Influence of the Vertical Support Stiffness on the Dynamic Behavior of High-Speed Railway Bridges

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À minha Avó Xanoca,
Abstract

When designing a bridge, the modelling of the soil and the soil-structure interaction is commonly far from reality. Furthermore, the studies undertaken by the European Rail Research Institute (ERRI) point out that it is essential to model the support conditions, if realistic predictions of the dynamic behavior are to be made. When a bridge is subjected to the loads of a high-speed train, the dynamic response of the structure is, obviously, influenced by the soil beneath the foundations and surrounding the bridge. The magnitude of that influence was studied, namely from the analysis of the variations obtained for the eigenfrequencies, displacements and accelerations. Greater attention was given to how changes in the vertical support stiffness influence the dynamic behavior of bridges subjected to loads travelling at a speed that induces the resonant response.

As a first approach to the problem, a theoretical analysis of two common railway bridges from the Bothnia Line (the new Swedish high-speed line) was made. Afterwards, as a case study, another railway bridge was analyzed using the Finite Element Method. The results obtained with this method were compared with data measured in situ, to validate the model. Theoretical estimation of the support stiffness and model updating were also performed.

The results obtained with the updated Finite Element model were found to be very satisfactory. The findings suggest that models with stiff supports can greatly underestimate the maximum responses of high-speed railway bridges and may not be reliable. Furthermore, it was concluded that simple 2D beam models are able to simulate reasonably well the behavior of real bridges. With the help of field measurements and model updating, these simulations can be increasingly accurate, and particularly meaningful for structures under constant monitoring.

Key words: bridge dynamics, mechanical vibration, resonance, vertical support stiffness, high-speed railway bridges
Resumo

Ao projectar uma ponte, a modelação do solo e a interacção solo-estrutura são geralmente menosprezados. No entanto, estudos conduzidos pelo European Rail Research Institute (ERRI) indicam que é essencial modelar as condições de apoio, de forma a avaliar de modo realista o comportamento dinâmico da estrutura. Quando uma ponte é sujeita às cargas dos eixos de um comboio de alta velocidade, a resposta dinâmica da estrutura é, obviamente, influenciada pelo solo sob as fundações. A importância dessa influência foi estudada através de análises de sensibilidade das frequências, deslocamentos e acelerações. Maior atenção foi dada à influência que alterações na rigidez vertical das fundações podem ter no comportamento dinâmico de estruturas sujeitas à acção de cargas móveis com velocidade que origine efeitos de ressonância.

Numa primeira abordagem ao problema, procedeu-se a uma análise numérica e teórica de duas pontes ferroviárias correntes da Bothnia Line (a nova linha ferroviária de alta velocidade da Suécia). Em seguida, como caso de estudo, outra ponte ferroviária, mais complexa, foi analisada usando o Método dos Elementos Finitos. Os resultados obtidos com este método foram comparados com medições obtidas in situ, para validar o modelo. Foram também efectuadas estimativas teóricas da rigidez vertical das fundações, utilizadas na actualização do modelo.

Os resultados obtidos com o modelo actualizado de Elementos Finitos foram muito satisfatórios. As conclusões sugerem que modelos numéricos com apoios rígidos podem seriamente menosprezar as respostas máximas de pontes ferroviárias de alta velocidade e não devem ser utilizados, sob prejuízo de avaliar erradamente o comportamento estrutural. Concluiu-se ainda que modelos simples bi-dimensionais podem simular relativamente bem o comportamento de estruturas reais. Com a ajuda de medições in situ e actualização do modelo, estas simulações podem ser progressivamente mais exactas, e particularmente úteis para pontes sob monitorização constante.

Palavras chave: dinâmica estrutural, vibração mecânica, ressonância, rigidez vertical dos apoios, pontes ferroviárias de alta velocidade
Preface

The research presented in this dissertation was carried out at the Department of Structural Engineering, Structural Design and Bridges, at the Royal Institute of Technology (KTH) in Stockholm, Sweden. The study was conducted under the supervision of Associate Professor Raid Karoumi and PhD Student Mahir Ülker.

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<td>BE</td>
<td>Boundary Element</td>
</tr>
<tr>
<td>BEM</td>
<td>Boundary Element Method</td>
</tr>
<tr>
<td>DAF</td>
<td>Dynamic Amplification Factor</td>
</tr>
<tr>
<td>DOF</td>
<td>Degree of Freedom</td>
</tr>
<tr>
<td>ERRI</td>
<td>European Rail Research Institute</td>
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<tr>
<td>FE</td>
<td>Finite Element</td>
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<td>FEM</td>
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<tr>
<td>ORE</td>
<td>Office of Research and Experiments</td>
</tr>
<tr>
<td>PhD</td>
<td>Philosophiae Doctor (Doctor of Philosophy)</td>
</tr>
<tr>
<td>RAVE</td>
<td>Rede Ferroviária de Alta Velocidade</td>
</tr>
<tr>
<td>SASW</td>
<td>Spectral Analysis of Surface Waves</td>
</tr>
<tr>
<td>SDOF</td>
<td>Single Degree of Freedom</td>
</tr>
<tr>
<td>UIC</td>
<td>International Union of Railways</td>
</tr>
<tr>
<td>WIB</td>
<td>Wave Impeding Barrier</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Background and motivation

Nowadays, in modern societies, the need to move people and goods is growing fast, both in number of transactions and in traveled distance. Therefore, transport infrastructures are being built all over the world, to cope with the market claim.

The railways have been used for many decades to assure the conveyance of people and goods. With the launch of the high-speed trains (HST), this way of transportation became even more useful. From the early 1980’s, when the Paris-Lyon railway was built, with a total distance of 410 km, the high-speed railway (HSR) have grown and spread to all the world. The current HSR infrastructures, and the new lines planned can be seen in figure 1.1, and totalize more than 10000 km.

![European high-speed network](https://www.uic.asso.fr)

Figure 1.1: European high-speed network (from www.uic.asso.fr).
With the appearance of the TransRapid05, in 1979, a new type of HST was born, the Magnetic Levitation Train or Maglev. The first commercial Maglev was opened in 1984 in Birmingham, covering 600 meters between its airport and railhub, but was eventually closed in 1995 due to technical problems. At the time of this dissertation, the only operating high-speed maglev line of note is the Initial Operating Segment (IOS) demonstration line of Shanghai, that transports people through 30 km to the airport in just 7 minutes 20 seconds, achieving a top velocity of 431 km/h and averaging 250 km/h.

The oriental civilizations have always been in the front edge of the HSRs. In Japan, the Maglev experimental trains have achieved, in 2003, 581 km/h, but the high cost of its tracks makes it unprofitable for conventional passenger lines. While engineers are trying to lower the expenses involved with Maglev trains, the Shinkansen (figure 1.2) spreads its tracks around Japan. Since the initial Shinkansen opened in 1964 running at 210 km/h, the network (2,459 km) has expanded to link most major cities with running speeds of up to 300 km/h, in an earthquake and typhoon prone environment.

Despite not being in the front edge, countries like Sweden and Portugal are now starting to implement HSR networks. The Bothnia Line, the new Swedish railway from Nyland, north of Sundsvall, to Umeå, will provide a direct rail link for the first time between Sundsvall, Örnsköldsvik and Umeå, serving about 350,000 people. It will also double the rail capacity between central and northern Sweden. This HSR consists of 190 km of railway with 150 bridges and 30 km of tunnels and it is designed for operation by 120 km/h freight trains and 250 km/h passenger trains, making this Sweden’s first line capable of this speed. The line will be single track with 22 two or three-track, 1 km long, passing loops.

In Portugal, the Rede Ferroviária de Alta Velocidade (RAVE) was constituted to
build and manage the Portuguese HSR, in 2000. After the Iberian Meetings with Spain, in 2003 and 2005, it was decided to make 4 international HSR connections: Lisboa-Madrid, opening in 2013; Porto-Vigo, to be started after 2009; Aveiro-Salamanca, after 2015; and Évora-Faro-Huelva, in the 3rd decade of the century. The connection Lisboa-Porto, very important for the Portuguese transport network will allow people to commute between the 2 cities in 1 hour and 35 minutes.

Both the networks will need a large number of bridges and viaducts. In the case of trains running at high-speed, the risk of resonance in the structures is larger than classical trains, and assessment of vibration problems in the high-speed railway bridge is required during its design, to guarantee the safety of the crossing train, which is subordinated to strict crossing conditions. Therefore, performing dynamic analysis that investigates resonance of the bridge induced by the bridge-train interaction constitutes an essential element of the design.

1.2 Aims and scope

When designing a bridge, the modelling of the soil and the soil-structure interaction is commonly far from reality. Furthermore, the studies undertaken by ERRI and Committee D214 (see section 2.3) point out that it is essential to model the support conditions, if realistic predictions of the dynamic behavior are to be made. Hereby, the importance of the accurate estimation of the vertical support stiffness, on the dynamic behavior of the bridge, is the subject of this MSc dissertation.

When a bridge is subjected to the loads of a high-speed train, the dynamic response of the structure is, obviously, influenced by the soil beneath the foundations and surrounding the supports. The magnitude of that influence is going to be studied, namely from the analysis of the variations obtained for the eigenfrequencies, displacements and accelerations. Greater attention will be given to the effect that changes in the vertical support stiffness have on the dynamic behavior of structures subjected to loads travelling at a speed that induces the resonant response.

1.3 General structure of the dissertation

The dissertation will be divided in chapters, sections and subsections, properly enumerated.

In chapter 2, a review of the state-of-the-art investigation on the subject of high-speed railway (HSR) will be presented. Most of the information discussed in that chapter will not be used in this dissertation, but the goal is to give general and historical approach to the whole problem.

Important results achieved in the past, but essentially related to the studies undertaken in this dissertation, will be presented in chapter 3.
In the 4th chapter, a theoretical analysis of two common railway bridges will be done. One single span and one double span composite bridge from the Bothnia Line (the new Swedish HSR) will be used for the study. No comparison with experimental measurements will be made at this stage.

To adjust the theoretical model to reality, a case study with the Sagån bridge will be focused on chapter 5. Theoretical predictions will be compared with the results from the experimental measurements in situ.

Chapter 6 states the general conclusions inferred from the studies undertaken and recommends some directions for further investigation on the subject.
Chapter 2

State-of-the-art Review

In this chapter, a state-of-the-art on the subject of high-speed railway (HSR) will be presented. Most of the information discussed here will not be used in this dissertation, but the goal is to give general and historical background to the whole problem.

The chapter will be partitioned in three sections. The first will refer to the main results on high-speed structural behavior, obtained in the last years. The second section will be reserved to investigation involving the soil-structure interaction and the modelling of the soil. The last part will be dedicated exclusively to the work developed by the UIC, that greatly extended the knowledge on the high-speed subject, all over the world.

Important results achieved in the past but essentially related to the studies undertaken in this dissertation will be presented in chapter 3.

2.1 General studies on the subject

Since the 19th century, when the first accidents with metal railway bridges occurred, many scientist and engineers tried to explain the phenomenon, both through experimental and analytical studies. The first publications about the dynamic behavior of bridges came from the mid-nineteenth century, following the works of Willis [60] in investigating the collapse of the Chester rail bridge, over the river Dee, in England, 1847, the first case of collapse of a railway bridge in history. In this pioneer work, the inertial effect of the beam was ignored, and the vehicle was modeled as a concentrated moving mass travelling at constant speed. Although for this particular case, an exact solution could be obtained, its applicability remains rather limited, due to the omission of the inertial effect of the beam. Nevertheless, the contribution of Willis is considered historical, since he is among the first to bring the problem of vehicle impacts to the design desks of bridge engineers.

In the beginning of the 20th century, the theoretical view of Timoshenko [55] gave a complete solution for the problem of the dynamic behavior of a prismatic bar
acted upon by a moving harmonic pulsating force moving with a constant velocity. The publication of Inglis [35], also provided extremely important basis for the study of the dynamic behavior of high-speed railway bridges. Later in the 20th century, the investigations in that field by Ladislav Frýba (see Frýba [26] and Frýba [27]) have greatly developed the knowledge on the subject. Thus, and with the growing number of high-speed railway lines, all across the world, the dynamical problems associated with moving of the loads across the structure have caught the attention of both investigators and designers.

Ladislav Frýba, in the past decade, published some very interesting analytical solutions (Frýba [28]) for a simply supported beam subjected to any set of loads, travelling to a certain speed. The complete derivation of that solution and the definition of all the variables used will be deeply discussed in section 3.4.1. In the same article, there are also some very interesting propositions for the calculation of the interoperability, defined as the capability of a bridge to carry a particular train or vehicle running at certain speed or, in other way, the technical conditions which ascertain that the train could move on a given railway line, including bridges, at the designed speed. Using both definitions, Frýba proposed ways to calculate the interoperability constant of the bridge (B1) and the vehicle (V1 and V2), separately. For bridges with ballast, the quantification of these constants is such that the maximum vertical bridge deck acceleration is less than the limits conducting to the destabilization of ballast. The maximally accepted values for the acceleration of the bridge deck, $a_{ult}$, are specified by [21]:

$$a_{ult} = 3.5 \text{ m/s}^2 \text{ or } a_{ult} = 5 \text{ m/s}^2$$ (2.1)

for bridges with ballast or without ballast, respectively.

The formulas suggested for the amplitudes of the deflection, bending moment and acceleration include the most important parameters: speed, span, natural frequency, damping, length of vehicles axle load and permanent load of the bridge. As they depend on the square of the speed it is, therefore, explained why the resonance vibration appears at high speeds only. Furthermore, the results obtained by Frýba point that the amplitudes of resonance vibration depend on the square of speed and on the span of the bridge and inversely on damping, vehicle length and bridge rigidity. Moreover, the acceleration depends also on the ratio of the axle load to the permanent load of the bridge.

Furthermore, two reasons for the resonance vibration of railway bridges on high-speed lines were discovered: repeated action of axle loads and loss of stability under moving forces. While the first reason appears actually on high-speed lines at today’s speeds, the second one is not yet actual.

Apart from Ladislav Frýba, there is another investigator that has contributed for the scientific community with a large number of articles about the dynamic behavior of railway bridges. He Xia, from the Beijing Jiaotong University, has investigated, both analytically and experimentally, the dynamic phenomena occurring in bridges crossed by HSTs, namely in Xia et al. [61, 62, 63, 65, 66] and and Xia and Zhang
His partnership with Guido de Roeck and Nan Zhang has been particularly productive.

In Xia et al. [61, 66], the Thalys articulated train (figure 2.2) passing along the Antoing bridge on the Paris–Brussels high-speed railway line was analyzed. The train was modeled as 17 rigid bodies and 85 degrees-of-freedom (DOFs) in total (figure 2.1). With the 30 DOFs of the two locomotives, the total number of the DOFs of the whole train model is 115. The bridge was modeled by finite elements.

![Dynamic model of articulated vehicles](from Xia et al. [66]).

The Newmark-β Method was used in the step-by-step integration of the combined vehicle and bridge system.

After the experimental and analytical results comparison, it was concluded that:

- The dynamic analytical model of the bridge-articulated train system and the computer simulation method proposed could well reflect the main vibration characteristics of the bridge and the articulated train vehicle;

- The calculated results were well in accordance, both in response curves, in amplitudes and in distribution tendencies, with the *in situ* measured data, which verified the effectiveness of the analytical model and the computer simulation method;

- The articulated train vehicles have a rather smooth running at high-speed, which also helps to reduce the impact on the bridge structures.

![High speed Thalys train composition](from Xia and Zhang [64]).

In Xia et al. [62], Xia and Zhang [64] the same type of analysis was used, and some assumptions are referred. Regarding the vehicle, it was assumed that:
- The effects of the elastic deformations of the car bodies and the wheel sets was negligible;

- All vehicles could be simplified into the suspension system and the single-set-springs and dashpots and its coefficients divided over the wheels;

- The configuration of each vehicle body could be specified by 5 DOFs: lateral movement, rolling, yawing, floating and nodding; and each wheel had 3 DOFs: lateral moving, rolling and floating. Hence, a six-axle locomotive represented 23 DOFs and each regular coach 17 DOFs.

For the bridge model, it was assumed that:

- There was no displacement between the track and the bridge deck, and the rail pad effect could be neglected;

- The vibration modes of the bridge girders were the same as the modes of the bridge deck;

- The influence of the masses of the vehicles was much lower that the bridge weight and, therefore, negligible;

- The cross-section deformation of the girder was negligible, thus its movement could be expressed with 3 DOFs: lateral and vertical displacement and rotation.

Figure 2.3: Dynamic interaction of vehicle and bridge (from Xia and Zhang [64]).

These assumptions can be very important for the modelling and dynamic analysis of bridges in countries where the high-speed railway is starting, as happens in Portugal and Sweden. From the dynamic studies, it was concluded that the models, although with the simplifications, could rather well reflect the main vibration characteristics of
the train-bridge interaction system. The calculated results were well in accordance, regarding the response curves, amplitudes and distribution tendencies, with the experimental data, which verified the effectiveness of the analytical model and the computer simulation method.

In Xia et al. [63], a three-dimensional finite element model was used to represent a long suspension bridge and each 4-axle vehicle in a train was modeled by a 27 DOFs dynamic system. The measured track irregularities and the wheel hunting described by a sinusoid function were used to represent the two most important self-excitation in the coupled train bridge system. The degrees-of-freedom for all wheels were eliminated from the basic coupled equations of motion to reduce computation efforts and the Mode Superposition technique was then applied, only to the bridge. By using the Mode Superposition technique it was assumed that the bridge was operating in a linear range.

Similar studies on the dynamic behavior of bridges subjected to high-speed loads have been made in Korea. Kwark et al. [39] presents a study of the amplified dynamic responses of bridges crossed by the Korean high-speed train (KHST).

The bridge used was representative of the bridges adopted for the Korean HSR: a simple box continuous concrete bridge of 80 m length constituted by two-span of 40 m length. Three dimensional space frame elements constituted by two-nodes, each node with 6 degrees-of-freedom, were established to numerically model the bridge. First order Lagrange interpolation function and third order Hermite Interpolation function as displacement shape functions were used for axial directional degrees-of-freedoms and flexural degrees-of-freedoms, respectively. Applying classical finite element method, the solution of the equation of motion of the bridge was obtained easily.

The equation of motion of the high-speed train could be expressed in matrix form such as equation 2.2.

\[ M \ddot{u}(t) + C \dot{u}(t) + K u(t) = P_t \quad (2.2) \]

where \( u \) represents the displacement, \( \dot{u} \) stands for the velocity and \( \ddot{u} \) represents the acceleration. \( M, C, K \) and \( P_t \) represent the mass, damping and stiffness matrices, and the external force vector which includes the interaction vectors of vehicles, respectively. In order to solve equation of motions for the bridge-vehicle system, the magnitude of interacting forces at the end of each time interval should be determined. For the vehicle model used, the interacting force between a given axle and bridge surface was a function of the stiffness of its suspension spring assembly and the deformation that was applied to the spring assembly. The displacement, velocity and acceleration of each axle in previous time step was used as the initial condition for the next time step.

Using the proposed method, the dynamic behavior of the bridge according to the modelling method of the train system was investigated. Effects of the idealization method of the train were studied by comparing bridge responses resulting from
the simplest idealization method, i.e., using constant moving forces, and the three-dimensional model, which takes into account the bouncing, pitching and rolling of each component of the train. To obtain the solution of the bridge-train interaction problem, a direct integration method: the Newmark-$\beta$ method, adopting a predictor-corrector iteration scheme, was applied.

The conclusions achieved from the analysis were the following:

- The dynamic response of the bridge crossed by trains running at high-speed is significantly amplified at the vicinity of the critical speed, itself closely related to the fundamental natural frequency of the bridge and the effective beating interval produced by the train. Consequently, safety verification related to the dynamic behavior at speeds close to the critical speed shall be necessary performed for bridges intended to be crossed by trains running at high-speed;

- During the analysis of dynamic behavior of structures, damping is essential. Especially, when resonance occurs, responses show very sensitive variations. During the verification of the dynamic behavior of the bridge expected to operate at resonance under the crossing of the high-speed train, damping shall necessarily be selected carefully and rationally;

- Compared with the actual field test results, the proposed numerical analysis method led to reasonable results. As the use of three-dimensional models without vehicle-bridge interactions may produce conservative results, it seems advisable to use a model that takes into account interactions during the verification of the dynamic behavior of the bridge;

- Bridge-train interaction effects appeared significantly at every speed of the KHST;

- The maximum deflection of the bridge being generally produced by locomotives heavier than coaches, the passengers loading of the coaches does not affect the maximum deflection. When resonance occurs due to train running at the critical speed, the resonance induced by the coaches may amplify the maximum deflection produced by the locomotives. Such amplification depends on the number of coaches, that is, the duration of resonance, and the intensity of the loading.

Oscarsson [48] took the next step by studying properties such as rail pad stiffness, ballast stiffness, dynamic ballast-subgrade mass and sleeper spacing, and their influence on the dynamic behavior of the vehicle-bridge interaction. A perturbation technique was used to investigate the influence of the selected track properties. The train-track interaction problem was numerically solved by use of an extended state-space vector approach in conjunction with a complex Modal Superposition for the whole track structure. All numerical simulations were carried out in the time-domain with a moving mass model.
2.1. GENERAL STUDIES ON THE SUBJECT

The non-stochastic parameters (figure 2.4) were studied in situ and in laboratory, and the values obtained are expressed in table 2.1. A stochastic model was used to find the remaining parameters, shown in table 2.2.

Table 2.1: Non-stochastic track parameters used in the numerical simulations (from Oscarsson [48]).

<table>
<thead>
<tr>
<th>Track Element</th>
<th>Parameter</th>
<th>Parameter Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail Pad Damping [kNs/m]</td>
<td>$c_p$</td>
<td>20</td>
</tr>
<tr>
<td>Ballast Damping [kNs/m]</td>
<td>$c_b$</td>
<td>500</td>
</tr>
<tr>
<td>Subgrade Stiffness [MN/m]</td>
<td>$k_s$</td>
<td>600</td>
</tr>
<tr>
<td>Subgrade Damping [kNs/m]</td>
<td>$c_s$</td>
<td>650</td>
</tr>
<tr>
<td>Subgrade Shear Stiffness [MN/m]</td>
<td>$k_{ss}$</td>
<td>700</td>
</tr>
<tr>
<td>Subgrade Shear Damping [kNs/m]</td>
<td>$c_{ss}$</td>
<td>150</td>
</tr>
</tbody>
</table>

Table 2.2: The mean values (Mean) and the standard deviations (S.D.) of the stochastic variables evaluated at the two different test sites, Gåsakulla and Grundbro (from Oscarsson [48]).

<table>
<thead>
<tr>
<th>Track Element</th>
<th>Gåsakulla Mean</th>
<th>Gåsakulla S.D.</th>
<th>Grundbro Mean</th>
<th>Grundbro S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sleeper Spacing [m]</td>
<td>0.652</td>
<td>0.017</td>
<td>0.650</td>
<td>0.020</td>
</tr>
<tr>
<td>Ballast Stiffness [MN/m]</td>
<td>255</td>
<td>16</td>
<td>186</td>
<td>22</td>
</tr>
<tr>
<td>Ballast-Subgrade Mass [kg]</td>
<td>-</td>
<td>-</td>
<td>11600</td>
<td>5520</td>
</tr>
</tbody>
</table>

In the Iberian Peninsula, the research about high-speed railway effects have been particularly intensive in the Faculdade de Engenharia da Universidade do Porto, with the work of professors Rui Calçada and Raimundo Delgado, and in the Universidad Politécnica de Madrid, through professors José Mª Goicolea and Felipe Gabaldón Castillo.

After the first approach of dos Santos [25] in 1994, the work of Delgado and dos Santos [24] started the discussion in Portugal about the HST effects on structures,
even though the rail traffic in the country could not achieve high-speeds. The
differential equation for the models with and without train-structure interaction
were obtained and solved with the Newmark Method. Track irregularities were
studied as well and various parameters were identified, such as: the stiffness and
length of the bridge, the existence of ballast or the structural damping. The results
suggested that the stiffness of the structure and track irregularities were responsible
for greater changes in the response. Roughness in the track was found to increase
the amplifications as well, especially if there was a coincidence in the periods of the
bridge or train. In the beginning of the 21\textsuperscript{th} century, the work of Ribeiro [50] and
Pinto [49] carried on with the research on the subject.

In Spain, Goicolea et al. [31] suggested new dynamic analysis methods for railway
bridges to cover the possibility of resonance in the structure. The suggestions of the
new Eurocode [10] and the Spanish norms were studied and compared with other
European norms. The results obtained suggested that:

1. The design of high-speed railroad bridges, because of the real possibility of
resonance, require consideration of the dynamic vibration under moving loads;

2. It is essential to apply dynamic analysis methods in order to improve the
knowledge about the dynamic response of the bridges from the designer point
of view, as well as to be able to develop engineering design methods and codes
which are sufficiently practical, secure and simple to use;

3. The final draft of Eurocode 1 of actions in bridges [10] cover adequately the
necessity of dynamic analysis for the high-speed lines.

