Turbulence modulation and rotational dynamics of large nearly neutrally buoyant particles in homogeneous isotropic turbulence

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This paper is an experimental investigation of turbulence modulation effects by Taylor-scale size particles in the dilute regime. Experiments are performed on a turbulence tank able to provide Homogeneous Isotropic Turbulence at $Re_\lambda \approx 270$. A novel experimental technique capable of simultaneously measuring rotational rates of arbitrarily shaped particles and fluid velocity using standard Stereoscopic Particle Image Velocimetry (Stereo-PIV) and Index-of-Refracion matching is presented here. Particles of the same IoR of water with embedded tracers allowed the measurement of the velocity of the portion of particles in the measurement plane. A novel algorithm based on the assumption of solid body rotation, is then used to extract particle rotation rates. We compare the results from two particle shapes to the single phase measurements: spherical and ellipsoidal particles with aspect ratio 2. It is found that spherical particles provide a 15\% turbulence reduction, about five times more than what is provided by ellipsoidal particles at the same volume fraction ($\phi_v \approx 0.1\%$), and with less particle surface area available. These result suggest that there might be an turbulence production mechanism for ellipsoidal particles that is not present for spheres. This hypothesis is supported by spectral analysis. Pivoting effect is observed for both spherical and ellipsoidal particles, but for the latter, the reduction in the small wavenumber region is less evident. Preliminary results of statistics of rotational rates shows that ellipsoidal particles tend to have an enhanced rotational velocity as compared to spheres.

1. Introduction

Particle-turbulence interaction is central to many processes, both occurring in nature and in industrial applications such as sediment and nutrient transport in rivers and oceans, pollutant dispersion in the atmosphere, fuel mixing in combustion engines, flow of fiber suspension in papermaking, etc. Recent studies on particle-flow interaction have led to significant advances in predicting
small particles in high-Reynolds turbulence. These advances unite the classical theoretical work of Stokes andBatchelor with the new capabilities enabled by direct numerical simulation of turbulence. For particles smaller than the Kolmogorov scale, the flow around the particles does not need to be resolved, but rather simulations can use the equations of motion derived by Maxey & Riley (1983) for spheres and Jeffery (1922) for ellipsoidal particles.

Numerical studies using Eulerian-Lagrangian tracking method with point-particle approximation have shown that even moderate volume fractions ($\phi_v > 0.01\%$) of small particles can affect the turbulence in the carrier phase (Squires & Eaton 1990; Elghobashi & Truesdell 1993; Elghobashi 2003; Ferrante & Elghobashi 2004). Both the production and dissipation of turbulent kinetic energy (TKE) are altered by the particles. The kinetic energy spectrum shows increased energy at large wave numbers and decreased energy at small wave numbers. Lucci et al. (2010) explained this with the fact that finite-size particles destroy the organization of large vortices and introduce smaller structures of half the size of the particles. This spectral pivoting (i.e. increased energy to small scales and damping of large scales) has been recently confirmed experimentally in several studies (Geiss et al. 2004; Yang & Shy 2005; Schreck & Kleis 1993; Poelma & Westerweel 2007).

Stokes number is a good indicator to predict turbulence modulation effect for small particles $St \approx 1$ (see Eaton 1991). However very often the dispersed particles are larger than the Kolmogorov scales, thus scale separation can no longer be applied and the details of the flow around the particles has to be taken into account (Balachandar & Eaton 2010).

Therefore there is a need for fully resolved experiments and simulations to identify what are the fundamental mechanisms of interphase coupling for finite-size particles.

1.1. Finite-size effects

Finite size effect of spherical particles have been investigated in detail in recent numerical simulations by Burton & Eaton (2005) and Lucci et al. (2010). In their simulations, they flow around particle surface was fully resolved. Thanks to this approach they found that the strain rate, which is directly connected to turbulent dissipation, increases close to the particle surface due to the no-slip condition. Thus particles act as localized dampers of turbulent kinetic energy, and turbulence modulation occurs due to the local distortion of the flow field.

The balance of the effects was to reduce the TKE, for the additional dissipation far exceeded the additional production. Tanaka & Eaton (2010) performed Particle Image Velocimetry with sub-Kolmogorov resolution around finite-size spheres and their results confirmed to a large extent what had been observed in DNS by Lucci et al. (2010).
In this regime the Stokes number is not as useful as in the case of small particles as an indicator of turbulence modulation by particles. In fact, according to Lucci et al. (2011), the determination of a particle time response is not straightforward due to the difficulty to identify a free-stream velocity around a given particle. Furthermore, the particle Reynolds number can become so high that inertial effects can be relevant and the assumption of Stokes flow is no longer valid. Indeed the authors have shown that particles with the same Stokes number can have different effects on turbulence. Instead they found the total wet surface and the particle inertia are the dominant parameters.

