



**KTH Architecture and
the Built Environment**



An innovative ground storage

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Master Thesis Project

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2007-2008

Acknowledgements

This master thesis is the result of my work at the Division of Building Technology, Department of Civil and Architectural Engineering at the Royal Institute of Technology, Stockholm, during the period between September 2007 and may 2008. I would like to express my gratitude to my supervision Prof. Gudni Johannesson for leading me in this research work with his experience and his creativity. I wish to thank also the PhD Student Marco Molinari for helping me during the whole period I worked on the project.

Stockholm, June 2008

Alberto Lazzarotto

Abstract

Solar energy is a very important source for the future development and has a very high potential. The main problems connected to its utilization are its low intensity and availability. Utilization of solar energy in the building sector is very interesting cause the main energy needs are for heating and cooling. By using low temperature heating system and high temperature cooling system the quality of the energy needed from the building can match the one that can be given by using solar energy. Then, the main issue is to manage to make the energy coming from the solar available when is needed. In order to make it possible, storage system are necessary. This master thesis aim is to make a study of a ground heat storage. The study consists in the development of a configuration for a ground heat storage, its modeling, and then the implementation of a computer program for the calculation of the performances of the system. For the modeling, has been important mainly the frequency domain approach that has permitted the implementation of fast programs. Two programs will be presented. The first one very fast, permits a rough analysis of the configuration and a first estimation of the system behavior; the second one, that is slower, permits to go more in detail, gives the possibility to change a lot of parameters that can have a big influence on the system performances. In this second program a lot of interest has been given to methods for the solution of large nonlinear system with the General minimal residual method. For the implementation of the programs and of the finite element simulation have been used Comsol Multiphysics and Comsol Script. The results obtained are interesting and they encourage the development of the tool and future investigation on this kind of system.

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Chapter 1

Introduction

This master thesis is focused on the investigation of the ground storage system. A brief introduction is given in this chapter to introduce the topic.

1.1 Ground heat storage

Ground temperature below a certain depth is almost constant all over the year. This is due basically to the fact that the soil mass is very large and then its thermal capacity is large as well. As a consequence, the temperature fluctuation at a certain depth is decreasing compared with the one of the surface. Moreover there is a time lag between these two temperature waves. That means that the soil temperature is higher than the air temperature in winter and is lower in summer [1]. For this reason the idea to use the soil as heat exchanger to preheat a working fluid in winter and precool it in summer has been developed. In the following sections it will be presented some results obtained in previous studies about thermal behavior of the ground and an overview of the existent kind of heat exchanger designed to exploit this source.

1.1.1 Ground thermal behavior

Several studies to investigate the thermal behavior of the ground have been led; both numerical [2, 3] and experimental [4] data are available in literature. Popiel performed experimental measurement with thermocouple in the ground for the period between summer 1999 and spring 2001 in Poznan, Poland. The conclusion drawn starting from the measurements are that three zones in the ground can be distinguished:

1. *Surface zone* from 0 to 1 m of depth; the soil temperature is sensitive to short time changes of outside temperature.
2. *Shallow zone* from 1 to 8 m (for dry soils) of depth or 20 m (for moist heavy soils); the ground temperature is almost constant and close to the average annual temperature.
3. *Deep zone* below 8m or 20m; the ground temperature is constant and rising very slow with depth because of the geothermal gradient.

1.1.2 Types of ground heat exchanger

To exploit the ground thermal capacity it is necessary an heat exchanger. Basically it consists in buried pipes in the ground. The useful zones, following previous description, are the shallow zone and the deep zone. To exploit the deep zone is necessary to drill the ground with very large vertical borehole and that is worthy just if the geothermal gradient of temperature is significant, otherwise it's a very expensive practice. The other possibility is to exploit the shallow zone. In this second case, the way to exploit it is by burying horizontal pipes in the ground. The system ground pipes behaves like an heat exchanger. In general there are two types of those ground heat exchanger. The classification is between open and closed systems [1]. In an open system the working fluid that is exchanging heat with the ground is the same that after can be used for space heating or cooling. In the closed system the media is used as a source to exchange heat with another fluid. This way has some drawbacks cause another heat exchanger is required, and this introduce new irreversibility then the energy that can be exploited is less then the one of a direct system. On the other hand, by using an indirect system air quality in the secondary loop is not needed. Moreover, the air can be recirculated in the loop with a large influence on the overall performance of the system. By recirculating air in the loop during winter, low inlet temperature in the system are avoided, and then it is realized the possibility to make the system able to supply air at interesting temperature for a longer period.

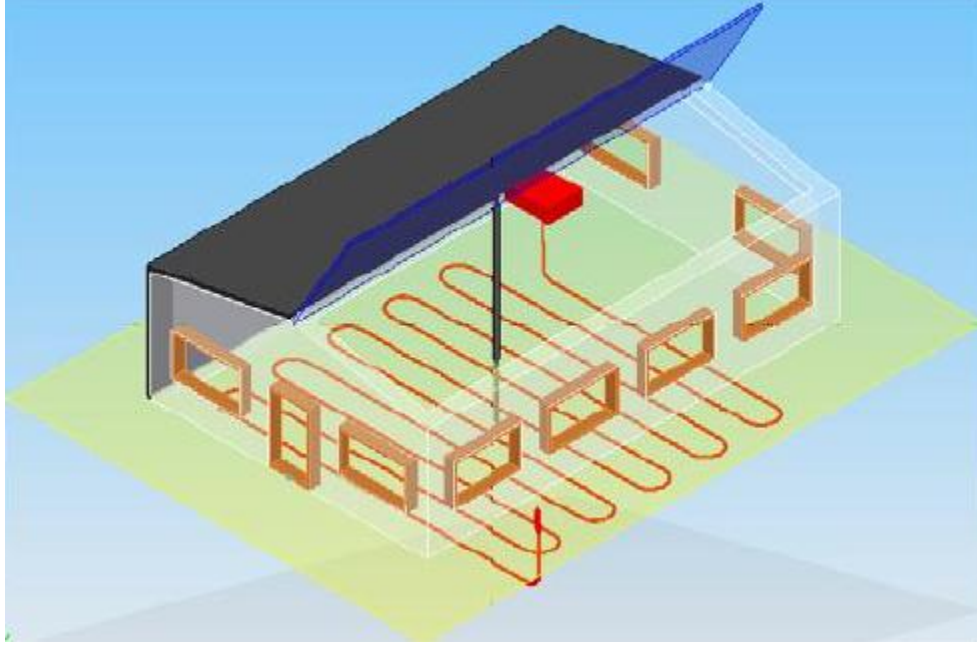


Figure 1.1: System studied by Jorge Noguera[5]: air solar collector to collect heat, and horizontal piping in the ground to store it in the ground.

1.1.3 Heat pumps

Heat pumps are system designed to transfer heat from a colder source at a temperature T_{source} to a warmer one at temperature T_{sink} , as a consequence of a work input. In order to do that a thermodynamical cycle between the temperature T_1 and T_2 is necessary. The efficiency of a thermodynamical system is upper limited by the Carnot efficiency that for a heat pump can be expressed as follow:

$$\eta_C = \frac{T_1}{T_1 - T_2}$$

The system efficiency is highly dependent on T_1 and T_2 , and is much more efficient if T_2 is close to T_1 . T_1 is fixed by the chosen application: if the aim of the heat pump is to warm air at the temperature of $30C$, the temperature T_1 will be slightly higher than this. T_2 , on the other hand, is depending on the available heat source. It is very common to couple a heat pump with a ground thermal storage. Heat pumps are very efficient when they works in nominal condition, but if they are working in a different way the COP decreases a lot [6] .

Then, it is very important to have a stable source. Geothermal source heat pumps usually work in this way. During summer the ground is colder than the outdoor air then the heat is pumped in the soil; in winter there is the opposite situation, then the heat is pumped from the ground to the building. The heat pump in summer charge the ground with the heat pumping heat from the house to the soil; and in winter the heat pump discharge the ground to heat the house. In case like Sweden, in which there is almost no need for air conditioning not a big amount of heat can be pumped in the ground from the house then a system to replace that case can be interesting.

1.1.4 Why air?

In the present study the configuration is an horizontal loop of buried pipes. For this first study the working fluid will be air. The choice of air is due to different reasons, but mainly because of his very low density and then because of the possibility to drive it inside the pipes with much lower energy consumption compared with other fluids that has higher density like water. On the other hand the cp of air is very low then a larger mass is necessary to transfer the same heat quantity. Moreover, air is very clean and if moisture formation is prevented there is no danger of corrosion. It's preferable the option of closed system because sanitary problem can be connected to the direct use of the air injected in the pipes and also, as said before, it can be advantageous because in that way air can be recirculated.

1.1.5 Air solar collector

The target to achieve is storing some heat in the soil, therefore a possible solution to gain the performance of the system can be couple a solar collector with the ground. The solar collector can be installed in the south facade of the house and in the south side of the roof. Therefore, the surface that can be exploited is very high. From the wideness of the collector depends the amount of solar radiation that can be exploited. It's hard to

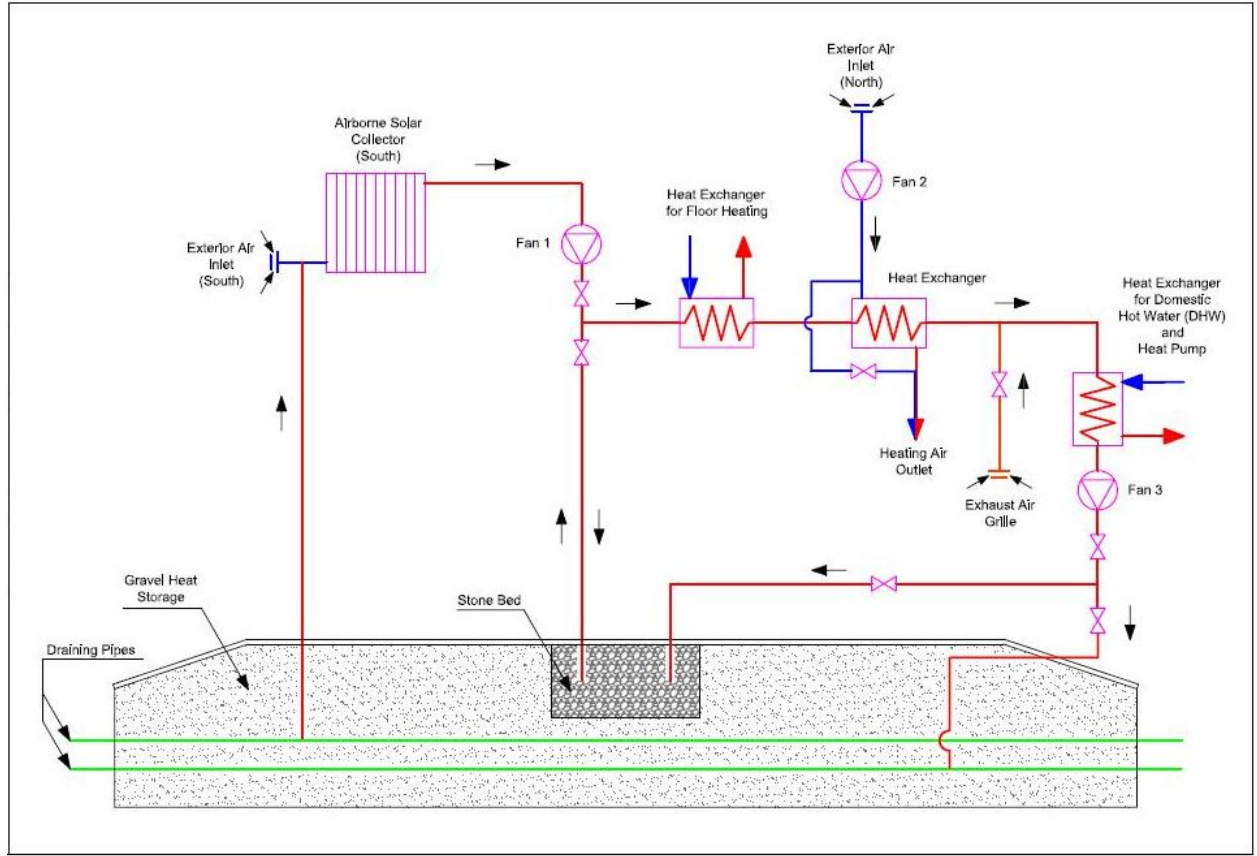


Figure 1.2: Calidhogar house by Jorge Noguera [5]

enhance performance of solar collector but it's easy to increase the receiving surface, then, it's always convenient to make it larger. In the present work the attention will be focused on a system in which a solar collector is coupled with a ground heat storage. A previous study of such a system has been done by Noguera [5]. In figure 1.2 is presented the system configuration of the Calidhogar house an example of low-energy building. The attention in the next chapters will be focused on the study of a similar configuration.

Chapter 2

The ground heat storage model

2.1 Objective and methodology

In order to investigate the ground storage performances has been chosen the development of a model able to describe the thermal behavior of the ground. That could be very interesting mainly to study thermal losses, and to investigate how much the thermal inertia of the ground can be exploited. The analysis of a ground storage involve a big volume in the ground therefore it is very expensive from a computational point of view to make an analysis in the time domain. And if it is considered that basically the heat transfer in the ground is by conduction, the partial differential equation that describe the phenomenon is linear. This permits to apply the superposition method and then the Fourier analysis as it will be explained hereinafter. The configuration of the case studied is a pipe embedded in the ground with insulation on top. To reduce the computational costs first it is performed a bi-dimensional analysis of the cross section of the ground in the frequency domain. Using this kind of analysis it is possible to get the parameter of the system (admittances and transmittances); and therefore describing the phenomenon with different input signals. Before starting the description of the model it is necessary to give few pieces of information about the frequency domain analysis.

2.2 Linear system and Fourier analysis

2.2.1 LTI systems

The choice of the analysis typology depends on the characteristics of the problem to face. Basically, these are related to the kind of differential equations that characterize the system. The simplest system is the LTI system (linear, time invariant)[7]. This is characterized by two features:

- linearity: the system satisfies the superposition property. For instance, if $x_1(t)$ and $x_2(t)$ are two input signals and $y_1(t)$ and $y_2(t)$ are the output then $y(t) = y_1(t) + y_2(t)$ is the output signal of $x(t) = x_1(t) + x_2(t)$.
- time invariance: it means that, if an input signal is applied at the instant 0 or T second later the output will not be affected and will be identical except for the time delay.

For a system that accomplishes those properties it is sufficient a single function to define it. This function is the impulse response $h(t)$. If $h(t)$ is known, it is possible to find the response of the system $y(t)$ to an input signal $x(t)$ by the operation of convolution between $x(t)$ and $y(t)$.

$$y(t) = h(t) * x(t)$$

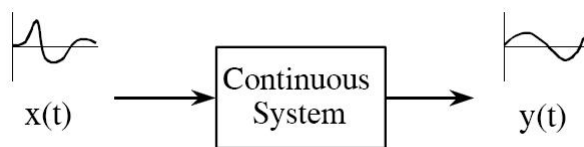


Figure 2.1: A system is any process that causes an output signal as a response of an input signal

The same thing can be done in the frequency domain. The system will be characterized by the transfer function $H(s)$ where s is a complex number. This is the Laplace transform

of $h(s)$. To get the output signal is sufficient to multiply the Laplace transform of the input signal $X(s)$ by the transfer function $H(s)$.

$$Y(s) = H(s) \cdot X(s)$$

If the complex variable s is $i \cdot \omega$ the transform is a Fourier transform. Basically, the advantage in the frequency domain is that to find the response of the system instead of the operation of convolution that was used in time domain, multiplication is used, and of course it's much easier to deal with multiplication. To go back in the time domain the inverse Fourier transform is used.

2.2.2 Fourier analysis

The Fourier analysis is the decomposition of a signal as a sum of sinusoidal waves with different frequency.

The Fourier analysis can be applied to continuous and discrete signals, both periodic and

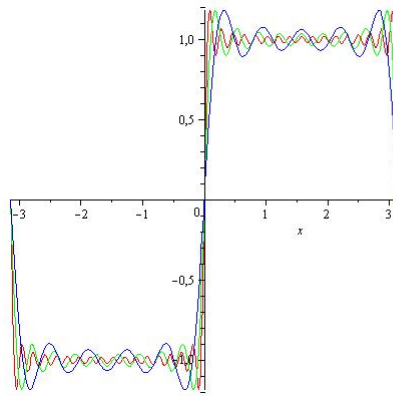


Figure 2.2: Fourier series of a square wave

aperiodic. In the present work the analyzed signals are periodic and discrete. Therefore, we will concentrate on it but it is useful to briefly introduce the continuous signals as well.

Continuous periodical function can be expressed as Fourier series.

$$f(t) = \frac{1}{2} \cdot a_o + \sum_{n=1}^{\infty} [a_n \cdot \cos(\omega_n \cdot t) + b_n \cdot \sin(\omega_n \cdot t)]$$

Where $\omega_n = n \cdot 2 \cdot \pi / T$ with $(n = 0, 1 \dots)$. ω_1 is the first harmonic and all the other harmonics are multiple integer of the first one. For each harmonic the knowledge of the coefficient a_n and b_n make it possible to rebuild the original signal.

For each harmonic the term is valid the expression

$$a_n \cdot \cos(\omega_n \cdot t) + b_n \cdot \sin(\omega_n \cdot t) = C_n \cdot \cos(\omega_n \cdot t + \varphi_n)$$

For a given ω_n the wave can be represented with a complex number that give information about the phase and the magnitude. By working in this way the result will be one complex number for each harmonic. It is possible to represent the module and the phase as a function of the frequency. For example in figure 2.3a it is represented the function $2 \cdot \sin(x - 0.5) + 0.5 \cdot \sin(3 \cdot x + 0.2)$ and in figure 2.3b there is its representation, module and phase, in the frequency domain.

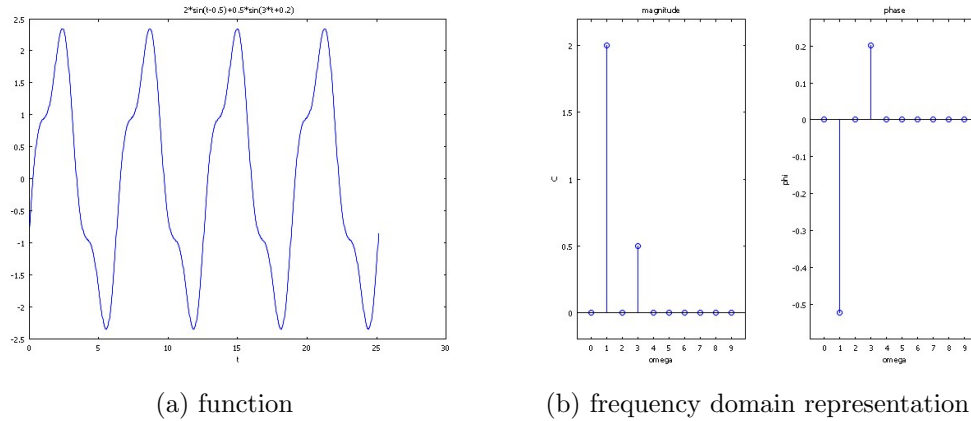


Figure 2.3: time domain and frequency domain representation

2.2.3 Discrete Fourier transform

The discrete Fourier transform (DFT) is the tool necessary to work with discrete periodical functions. DFT is a transform that given a series of number, that usually are samples taken with a frequency f_s from a continuous function, gives a series of complex coefficient that represents an approximation of the coefficient of the Fourier series of the continuous signal. The number of frequencies that can be analyzed is limited theoretically from the sampling frequency. It is not possible to go further than the Nyquist frequency that is $f_s/2$ for the Nyquist theorem because otherwise the result signal will be affected by the aliasing phenomenon.

2.3 Formulation of the problem

The formulation that describes the conduction in non steady-state condition is the following

$$\rho \cdot c_p \cdot \frac{\partial \theta}{\partial t} = k \cdot \nabla^2 \theta + \varphi \quad (2.1)$$

it can be expressed also introducing diffusivity as

$$\frac{\partial \theta}{\partial t} = a \cdot \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) + \frac{\varphi}{\rho \cdot c_p} \quad (2.2)$$

considering that diffusivity is $a = k/(\rho \cdot c_p)$.