Museros et al. [45] focused only in the analysis of short span railway bridges. The
moving loads model is not well suited for the study of short bridges ($L \leq 20-25$ m)
since the results it produces (displacements and accelerations) are much greater than
those obtained from more sophisticated ones (see [17] for more information). The
investigation was undertaken in order to decide on the influence of the distribution of
the loads due to the presence of the sleepers and ballast layer, and the train-bridge
interaction. The results shown that the maximum accelerations of the deck are not
significantly affected by the load distribution through the sleepers and ballast layer,
extcept for the shorter values of the wavelength. However, the train-bridge interaction
causes reductions of considerable importance in the maximum displacements
and accelerations of short bridges, since it decreases the concentration of the loads.
It has been found that the reductions obtained in bridges with different natural fre-
quency and moment of inertia are nearly proportional to each other. Nevertheless,
further study is required in order to prove the validity of the procedure for different
trains and span lengths.
2.2 Soil-structure interaction and soil modeling

Soil-structure interaction has been described as the group of phenomena influencing the dynamic behavior of structures while interacting with the soil, induced by the application of loads in the system. In the past years, some work on this topic has been made, specially concerning railway induced vibrations.

According to Bayoglu [9], one of the key points in soil structure interaction is to evaluate the behavior of the soil under external loads. For designing purposes, the stresses and strains in the soil around the structures to be designed are more important than the behavior of the whole medium. Hence, the tendency is to model the soil by surface deflection due to external forces.

The simplest one of these linear elastic models is the Winkler soil model, where the soil is idealized with springs. The spring stiffness $k$ is known as the modulus of subgrade reaction and the pressure at soil surface, $p$, is:

$$p = k \cdot z$$

where $z$ is the deformation at the foundation surface.

However, non-linear elastic soil models are considered to be more realistic. To use this approach, the material parameters from the theory of linear elasticity should be modified through increments, to simulate the non-linear behavior. The material parameters are often taken to be function of the current stress state.

Nowadays, numerical methods make it easier to apply the rules of mechanics to soil-structure interaction of a very complex medium. Most of that analysis involves the following methods:

- Finite Element Method (FEM): The soil is divided into discrete elements, with specified material properties and deformation behavior. Individual element stiffness are derived and assembled to yield a global system of equations, from which displacements, strains and stresses are obtained. The FEM is very flexible and can be applied to more generalized soil models. However, when the material is linear elastic, the boundary conditions are known and the stresses/displacements fields in the interior can be found easily, making it unnecessary to use FE;

- Boundary Element Method (BEM): The governing differential equation are transformed to integral equations, which are solved numerically on the boundary regions. The number of physical dimensions to be considered is reduced by one, resulting in a smaller system of equation and a more efficient solution. The numerical technique is very similar to the one used with FEM. However, the boundary functions are approximated and solved and the stress/displacement vectors on the boundary are primary unknowns hence, of equal accuracy. BEM
is best suited for homogeneous bodies, and can simulate the behavior of infinite and semi-infinite domains;

- Combined FEM and BEM: Coupling of FE and BE is a very used technique when considering large or infinite domains with mostly linear behavior, except in a small portion. The modelation is easier, the computation faster and the results more accurate (see figure 2.5).

![Figure 2.5: Coupling of finite elements with a boundary element domain. Vector $\mathbf{n}_j$ is the unit normal of element $j$ representing the subsurface $S_j$ of the BE domain. $V_k$ is the volume represented by finite element $k$ (from Andersen et al. [3]).](image)

Lombaert et al. [42] and Takemiya [54] both present very interesting models to predict the train-track and nearby ground-borne vibrations, and compare the results with experimental data. Lombaert et al. [42] used measurements from the new HST track on the line L2 between Brussels and Köln to measure the soil transfer functions, the track-soil transfer functions and the track and free field vibrations during the passage of a Thalys high-speed train. Ground-borne railway induced vibrations are generated by a large number of excitation mechanisms. For HST tracks on soft soils, the train speed can be close to or even larger than the critical phase velocity of the coupled track-soil system. In this case, the quasi static contribution of the load is important for both the track and the free field response. High vibration levels and track displacements are obtained, affecting track stability and safety.

For the model, the rails were assumed to behave as Euler-Bernoulli beams and the sleepers were supposed to be rigid in the plane of the track cross-section, so that the vertical sleeper displacements along the track were determined by the vertical displacement $u_{sl}(y, t)$ at the center of gravity of the sleeper and the rotation $\beta_{sl}(y, t)$ about this center. The sleepers were presumed not to contribute to the longitudinal stiffness of the track, so that they can be modelled as a uniformly distributed mass along the track. The track-soil interface was assumed to be rigid in the plane of the track cross-section. The model used for the numerical calculations can be seen in figure 2.6 and alternative models are shown in figure 2.7.

A boundary element method is used to calculate the soil tractions at the track-soil interface, assuming that the track is located at the soil's surface. The bound-
2.2. Soil-Structure Interaction and Soil Modeling

Figure 2.6: Cross section of a ballasted track model (from Lombaert et al. [42]).

Any element formulation is based on the boundary integral equations in the frequency-wavenumber domain, using the Green’s functions of a horizontally layered soil. Each layer in the half-space model is characterized by its thickness $d$, the dynamic soil characteristics $E$ and $\nu$ or the longitudinal and transversal wave velocities $C_p$ and $C_s$, the material density $\rho$ and a material damping ratio $\beta_p$ and $\beta_s$ in volumetric and deviatoric deformation, respectively.

An elaborate measurement campaign was performed to validate the numerical model. The measurement campaign consisted of experiments that were performed to identify model parameters and experiments that were used to validate the numerical model. A Spectral Analysis of Surface Waves (SASW) test and a track receptance test were used to determine the dynamic soil and track parameters. The transfer functions between a steel foundation and the free field and between the track and the free field were subsequently used to validate the numerical model.

With the analysis, it was shown that the experimental and numerical track-free field transfer functions show a relatively good agreement, although at small distances an overestimation of the experimental response is observed. The sleeper response and the free field vibrations due to the passage of the Thalys HST was also predicted and validated for two train speeds. The results emphasize the crucial role of the dynamic soil properties. Given the large number of modelling uncertainties, the numerical results of the free field vibrations shown good agreement with the experimental results.

Takemiya [54] studied the train-track and nearby ground-borne vibrations produced by the Swedish high-speed train X-2000 at Ledsgard. As the result of the soft soil deposits at Ledsgard, the high train speed is almost in the trans-Rayleigh wave state so that a large amplified track response appeared due to the resonance between the track behavior and the Rayleigh wave propagation in the ground. Field measurement at the low train speed of 70 km/h shown that the track and ground deformations were small, while at the high train speed of 200 km/h, they were very large, indicating that appreciable non-linear soil behavior had occurred at the soft clay layers.
Kaynia et al. [38] investigated the track behavior by increasing the rigidity of the track as a potential remedy. An alternative measure is to improve the soft subsoil beneath the track embankment. Takemiya et al. [53] proposed the wave impeding barrier (WIB) procedure for the mitigation of track vibration and proved its effectiveness.

According to Takemiya [54], a series of impact forces act on the rails at the moment when the train wheels roll over them, with specified time delays occurring according to the train geometry, the sleeper spacing, and the speed of motion. These axle loads are transmitted through the rails to the ballast bed and then into the underlying ground. Depending on the train speed, an inertia force can be generated in the whole track including the ballast bed and embankment, as a result of the dynamics of the track-ground system. The consequence is the wave fields in the whole system, i.e., rail, track and ground. These train-induced vibrations at the track differ significantly depending on train geometry and speed. At low speeds the response is quasi static so that the track response due to train axle loads appears mostly downward at the point of their action. On the other hand, at high-speeds the train-induced response becomes dynamic due to the inertia generated in the track-ground system, so that the track vibrations appear evenly in both upward and downward directions.

Through computer simulations the train-induced responses were interpreted in a quasi static sense for the low speed (70 km/h), while in a dynamic sense for the high-speed (200 km/h). At the low speed, the response appeared to the axle loads at the instantaneous position so that the moving effect on track was given by the time shifting of the static response configuration accordingly. At the high-speed, on the other hand, wave motions occur due to the inertia generated in the track and ground system. The time histories of ground responses are reasonably different at low and high train speeds. For low speed trains, a clear-cut individual axle load effect appears. However, as the distance from the track to the observation location increases, those corresponding displacements disappear, so that the response turns
out to be a smooth long-span displacement due to the total train weight. The velocity response follows such a change of displacement with distance. A similar tendency also holds true for the acceleration response. For high-speed trains, on the other hand, due to a true wave field generation in the track-ground system, the vibration appears in all response quantities at both the track and the nearby ground locations.

A crude model was investigated in which a soft layer was replaced by an equivalent stiff layer with the WIB installation. The results suggested that a significant response reduction can be expected in the case where the WIB is installed over a substantial area across the transverse section. The improved soil model has led to a dramatic response reduction. Surprisingly, the response features were brought to similar values to those experienced in the quasi static state for low train speeds so that the wave propagation disappeared as the distance from the track increased. This suggested that installing the WIB may be a very promising method for vibration mitigation.

A different investigation was made by Andersen et al. [3] and Ju [36], where numerical solutions were provided, using finite elements. Andersen et al. [3] used a finite-element time-domain analysis in convected coordinates with a simple upwind scheme, including a special set of boundary conditions permitting the passage of outgoing waves in the convected coordinate system. The modification of frequency-dependent damping to convected coordinates is described, and the convected formulation of boundary elements is presented and used for illustrating the effect of high-speed motion.

Figure 2.8: Rayleigh wave propagation from a vertical harmonic rectangular load (the encircled area) moving along the surface of an elastic half-space in the direction indicated by the vector and at the speeds: (a) $v = 0$ m/s, (b) $v = 100$ m/s and (c) $v = 200$ m/s. Dark and light shades of grey indicate negative and positive vertical displacements, respectively (from Andersen et al. [3]).

Figure 2.8 shows the results for a load moving along the surface of a homogeneous half-space. The half-space has a mass density of 1550 kg/m$^3$, and the P- and S-wave speeds are 539 and 308 m/s, respectively. The load is applied vertically at the frequency 40 [Hz] and is distributed uniformly over a $3 \times 3$ m$^2$ rectangular area with a total intensity of 1 N.

As an alternative to the implementation of local transmitting boundary conditions in a finite-element scheme for an infinite domain, the FE scheme may be coupled with a boundary-element scheme, as said before. In this way, a computation model is
CHAPTER 2. STATE-OF-THE-ART REVIEW

achieved that combines the adaptability of the FEM with the radiation capabilities of the BEM. However, the coupling is not straightforward, since exterior loads are applied as nodal forces in the FEM and as surface traction in the BEM.

Ju [36] used finite element analysis to investigate the behavior of the building vibration induced by high-speed trains moving on bridges. The model included the bridge, nearby building, soil and train.

![Finite element mesh of the bridge and foundations (from Ju [36]).](image)

A 3D semi-infinite soil profile \((-\infty < x < \infty; -\infty < y < \infty; -\infty < z < 0\) supported a continuous railroad bridge along the \(x\)-axis with pile foundations. The \(y\)-axis is perpendicular to the railroad bridge, and the negative \(z\)-axis is the soil depth direction. The top surface of piles is connected by a reinforced concrete cap with, which is buried underground. The simple bridge beam with the cross-section is supported on the rectangular pier using four bearing plates (figure 2.9). Other than the beam mass itself, the bridge beam supports an extra-mass for the railway, parapet and devices. The high-speed train investigated in the study was the modified Japan SKS-700.

The results (partially shown in figure 2.10) demonstrated that trainload frequencies are more important than the natural frequencies of bridges and trains for building vibrations. If the building natural frequencies approach the trainload frequencies, which equal an integer times the train speed over the compartment length, the resonance occurs and the building vibration will be large. Moreover, the vibration shape is similar to the mode shape of the resonance building frequency.

Three common types of the foundation were used to isolate the building vibration induced by moving trains. Retaining walls cannot reduce significantly either the horizontal or the vertical vibration. Pile foundations were found to reduce the vertical vibration somewhat but could not reduce the horizontal vibration successfully. Soil improvement around the building reduced the building vibration effectively both in horizontal and vertical directions. Furthermore, the last method can be constructed after the building is built.
2.3 The UIC reports

With the objective of defining the standards for the dynamic amplification factors on the high-speed railway bridges, in the early 1970’s, the International Union of Railways (UIC), through its Office of Research and Experiments (ORE), studied the dynamic behavior of bridges, subjected to high-speed loads. During that decade, more than 350 measurements on 37 bridges, using different trains, were made, together with studies on bridge models loaded with scaled prototypes. The results were filtered and numerical simulations were performed to extrapolate scenarios not predicted in the measurements. From these studies, an expression for the dynamic amplification factor (DAF) was obtained:

\[ 1 + \varphi = 1 + \varphi' + \lambda \varphi'' \] (2.3)

where \(1 + \varphi'\) is the DAF in an ideal track:

\[ \varphi' = \frac{K}{1 - K + K^4};\quad K = \frac{v}{2L_\Phi n_0} \] (2.4)

\(\varphi''\) includes irregularities in the track, and is the following, for speeds greater than 22 m/s:

\[ \varphi'' = \frac{1}{100} \left[ 56e^{-\left(\frac{L_\Phi}{80}\right)^2} + 50 \left(\frac{L_\Phi n_0}{80} - 1\right)e^{-\left(\frac{L_\Phi}{80}\right)^2} \right] \] (2.5)

\(\lambda\) is a function related to the type of maintenance of the track, \(v\) is the speed of the train, \(n_0\) the first natural frequency of the structure and \(L_\Phi\) the determinant length. For \(n_0\) between certain limits, as a function of \(L_\Phi\), these coefficients were established in the UIC leaflet 776-1R [57]. Whenever these limits were not verified, a complete dynamic analysis would have to be performed for the structure. From the effort of
trying to simplify the design calculation, the train model LM71 (see section 3.3 for details) was developed. Using the DAF and the static responses obtained with the LM71, the dynamic displacements were reasonable for the train speed achieved in that decade.

With increasing train speeds, the dynamic amplifications started to become much higher than predicted. In 1992, due to resonance in the structure, a bridge with 44 m between Hannover and Würzburg registered dynamic displacements 35% higher than the static ones. The reasons founded to explain this behavior were the following:

- Amongst the trains used by UIC to study the DAF, only one could achieve speeds up to 300 km/h. This train, called the *Turbo Train*, was 38.4 m long with loads of 170 kN per axle. Nowadays, almost all the HST can achieve that speed and are much longer, causing the axles to excite the structure in a frequency close to resonance. The resulting amplification depends on the number of coaches, i.e., the duration of excitation, and the intensity of the loading;

- The damping on modern structures is lower than what was common on old bridges.

Similar problems were experienced in short span bridges, where high-speed traffic affected the ballast stability. Subsequent testing and analysis demonstrated that the ballast was subjected to accelerations of \(0.7 - 0.8g\) (\(7 - 8 \text{ m/s}^2\)).

Acknowledging that the safety of the structures and the passengers was not assured for the modern trains, the UIC decided to study the dynamic behavior of structures subjected to high-speed trains in detail. Therefore, in 1999, the Specialist Sub-Committee D214 was formed by the, then known as, European Rail Research Institute (ERRI). Its aim was to give guidance on the additional criteria to be satisfied to ensure that the performance of structures on or about high-speed lines met the necessary safety criteria to ensure satisfactory dynamic performance in service at speeds up to 300 km/h. From the studies conducted by the Committee, 9 reports were produced. The subject and findings of these reports will be discussed in the following.

One of the studies undertaken by the D214 Committee was the definition of unified models that could cover the dynamic effects caused by modern trains. The first try was made with new Univ-A load model, obtained from the Eurostar with different distances between axles. However, the dynamic signature, \(S_0\), obtained with the Univ-A was not high enough to cover the responses obtained with the Talgo and Virgin trains. Hence, a new envelope was developed and adopted for the Eurocode [10]. This new envelope (figure 2.11) contemplated the possibility of new Virgin trains being built with different spacing between axles, adding a constant value of 5700 kN for wavelengths between 24 and 27 m. The comparison between the new envelope and the dynamic signatures of common HSTs is shown in figure 2.12.

Subsequent studies were started with the objective of defining load models that would produce a signature as close as possible to that envelope. The result was the
2.3. THE UIC REPORTS

Figure 2.11: Eurocode envelope.

Figure 2.12: Comparison of the Eurocode envelope with real trains.

Figure 2.13: Comparison between the Eurocode envelope and the HSLM’s signature.
HSLM-A1 to A10 that will be properly defined in section 3.3. The signatures of the ten trains are compared with the Eurocode envelope in figure 2.13.

The HSLM-A was developed for almost every bridge, except short simply supported bridges, for which was developed yet another load model, the HSLM-B. Once more, this model will be suitably defined in section 3.3.

The studies undertaken covered a large number of aspects concerning the dynamic behavior of railway bridges. The study of the dynamic behavior can be made considering a single moving force, a single moving mass, moving forces, or modelling the complex vehicle-structure interaction. From the work of the Committee, it was concluded that greater accuracy in the prediction of resonance and dynamic effects of HST travelling over typical bridges is obtained if the nature of the spacing of the axles is taken into account. Where the engineer wishes to investigate mass interaction, structure finite element updating techniques are more suitable for real trains.

The first report published [14] is a literature survey and report 2 gives recommendations for calculating bridge deck stiffness [15]. In the third report [16], the importance of damping was studied. The measured damping results show that:

- Reducing the damping to a single value is an over-simplification;
- Increasing the number of bridge categories does not produce satisfactory results;
- A rule that takes into account the span of the bridge allow the estimates to be refined.

The values of critical damping to be used for design are shown in table 2.3. The lower limits should be used for design.

<table>
<thead>
<tr>
<th>Span</th>
<th>Steel and Composite L&lt;20 m</th>
<th>Prestressed Concrete L&lt;20 m</th>
<th>Reinforced Concrete and Filler Beams L&lt;20 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limit of ξ (%)</td>
<td>$\xi = 0.5 + 0.125(20 - L)$</td>
<td>$\xi = 1.0 + 0.07(20 - L)$</td>
<td>$\xi = 1.5 + 0.07(20 - L)$</td>
</tr>
<tr>
<td>L&gt;20 m</td>
<td>$\xi = 0.5$</td>
<td>$\xi = 1.0$</td>
<td>$\xi = 1.5$</td>
</tr>
<tr>
<td>L&gt;20 m</td>
<td>$\xi = 0.5$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The 4th report [17] focused on structure-vehicle interaction. According to this report, the effects of this phenomena are reduced by the presence of the vehicle suspension. Typical primary and secondary suspensions limit approximately 90% of the vehicle mass to accelerations around 0.1g. The full complex behavior of the vehicle primary and secondary suspension effect and associated masses was modeled, and its dynamic behavior studied. The results obtained shown that:
- Away from resonance, including or excluding vehicle mass interaction phenomena in the analysis made negligible difference;

- At resonance, the analytical techniques based on the interaction predict reduced dynamic responses in the structure, when compared to moving force models. Both peak accelerations and deflection are reduced;

- The reduction in peak effect displayed by interactive models, when compared with moving force models, at resonance, for continuous spans and for a sequence of simply supported spans, is less than the reduction for a simply supported span;

- The interaction models predict that resonance effects will occur at slightly lower speeds than travelling force models;

- The effects of interaction diminish with spans over 30 m.

Furthermore, the studies indicated that the use of an increase in structural damping, to simulate the interaction, was feasible. It is also mentioned that the single mass calculation techniques fail to represent the effects of a train of axles with varying spacing, and therefore are not recommended. The more complex mass interaction models show only a small advantage over travelling force models. Concerning the approximation provided by equation 2.3, it was concluded that, at train speeds away from resonance, the over estimation of $\varphi'$ compensates the underestimation of the dynamic effects due to track irregularities $\varphi''$, validating the formula for the vast majority of cases.

The structure-rail interaction and the load distribution due to the track structure were briefly studied. Because of the composite action between track, ballast and bridge, the overall effective stiffness of the structure is increased above that due to the structure alone. As a result, the natural frequencies increase. This effect is more pronounced for short spans. The same is believed to happen with the load distribution phenomena, which is more significant for spans of less than 20 m (according to Museros et al. [45]).

The studies undertaken show that it is essential to have an accurate estimate of the natural frequencies and mode shapes of the structure if realistic prediction of the dynamic behavior are to be made. Modeling support conditions, skew and bending stiffness accurately is, therefore, mandatory.

The design criteria, requires that:

- The verification of maximum peak deck acceleration shall be regarded as a serviceability limit state for traffic safety for the prevention of track and ballast instability;

- The static effects of loads should be enhanced by dynamic factor to allow for dynamic increment due to impact, resonance and track irregularities. Normally, the DAF (from equation 2.3) should be used;
- A check shall be carried out to ensure that the dynamic loading effects including resonance are covered by the DAF. If not, the greater effects shall be used in all calculations;

- A check should be carried out to ensure that the fatigue loading at resonance is covered by consideration of the stresses due to the load effects, considering the DAF.

A flow chart that summarises simple conservative checks is given in figure 2.14.

As mentioned previously, structures operating at resonance show extremely sensitive dynamic responses, depending on their damping level. Damping in structures occur because of energy losses during cycles of oscillation. As a result, the free vibrations of structures diminish with time. All structures exhibit damping, and it is mainly due to:

- Energy dissipation through bending of materials;
- Friction at supports and along structural boundaries;
- Energy dissipation from soil-structure interaction at the ends of bridges;
- Energy dissipation in ballast;
- Opening and closing of cracks in the material (especially concrete).

The 5th report [18] concentrated on the effect of track irregularities. These irregularities affect the dynamic behavior of railway bridges and can increase the dynamic load effects. The increase in the dynamic loading due to track irregularities increases with speed and decreases for longer bridge spans, and is mainly due to the loading effects developed in the unsprung axle masses of the vehicles traversing the track. In order to avoid complex bridge and track profile specific dynamic calculations, railway bridge engineers increase the live load static effects by a factor of $\varphi''$ as shown in equation 2.3. The prime purpose of the study was to investigate if the formula proposed by ORE Committees D23 and D128 was valid for the dynamic effects resulting from HST, due to resonance phenomena, on modern structures.

During the course of the D128 studies, the importance of damping was realized and a number of calculations were carried out with zero damping as a lower bound value. However, some of the results obtained from such calculations were considered to be unrepresentative, as the measured damping values of existing structures were significantly higher. Recent test results indicate that lower damping values should be taken into account than those assumed in the derivation of UIC 776-1R [57] criteria for the design of bridges.

The validity of equation 2.5 was studied using numerical methods with and without track defects at resonance. The values provided by equation 2.5 were compared
Figure 2.14: Flow chart to determine whether a dynamic analysis is necessary (from [22]).
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with the calculated increase in deflection and acceleration due to a track defect, respectively:

\[
\varphi'' = \frac{\delta_{\text{max, dip}}}{\delta_{\text{max}}} - 1; \quad \varphi''_{\text{acc}} = \frac{\text{acc}_{\text{max, dip}}}{\text{acc}_{\text{max}}} - 1
\]  

(2.6)

where \(\delta_{\text{max, dip}}\) and \(\text{acc}_{\text{max, dip}}\) represent the maximum responses of the beam with a track dip, while \(\delta_{\text{max}}\) and \(\text{acc}_{\text{max}}\) represent the same responses without the dip.

From the comparison of the values, it was concluded that the displacements from the trials were always below or comparable to the UIC values. For the accelerations, the calculated values where also below or comparable to the values obtained with equation 2.5, in the vicinity of train speed corresponding to resonance. Away from resonance, the values appear to increase with decreasing speed. However, at low speeds, the deck acceleration caused by the impact of the heaviest axles is usually less than at higher speeds. Thus, any errors obtained from the usage of the UIC formula will be less significant, providing the standard of track maintenance is sufficient to prevent wheel lift off.

However, from the studies undertaken and presented in this report, it may be concluded that the existing criteria for \(\varphi'\) severely underestimates the dynamic factor for an individual train when resonance phenomena occurs. Therefore, it is recommended that the limit of validity of \(\varphi'\) should be taken as 200 km/h. Specific checks shall be made for trains travelling at higher speeds.

The effect of track irregularities were found to be very significant and should be taken into account in the designing of railway bridges for speeds up to 350 km/h. From the calculations undertaken, it was shown that:

- Wheel lift off is likely to occur when large track defects are present, for speeds above 260 km/h, which causes high frequency bridge deck accelerations to increase considerably due to the impact from the unloaded wheel regaining contact with the rail;

- Calculated values for \(\varphi''\) are always below or comparable to the values provided by equation 2.5, providing the standard of track maintenance is sufficient to prevent wheel lift off.