1.2. Non-spherical particles
The studies mentioned above focus on spherical particles, as the vast majority of investigations. However, also non-spherical particles are of immense practical concern in numerous scientific and engineering areas. The dynamics of elongated particles and ellipsoids are of particular interest for the pulp and paper industry (e.g. Lundell et al. 2010). Furthermore, pollutant and particle dispersion in the atmosphere (e.g. Volcano ashes) and in the ocean (e.g. plankton and marine snow).

The dynamical behavior of non-spherical particles is far more complicated than that of spherical ones due to the coupling between translational and rotational dynamics. The drag force is also a function of the orientation and rotation rate. This coupling can give rise to a complex behavior such as particle aggregation mechanisms that are not present for spheres (Koch & Shaqufeh 1989). Another implication of the dependency of the drag to particle orientation is that to determine the particle response time assumptions on the orientation distribution have to be made. Based on the assumption of Stokes flow, Zhang et al. (2001) derived an expression for randomly oriented spheroids as a function of the particle aspect ratio. In order to completely characterize dynamics, also rotational relaxation times have to be considered. Mortensen et al. (2007) have shown that the rotational response time of spheres is one third of the translational response time. For non spherical particles the relaxation time strongly depends on the axis about which the particle rotates, so the definition of a single response time is not enough. The torque of non-inertial ellipsoids in creeping shear flow have been analytically derived by Jeffery (1922). Based on this solution several numerical simulations of small ellipsoidal particles in turbulent flows have been performed (Zhang et al. 2001; Mortensen et al. 2008b,a).

However, as for spherical particles, the results obtained under these assumptions do not hold for finite-size and inertial particles. Recent work of Lundell (2011) has shown that particle inertia can lead to chaotic rotational behavior of ellipsoids even in creeping flow. Present models do not include the effect of fluid inertia, hence, again, there is a need for fully resolved DNS or experiments to understand the effect of shape on particle fluid interaction. Some
work in this direction has been done by Khoury et al. (2010), which revealed a complex wake behind a ellipsoid with aspect ratio 6 at high particle Reynolds number. However measurements of interaction of finite-size non-spherical particles and turbulence, where both particle and fluid inertia are important have not been reported yet.

1.3. Present work

Here we report an experimental technique with which we can simultaneously measure rotational rates of arbitrarily shaped particles and fluid velocity by combining standard stereoscopic Particle Image Velocimetry (S-PIV) and particles that have the same refractive index as the surrounding fluid. Tracers embedded in the particles allows us to measure their linear and angular velocities. This technique is applied to study the interaction of Taylor-scale spherical and non-spherical particles in Homogeneous Isotropic Turbulence (HIT).

In this work we consider particles that are larger than the Kolmogorov lengthscale. Both spherical and ellipsoidal particles are considered. For these, we measure the effects on the turbulent flow (e.g. spectral pivoting) and the rotation rates of the particles. The particles are selected to be very near neutral buoyancy (SG = 1.02) to remove the influence of inertia and to achieve relevance to certain particles of environmental interest: namely plankton and cohesive sediment aggregates (“flocs”).

1.4. Structure of the paper

In section 2 the experimental facility, particles, experimental method and techniques are presented. Then single flow measurements are discussed. Results for turbulent modulation by particles are shown in section 4 and statistics particle rotation rates are in section 4. In section 6 the results are discussed. Finally, conclusions are summarized in section 7.

2. Experimental Method

2.1. Turbulence tank

Experiments are performed in a rectangular tank of dimensions 80×80×162 cm$^3$. The tank is filled with tap water, which is filtered to 5 micron and purified by a flow-through UV filter when experiments are not being run. Turbulence is generated by two arrays of pumps located at both ends of the tank. Each pump array has 64 individual pumps that are triggered in a random sequence (see figure 1a). The pumps follow a randomizing algorithm developed by (Variano & Cowen 2008) which maximizes the shear production of turbulence. The test section is centered at the point of symmetry between them (figure 1b). This symmetry is a primary contributor to isotropy in the flow; the other main contributor is the use of a stochastic stirring pattern. By preventing mean velocity gradients from persisting, the stochastic stirring greatly reduces
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Figure 1. Experimental setup. Each of the 16 clusters seen in the pump array photo (a) has four synthetic jets, and the jets make an 8x8 cartesian grid.

tank-scale circulation. Such circulation is an unwanted effect of any turbulence generation method. By reducing the tank-scale circulation, the flow we measure has a mean velocity close to zero.

