)
integer j, Nq
real*8 xs, ys, x, y, w0, xr, pi, lambda, rv, k, gau_shift
complex*16 gaussian, ci, q, q0, eg, fact
parameter(Ci = (0.d0, 1.d0), pi = 3.141592653589793D0)

lambda = 2*pi/k
xr = pi*w0**2/lambda
q = x + ci*xr
q0 = ci*xr
fact = sqrt(2/pi)*q0/w0
rv = y - (ys + gau_shift*(j-Nq/2.d0)) !symmetrical points
eg = fact/q*exp(-ci*k*(0.5d0/q*rv**2 + x))
gaussian = eg
return
end

function ant_pat(k,xs,ys,x,y,FNBW)  !decays with x
real*8 xs,ys,x,y,k,FNBW,wL
complex*16 ant_pat,Ci,fact
parameter(Ci= (0.d0,1.d0))

wL= x*FNBW/2.d0
fact= exp(-Ci*k*x)/x
if(abs(y-ys) .LT. wL) then
  ant_pat= fact*(1.d0-((y-ys)/wL)**2)
else
  ant_pat= (0.d0,0.d0)
endif

return
end
Appendix C: The FORTRAN program pol

program pol
integer i,j,points,N
real*8 xi,yi,x_start,x_stop,thick,sum,alfa(10),beta(10)
complex*16 z(10)

open(1,file='poly.dat')
open(2,file='poly.pol')

x_start= 0.d0
x_stop= 550.d0
thick= -10.d0
points= 1000
if(points .LT. 5) stop 'points too small'
write(6,*)'x_start=', sngl(x_start)
write(6,*)'x_stop=', sngl(x_stop)
write(6,*)'thick=', sngl(thick)
write(6,*)'points=', points

c Define hills, pole point, exponent, coefficient

N= 4
z(1)= (80.,-24.0); alfa(1)= 1.0 ; beta(1)= 600.
z(2)= (180. , -16.0); alfa(2)= 1.0 ; beta(2)= 600.
z(3)= (250. , -22.0); alfa(3)= 1.0 ; beta(3)= 600.
z(4)= (400. , -17.0); alfa(4)= 1.0 ; beta(4)= 1000.

c Write polygon to files

write(1,*)sngl(x_stop), sngl(thick)
write(2,*)sngl(x_stop), ', ', sngl(thick)
do i=points-3,1,-1
xi= x_start+ (x_stop-x_start)*(i-1)/dfloat(points-4)
sum= 0.d0
do j=1,N
sum= sum+ beta(j)/abs(xi-z(j))**alfa(j)
endo
yi= sum
write(1,*)sngl(xi), sngl(yi)
write(2,*)sngl(xi), ' ; ', sngl(yi)
endo

write(1,*)sngl(x_start), sngl(thick)
write(2,*)sngl(x_start), ', ', sngl(thick)
write(1,*)sngl(x_stop), sngl(thick)
write(2,*)sngl(x_stop), ', ', sngl(thick)
write(6,*)'
write(6,*)'file poly.dat for Mathematica'
write(6,*)'file poly.pol for wxGBTool'
stop
end