It is not easy to find a solution of this equation in general conditions. In this section it will be presented a way to solve it in a bi-dimensional domain limiting the temperature variation at the boundary to harmonic functions and considering that there is no heat generation ($\varphi = 0$). The equation is linear, and therefore the variation of temperature inside the system will be an harmonic function with the same frequency as the harmonic waves imposed at the boundary.

$$\theta = (u + i \cdot v) \cdot e^{i \cdot \omega \cdot t}$$

And introducing this in (2.2)

$$i \cdot \omega \cdot (u + i \cdot v) \cdot e^{i \cdot \omega \cdot t} = a \cdot (\nabla^2 u + i \cdot \nabla^2 v) \cdot e^{i \cdot \omega \cdot t} \quad (2.3)$$

$$\frac{i \cdot \omega \cdot u}{a} - \frac{\omega \cdot v}{a} = \nabla^2 u + i \cdot \nabla^2 v$$

Simplifying the term $e^{i \cdot \omega \cdot t}$ in (2.3) the equation is not time dependent anymore, and the problem is reduced to a steady state problem. Then, it is possible to solve separately the equation for the real part and for the imaginary part. Therefore, the problem will be a system of two differential equation with two unknown quantities that are the real and the imaginary part of the temperature.

$$\begin{cases} \nabla^2 u = -\frac{\omega \cdot v}{a} \\ \nabla^2 v = \frac{\omega \cdot u}{a} \end{cases} \quad (2.4)$$

With this formulation [8] the problem can be solved with a finite element code capable to couple two differential equations. The solution of the problem will be the field of the real part and the imaginary part of the temperature. This means that for each point of the field it is known the magnitude and the phase of the sinusoidal wave that represents the temperature. These sinusoidal waves, as said before, have the same frequency as the sinusoidal variation of temperature imposed at the boundary. Therefore, for each point of the field, it is possible to rebuild a time domain solution. This bi-dimensional formulation can be suitable in the analysis of the ground. In the chosen situation there is a pipe embedded in the ground. As long as with such a configuration the temperature gradient

along the pipe axis is not that high compared with the one in radial direction we can do the assumption of radial heat transfer from the pipe to the ground. Under these conditions, the bi-dimensional formulation just showed is suitable for the description of the heat losses to the ground.

2.4 Cross section study

The cross section study consists in the quantitative evaluation of the heat flux that can be exchanged between embedded pipes and ground. In order to get a solution it is necessary to have a well defined finite element problem. This means imposing a formulation to define the problem inside the domain studied and at the boundaries. By doing that, the solution of the problem will be unique. For the domain formulation the problem has been defined in the previous sections. Regarding boundary conditions, they have to be defined being coherent with the physics of the problem therefore some consideration about the configuration and the consequent conditions that can be assumed have to be done. We can start by noticing that we are studying a serpentine, and instead of studying the whole cross section of the ground with all the loops, the best thing is to study just one pipe and consider thermal insulation on the lateral boundaries. This is a good approximation, in fact as said just before the magnitude of the temperature differences along the pipe in one loop is not very big, and that means that at a certain distance from the pipes there is no temperature gradient and therefore no heat flux. Moreover, the temperature at a depth of 6 meters can be considered more or less constant, and equal to the average external temperature of the year. To simulate this condition, considering the fact that we are going to make the study in the frequency domain for ω equal to 0 the condition is constant temperature equal to the average external temperature during the whole year and for other frequencies is just thermal insulation. In figure 2.4 is showed the model used.

The boundaries that are missing are the upper one, between the ground and the air that is on top and the internal boundary, the internal circle that represent the pipe cross

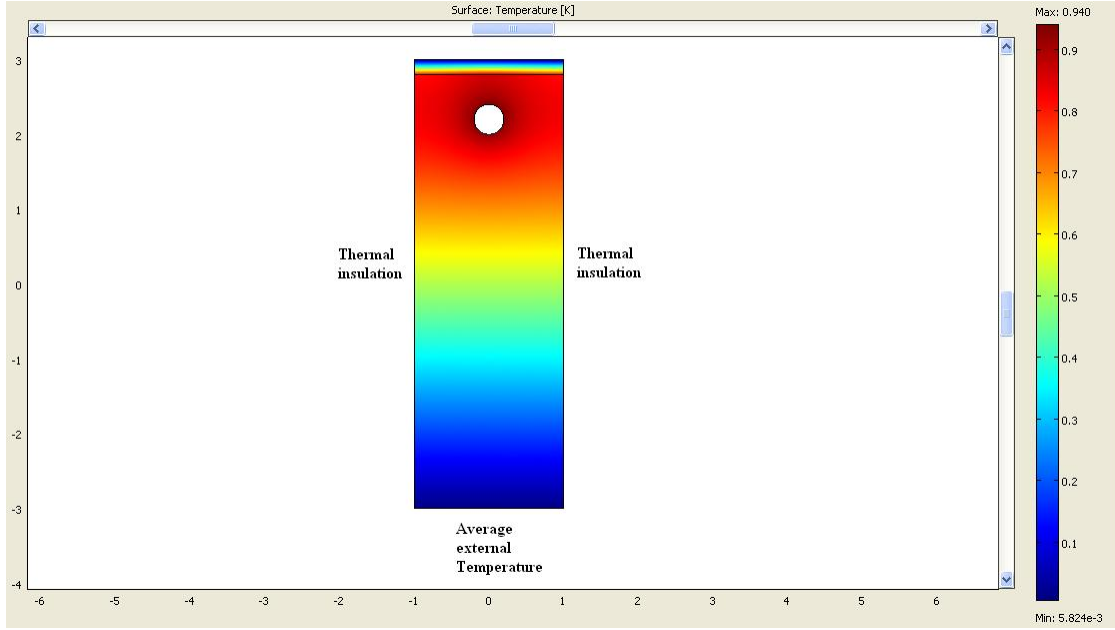


Figure 2.4: Model used

section. Therefore, the boundary condition on the upper side is related to the outside air temperature, and the internal one is related to the temperature of the air inside the pipe. The boundary condition chosen is always a Neumann condition, that means that the external ambient temperature and the heat transfer coefficient are assigned and in this way information about heat transfer through the boundary are given.

$$q = h \cdot (T_{amb} - T_{boundary})$$

The outside temperature and the temperature along the pipe are actually the two variables that influence the heat transfer phenomenon in the study case. Therefore, those two variables can be seen as our input signals.

Another aspect that plays an important role in the cross section analysis is the linearity of the problem. This is exploited in different ways:

- Separations of the effects: The effect of the two input signals are studied separately and afterward the total result is found by superposition.

- FFT: The input functions are just sampled continuous function. The whole continuous function is not available. By using the Fast Fourier transform the transformation of the signals in the frequency domain is achieved. The finite element study can be done for each frequency separately, and the time depending problem is reduced to a finite number of steady state problems.
- Usually, in a certain instant, the external temperature is constant along the all pipe, but the temperature inside the pipe is not constant at all. To have a general description of the heat flux trough the pipe boundary, independent from the magnitude of the temperature inside of it, the only way it is to use a unitary input signal for each frequency. Thanks to the linearity of the problem, the response to the unitary input signal multiplied by the Fourier coefficients gives as result the heat flux due to the real input signal. Moreover, that kind of approach is used for both the input signals, because thereby, the same configuration of the system can be studied under different conditions (different input signals), with just one finite element simulation.

Summarizing, for each frequency analyzed two simulations are run: one with $T_{out} = e^{i \cdot \omega \cdot t}$ and $T_x = 0$ where T_{out} is the external temperature and T_x the temperature inside the pipe at a certain length x , and the other one with $T_{out} = 0$ and $T_x = 1e^{i \cdot \omega \cdot t}$. From the FEM simulation, as a result, we can get temperature field, both real and imaginary part. By integrating the heat flux on the pipe boundary we get the heat flux per meter through the pipe at certain length x of it. In figure2.5 there is an example of such a kind of simulation for one harmonic. In the two upper figure there is a representation of the fields of real and imaginary part of the temperature when there is a wave of temperature with module one just inside the pipe, whereas the other two pictures are the same temperature fields but when the input wave is just outside the ground. From the integration of the heat flux field along the pipe boundary two parameters are founded. In the following work these two parameter will be called E and F . E is the heat flux trough the pipe when the temperature inside the pipe is a sinusoidal wave with unitary module and the outdoor

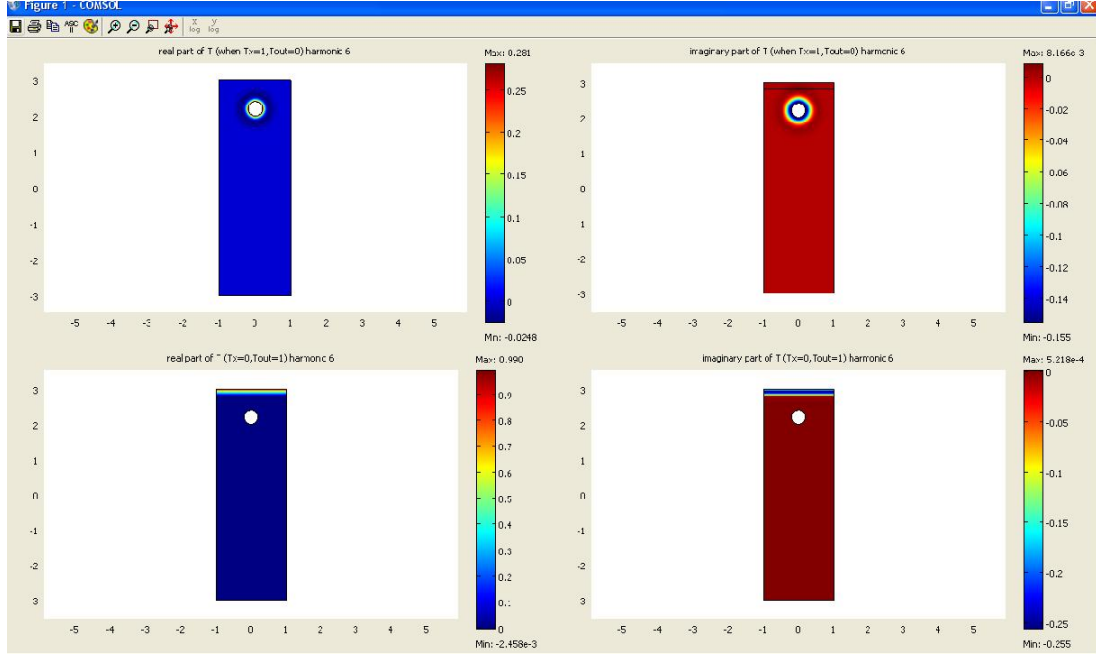


Figure 2.5: Example of the result of one simulation with these kind of conditions in this case for the 6th harmonic

temperature is constant and equal to zero. F is the heat flux through the pipe when the outdoor temperature is a sinusoidal wave with unitary module and the temperature inside the pipe is zero. Both E and F are complex numbers. For a given ω with such parameters the calculation of the total flux can be done as it follows

$$Q(\omega) = T_{in}(\omega) \cdot E(\omega) + T_{out}(\omega) \cdot F(\omega)$$

2.5 Analysis along the pipe

Once that the parameters E and F are calculated we can make a heat balance in a control volume as follows.

$$u \cdot \rho \cdot c \cdot A \cdot (T_{x+dx} - T_x) + i \cdot \omega \cdot \rho \cdot c \cdot A \cdot dx + dx \cdot E \cdot T_x + dx \cdot F \cdot T_{out} = 0 \quad (2.5)$$

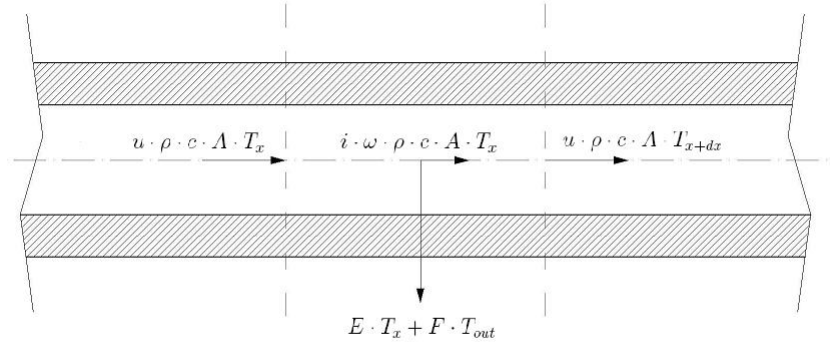


Figure 2.6: Heat balance in the control volume

$$T_{x+dx} = T_x + \frac{\partial T_x}{\partial x} \cdot dx$$

$$u \cdot \rho \cdot c \cdot A \cdot \frac{\partial T_x}{\partial x} + i \cdot \omega \cdot \rho \cdot c \cdot A \cdot T_x + E \cdot T_x + F \cdot T_{out} = 0 \quad (2.6)$$

$$\frac{\partial T_x}{\partial x} = \frac{-F \cdot T_{out}}{(i \cdot \omega \cdot \rho \cdot c \cdot A + E) \cdot T_x}$$

The general integral is

$$T_x(x) = C \cdot \exp \left\{ \frac{-F \cdot T_{out}}{(i \cdot \omega \cdot \rho \cdot c \cdot A + E)} \cdot x \right\} + D$$

By putting the initial condition $T_x(0) = T_0$ it follows that

$$C \cdot \exp \left\{ \frac{-F \cdot T_{out}}{(i \cdot \omega \cdot \rho \cdot c \cdot A + E)} \cdot 0 \right\} + D = C + D = T_0$$

For $x \rightarrow \infty$ it follows that $\frac{\partial T_x(\infty)}{\partial x} = 0$

$$i \cdot \omega \cdot \rho \cdot c \cdot A \cdot T_x(\infty) + E \cdot T_x(\infty) + F \cdot Tout = 0$$

$$T_x(\infty) = \frac{-F \cdot Tout}{i \cdot \omega \cdot \rho \cdot c \cdot A + E} = D$$

$$C = T_0 - \frac{-F \cdot Tout}{i \cdot \omega \cdot \rho \cdot c \cdot A + E}$$

By replacing the constants in the general Integral it follows the Analytical solution of the problem

$$T_x(x) = (T_0 + \frac{F \cdot Tout}{i \cdot \omega \cdot \rho \cdot c \cdot A + E}) \cdot e^{\frac{-F \cdot Tout}{(i \cdot \omega \cdot \rho \cdot c \cdot A + E)} \cdot x} + \frac{-F \cdot Tout}{i \cdot \omega \cdot \rho \cdot c \cdot A + E} \quad (2.7)$$

By having an analytical solution of such a kind of problem it follows that the whole computational problem is getting the parameters E and F by Finite Element simulation, and once that those are known the temperature T_x can be calculated in a period of time that can be neglected. The analytical solution shown now permit therefore the calculation of the Fourier coefficient of the signal T_x . By having the frequency domain representation of the signal the time domain is calculated by Inverse Fast Fourier Transform.

Chapter 3

Utilization of Multiphysic

3.1 Introduction

Multiphysic is a Finite Element Method code. The program basically permits to solve Ordinary differential equation and Partial differential Equation. It can perform either bi-dimensional or three-dimensional analysis. Moreover, it can also solve system of equations [10]. In this work Multiphysics has been used for the cross section study. The geometry is bi-dimensional and the drawing of the geometry has been done with a CAD integrated in Multiphysic. The draw is a rectangular with a hole, as seen in the first chapter. Inside this domain will be calculated numerically the solution of equation 2.4. The domain is divided in two zones. They are called subdomains. In figure 3.1 is shown how the geometry looks like and it can be noticed the division in subdomains.

For each subdomain different conditions can be set up. In this case the upper rectangle is the layer of insulation between the ground and the external ambient. The other subdomain is the remaining part and represents the ground. The equations solved are the same in both the subdomains. The differences are the material properties. Density , specific heat coefficient and conductivity are specified to distinguish the behavior of the insulation layer to the one of the ground.

In figure 3.2 there is one example of how has been done the subdomain setting. The equations to solve are the following.

$$\begin{cases} k \cdot \nabla^2 u = -\rho \cdot cp \cdot \omega \cdot v \\ k \cdot \nabla^2 v = \rho \cdot cp \cdot \omega \cdot u \end{cases} \quad (3.1)$$

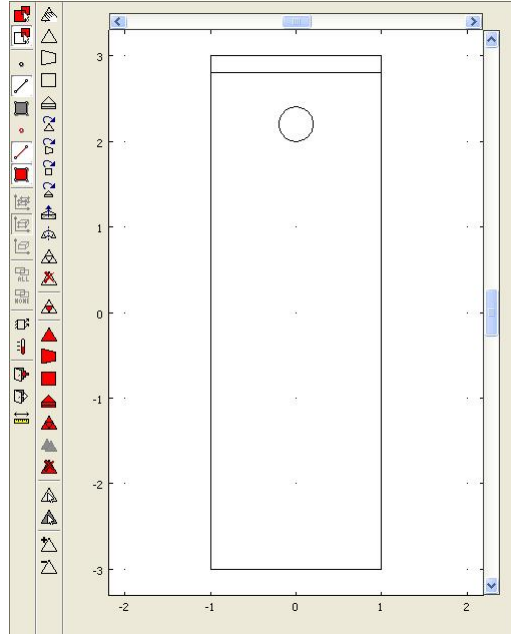


Figure 3.1: Geometry

This is a system of equations and in order to solve it, two Multiphysics models are necessary. One with u as independent variable and the other with v . They are real and imaginary part of the temperature for a given frequency $T = u + i \cdot v$. The program works in parallel and the two fields are found at the same time. Comsol Multiphysics is basically a PDE solver but inside the program there are different features that make it more user-friendly for a direct application to a specific field. The application mode chosen has been 'Heat transfer by conduction in steady state' [11, 12]. In this mode basically the equation solved in the subdomains is

$$k \cdot \nabla^2 u = Q$$

where Q is a heat source and T is the unknown quantity. The two Multiphysics models are set up in in order to have u as unknown quantity in one of them and v in the other

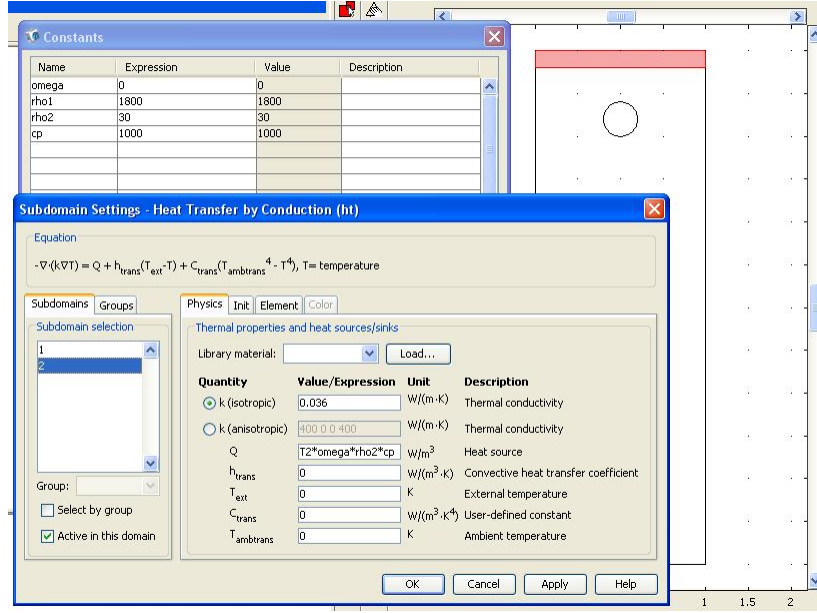


Figure 3.2: Subdomain Setting example

one. And, by setting the heat source term in the first case equal to $-\rho \cdot cp \cdot \omega \cdot v$ and in the second case equal to $\rho \cdot cp \cdot \omega \cdot u$, the two fields are linked and the program will solve the both at the same time.

3.2 Verification

The problem of working with simulation tools like finite element codes is that is very easy to make mistakes and it is always very important to ensure the propriety of the results. Moreover, the results of one simulation at a given frequency are the fields of real and imaginary part, therefore the field of Fourier coefficient for that frequency. This result is not really suitable for interpretation because it's abstract. A time domain representation gives us more information than it, even if the information contained in is the same. In order to understand if what we are doing is correct the best thing is to start from a very simple model and verify the numerical solution with an analytical one.