In addition to large scale isotropy, this turbulence tank has a large homogeneous region in the tank center. The size of this region is estimated to be of about 30×30×30 cm³. This is much larger than the turbulent integral length scale (6 cm), which means that the statistics measured there will be unbiased by spatial gradients, even in the most energetic turbulent events. The details of the flow will be discussed in section 3.

The Reynolds number achieved here is $R_\lambda \approx 270$ (defined with respect to the Taylor microscale) or equivalently $Re_\Lambda \approx 1300$ (defined with respect to the integral length scale). This Reynolds number is high enough to establish a clear inertial subrange in the two-point fluid velocity statistics.

2.2. Particles

Particles are made of a hydrogel that is 99.5% water (Agarose 5 g/L), and they are manufactured by injection molding. This particular hydrogel solution has been chosen because its physical and optical properties are extremely similar to that of the suspending medium. The index of refraction is 1.337 at 20°C, and the density $\rho_p = 1020\text{kg/m}^3$.

The hydrogel particles are nearly transparent, thus to visualize particle motion, 50µm glass spheres are embedded in the hydrogel solution.

Two shapes are considered here: spheres of diameter $d_s = 8\text{ mm}$, and ellipsoids whose polar and equatorial axes are $l_e = 16$ and $d_e = 8\text{ mm}$, respectively. As discussed above, defining a characteristic relaxation time is nontrivial when particles are larger than the Kolmogorov scale. However, for comparisons with
other studies we report here estimates obtained based on Stokes flow assumptions. The time response of the spherical particle is \( \tau_{p(sp)} = \frac{\rho_p d_s^2}{18 \mu} = 3.64s \), where \( \mu \) is the fluid viscosity. The response time of the ellipsoidal particles is computed using the expression derived by Zhang et al. (2001) for randomly oriented ellipsoidal particles:

\[
\tau_{p(el)} = \frac{\tau_{p(s)}}{\lambda \log(\lambda + \sqrt{\lambda^2 - 1}) \sqrt{\lambda^2 - 1}},
\]

where \( \lambda = l_e/d_e \) is the particle aspect ratio. In this case, \( \lambda = 2 \), which gives \( \tau_{p(el)} = \frac{1}{1.5} \tau_{p(s)} = 5.46s \).

2.3. Measurement technique and phase discrimination

Simultaneous fluid and particle phase velocity measurements are performed using stereoscopic PIV. Two cameras (Imager PRO-X, 2048 × 1024 pixels, both fitted with a 50 mm Nikkor lens and Scheimpflug adapter) view the measurement volume at a separation angle of about 70°. To avoid image distortion due to the air-water interface, two 35° prisms filled with water are attached to the walls (see figure 2).

The fluid phase is seeded with 10 \( \mu \)m silver coated glass particles, which are illuminated by a dual head, 135 mJ/pulse @ 532 nm Nd:Yag laser. The laser sheet thickness is approximately 1 mm.

The commercial software Davis from LaVision GmbH is used for image acquisition and processing of the PIV images. The algorithm uses continuous window deformation and reduction with an initial interrogation window of 64×64 pixels and final interrogation area of 32×32 pixels with 50% overlap and Gaussian sub-window weighting (see Stanislas et al. (2008) for performance of Davis algorithm). The output is a 2-dimensional array of 3-dimensional velocity vectors, with 61 × 109 measurement points equally spaced at 1.3635 mm grid size. This grid spacing is the same order of magnitude of the laser thickness. The total field of view is thus 147×81 cm², more than twice the estimated integral length scale \( \Lambda_x \). The laser sheet defines the plane \( z = 0 \), so that the velocity vector is defined as: \( \mathbf{U} = \{U(x_i, y_j, 0, t_n), V(x_i, y_j, 0, t_n), W(x_i, y_j, 0, t_n)\} \)

where boldface type indicates vector quantities.

For particle measurements, a known amount of particles is injected from the top of the tank through a removable ceiling panel. Measurements start after the panel has been restored and the turbulence has dispersed the particles.