Moreover, and in accordance with UIC 776-1R [57], when the track maintenance is sufficiently exhaustive, the usage of \(\varphi''/2\) (\(\lambda = 0.5\) in equation 2.3) is permitted, both for the dynamic deflection and for the bridge deck acceleration. Dynamic Amplification Factors such as \(\Phi_2\) (for very well maintained tracks) and \(\Phi_3\) (for common tracks) have also been suggested by past UIC studies, and are defined as:

\[
\Phi_2 = \frac{1.44}{\sqrt{L_\Phi} - 0.2} + 0.82
\]

(2.7)

\[
\Phi_3 = \frac{2.16}{\sqrt{L_\Phi} - 0.2} + 0.73
\]

(2.8)
where the equivalent span (or determinant length) \( L_\Phi \) coincides with the real one for a simply supported isostatic element, and an equivalence table is provided for other structural types. These factors have been developed in conjunction with LM71 and must only be used with their associated load model. \( \Phi_n \) represents the increase in LM71 necessary to cover the dynamic effects of all trains on all spans. Thus, it must not be applied to an individual load train, as it would most likely underestimate the dynamic effects of that particular train. On the other hand, dynamic factors such as \( \varphi' \) and \( \varphi'' \) have been specifically developed to allow the engineer to estimate the likely maximum dynamic effects of a series of forces representing the axle loads of a particular train.

It is suggested that the dynamic analysis of a structure should consider at least the first 3 or 4 modes of vibration. Filtering at a cut-off frequency of 20 Hz may not identify critical behaviors.

Report 6 [19] studies the influence of the mass and the stiffness of the bridge, on the magnitude of the displacement and acceleration. The results indicate that the maximum response of the structure at resonance is inversely proportional to its distributed mass. The resonance condition produced by a series of forces at equidistant distances \( d \), is calculated from the time necessary for crossing the distance \( d \) at speed \( c \) which is equal to the \( k \)-multiple of the period of natural vibration \( 1/f_j \):

\[
\frac{d}{c} = \frac{k}{f_j}, j = 1, 2, 3..., k = 1, 2, 3...
\]  

Equation 2.9 provides the critical speeds:

\[
cr = \frac{df_j}{k}, j = 1, 2, 3..., k = 1, 2, 3...
\]  

Figures 2.15 and 2.16 show the influence of the mass on the responses of a single-track bridge, for the \( k \)-multiple of the period of natural vibration of the structure. Therefore, it is recommended for designers to estimate the lower bound of the mass.
Figure 2.16: Maximum mid-span displacement on a single-track bridge with different mass (from [19]).

Figure 2.17: Maximum mid-span acceleration on a single-track bridge with different bending stiffness (from [19]).

Figure 2.18: Maximum mid-span displacement on a single-track bridge with different bending stiffness (from [19]).
2.3. THE UIC REPORTS

considering the maximum deck accelerations permitted, and the higher bound considering the lowest speeds at which resonant effects are likely to occur.

Concerning the stiffness, the relation is proportional, i.e., increasing the stiffness of the structure is beneficial because it raises the critical speeds at which resonance effects occur. Figures 2.17 and 2.18 show the influence of the stiffness on the a single-track bridge response, \( L \) being the span and \( f \) the deflection at mid-span. However, increasing the stiffness has a major impact upon cost. Hence, for economic design, it is necessary to make an accurate prediction of the correct stiffness for the structure.

Central to achieving a sufficiently accurate understanding of the behavior of any structure is the employment of realistic values in the equations for bending and torsional stiffness. Yet, accuracy of calculation of reinforced concrete structures is complicated when significant amounts of cracking occur. Where possible, in the design of new structures, the strategy should be to ensure that tensile stresses are controlled to the extent that cracked sections need not be taken into account.

The 7th report [20] undertakes studies related with the calculation of very complex structures and suggests programs to solve the problem. For structures being designed that show unsatisfactory dynamic behavior, the designer should consider the following options:

- Using a more refined analysis technique, that could overcome some simplifications adopted in the previous analysis;
- Using added damping to simulate vehicle-bridge interaction;
- Using a program which allows for vehicle-bridge interaction;
- Using a better estimate for damping, based on tests on identical structures;
- Increasing the mass of the structure;
- Allowing for the distribution of point axle forces by the track by using moving distributed loads (particularly important in short spans);
- Increasing the stiffness of the structure, through a reduction on the extend of the cracking in concrete, increasing the quality of the concrete and its modulus of elasticity or increasing the cross-section;
- Changing the structure;
- Restraining trains speeds.

In the assessment of existing lines for high-speed, the Train Signature Method enables the dynamic effects of different trains at resonance and away from resonance to be compared without reference to bridge or track characteristics. The train signature represents the dynamic excitation characteristics of a particular trains and is independent of the structure. If the magnitude of the train signature for a new train is less than that of the trains using the structure, then it will be satisfactory for the new train. Furthermore, the requirements to ensure traffic safety when assessing an existing structure are similar to the requirements for designing a new structure:
1. A check should be carried out to ensure that the DAF including resonance and excessive vibration effects is covered by the load effects from $\Phi \cdot \text{LM71}$ or $\left(1 + \varphi\right) \cdot \text{HSLM}$. When the load effects of particular trains exceed the load effects previously referred, the greater effects shall be used in all of the assessment;

2. The verification of maximum peak deck acceleration shall be regarded as a serviceability limit state for traffic safety, to prevent track instability;

3. A check should be carried out to ensure that the fatigue loading at resonance is covered by the stresses due to the static load effect, increased by the DAF.

Report 8 [21] focuses on the maximum permitted bridge deck accelerations. From the results and the observed behavior of track and ballast subjected to resonant loading, it can be seen that critical behavior starts for bridge deck accelerations of $0.7 - 0.8g$. Applying a factor of safety of 2, the maximum permitted deck acceleration was fixed at $3.5 \text{ m/s}^2$ for ballasted tracks. For direct fastening decks with track and structural decks designed for high-speed traffic, that limit is $5 \text{ m/s}^2$.

Once again, it was concluded that the 20 Hz frequency limit for cut off is not valid for a large number of structures. It is recommended that bridge deck acceleration should be checked for all elements supporting the track at frequencies up to 30 Hz or 2 times the frequency of the first mode of vibration.

![Figure 2.19: Transfer function between bridge deck acceleration ($B_{m}$) and ballast acceleration ($B_{a}$) (from [21]).](image_url)

An amplification factor was defined as the ratio between the maximum acceleration within the ballast layer and the applied acceleration at the bottom of the ballast layer and is shown in figure 2.19. This transfer function shows the non-linearity of the ballast bed for bridge deck accelerations higher that $0.6g$. The existence of ballast mats increases the amplifying effects: the maximum amplification factor was 15% without mats and 60% with ballast mats.
2.3. THE UIC REPORTS

The minimum ballast depth recommended is 250 mm, to ensure the maintainability of the track, a satisfactory transition between formation stiffness on and off the bridge and other items relevant for the satisfactory performance of the track structure. However, it may be noted that on a number of high-speed lines, a ballast depth of 350 mm beneath the underside of sleepers has been adopted with satisfactory results.

The last report [22], synthesizes the results obtained by the D214 Research Committee and proposes further research in the area.

With the studies and experiments developed in report D214, the UIC, through that Committee, found the answer for numerous problems related to HSTs and placed the foundations for the norms used nowadays in the vast majority of the European countries, namely, the Eurocodes.
Chapter 3

Important Theoretical Concepts

The aim of this chapter is to introduce the theoretical concepts that will be used in the investigations undertaken in chapters 4 and 5.

The two first sections bring the most important concepts for understanding the work undertaken in this dissertation. The differences between the static and the dynamic behavior of a structure will be referred in the first section, so that the reader can fully understand the influence of the dynamic loads and the occurrence of resonance. Section 3.2 will explain how to estimate the stiffness of bridge supports, used for the stiffness of the springs in the FE analysis of chapters 4 and 5.

Afterwards, the most common high-speed load models will be defined in section 3.3. Section 3.4 will shortly present the theoretical approach to the 2D beam element, that gave the basis to the 2D finite beam element. The analytical solution proposed by Fryba and used for the theoretical studies undertaken in chapter 4 will be presented in subsection 3.4.1.

The last three sections will be focused on the methods used (Mode Superposition and FEM) for the numerical calculations obtained with ABAQUS/BRIGADE.

3.1 Dynamic behavior of HSR bridges

From the late 1950’s, the train speed record has been increasing continuously and the experimental maximum speed achieved by the TGV train, in France, is already 515 km/h. However, while the speed increases, the safety and the comfort of passenger trains have to be kept at the same high level. To do so, the interaction between the high-speed train and the structure has to be deeply studied. This interaction can be seen in both ways: the dynamic impact of the running axles affects the service life and the working state of the structure and, at the same time, the vibration of the structure has an effect on the stability and the safety of the train. The way by which the two subsystems interact with each other is determined primarily by the inherent frequencies of the two subsystems and the driving frequency of the moving vehicles.
Occurrence of resonance phenomenon in bridges can be observed for trains running at speeds close to the critical speed. As the passengers loading is not a deterministic quantity and it is believed to be low, the bridges response obtained by means of analytical method ignores such loading and considers only the load induced by the train itself. Generally, the maximum response induced by the crossing train is produced by the axles located between the heaviest locomotive and motorized car and the increase of response brought by the additional loading due to passengers does not affect the maximum response of the bridge. However, in the case of trains crossing at critical speed, resonance, occurring due to the repeated loading generated by wheel loads of the coaches crossing continuously at regular interval, amplifies significantly the maximum response of the bridge produced by the locomotive and motorized car.

Consequently, when resonance due to train running at the critical speed occurs, the maximum response generated by the locomotive is amplified to a scale depending on the duration of the resonance produced by the coaches. Furthermore, structures operating at resonance show extremely sensitive dynamic responses according to their damping level.

When a SDOF system is subjected to periodic excitation with a frequency similar to the natural one of the system \((\beta = w/p \approx 1)\), resonance will occur and the static response will be amplified up to \(u_{st}/2\xi\sqrt{1-\xi^2}\) (figure 3.1), depending on the damping, where \(u_{st}\) is the static response, as can be seen in equation 3.2. From Chopra [11] we have that:

\[
DAF = \frac{u_{dynam}}{u_{static}} = [(1 - \beta^2)^2 + (2\xi\beta)^2]^{-\frac{1}{2}}
\]  

(3.1)

where \(\beta\) represent the ratio between the applied load frequency and the natural free vibration frequency of the bridge. Hence:

\[
\frac{\partial}{\partial \beta} DAF = 0 \iff \beta = \sqrt{1 - 2\xi^2}
\]

Figure 3.1: Dynamic amplification factor as a function of the frequency ratio \(\beta\) (from Clough and Penzien [12]).
Thus:

\[ DAF(\beta = \sqrt{1 - 2\xi^2}) = \frac{1}{2\xi \sqrt{1 - \xi^2}} \]  

(3.2)

The dynamic response will reach the limit value, steady-state amplitude, for a certain time, depending on the damping. The more the damping increases, the less time is needed to reach the steady-state amplitude. Therefore, the more numbers of coaches are arranged successively with equally spaced axles, the more the response will be amplified up to \( u_{st}/2\xi \) (considering low damping). Therefore, during the design of high-speed railway bridges, studying the dynamic behavior of bridges arises as an essential issue to guarantee safety of passengers and riding conditions.

## 3.2 Theoretical estimation of the support stiffness

The estimation of the support stiffness as a function of the dynamic proprieties of the soil is a very complex problem that was deeply studied by Richart and Lysmer in the decade of 1960 (Lysmer and Richart [43] and Richard et al. [51]), giving very important contribution on the subject for the scientific community. In 1962, Barkan [7] had already suggested one expression to calculate the vertical spring stiffness for rigid rectangular footings resting on elastic half-space:

\[ K_z = \frac{G}{1 - \nu} \beta_z \sqrt{A_b} \]  

(3.3)

where \( A_b \) represents the area of footing, \( G \) the soil shear modulus, \( \nu \) the soil Poisson’s ratio and \( \beta_z \) is given in figure 3.2.

![Figure 3.2: Coefficients \( \beta_x \), \( \beta_z \) and \( \beta_\psi \) for rectangular footing (from Whitman and Richart [58]).](image-url)
Before that, Timoshenko [56] had derived the vertical spring stiffness if the rigid footing was circular, also resting on elastic half-space:

\[ K_z = \frac{Gr}{1 - \nu} \]  

(3.4)

However, if one defines \( r \) as the equivalent radius:

\[ r = \sqrt{\frac{A_b}{\pi}} \]

it is possible to estimate the stiffness of rectangular footing using Timoshenko’s expression.

In the beginning of the 21\textsuperscript{th} century, George Gazetas [29, 30] studied the dynamic interaction of spread footings and pile groups, suggesting expressions, not only for the static stiffness, but also for the frequency dependent stiffness. For the surface foundation on homogeneous half-space, Gazetas suggested that the static stiffness should be obtained from:

\[ K_z = \frac{2GL}{1 - \nu} \left( 0.73 + 1.54 \left( \frac{B}{L} \right)^{0.75} \right) \]  

(3.5)

where \( B \) and \( L \) represent the semi-width and semi-length of the circumscribed rectangle, \( L > B \).

The presence of bedrock at shallow depth, rather than having natural soil deposits extending to practically infinite depth as the homogeneous half-space implies, modifies the static stiffness. Particularly sensitive to variations in the depth of the bedrock is the vertical stiffness. Thus, for footings lying on an homogeneous stratum with height \( H \), over a bedrock, the expression is:

\[ K_z = \frac{2GL}{1 - \nu} \left( 0.73 + 1.54 \left( \frac{B}{L} \right)^{0.75} \right) \left( 1 + \frac{B/H}{0.5 + B/L} \right) \]  

(3.6)

To obtain a dynamic stiffness, Gazetas suggested that the values for the static stiffness, \( K_z \), should be multiplied by a factor depending on the frequency.

The expression for the calculation of the dynamic stiffness and dashpot for arbitrary shaped foundations on homogeneous half-space is shown in table 3.3, together with the graphs in figure 3.4. In table 3.4 and figure 3.5, the solutions for the dynamic stiffness and dashpot coefficients for surface foundations on homogeneous stratum over bedrock are presented. For the usage of the tables: \( G, \nu, A_b, B, L \) and \( H \) are as already defined and \( I_{aa} \) is the area moments of inertia about the \( a \)-axis of the actual soil-foundation contact surface. The solution for the partially and fully embedded foundations and for the foundation on soil stratum over half-space were also found, but will not be discussed in this study.
### 3.2 THEORETICAL ESTIMATION OF THE SUPPORT STIFFNESS

Figure 3.3: Dynamic stiffness and dashpot coefficients for arbitrary shaped foundations on homogeneous half-space surface (from Gazetas et al. [30]).

<table>
<thead>
<tr>
<th>Vibration Mode</th>
<th>Static Stiffness $K$ (with equivalent rectangle $2L \times 2B$, $L\geq B$)</th>
<th>Dynamic Stiffness $K$</th>
<th>Dynamic Stiffness $C$</th>
<th>Radiation Dashpot Coefficient $C_r$</th>
<th>Dashpot Coefficient $C_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical, $z$</td>
<td>$K_x = 2.24L \left( \frac{L}{B} \right)$</td>
<td>$k_x = 0.04G_{eq}$</td>
<td>$c_x = \rho v_s A_b$</td>
<td>$C_x = \rho v_s A_b$</td>
<td>$C_x = \rho v_s A_b$</td>
</tr>
<tr>
<td>Horizontal, $y$ (lateral direction)</td>
<td>$K_y = 2.04L \left( 2 + 2.5 \frac{L}{B} \right)$</td>
<td>$k_y = 0.04G_{eq}$</td>
<td>$c_y = \rho v_s A_b$</td>
<td>$C_y = \rho v_s A_b$</td>
<td>$C_y = \rho v_s A_b$</td>
</tr>
<tr>
<td>Horizontal, $x$ (longitudinal direction)</td>
<td>$K_x = K_y - \frac{1}{2} \mu R x G L (1 - \frac{L}{B})$</td>
<td>$k_x = 0.04G_{eq}$</td>
<td>$c_x = \rho v_s A_b$</td>
<td>$C_x = \rho v_s A_b$</td>
<td>$C_x = \rho v_s A_b$</td>
</tr>
<tr>
<td>Rocking, $m$ (around $x$ axis)</td>
<td>$K_m = \frac{K_x + K_y}{2}$</td>
<td>$k_m = 0.04G_{eq}$</td>
<td>$c_m = \rho v_s A_b$</td>
<td>$C_m = \rho v_s A_b$</td>
<td>$C_m = \rho v_s A_b$</td>
</tr>
<tr>
<td>Rocking, $n$ (around $y$ axis)</td>
<td>$K_n = \frac{K_x + K_y}{2}$</td>
<td>$k_n = 0.04G_{eq}$</td>
<td>$c_n = \rho v_s A_b$</td>
<td>$C_n = \rho v_s A_b$</td>
<td>$C_n = \rho v_s A_b$</td>
</tr>
<tr>
<td>Torsional</td>
<td>$K_t = 8.3G_{eq}B^3$</td>
<td>$k_t = 0.04G_{eq}$</td>
<td>$c_t = \rho v_s A_b$</td>
<td>$C_t = \rho v_s A_b$</td>
<td>$C_t = \rho v_s A_b$</td>
</tr>
</tbody>
</table>

Note: $G_{eq}$ is the equivalent shear modulus, $\mu$ is the Poisson's ratio, $R_x$ and $R_y$ are the moments of inertia of the foundation about the $x$ and $y$ axes, respectively.
Figure 3.4: Graphs accompanying table 3.3 (from Gazetas et al. [30]).
### 3.2. THEORETICAL ESTIMATION OF THE SUPPORT STIFFNESS

#### Figure 3.5: Dynamic stiffness and dashpot coefficients for surface foundations on homogeneous stratum over bedrock (from Gazetas et al. [30]).

<table>
<thead>
<tr>
<th>Foundation Shape</th>
<th>Circular Foundation of Radius $B = R$</th>
<th>Rectangular Foundation $2B$ by $2L$ ($L &gt; B$)</th>
<th>Strip Foundation $2L \to \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Static stiffness $K$</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical, $z$</td>
<td>$K_z = \frac{4GRI}{1-\nu^2} \left(1 + \frac{1.3\nu^2}{3}\right)$</td>
<td>$K_z = \frac{2G}{1-\nu^2} \left[0.73 + 1.54 \left(\frac{B}{L}\right)^{3/5}\right] \left(1 + \frac{B^2}{0.05B^2}\right)$</td>
<td>$K_z = \frac{2G}{1+\nu^2} \left(1 + 3.5\nu^2\right)$</td>
</tr>
<tr>
<td>Horizontal, $y$</td>
<td>$K_y = \frac{4GRI}{\nu(1-\nu)} \left(1 + 0.5\nu^2\right)$</td>
<td>$*$</td>
<td>$K_y = \frac{G}{1-\nu^2} \left(1 + 2\frac{B}{L}\right)$</td>
</tr>
<tr>
<td>Horizontal, $x$</td>
<td>$K_x = K_y$</td>
<td>$*$</td>
<td>$-$</td>
</tr>
<tr>
<td>Rocking, $rx$</td>
<td>$K_{rx} = \frac{8GRI}{3(1-\nu^2)} \left(1 + 0.17\frac{B}{L}\right)$</td>
<td>$*$</td>
<td>$K_{rx} = \frac{G}{2(1-\nu^2)} \left(1 + 0.2\frac{B}{L}\right)$</td>
</tr>
<tr>
<td>Rocking, $ry$</td>
<td>$K_{ry} = K_{ry}$</td>
<td>$*$</td>
<td>$-$</td>
</tr>
<tr>
<td>Torsional, $t$</td>
<td>$K_t = \frac{16GRI}{3} \left(1 + 0.10\frac{B}{L}\right)$</td>
<td>$*$</td>
<td>$-$</td>
</tr>
<tr>
<td><strong>Dynamic stiffness coefficient $k(\omega)$</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical, $z$</td>
<td>$k_z = k_z(H/R, \alpha_0)$ is obtained from Graph III-1</td>
<td>$k_z = k_z(H/B, L/B, \alpha_0)$ is plotted in Graph III-2 for rectangles and strip</td>
<td>$-$</td>
</tr>
<tr>
<td>Horizontal, $y$ or $x$</td>
<td>$k_y = k_y(H/R, \alpha_0)$ is obtained from Graph III-1</td>
<td>$-$</td>
<td>$k_y = k_y(H/B, \alpha_0)$ is obtained from Graph III-3</td>
</tr>
<tr>
<td>Rocking, $rx$ or $ry$</td>
<td>$k_{ux} = k_d(\infty)$</td>
<td>$*$</td>
<td>$*$</td>
</tr>
<tr>
<td>Torsional, $t$</td>
<td>$\alpha = rx$, $ry$, $t$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Radiation dashpot coefficient $C(\omega)$

- $C_z(H/B) = 0$ at $t < t_e$; regardless of foundation shape
- $C_z(H/B) = 0.8C_z(\infty)$ at $t \geq 1.5t_e$

At intermediate frequencies: interpolate linearly.
$$t_e = \frac{V_{sa}}{4H}, \quad V_{sa} = \frac{34V_s}{(1-\nu)^2}$$

- $C_y(H/B) = 0$ at $t < \frac{t}{t_e}; \quad C_y(H/B) = C_y(\infty)$ at $t > \frac{t}{t_e}$
- Similarly for $C_x$

At intermediate frequencies: interpolate linearly.
$$t_e = \frac{V_{sa}}{4H}, \quad V_{sa} = \frac{34V_s}{(1-\nu)^2}$$

- $C_{ux}(H/B) = 0$ at $t < t_e$; $C_{ux}(H/B) = C_{ux}(\infty)$ at $t > t_e$
- Similarly for $C_{uy}$

- $C_{t}(H/B) = C_{t}(\infty)$
Figure 3.6: Graphs accompanying table 3.5 (from Gazetas et al. [30]).
3.3 Load models

Whenever the designer knows in advance which trains will load the track or bridge structure, he or she should use the axle load induced by that train for design purpose.

Nowadays, the vast majority of HSTs are grouped in three train architectures:

1. Conventional, where each carbody is supported by two independent bogies. Two high-speed trains of this type are the VIRGIN and the ICE2;

   ![Figure 3.7: Conventional Train.](image)

2. Articulated with single-axle bogie, where each bogie, placed between two sections, has one single axle. The new Talgo is an example;

   ![Figure 3.8: Articulated train with single axle bogie.](image)

3. Articulated with Jakobs-type bogie, named after Wilhelm Jakobs (1858-1942), where each bogie is placed between two carbody sections. Thalys, Eurostar and TGV share this architecture.

   ![Figure 3.9: Articulated train with Jakobs-type bogie.](image)

Some HSTs are equipped with tilting mechanisms that enables increased speed on regular railroad tracks. As any vehicle rounds a curve at speed, independent objects inside it exert centrifugal force since their inherent momentum forward no longer lies along the line of the vehicle’s course. Tilting trains are designed to counteract this discomfort. This type of trains may be constructed such that inertial forces themselves cause the tilting, commonly referred to as passive tilt, or it may be actively induced by a computer-controlled mechanism, referred to as active tilt. This improvement is shared by the Talgo, the ICE-T, the Pendolino and the Swedish X2000, amongst others.
Although, sometimes, the train loading the structures is known, whenever the track is designed for different types of trains, load models, that produce an envelope large enough to contemplate the dynamic effects of all trains, have to be found. The historical background about the development of most commonly used load models was referred in 2.3. To perceive the static effects of normal traffic in continuous bridge and heavy train traffic, load models SW/0 and SW/2 were developed, respectively. The loads for the two load models are defined in table 3.1 and should be multiplied by a factor $\alpha$, depending on the type of trains the line will serve. For lines subjected to heavy train loads, $\alpha$ can be as high as 1.46. For light loaded lines, the value can decrease to 0.75.

![Figure 3.10: SW/0 and SW/2 load models.](image)

<table>
<thead>
<tr>
<th>Load Model</th>
<th>$q_{vk}$ [kN/m]</th>
<th>a [m]</th>
<th>c [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>SW/0</td>
<td>133</td>
<td>15.0</td>
<td>5.3</td>
</tr>
<tr>
<td>SW/2</td>
<td>150</td>
<td>25.0</td>
<td>7.0</td>
</tr>
</tbody>
</table>

Besides the two SW models, based on the effects observed in structures due to 6 real trains, the LM71 load model was developed (see 2.3 for more information). The loads, likewise happens with SW/0 and SW/2, should be multiplied by $\alpha$.

![Figure 3.11: LM71 load model.](image)

From the studies undertaken for the Eurocode [10], yet another load model was developed. The HSLM-A was defined with the architecture shown in figure 3.12, but different axle characteristics. The 10 models are defined in table 3.2. The HSLM-A should be used for almost every bridge, except short simply supported bridges, for which the HSLM-B should be used instead.
3.3. Load Models

Figure 3.12: High speed load model A axle configuration (from [10]).

Table 3.2: High speed load model A (from [10]).