3.2.1 Analytical solution

The case chosen is heat conduction in a semi-infinite slab making the hypothesis of no heat sources within the domain. The only input signal is at the boundary and it is a sinusoidal wave at frequency ω . The temperature inside the slab varies in time and along the x coordinate. The equation that describes the phenomenon is the Fourier law

$$\frac{\delta T(x, t)}{\delta t} = a \cdot \frac{\delta^2 T(x, t)}{\delta x^2} \quad (3.2)$$

Utilizing always the same strategy and using harmonic function as input signals at the boundary the equation can be reduced from PDE to ODE. The input signal is an harmonic function with frequency ω , therefore in all the points of the domain the solution will have the following form

$$T(x, t) = T_{Amp} \cdot e^{i \cdot (\omega \cdot t + \phi)} = (u + i \cdot v) \cdot e^{i \cdot (\omega \cdot t)} = T(x) \cdot e^{i \cdot (\omega \cdot t)}$$

The first derivative of an harmonic function is the initial function multiplied by $i \cdot \omega$

$$\frac{\delta T(x, t)}{\delta t} = i \cdot \omega \cdot T(x, t)$$

Replacing this term in equation 3.2

$$i \cdot \omega \cdot T(x, t) = a \cdot \frac{\delta^2 T(x, t)}{\delta x^2} \quad (3.3)$$

As done in chapter 2 simplifying the exponential term we can make the equation dependent just on the variable x .

$$i \cdot \omega \cdot T(x) \cdot e^{i \cdot \omega \cdot t} = a \cdot \frac{\delta^2 T(x)}{\delta x^2} \cdot e^{i \cdot \omega \cdot t}$$

$$\frac{\delta^2 T(x)}{\delta x^2} - \frac{i \cdot \omega \cdot T(x)}{a} = 0 \quad (3.4)$$

This equation is an ordinary differential equation in the variable x . The solution is in the form of $T = e^{\lambda \cdot x}$. It follows that

$$e^{\lambda \cdot x} \cdot \left(\lambda^2 - \frac{i \cdot \omega}{a} \right) = 0$$

The term $c = -\frac{i \cdot \omega}{a}$ is a constant complex quantity. The equation can be rewritten in this way

$$e^{\lambda \cdot x} \cdot (\lambda^2 + c) = 0 \Rightarrow \lambda = \pm i \cdot \sqrt{c}$$

$$\pm \sqrt{c} = \pm (1 - i) \cdot \sqrt{\frac{\omega}{2 \cdot a}}$$

In such a case the general integral of the differential equation has the following form

$$T = C_1 \cdot \cos \left((1 - i) \cdot \sqrt{\frac{\omega}{2 \cdot a}} \cdot x \right) + C_2 \cdot \sin \left((-1 + i) \cdot \sqrt{\frac{\omega}{2 \cdot a}} \cdot x \right)$$

$$T = C_1 \cdot \cos \left((1 - i) \cdot \sqrt{\frac{\omega}{2 \cdot a}} \cdot x \right) - C_2 \cdot \sin \left((1 - i) \cdot \sqrt{\frac{\omega}{2 \cdot a}} \cdot x \right)$$

Sinusoidal functions with a complex argument can be transformed to exponential function with complex argument by using the Euler formula

$$e^{i \cdot \omega x} = \cos x + i \cdot \sin x$$

Therefore, we can link a sinusoidal function with a complex argument to an exponential functions, and therefore to hyperbolic functions. In our case the relations in which we are interested, are the following

$$\cos(x) = \cosh(ix)$$

$$\sin(x) = -i \cdot \sinh(ix)$$

$$T(x) = C_1 \cdot \cosh \left((1+i) \cdot \sqrt{\frac{\omega}{2 \cdot a}} \cdot x \right) + C_2 \cdot i \cdot \sinh \left((1+i) \cdot \sqrt{\frac{\omega}{2 \cdot a}} \cdot x \right)$$

To find the constant of the general integral, we have to put the necessary boundary conditions on the problem. For $x = 0$ the initial condition is $T(0) = T_0$. Because of the properties of hyperbolic functions $\sinh(0) = 0$ and $\cosh(0) = 1$. It follows

$$T(0) = C_1 \cdot \cosh(0) + C_2 \cdot i \cdot \sinh(0) = C_1$$

Applying the fourier law

$$q(0) = -\lambda \cdot \left(\frac{\delta T}{\delta x} \right)_{x=0} = -\lambda \cdot C_2 \cdot i \cdot (1+i) \cdot \sqrt{\frac{\omega}{2 \cdot a}} \cdot \sinh \left((1+i) \cdot \sqrt{\frac{\omega}{2 \cdot a}} \cdot 0 \right) + \dots \sinh(0)$$

Therefore

$$C_2 = \frac{-q_0}{\lambda \cdot i \cdot (i+1) \cdot \sqrt{\frac{\omega}{2 \cdot a}}}$$

Now it is possible to express the temperature of the slab as function of the temperature and the heat flux on the surface

$$T(x) = T_0 \cdot \cosh \left((1+i) \cdot \sqrt{\frac{\omega}{2 \cdot a}} \cdot x \right) - q_0 \cdot \frac{i \cdot \sinh \left((1+i) \cdot \sqrt{\frac{\omega}{2 \cdot a}} \cdot x \right)}{\lambda \cdot i \cdot (i+1) \cdot \sqrt{\frac{\omega}{2 \cdot a}}} \quad (3.5)$$

In the case of semi infinite body, for $x \rightarrow \infty$, $T(x) \rightarrow 0$. In such a condition

$$q_0 = \frac{\lambda(i+1) \cdot \cosh \left((1+i) \cdot \sqrt{\frac{\omega}{2 \cdot a}} \cdot x \right)}{\sinh \left((1+i) \cdot \sqrt{\frac{\omega}{2 \cdot a}} \cdot x \right)}$$

Additionally for $x \rightarrow \infty$, $\sinh(x) \rightarrow \cosh(x)$. Then the expression for the heat flux at the surface become

$$q_0 = \lambda \cdot (i+1) \cdot \sqrt{\frac{\omega}{2 \cdot a}} \cdot x$$

Replacing this term in equation 3.5

$$\begin{aligned} T(x) &= T_0 \cdot \left\{ \cosh \left((1+i) \cdot \sqrt{\frac{\omega}{2 \cdot a}} \cdot x \right) - \sinh \left((1+i) \cdot \sqrt{\frac{\omega}{2 \cdot a}} \cdot x \right) \right\} \\ &\Rightarrow T(x) = T_0 \cdot e^{(1+i) \cdot \sqrt{\frac{\omega}{2 \cdot a}} \cdot x} \end{aligned} \quad (3.6)$$

Equation 3.6 is the analytical solution for of the present study case [13]. That result will be used to control the numerical solution in order to understand if the problem has been set up in the correct way.

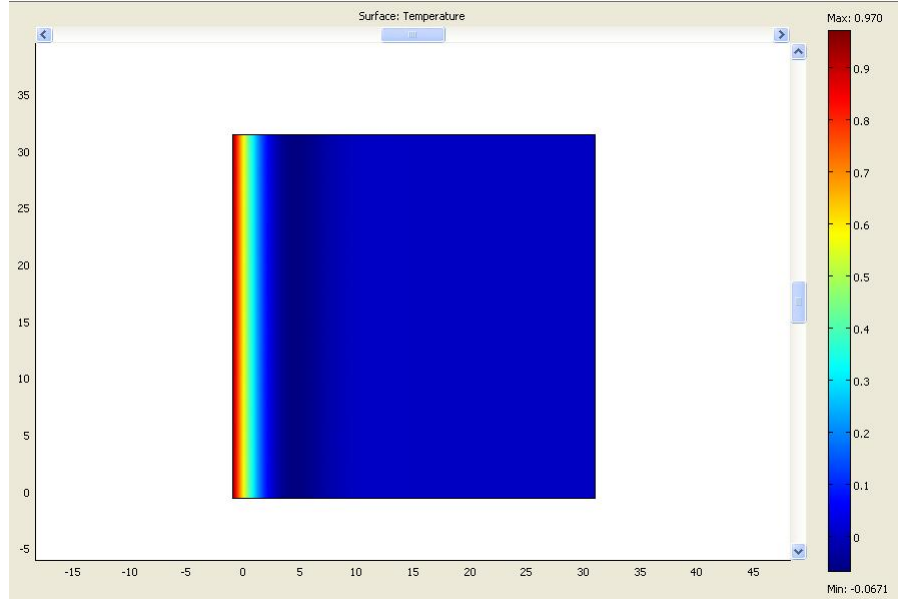


Figure 3.3: Slab analyzed

3.3 Test Model

To model a semi infinite slab we consider a big square with side of more than 30 m and we put a unitary sinusoidal wave at a given frequency on one side and thermal insulation on the other three sides.

The scheme is the one in figure 3.3. In this case we are interested in the temperature along the x coordinate. In order to find it, it can be used a postprocessing tool called cross section plot parameter. With that tool we get exactly the plot along a given straight line of the temperature.

In figure 3.4 there is a description of how it works. On the left, there is the 2D surface plot and a red line that represent our cross line. And on the right, there is the plot of the parameter along the red straight line. That is basically the configuration used for the test. The equation 3.6 can be written as follow to highlight real and imaginary part.

$$T(x) = T_0 \cdot e^{-\sqrt{\frac{\omega}{2 \cdot a}} \cdot x} \cdot \left[\cos \left(\sqrt{\frac{\omega}{2 \cdot a}} \cdot x \right) - i \cdot \sin \left(\sqrt{\frac{\omega}{2 \cdot a}} \cdot x \right) \right] \quad (3.7)$$

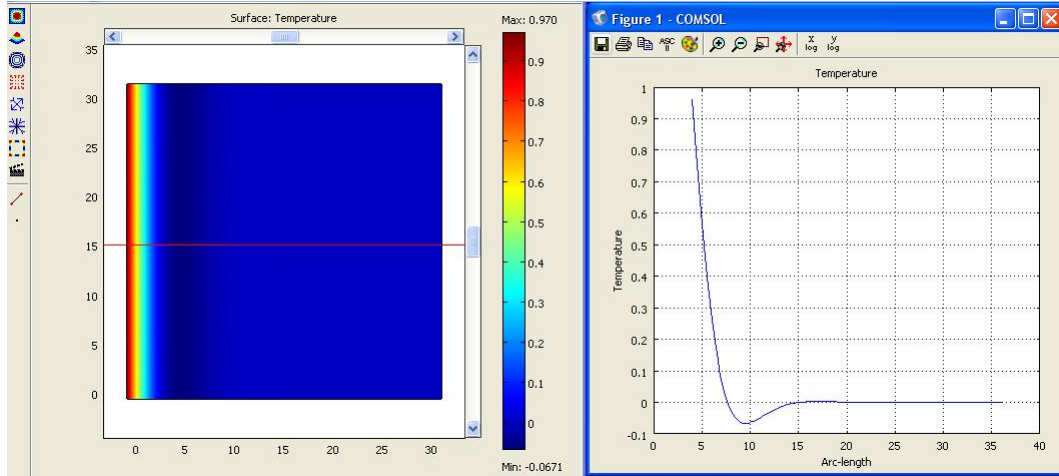


Figure 3.4: Plot section parameter

By using COMSOL we can plot functions. In figure 3.5 there is the plot of real part, imaginary part and of the module of the function described by equation 3.7.

To test the numerical solution we can plot in the same graph the numerical and the analytical solution. By doing this test, we can see from figure 3.6 that they are very close. The test has been done both for the real and for imaginary part.

3.4 Utilization of Comsol script

The package Comsol includes Comsol Multiphysics and Comsol Script. The first as seen before is a software with graphical interface for finite element simulations; the latter is an environment where it is possible to run scripts. The two programs are interfaced each other. The simulation in Multiphysics can be saved in '.mph' format or in '.m format' [10]. If a simulation is saved in the '.m' format it can be run with Comsol Script. This file can be open from a text editor. Inside of it there will be the code that contains the whole information regarding settings of the simulation (geometry, boundary condition, postprocessing parametr ecc.). Therefore, the set up of the problem can be done directly in

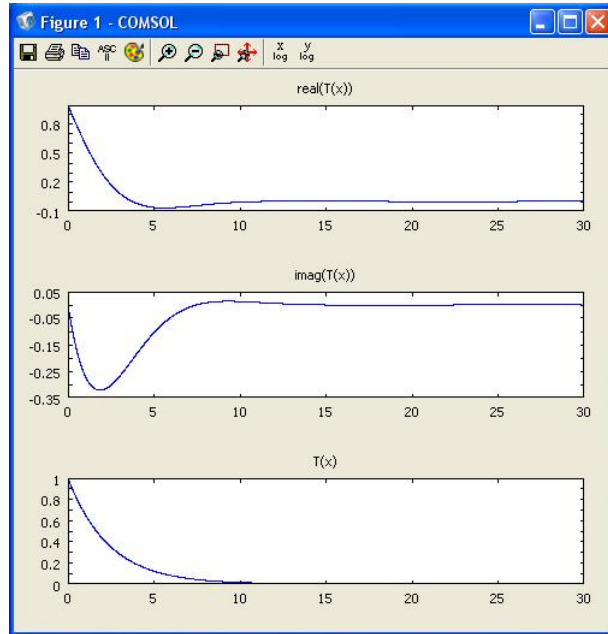


Figure 3.5: Plotting of the analytical solution

the code, and then run the program from Comsol Script [14]. That kind of approach can be very effective in our case because the harmonic analysis consists in different simulation where just the frequency is changing and it is sufficient a simple loop to run the whole simulation. In the following example there is a typical script used to manage different simulation at different frequencies. The simulation permit to calculate E and F for all the frequency in witch we are interested in.

`%Calculation of E and F coefficient`

`omega = 0;`

`serpentine;`

`E(1) = I1+i*I2;`

`F(1) = I3+i*I4;`

`omega = 2*pi/(365*24*3600);`

`for k=1:6`

`%serpentine.m is the file imported from`

`%Comsol Multiphysics of the FEM simulation`

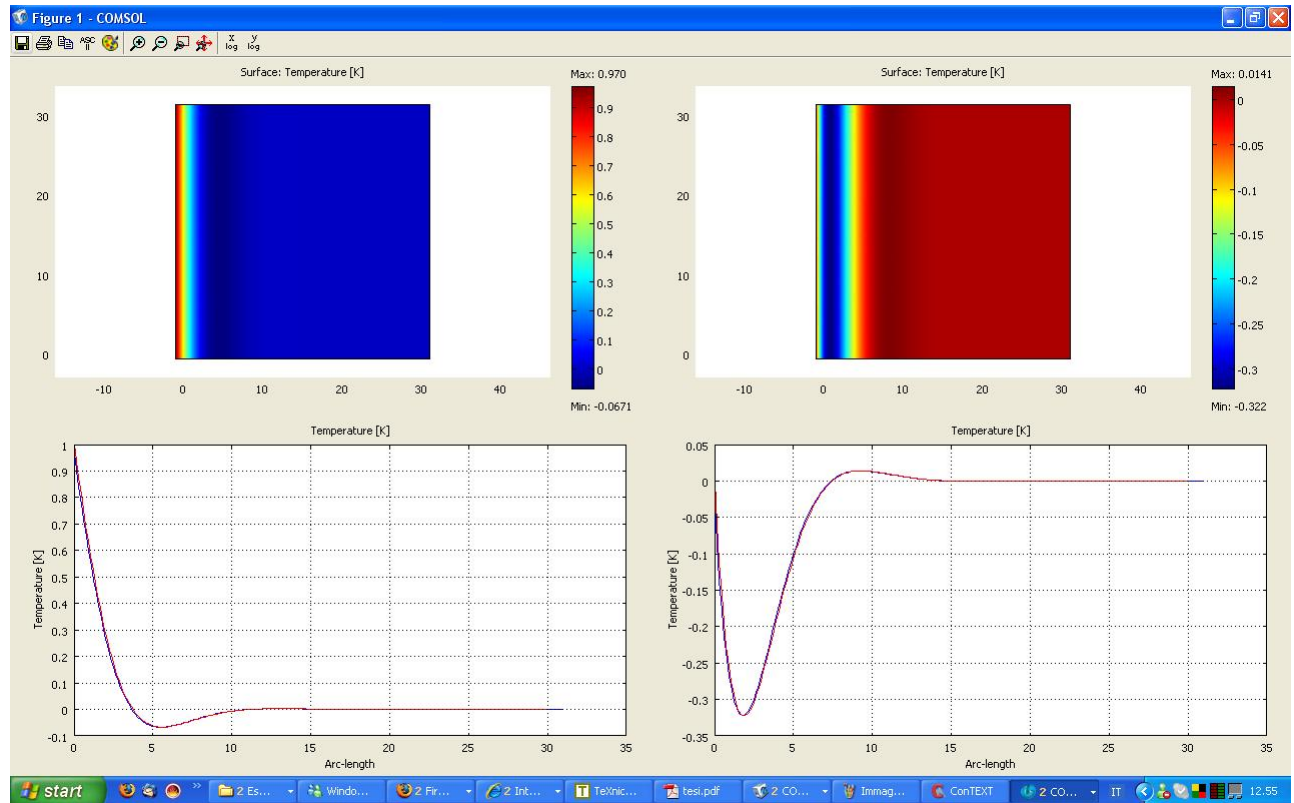


Figure 3.6: comparisons between analytical and numerical solution

```

serpentine;
E(k+1) = I1+i*I2;
F(k+1) = I3+i*I4;
omega = omega*(k+1)
end

dlmwrite('E.txt',E,' ');
dlmwrite('F.txt',F,' ');
E;
F;

```

This is just an example of application that can be useful for the resolution of this kind of problem, but a lot of possibility are offered by this program. Both Comsol Multiphysics and Script have been necessary for the realization of the present work.

Chapter 4

Realization of the calculation tool

4.1 Application

The result of the theory explained in the previous chapter is a computer program to make simulation of the ground storage. The program works with average monthly temperature and constant velocity. Its aim is to make a previous study to see the behavior of the system. The program is based on different scripts. Basically, the sequence of operations is the same that has been explained before. Therefore, there is a first group of scripts to calculate the parameters E and F , and after a second one to calculate the temperatures along the pipe during the year. In the next sections will be explained with examples some features and setting of the program.