Images include both particle phase and fluid phase. PIV is applied to each phase separately. The velocity vectors obtained for the particle phase are used to determine particle rotation, as described in section 2.4. The two phases are discriminated based on intensity, for the hydrogel particles scatter light more readily than water, and thus show up as bright regions in the images. Figure 3 (a) and (b) show a typical image where both fluid tracers and a large particle are visible. It can be seen that three areas can be identified, based on the
Figure 2. Stereo-PIV setup

Particles are considered to undergo solid body rotation. To determine their angular velocity $\Omega$ one can use the rigid body kinematic equation with the velocity measured at any two locations $X_M$ and $X_N$ in the particle’s interior:

$$U_N = U_M + \Omega \times (X_N - X_M).$$

Stereoscopic PIV provides the necessary values $U_M$, $U_N$, $X_M$ and $X_N$. However, since all points $M$ and $N$ are located in the same $z = 0$ plane, equation 2 is over-determined in $\Omega_z$ and under-determined in $\Omega_x$ and $\Omega_y$. Hence, to solve the system we need at least 3 non-aligned points. By including a third point $P$ in the analysis, all three components of $\Omega$ are determined. Because all three points lie in the plane $z = 0$, four estimates of $\Omega_z$ are obtained for each triplet of points considered. The mean of the four estimates is taken as the estimate of $\Omega_z$ for a given triplet.

For each particle measured in this experiment, we obtain the velocity of significantly more than three interior points. This additional data can be used to build a more solid estimate of $\Omega$ than is obtained from three points. The method used herein is to find a value of $\Omega$ using every possible triplet of points $M$, $N$ and $P$ where PIV data were available. From this ensemble of estimates of $\Omega$ we take the mode as our final estimate. A global optimization based on
Singular Value Decomposition was also tested, and performed less well when compared to known rotation values. These results, and the related benchmark tests, are discussed in Collignon & Variano (in preparation).

3. Single phase measurements

Measurements are done at the symmetry plane of the tank perpendicular to the pump axis. The origin of the coordinate system is at the center of the tank and is so that the $x$ and $y$ axes are the in the measurement plane and the $z$ is the out-of plane axis (as shown in figure 1a). The instantaneous velocity
Mean flow magnitude \( M \equiv \sqrt{U^2 + V^2 + W^2} \) \( 0.3 \pm 0.2 \times 10^{-2} \text{ms}^{-1} \)
Velocity fluctuations in \( x \) \( \sqrt{u_{rms}^2} \) \( 2.1 \pm 0.2 \times 10^{-2} \text{ms}^{-1} \)
Velocity fluctuations in \( y \) \( \sqrt{v_{rms}^2} \) \( 2.2 \pm 0.2 \times 10^{-2} \text{ms}^{-1} \)
Velocity fluctuations in \( z \) \( \sqrt{w_{rms}^2} \) \( 2.5 \pm 0.3 \times 10^{-2} \text{ms}^{-1} \)
Turbulent kinetic energy \( k = \frac{3}{2}(u_{rms}^2 + v_{rms}^2 + w_{rms}^2) \) \( 7.2 \pm 0.1 \times 10^{-4} \text{m}^2\text{s}^{-2} \)
Mean flow to turbulence ratio \( I = \frac{M}{\sqrt{2k}} \) \( 0.079 \pm 0.053 \)
Longitudinal Integral length scale \( \Lambda_x \) \( 59.5 \times 10^{-3} \text{m} \)
Taylor micro-scale \( \lambda_x \) \( 12.2 \times 10^{-3} \text{m} \)
Eddy turnover time \( T = \frac{\Lambda_x}{u_{rms}} \) \( 2.7 \text{[s]} \)
Dissipation rate (from \( \lambda_x \)) \( \epsilon = 15\nu u_{rms}^2/\lambda_x^2 \) \( 4.83 \times 10^{-5} \text{m}^2\text{s}^{-3} \)
Kolmogorov time scale \( \tau_\eta = (\nu/\epsilon)^{1/2} \) \( 0.14 \text{[s]} \)
Kolmogorov length scale \( \eta = (\nu/\epsilon)^{1/4} \) \( 0.38 \times 10^{-3} \text{m} \)
Reynolds number (based on \( \Lambda_x \)) \( \text{Re}_L = (\Lambda_x u_{rms})/\nu \) \( 1308 \)
Reynolds number (based on \( \lambda_x \)) \( \text{Re}_{\lambda_x} = (\lambda_x u_{rms})/\nu \) \( 269 \)

Table 1. Flow statistics from single phase measurements.
The parameters denoted by the overbar are average quantities over the entire measurement domains, and the intervals correspond to the 95% CI. The integral length scale and the Taylor micro-scale are computed from the longitudinal two-point correlation, whereas the dissipation rate and the Kolmogorov scales are computed using the definitions given in Pope (2000).