<table>
<thead>
<tr>
<th>HSLM</th>
<th>N</th>
<th>D [m]</th>
<th>d [m]</th>
<th>P  [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>18</td>
<td>18</td>
<td>2.0</td>
<td>170</td>
</tr>
<tr>
<td>A2</td>
<td>17</td>
<td>19</td>
<td>3.5</td>
<td>200</td>
</tr>
<tr>
<td>A3</td>
<td>16</td>
<td>20</td>
<td>2.0</td>
<td>180</td>
</tr>
<tr>
<td>A4</td>
<td>15</td>
<td>21</td>
<td>3.0</td>
<td>190</td>
</tr>
<tr>
<td>A5</td>
<td>14</td>
<td>22</td>
<td>2.0</td>
<td>170</td>
</tr>
<tr>
<td>A6</td>
<td>13</td>
<td>23</td>
<td>2.0</td>
<td>180</td>
</tr>
<tr>
<td>A7</td>
<td>13</td>
<td>24</td>
<td>2.0</td>
<td>190</td>
</tr>
<tr>
<td>A8</td>
<td>12</td>
<td>25</td>
<td>2.5</td>
<td>190</td>
</tr>
<tr>
<td>A9</td>
<td>11</td>
<td>26</td>
<td>2.0</td>
<td>210</td>
</tr>
<tr>
<td>A10</td>
<td>11</td>
<td>27</td>
<td>2.0</td>
<td>210</td>
</tr>
</tbody>
</table>

Figure 3.13: High speed load model B axle configuration (from [10]).

Figure 3.14: Parameters N and d as functions of L, for HSLM-B (from [10]).
3.4 2D beam theory

The structural analysis of bridges is usually based on the linear elasticity theory. The classic elasticity theory assumes a linear dependence between strain and stress, disregarding the non-linear behavior of some materials.

A beam in a plane frame structural model can be affected by transversal and longitudinal loadings. Standard equilibrium equations are thereby formulated comparing the internal forces $N$, $V$ and $M$ with the applied external loads $p_x$ and $p_y$.

![Figure 3.15: Beam element with internal forces and applied loading in positive direction.](image)

The equation of equilibrium can be obtained from figure 3.15 and are the following:

\[
\frac{\partial N}{\partial x} + p_x = 0 \quad (3.7)
\]
\[
\frac{\partial V}{\partial x} + p_z = 0 \quad (3.8)
\]
\[
\frac{\partial M}{\partial x} - V = 0 \quad (3.9)
\]

For the 2D analysis of a beam, two theories are commonly used: Euler-Bernoulli and Timoshenko beam theory. A brief description of the two theories will be presented in sections 3.4.1 and 3.4.2. Further information on the subject is available in Timoshenko [56].

### 3.4.1 Euler-Bernoulli beam

According to Hibbitt et al. [32], Euler-Bernoulli beams do not allow for transverse shear deformation, plane sections initially normal to the beam’s axis remain plane (if
there is no warping) and normal to the beam axis. They should be used only to model slender beams, i.e., the beam's cross-sectional dimensions should be small compared to typical distances along its axis (such as the distance between support points or the wavelength of the highest mode that participates in a dynamic response).

In ABAQUS/BRIGADE, the Euler-Bernoulli beam elements use a consistent mass formulation and cubic interpolation functions, which makes them reasonably accurate for cases involving distributed loading along the beam. Therefore, they are well suited for dynamic vibration studies, where the d’Alembert (inertia) forces provide such distributed loading. The cubic beam elements are written for small-strain, large-rotation analysis. They may not be appropriate for torsional stability problems due to the approximations in the underlying formulation and cannot be used in analysis involving very large rotations, quadratic or linear beam elements should be used instead.

Analytical solution for the Euler-Bernoulli beam

The analytical response of the Euler-Bernoulli beam was studied by Fryba [28], for a simply supported beam subjected to any set of loads, travelling to a certain speed. The derivation of that solution is included in the following.

If a beam of span $l$ is subjected to a row of $N$ axle forces $F_n$, moving at constant speed $c$, its behavior is described by the Euler-Bernoulli partial differential equation:

$$EI \frac{\partial^4 v(x,t)}{\partial x^4} + \mu \frac{\partial^2 v(x,t)}{\partial t^2} + 2\mu\omega_d \frac{\partial v(x,t)}{\partial t} = \sum_{n=1}^{N} \varepsilon_n(t) \partial(x - x_n)F_n$$  \hspace{1cm} (3.10)

where $EI$ is the bending stiffness, $\mu$ the mass per meter, $v(t, x)$ represents the vertical deflection of the beam, $\omega_d$ is the damped circular frequency, $F_n$ is the $n_{th}$ axle force and:

$$\varepsilon_n(t) = h(t - t_n) - h(t - T_n)$$  \hspace{1cm} (3.11)

with $h(t)$ being the Heaviside unit function:

$$h(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t \geq 0 \end{cases}$$  \hspace{1cm} (3.12)

$t_n$ and $T_n$ being, respectively, the instants $F_n$ enters and leaves the beam:

$$t_n = \frac{d_n}{c}$$  \hspace{1cm} (3.13)

$$T_n = \frac{l + d_n}{c}$$  \hspace{1cm} (3.14)

and $\partial(x)$ is the Dirac delta function, where:

$$x_n = ct - d_n$$  \hspace{1cm} (3.15)
with $d_n$ being the distance between the $n^{th}$ axle and the first axle of the train.

Taking into account the boundary conditions of the simply supported beam, and the initial values of the problem being zero, the natural frequencies, $\omega_j$, $f_j$ and shapes of natural vibration $v_j(x)$ are:

\[
\omega_j^2 = \frac{j^2\pi^4 EI}{l^4} \mu 
\]

(3.16)

\[
f_j = \frac{1}{2\pi} w_j 
\]

(3.17)

\[
v_j(x) = \sin \left( \frac{j\pi x}{l} \right) 
\]

(3.18)

Using the mutual relations of the Fourier integral transformation and the Laplace-Carlson integral transformation, the solution for the problem comes in the following form:

\[
v(x,t) = \sum_{j=1}^{\infty} \sum_{n=1}^{N} v_0 F_n F_j jw^2 \left[ f(t - t_n)h(t - t_n) - \right. \\
\left. -(-1)^j f(t - T_n)h(t - T_n) \right] \sin \left( \frac{j\pi x}{l} \right) 
\]

(3.19)

where (note that there is an error in the expression shown in Frýba [28]):

\[
f(t) = \frac{1}{\omega_j^2 D} \left[ \frac{\omega_j}{j\omega} \sin(j\omega t + \lambda) - e^{\omega_d t} \sin(\omega_d t + \varphi) \right] 
\]

(3.20)

with $v_0$ representing the deflection of a simple beam at its center due to the force $F = F_n$ place at the same point:

\[
v_0 \approx \frac{Fl^3}{48EI} 
\]

(3.21)

The first term between the parenthesis on equation 3.20 stands for forced vibration due to the moving loads and the second term represent the free damped vibration.

Knowing that the bending moment on the beam can be expressed by:

\[
M(x,t) = -EI \frac{\partial^2 v(x,t)}{\partial x^2} 
\]

(3.22)

the result can be calculated in a similar way, expressed as:

\[
M(x,t) = \sum_{j=1}^{\infty} \sum_{n=1}^{N} M_0 F_n \frac{F_j j^3 \omega_j^2}{F} \left[ f(t - t_n)h(t - t_n) - \right. \\
\left. -(-1)^j f(t - T_n)h(t - T_n) \right] \sin \left( \frac{j\pi x}{l} \right) 
\]

(3.23)
where $M_0$ is the bending moment at the center of a simple beam due to the force $F = F_n$ applied to the same point:

$$ M_0 \approx \frac{Fl}{4} \quad (3.24) $$

In a similar way, and knowing that the acceleration can be expressed as:

$$ a(x,t) = \frac{\partial^2 v(t,x)}{\partial t^2} \quad (3.25) $$

the acceleration comes as:

$$ a(x,t) = \sum_{j=1}^{\infty} \sum_{n=1}^{N} v_0 \frac{F_n}{F} j \omega_j^2 [\dot{f}(t-t_n)h(t-t_n) - (-1)^j \dot{f}(t-T_n)h(t-T_n)] \sin \left( \frac{j\pi x}{l} \right) \quad (3.26) $$

where:

$$ \dot{f}(t) = -\frac{\omega_j^2 - \omega_d^2}{\omega_j^2 - \omega_d^2} \left[ \frac{j \omega_j' \omega_d}{\omega_j^2 - \omega_d^2} \sin(j\omega t + \lambda) - e^{\omega_d t} \sin(\omega_j t + \varphi) \right] \quad (3.27) $$

In the response equations, the following relations were used:

$$ \omega = \frac{\pi c}{l} \quad (3.28) $$

$$ \omega_j' = \omega_j^2 - \omega_d^2 - 2i \omega_d \omega_j \quad (3.29) $$

$$ \Omega_j^2 = \omega_j^2 + \omega_d^2 \approx \omega_j^2 \quad (3.30) $$

$$ D^2 = (\Omega_j^2 - j^2 \omega^2)^2 + 4j^2 \omega^2 \omega_d^2 \approx (\omega_j^2 - j^2 \omega^2)^2 \quad (3.31) $$

$$ \lambda = \arctan \left( \frac{-2j \omega_d \omega_j}{\Omega_j^2 - j^2 \omega^2} \right) \quad (3.32) $$

$$ \varphi = \arctan \left( \frac{2 \omega_j' \omega_d}{\omega_d^2 - \omega_j^2 + j^2 \omega^2} \right) \quad (3.33) $$

$$ \varphi' = \varphi + \arctan \left( \frac{2 \omega_j' \omega_d}{\omega_j^2 - \omega_d^2} \right) \quad (3.34) $$

Note that the approximations used with $\Omega_j$ and $D$ are valid only for low damping systems.

With equation 3.19, 3.23 and 3.26, Frýba was able to describe the response of a simply supported beam to a train passing at constant speed $c$. 

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3.4.2 Timoshenko beam

Timoshenko beams allow for transverse shear deformation. They can be used for thick as well as slender beams. For beams made from uniform material, shear flexible beam theory can provide useful results for cross-sectional dimensions up to 1/8 of typical axial distances or the wavelength of the highest natural mode that contributes significantly to the response. The Timoshenko beams can be subjected to large axial strains but the axial strains due to torsion are assumed to be small. In combined axial-torsion loading, torsional shear strains are calculated accurately only when the axial strain is not large.

The FE beam elements used for most part of the research undertaken in this dissertation were based on the Timoshenko theory. For further research in the subject, one should study Cook et al. [13] or and W. Wunderlich [2]. ABAQUS assumes that the transverse shear behavior of Timoshenko beams is linear elastic with a fixed modulus and, thus, independent of the response of the beam section to axial stretch and bending. For most beam sections ABAQUS will calculate the transverse shear stiffness values required in the element formulation.

3.5 Mode Superposition Method

The Mode Superposition Method provide higher accuracy, and requires less computational effort than the traditional step-by-step integration solution technique. Besides, the user can choose the number of modes to use in the response. Other advantage is that modal damping can be used in addition to (or instead of) Rayleigh damping. When experimental data are available, modal damping gives a more accurate representation of the damping in the system.

However, Modal Superposition is not suitable for problems such as shock loads or impacts, where the higher frequency modes are excited. In these cases, many modes are required, and the cost to calculate these modes can significantly offset the saving in time. The method is best suited to structures where the lower frequencies dominate the response. Typically, 10 modes will provide good accuracy for these problems, although prior frequency analysis is required. Yet, the most serious disadvantage of this approach is that it is not capable of handling any non-linearity in the solution.

For its simplicity, this method was chosen as the basic method for solving the dynamic equations of the bridges studied, since it is a very powerful method used to reduce the number of equations to be solved in a dynamic response analysis. All types of loading can be accurately approximated by linear functions within a small time increment. An exact solution exists for this type of loading and this solution can be computed with a trivial amount of computer time for equal time increments. Using the Mode Superposition Method, responses of the individual modes of the structure are calculated separately and then combined to produce the total response of the structure. The method will be briefly presented here, for more information
one should see Bathe [8].

To solve the linear dynamic response of structures subjected to periodic loading it is only necessary to add a corrective solution to the transient solution for a typical
time period of loading. Hence, only one numerical algorithm is required to solve
a large number of different dynamic response problems in structural engineering.
Participating mass factors can be used to estimate the number of vectors required
in an elastic analysis. The use of mass participation factors to estimate the accuracy
of a non-linear analysis can introduce significant errors. This is mainly due to the
existence of internal non-linear forces, in opposite directions, that do not produce
a base shear. A dynamic load participation ratio is defined and can be used to
estimate the number of vectors required for other types of loading.

The dynamic force equilibrium equation can be written in the following form as a
set of $N$ second order differential equations:

$$
M \ddot{u}(t) + C \dot{u}(t) + Ku(t) = F(t) = \sum_{j=1}^{J} f_j \cdot g_j(t)
$$

(3.35)

All possible types of time-dependent loading, including wind, wave and seismic, can
be represented by a sum of $J$ space vectors $f_j$, which are not a function of time, and
$J$ time functions $g_j(t)$, where $J$ cannot be greater than the number of displacements
$N$. $M$, $C$, $K$ and $F$ represent the mass, damping and stiffness matrices, and the
external force vector; $u$ represents the displacement, $\dot{u}$ stands for the velocity and $\ddot{u}$
represents the acceleration.

The number of dynamic degrees-of-freedom is equal to the number of lumped masses
in the system. Many publications advocate the elimination of all massless displace-
ments by static condensation prior to the solution of equation 3.35. The static con-
densation method reduces the number of dynamic equilibrium equations to solve;
however, it can significantly increase the density and the bandwidth of the condensed
stiffness matrix.

The fundamental mathematical method that is used to solve equation 3.35 is the
separation of variables. This approach assumes the solution can be expressed in the
following form:

$$
u(t) = \Phi Y(t)
$$

(3.36)

Where $\Phi$ is an $N$ by $L$ matrix containing $L$ spatial vectors which are not a function
of time, and $Y(t)$ is a vector containing $L$ functions of time. From equation 3.36 it
follows that:

$$
\dot{u}(t) = \Phi \dot{Y}(t)
$$

(3.37)

$$
\ddot{u}(t) = \Phi \ddot{Y}(t)
$$

(3.38)
Prior to solution, we require that the space functions satisfy the following mass and stiffness orthogonality conditions:

\[ \Phi^T M \Phi = I \]  
\[ \Phi^T K \Phi = \Omega^2 \]  \hspace{1cm} (3.39, 3.40)

Where \( I \) is a diagonal unit matrix and \( \Omega^2 \) is a diagonal matrix which may or may not contain all the natural frequencies. It should be noted that the fundamentals of mathematics place no restrictions on these vectors, other than the orthogonality properties.

After substitution of equations 3.36, 3.37 and 3.38 into equation 3.35 and the premultiplication by \( \Phi^T \), the following \( L \) equations is produced:

\[ I \ddot{Y}(t) + D \dot{Y}(t) + \Omega^2 = \sum_{j=1}^{J} p_j \cdot g_j(t) \]  \hspace{1cm} (3.41)

Where \( p_j = \Phi^T f_j \) and are defined as the modal participation factors for time function \( j \). The term \( p_{nj} \) is associated with the \( n-th \) mode. For all real structures the \( L \times L \) matrix \( D \) is not diagonal; however, in order to uncouple the modal equations it is necessary to assume that there is no coupling between the modes. Therefore, it is assumed to be diagonal with the modal damping terms defined by:

\[ D_{nn} = 2 \xi_n \omega_n \]  \hspace{1cm} (3.42)

Where \( \xi_n \) is defined as the damping ratio in the \( n-th \) mode. For more information on the construction of matrix \( D \) see Karoumi [37].

A typical uncoupled modal equation, for linear structural systems, has the following form:

\[ \ddot{y}_n(t) + 2 \xi_n \omega_n \dot{y}_n(t) + \omega_n^2 y_n(t) = \sum_{j=1}^{J} p_j \cdot g_j(t) \]  \hspace{1cm} (3.43)

3.6 Modeling in ABAQUS/BRIGADE

The models of the three structures studied in chapters 4 and 5 were made in the graphical interface of ABAQUS. ABAQUS/CAE is a user-friendly template divided by different modules. Each module is a step in the modelling process, presented in logical sequence. The first modules, described in section 3.6.1, give shape and material properties to the structure. After the model is defined, the analysis starts, described in section 3.6.2. The 3\textsuperscript{rd} section describes how the results are visualized.
and the last section explain the advantages of the BRIGADE pre-processor. For detailed information on the subject, one should see Hibbitt et al. [32] and [1].

3.6.1 Basic modules

The first module of ABAQUS is the Part module, where the different parts of the structure are drawn (possibly with the help of the Sketch module), without connections between each other. In the Property module, the sections and profiles are defined, as well as the properties and material behavior (elastic, plastic, thermal, acoustic or electric, between others). The Assembly module assembles the different parts of the structure, with stiff connections or with connectors defined in the Interaction module. In this module almost all the real structural connectors can be simulated from a vast interaction library.

The next level is the definition of the applied loads and the boundary conditions, both done with the Load module. All kind of mechanical, electric or acoustic boundary conditions and applied loads are available in the pre-defined library. The last basic definition is done in the Mesh module. Different mesh types can be applied in all the structure or in partitioned parts of it.

3.6.2 Analysis type

The analysis type is defined in the Step module. There are two basic procedures to determine the response of the structure: general analysis steps, which can be used for both linear or non-linear analysis, and linear perturbation steps, which can be used only to analyze linear problems. Loading conditions are defined differently for the two cases, and the time steps are different as well. Therefore, the results should be interpreted differently.

General static analysis

A general analysis step is one in which the effects of any non-linearities present in the model can be included. The starting condition for each general step is the ending condition from the last general step, with the state of the model evolving throughout the history of general analysis steps as it responds to the history of loading. The non-linearity of the real structure can arise from: material non-linearity, geometric non-linearity and boundary non-linearity.

The general analysis procedures in ABAQUS offer two approaches for controlling incrementation: automatic control or direct user control. In non-linear problems, the challenge is always to obtain a convergent solution in the least possible computational time. In these cases automatic control of the time increment is usually more efficient because ABAQUS can react to non-linear response that you cannot predict ahead of time.
CHAPTER 3. IMPORTANT THEORETICAL CONCEPTS

Linear eigenvalue analysis

The frequency extraction procedure is a particular linear perturbation procedure that performs eigenvalue extraction to calculate the natural frequencies and the corresponding mode shapes of a system. The linear perturbation response has no effect as the general analysis is continued. The step time of linear perturbation steps, which is taken arbitrarily to be a very small number, is never accumulated into the total time. The frequency extraction procedure uses eigenvalue techniques to extract the frequencies of the current system, and two different algorithms are available: the Lanczos method and the subspace iteration. The first is generally faster when a large number of eigenmodes is required for a system with many degrees-of-freedom. The subspace iteration method may be faster when only a few (less than 20) eigenmodes are needed.

3.6.3 Visualization of the results

The Visualization module provides graphical display of finite element models and results. It obtains model and result information from the output database and provides control over the information that is placed in the output database, by modifying the output requests in the Step module. Amongst others, it is possible to plot:

- Undeformed and deformed shape;

- Contours: the values of an analysis variable such as stress or strain at a specified step and frame of your analysis;

- Symbols: the magnitude and direction of a particular vector or tensor variable at a specified step and frame of your analysis;

- X-Y data;

- Time history, scale factor and harmonic animation;

- Paths through the model, with the respective values.

3.6.4 BRIGADE add-ons

ABAQUS supplement BRIGADE\(^1\) adds 3 new modules to the graphical interface. Two of them give the possibility of adding static live loads and load combinations, but were not used in this dissertation. The other one, though, was very useful for the studies undertaken. It added a pre-processor, that automatically calculated the amplitude functions for each nodes of the mesh, for a train passing any given speed or group of speeds. It is called Dynamic Live Load module and has 3 main options:

\(^1\)Supplement developed by Scanscot Technology, see http://www.scanscot.se/
3.6. MODELING IN ABAQUS/BRIGADE

Track Manager, Vehicle Manager and Dynamic Live Loads Manager. The track
definition, with the entrance point of the train in the bridge, the possibility of using
2 or only 1 rail (for 2D analysis), the delay of the rail loading (to simulate curves)
and the load direction, are set in the Track Manager. The Vehicle Manager has the
most common load models already defined but allows user defined load models. The
user can define any new train by adding each axle load and its distance to the first
axle. Finally, the Dynamic Live Loads Manager puts together the track and the
load model, for a speed or a span of speeds defined by the user. One can also choose
the analysis procedure: modal or direct time integration and the time period of the
trial. If Mode Superposition is chosen, the time increment, the damping (direct or
composite) and the number of eigenmodes to account for during the analysis can
be changed. On the other hand, if it was chosen to analyze the model with direct
time integration, the characteristics of the increment should the defined, and the
possibility of non-linear geometry is added.
Chapter 4

Theoretical Behavior of Simple Bridges

In this chapter, a theoretical analysis of two common railway bridges will be presented. One single span and one double span composite bridges from the Bothnia Line (the new Swedish HSR) will be used for the study. No comparison with experimental measurements will be made at this stage.

The study of the two bridges will not follow the same pattern. The dynamic behavior of a bridge is such a large subject, that it was chosen to present the results in different ways. Both studies will be mainly based on the FEM but, for the single span bridge, a comparison with the analytical solution for the Euler-Bernoulli beam will be performed.

The FE program used is ABAQUS with a pre-processing unit for the dynamic live loads: BRIGADE. The pre-processor will be used for the definition of the amplitude functions of the dynamic loads. To solve the numerical equations of the analytical solutions and find the maximum responses of the FEM matrices, the commercial software MATLAB is used (see MathWorks [44] and Ortigueira [47]).

4.1 Single span bridge

4.1.1 The bridge

The Banafjäl bridge is a 42 m long, simply supported bridge, which carries one ballasted track. The bridge is a composite structure, with an ordinary reinforced concrete deck supported by two steel beams, and has the following physical properties:

- Mass of the composite section, $m = 10700 \text{ kg/m}$;
- Density of ballast, $\rho_{\text{ballast}} = 2000 \text{ kg/m}^3$;
- Thickness of ballast, \( h_{\text{ballast}} = 0.6 \) m;
- Width of ballast, \( b_{\text{ballast}} = 6.2 \) m.

The composite cross-section was homogenized, so that the material properties of steel could be used for the calculations. After the homogenization, the following characteristic values were used on the model:

- Modulus of elasticity, \( E = 210 \cdot 10^9 \) Pa;
- Moment of inertia, \( I = 0.62 \) m\(^4\);
- Area, \( A = 0.57 \) m\(^2\);
- Linear mass, \( \mu = 1814 \) kg/m;
- Density, \( \rho = 31824.6 \) kg/m\(^3\).

### 4.1.2 FE model

The FE model of the bridge was developed using ABAQUS FE graphical interface, with the pre-processor BRIGADE. Information about the general usage of the programs is described in section 3.6.

The cross-section of the single span beam model was a generalized profile with the properties of the Banafjäl bridge. The supports were, initially, considered stiff to study its dynamic behavior and so that the results could be compared with the analytical solution. The first model is shown in figure 4.1, represented with stiff supports.

![Figure 4.1: FE model of the single span bridge.](image_url)
As can be seen in figure 4.1, only the permanent parts of the structure were modeled. International System units were used in the model definition and analysis. The damping was considered as direct damping (see Hibbitt et al. [32] for more information on the subject) and equal for the whole beam. The value chosen was $\xi = 0.5\%$, according to [16] and table 2.3.

The embankments were considered stiff enough to lock the longitudinal DOFs at the ends of the beam. Nevertheless, a study was made without restraining the longitudinal displacement, and the discrepancies in the results were negligible. Because the bridge is simply supported, the rotations at the ends were defined as free.

For the sensitivity study, the supports were replaced by springs connected to the soil, representing the foundations. The springs permitted free rotation and horizontal displacement of the nodes, and the stiffness was only noted for vertical displacement.

The dynamic loads were added in the Dynamic Live Load module of BRIGADE, where the HSLM-A1 (see section 3.12 and table 3.2) was already defined. It was decided to run only the HSLM-A1, because the objective of the study was a comparison of the responses for different support condition, and not a complete study of the structural behavior.

### 4.1.3 Frequency analysis and convergence study

The eigenfrequencies extraction was made using both the FE model and the analytical solution proposed by Fryba (described in 3.4.1). To solve the numerical equations, the commercial software MATLAB was used.

The exact circular bending frequencies, for mode $j$, of a simply supported Euler-Bernoulli beam are given by the equation:

$$w_j = \sqrt{\left(\frac{j\pi}{l}\right)^4 \frac{EI}{\mu}}$$

(4.1)

For the Banafjal bridge, the exact eigenfrequencies [Hz] for the first ten bending modes, using equation 4.1, were calculated as:

$$f_{exact} = \begin{bmatrix}
2.39 \\
9.54 \\
21.47 \\
38.17 \\
59.64 \\
85.88 \\
116.90 \\
152.68 \\
193.24 \\
238.57 \\
\end{bmatrix}$$

57
To find convergence in the responses, different parameter studies were made, using both FEM calculations and the Fryba analytical solution. The first study contemplates the number of modes needed to get convergence in the responses, for a single point load moving on the bridge. For the subsequent studies, the bridge was loaded with HSLM-A1 model proposed by the Eurocode [10].

**Number of modes**

Mode Superposition Method gives the possibility of using only a defined number of modes in the analysis. Yet, the accuracy of the response depends on the number of modes used. Therefore, a brief study on the number of modes needed to get convergent results was made.