4.2 Parameter setting

4.2.1 Geometrical parameters

The first step is the geometry and mesh setting. Those are performed by changing the values of some parameters in the file 'parametergeometry.m'. The parameters that can be controlled are for instance the position of the pipe in the ground, the diameter of the pipe, the width of the insulation etc. Moreover is very important to adapt the mesh to the geometry chosen, therefore to remesh in the critical position that are basically the insulation and the zone that surrounds the pipe. This operation is very important because the precision of the E and F depends on it. The geometrical parameters setting can be

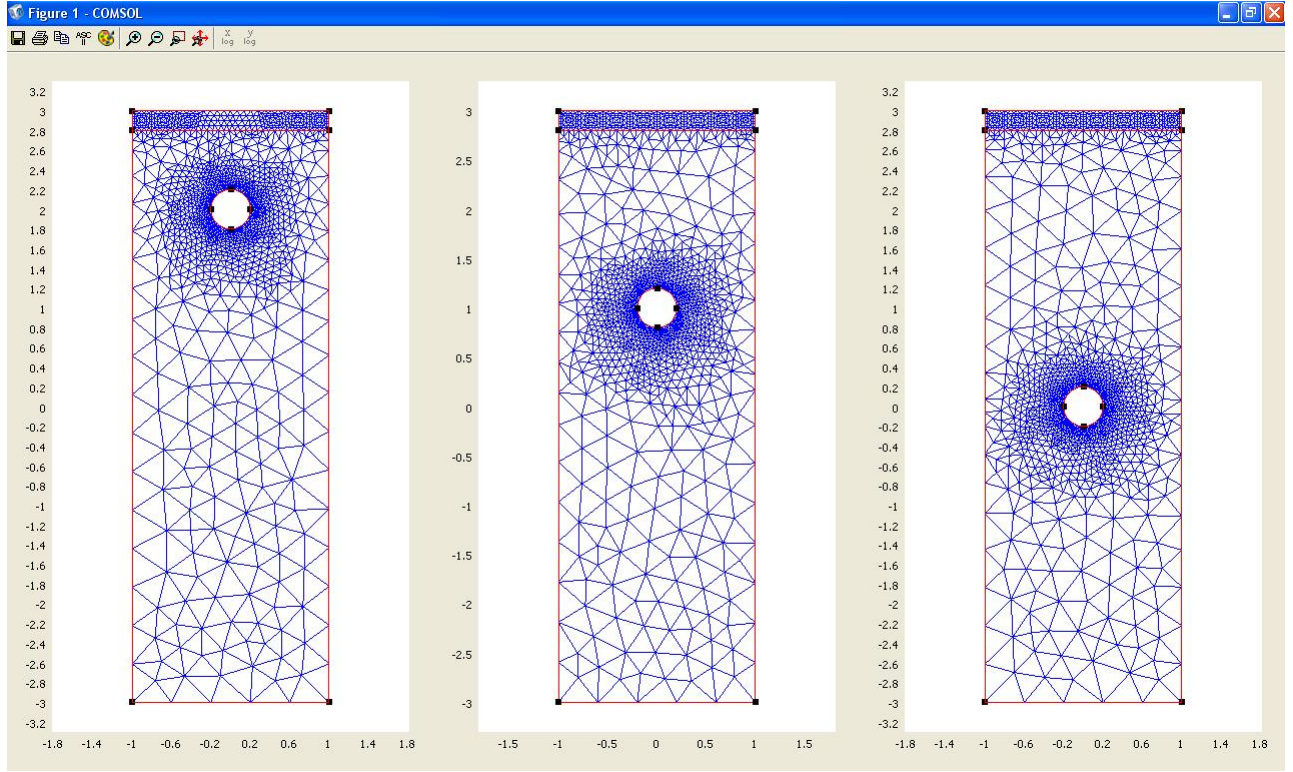


Figure 4.1: Result of the function 'checkgeometry' for three different setting

checked by using the function 'checkgeometry'. This function performs the plotting of the domain and of the mesh. In figure 4.1 there is an example of utilization of this function in three different case. In the three cases has been varied the position of the pipe in the ground and consequently the remeshing that surround it.

4.2.2 Materials properties

The second step is setting the material properties. As said before there are two subdomains that correspond to the insulation and to the ground. The properties that have to be set up are the thermal conductivity λ , the density ρ and the specific heat coefficient c_p . A typical setting can be

- For the insulation

- $\lambda = 0.0036 \text{ W/(mK)}$
- $\rho = 30 \text{ m}^3/\text{kg}$
- $c_p = 1000 \text{ J/(kgK)}$
- For the ground
 - $\lambda = 1.6 \text{ W/(mK)}$
 - $\rho = 1800 \text{ m}^3/\text{kg}$
 - $c_p = 1000 \text{ J/(kgK)}$

By changing the values of the properties there is the possibility of analysing different kind of insulation or different kind of ground.

4.3 Cross section study

The cross section study consist on the determination of the parameters E and F for each frequency. The input data are the average monthly temperatures at the beginning of the pipe and outdoor temperatures. This means that 12 samples of those function are known and the period is one year. As seen previously, the operation that has to be performed is to get the input signals, to utilise the frequency domain representation and to study the system. In our case we don't have a continuous function as input but just a finite number of samples. The Shannon-Nyquist theorem provides a condition under witch the perfect reconstruction of the signal is possible. The sampling frequency must be more than twice than the maximum frequency contained in the signal. The period is one year. The first harmonic has a frequency of $f_1 = 1/T = 1/(365 \cdot 24 \cdot 3600)$. The sampling frequency is $f_s = 1/(12 \cdot 365 \cdot 24 \cdot 3600)$. The band limit is twice the sampling frequency that correspond to the sixth harmonic. To have a perfect reconstruction of the periodical signal in the case of twelve samples we have to go till the sixth harmonic. Therefore E and F will be calculated till the sixth harmonic. The calculation of E and F is performed with a postprocessing

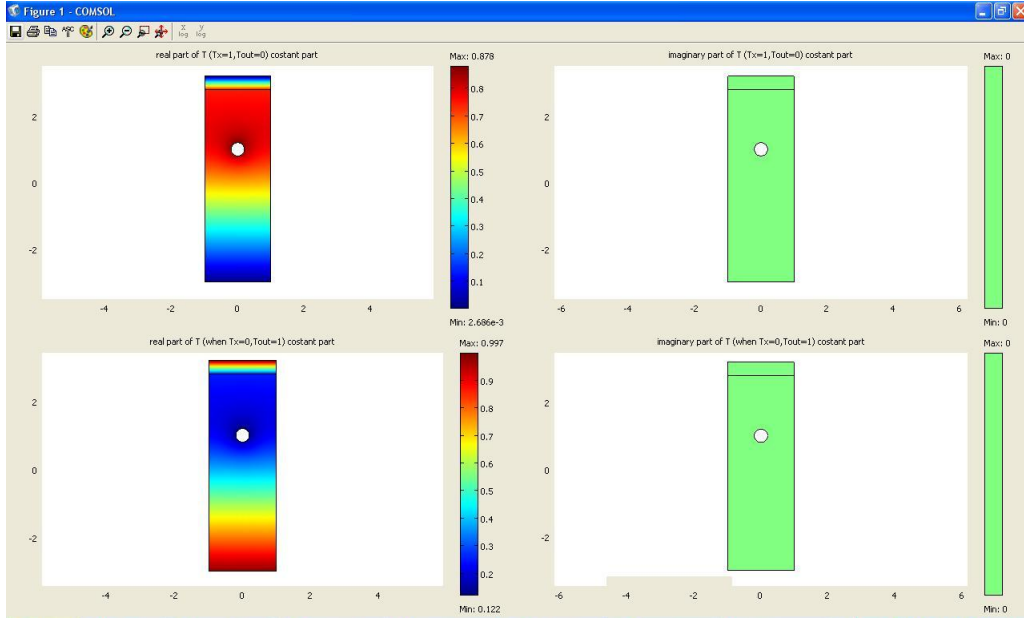


Figure 4.2: Field of temperatures constant part

operation, that is the integration of the heat flux along the border of the pipe, after the determination of the temperature fields. In figures 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8 there is the result of a whole simulation: all the fields of temperature necessary to determine the parameters E and F for each harmonic.

4.4 Study along the pipe

Once that the parameters E and F are determined the temperature at a given length of the pipe is found by using equation 2.7. In this way we find the frequency domain representation of the signal and after by Inverse Fast Fourier Transform we can get the correspondent time domain one. In figure 4.10 there is a plot of a typical result obtained with the program. On the left has been plot the configuration studied and, on the right, the temperatures that are object of study: the temperature on the top of the ground (green line), the temperature at the beginning of the pipe (blue line) and the temperature after at a given length (red line). The program has been developed to study the behavior of a serpentine in the ground. The

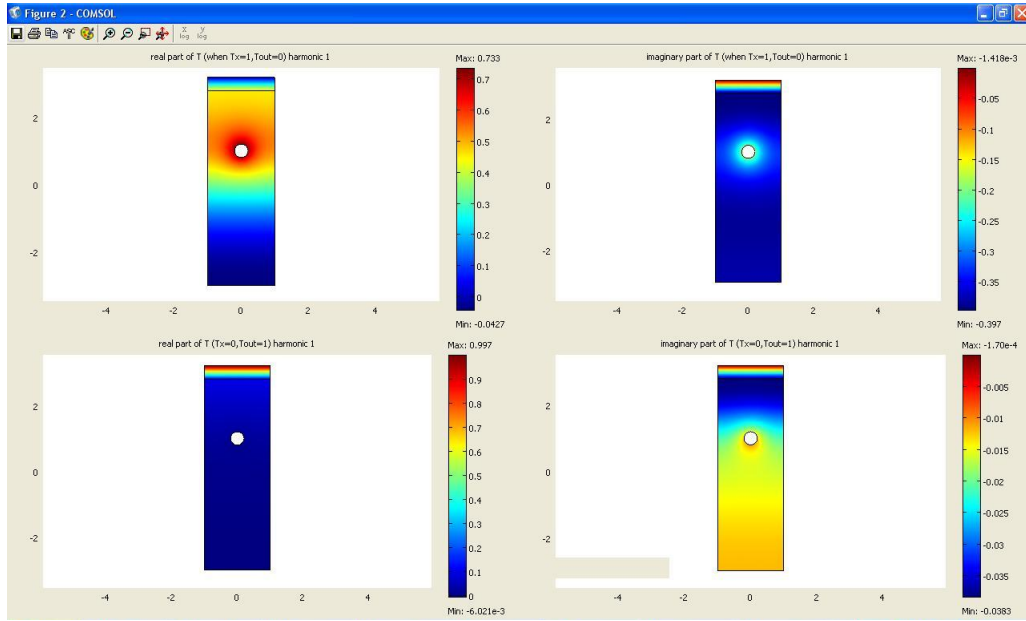


Figure 4.3: Fields of temperature first harmonic

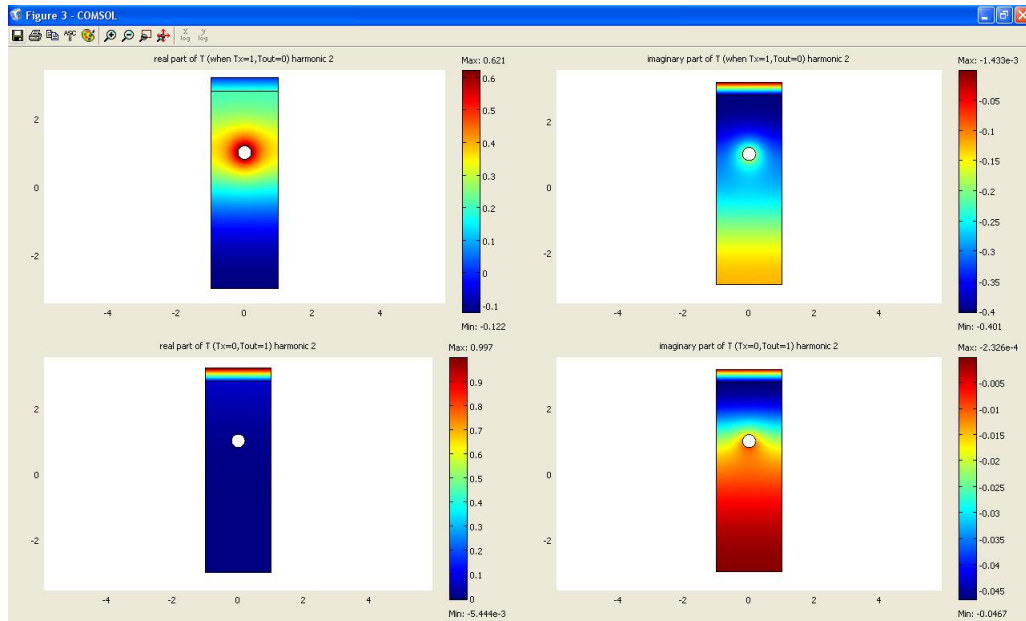


Figure 4.4: Fields of temperature second harmonic

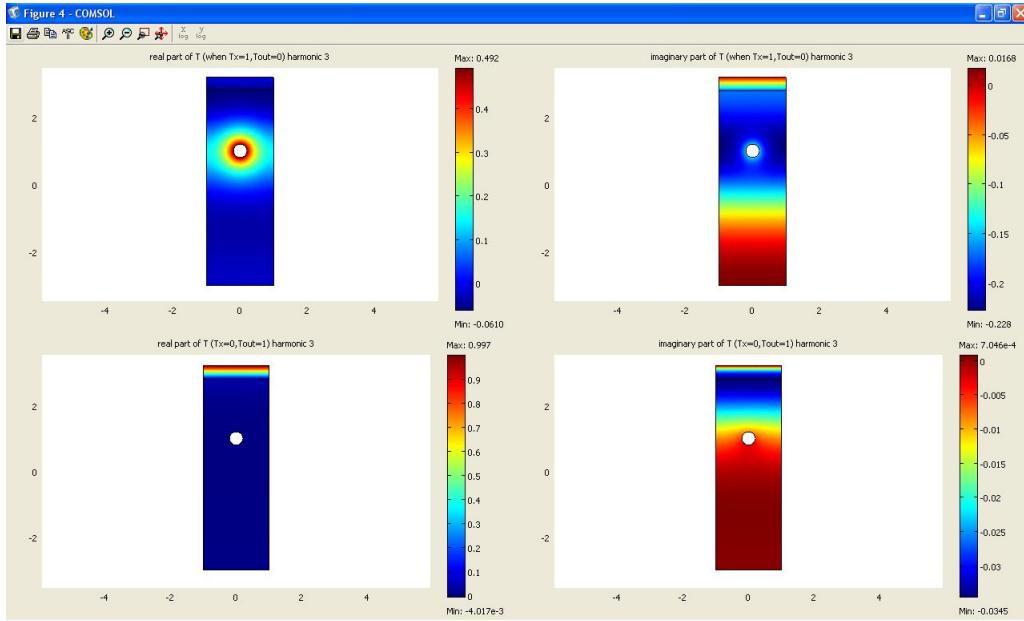


Figure 4.5: Fields of temperature third harmonic

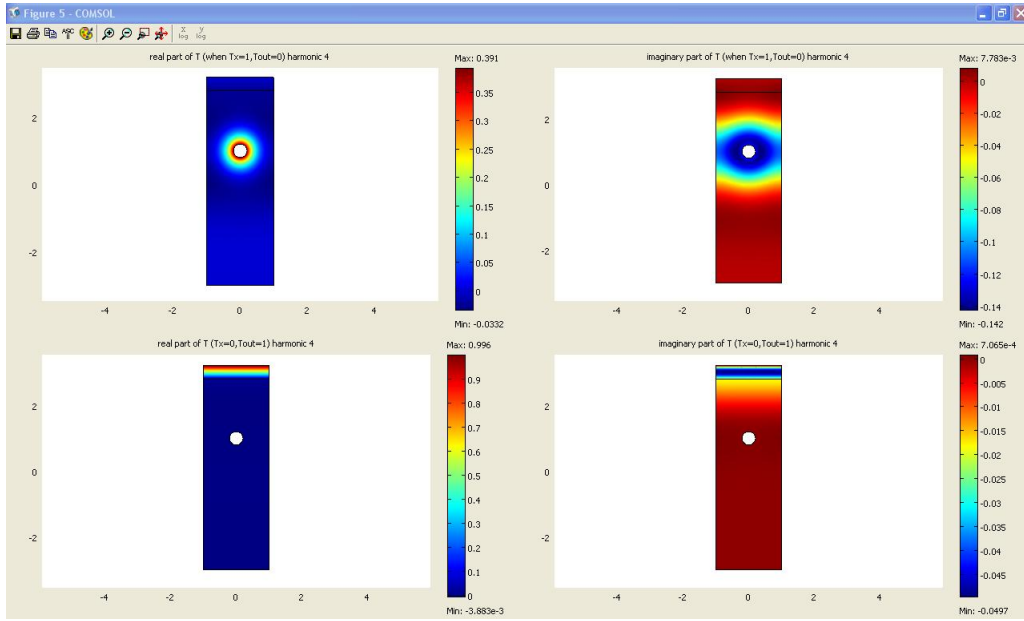


Figure 4.6: Fields of temperature fourth harmonic

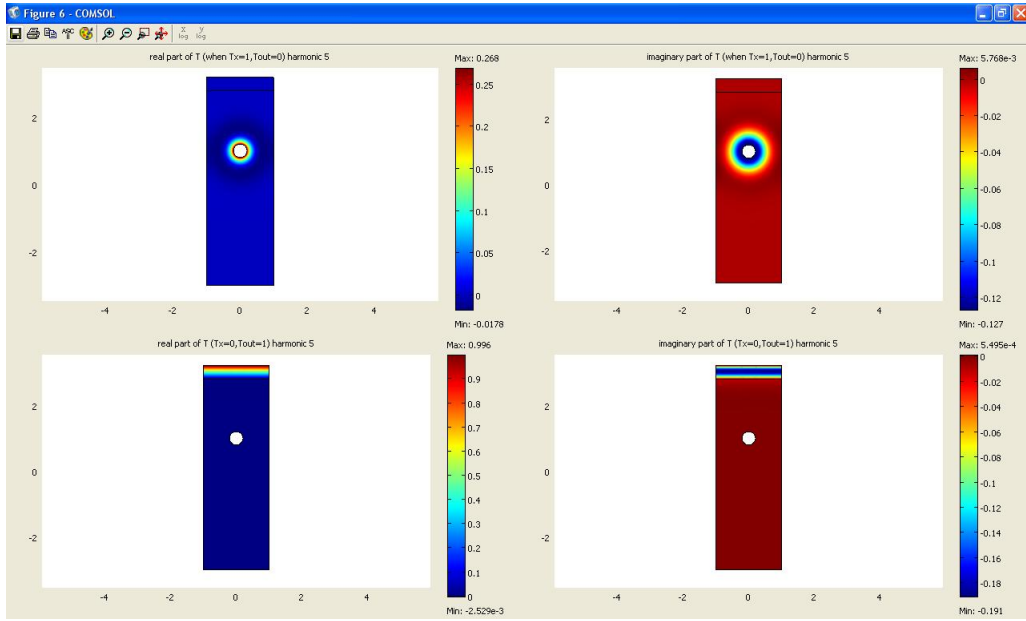


Figure 4.7: Fields of temperature fifth harmonic

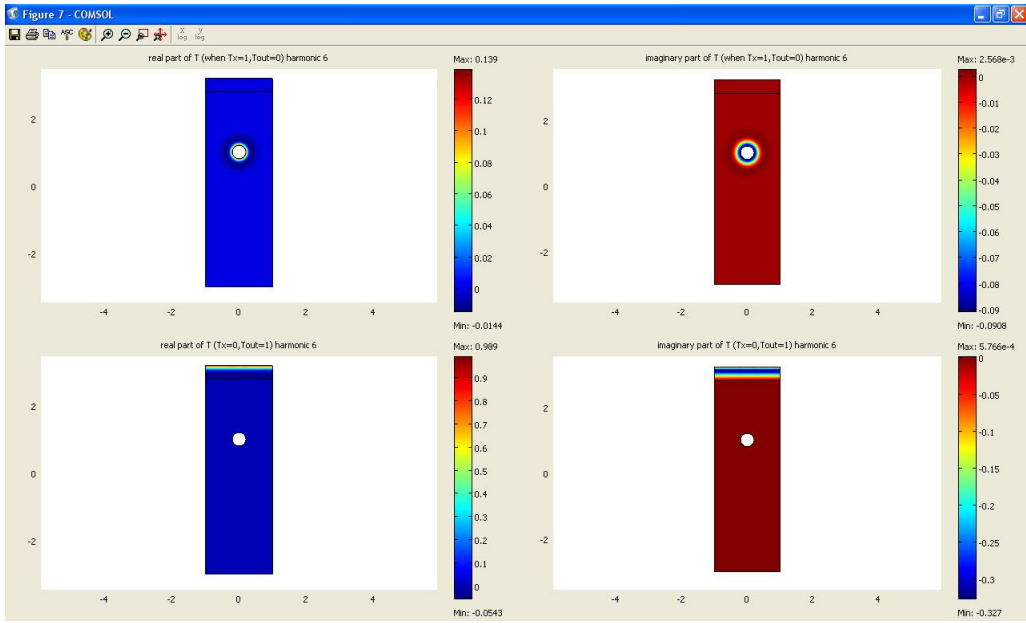


Figure 4.8: Fields of temperature sixth harmonic

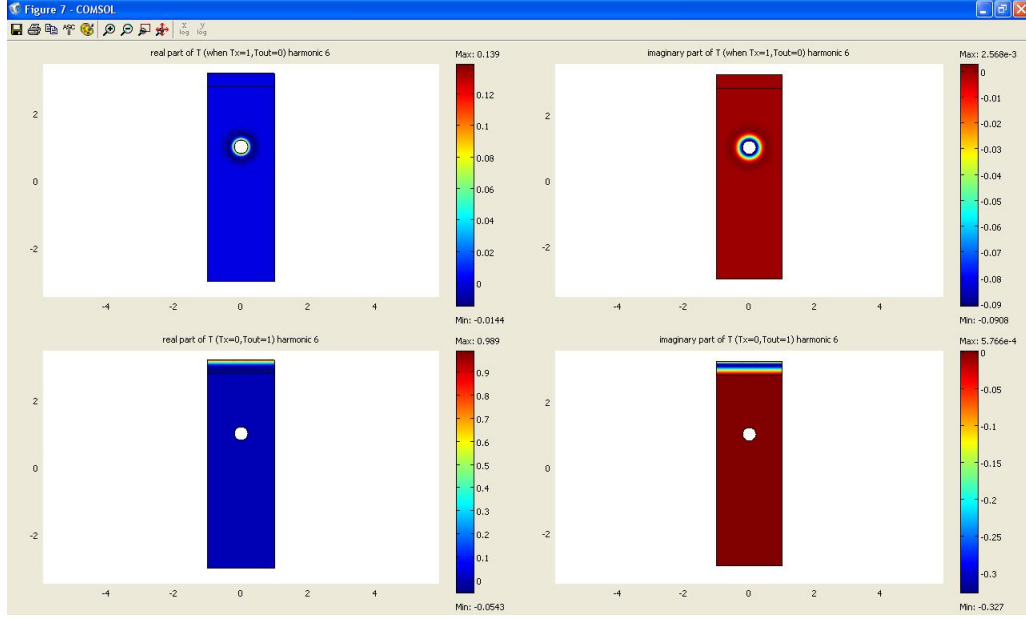


Figure 4.9: Fields of temperature sixth harmonic

applications are different, but the study case to investigate is an airborne solar collector to heat the ground and store this heat for the winter. To simulate this situation at first the incoming temperature has been approximated with a correlation between the month mean outside temperature and the monthly mean Irradiation. For an initial test, the solar collector won't be simulated. The objective is utilize some reasonable data for the inlet temperature to the system. The relation used for the input temperature to the system is the following

$$t_{in} = t_{out} + 0.07 \cdot I$$

From the result of the simulation we can see the effect of the inertia of the ground the wave of temperature after passing through the system is reduced in module and shift in phase.