Statistics of the flow field are computed from a series of 510 independent snapshots acquired at 0.5Hz (which corresponds to 17 minutes or 378 eddy turnover times).

At first we analyze properties of homogeneity and isotropy of the flow. A way to show flow isotropy is to analyze the probability density functions (p.d.f.) of the three velocity components. These are calculated at every point in the plane, and all values are combined together following the homogeneity assumption. Flow isotropy implies that the three p.d.f. should be nearly gaussian, un-skewed and with the same variance \( \sigma^2 \). From figure 4(a), showing the p.d.f. of the three components \( p(U), p(V) \) and \( p(W) \), we can see that these conditions are well satisfied. Only the variance of \( W \) \( (\sigma_W^2) \) is slightly larger than the other two components, likely due to the additional noise that S-PIV measurements exhibit in the out-of-plane direction. In figure 4(b) the p.d.f. of the normalized variable \( Z_x = \frac{X - \mu_x}{\sigma_x} \) (where \( \mu_x \) and \( \sigma_x \) are the mean vector \( U(x, y, z, t) = (U, V, W) \) is defined so that \( U, V \) and \( W \) are aligned with the \( x, y \) and \( z \) axes, respectively.

\( \text{Re}_L = (\Lambda_x u_{rms})/\nu \) \( 1308 \)
\( \text{Re}_{\lambda_x} = (\lambda_x u_{rms})/\nu \) \( 269 \)
Figure 4. Probability density functions of the \( U \) (\( \circ \)), \( V \) (\( \square \)) and \( W \) (\( \triangle \)) velocity components in dimensional (a) and normalized (b) form. The measured mean are: \( \mu_U = 0.02 \cdot 10^{-2} \), \( \mu_V = 0.22 \cdot 10^{-2} \) and \( \mu_W = -0.21 \cdot 10^{-2} \) m/s. The standard deviations are: \( \sigma_U = 2.02 \cdot 10^{-2} \), \( \sigma_V = 2.05 \cdot 10^{-2} \) and \( \sigma_W = 2.51 \cdot 10^{-2} \) m/s.

and standard deviation of the stochastic variable \( x \in [U,V,W] \) for the three velocity components is compared to the normal distribution \( \phi_n = \frac{1}{\sqrt{2\pi}}e^{-x^2/2} \). The \( y \) axis is on a semi-logarithmic scale to highlight the behavior of the tails. It can be seen that there is a good agreement until about 2.5 times the standard deviation, then the tails deviate from the normal distribution to show the super-gaussian behavior often seen in turbulence see (e.g. Tsinober 2004).

We now look at the spatial distribution of the flow statistics in order to investigate homogeneity and mean circulation. To do this we decompose the velocity field into a temporal mean \( \overline{U}(x) \) and a fluctuating component \( u'(x,t) \), where the overbar denotes the linear time-average operator. The magnitude of the turbulent velocity fluctuations is defined as \( u_{rms}(x) = \sqrt{u'^2(x,t)} \). Here ergodicity property of the flow statistics is used to estimate the expected value \(< \cdot >\) by the time average.

The horizontal profiles averaged in \( y \) of \( u_{rms}(x) \) and \( \overline{U}(x) \) are shown in figure 5. Uncertainty in mean and variance is determined using a students distribution applied to the timeseries at one location [cite Bendat & Piersol].
It can be seen that profiles of \( u_{rms} \) are homogeneous across the entire measurement domain (with \( w_{rms} \) slightly larger than \( u_{rms} \) and \( v_{rms} \) for the reason explained above). It can also be noted that there is a non-zero mean circulation in the tank, in the form of a slow clockwise rotation around the \( y \) axis, but the magnitude of the mean flow is one order of magnitude smaller than that of the turbulent fluctuations. Mean flow strength can be quantified by comparing the quantity \( M^2 = U^2 + V^2 + W^2 \) to the turbulent kinetic energy (TKE) \( k = \frac{1}{2}(u^2 + v^2 + w^2) \). Computing the quantity \( I = M/\sqrt{2k} \) across the entire domain and averaging it, one obtains that the mean flow strength is not larger than 7-8% of the TKE.