Using the beam response to a single point load of 170 kN, moving along a simply supported Euler-Bernoulli beam, proposed by Fryba [28], and already described in section 3.4.1, the number of bending modes needed to get convergent results, was studied. To make this, the analytical responses were calculated, using an increasing number of modes. The study was undertaken from the moment the first axle of the load model enters the bridge, to the moment the last axle is at a distance of two spans from the structure.

The results for a single point load, moving on the beam at 250 km/h, are shown in figures 4.3, 4.4 and 4.5. The $x$-axis represent the longitudinal position along the beam, and the $y$-axis show the maximum magnitude of the response, in time. The normalized results against each maximum are shown in figure 4.2.

It is clear that the displacement converges to the correct values for a small number of modes and the bending moment converges, in a well behaved way, some modes after. The accelerations were only roughly approximated if less than 100 modes were considered. Its convergence has many fluctuations and higher modes have an important role in the final values, unlike the displacement and the bending moment.

![Figure 4.2: Maximum responses for the first 100 modes (left) and zoomed in the first 25 modes (right).](image)
4.1 SINGLE SPAN BRIDGE

Figure 4.3: Maximum displacement of the whole beam (left) and zoomed in mid-span (right), with increasing mode number as parameter.

Figure 4.4: Maximum bending moment of the whole beam (left) and zoomed in mid-span (right), with increasing mode number as parameter.

Figure 4.5: Maximum acceleration of the whole beam (left) and zoomed in mid-span (right), with increasing mode number as parameter.
Analyzing the convergence of the values, the number of modes needed to get a good approximation of the response, was chosen, as follows:

- Displacement: 5 modes;
- Bending moment: 25 modes;
- Acceleration: 32 modes.

Time step

The choice of the correct time step size is important to ensure that the complete response of the structure is captured by the solution and that the solution is stable and free from divergence. Usually, the smaller the time step the more accurate the solution. However, there are practical limits on how small the step can be, as more solution steps will be required for a smaller time step and the required computational time will increase accordingly. Therefore, a limit may be set by the time required for the solution. The size of the time step is also limited by the number of sets of results that can be physically stored.

If the time step is too large then much of the higher frequency response of the structure will be missed and the solution may not adequately represent the real behavior of the structure as shown in figure 4.6.

![Figure 4.6: Calculated response with a time step too large.](image)

The analytical responses for a set of point loads, using different time steps, was found, and the convergence of the solutions was studied. The point loads corresponded to the HSLM-A1 axle loads proposed by the Eurocode [10] and described in section 3.3. Three time steps were considered: 0.0002 s, 0.002 s and 0.02 s. The number of modes used was chosen depending on the response being studied and according to the earlier performed mode convergence study.
4.1. SINGLE SPAN BRIDGE

Figures 4.7 and 4.8 show very high responses for the same train speed. That speed corresponds to the resonant speed of the bridge for the HSLM-A1, and is the same for the displacement, acceleration and bending moment. The resonance response is usually dominated by the first frequency. Hence, an expression to determine the resonant speed for the bridge, is to notice that the frequency induced by the axles is obtained by the following relation:

\[ f = \frac{v}{\lambda} \]  \hspace{1cm} (4.2)

where \( \lambda \) is the distance between axles.

Thus:

\[ v = f \cdot \lambda = 2,39 \cdot 18 = 43.02 \text{ m/s} \approx 155 \text{ km/h} \]

it is interesting to notice that the discrepancies in the plots for the displacement and bending moment were almost negligible. That is due to the small influence of
higher modes of vibration. The lower modes were responsible for the large majority of the displacement and the bending moment. However, for accelerations, the dissimilarities between the 3 time steps are evident. This is explained by the influence of higher modes of vibration on the accelerations of the beam. These modes have higher frequency rates that were not fully captured by the larger time steps.

The responses, for the different time steps are shown in table 4.1. The train speed [km/h], at which those responses were achieved, is given in parenthesis.

<table>
<thead>
<tr>
<th>Time Step [s]</th>
<th>Displacement [cm]</th>
<th>Acceleration [m/s²]</th>
<th>Moment [MNm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>2.28 (155)</td>
<td>4.03 (155)</td>
<td>20.3 (155)</td>
</tr>
<tr>
<td>0.002</td>
<td>2.28 (155)</td>
<td>4.32 (155)</td>
<td>20.5 (155)</td>
</tr>
<tr>
<td>0.0002</td>
<td>2.28 (155)</td>
<td>4.38 (155)</td>
<td>20.5 (155)</td>
</tr>
</tbody>
</table>

After the analysis of the results, it was decided to proceed the studies using a time step of 0.02 s for the displacement and the bending moment. For the accelerations, a time step of 0.002 s was considered to be accurate enough, without increasing the computational time of the trials too much.

**Speed step**

To see how sensitive the dynamic responses peaks were to the speed step used, the analytical calculations of the Fryba solution were made with increments of 2.5 km/h and compared with the results using a speed step of 5 km/h. The span of the speeds was from 100 km/h to 300 km/h in both cases. Results can be seen in figures 4.9 and 4.10. Normalized amplitudes are compared in figure 4.11.

Examining the figures, the similarity between the plots of the displacement and the bending moment is obvious. Furthermore, one can notice that, for the displacement and bending moment, the influence of the smaller speed step is not important. However, for the accelerations, the differences between the two speed steps were evident. That can be explained, once more, by the influence of higher modes of vibration on the accelerations of the beam. As a result, higher frequencies are excited when the train runs at different speeds, and the influence of higher modes generate higher accelerations. Nevertheless, these fluctuations occur mainly for the higher speeds, and the main resonant speed of 155 km/h was not affected.

The maximum responses, for the different speed steps, were the same. Thus, one can conclude that the resonant speed is closer to a multiple of 5 km/h than 2.5 km/h and, therefore, it was decided to proceed the study with the speed step of 5 km/h.

**Mesh type**

The influence of the mesh type on eigenfrequencies was studied as well. For the FE model, four meshes were tested: Timoshenko beam elements of 1 and 2 m and
Figure 4.9: Maximum displacement (left) and bending moment (right) on the beam, with speed step as parameter.

Figure 4.10: Maximum acceleration on the beam, with speed step as parameter.

Figure 4.11: Maximum responses normalized, for speed steps of 5 km/h (left) and 2.5 km/h (right).
Euler-Bernoulli beam elements of 1 and 2 m. The values obtained were compared with the exact frequencies provided by the exact solution of an Euler-Bernoulli beam (see Frýba [28]).

Figure 4.12 shows the values [Hz] of the eigenfrequencies for the first ten bending modes, obtained using different elements and mesh sizes. The same frequencies are shown in table 4.1.

![Graph showing frequency vs mode for different beam elements and mesh sizes.](image)

**Figure 4.12:** First 10 bending frequencies (left) and zoomed in frequencies 7 to 10 (right) of the Banafjäl bridge, with mesh type as parameter.

**Table 4.2:** First 10 bending frequencies [Hz] of the Banafjäl bridge, with element type and size as parameters.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Exact</th>
<th>Timoshenko Beam 1 [m]</th>
<th>Timoshenko Beam 2 [m]</th>
<th>Euler-Bernoulli Beam 1 [m]</th>
<th>Euler-Bernoulli Beam 2 [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\textsuperscript{st}</td>
<td>2.39</td>
<td>2.36</td>
<td>2.36</td>
<td>2.39</td>
<td>2.39</td>
</tr>
<tr>
<td>2\textsuperscript{nd}</td>
<td>9.54</td>
<td>9.17</td>
<td>9.18</td>
<td>9.54</td>
<td>9.54</td>
</tr>
<tr>
<td>3\textsuperscript{rd}</td>
<td>21.47</td>
<td>19.73</td>
<td>19.75</td>
<td>21.47</td>
<td>21.47</td>
</tr>
<tr>
<td>4\textsuperscript{th}</td>
<td>38.17</td>
<td>33.20</td>
<td>33.23</td>
<td>38.17</td>
<td>38.17</td>
</tr>
<tr>
<td>5\textsuperscript{th}</td>
<td>59.64</td>
<td>48.80</td>
<td>48.79</td>
<td>59.64</td>
<td>59.65</td>
</tr>
<tr>
<td>6\textsuperscript{th}</td>
<td>85.88</td>
<td>65.89</td>
<td>65.74</td>
<td>85.89</td>
<td>85.92</td>
</tr>
<tr>
<td>7\textsuperscript{th}</td>
<td>116.90</td>
<td>84.00</td>
<td>83.51</td>
<td>116.90</td>
<td>116.98</td>
</tr>
<tr>
<td>8\textsuperscript{th}</td>
<td>152.68</td>
<td>102.77</td>
<td>101.67</td>
<td>152.70</td>
<td>152.86</td>
</tr>
<tr>
<td>9\textsuperscript{th}</td>
<td>193.24</td>
<td>121.93</td>
<td>120.67</td>
<td>193.27</td>
<td>193.59</td>
</tr>
<tr>
<td>10\textsuperscript{th}</td>
<td>238.57</td>
<td>142.58</td>
<td>137.77</td>
<td>238.62</td>
<td>239.23</td>
</tr>
</tbody>
</table>

After analyzing the frequencies, one can notice that the model with the Timoshenko elements provides solutions lower than the Euler-Bernoulli solution. As expected, the Euler-Bernoulli beam elements provide a solution very close to the exact analytical frequencies (also based on the Euler-Bernoulli theory), approaching them from higher values. For the model with the 1 m long Euler-Bernoulli beam elements, the precision of the results is greater than 0.1 Hz, for the 10\textsuperscript{th} mode of vibration. The models meshed with Timoshenko beam elements show a relation almost linear with
the mode number. On the other hand, the Euler-Bernoulli solution has a quadratic relation with the mode number, as can be seen from equation 4.1:

\[ w_j = \sqrt{\left( \frac{j\pi l}{l} \right)^4 \frac{EI}{\mu} = j^2 \cdot \text{constant}} \] (4.3)

Therefore, the difference between the Timoshenko models and the Euler-Bernoulli analytical solution increases very rapidly with the number of modes and is around 100 Hz for the 10th bending frequency. Comparing the mesh size for the same beam elements, the differences are only visible for higher modes, and can be considered negligible.

The degree of the interpolation functions has a great influence on the computational time needed to compute one trial. To run trials with the Euler-Bernoulli theory and the cubic interpolation functions, very fast processors are needed to get results in reasonable time. Hence, and because this study focus on the influence of the vertical support stiffness, it was decided to proceed the study meshing the model with Timoshenko beam elements of 1 m. It permitted to simulate the transverse shear deformation of the girders and lower the computational time, through the use of linear interpolations functions.

### 4.1.4 Comparison of the methods

To validate the FEM solution, the responses obtained from the FE model were compared with the analytical solution based on Fryba’s equations.

Figures 4.13 and 4.14 show the responses using the analytical and the FEM numerical solutions, with infinite support stiffness. The responses were calculated using the number of modes and the time step defined during the parametric study (section 4.1.3), both for the analytical solution and the FEM solution. The analytical solution was numerically solved in MATLAB using the same beam properties of the FE model.

The similarities are clear, even though the FE model uses Timoshenko beam theory, and the analytical solution uses the Euler-Bernoulli principles. The plots on the left show this comparison for the case with crawling speed, and the values are very close, both in magnitude and time. For the train running at resonant speed, the agreement is, once more, very good, both in magnitude and time. However, one may notice that the analytical solution shows more damping, and the response seems to lag behind, for the free vibration, when the train has left the beam. That instant can be acknowledged in the displacement plot, when its magnitude becomes a reflection in the x-axis. The very good agreement between the FEM and the analytical method can also be concluded from the observation of figure 4.15, where the maximum displacement for the different train speeds is shown.
CHAPTER 4. THEORETICAL BEHAVIOR OF SIMPLE BRIDGES

Figure 4.13: Vertical displacement at mid-span, for HSLM-A1 running at crawling (left) and resonant (right) speed.

Figure 4.14: Acceleration at mid-span, for HSLM-A1 running at crawling (left) and resonant (right) speed.

Figure 4.15: Maximum displacement on the beam, for different train speeds.
4.1.5 Resonance effects

The influence of the dynamic interaction is clear when the resonant responses (train moving at 155 km/h) are compared with the responses with the train running at crawling speed (5 km/h). The trials were conducted until the train was one span away from the bridge, to allow the identification of the free vibration.

Plots of the displacement, bending moment and acceleration on the bridge, both in time and space, are shown in figures 4.16 to 4.21.

Analyzing figure 4.16, one can notice that the maximum displacement increases from 1 cm to, approximately, 2.5 cm, when the resonance is reached. Furthermore, it is interesting to note from figure 4.17 that, although for the first 5 s the displacements were all negative, this behavior changed for the next seconds, and values up to 1 cm were obtained for the upwards displacement, when the train has already passed the bridge.

The plots for the bending moment show almost the same relation of the displacement, as expected. The maximum bending moment occurs always at mid-span, and the maximum negative bending moment is obtained in the final seconds, when the train has already left the bridge. The values for the positive bending moment (corresponding to the downwards displacement) rise up to $20 \times 10^9$ Nm. For the quasi static case, that value is around $8 \times 10^9$ Nm.

The accelerations are, on the other hand, far more complex. Yet, the resonance is clear, with the magnitudes rising very rapidly, until the train leaves the bridge. At the resonant speed, the acceleration of the bridge deck rises up to $4 \text{ m/s}^2$, and it would be greater for longer trains. This peak acceleration, around $0.4g$, can cause destabilization of the ballast, as it is higher than the valued recommended in the ERRI D214 reports [21]: $a_{ult}=3.5 \text{ m/s}^2$ for bridges with ballast.

One can also notice that the maximum acceleration is not always registered at mid-span, as might be expected. In subsequent studies, namely when the acceleration sensitivity to the vertical displacement is studied, this phenomenon will be further discussed.

4.1.6 Sensitivity to vertical support stiffness

In the following, the effect of the support stiffness on the dynamic behavior of the simply supported bridge is studied. The vertical support stiffness was varied according to the values found to provide changes in the behavior of the structure: from $1 \times 10^8$ N/m, for soft clays, to $1 \times 10^{11}$ N/m, for foundation on solid bedrock or pile groups. To achieve that effect on the FE model, the stiff boundary conditions at the end of the beam were replaced by non-homogeneous boundary condition, representing the soil-foundation interaction. The stiffness of the springs was then varied within the chosen interval ($1 \times 10^8$ to $1 \times 10^{11}$ N/m).

To see how the vertical stiffness of the supports affected the dynamic behavior of
Figure 4.16: Vertical displacement, over time and space, for HSLM-A1 running at crawling speed (5 km/h).

Figure 4.17: Vertical displacement, over time and space, for HSLM-A1 running at resonant speed (155 km/h).
4.1. SINGLE SPAN BRIDGE

Figure 4.18: Bending moment, over time and space, for HSLM-A1 running at crawling speed (5 km/h).

Figure 4.19: Bending moment, over time and space, for HSLM-A1 running at resonant speed (155 km/h).
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Figure 4.20: Vertical acceleration, over time and space, for HSLM-A1 running at crawling speed (5 km/h).

Figure 4.21: Vertical acceleration, over time and space, for HSLM-A1 running at resonant speed (155 km/h).
4.1. SINGLE SPAN BRIDGE

A bridge crossed by a HST, numerous trials were made and are presented in the following.

Eigenfrequencies

With the decrease of the vertical stiffness at the end supports, the bending frequencies started to decrease. Table 4.3 shows the effect of the supports stiffness on the eigenfrequencies. The last column shows the ratio between the frequencies with the less stiff model and with the stiffest model. The same relations are also shown in figure 4.22, with the normalized frequencies in the left plot.

Table 4.3: Effect of the end support stiffness in the frequencies [Hz] of the first ten bending modes of vibration of the Banafjäl bridge.

<table>
<thead>
<tr>
<th>Support stiffness [N/m]</th>
<th>$1 \cdot 10^{11}$</th>
<th>$1 \cdot 10^{10}$</th>
<th>$5 \cdot 10^{9}$</th>
<th>$1 \cdot 10^{9}$</th>
<th>$5 \cdot 10^{8}$</th>
<th>$1 \cdot 10^{8}$</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st frequency</td>
<td>2.36</td>
<td>2.35</td>
<td>2.35</td>
<td>2.28</td>
<td>2.21</td>
<td>1.79</td>
<td>76%</td>
</tr>
<tr>
<td>2nd frequency</td>
<td>9.16</td>
<td>9.05</td>
<td>8.94</td>
<td>8.07</td>
<td>7.16</td>
<td>4.14</td>
<td>45%</td>
</tr>
<tr>
<td>3rd frequency</td>
<td>19.68</td>
<td>19.22</td>
<td>18.71</td>
<td>15.00</td>
<td>12.28</td>
<td>7.44</td>
<td>38%</td>
</tr>
<tr>
<td>4th frequency</td>
<td>33.07</td>
<td>31.83</td>
<td>30.32</td>
<td>22.01</td>
<td>18.48</td>
<td>14.85</td>
<td>45%</td>
</tr>
<tr>
<td>5th frequency</td>
<td>48.54</td>
<td>45.85</td>
<td>42.57</td>
<td>30.98</td>
<td>28.38</td>
<td>26.43</td>
<td>54%</td>
</tr>
<tr>
<td>6th frequency</td>
<td>65.45</td>
<td>60.48</td>
<td>54.85</td>
<td>43.38</td>
<td>41.83</td>
<td>40.73</td>
<td>62%</td>
</tr>
<tr>
<td>7th frequency</td>
<td>83.33</td>
<td>75.12</td>
<td>67.68</td>
<td>58.53</td>
<td>57.58</td>
<td>56.88</td>
<td>68%</td>
</tr>
<tr>
<td>8th frequency</td>
<td>101.83</td>
<td>89.76</td>
<td>81.93</td>
<td>75.38</td>
<td>74.75</td>
<td>74.28</td>
<td>73%</td>
</tr>
<tr>
<td>9th frequency</td>
<td>120.69</td>
<td>104.65</td>
<td>97.82</td>
<td>93.26</td>
<td>92.82</td>
<td>92.48</td>
<td>77%</td>
</tr>
<tr>
<td>10th frequency</td>
<td>141.15</td>
<td>139.70</td>
<td>120.28</td>
<td>114.97</td>
<td>111.74</td>
<td>111.42</td>
<td>79%</td>
</tr>
</tbody>
</table>

Figure 4.22: Effect of the end supports vertical stiffness in the bending frequencies of the first ten bending modes of vibration of the Banafjäl bridge.

The plot on the left of figure 4.22 shows the eigenfrequencies plotted against the end supports stiffness. The values are normalized against the frequency of the stiff model. The plot to the right shows the different modes on the $x$-axis, and the various colors represent each vertical stiffness.
From the analysis of the plots, one can notice that there is hardly any change in the first three eigenfrequencies, for stiffness higher than $5 \cdot 10^9$ N/m. However, the $3^{rd}$ and $4^{th}$ eigenfrequencies change very much between support stiffness of $5 \cdot 10^9$ N/m and $1 \cdot 10^8$ N/m. The same type of behavior is observed for frequencies above the $4^{th}$: barely altered for the higher stiffness, and with greater variations for stiffness lower than $5 \cdot 10^9$ N/m. On the other hand, the first two modes shows a different behavior, with large changes happening only when the stiffness decreases from $5 \cdot 10^8$.

**Resonant speed**

As seen before, the resonant speed is a function mainly of the $1^{st}$ eigenfrequency of the structure. Therefore, with the highest variations of the $1^{st}$ eigenfrequency starting for vertical stiffness lower than $1 \cdot 10^9$ N/m, the highest variations on the resonant speed were expected for lower stiffness. Table 4.4 shows the different resonant speeds, obtained not from the FE model but from equation 4.2.

<table>
<thead>
<tr>
<th>Support stiffness [N/m]</th>
<th>$1^{st}$ Frequency [Hz]</th>
<th>Resonant speed [km/h]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\infty$</td>
<td>2.36</td>
<td>153</td>
</tr>
<tr>
<td>$1 \cdot 10^{11}$</td>
<td>2.36</td>
<td>153</td>
</tr>
<tr>
<td>$1 \cdot 10^{10}$</td>
<td>2.35</td>
<td>152</td>
</tr>
<tr>
<td>$5 \cdot 10^9$</td>
<td>2.34</td>
<td>152</td>
</tr>
<tr>
<td>$1 \cdot 10^9$</td>
<td>2.28</td>
<td>148</td>
</tr>
<tr>
<td>$5 \cdot 10^8$</td>
<td>2.21</td>
<td>143</td>
</tr>
<tr>
<td>$1 \cdot 10^8$</td>
<td>1.78</td>
<td>116</td>
</tr>
</tbody>
</table>

The results of table 4.4 can be compared with the plots of the responses, for different train speeds. Figures 4.23 to 4.25 show the maximum displacement of the beam, at mid-span. The trials were run in ABAQUS/BRIGADE, the number of modes used was 5 and the time step 0.02 s, in accordance to the study on mode convergence (section 4.1.3). The frequency for the $5^{th}$ mode of vibration is, approximately, 50 Hz, and its period around 0.02 Hz. Hence, the time step is small enough to capture frequencies lower than that. The speed step is 5 km/h, since it gives acceptable results, without increasing the computational time too much.

The trials were divided in groups of two, to make the presentation of the results clear. The response for the model with infinite support stiffness is shown in all figures, for the purpose of comparison. The resonant speeds obtained from the trials are shown in table 4.5, and compared the ones obtained with expression 4.2.

The first similarity one can see is between the resonant speed obtained from equation 4.2 and from the FEM solution. The match is very good for all the cases, within the reach of the speed step. From the analysis of figure 4.23, it can also be seen that the displacement is hardly affected by the decrease of the vertical support stiffness, from $\infty$ to $1 \cdot 10^{10}$. Only a slight decrease in the maximum displacement is registered,
4.1. SINGLE SPAN BRIDGE

Figure 4.23: Maximum downwards (left) and upwards (right) displacement of the beam, as function of the train speed.

Figure 4.24: Maximum downwards (left) and upwards (right) displacement of the beam, as function of the train speed.

Figure 4.25: Maximum downwards (left) and upwards (right) displacement of the beam, as function of the train speed.
which will be analyzed in the next section.

For lower values of stiffness, major changes start to appear, not only in the resonant speed, but also in the magnitude of the response. Changes in the resonant speed were expected. However, the decrease of the maximum displacement on resonance, for lower vertical stiffness, was an interesting finding. Moreover, the resonance is not as clear as it appears for higher stiffness. Instead, for a stiffness of $5 \cdot 10^8$ N/m, the relation between the displacement in resonance and the displacement for speeds up to 200 km/h decreases significantly. In fact, almost no resonance effects appear on displacements.

### Displacement

To compare the displacements, with different support stiffness, a point at mid-span was studied. The bridge was loaded with the HSLM-A1 train running at resonant speed. The resonant speed of each trial depended on the vertical stiffness of the supports and is shown in table 4.5. The time step used was 0.02 s and 5 bending modes were included, in accordance to the convergence study (section 4.1.3). Since there was one longitudinal mode between the first 5 bending modes, the trials were run with a total of 6 modes.

The influence of the vertical stiffness of the supports on the displacements due to the HSLM-A1, running at crawling speed (5 km/h), is shown in figures 4.26, 4.27 and 4.28. The results for resonant speed are shown in figures 4.29 and 4.30.

From the analysis of figure 4.26, it is clear that, as suggested from figure 4.23, the maximum displacement at mid-span, with the train running at crawling speed, is only slightly affected with the decrease of vertical support stiffness.

Figure 4.27 shows different results. For the displacement with the train running at crawling speed, although no major changes take place for lower stiffness, one can see that, for a stiffness of $1 \cdot 10^9$ N/m, the vibrations were not only due to bending modes, but also to rigid body movements. These rigid body movements were produced by the vibration of the whole bridge over the support springs. However, for the resonant response, the displacement decreases with the stiffness, as it was already discussed.
Figure 4.26: Displacement at mid-span, for HSLM-A1 running at 5 km/h. Whole time span (left) and zoomed in the high frequency vibrations (right).

Figure 4.27: Displacement at mid-span, for HSLM-A1 running at 5 km/h. Whole time span (left) and zoomed in the high frequency vibrations (right).

Figure 4.28: Displacement at mid-span, for HSLM-A1 running at 5 km/h. Whole time span (left) and zoomed in the high frequency vibrations (right).
when analyzing the speed trials (previous section). The plot on the right, from figure 4.29, shows some decrease on the displacement for the $5 \cdot 10^9$ N/m stiffness and a large reduction for $1 \cdot 10^9$ N/m. For this last case, one can also notice the lower frequency of the response, due to the lower speed of the load model (148 km/h, see table 4.5).

When the stiffness decreases below $1 \cdot 10^9$ N/m, the displacement at crawling speed increases very much, with a greater contribution of modes with higher frequencies, as can be seen in 4.28. The maximum static displacement is almost 2 times greater for a vertical support stiffness of $1 \cdot 10^8$ N/m than for infinite stiff supports.