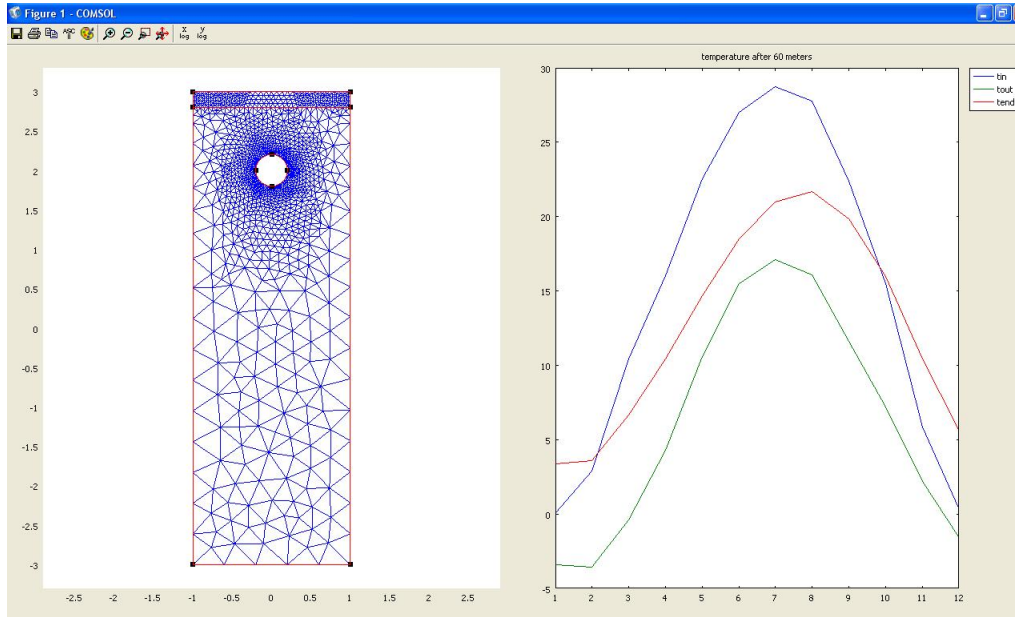


Figure 4.10: Typical result of a simulation

4.5 Example

In this section will be test the program to see the effect of the different parameters on the system. The target is to increase the temperature at the end of the pipe in winter when heat is needed. Different situation will be analyzed in order to understand the best set up for the system.

4.5.1 Base case

- Geometrical features
 - thickness of the insulation: 0.2 m
 - diameter pipe: 0.4 m
 - depth of the pipe: 1 m
- Material feature

- $\lambda_{ground} = 1.6 \text{ W/(mK)}$
- $\rho_{ground} = 1800 \text{ m}^3/\text{kg}$
- $cp_{ground} = 1000 \text{ J/(kgK)}$
- $\lambda_{insulation} = 0.0036 \text{ W/(mK)}$
- $\rho_{insulation} = 30 \text{ m}^3/\text{kg}$
- $cp_{insulation} = 1000 \text{ J/(kgK)}$
- Input temperatures
 - temperature of the air coming into the system: $t_{in} = t_{out} + 0.07 \cdot I$
 - temperature on top of the insulation: outdoor temperature of Stockholm t_{out}
- working fluid: air
 - $\rho = 1.2 \text{ m}^3/\text{kg}$
 - $cp = 1000 \text{ J/(kgK)}$

And of course, the program can be run for different pipe length. The configuration and the result for a pipe length of 60 m is the one in figure 4.10.

4.5.2 Pipe length

By using the configuration that has been presented now we can see what happened by changing the pipe length. Figure ?? shows the result of the calculation for a 20, 40, 60, and an 80 m pipe.

The longer is the the pipe the more is effective the system. The ground works like an heat exchanger with a kind of storage due to thermal capacity of the soil. A high air temperature can be reached by using an air solar collector; The inlet air in summer will be at higher temperature compared with the outdoor temperature, therefore the air entering the pipe buried will exchange some heat with the surrounding ground and will warm it

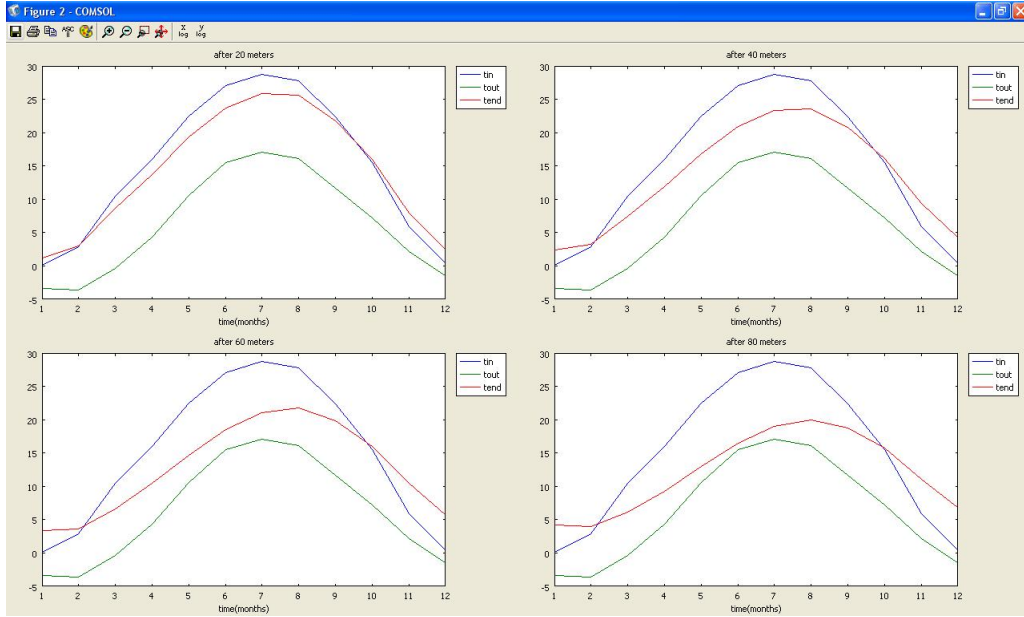


Figure 4.11: test with different length

up. The opposite situation will happen in winter: the outside temperature will be very low and by passing through the system will increase its temperature. From this simulation with an average outside temperature of $-5\text{ }^{\circ}\text{C}$ in winter with a this configuration is possible to reach a condition of almost $5\text{ }^{\circ}\text{C}$ with a temperature difference of more or less $9 \div 10\text{ }^{\circ}\text{C}$.

4.5.3 Insulation thickness

Keeping 80 meters length and varying just the insulation thickness we can make a comparison to see how this feature characterizes the behaviour of the system. The picture in figure 4.12 shows the results obtained from a test done for insulation thickness of 0 cm, 5 cm, 10 cm and 15 cm. From the result we can see that by using insulation there is a certain gain on the temperature at the end of the pipe in winter. The results of the simulation proof also that an insulation of 15 cm is effective and that by adding extra insulation there is not that much gain. The $15 \div 20\text{ cm}$ of insulation is enough for this kind of system.

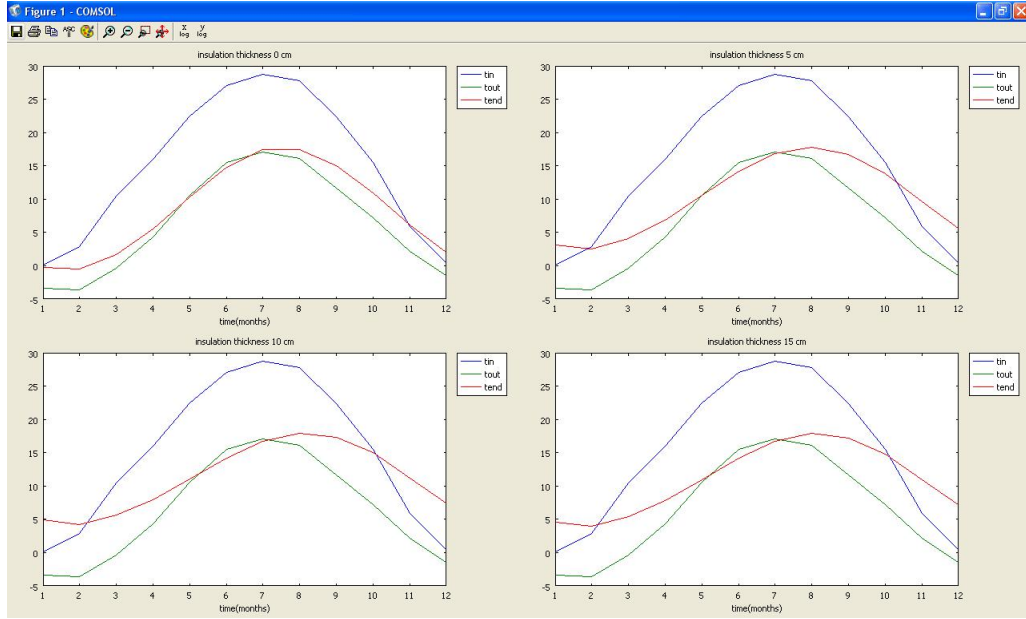


Figure 4.12: Test of the system with different insulation thickness

4.5.4 Position of the duct

The effect obtained by changing the pipe depth shows that there is a little gain if the pipe is at 2 *m* depth, and it seems that is not worthy to put it at 3 *m* of depth cause there is no more gain. The simulation has been done for the three situation in figure 4.1 for 1, 2, and 3 *m*.

4.5.5 Pipe diameter

To compare what happened by changing the diameter of the pipe we consider constant air flow and constant heat exchange surface. By keeping the heat exchange surface constant the pipe material will be more or less constant and it will also be possible a comparison on the heat exchange performances. Case 1 has the following characteristics:

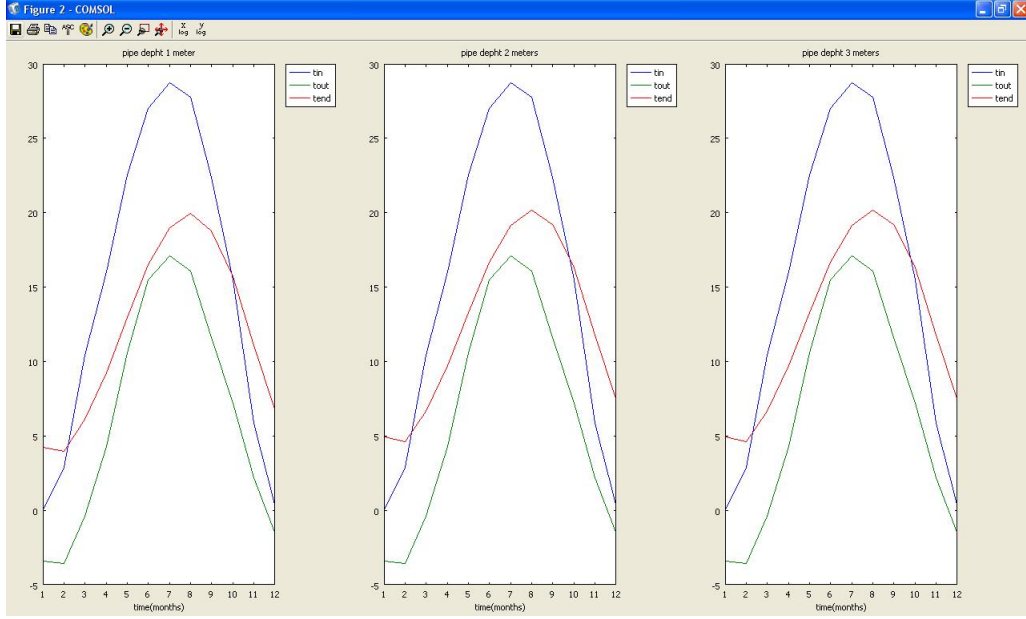


Figure 4.13: Test of the system for different pipe depth

case	1
length	80 m
diameter	0.4 m
velocity	1 m/s
volume flow	0.1256 m ³ /s
heat exchange surface	100.53 m ²

The volume flow and the heat exchange surface have respectively the following expression:

$$\dot{V} = u_i \cdot \frac{\pi \cdot d_i^2}{4} = cost$$

$$A_{exc} = \pi \cdot d_i \cdot l_i = cost$$

By using these formulas we can change the diameter and find the correspondent values for the length and the velocity that satisfy the condition of constant volume flow and constant heat exchange surface. Using these procedure we found the data of the following table:

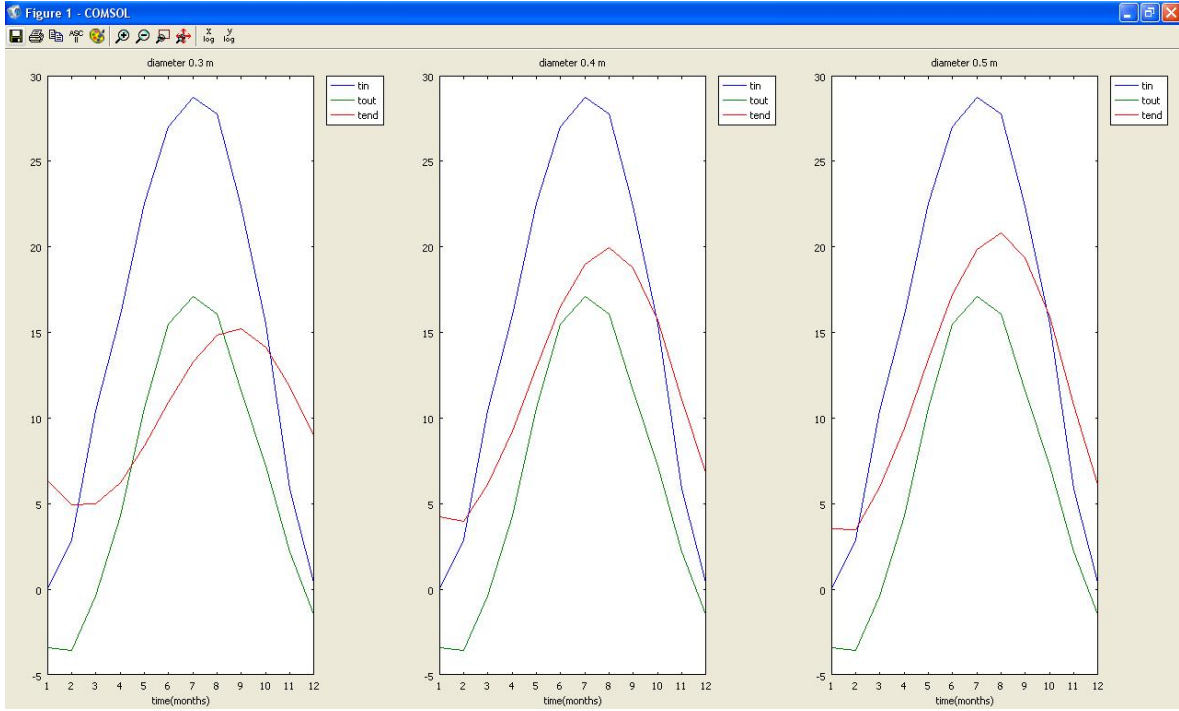


Figure 4.14: Test of the system for different pipe diameters

case	1	2	3
diameter	0.4 m	0.3 m	0.5 m
length	80 m	106.66 m	64 m
velocity	1 m/s	1.77 m/s	0.64 m/s
volume flow	0.1256 m ³ /s	0.1256 m ³ /s	0.1256 m ³ /s
heat exchange surface	100.53 m ²	100.53 m ²	100.53 m ²

In figure 4.14 there is the result obtained by using these data of pipe length and air velocity. The cases analyzed are pipe diameters of 0.3, 0.4 and 0.5 meters. From the results we can notice that by increasing the diameter there is a decreasing of the heat transfer performance. This is due to the fact that the velocity will be higher for the smaller pipe with a higher heat transfer coefficient as a consequence. On the other hand, to size the pipe diameter has to be taken into account the power that the fan has to supply. To reduce this power the pressure losses have to be taken into account. Those can be calculated by

using the Darcy-Weisbach relation:

$$\Delta p = f \frac{L}{D} \frac{w^2}{2g} \quad (4.1)$$

with L the length the duct, f loss coefficient, D the diameter of the duct, g gravity acceleration and w velocity of the fluid. The velocity in a duct can be express as follow:

$$w = \frac{4\dot{V}}{\pi D^2}$$

Therefore

$$\Delta p = f \frac{8L\dot{V}^2}{g \cdot \pi^2 D^5} \quad (4.2)$$

The expression shows that with a small duct we have a high pressure drop since that is inversely proportional to the fifth power of the diameter. Then, in this case it's better to size larger pipes because the power that has to be supplied by the fan is electrical power with a high exergy content, and what we want to take out from the system is low temperature heat with a low exergy content. The quotient between the energy from the fan and the energy that we will manage to take out from the system has to be very low.

Chapter 5

Nonlinear formulation

5.1 Problems

In the previous chapters all the analysis have been done thinking about average monthly temperatures and constant velocity in the pipe. This kind of analysis gives us the idea of the behavior of the system. The next step is to perform a deeper analysis that makes it possible to work with daily temperature variation and that permits to variate the air velocity inside the pipe. The regulation has a very important role to understand the real performance of the system. For example during summer the temperature during day time is greater than the ground temperature but during the night it can be less than it; therefore, if the air velocity is constant, during the day we will be heating the ground and during the night we will be cooling down it. For this new problem the idea is to try to use the same approach used for the previous work and then, at first a cross section analysis to obtain the system parameters to get the transfer function from a finite element analysis and after a finite difference analysis along the pipe to see the evolution of the temperature profile with the increasing of the length of the pipe. The hardest issue is that the introduction of variable velocity, as it will be explained afterward, means to solve a system of nonlinear equation of large dimension. To solve it will be used The GMRES (General Minimal Residual) Method [16] that permits to decrease the time if compared with the Newton-Raphson method.