In homogeneous isotropic turbulence, many information about the turbulent scales can be extracted from the longitudinal two-point autocovariance, which is defined in equation (3):

\[
R_{11}(x, \delta_x) = \langle u'(x, t) u'(x + \delta_x, t) \rangle,
\]

where \( \delta_x \) is the separation in \( x \). As before, since the flow is statistically stationary, the expected value by the time average can be estimated so that eq. (3) is computed as:

\[
R_{11}(x, \delta_x) = \frac{1}{N_t} \sum_{i=1}^{N_t} u'(t_i, x) u'(t_i, x + \delta_x),
\]

\[ (U,V,W), (u_{rms},v_{rms},w_{rms}) \ [m/s] \]

**Figure 5.** \( u_{rms} \) (○), \( v_{rms} \) (□), \( w_{rms} \) (△) (filled markers), \( \overline{U}, \overline{V}, \overline{W} \) (open markers). Dashed line is 95% confidence intervals. For clarity, only every third sample in space is shown here.
where \( N_t \) is the number of snapshots. Due to homogeneity of the flow we can average \( R_{11} \) over space. The result is shown in 6(a). By introducing the autocorrelation function \( f(\delta) = R_{11}(\delta) / u_{\text{rms}}^2 \), (shown in 6(b)) we can compute the integral length scale \( L_{11} = \lim_{L \to \infty} \int_0^L R_{11}(\delta_x) d\delta_x \). In our measurements, \( L \) is determined by the length of the measurement domain in \( x \).

The Taylor length scale \( \lambda_x \) is determined from the autocorrelation function as \( \lambda_x = [-0.5 f''(0)]^{-0.5} \). From the estimated Taylor length scale we can determine the dissipation rate \( \epsilon = 15\nu u_{\text{rms}}^2 / \lambda_x^2 \), where \( \nu \) is the kinematic viscosity, and from that all the other turbulent scales which are summarized in table 1.

Following Pope (2000), we can define the longitudinal wavenumber spectrum \( E_{11}(\kappa) \) as:

\[
E_{11}(\kappa) = \frac{2}{\pi} u_{\text{rms}}^2 \int_0^\infty f(\delta_x) \cos(\kappa \delta_x) d\delta_x.
\]  

Particular care is needed when computing spectra from a window of finite-size such as in PIV measurements (where \( \delta_x \in [0, L] \)). This is because spectral distortion can occur due to side-lobe leakage (Bendat & Piersol 2010). In order to suppress that, before computing the spectrum according to eq. (5), the correlation function is premultiplied by a linear function \( u_t(\delta_x) \) such that
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\( u_t(0) = 1 \) and \( u_t(L) = 0 \). Thus the spectrum \( E_{11} \) is computed as:

\[
E_{11}(\kappa) = \frac{2}{\pi} u_{\text{rms}}^2 \int_0^L u_t(\delta_x) f^c(\delta_x) \cos(\kappa \delta_x) d\delta_x, \tag{6}
\]

where \( f^c(\delta_x) \) is the circular correlation function. From a mathematical point of view, equation (6) is equivalent to computing the spectrum from:

\[
E_{11}(\kappa) = \frac{1}{N_t} \sum_{i=1}^{N_t} \frac{1}{L} (X_i^*(\kappa) X_i(\kappa)), \tag{7}
\]

where \( X_i(\kappa) \) is the Fourier Transform of \( u'(x,t_i) \). Although estimating the spectra from eq.(7) is computationally more efficient, it requires the interpolation of the missing vectors in the velocity field, which can cause significant distortion if the number of interpolated vectors is larger than 10% of the total (see Poelma et al. (2006)). This is usually not a problem for single phase measurements, where typically the number of missing vectors is lower than 5%, however the distortion can become significant when analyzing the particle laden cases, since the number of missing vectors will be on the order of 10% due to the large gaps in the velocity fields left by the particles. Instead, computing the two-point correlation does not require any interpolation of the velocity fields, since missing vectors can simply be ignored (a large number of missing vectors will just imply that more snapshots are needed for statistics to converge), thus here we compute the spectra using equation (6).