What happens at resonant speed is contradictory (figures 4.29 and 4.30). On one hand, when the vertical support stiffness decreases to $1 \cdot 10^8$ N/m, the displacement rises more than double, when compared to the stiff support model. On the other hand, the displacement, for a stiffness of $5 \cdot 10^8$ N/m, is less than the displacement with stiff supports. In fact, this is in accordance with figure 4.25, where the resonant is almost impossible to identify, for a stiffness of $5 \cdot 10^8$ N/m. Table 4.6 summarizes the results.

<table>
<thead>
<tr>
<th>Support stiffness [N/m]</th>
<th>Maximum displacement [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\infty$</td>
<td>Crawling speed</td>
</tr>
<tr>
<td>$1 \cdot 10^{11}$</td>
<td>0.99</td>
</tr>
<tr>
<td>$1 \cdot 10^{10}$</td>
<td>0.99</td>
</tr>
<tr>
<td>$5 \cdot 10^9$</td>
<td>1.00</td>
</tr>
<tr>
<td>$1 \cdot 10^9$</td>
<td>1.08</td>
</tr>
<tr>
<td>$5 \cdot 10^8$</td>
<td>1.15</td>
</tr>
<tr>
<td>$1 \cdot 10^8$</td>
<td>1.83</td>
</tr>
</tbody>
</table>

Figure 4.29: Vertical displacement at mid-span, for HSLM-A1 running at resonant speed.
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As it was mentioned previously and can be seen in figure 4.21, the maximum acceleration is not always found at mid-span. Hence, the accelerations for different values of support stiffness were studied at each node along the bridge model. The convergence study (section 4.1.3) shown that 32 bending modes and a time step of 0.002 s were need to get reliable results. However, there were 30 longitudinal modes between the first 32 bending modes and, therefore, to ensure the convergence of the accelerations, a total of 62 modes were included for the FE trials.

Unfortunately, because such a small time step increases the computational time very much, it was not possible to present plots with the maximum acceleration for different train speeds. Hence, and based on figure 4.11 and the studies undertaken in the model with stiff supports, it was assumed that the resonant speed for the acceleration was the same as for the displacement. Hereby, the trials were made with the train running at a speed that excited the first frequency of the structure. That speed is shown in table 4.5.

It was seen from figure 4.21 that the maximum acceleration is not always found at mid-span. Hereby, it was chosen to plot the maximum accelerations, in time, for each one of the nodes along the bridge model. Figures 4.31, 4.32 and 4.33 show the variation of the maximum acceleration. Table 4.7 summarizes the maximum acceleration obtained for different stiffness.

From the analysis of figure 4.31, it can be seen that, for vertical support stiffness lower than $1 \cdot 10^{11}$ N/m, the higher accelerations were found at the supports. This
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Figure 4.31: Maximum downwards (left) and upwards (right) acceleration, for HSLM-A1 running at resonant speed.

Figure 4.32: Maximum downwards (left) and upwards (right) acceleration, for HSLM-A1 running at resonant speed.

Figure 4.33: Maximum downwards (left) and upwards (right) acceleration, for HSLM-A1 running at resonant speed.
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is due to the oscillation of the support springs of the FE model. The acceleration registered for the model with infinite stiffness and for the model with stiffness of \(1 \cdot 10^{11} \text{ N/m}\), were similar, growing from the supports to mid-span, but usually less than the limit of 3.5 \(\text{m/s}^2\) proposed in the ERRI reports [21]. For stiffness of \(1 \cdot 10^{10} \text{ N/m}\) and \(5 \cdot 10^9 \text{ N/m}\), the maximum accelerations were greater, exceeding the limit proposed for bridges, even without ballast (5 \(\text{m/s}^2\)). For lower values of stiffness, as shown in figure 4.32 and 4.33, the acceleration in the supports were greater than gravity, which may definitely cause destabilization of the ballast and the rails, if proved that these magnitude of accelerations are possible to appear in reality.

<table>
<thead>
<tr>
<th>Support stiffness [N/m]</th>
<th>Maximum acceleration [m/s²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>∞</td>
<td>3.22</td>
</tr>
<tr>
<td>(1 \cdot 10^{11})</td>
<td>3.58</td>
</tr>
<tr>
<td>(1 \cdot 10^{10})</td>
<td>6.60</td>
</tr>
<tr>
<td>(5 \cdot 10^9)</td>
<td>7.75</td>
</tr>
<tr>
<td>(1 \cdot 10^9)</td>
<td>10.2</td>
</tr>
<tr>
<td>(5 \cdot 10^8)</td>
<td>10.9</td>
</tr>
<tr>
<td>(1 \cdot 10^8)</td>
<td>10.4</td>
</tr>
</tbody>
</table>

4.1.7 Discussion of the results

The study on the single span bridge started with a convergence study on important parameters. The results show, on one hand, that accurate values for the displacement could be found with only 5 bending modes and a time step of 0.02 s. On the other hand, 32 bending modes and a time step of 0.002 were essential to get convergence for accelerations. The reason for this is that the displacement is dominated by the lower modes, while the maximum accelerations have greater contributions from higher modes.

The study on the influence of the element type and size provided interesting conclusions. Results using Timoshenko beam theory were compared with the solutions provided by the Euler-Bernoulli theory. Timoshenko beam elements allow for transverse shear deformations, can be subjected to large axial strains and use linear interpolations functions. Euler-Bernoulli beams do not allow for transverse shear deformation, use a consistent mass formulation and cubic interpolation functions, which makes them reasonably accurate for cases involving distributed loading along the beam. However, the cubic interpolation functions greatly increase the computational time. From the study, it was seen that Timoshenko beam elements of 1 m provided accurate results, for the bridge in question, reducing the computational time. The accuracy of the FEM solutions was found to be very good, when compared to the analytical solutions for the Euler-Bernoulli beam, even though the FE model was meshed with Timoshenko beam elements.
The study on the influence of the vertical support stiffness provided the most important result: it is mandatory to model the stiffness of the supports if realistic predictions of the dynamic behavior are to be made. Large changes are obtained in the eigenfrequencies and mode shapes, resonant speeds of the trains and magnitude of displacements and accelerations.

For a vertical stiffness of $5 \times 10^8$ N/m in the supports, resonance effects are not observed. In fact, the maximum displacement at resonant speed, for stiffness of $5 \times 10^8$ and $1 \times 10^9$ N/m, is lower than the displacement with stiff supports. This fact was surprising and its reasons should be deeply investigated in future studies.

In what concerns the maximum accelerations, they were always found at the supports, for models with stiffness lower than $1 \times 10^{11}$ N/m. This is due to the oscillation of the support springs in the FE model, but should be validated with further research. It is uncertain if this behavior happens in real structures.

4.2 Double span bridge

4.2.1 The bridge

The bridge over Lögdeälv is a two span composite bridge along the Bothnia Line. The concrete deck is supported by two steel girders, connected to each other by transversal steel beams, weighting 0.2 N/m. Each of the girders is composed of 4 different beam cross-sections, with variations in cross-sectional area and moment of inertia. A sketch of the bridge is shown in figure 4.34 and the properties of the girders are presented in tables 4.8 and 4.9. The weight of the ballast can be considered roughly constant with the value of 30 N/m and the weight of the concrete slab is 30.8 N/m.

<table>
<thead>
<tr>
<th>Beam type</th>
<th>$g_{\text{beam}}$ [N/m]</th>
<th>$g_{\text{transc}}$ [N/m]</th>
<th>$g_{\text{conc}}$ [N/m]</th>
<th>$g_{\text{bal}}$ [N/m]</th>
<th>$g_{\text{tot}}$ [N/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.2</td>
<td>0.2</td>
<td>30.8</td>
<td>30.0</td>
<td>69.2</td>
</tr>
<tr>
<td>2</td>
<td>10.0</td>
<td>0.2</td>
<td>30.8</td>
<td>30.0</td>
<td>71.0</td>
</tr>
<tr>
<td>3</td>
<td>9.7</td>
<td>0.2</td>
<td>30.8</td>
<td>30.0</td>
<td>70.7</td>
</tr>
<tr>
<td>4</td>
<td>16.0</td>
<td>0.2</td>
<td>30.8</td>
<td>30.0</td>
<td>77.0</td>
</tr>
</tbody>
</table>

Table 4.8: The weight of the Lögdeälv bridge.

<table>
<thead>
<tr>
<th>Beam type</th>
<th>$\rho$ [kg/m$^3$]</th>
<th>$E$ [Pa]</th>
<th>$A$ [m$^2$]</th>
<th>$I$ [m$^4$]</th>
<th>$2 \cdot A$ [m$^2$]</th>
<th>$2 \cdot I$ [m$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>34343</td>
<td>$2.10 \times 10^{11}$</td>
<td>0.205</td>
<td>0.177</td>
<td>0.411</td>
<td>0.355</td>
</tr>
<tr>
<td>2</td>
<td>31223</td>
<td>$2.10 \times 10^{11}$</td>
<td>0.232</td>
<td>0.220</td>
<td>0.464</td>
<td>0.440</td>
</tr>
<tr>
<td>3</td>
<td>32059</td>
<td>$2.10 \times 10^{11}$</td>
<td>0.225</td>
<td>0.209</td>
<td>0.450</td>
<td>0.419</td>
</tr>
<tr>
<td>4</td>
<td>25509</td>
<td>$2.10 \times 10^{11}$</td>
<td>0.308</td>
<td>0.312</td>
<td>0.615</td>
<td>0.624</td>
</tr>
</tbody>
</table>

Table 4.9: The material and sectional properties for the Lögdeälv bridge.
4.2. DOUBLE SPAN BRIDGE

The bridge was modeled as a double span beam, supported at the ends and at the middle node. The bending stiffness of the beam was assumed to be twice the bending stiffness of one girder, the same happening with the area. Four different beam cross-sections were defined, with changes in the moment of inertia and area, in accordance with table 4.9 (see also figure 4.35).

The model of the bridge is shown in figure 4.35, with different colors representing different type of girders. The model shows the supports, later replaced by springs.

The stiffness of the supports was made variable with the substitution of the stiff boundary conditions for spring connected to the soil, representing the foundations of the structure. The springs, located at the ends of the beam and mid-span, permitted free rotation and horizontal displacement of the nodes, and the stiffness was only noted for vertical displacement.

The effective axial stiffness of the main column was neglected (it was believed to be
much greater than the values in question) and only the support stiffness, i.e., the stiffness of the foundation and soil beneath, was modeled. The deformation of the bearing was neglected as well. The bending stiffness of the column was considered very small and, therefore, the middle node was free to move longitudinally. The rotation was also permitted at the center of the beam, because the Lögdeälv bridge had a bearing over the main column.

To simulate the transverse shear deformation of the girders, the Timoshenko beam elements were used, with linear interpolations functions (more information can be found in section 3.4.2).

The damping considered was, according to [16] and table 2.3, $\xi = 0.5\%$. This values was the same for every part of the structure, and was defined as direct damping (see Hibbitt et al. [32] for more information on the subject).

### 4.2.3 Convergence study

To achieve convergent results, different parametric studies were made using the FE model developed in ABAQUS/BRIGADE. All the trials were run using the HSLM-A1 and the responses registered until the load model was, at least, one span away from the bridge, to ensure the high accelerations produced immediately after the train has left the bridge were included.

**Mesh elements**

The size of the mesh elements needed to get convergence for displacement and acceleration was studied, using ten vibrating modes. Since the eigenfrequency of the 10th mode was, approximately, 33 Hz, a time step of 0.02 s was considered small enough to get accurate results and was, therefore, used. Figures 4.36 and 4.37 show the responses as functions of train speed, with element length as parameter.

![Figure 4.36: Maximum upwards (left) and downwards (right) displacement as function of train speed, with element length as parameter.](image_url)
4.2. DOUBLE SPAN BRIDGE

Figure 4.37: Maximum upwards (left) and downwards (right) acceleration as function of train speed, with element length as parameter.

Table 4.10: Maximum responses, with element length as parameter.

<table>
<thead>
<tr>
<th>Mesh size [m]</th>
<th>Maximum displacement [cm]</th>
<th>Maximum acceleration [m/s²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.05</td>
<td>1.53</td>
</tr>
<tr>
<td>2</td>
<td>2.04</td>
<td>1.53</td>
</tr>
<tr>
<td>4</td>
<td>2.05</td>
<td>1.55</td>
</tr>
</tbody>
</table>

Figure 4.38: Eigenfrequencies, with element length as parameter.
Table 4.10 summarized the results from figures 4.36 and 4.37, showing the maximum displacement and acceleration for the different trials. The influence of the element size in the eigenfrequencies of the first ten modes was studied and is shown in figure 4.38. The differences are negligible.

From the analysis of the results, one can see that the displacement almost does not show any sensitivity to the mesh size, and the changes in the acceleration were less than to 0.1 m/s². The influence on the eigenfrequencies is negligible as well. Thus, it was decided to proceed the studies with an element length of 2 m.

**Time step and number of modes**

In contrast to what happened in the single span bridge study (section 4.1), the number of modes and the time step convergence were studied together. The FE solution uses Timoshenko beam elements of 2 m and a speed step of 5 km/h was used. Three trials were run: 0.02 s and 10 modes, 0.01 s and 25 modes, 0.005 s and 40 modes. The time step was chosen so that it would be less than the period of the highest mode included.

The results are shown in figure 4.39 to 4.40 and table 4.11.

![Figure 4.39: Maximum upwards (left) and downwards (right) displacement as function of train speed, with time step and number of modes as parameter.](image)

<table>
<thead>
<tr>
<th>Trial number</th>
<th>Maximum displacement [cm]</th>
<th>Maximum acceleration [m/s²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.05</td>
<td>1.55</td>
</tr>
<tr>
<td>2</td>
<td>2.06</td>
<td>1.56</td>
</tr>
<tr>
<td>3</td>
<td>2.06</td>
<td>1.56</td>
</tr>
</tbody>
</table>

One can notice that the time step and the number of modes does not affect the maximum displacement, assuring the convergence for the first trial. Therefore, for
studying the maximum displacement of the bridge, 10 modes and a time step of 0.02 s were used.

For the acceleration, the differences between trial 2 and 3 were not large, so it was considered that the response for trial 3 were close enough to the exact. Hence, for the subsequent studies concerning acceleration, a time step of 0.005 s was used and 40 modes were considered.

**Speed step**

From the time step study, one can notice that the peaks of the responses were not obtained, for a speed step of 5 km/h. Hence, trials with a speed step of only 1 km/h were made, only for the speeds around the resonant peaks. The same three trials were run: 0.02 s and 10 modes, 0.01 s and 25 modes, 0.005 s and 40 modes. The structure responses are shown in figures 4.41 and 4.42 and table 4.12.

Figure 4.40: Maximum upwards (left) and downwards (right) acceleration as function of train speed, with time step and number of modes as parameter.

Figure 4.41: Maximum upwards (left) and downwards (right) displacement as function of train speed, with time step and number of modes as parameter.
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Figure 4.42: Maximum upwards (left) and downwards (right) acceleration as function of train speed, with time step and number of modes as parameter.

Table 4.12: Maximum responses, with speed step as parameter.

<table>
<thead>
<tr>
<th>Speed step</th>
<th>Trial number</th>
<th>Maximum displacement [cm]</th>
<th>Maximum acceleration [m/s²]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Downwards</td>
<td>Upwards</td>
</tr>
<tr>
<td>5 km/h</td>
<td>1</td>
<td>2.05</td>
<td>1.55</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.06</td>
<td>1.56</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2.06</td>
<td>1.56</td>
</tr>
<tr>
<td>1 km/h</td>
<td>1</td>
<td>2.34</td>
<td>1.84</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.35</td>
<td>1.84</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2.35</td>
<td>1.84</td>
</tr>
</tbody>
</table>

It can be seen that both the displacement and the acceleration are affected by the speed step. Hence, it was decided, for subsequent studies, to run trials with 5 km/h for the whole range of speeds, and then analyze the resonant areas with a smaller speed step of 1 km/h.

4.2.4 Sensitivity to vertical support stiffness

After the analysis of the results provided by the study of the single span bridge, it was decided to perform the sensitivity study on the double span bridge only for stiffness varying between \(5 \cdot 10^{10}\) and \(5 \cdot 10^8\) N/m, since this interval of values provided very interesting results in the first study. Therefore, it was settled to run 9 trials on the bridge. Each trial is identified by the case number, and the cases are defined in table 4.13.

Table 4.13 is organized as a matrix. For instance, case 4 has a vertical stiffness of \(5 \cdot 10^9\) N/m for the main column and \(5 \cdot 10^{10}\) N/m for the end supports. The model with infinite stiffness both in the end supports and in the center column is referred to as case 0.
4.2. DOUBLE SPAN BRIDGE

Table 4.13: Definition of the cases for the study.

<table>
<thead>
<tr>
<th>Center column stiffness [N/m]</th>
<th>End supports stiffness [N/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 \cdot 10^{10}</td>
<td>Case 1 Case 2 Case 3</td>
</tr>
<tr>
<td>5 \cdot 10^{9}</td>
<td>Case 4 Case 5 Case 6</td>
</tr>
<tr>
<td>5 \cdot 10^{8}</td>
<td>Case 7 Case 8 Case 9</td>
</tr>
</tbody>
</table>

Eigenfrequencies

With the element size of 2 m, the influence of the support stiffness on the eigenfrequencies was studied. Table 4.14 shows the frequencies for the FE trials with different supports conditions (cases 0 to 9). The same information is shown in figure 4.43.

Table 4.14: Effect of the vertical support stiffness in the frequencies [Hz] of the first ten bending modes of vibration of the Lögdeälv bridge.

<table>
<thead>
<tr>
<th>Case</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
<th>7th</th>
<th>8th</th>
<th>9th</th>
<th>10th</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.11</td>
<td>3.43</td>
<td>8.16</td>
<td>10.45</td>
<td>15.51</td>
<td>15.51</td>
<td>17.72</td>
<td>20.59</td>
<td>30.16</td>
<td>33.33</td>
</tr>
<tr>
<td>1</td>
<td>2.12</td>
<td>3.49</td>
<td>8.41</td>
<td>10.93</td>
<td>15.51</td>
<td>18.93</td>
<td>22.27</td>
<td>29.84</td>
<td>33.74</td>
<td>37.52</td>
</tr>
<tr>
<td>2</td>
<td>2.12</td>
<td>3.44</td>
<td>8.41</td>
<td>10.38</td>
<td>15.51</td>
<td>18.93</td>
<td>19.63</td>
<td>29.84</td>
<td>30.52</td>
<td>33.74</td>
</tr>
<tr>
<td>3</td>
<td>2.12</td>
<td>2.96</td>
<td>6.86</td>
<td>8.41</td>
<td>14.14</td>
<td>15.51</td>
<td>18.93</td>
<td>26.36</td>
<td>29.84</td>
<td>33.74</td>
</tr>
<tr>
<td>4</td>
<td>2.12</td>
<td>3.48</td>
<td>8.34</td>
<td>10.81</td>
<td>15.51</td>
<td>18.58</td>
<td>21.8</td>
<td>29.84</td>
<td>32.59</td>
<td>36.13</td>
</tr>
<tr>
<td>5</td>
<td>2.12</td>
<td>3.43</td>
<td>8.34</td>
<td>10.29</td>
<td>15.51</td>
<td>18.58</td>
<td>19.34</td>
<td>29.79</td>
<td>29.84</td>
<td>32.59</td>
</tr>
<tr>
<td>6</td>
<td>2.12</td>
<td>2.96</td>
<td>6.83</td>
<td>8.34</td>
<td>13.96</td>
<td>15.51</td>
<td>18.58</td>
<td>25.69</td>
<td>29.84</td>
<td>32.59</td>
</tr>
<tr>
<td>7</td>
<td>2.08</td>
<td>3.37</td>
<td>7.63</td>
<td>9.59</td>
<td>15.05</td>
<td>15.51</td>
<td>17.32</td>
<td>25.02</td>
<td>28.09</td>
<td>29.84</td>
</tr>
<tr>
<td>8</td>
<td>2.08</td>
<td>3.32</td>
<td>7.63</td>
<td>9.25</td>
<td>15.05</td>
<td>15.51</td>
<td>16.14</td>
<td>24.31</td>
<td>25.02</td>
<td>29.84</td>
</tr>
<tr>
<td>9</td>
<td>2.08</td>
<td>2.9</td>
<td>6.52</td>
<td>7.63</td>
<td>12.06</td>
<td>15.05</td>
<td>15.51</td>
<td>20.09</td>
<td>25.02</td>
<td>29.84</td>
</tr>
</tbody>
</table>

Figure 4.43: Effect of the vertical support stiffness on the frequencies of the first ten modes of vibration of the Lögdeälv bridge.
From the analysis of the results, one can conclude that the greatest changes occur when the stiffness of the end supports decreases to $5 \cdot 10^8$ N/m (cases 3, 6 and 9). For the stiffer support conditions (cases 0, 1 and 2) the lower frequencies show very small changes. However, for higher frequencies, namely the 7th and the 8th, the differences between cases 0 and 1 are clear. The reason why, for higher modes (from the 6th), the eigenfrequencies are higher in the models with spring than in the model with stiff supports is that there are two modes shapes with the same frequency (5th and 6th, both longitudinal modes, but in opposite directions: 15.51 Hz) in the stiffer model. For the models with springs, that frequency is only due to the 6th mode shape, thus disarranging the correspondence between models.

One can also obtain the speed of excitation for the first frequencies. Table 4.15 shows the different resonant speeds, obtained from equation 4.2.

<table>
<thead>
<tr>
<th>Case</th>
<th>1st mode Frequency [Hz]</th>
<th>Speed [km/h]</th>
<th>2nd mode Frequency [Hz]</th>
<th>Speed [km/h]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.11</td>
<td>137</td>
<td>3.42</td>
<td>222</td>
</tr>
<tr>
<td>1</td>
<td>2.12</td>
<td>137</td>
<td>3.49</td>
<td>226</td>
</tr>
<tr>
<td>2</td>
<td>2.12</td>
<td>137</td>
<td>3.44</td>
<td>223</td>
</tr>
<tr>
<td>3</td>
<td>2.12</td>
<td>137</td>
<td>2.96</td>
<td>192</td>
</tr>
<tr>
<td>4</td>
<td>2.12</td>
<td>137</td>
<td>3.48</td>
<td>226</td>
</tr>
<tr>
<td>5</td>
<td>2.12</td>
<td>137</td>
<td>3.43</td>
<td>222</td>
</tr>
<tr>
<td>6</td>
<td>2.12</td>
<td>137</td>
<td>2.96</td>
<td>192</td>
</tr>
<tr>
<td>7</td>
<td>2.08</td>
<td>135</td>
<td>3.37</td>
<td>218</td>
</tr>
<tr>
<td>8</td>
<td>2.08</td>
<td>135</td>
<td>3.32</td>
<td>215</td>
</tr>
<tr>
<td>9</td>
<td>1.08</td>
<td>135</td>
<td>1.90</td>
<td>188</td>
</tr>
</tbody>
</table>

### Displacement

To analyze the influence of the vertical stiffness on the maximum displacement of the bridge deck, trials were made in ABAQUS/BRIGADE, for train speeds from 100 to 300 km/h. The model was meshed using Timoshenko beam elements with 2 m and the speed step was 5 km/h. 10 modes were used, with a time step of 0.02 s, because the parameter study (section 4.2.3) shown it was enough to get convergence for the displacements. Figures 4.44 to 4.46 show the maximum displacement as function of the train speed, for the 10 cases.

Analyzing the plots in figures 4.44 to 4.46, one can see that the peaks were not always found, due to an excessively coarse speed step (5 km/h). Hence, the trials were rerun for the speed interval were the resonance occurs. The speed step used was 1 km/h and the results are presented in figures 4.47 (cases 0 to 6, speed range: 135 to 145 km/h), 4.48 (cases 7 and 8, speed range: 210 to 220 km/h) and 4.49 (case 9, speed range: 185 to 195 km/h).

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Figure 4.44: Maximum upwards (left) and downwards (right) displacement of the beam (cases 1 to 3).

Figure 4.45: Maximum upwards (left) and downwards (right) displacement of the beam (cases 4 to 6).

Figure 4.46: Maximum upwards (left) and downwards (right) displacement of the beam (cases 7 to 9).
Figure 4.47: Maximum upwards (left) and downwards (right) displacement of the beam (cases 0 to 6 with speed step of 1 km/h).

Figure 4.48: Maximum upwards (left) and downwards (right) displacement of the beam (cases 7 and 8 with speed step of 1 km/h).

Figure 4.49: Maximum upwards (left) and downwards (right) displacement of the beam (case 9 with speed step of 1 km/h).
Table 4.16 show the values of displacements and between parenthesis the speeds [km/h] at which these maximum were obtained. With the speed increment of 1 km/h, more accurate results for the displacement were obtained from the trials, with an increase of about 5%.