5.2 New formulation

To solve the problem in the new conditions, the FEM analysis to calculate system parameter has to be performed in a slightly different way compared with the one used in the first program. In that case, it was chosen for the system study to use as input signals the outside temperature T_{out} and the temperature inside the pipe T_x , and then it was assigned the value of the global heat transfer coefficient h depending on the velocity chosen. In this case, it is not possible to assign this kind of conditions because to make the frequency domain analysis, it is important to keep on the system linear. The aim is to find a formulation that permits us to find a solution of the problem in a similar way compared to the one used in the first program. Then, it is very important to use the linear analysis of the cross section. The problem introduced by using variable velocity is that the heat transfer coefficient h is not constant anymore. What can be done is to assign a temperature to the pipe surface and verify that the heat transfer between the working fluid and the pipe surface is the same as the heat transfer between the pipe surface and the ground. Once that is known the surface temperature the calculation of the heat transfer between the surface and the ground is a linear problem. By supposing the surface temperature more or less homogeneous, we can define two new parameters E^* and F^* in the same way as it has been done in the previous program with the only difference that this time for the definition of E^* the temperature oscillation is directly imposed on the border of the pipe. Then, the pipe can be divided in a finite number of element in which the temperature of the surface $T_s(t)$ is supposed to be function of the time but not of the space. If it is known the pipe surface temperature $T_s(t)$ in one of these elements, then its Fourier series coefficient $\tilde{T}_s(\omega)$ is known, and the heat flux $\tilde{Q}(\omega)$ can be calculated. Afterward through inverse Fourier transform the time domain representation $q_1(t)$ can be calculated.

$$\tilde{Q}(\omega) = E^*(\omega) \cdot \tilde{T}_s(\omega) + F^*(\omega) \cdot \tilde{T}_{out}(\omega) \quad (5.1)$$

The heat flux can be expressed also as

$$q_2(t) = h(t) \cdot (T_x(t) - T_s(t)) \quad (5.2)$$

The solution of the problem is to find $T_s(t)$ that satisfy the relation

$$q_1(t) = q_2(t) \quad (5.3)$$

When the temperature of the surface $T_s(t)$ is known in the element, we need one relation to link the temperature inside the pipe to the next element. From the heat balance we have that

$$h(t) \cdot (T_x(t) - T_s(t)) + \frac{\partial T_x(t)}{\partial x} \cdot \rho \cdot cp \cdot u \cdot A = 0$$

And then

$$\frac{\partial T_x(t)}{\partial x} = -\frac{h(t) \cdot (T_x(t) - T_s(t))}{\rho \cdot cp \cdot u \cdot A}$$

By knowing the derivative $\partial T_x(t)/\partial x$ we find the new value $T_{x+1}(t)$ as

$$T_{x+1}(t) = T_x(t) + \frac{\partial T_x(t)}{\partial x} \cdot \Delta x$$

This kind of approach works only if the velocity u is not zero. Later on an approach to solve the problem even in this last case will be presented. In our problem the input signals are discrete and therefore the output signal will be a discrete function $T_s(t)$ with the same

number of samples as the input signals. The solution of the problem is the discrete signal $T_s(t)$ that satisfies the equation 5.3 for each sample $t_s(t)$. Basically this is the way followed even if there are some other things that will be explained later on.

5.3 GMRES

Practically, given the temperature $T_x(t)$ to find $T_s(t)$, it is necessary to solve a nonlinear system of n equations in n unknown quantities where n is the number of samples $t = 1, 2, \dots, n$. A method to solve nonlinear problems is the Newton-Raphson iteration.

$$F(x) = 0$$

$$F(x) = F(x_0) + \frac{\partial F(x_0)}{\partial x} \cdot \partial x$$

$$\partial x = -\frac{F(x_0)}{\frac{\partial F(x_0)}{\partial x}}$$

If $F(x) = f_1(x), f_2(x), \dots, f_n(x)$ and $x = x_1, x_2, \dots, x_n$ then the term $\frac{\partial F(x_0)}{\partial x}$ is the Jacobian J

$$J = \begin{pmatrix} \frac{\partial f_1(x_0)}{\partial x_1} & \frac{\partial f_1(x_0)}{\partial x_2} & \dots & \frac{\partial f_1(x_0)}{\partial x_n} \\ \frac{\partial f_2(x_0)}{\partial x_1} & \frac{\partial f_2(x_0)}{\partial x_2} & \dots & \frac{\partial f_2(x_0)}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n(x_0)}{\partial x_1} & \frac{\partial f_n(x_0)}{\partial x_2} & \dots & \frac{\partial f_n(x_0)}{\partial x_n} \end{pmatrix} \quad (5.4)$$

Therefore

$$\partial x_1 = -\frac{f_1(x_0)}{\frac{\partial f_1(x_0)}{\partial x_1}} + \frac{f_2(x_0)}{\frac{\partial f_1(x_0)}{\partial x_2}} + \dots + \frac{f_n(x_0)}{\frac{\partial f_1(x_0)}{\partial x_n}}$$

$$\partial x_2 = -\frac{f_2(x_0)}{\frac{\partial f_2(x_0)}{\partial x_1}} + \frac{f_2(x_0)}{\frac{\partial f_2(x_0)}{\partial x_2}} + \dots + \frac{f_2(x_0)}{\frac{\partial f_2(x_0)}{\partial x_n}}$$

And so on till ∂x_n . This method permits to get quadratic convergence, if the initial guess x_0 is quite close to the solution and the system does not diverge. The problem of this method is that for each iteration it is necessary to calculate the Jacobian Matrix and if the system to solve is very large, it takes a long time to calculate it and that makes it impossible to find the solution in a reasonable time. The method that will be used to solve that kind of problem is the GMRES General minimal residual Method for nonlinear system. This is a Newton-Krilov method and mixes the Krilov method for the solution of linear system with the Newton Iteration. That is done solving the linear system of the Newton iteration with the GMRES method for linear system [16, 17, 18]. Given an initial guess x_0 we have to find the δ that fulfills the relation

$$J \cdot \delta = -F(x_0) \quad (5.5)$$

If δ_0 is an initial guess for the true solution, then letting $\delta = \delta_0 + z$ we have an equivalent system

$$J \cdot z = r_0 \quad (5.6)$$

where $r_0 = -F - J \cdot \delta_0$. Let K_m be the Krilov subspace

$$K_m = span \{r_0, J \cdot r_0, \dots, r_0 \cdot J^{m-1}\}$$

The iteration is

$$\delta_m = \delta_0 + z_m \quad (5.7)$$

where $z_m \in K_m$ such that $(r_0 - J \cdot z_m) \perp K_m$ (that is equivalent to $(-F - J \cdot \delta_m) \perp K_m$). In other words we are looking for the δ_m that minimizes the $\|F + J \cdot \delta_m\|$; that δ_m is the orthogonal projection of δ in the Krilov subspace of dimension m .

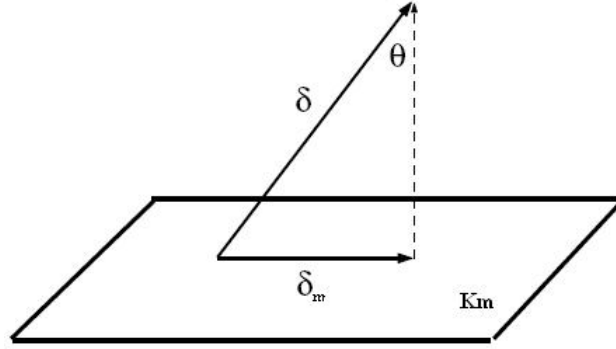


Figure 5.1: Orthogonal projection of δ in the Krilov subspace K_m [19]

In this way we can decide a maximum dimension m of the Krilov subspace in which we are going to calculate the δ_m . As the value of m increase, decreases the value of $\delta - \delta_m$ but increases the complexity of the problem at the same time, because we have to look for the minimum of the function $\|F + J \cdot \delta_m\|$ inside a bigger space. It results convenient to limit the dimension of the Krilov subspace and after one iteration calculate the new value of r and build on it a new Krilov subspace and do a new iteration.

5.4 Algorithm

1. Start with an initial guess x_0 , a tolerance ϵ_0 , and m_{max} , the maximum dimension of the Krilov subspace.
2. *Arnoldi process*: Build an orthonormal base of the Krilov subspace by the Gram-Schmidt process.
 - for an initial guess δ_0 , form $r_0 = -F(x_0) - J(x_0) \cdot \delta_0$

- Compute $\beta = \|r_o\|_2$ and $v_1 = r_o/\beta$
- for $j = 1, 2, \dots, m$
 - Form $J \cdot v_j$ and orthogonalized it to the previous v_1, v_2, \dots, v_j

$$h_{i,j} = (J \cdot v_j, v_i), \quad i = 1, 2, \dots, j$$

$$\widehat{v_{j+1}} = J \cdot v_j - \sum_{i=1}^j h_{i,j} \cdot v_i$$

$$h_{j+1,j} = \|\widehat{v_{j+1}}\|_2$$

$$v_{j+1} = \widehat{v_{j+1}}/h_{j+1,j}$$

- Compute the residual norm $\rho_j = \|F + J \cdot \delta_j\|$ of the solution δ_j that would be obtained if we stopped at this step.
 - if $\rho_j \leq \epsilon$ or $m > m_{max}$ go to the next step
3. *Form the approximate solution:* Define H_m to be the Hessemberg matrix $(m+1) \times m$ whose nonzero entries are the coefficients $h_{i,j}$, $1 \leq i \leq j$, $1 \leq j \leq m$ and define $V_m = [v_1, v_2, \dots, v_m]$.
- Find the vector y_m that solves the linear system $\|\beta \cdot e_1 - H_m \cdot y\|$ over all vectors in R^m where $e_1 = [1, 0, \dots, 0]^T$.
 - Compute $\delta_m = \delta_0 + z_m$ where $z_m = V_m \cdot y_m$, and $x_{n+1} = x_n + \delta_m$.
4. if x_{n+1} is a good enough approximation to a root then stop, else restart the process with $x_0 = x_n$ [19].

Steps 2 and 3 are the core of the Algorithm; first there is the Arnoldi process to create an orthonormal base of the Krilov subspace and the calculation of the vector z that

minimize the $\|F + J \cdot \delta_m\|_2$. In step 2 the exit condition is $j > m$ or $r_j < tolerance$. The last one implies that we have to calculate r_j for each iteration as the dimension of the Krilov subspace is growing. As seen $\|r\| = \|F_j + \delta\|$, then to get it for each iteration it is necessary to solve the problem of minimization that is the problem of step 3. This, at a first glance can look like a very expensive process, but due to the features of the matrix that are involved it results simplified.

Before going forward let's describe more in detail what we are doing in step 3. The minimization of the residual norm inside the Krilov subspace is obtained by imposition of the orthogonality between the the vector r and the the Krilov subspace.

$$\min_{\delta \in \delta^{(0)} + K_m} \|F_j + \delta\|_2 = \min_{z \in K_m} \|r^{(0)} - J \cdot z\|_2$$

Finding the minimum of these expression means to solve the system of equation

$$\|r^{(0)} - J \cdot z\|_2 = 0 \quad \text{with } z \in K_m$$

The condition $z \in K_m$ is obtained by building the vector as a linear combination of the basis vector of the Krilov subspace. The matrix $V_m = [v_1, v_2, \dots, v_m]$ has as columns the vector basis, then we can say that a vector z obtained as it follows is inside the Krilov subspace.

$$z = V_m \cdot y \quad \text{with } y \in R^m$$

Therefore

$$\|r^{(0)} - J \cdot z\|_2 = \|\beta \cdot v_1 - J \cdot V_m \cdot y\|_2$$

$$J \cdot V_m = V_m \cdot H_m + \widehat{v_{m+1}} \cdot e_m^T$$

where $e_m^T = [0, \dots, 1]^T \in R^m$. And this can be rewritten as

$$J \cdot V_m = V_{m+1} \cdot \bar{H}_m$$

Now by substituting this one we obtain

$$\|V_{m+1} \cdot (\beta \cdot e_1 - \bar{H}_m \cdot y)\|_2 = 0$$

That is equivalent to

$$\|(\beta \cdot e_1 - \bar{H}_m \cdot y)\|_2 = 0 \quad (5.8)$$

The problem of minimization is reduced to the solution of a linear system. As previously said, to get the exit condition $\|r\| > \text{toll}$ we should calculate r while the Krilov subspace is growing ($j = 1, 2, \dots, m$), then we should solve the system 5.8 for each iteration. This is an $((m+1) \times m)$ system and is overdetermined. Such a kind of problem is called least squares problem. To solve it efficiently taking into account that the matrix \bar{H}_j is orthonormal we can use the QR decomposition and the Givens rotation. Using this tools we can reuse much information from the step $j - 1$ and save a lot of time [16].

5.4.1 QR decomposition

The QR decomposition is the factorization of a matrix A into an orthogonal matrix Q and an upper triangular matrix R . The method that will be used is the Givens rotation. It consists in multiplying the matrix A times rotations matrix G_j in order to zero the lowest and left elements and obtain the R matrix [20]. For each rotation we will have one element that becomes zero. The final result will be $G_1 \times \dots \times G_j \times A = R$ Let's consider a matrix A and we want to find the matrix Q and R

$$A = QR \quad (5.9)$$

$$A_{m,n} = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{pmatrix} \quad (5.10)$$

The rotation matrix $G_{i,k,\theta}$

$$G_{i,k,\theta} = \begin{pmatrix} 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & & \vdots & & \\ 0 & \cdots & \cos(\theta) & \cdots & \sin(\theta) & \cdots & 0 \\ \vdots & & \vdots & \ddots & \vdots & & \\ 0 & \cdots & -\sin(\theta) & \cdots & \cos(\theta) & \cdots & 0 \\ \vdots & & \vdots & & \vdots & \ddots & \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{pmatrix} \quad (5.11)$$

$$g_{i,i} = \cos(\theta)$$

$$g_{i,k} = \sin(\theta)$$

$$g_{k,i} = -\sin(\theta)$$

$$g_{k,k} = \cos(\theta)$$

When a Given rotation matrix $G_{i,k,\theta}$ is multiplied times another matrix only the rows i and k are affected.

$$\begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} a_{i,i} \\ a_{k,i} \end{pmatrix} = \begin{pmatrix} r_{i,i} \\ 0 \end{pmatrix} \quad (5.12)$$

The angle that gives this result is

$$r = \sqrt{a_{i,i}^2 + a_{k,i}^2}$$

$$\cos(\theta) = a_{i,i}/r_{i,i}$$

$$\sin(\theta) = a_{k,i}/r_{i,i}$$

$$\tan(\theta) = a_{i,i}/a_{k,i}$$

The expression used to find the rotation that have to be applied in order to zero the element $r_{i,i}$ is the last one. For each Givens rotation one element will be zero, then is necessary to apply more rotations to get the upper triangular matrix R . In the present condition, at step j , we want to calculate the matrix Q_j and R_j . If we look at how has been built the \bar{H}_j matrix, we can see that there is just one diagonal that has to be zero because the other elements that should be zero by Givens rotation are already zero. That means that for each step j , it is necessary just one Givens rotation to gain the decomposition [20]. Let's indicate with $G_j(k)$ the matrix in step j that eliminates $h_{j+1,j}$, then $Q_j = G_j(1)^T \times \dots \times G_j(j)^T$. As a result coming back to the problem of equation 5.8

$$\|(\beta \cdot e_1 - \bar{H}_j \cdot y)\|_2 = \left\| \left(\beta \cdot e_1 - Q_j \begin{pmatrix} R_j \\ 0 \end{pmatrix} \cdot y \right) \right\|_2 \quad (5.13)$$

$$\begin{pmatrix} R_j \\ 0 \end{pmatrix} \cdot y = Q_j^T \beta \cdot e_1 = G_j(1) \times \dots \times G_j(j) \beta \cdot e_1 \quad (5.14)$$

$$\begin{pmatrix} g_j \\ \rho_{j+1} \end{pmatrix} = Q_j^T \beta \cdot e_1 \quad (5.15)$$

Then gathering information from eq 5.14 and 5.15 we obtain

$$\begin{pmatrix} R_j \\ 0 \end{pmatrix} \cdot y = \begin{pmatrix} g_j \\ \rho_{j+1} \end{pmatrix} \quad (5.16)$$

The least square problem has been simplified to the solution of the linear system of equation $R_j \cdot y = g_j$ [20, 18] and residual norm $\|r_j\|$ is

$$\|r_j\| = |\rho_{j+1}|$$

This result is very good because we can manage to get the residual norm without calculating directly the function. Therefore, it is possible to decide to iterate or not saving a lot of computational time. The R_j matrix is update column wise step by step. In order to do that all the sines and cosines from the Givens matrix have to be stored. However g_j can be easily updated from one step to the other by using the following expression

$$\rho_1 = \|r_0\|$$

$$\rho_j = -\rho_{j-1} \cdot \sin(\theta_j)$$

$$g_0 = \oslash$$

$$g_j = \begin{pmatrix} g_{j-1} \\ \rho_j \cdot \cos(\theta_j) \end{pmatrix}$$

5.4.2 Advantages of GMRES

The advantage of this method is basically that the Jacobian J is never explicitly necessary but we have to calculate the product of the Jacobian times a vector [16, 17, 18]. This can be approximated as follow

$$J(x) \cdot v = \frac{F(x + \sigma \cdot v) - F(x)}{\sigma}$$

Instead of calculating each iteration $(n \times n)$ elements, are necessary only two matrix: the Hessemberg matrix H $(m + 1 \times m)$ and the matrix with the basis vector V $(n \times m)$. And it is easy to understand that keeping low the number of m_{max} , the number of elements that should be calculated is reduced a lot.

5.4.3 Line search method

Newton method by itself can fail if the initial guess is far away from the solution. To increase the robustness of the GMRES a strategy to obtain global convergence will be used [16]. The method explained in this chapter combines the Newton iteration with the GMRES method for the solution of linear system. During one iteration, once the linear system $J \cdot \delta = -F$ has been solved by using the GMRES, the following step is to update the value $x_{k+1} = x_k + \delta_k$. The term δ_k represent the step between the points x_k and x_{k+1} , in the n dimensions space. By using the line search method we can ensure that x_{k+1} will be nearer to the solution of the nonlinear system compared to the point x_k . The line search method is based on the definition of an objective function as follow

$$f(x) = f(x_1, \dots, x_n) = \frac{1}{2} F(x)^T F(x) \quad (5.17)$$

This function is semi-positive defined, and it means that $f(x) \geq 0$. A descendant direction for f at the current approximation x is any vector p such that

$$\nabla f(x)^T p < 0$$

The derivative of the function $f(x)$ is

$$\nabla f(x) = J(x)^T F(x)$$

Therefore p is a descendant direction if

$$F(x)^T J(x)p < 0$$

For this direction exists a coefficient $\alpha_0 > 0$ such that $f(x + \alpha p) < f(x)$ for all the $0 < \alpha < \alpha_0$. If $\tilde{\delta}$ is an approximate solution of the problem $J\delta = -F$ it follows that

$$\tilde{r} = -J(x)\delta - F(x) \quad (5.18)$$

$$F^T J(x)\tilde{\delta} = -F^T F - F(x)\tilde{r} \quad (5.19)$$

A sufficient condition for having a descendant direction $\tilde{\delta}$ is $\|r\|_2 < \|F\|_2$. Such a condition can be verified practically calculating the value of $\|r\|_2$ and $\|F\|_2$. Once that we now that the direction is a descendant direction for the function $f(x)$ the iteration that has to be used is the following

$$x_{k+1} = x_k + \alpha \cdot \tilde{\delta} \quad \text{with } \alpha > 0 \quad (5.20)$$

The scalar α is a coefficient that determines the step lenght in the direction $\tilde{\delta}$ [18]. Ideally, we want to have the α that minimizes the function $f(x_{k+1})$. In figure 5.2 there is the picture of a possible situation: moving along the direction $\tilde{\delta}$ at first the function decreases but, after that it increases again.