The one-dimensional power spectrum is shown in figure 7. The spectrum shows a power-law decay with \(-5/3\) slope which extends for more than one decade, until \( \kappa \eta < 0.1 \), where there is a transition to the exponential decay range, as predicted by the K41 theory for \( Re_{\lambda_x} = 270 \). (cite Pope model spectrum) When it comes to evaluating the effect of particles it is important to keep in mind the accuracy of the spectral estimate. This can be quantified by the random error \( \epsilon_r = \sqrt{1/N_t} \), where \( N_t = 510 \) in our case we get a random error of about 5% (cite Bendat and Piersol).

4. Turbulent modification by particles

Here we compare the results for two particle laden cases with the same volume and mass fraction but with different particle shapes to the single phase case.

The aim here is to measure the effect of particle shape only on turbulence modulation. Thus for each case, the same volume of particles (1.5l) are prepared and dispersed in the 1037l of the turbulence tank. This gives a volume fraction of about \( \phi_v = 0.14\% \). For such low volume fraction, the suspension regime is considered \textit{dilute}, which means that particle-particle interactions can be neglected, but two-way coupling effects are indeed expected for very small particles at that volume fraction (Elghobashi 1994).
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Figure 7. Longitudinal spectrum calculated using equation 6. The dashed lines show the magnitude of the random error for the spectral estimate.

In this experiment, the number of particles is selected so that the volume fraction of ellipsoids is identical to that of the spheres. Given the ellipsoid shape, the number density of ellipsoids is half that of the spheres, and the total surface area of ellipsoids in the tank is 50% larger than that for spheres.

Physical and geometrical characteristics of the particles have been described in 2.2. As previously mentioned, the density ratio between particles and water is close to unity to avoid concentration gradients in the tank due to sedimentation, whereas particle size has been chosen for the particle response time to be approximately equal to the eddy-turnover time $T$.

<table>
<thead>
<tr>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle shape</td>
<td>Single phase</td>
<td>Spherical ($\lambda = \frac{d_p}{l_p} = 1$)</td>
</tr>
<tr>
<td>Volume fraction $\phi_v$</td>
<td>0</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>$(d_p, l_p)/\eta$</td>
<td>21, 21</td>
<td>21, 42</td>
</tr>
<tr>
<td>$(d_p, l_p)/\lambda_x$</td>
<td>0.65, 0.65</td>
<td>0.65, 1.3</td>
</tr>
<tr>
<td>$\tau_p/\tau_\eta$</td>
<td>26</td>
<td>39</td>
</tr>
<tr>
<td>$\tau_p/T$</td>
<td>1.3</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 2. Definition of the three experimental cases A, B, C.
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For simplicity, the single-phase flow case, and the two particle-laden cases will be referred to as: Case A (single phase), Case B (spherical particles) and Case C (ellipsoidal particles). The summary of the three cases and a comparison between particles and turbulence time and length scales is shown in table 2.

4.1. Effect of particles on one-point statistics

Statistics for the particle-laden cases are computed from 680 vector fields, compared to the 510 fields used for single phase measurements. This increase (+25%) was sufficient compensate for data losses due to the presence of the particles (that is no more than 10-15%), and get converged statistics.

Homogeneity and isotropy of the flow are not notably affected by the particles. This is seen in figure 8, which, similar to fig. (5), shows the spatial distribution of mean velocity and r.m.s. profiles for the three cases. As it can be seen, there is some indication that the magnitude of the mean flow is increased by the particles’ presence. Clearly, the turbulent kinetic energy is decreased in the particle-laden cases. This is likely due to the increased dissipation rate at particle surface as discussed by Tanaka & Eaton (2010); Lucci et al. (2010). By averaging over space for Case B and Case C and comparing it to Case A, we found that spherical particles provide a 15% reduction of TKE, while only a 3% reduction is observed for ellipsoidal particles. In table 3 statistics of whole field flow modifications by particles are summarized.

If the tank was driven so as to keep TKE constant, then the results could only show modifications to the spectral shape. These modifications might also be biased by the interaction of the particles with the driving scheme Lucci et al. (2010). In this flow, the measurement area is spatially far removed from the driving elements, and there is no interaction between the particles and the generation mechanism. Thus changes to the TKE can be observed (due to the accumulated effects of particles acting on the flow throughout the entire tank volume, and the spectral shift represents only the interaction between the particles and turbulence.