<table>
<thead>
<tr>
<th>Case</th>
<th>Downwards [cm]</th>
<th>Upwards [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5 km/h</td>
<td>1 km/h</td>
</tr>
<tr>
<td>0</td>
<td>2.04 (140)</td>
<td>2.34 (139)</td>
</tr>
<tr>
<td>1</td>
<td>2.24 (140)</td>
<td>2.34 (139)</td>
</tr>
<tr>
<td>2</td>
<td>2.24 (140)</td>
<td>2.35 (139)</td>
</tr>
<tr>
<td>3</td>
<td>2.37 (140)</td>
<td>2.47 (139)</td>
</tr>
<tr>
<td>4</td>
<td>2.18 (140)</td>
<td>2.30 (139)</td>
</tr>
<tr>
<td>5</td>
<td>2.17 (140)</td>
<td>2.31 (138)</td>
</tr>
<tr>
<td>6</td>
<td>2.31 (140)</td>
<td>2.42 (139)</td>
</tr>
<tr>
<td>7</td>
<td>2.23 (220)</td>
<td>2.33 (218)</td>
</tr>
<tr>
<td>8</td>
<td>2.30 (215)</td>
<td>2.30 (215)</td>
</tr>
<tr>
<td>9</td>
<td>2.08 (190)</td>
<td>2.24 (188)</td>
</tr>
</tbody>
</table>

### Acceleration

To analyze the influence of the vertical stiffness on the maximum acceleration of the bridge deck, trials were made in ABAQUS/BRIGADE, for train speeds from 100 to 300 km/h. In the first approach, the speed step was 5 km/h, with only 10 modes being used, and a time step of 0.02 s. This rough approach will later be refined, for the critical train speeds. The figures 4.50 to 4.52 show the first results.

Figures 4.50 and 4.51 are very similar. They show a decrease in the maximum acceleration for case 3 and 6, when the stiffness of the central column is decreased. Furthermore, the resonant peaks were obtained for the same trains speeds. Figure 4.52 shows a very different behavior. The resonant speeds were many more and the accelerations change from about 3 to 8 m/s². For these cases (with end supports stiffness of $5 \cdot 10^8$ N/m), the maximum acceleration is obtained at the ends of the beam, where the springs, with low stiffness, were located. This behavior, present in the model, needs to be validated with experimental results from bridges with very low support stiffness (if such bridges can be found). Therefore, cases 7 to 9 will not be followed in the subsequent studies.

The maximum acceleration is summarised in table 4.17, with the speed at which the corresponding value was achieved between parenthesis. To get more reliable results, the number of modes considered should be at least 40, according to the parameter study made before. Therefore, the trials were rerun (from cases 0 to 6) with a speed step of 1 km/h, time step of 0.005 s and considering 40 modes. The results are presented in figures 4.53 (cases 0 to 2, 4 and 5, speed range: 215 to 230 km/h) and 4.54 (cases 3 and 6, speed range: 185 to 200 km/h).
Figure 4.50: Maximum upwards (left) and downwards (right) acceleration of the beam (cases 1 to 3).

Figure 4.51: Maximum upwards (left) and downwards (right) acceleration of the beam (cases 4 to 6).

Figure 4.52: Maximum upwards (left) and downwards (right) acceleration of the beam (cases 7 to 9).


Table 4.17: Maximum acceleration \(\text{[m/s}^2\text{]}\) (using the speed step of 5 km/h).

<table>
<thead>
<tr>
<th>Case</th>
<th>Downwards</th>
<th>Upwards</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.13 (220)</td>
<td>4.26 (220)</td>
</tr>
<tr>
<td>1</td>
<td>4.41 (225)</td>
<td>4.43 (225)</td>
</tr>
<tr>
<td>2</td>
<td>3.92 (220)</td>
<td>3.95 (220)</td>
</tr>
<tr>
<td>3</td>
<td>3.14 (190)</td>
<td>3.02 (190)</td>
</tr>
<tr>
<td>4</td>
<td>4.52 (225)</td>
<td>4.54 (225)</td>
</tr>
<tr>
<td>5</td>
<td>4.24 (220)</td>
<td>4.20 (220)</td>
</tr>
<tr>
<td>6</td>
<td>3.35 (190)</td>
<td>3.22 (190)</td>
</tr>
<tr>
<td>7</td>
<td>7.83 (160)</td>
<td>7.89 (140)</td>
</tr>
<tr>
<td>8</td>
<td>7.28 (175)</td>
<td>7.16 (150)</td>
</tr>
<tr>
<td>9</td>
<td>7.81 (130)</td>
<td>7.93 (130)</td>
</tr>
</tbody>
</table>

Figure 4.53: Maximum upwards (left) and downwards (right) acceleration of the beam (cases 0 to 2, 4 and 5 with speed step of 1 km/h).

Figure 4.54: Maximum upwards (left) and downwards (right) acceleration of the beam (cases 3 and 6 with speed step of 1 km/h).
Cases 1 to 3 shown a good behavior, with the maximum being obtained for the predicted resonant speed. For cases 4 to 6, however, the results were not the expected. Instead of resonant peaks similar to what were obtained using 10 modes, the acceleration show diffuse peaks at different speeds, analogous to the behavior obtained for cases 7 to 9. To get the bigger picture of the problem, trials with 40 modes and a time step of 0.005 s were made for speeds between 150 and 250 km/h. The results are shown in figure 4.55. Analyzing the figure, it becomes clear that a phenomenon similar to what happened for cases 7 to 9 is noted here. The peaks become diffuse and one cannot define a clear resonant speed. That is due to the influence of higher modes. The importance of higher modes is demonstrated by the different maximum accelerations obtained, when 10 and 40 modes were considered.

![Figure 4.55: Maximum upwards (left) and downwards (right) acceleration of the beam with different number of modes (cases 4 to 6 with speed step of 5 km/h).](image)

### 4.2.5 Discussion of the results

From the convergence study undertaken in the beginning of section 4.2, it was possible to conclude that mesh elements of 2 m were short enough to get convergent results for displacements and accelerations. The influence of the element size on the eigenfrequencies was found to be negligible. The same was inferred for the influence of the time step and number of modes on the displacements. 10 modes and a time step of 0.02 s were found to be good enough. However, to ensure the convergence of the accelerations, a time step of 0.005 s was used and 40 modes were considered.

After the study on the influence of the support stiffness one can infer, once more, that it is essential to consider the vertical stiffness of the supports, when designing...
or studying HSR bridges. The vertical stiffness of the supports, alone, is responsible for major alterations in the dynamic behavior of the structure. Regarding the eigenfrequencies, it was seen that large differences occur when the stiffness of the end supports decrease to $5 \cdot 10^8$ N/m. The influence of the central column stiffness is not so clear, when analyzing figure 4.43. Even though, the 10th eigenfrequency, for instance, decreased around 4% between stiffness $5 \cdot 10^9$ and $5 \cdot 10^8$ N/m and 10% between stiffness $5 \cdot 10^9$ and $5 \cdot 10^8$ N/m.

Changes in maximum displacement and maximum acceleration were obvious. Table 4.18 shows the maximum values obtained for displacement and acceleration, with the HSLM-A1 running at resonance, whenever resonant speed was found.

**Table 4.18: Maximum responses for the cases where resonance was found (resonant speed [km/h] is indicated between parenthesis).**

<table>
<thead>
<tr>
<th>Case</th>
<th>Maximum displacement [cm]</th>
<th>Maximum acceleration [m/s$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Downwards</td>
<td>Upwards</td>
</tr>
<tr>
<td>0</td>
<td>2.34 (139)</td>
<td>1.84 (139)</td>
</tr>
<tr>
<td>1</td>
<td>2.34 (139)</td>
<td>1.82 (139)</td>
</tr>
<tr>
<td>2</td>
<td>2.35 (139)</td>
<td>1.82 (139)</td>
</tr>
<tr>
<td>3</td>
<td>2.47 (139)</td>
<td>1.69 (139)</td>
</tr>
<tr>
<td>4</td>
<td>2.30 (139)</td>
<td>1.76 (138)</td>
</tr>
<tr>
<td>5</td>
<td>2.31 (138)</td>
<td>1.77 (139)</td>
</tr>
<tr>
<td>6</td>
<td>2.42 (139)</td>
<td>1.64 (138)</td>
</tr>
<tr>
<td>7</td>
<td>2.33 (218)</td>
<td>1.58 (218)</td>
</tr>
<tr>
<td>8</td>
<td>2.30 (215)</td>
<td>1.57 (214)</td>
</tr>
<tr>
<td>9</td>
<td>2.24 (188)</td>
<td>1.56 (187)</td>
</tr>
</tbody>
</table>

The reason why table 4.18 does not show the maximum accelerations for cases 4 to 9 is that no simple resonance peak was clear as for the other cases. In cases 4 to 6, although these peaks were found using 10 modes, they were not clear with 40 modes, as can be seen in figure 4.55. That is due to the importance of higher modes on the maximum accelerations. The values from table 4.18 are plotted in figure 4.56. The downwards responses are shown reflected over the abscissa (absolute values).

One can see the influence the vertical stiffness of the supports has on the responses of the bridge, especially on the maximum accelerations. When the stiffness of the end supports decreases, the acceleration drops from almost $4.9$ m/s$^2$ to $3.5$ m/s$^2$ (case 3). The influence of the main column stiffness is not that evident. For cases 1 to 2, the decrease of the maximum acceleration is only around 3%. The resemblance between the downwards and upwards accelerations is also clear. The influence to the support stiffness is very similar on both.

Large changes in the displacement, due to the supports stiffness, can also be seen, especially from cases 2 to 3 and 5 to 6. When the stiffness of the end supports decreases, the downwards and the upwards displacement show opposite behavior. The less stiff the supports are, the more settlement will occur and, therefore, the absolute value of the downwards displacement will rise while the upwards displacement will
CHAPTER 4. THEORETICAL BEHAVIOR OF SIMPLE BRIDGES

Figure 4.56: Maximum displacement (cases 1 to 9) and acceleration (cases 0 to 3) of the beam (with speed step of 1 km/h).

decrease. Another fact to be noted is the influence of the first and second modes to the response of the structure. Table 4.19 shows the resonant speed obtained from equation 4.2, using the two first eigenfrequencies, and the resonant speeds obtained from the trials run in ABAQUS.

Table 4.19: Resonant speeds [km/h] obtained from equation 4.2 and with the FEM.

<table>
<thead>
<tr>
<th>Case</th>
<th>1st mode</th>
<th>2nd mode</th>
<th>from displacements</th>
<th>from accelerations</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>137</td>
<td>222</td>
<td>139</td>
<td>225</td>
</tr>
<tr>
<td>1</td>
<td>137</td>
<td>226</td>
<td>139</td>
<td>225</td>
</tr>
<tr>
<td>2</td>
<td>137</td>
<td>223</td>
<td>139</td>
<td>222</td>
</tr>
<tr>
<td>3</td>
<td>137</td>
<td>192</td>
<td>139</td>
<td>191</td>
</tr>
<tr>
<td>4</td>
<td>137</td>
<td>226</td>
<td>139</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>137</td>
<td>222</td>
<td>138</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>137</td>
<td>192</td>
<td>139</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>135</td>
<td>218</td>
<td>218</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>135</td>
<td>215</td>
<td>215</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>135</td>
<td>188</td>
<td>188</td>
<td>-</td>
</tr>
</tbody>
</table>

As can be seen in table 4.19, for case 0 to 6, the maximum displacement is clearly due to the first mode. However, for lower stiffness at the central column (cases 7 to 9), the second mode becomes the most important. This can be explained from the analysis of the mode shapes (appendix B), where one can see that the vertical stiffness of the main column has a great influence on the 2nd mode shape. The comparison of the resonance can also be seen in figures 4.44 to 4.46, where the peaks occur for the resonant speeds corresponding to the modes referred.

In the first 3 cases, where resonance speed for acceleration could be established, the second mode dominated the response. For cases 4 to 9, higher modes (both bending modes and rigid body modes) have a great influence on the response and the resonance peaks were not clear for the studied speed interval.
Chapter 5

Case Study: the Sagån Bridge

In this chapter, one railway bridge will be analyzed using the FE program ABAQUS with a pre-processing unit for the dynamic live loads: BRIGADE, with special empathy in the influence of the vertical support stiffness. The results obtained from the FEM will be compared with data measured \textit{in situ}, to validate the model. Theoretical estimation of the support stiffness and model updating will also be performed.

5.1 The bridge

The bridge over Sagån is located in Sweden, on the railway connecting Enköping to Västerås and serving the airport.

Figure 5.1: The Sagån bridge, view from northwest (a column belonging to an older bridge behind the studied bridge can be seen in the middle).

The structure is a three-span post-tensioned concrete girder bridge, carrying one
ballasted track. The main span is 26 m long and the two side spans are 18.5 m long. Between the short columns and the embankments, on both sides, there is a span of 2.2 m. Therefore, the total length of the bridge is 67.4 m.

The end supports are not structural and the facing walls only support the abutments. The loads are distributed between the two main columns and the two short columns near the embankments (figure 5.3). This fact will be further discussed in the model section.

Figure 5.2: The Sagån bridge, view from east.

Figure 5.3: The short columns (left) and the embankments (right) of the Sagån bridge.
5.2 FE Model

For the modelling and the results extraction, using the Finite Element Method, the FE program ABAQUS/BRIGADE was used. The general usage of the program is described in section 3.6.

The model consisted only of the permanent parts of the bridge: two main columns supporting the continuous beam. The beam was placed along the $x$-axis (or axis 1), which was directed from west to east. The main columns had the direction of the $y$-axis (or axis 2). The $z$-axis (or axis 3) from the global cartesian coordinate system, points in the transversal direction.

For the main beam, it was used a generalized profile with the characteristics of the Sagån bridge cross-section (see figure 5.4). The material of the beam was concrete and the physical properties were the following:

- Specific weight, $\gamma=25$ kN/m;
- Modulus of elasticity, $E = 32 \cdot 10^9$ Pa;
- Moment of inertia, $I_{xx} = 1.74 \text{ m}^4$;
- Area, $A = 6.3 \text{ m}^2$;
- Poison’s ratio, $\nu = 0.25$.

The concrete slab was covered with $3.6 \text{ m}^2$/m of ballast, with a weight, $\gamma$, of 20 kN/m$^3$. It was decided to increase the density of the concrete, to include the additional weight of the ballast. Thus, the density of the cross-section was considered as 3600 kg/m$^3$. The section of the columns (see appendix D) was modeled as a rectangular shape of $4.8 \cdot 1.2 \text{ m}^2$, with a density of 2500 kg/m$^3$. The left column was modeled as 7.95 m high and the right column with 6.7 m of height.
To simulate the behavior of the bearings, joins were used to constrain the interaction between the main columns and the beam. The short columns were modeled as supports and, therefore, no connectors were necessary. The joins (or connectors) are defined in the Interaction Module of ABAQUS, and the interaction diagram of a join connector can be seen in figure 5.5.

![Join connector used to connect the beam to the columns.](image)

The interaction of the structure with the soil was, initially, considered stiff. The behavior of the bridge-embankment interaction was discussed and it was decided to restrain only the vertical displacement on the end of the beam. The rotation and longitudinal displacement were permitted because it was believed that the facing walls of the embankment were not thick enough to constrain the corresponding DOFs. Nevertheless, a study was made on the subject, and it was concluded that those constraints had negligible influence on the bridge behavior, for stiff supports.

The bending stiffness of the short columns was considered infinite, since the longitudinal forces were not believed to be strong enough to change the behavior of the structure. Thus, the longitudinal displacement at the short columns supports was locked. The foundation of the main columns was believed to be stiff enough to restrain the longitudinal displacement, and that DOF was also locked. The rotation of those foundations probably allow some rotation. However, it was decided not to include rotational springs in this study. Therefore, the rotation at the lower end of the columns was restrained.

The model was meshed with Timoshenko beam elements of 0.3 m on the main beam and 1 m on the columns. The interpolations functions used were linear. The damping was considered as direct damping and equal for the whole model. The value chosen was, according to [16] and table 2.3, $\xi = 1\%$.

The final model, with stiff supports, can be seen in figure 5.6.

The loads of the Gröna Tåget presented in section 5.3.1 were used to define a new train in the Dynamic Live Load model of BRIGADE. The load model was then run over the structure, at different speeds.
5.3  

**In situ measurements**

5.3.1  The Gröna Tåget

The train used for the measurements was a testing train, called the Gröna Tåget (or Green Train), property of Banverket, the authority responsible for administrating all railway traffic in Sweden. It is a modified Regina train, capable of achieving speeds up to 300 km/h and designed specifically for the Nordic climate and Nordic tracks.

Figure 5.7: 3D render (left) and picture of the interior (right) of the Gröna Tåget test train.
The special equipment of the test train includes:

- Traction motors on all 8 axles;
- Motor reduction gears for higher speeds;
- Speedometers and pacesetters for speeds up to 300 km/h;
- Pantograph tops for higher speeds on existing catenary;
- Track-friendly bogies for improved running stability as well as low track forces and wheel-rail wear;
- Brake pads for higher temperatures and speed;
- Bogie fairings for reduced external noise and improved aerodynamics.

The high-speed tests were run on the railway lines between Skövde-Töreboda and Enköping-Västerås. In this last line, the train crossed the Sagån bridge, and the effects on the structure were measured. During the test periods, the behavior of the train and the track is constantly monitored, namely:

- Running stability;
- Lateral and vertical track forces;
- Forces and motions on pantograph;
- External noise: influence of bogie skirts and low noise barriers;
- Internal noise and vibration;
- Passenger comfort;
- Air flow under the train;
- Behavior on curves;
- High speed behavior;
- Bogie fairings for reduced external noise and improved aerodynamics;
- Aerodynamics and pressure variations in tunnels.

The loads induced by the axles of the Gröna Tåget, and the distances between axles are shown in table 5.1.
### Table 5.1: Gröna Tåget axle loads and distances.

<table>
<thead>
<tr>
<th>Axle number</th>
<th>Axle load [N]</th>
<th>Axle distance [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>179 000</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>177 000</td>
<td>2.7</td>
</tr>
<tr>
<td>3</td>
<td>176 000</td>
<td>19.0</td>
</tr>
<tr>
<td>4</td>
<td>175 000</td>
<td>21.7</td>
</tr>
<tr>
<td>5</td>
<td>162 000</td>
<td>26.6</td>
</tr>
<tr>
<td>6</td>
<td>161 000</td>
<td>29.3</td>
</tr>
<tr>
<td>7</td>
<td>177 000</td>
<td>45.6</td>
</tr>
<tr>
<td>8</td>
<td>178 000</td>
<td>48.3</td>
</tr>
</tbody>
</table>

### 5.3.2 Instrumentation of the bridge

The instrumentation of the bridge consisted of (see figure 5.9):

- 4 accelerometers: micro electro mechanical systems (MEMS) based upon a bulk micro-machined silicon element. The core consists of a 3-terminal variable capacitance sensor in a custom ceramic package sealed under vacuum (see figure 5.8, left). Applied acceleration or tilt to the sensitive axis, changes the inertia, causing the mass to move between the upper and lower electrodes, which results in a change to the values of the capacitors. As differential variation of the sensing capacitors is measured, a restoring electrostatic force is applied to maintain the proof mass in a central (neutral) position. The output signal of the sensor is derived directly from the correction signal used to keep the center-mass in the neutral position. This correction signal is linearly proportional to the acceleration applied (by the ground motion) to the sensor. This type of closed loop design generally provides a better degree of output linearity than open-loop sensors;

- 4 linear variable differential transformers (LVDTs): electric transformers used for measuring linear displacements. Each transformer contains three solenoidal coils placed end-to-end around a tube. A cylindrical ferromagnetic core, attached to the object whose position is to be measured, slides along the axis of the tube (see figure 5.8, right). The magnitude of the output voltage is proportional to the distance moved by the core, which is why the device is described as linear. The phase of the voltage indicates the direction of the displacement. Because the sliding core does not touch the inside of the tube, it can move without friction, making the LVDT a highly reliable device. The absence of any sliding or rotating contacts allows the LVDT to be completely sealed against the environment.

The placement of the instrumentation can be seen on figure 5.9. The four accelerometers were placed on the edges of the beam and were later used to determine the eigenfrequencies. The LVDTs were used to measure the vertical displacement of the first span, the longitudinal displacement over one of the bearings at the first support
and the rotation at the first support. In addition to the mentioned sensors, strain transducers and geophones were also mounted on the bridge, but not used in this study.

### 5.3.3 Measurement data

The eigenfrequencies were identified using the Peak Picking Method from figure 5.10 (information about this method and the identification of the mode shapes can be found in Ülker [40]). The mode types can be seen in table 5.2 and have been named in the following way: vertical bending modes (VB), transversal bending modes (TB) and torsional modes (T), followed by the number of the mode. Closely spaced modes with similar shape have been labeled with +. The mean value and the standard deviation of the 6 passages of the Gröna Tåget are displayed in the bottom of the table.
### 5.3. IN SITU MEASUREMENTS

![Figure 5.10: PSD estimated from accelerations, when the Gröna Tåget crosses the Sagån bridge (from Ülk er [40]).](image)

### Table 5.2: Estimation of the eigenfrequencies, when Gröna Tåget crosses the Sagån bridge (from Ülk er [40]).

<table>
<thead>
<tr>
<th>Time</th>
<th>VB 1</th>
<th>TB 1</th>
<th>TB 1+</th>
<th>TB 2</th>
<th>VB 2</th>
<th>VB 2+</th>
<th>VB 3</th>
<th>T 1</th>
<th>T 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.51</td>
<td>5.76</td>
<td>6.6</td>
<td>−</td>
<td>9.17</td>
<td>9.76</td>
<td>10.3</td>
<td>12.9</td>
<td>14.5</td>
<td></td>
</tr>
<tr>
<td>2.08</td>
<td>5.79</td>
<td>6.61</td>
<td>7.35</td>
<td>7.79</td>
<td>9.15</td>
<td>9.72</td>
<td>10.5</td>
<td>12.7</td>
<td>14.3</td>
</tr>
<tr>
<td>2.26</td>
<td>5.76</td>
<td>6.6</td>
<td>7.15</td>
<td>7.73</td>
<td>9.24</td>
<td>9.73</td>
<td>10.3</td>
<td>12.9</td>
<td>14.5</td>
</tr>
<tr>
<td>3.08</td>
<td>5.76</td>
<td>6.6</td>
<td>7.16</td>
<td>7.71</td>
<td>−</td>
<td>9.76</td>
<td>10.3</td>
<td>13</td>
<td>14.5</td>
</tr>
<tr>
<td>3.25</td>
<td>5.84</td>
<td>6.61</td>
<td>7.33</td>
<td>7.79</td>
<td>9.2</td>
<td>9.74</td>
<td>10.5</td>
<td>12.6</td>
<td>14.3</td>
</tr>
<tr>
<td>4.05</td>
<td>−</td>
<td>6.62</td>
<td>7.34</td>
<td>7.79</td>
<td>9.15</td>
<td>−</td>
<td>10.5</td>
<td>12.7</td>
<td>14.3</td>
</tr>
</tbody>
</table>

| m   | 5.78 | 6.61 | 7.27  | 7.76 | 9.18 | 9.74  | 10.4 | 12.8| 14.4|
| σ   | 0.03 | 0.01 | 0.1   | 0.04 | 0.04 | 0.02  | 0.1  | 0.2 | 0.1 |

The eigenfrequencies chosen and later compared with the FEM results were the ones identified with VB 1, VB 2+, and VB 3, respectively, 5.78, 9.74 and 10.40 Hz. The displacements were obtained from the LVDTs and are shown in figure 5.11, for the Gröna Tåget passing, approximately, at 230 km/h.

![Figure 5.11: Vertical displacements measured with the LDVTs.](image)

The rotation of the bridge deck over the east bearing was calculated, based on the displacement registered at east and west of the bearing and the distance between the
LVDTs. Figure 5.12 shows the rotation over the east bearing, for the same Gröna Tåget run.

![Figure 5.12: Rotation over east bearing, calculated using the signal of the two LVDT sensors.](image)

### 5.4 Influence of the vertical support stiffness

To study the behavior of the modeled structure and the influence of the vertical support stiffness, and afterwards compare the numerical response with the measured values, the FE model was modified. All vertical supports were replaced by springs, and their stiffness varied. The formulas 3.3, 3.4, 3.5 and 3.6 suggested by Richart and Lysmer in 1966 [43, 51] and Gazetas [29, 30] are fully explained in chapter 3.2 and were used in the following. The new model is shown in figure 5.13, where the spheres represent the springs.

![Figure 5.13: Modified FE model of the Sagån bridge (with spheres representing the springs).](image)
5.4. INFLUENCE OF THE VERTICAL SUPPORT STIFFNESS

5.4.1 Eigenfrequencies sensitivity

The sensitivity of the eigenfrequencies to the stiffness of the supports was studied. The supports conditions were grouped in 3 sets: end supports, short columns and main columns. From the results obtained in chapter 4, it was decided to study a span of stiffness between $5 \cdot 10^8$ and $5 \cdot 10^{10} \text{ N/m}$. The stiffness of each set of supports was changed, while the others were maintained stiff.

Figure 5.14 shows the variation of the frequencies of the first ten modes of vibration, for a support stiffness changing within the defined interval.

Figure 5.14: Effect of the end supports stiffness in the frequencies of the first ten (left) and three (right) modes of vibration of the Sagan bridge.