Finding the α that minimizes the function f can be, from a computational point of view very demanding. Then, in the present work we decide to start from a certain value of α and verifying the condition

$$f(x_k + \alpha \cdot \tilde{\delta}) < f(x_k)$$

If the condition is true we choose this value of α and we start the new iteration; if not the alpha will be reduced and the verification will be iterated until a suitable α will be

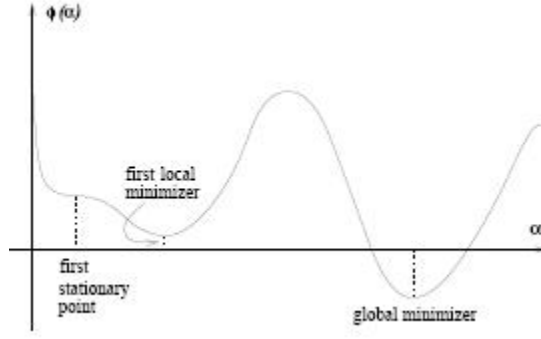


Figure 5.2: Ideal step length to minimize the global function

found. This approach is not optimized because we are doing more steps than the ones that we would do finding each iteration the optimum α , but on the other hand we have faster iteration.

An example of the efficiency of the combination of GMRES and line search method is proposed in figure 5.3. The system analyzed is a system of 8760 equation in 8760 unknown quantities and is the typical case used in the present work. The unknown quantities represent the temperature t_x for each hour of the year, and the equation are the relative balance relations. The plot is a profile of convergence of the norm $|f|$ of the global function of the system. In just 18 iterations the algorithm reaches a value

The $|f|$ is $\sqrt{f_1^2 + f_2^2 + \dots + f_n^2}$ then, in this case with $n = 8760$ to have $|f| < 10^{-5}$, the average value of f is 10^{-7} , that means that it is possible to reduce a bit the precision and decrease the time of calculation.

5.5 Variable velocity

The program has been done in order to make it possible air velocity changes Basically two cases are object of study:

- velocity is not zero, then the system of equation 5.3 is solvable. It is necessary to

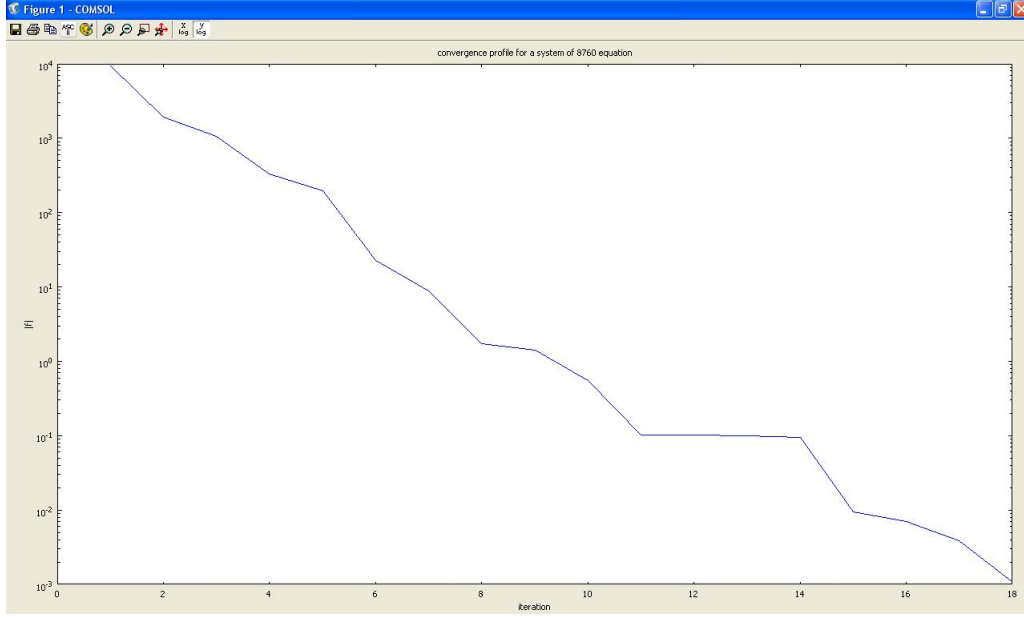


Figure 5.3: Convergence profile for a system of 8760 equations

adapt the heat transfer coefficient $h(t)$ depending on the velocity $u(t)$.

- velocity is zero. Then we have to find a way to model the behavior in this case, because the relations studied until this moment are not taking into account this situation.

5.5.1 case 1: non zero velocity

When the system is working we assume that the amount of mass flow needed require a velocity such as the Reynolds number inside the pipe is in the range of turbulent flow. For turbulent flow the Dittus Boelter relation can be used [21].

$$Nu = 0.023 \cdot Re^{0.8} \cdot Pr^{0.4} \quad (5.21)$$

Knowing the Nusselt number the heat transfer coefficient by convection can be determined.

$$h = \frac{Nu \cdot \lambda}{d}$$

5.5.2 case 2: velocity equal to zero

The second case is velocity imposed equal to zero. We are in this case when the flow inside the duct is undesired. For instance, in summer when the target is to harvest heat, it is preferable not to have flow if the air temperature is lower than the duct temperature, because in this case the air flowing will cool down the duct and then the ground. Therefore, a typical condition in which we are interested in is: air temperature lower than the duct temperature, the fan that forces the air flow switch off and a transient condition takes place until air and duct temperatures come to an equilibrium situation. This case is rather complex because the difference of temperature along the pipe will drive the mass, then there will be some natural convection between air and pipe. To make the analysis we have to simplify the problem and make some assumption. We suppose that air mass flow inside the pipe in natural convection is not that much because even if we can have a quite large temperature difference between beginning and end of pipe the fact that the pipe is horizontal limit the increasing of speed. For these reasons we don't take into account the air mass flow along the pipe when the air is not forced by fans. In the present work the transient evolution of the temperature is not of interest. The interesting quantity is the equilibrium temperature air-duct. Considering now a single element of length Δl and comparing the air mass contained in the volume inside duct and the ground mass that surrounds it, we can notice that they are not comparable and the ground is much larger than the air mass. Then we can suppose that the thermal capacity of the air is neglectable compared with the one of the ground and then as that the equilibrium air temperature after transient will be the initial surface duct temperature. Then the relation used in this case are the following

$$T_s(t - 1) = T_s(t) = T_x(t)$$

At the instant t the surface maintain the temperature of the instant $t - 1$ and the air. All the tools needed to build a program to make a detailed analysis of the ground storage are now available, therefore a practical implementation is possible.

Chapter 6

Solar collector model

The simulation of the ground storage needs as input data hourly temperature from the solar collector. The data required are quite detailed and influence a lot the performance of the whole system. Therefore, it is necessary to make a simple model of the solar collector to get the input data for the ground storage.

6.1 Configuration

The solar collector of the present case is an air solar collector. The idea is to integrate it in the south facade of the building and in the roof. The model has to take into account the solar radiation coming to the collector and evaluate by an energy balance the heat flux that can be extract by the air that is flowing inside the channel. The data available are the hourly direct solar radiation, hourly diffuse solar radiation, hourly outdoor temperature. The useful radiation is the direct solar radiation that strikes perpendicularly the surface of the collector. We can add to this term the diffuse solar radiation. This is not all useful but it's much less than the direct part, then it doesn't influence so much the result. We assume that all the diffuse radiation is striking the surface of the collector. In order to know the useful direct solar radiation we have to know the hourly sun position with the respect of a certain location of the earth.

6.2 Sun's angle

The direction of the sun's beam can be described as a function of:

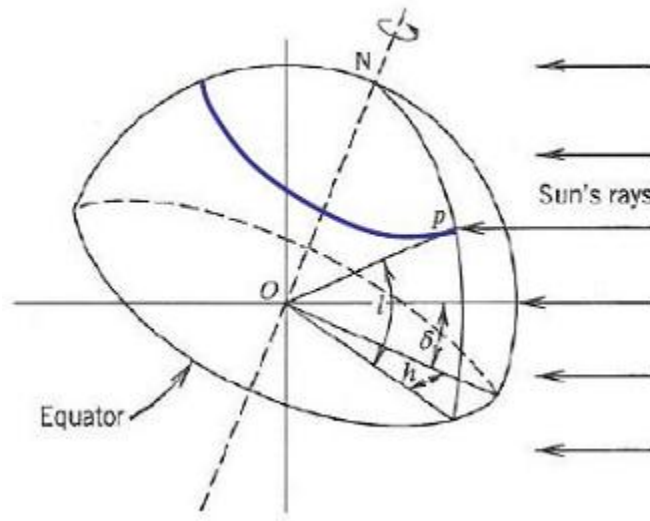


Figure 6.1: latitude , hour angle and declination angle

- Location on the earth surface
- Time of the day
- Day of the year

The corresponding parameters are:

- Latitude (l)
- hour angle (h)
- Sun's declination (δ)

In figure 6.1 an image shows those parameter. In the following part the hour angle and sun's declination will be briefly discussed.

6.2.1 Hour angle

It is defined as the angle between the plane of the meridian containing the point of interest and the meridian that touches the earth sun line. The hour angle is zero at solar noon. It

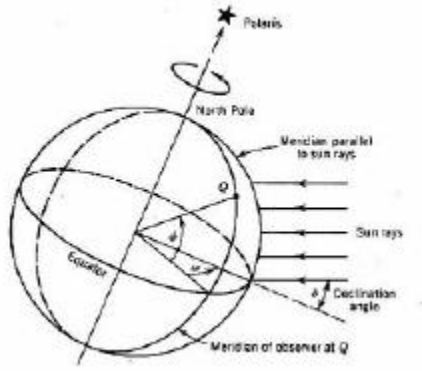


Figure 6.2: solar angle

increases by 15 degrees every hour. An expression for the hour angle is

$$h = 15 \cdot (t_s - 12) \quad (6.1)$$

where t_s is the solar time in hours.

Therefore, in order to calculate h , the local solar time is required. This depends on the local civil time, on the longitude of the location and on the equation of time(EOT). The equation of time is a relation that permits to take into account the irregularity of the sun motion due to its inclination and its eccentric orbit [21].

$$t_s = LCT - 4 \cdot (L_{st} - L_{loc}) + EOT \quad (6.2)$$

- L_{st} is the standard meridian for local time zone
- L_{loc} is the longitude of the location in question in degrees west. Every degree west with respect to the reference location is 4 minutes less int the solar time.
- EOT Equation of time. The expression for the equation of time is the following

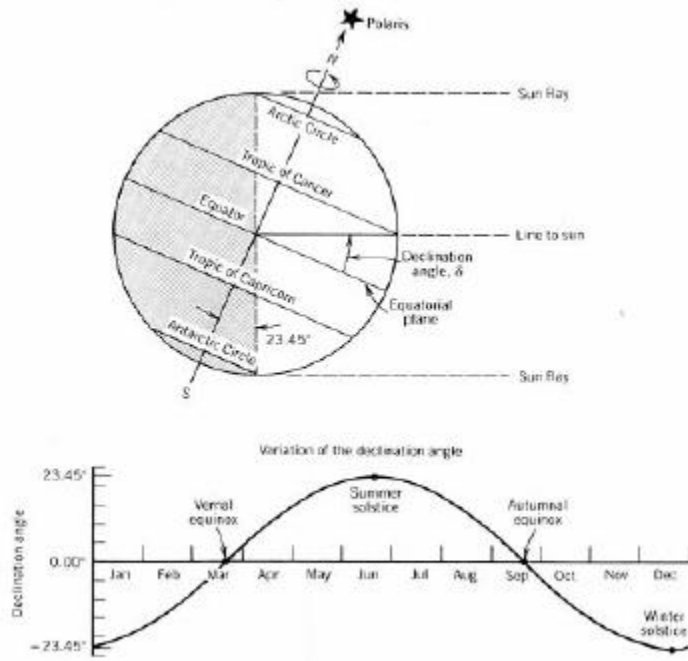


Figure 6.3: Declination angle

$$EOT = 229.2 \cdot (0.000075 + 0.001868 \cdot \cos(N) - 0.032077 \cdot \sin(N) - 0.04089 \cdot \sin(2N)) \quad (6.3)$$

$$N = (n - 1) \cdot \frac{360}{365} \quad \text{with } 1 < n < 365$$

6.2.2 Sun's declination

The Earth's axis is tilted $23 \frac{1}{2}$ degrees from being perpendicular to the plane of the ecliptic. The axis of rotation remains pointing in the same direction as it revolves around the Sun, pointing toward the star Polaris. The constant tilt and parallelism causes changes in the angle that a beam of light makes with respect to a point on Earth during the year. The declination angle δ is defined as the angle between a line connecting centers of earth and the projection of that line on the equatorial plane [21]. The equation that describes the declination angle is the following.

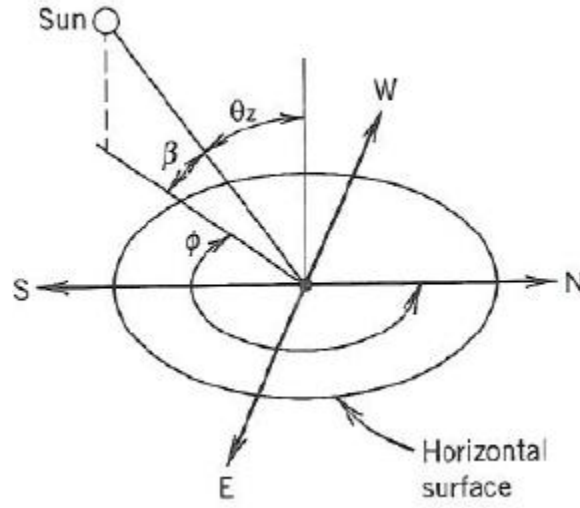


Figure 6.4: solar altitude angle β and solar azimuth Φ

$$\begin{aligned} \delta = & (0.006918 - 22.9132745 \cdot \cos(N) + 4.0254304 \cdot \sin(N) - 0.3872050 \cdot \cos(2N) \\ & + 0.05196728 \cdot \sin(2N) - 0.1545267 \cdot \cos(3N) + 0.08479777 \cdot \sin(3N)) \end{aligned} \quad (6.4)$$

$$N = (n - 1) \cdot \frac{360}{365} \quad \text{with } 1 < n < 365$$

6.2.3 Sun's position

For a given location, the sun position in the sky with the relation to an horizontal surface can be defined by knowing two angles: the solar altitude β and the hour angle h . Those are described as follow:

$$\sin(\beta) = \cos(l) \cdot \cos(h) \cdot \cos(\delta) + \sin(l) \cdot \sin(\delta) \quad (6.5)$$

$$\cos(\phi) = \frac{\sin(\delta) \cdot \cos(l) - \cos(\delta) \cdot \sin(l) \cdot \cos(h)}{\cos(\beta)} \quad (6.6)$$

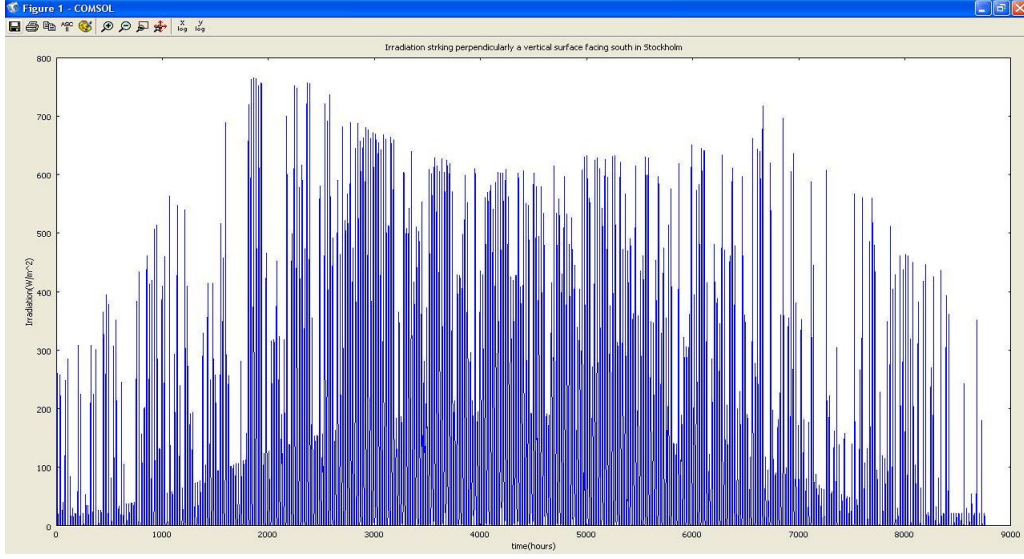


Figure 6.5: Irradiation striking a vertical surface facing south in Stockholm

Now, all the data necessary to determine the Irradiation striking a surface are available. We can consider the vertical surface facing south of the solar collector. The sun see the surface if β is higher than 0 degrees and if $\cos(\phi) < 0$. When this conditions are verified, given the direct irradiation, the amount of it striking perpendicularly the collector is:

$$I_{\perp} = \cos(\beta) \cdot (-\cos(\phi)) \cdot I_{direct}$$

The same calculation can be done for a tilted surface facing south. The radiation availability conditions are the same but this time the angle tilt angle γ has to be taken into account to get the right radiation striking the surface

$$I_{\perp} = \cos(\beta - (90 - \gamma)) \cdot (-\cos(\phi)) \cdot I_{direct}$$

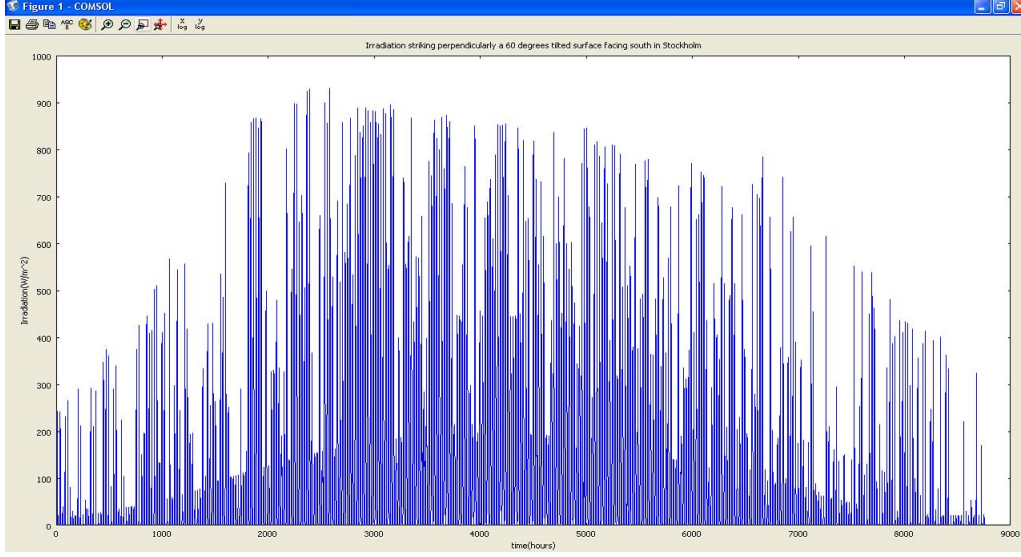


Figure 6.6: Irradiation striking a 60 degrees tilted surface facing south in Stockholm

6.3 Heat balance over the collector

Once the irradiation that strikes the surface is known, the heat balance of the collector can be done. Basically, of this radiation one part will be reflected than the useful part is the part that will be transmitted by the glass and absorbed by the absorber. Then, if I is the Irradiation striking the collector, the useful part will be $I \cdot \tau \cdot \alpha$. This heat flux partly will be absorbed by the air flowing in the channel, and partly will be lost due to the temperature difference between the collector surface and the outside temperature.

$$P_{in} - P_{losses} = P_{gain} \quad (6.7)$$

$$F_r \cdot (I \cdot \tau \cdot \alpha \cdot A - U \cdot A \cdot (t_{surface} - t_{out})) = \dot{m} \cdot c_p \cdot (t_2 - t_1) \quad (6.8)$$

Where t_{out} is the outdoor temperature, t_1 is the inlet temperature in the collector, t_2 is the outlet temperature, and F_r is the heat removal factor, a coefficient that take into account that not the whole heat available is useful [22, 23, 24]. By assuming that the

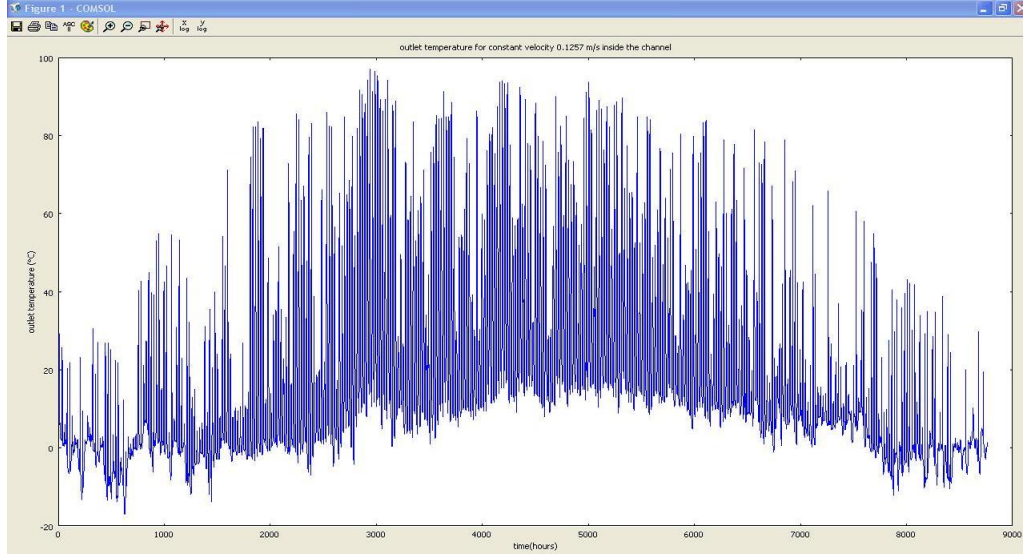


Figure 6.7: outlet temperature in the study case and with velocity of 0.1257 corresponding to a velocity of 1 m/s in a pipe of 0.4m diameter

surface temperature is the average temperature between inlet and outlet of the collector
the calculation of the outlet temperature is as follows:

$$t_2 = (A \cdot I \cdot \tau \cdot \alpha \cdot F_r + \dot{m} \cdot c_p \cdot t_1 + U \cdot A \cdot t_{out} \cdot F_r - U \cdot A \cdot F_r \cdot t_1 / 2) / (\dot{m} \cdot c_p + U \cdot A \cdot F_r / 2)$$

Let's consider an example to see some results from this model. Assuming:

- A vertical surface area of $5 \cdot 3.5 \text{ m}^2$ corresponding to the wall surface.
- A 60 degrees tilted surface of $5 \cdot 2.5 \text{ m}^2$ corresponding to the roof surface.
- The product $\tau \cdot \alpha$ equal to 0.75.
- The heat removal factor equal to 0.8
- Considering a double glazed surface the U value can be considered $3 \text{ W}/(\text{m}^2 \text{K})$
- An air flow of $0.125 \text{ m}^3/\text{s}$ corresponding to $452 \text{ m}^3/\text{hour}$

Chapter 7

Examples

In this chapter some application of the model explained previously will be showed. At first will be explained how to set up the input data and later on some results with relative comments will be given.

7.1 Input data

The input data required are the same needed for the program working with average monthly temperatures, but in this case will be considered hourly temperature of the whole year. The temperatures that have to be given are, as always the outside temperature and the temperature at the beginning of the pipe. The number of samples correspond to the number of hours in one year that is 8760. For this number of samples the frequencies needed to solve the linear part of the problem are $n/2 + 1$ corresponding to 4381. Then, in the system study, fem simulation performed in cross section, E and F coefficients have to be calculated for all these frequencies. For the calculation of inlet temperature when a solar collector is utilized, has been used the procedure explained in the previous chapter to find how much does the temperature can gain by passing through the solar collector.

to size the solar collector, has been considered that is always better to build a collector with a large surface in order to utilize as much solar radiation as possible. Therefore for the solar collector surface area has been considered a whole wall (south wall), and the south side of the roof. It has been considered then $(6 \times 4 m^2 + 6 \times 2.5 m^2)$. By reducing the velocity of the air the temperature in the collector rise but it results to be less efficient and heat transfer coefficient in the pipe is less. On the other hand, we want to keep the velocity

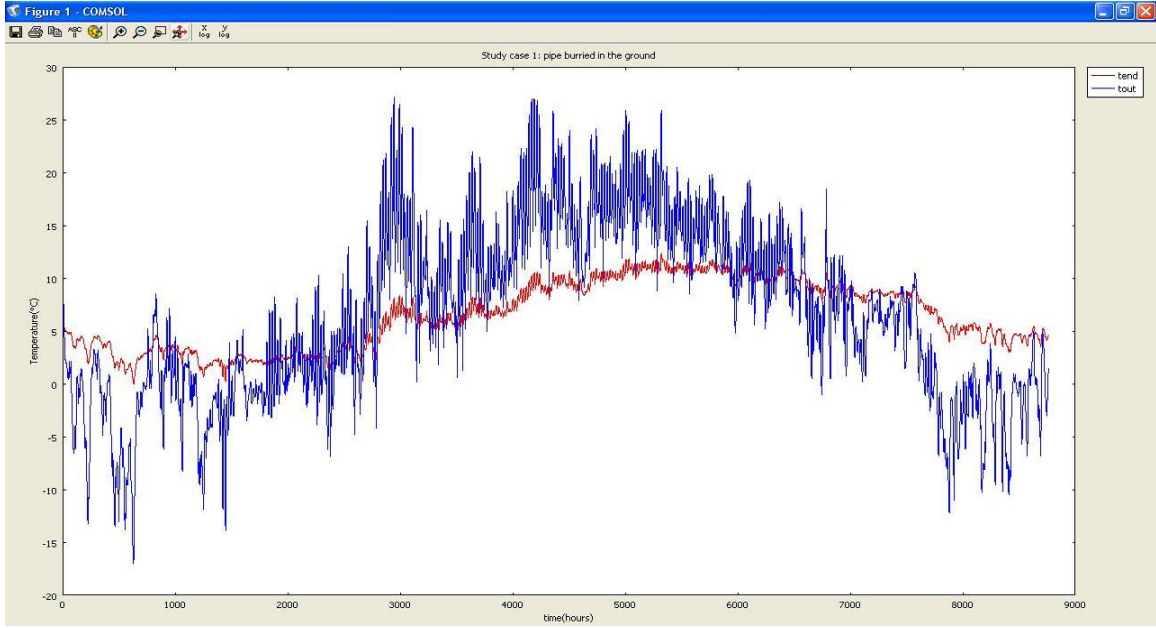


Figure 7.1: Outside temperature and inside temperature after 80 meters pipe in the case taking into account just the system pipe burried in the ground

low to reduce the power that has to be supplied to run the fans. Hence, the decision is to maintain the velocity equal to 1 m/s in winter and 2 m/s in summer.

7.2 Examples of analysis

The study that is going to be performed wants to show the effectiveness of the system. In order to do that, we will start from a basic case and after some features will be added to increase the performance. The first simulation is just to estimate what can be taken out from the ground without solar panel. The configuration of the system is 20 cm insulation and a pipe of 40 cm diameter buried 2 meters deep in the ground. With this system the temperature in winter after 80 meters is rather constant, variation of more or less 5 C between 0 and 5 degrees. The result of the simulation with such condition is given in figure 7.1.

The idea now is to verify what kind of results can be obtained by using a solar collector

before the piping system. The simulation has been lead using a velocity of 1 m/s in winter and 2 m/s in summer. During summer time the system is run just during the day because the aim is to charge the ground and the air temperature during the night is rather low, and it can't gain the earth temperature. The results of that simulation is showed in figure 7.2; from that simulation we can see that compared with the case of just ground there is a temperature gain due to the utilization of the solar collector. On the other hand, we can see that by using this configuration, the heat stored during summer is wasted very quickly in winter. That is due the fact that in winter the outdoor air temperature can reach -15 C and it is necessary a lot of power to increase this very low temperature to interesting temperature. The system can be regarded as a battery that is charged in summer and discharged in winter. To exploit the system effectively, it has to be charged as much as possible during summer and discharged slowly during winter in order to keep temperature in the system at a usefull level for the longest time.

For this reason, it is interesting to exploit the advantage of using indirect system. The main advantage of this configuration is that recirculation can be applied. By using it, the input temperature in the system will be always higher than a certain temperature level, and the heat dischargement is delayed much more than using the outdoor air. The problem to make a simulation of recirculation with the program implemented is that it is working in frequency and not on time. With a time domain simulation the output temperature of the previous time step can be used to quantify the input temperature of the present time step. The program implemented is working in frequency domain. That means that the all input signal has to be given and there is no way to make the simulation as would have been done in time domain. To achieve some results that can be close to the obtainable temperature by using recirculation, a strategy has to be set up. This will be based on some hypothesis and the validity of the results will be verified by checking their coherence with the assumptions made.

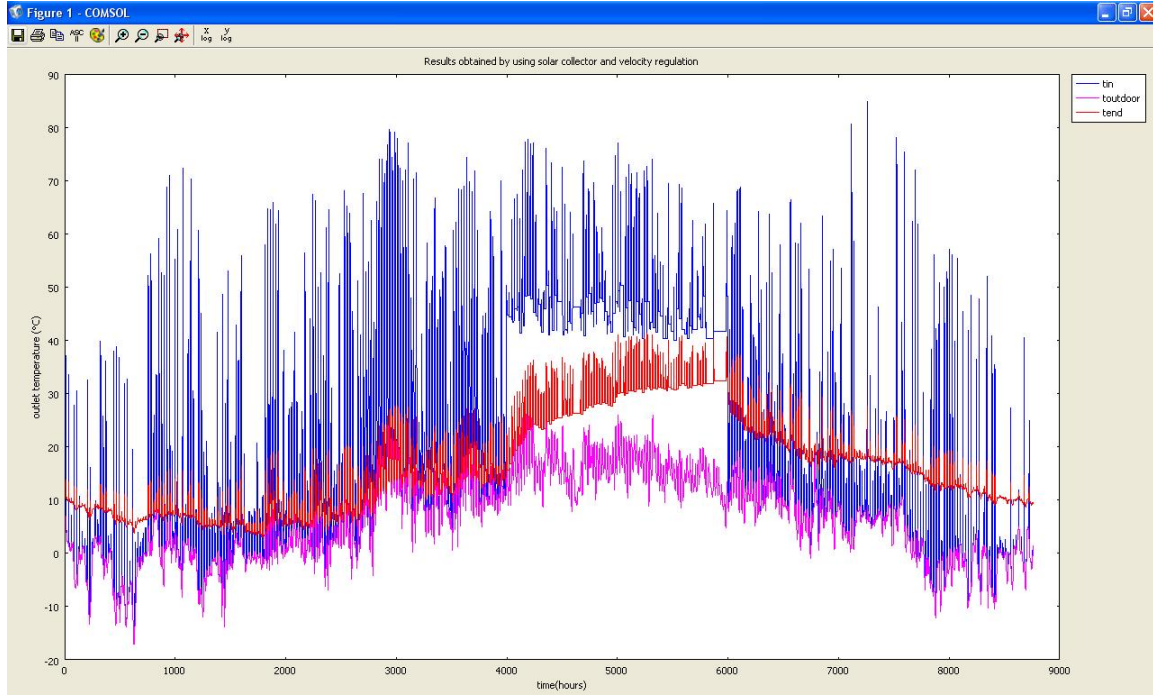


Figure 7.2: Results of simulation obtained using a solar collector before the storage

7.3 Recirculation

First we begin explaining what is meant by recirculation. The output air from the system has a certain temperature level that can be exploited as a heat source. After being used in this way the air will have a lower temperature level, but still higher than the outdoor air during winter time; therefore instead of using outdoor air, the air can be recirculated in a close loop. Hence, the hypothesis that has to be accomplished is that a temperature lift between the output and the recirculated air is always present. That condition means that the air, before being recirculated, will loose some of its exergy content and therefore temperature level, by passing through heat exchangers. The problem is that we don't know the input temperature in this case. What can be done is starting with a first temperature profile and verify that the conditions explained are verified. The first hypothesis is that the input temperature will be higher compared to the case of non recirculation. We have to decide what kind of temperature profile has to be given as input to the program. One idea

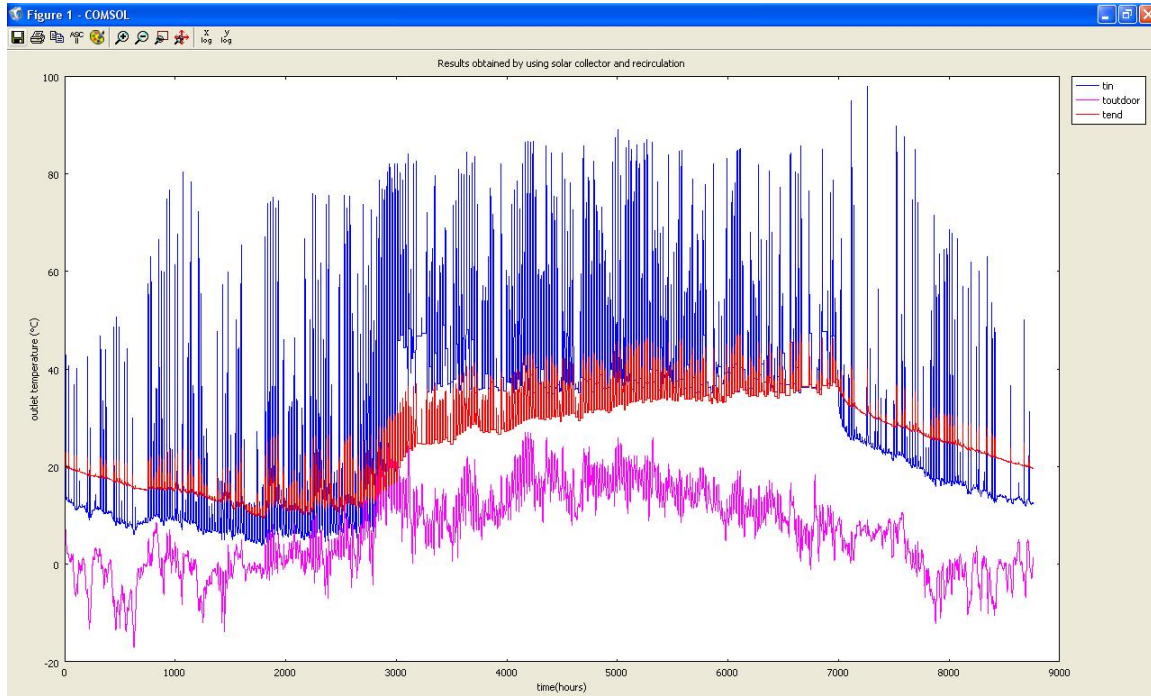


Figure 7.3: Results obtained by using solar collector and recirculation

can be to chose the output temperature of the simulation without recirculation in figure 7.2 as input temperature. The result of a simulation run in figure 7.3 shows that there is always a temperature gap of 10 degrees between the inlet and the outlet of the system. This temperature difference it's a good temperature to be used in a heat exchanger. It has been verified that not all the temperature profile are compatible to the hypothesis made; By increasing the temperature of the input profile the gap decreases, and if it is to high it disappear. That condition corresponds to absence of heat exchange between the ground and the air, and as a consequence the output air cannot be used to release heat to other fluids in a heat exchanger.

To simulate recirculation we have to take into account also the fact that we can decide to recirculate in the solar collector or directly in the ground. If the solar radiation is not enough there's no point in recirculating in the solar collector cause there will be just losses and the input temperature will drop down instead of increasing. In figure 7.4 there is an

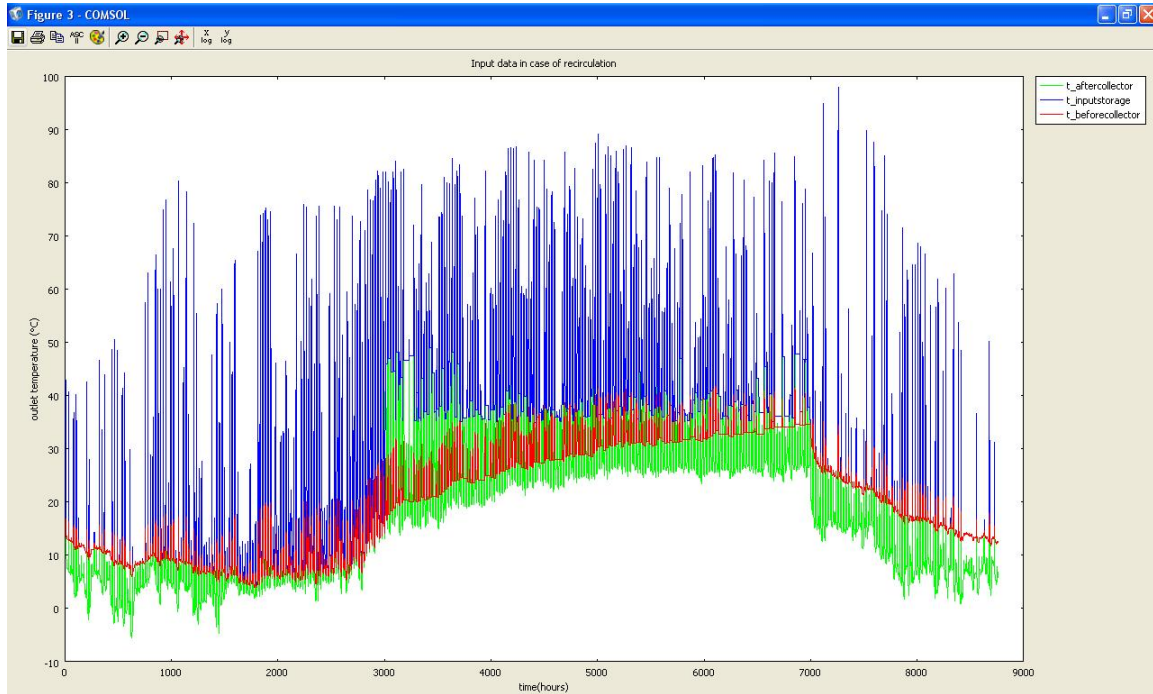


Figure 7.4: Data for the case of ricirculation

example of this two conditions.

7.4 System possibility

The next step is to have an overview of the possible practical utilization of the system. The target is to reduce the energy utilization for heating. The first thing that has to be taken into account is insulation. It's very important, and it permits to reduce heat losses from the internal to the external environment. If the house is well insulated there must be a ventilation system that supplies the air exchange per hour necessary to keep the air quality higher than a minimum level. A possible configuration for this house can be:

- Ventilation system: the air is taken from outside and pass through an exhaust-supply air heat exchanger. After that, if the ground storage can provide air with a sufficient temperature level to heat or pre-heat, then the supply air goes through an heat

exchanger to gain some power stored in the soil; otherwise a heat pump is used to supply the latter energy.

- Solar collector coupled with ground storage. Air extract energy passing in pipe buried in the soil and after release some heat first directly with the supply air to preheat it and after as heat source for the heat pump.

The temperature levels reached with the simulation are of technical interest. The air lasts over 20 C for a long time during winter. We can think about of using directly some of the air coming from the solar collector. In this way we would have some air at rather constant available for the whole day and some other warmer air available some hours a day. With this temperature levels heating and pre-heating for a systems working with low temperature can be used. In system like TermoDeck [25] for a well-insulated building heating can be achieved with air at 30-35 C. One part of the heat load can be supplied by the ground heat storage and the solar collector, and the latter one by a heat pump that is working with a very stable and good heat source, and therefore with a constant and high COP.

Chapter 8

Conclusion

Two programs to calculate temperature profile of a earth to air heat exchanger has been developed. The results that have been found are interesting and they give a first proof that the ground storage for solar energy is an interesting solution that can be exploited in the building sector. The two programs are complementary cause the first one is fast but the results are rough, while the second one is more detailed but slower. By using them together a high knowledge of the ground heat storage can be achieved.

From the results, it have been found that by using air as working fluid the system can have good performances and can be used as a good source for a heat pump and also for a preheating system supplying directly part of the heat needed from the house. Ideally, we would like to supply the whole heat needed by the house with this system without the help of the heat pump; but with the configuration studied and air as a working fluid, we found that is not possible to reach this result. Anyway, part of the heat can be supplied directly, and part from a heat pump working in ideal conditions.

A further investigation with the tool should be done by doing some tests and finding out the best working configurations for the system. It can be interesting to make tests with other working fluids like water or other secondary fluids. An experimental test should be done in order to proof on practice the potential of the system.

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