For end support stiffness of $5 \cdot 10^{10} \text{ N/m}$, the eigenfrequencies are very similar to the model with stiff supports. For lower stiffness, the main changes occur for the 2nd and 3rd modes and for the 6th and 7th. This can be explained by the analysis of the modes shapes shown in appendix C. The 6th and 7th mode shapes imply higher displacements for the nodes at the beam ends. Since lower vertical stiffness on the supports unlocks that degree-of-freedom, the beam can vibrate at low frequencies.

A similar behavior happens for variations in the stiffness of the short columns, that can be seen in figure 5.15. Once again, when the stiffness is reduced to $5 \cdot 10^{10} \text{ N/m}$, the frequencies are very close to the model with stiff supports. Moreover, the same modes are more affected by the reducing of the stiffness. This behavior can also be explained because modes 2, 3, 6 and 7 are responsible for the most part of the displacement at the beam ends. To conclude, it was seen that decreasing the stiffness of the short columns has similar influence to the reduction of the stiffness on the end supports. However, the differences in the value of the frequencies, is much greater for the 2nd study.

Figure 5.16 shows the influence, on the same frequencies, of the vertical stiffness of the supports/soil below the main columns. The differences between the model with stiff supports and the new model with vertical stiffness under the main columns of $5 \cdot 10^{10} \text{ N/m}$ are only noted for the 7th and 8th modes. For the second model, with a stiffness of $5 \cdot 10^9 \text{ N/m}$, all the frequencies decrease, expect the 2nd mode. This
Figure 5.15: Effect of the short columns stiffness in the frequencies of the first ten (left) and three (right) modes of vibration of the Sagån bridge.

happens because that mode shape (see appendix C) has nodes very close to the joins of the beam with the main columns. Therefore, because the displacement of that points is very small, the influence of the stiffness of the main columns supports is not relevant for that frequency. All the other frequencies decrease, specially from mode 6 to 8. In the try with less stiff supports, all the frequencies show great variations, and even the mode shapes change because the supports are too soft to restrain the displacements at the lower end of the columns.

Figure 5.16: Effect of the main columns stiffness in the frequencies of the first ten (left) and three (right) modes of vibration of the Sagån bridge.

5.4.2 Model with theoretical support stiffness

After the study on the effect that support stiffness had on mode shapes and frequencies of the structure, the effective stiffness of the bridge supports was estimated.

Some of the proprieties of the soil were not obtained from the in situ measurements and were assumed between the usual span of values. That span and the values as-
assumed were discussed with Associate Professor Anders Bodare, from the Division of Soil and Rock Mechanics of KTH, who commented and gave valuable suggestions from his professional experience. Since this dissertation only tries to show the influence of the support stiffness on the behavior of the structure, these assumptions are not very important. However, when designing a bridge, preliminary studies to evaluate the dynamic response of the soil should be performed, for greater accuracy of the model.

The soil supporting the foundations is a rock subtract partially covered by sand. To evaluate the theoretical stiffness of the supports, we can assume that the density of the water, $\rho_w$, is 1000 kg/m$^3$, the compact density of sand, $\rho_s$, 2650 kg/m$^3$ and the void ratio, $e$, 0.5. The effective density [kg/m$^3$] was obtained using formulae 5.1 for dry material and 5.2 for saturated material:

$$\rho = \frac{\rho_s}{1 + e} = 1767 \quad (5.1)$$

$$\rho = \frac{e \cdot \rho_w + \rho_s}{1 + e} = 2100 \quad (5.2)$$

The shear modulus, $G$, is:

$$G = \rho \cdot c_s^2 \quad (5.3)$$

The shear wave velocity in the soil, $c_s$, was assumed as 250 m/s and the height of the soil layer above the rock subtract 0.8 m. The foundation dimensions can be obtained from the bridge drawings in appendix D. Hence, using formulae 3.3, 3.4, 3.5 and 3.6, the values of the shear modulus, $G$ [Pa], and vertical stiffness of the supports, $K_z$ [N/m], can be evaluated (see table 5.3).

<table>
<thead>
<tr>
<th>Condition of the soil</th>
<th>Richart and Lysmer</th>
<th>Gazetas</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Circular foundation</td>
<td>Rectangular foundation</td>
</tr>
<tr>
<td>dry</td>
<td>$1.10 \cdot 10^8$</td>
<td>$2.3 \cdot 10^9$</td>
</tr>
<tr>
<td>saturated</td>
<td>$1.31 \cdot 10^8$</td>
<td>$2.7 \cdot 10^9$</td>
</tr>
</tbody>
</table>

The formulas referred by Richard and Lysmer and the estimation by Gazetas for the vertical stiffness of a surface foundation on homogeneous half-space, do not contemplate the existence of a rock subtract underneath the soil layer. Therefore, the theoretical vertical stiffness of the main columns and short columns supports was roughly estimated from the last column of table 5.3, as $1 \cdot 10^{10}$ N/m. For the end supports, to simulate the influence of the bridge-embankment interaction, a stiffness of $1 \cdot 10^9$ N/m was assumed. Using these stiffness, a new FE model was made (designated by model 2) and its eigenfrequencies were compared with the eigenfrequencies obtained from the model with stiff supports (designated from this
point forward by model 1) and with the measured frequencies. Figure 5.17 shows that relation.

![Graph showing frequencies of modes](image)

Figure 5.17: Frequencies of the first ten (left) and three (right) modes of vibration of the Sagån bridge (model 1 and 2).

As expected, comparing model 1 with model 2, all the frequencies decrease a lot, except the 5th. This is because the 5th mode is a longitudinal mode of vibration. The relation between the first 3 frequencies is very similar to the relation between the measured frequencies, although all the values are lower. This can be explained by a low estimation of the modulus of elasticity or an excessively high value assumed for the mass of the bridge, but this will be further discussed in the next section.

### 5.4.3 Model updating

In the last section, the resemblance between the experimental values and the numerical values of the model with springs (model 2), on what concerns the first frequencies, was discussed. In fact, the values are lower, but that is probably due to an unprecise estimation of important properties such as the mass of the ballast or the modulus of elasticity of the material. Hence, it was decided to increase the $E$ to 42 GPa increasing, therefore, the bending stiffness of the beam. With this increase on the stiffness of the model, the frequencies for the first modes became very close to the measured values (figure 5.18). However, due to the very low stiffness of the end supports, the nodes at the beam ends show large displacements, whenever a new train axle enters the bridge (see figure 5.20), demonstrating that the model should be updated. For model 1, with stiff supports, no changes were made to the modulus of elasticity, since it would stiffen the structure even more, thus reducing the, already too low, displacements. These displacements will be compared in figures 5.19 to 5.22, illustrating why the bending stiffness was not changed for model 1.

FE model updating is a very useful tool for designers and investigators. In 1995, Imregun et al. studied the problem of model updating and the accuracy of experimental data required [33, 34]. In the beginning of this century, Sinha et al. [52] and Zapicoa et al. [67], followed the studies undertaken by M. Imregun and, in 2007, the
5.4. INFLUENCE OF THE VERTICAL SUPPORT STIFFNESS

In the past years, the software company SDTools developed a Structural Dynamics Toolbox to work together with FEMLink and expand the possibilities of MATLAB into the FEM analysis. More information on the subject can be found in Balmès et al. [6]. Although powerful algorithms have already been implemented for computer aided model update, the usage of that algorithms is not a subject of this study and, therefore, the update was made by trying a reduced number of pre-established combinations of stiffness, and studying the variations. The study of the eigenfrequencies sensitivity to changes in the different supports stiffness (section 5.4.1) was particularly useful for the update, since it gave a clear idea on how the bridge would respond and what stiffness should be used. After 6 trials, a FE model was found that gave sufficiently accurate results for displacement. That model (designated by model 3) had the following properties:

- Modulus of elasticity: 42 GPa;
- End supports stiffness: $3 \times 10^9$ N/m;
- Short columns stiffness: $2 \times 10^{10}$ N/m;
- Main columns stiffness: $1 \times 10^{10}$ N/m.

The vertical support stiffness values were reasonable, considering the results from table 5.3. Moreover, recent experimental studies undertaken on the Sagån bridge columns by PhD Student Mahir Ülker, from the Division of Structural Design and Bridges of KTH, for estimating the support stiffness, suggested a stiffness around $9 \times 10^9$ N/m, very close to the values adopted for model 2 and 3. However, the experimental method used for that calculation needs further investigation and validation. The report explaining the method and showing the results will be available through KTH, as soon as complete (Ülker [41]).

The influence of the updating on the frequencies of the first ten modes of vibration is shown in figure 5.18. Table 5.4 shows the eigenfrequencies of the different models and between parenthesis the relations between them and the measured frequencies. The displacements of the FE models were compared with the measured values and are shown in figures 5.19 to 5.21.

One can see that the eigenfrequencies for model 2, with $E=42$ GPa, are very close to the measured frequencies, confirming the idea that the modulus of elasticity of the concrete was initially assumed too low. For model 3, the $2^{nd}$ and $3^{rd}$ frequencies are higher than model 2, and not so close to the measured values. However, as can be seen in figures 5.19 to 5.22, the displacements and the rotations show very good accuracy.
Figure 5.18: Frequencies of the first ten (left) and three (right) modes of vibration of the Sagån bridge (with the updated model).

Table 5.4: Frequencies [Hz] of the first ten modes of vibration with the percentages, comparing with the measured values, between parenthesis (with the updated model).

<table>
<thead>
<tr>
<th>Mode</th>
<th>Measured</th>
<th>Model 1 (E=32 GPa)</th>
<th>Model 2 (E=42 GPa)</th>
<th>Model 3 (E=42 GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.78 (100%)</td>
<td>5.34 (92%)</td>
<td>5.66 (98%)</td>
<td>5.78 (100%)</td>
</tr>
<tr>
<td></td>
<td>9.74 (100%)</td>
<td>11.10 (114%)</td>
<td>9.82 (101%)</td>
<td>10.53 (108%)</td>
</tr>
<tr>
<td></td>
<td>10.40 (100%)</td>
<td>12.45 (120%)</td>
<td>10.76 (103%)</td>
<td>11.40 (110%)</td>
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<td></td>
<td>-</td>
<td>17.90</td>
<td>17.70</td>
<td>17.72</td>
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<tr>
<td></td>
<td>-</td>
<td>23.23</td>
<td>26.44</td>
<td>26.61</td>
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<tr>
<td></td>
<td>-</td>
<td>31.21</td>
<td>26.61</td>
<td>26.90</td>
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<tr>
<td></td>
<td>-</td>
<td>48.21</td>
<td>48.51</td>
<td>48.68</td>
</tr>
</tbody>
</table>

Figure 5.19: Vertical displacement at mid-span of span 1 (see figure 5.9).
5.4. **INFLUENCE OF THE VERTICAL SUPPORT STIFFNESS**

![Graph](Image)

**Figure 5.20:** Vertical displacement over east bearing (east).

![Graph](Image)

**Figure 5.21:** Vertical displacement over east bearing (west).

![Graph](Image)

**Figure 5.22:** Rotation over bearing.
From the analysis of the displacement at mid-span 1 (figure 5.19), it is possible to see that all the models show reasonable agreement with the measured values. For model 1, with stiff supports, the displacements are lower, as expected. The opposite happens for model 3, with the less stiff supports.

The displacements over the bearing (figures 5.20 and 5.21) are 10 times lower than the observed displacement at mid-span, and show greater sensitivity whenever a train axle enters the bridge. Model 1 shows almost no displacement, implying that a model with stiff supports does not correctly simulate the behavior of the structure. For model 2, the computed displacements are higher than the experimental and show very high variations for the east LDVT. The reason for this is believed to be that the very low stiffness of the end supports leave the ends of the beam too loose, contrary to what happens in reality.

The rotation over the bearing (figure 5.22) is, once again, very low for model 1 and model 2 shows values greater than the measured. However, model 3 shows perfect agreement, the same happening for the displacements. Table 5.5 shows the maximum responses of the structure, for different support conditions. Between parenthesis are the percentages, comparing with the measured values.

| Table 5.5: Maximum responses for the 3 models (between parenthesis are the percentages, comparing with the measured values). |
|---|---|---|---|
| Rotation over bearing | Displacement at mid-span 1 | Displacement over bearing (east) | Displacement over bearing (west) |
| [10^{-3} rad] | [mm] | [mm] | [mm] |
| Measured | 0.06(100%) | 0.46(100%) | 0.02(100%) | 0.08(100%) |
| Model 1 | 0.02(34%) | 0.39(85%) | 0.01(25%) | 0.02(26%) |
| Model 2 | 0.09(141%) | 0.53(117%) | 0.05(216%) | 0.11(148%) |
| Model 3 | 0.06(96%) | 0.46(101%) | 0.02(103%) | 0.07(95%) |

5.5 Discussion of the results

The objective of chapter 5 was to show the significance of the supports stiffness on the dynamic behavior of HSR bridges. From the formulas suggested by Lysmer and Richart [43], Richard et al. [51] and Gazetas et al. [29, 30], values for the vertical supports stiffness of the foundations of the Sagan bridge were estimated. The model with theoretical support stiffness was compared with the experimental displacements and frequencies obtained in situ with the Gröna Tåget at 230 km/h, and with a model with stiff supports. From the comparison, the first conclusion was that a model with stiff supports may not show the actual behavior of the real structure. The values of the frequencies obtained from the model with stiff supports and, even more important, the relation between them, were far from the measured. For instance, the 3rd mode of vibration had a frequency 2 Hz (20%) greater than the measured one. Moreover, the maximum vertical displacement for the model with
5.5. DISCUSSION OF THE RESULTS

Stiff supports (model 1) was approximately between 70% and 15% lower than the measured values.

The relation between the first 3 frequencies, for the model with theoretical stiffness, was very similar to the relation between the measured frequencies, although the values were somewhat lower. Lower eigenfrequencies are believed to be due to an underestimation of the bending stiffness of the beam. It is common that the concrete supplied by the contractor has a modulus of elasticity greater than that stated on drawings. Hereby, the stiffness of the beam was increased, considering a modulus of elasticity for concrete of 42 GPa. After this change, the eigenfrequencies became very close to the 3 measured frequencies, with an error of around 2%. However, when the displacements were compared, the results were not as good. In fact, for the sensor on the left of the bearing, the FE model 2 shown very high oscillations, whenever a train axle entered the bridge. That was found to be due to the very low vertical stiffness assumed for the end supports (1 \cdot 10^9 N/m). Using knowledge of model updating and the know-how provided by previous studies on the bridge, a 3rd model was proposed, with a support stiffness of 3 \cdot 10^9 N/m for the bridge-embankment interaction.

The 3rd model had eigenfrequencies 8 to 10% higher than the measured, but shown a very good agreement for the displacement and the rotation over bearing. The maximum displacement at mid-span 1 was only 1% higher than the displacement registered with the LVDT and the rotation at east bearing was 4% lower.

With this case study is was shown that, on one hand, a model with stiff supports may not be reliable if realistic predictions of the dynamic behavior are to be made. On the other hand, that with a simple 2D beam model it is possible to simulate very well the behavior of the structure. Furthermore, with the help of field measurements and model updating, these simulations can be increasingly accurate.

However, one should be conscious of the simplifications made to the structure. The bridge-embankment interaction clearly represents an elastic rotational spring at the end of the bridge deck, that was not accounted for in the model. Rotational springs are also more accurate to simulate the interaction at the lower end of the columns, instead of locking the rotational DOFs. More investigation about this subject should be done. Moreover, a 2D beam analysis does not take into account transversal or torsional modes of vibration. This fact is not so harmful because the bridge is transversally symmetrical, with a single track, but a 3D analysis of the bridge should be made. Furthermore, the train-structure interaction and the distribution of the axle loads through the ballast were neglected, the same happening with the flexibility/stiffness of the bearings. The track-ballast bed may also give some extra stiffness to the structure, that was not accounted for. Studies on these questions seem very interesting and should be undertaken in the future.

Finally, there are uncertainties inherent to any experimental study. Uncertainties, on the measured train speeds or LVDTs measurements, are always expected and hard to avoid. Nevertheless, the results of the study were found to be very satisfactory and hopefully helpful for future investigations on the subject.
Chapter 6

Conclusions and Suggestions for Further Research

6.1 Conclusions

The work undertaken has presented the influence of vertical support stiffness on the behavior of high-speed railway bridges. Theoretical and numerical studies on two common bridges: one single span simply supported bridge and one double span continuous composite bridge on the Bothnia Line, were conducted. For the simply supported bridge, the exact responses proposed by Fryba [28] for an Euler-Bernoulli beam were used, and compared with the results obtained from the FEM. For the double span bridge, no analytical results were extracted, and only FEM was used in the study. From the two studies undertaken, the following conclusions can be made:

- Filtering at a cut-off frequency of 20 Hz may not identify critical behaviors. Even the value of 30 Hz or 2 times the frequency of the first mode of vibration, suggested in the D214 reports; [21], seems too low to include relevant contributions of higher modes of vibration for the acceleration response;

- The influence of the vertical stiffness of the supports on the eigenfrequencies is very large, and most of all, the relation between the frequencies change significantly for different support stiffness;

- Increasing the vertical stiffness of the supports moves the resonance peaks to higher speeds, possibly outside the speed range of the train, decreasing the probability of resonant behavior of the structure;

- Flexible supports bring the resonance peaks to lower speeds, but increase the mass participation. Furthermore, mode shapes change and new rigid body modes can occur;

- Large settlements may occur at the supports with low vertical support stiffness, when loaded systematically with train axles, resulting in opening of cracks in the concrete and fatigue related problems;
• Accelerations may be greatly underestimated by a model with stiff supports, which may mislead the designer to think that the bridge-track is stable and follows the ERRI recommendations on bridge deck accelerations [21]. This can, afterwards, instigate ballast and track destabilization, when the structure is at service and loaded with HSTs;

• Train resonant speeds are highly influenced by the stiffness of the supports. Using a simple formula, designers are able to check the train speed at which resonance effects may occur. Moreover, increasing the stiffness of the structure and of the supports, the resonant speeds can be altered;

• Resonance speeds may become more difficult to distinguish, if the stiffness of the supports is too low, due to the greater influence that higher modes (both bending modes and rigid body modes) have on the structural behavior;

• Most of all, it should be mandatory to model the stiffness of the supports when designing or studying HSR bridges.

In the 2\textsuperscript{nd} part of the study, an investigation was conducted to investigate the agreement of the FE models with experimental records. The Sagän bridge, a three-span post-tensioned concrete girder bridge, carrying one ballasted track was used as a case study. Studies on how sensitive were the eigenfrequencies, displacements and accelerations, to changes in the supports stiffness were performed. From the results, one can conclude:

• Models with stiff supports can greatly underestimate the maximum responses of HSR bridges and may not be reliable if realistic predictions of the dynamic behavior are to be made;

• The formulas suggested by Lysmer and Richart [43], Richard et al. [51] and Gazetas et al. [29, 30] give reasonably accurate estimations for the vertical stiffness of the supports;

• Model updating is a very useful tool for designers and researchers. Powerful algorithms have already been implemented for computer aided model updating, with reasonable results, speeding the updating process;

• Very simple 2D beam models are able to simulate reasonably well the behavior of real structures. Furthermore, with the help of field measurements and model updating, these simulations can be increasingly accurate, and particularly meaningful for structures under constant monitoring.

The results obtained with the updated FE model were found to be very satisfactory. Nevertheless, one should be aware of the simplifications in the model. Moreover, a 2D analysis may excessively underestimate the complexity of the structure, and provide misleading results, especially for bridges carrying more than one track.
6.2 Suggestions for further research

Some problems occurred when performing this study, mainly related to computational processing of the information. All the analysis done with FEM used the Mode Superposition Method to reduce the number of unknowns and uncouple the modal equations. However, for complex structures and if a large number of modes is to be taken into account, computational time increases. Moreover, the physical hard disk and RAM space needed to compute so much information, is sometimes unbearable. Problems with trials, that did not seem to end, happened some times and were the reason why some interesting plots are not shown, namely concerning accelerations. Hence, the first suggestion for further research concerns the influence of the supports stiffness on the resonant speeds for accelerations. Plots of the maximum accelerations, for speeds from 100 to 300 km/h (or more), using at least 40/50 modes and a time step ensuring the convergence of the values, would be very interesting to analyze.

Interesting results found in this investigation can also be helpful for further research. The fact that no resonance effects happened for vertical support stiffness of $5 \cdot 10^8$ N/m, in the Banafjäl bridge, even though very high resonance was found for stiffness of $1 \cdot 10^8$ N/m, was very surprising and more investigation on the subject should be undertaken. Moreover, the importance of higher modes of vibration in the magnitude of accelerations, as was obtained for trial cases 4 to 6 of the Lögdeälv bridge, may be a very interesting field of study.

The effect of the large amount of simplifications used for this study could be interesting to research, as well. Even though the Sagån bridge is transversally symmetrical, with one single track, a 3D analysis, accounting for transversal and torsional modes of vibration, could be used to compare with the results obtained in this study. Furthermore, as it was mentioned before, the bridge-embankment interaction clearly represents an elastic rotational spring at the end of the bridge deck. Investigations about the stiffness of that spring and the effect that rotational springs may have on the behavior of HSR bridges should bring up enough questions for a new MSc dissertation. Springs at the lower ends of the columns should also be accounted for. Moreover, accelerations, both on the bridge deck and on the ballast bed, and strains in the concrete were also registered in situ for the Sagån bridge. Studies on the accuracy of FE models to predict these field results can also be undertaken.

Other studies, such as the influence of vehicle-bridge interaction, distribution of axle loads through the ballast bed or the increase in structural stiffness due to the track-sleepers-ballast interaction, are ideas to try in any of the three bridges studied here, or even in others.

Nevertheless, the author hopes that the study undertaken here and the results obtained can be helpful to everyone interested in bridge dynamic response due to high-speed trains.
References


REFERENCES


REFERENCES


REFERENCES

[57] UIC. Code 776-1 R: Loads to be considered in railway bridge design, 1979.


Appendix A

Mode Shapes of the Banafjäl Bridge

![Stiff model (Mode 1: 2.36 Hz)]
- $1 \times 10^{10}$ N/m (Mode 1: 2.35 Hz)
- $1 \times 10^{9}$ N/m (Mode 1: 2.28 Hz)
- $1 \times 10^{8}$ N/m (Mode 1: 1.79 Hz)

![Stiff model (Mode 2: 9.17 Hz)]
- $1 \times 10^{10}$ N/m (Mode 2: 9.05 Hz)
- $1 \times 10^{9}$ N/m (Mode 2: 8.07 Hz)
- $1 \times 10^{8}$ N/m (Mode 2: 4.14 Hz)

![Stiff model (Mode 3: 19.73 Hz)]
- $1 \times 10^{10}$ N/m (Mode 3: 19.22 Hz)
- $1 \times 10^{9}$ N/m (Mode 3: 15 Hz)
- $1 \times 10^{8}$ N/m (Mode 3: 7.44 Hz)

![Stiff model (Mode 4: 30.57 Hz)]
- $1 \times 10^{10}$ N/m (Mode 4: 30.57 Hz)
- $1 \times 10^{9}$ N/m (Mode 5: 30.57 Hz)
- $1 \times 10^{8}$ N/m (Mode 6: 30.57 Hz)

Figure A.1: Modes shapes of the Banafjäl bridge (common to all models).

Figure A.2: Modes shapes of the Banafjäl bridge (common to all models).
Figure A.3: Modes shapes of the Banafjäl bridge (common to all models).

Figure A.4: Modes shapes of the Banafjäl bridge (only present in the stiffer models).

Figure A.5: Modes shapes of the Banafjäl bridge (only present in the stiffer models).
Figure A.6: Modes shapes of the Banafjäl bridge (only present in the less stiff models).

Figure A.7: Modes shapes of the Banafjäl bridge (only present in the less stiff models).
Appendix B

Mode Shapes of the Lögdeälv Bridge

Figure B.1: Modes shapes of the Lögdeälv bridge (common to all models).

Figure B.2: Modes shapes of the Lögdeälv bridge (common to all models).
Figure B.3: Modes shapes of the Lögdeälv bridge (common to all models).

Figure B.4: Modes shapes of the Lögdeälv bridge (only present in the stiffer models).

Figure B.5: Modes shapes of the Lögdeälv bridge (only present in the stiffest model).
Figure B.6: Modes shapes of the Lögdeälv bridge (only present in the less stiff models).

Figure B.7: Modes shapes of the Lögdeälv bridge (only present in the less stiff model).

Figure B.8: Modes shapes of the Lögdeälv bridge (only present in the least stiff model).
Appendix C

Mode Shapes of the Sagån Bridge

Figure C.1: Modes shapes of the Sagån bridge (common to all models).

Figure C.2: Modes shapes of the Sagån bridge (common to all models).
Figure C.3: Modes shapes of the Sagån bridge (common to all models).

Figure C.4: Modes shapes of the Sagån bridge (common to all models).

Figure C.5: Modes shapes of the Sagån bridge (only present in model 1).
Figure C.6: Modes shapes of the Sagån bridge (only present in model 1).

Figure C.7: Modes shapes of the Sagån bridge (only present in model 2 and 3).

Figure C.8: Modes shapes of the Sagån bridge (only present in model 2 and 3).
Appendix D

Drawings of the Sagån Bridge