<table>
<thead>
<tr>
<th></th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean flow magnitude, $M$</td>
<td>$3 \pm 0.2$</td>
<td>$0.5 \pm 0.2$</td>
<td>$0.4 \pm 0.2$</td>
</tr>
<tr>
<td>$[\times 10^{-2} \text{ms}^{-1}]$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TKE, $k$</td>
<td>$7.2 \pm 0.1$</td>
<td>$6.2 \pm 0.1$</td>
<td>$7.0 \pm 0.1$</td>
</tr>
<tr>
<td>$[\times 10^{-4} \text{m}^2\text{s}^{-2}]$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TKE variation relative to Case A</td>
<td>0%</td>
<td>-15%</td>
<td>-3%</td>
</tr>
</tbody>
</table>

Table 3. Summary of single-point velocity statistics for the three cases.
More information about the structure of turbulence can be obtained by considering the autocorrelation function and the turbulence power spectra. Correlations and spectra are computed following the procedure described in section 2.4, and are shown in figures 9 and 10.

Figures 9a shows the longitudinal two-point correlation in dimensional form so that the difference in energy between the three cases can be observed. As already discussed, Case B and C have a lower variance than for the single phase. The autocorrelation function is shown in figure 9(b). From this plot one can see that for short separations, Case B and C decay faster than Case A, but than they have a slower decay for large separations. This more evident for Case B.

From the two-point correlation functions, the turbulence quantities can be determined as discussed for the single-phase measurements. The results are summarized in table 4.

The longitudinal power spectra in figure 10 are premultiplied by $\kappa \eta$ so that equal areas correspond to equal velocity variance. From these it can be
Table 4. Summary of modifications of turbulence quantities by particles.

<table>
<thead>
<tr>
<th></th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Lambda_x ) ( \times 10^{-3} \text{ m} )</td>
<td>59.5</td>
<td>64.4</td>
<td>58.0</td>
</tr>
<tr>
<td>( \lambda_x ) ( \times 10^{-3} \text{ m} )</td>
<td>12.2</td>
<td>11.2</td>
<td>10.2</td>
</tr>
<tr>
<td>( T = \Lambda_x / u_{\text{rms}} ) [s]</td>
<td>2.7</td>
<td>3.3</td>
<td>2.8</td>
</tr>
<tr>
<td>( \epsilon = 15 \nu u_{\text{rms}}^2 / \lambda_x^2 ) ( \times 10^{-5} \text{ m}^2 \text{s}^{-3} )</td>
<td>4.83</td>
<td>4.5</td>
<td>5.9</td>
</tr>
<tr>
<td>( \tau_\eta = (\nu / \epsilon)^{1/2} ) [s]</td>
<td>0.14</td>
<td>0.15</td>
<td>0.13</td>
</tr>
<tr>
<td>( \eta = (\nu^3 / \epsilon)^{1/4} ) ( \times 10^{-3} \text{ m} )</td>
<td>0.38</td>
<td>0.39</td>
<td>0.36</td>
</tr>
<tr>
<td>( \text{Re}<em>L = (\Lambda_x u</em>{\text{rms}}) / \nu ) [-]</td>
<td>1308</td>
<td>1252</td>
<td>1174</td>
</tr>
<tr>
<td>( \text{Re}<em>{\lambda_x} = (\lambda_x u</em>{\text{rms}}) / \nu ) [-]</td>
<td>269</td>
<td>218</td>
<td>206</td>
</tr>
</tbody>
</table>

Figure 9. Two point correlation \( R_{11} \) (a) and two point correlation function \( f \) (b) for the three cases.

seen that the slope decreases for the particle-laden cases. This change in slope results in a clear pivoting effect for both particle laden cases. For the case of spherical particles, however, the pivoting effect is coupled to a clear decrease of energy in the low wavenumber region, decrease that is not as evident for the case of ellipsoidal particles, at least in the range resolved here.

Comparing the spectra for case B, the spheres reduce TKE at all scales, while ellipsoids only attenuate TKE at the small scales. This can be due to
a number of physical processes. Following the phenomenology described by Tanaka & Eaton (2010) and Lucci & Elghobashi (2010), we consider that both particle types increase dissipation (through action at the small scales) and production (through action at the large scales). For ellipsoids, the additional production nearly balances the additional dissipation. For spheres, the additional dissipation exceeds the additional production, and this is evidenced by a decline in energy at small wavenumber.

Another useful comparison can be made by looking at the velocity p.d.f.’s (figure 11). Here the main difference occurs in the tails, for \(|z| > 3\), as the tails for Case B seem to decay slightly faster than for Case A. This means that rare, high speed events are dampened by the presence of spherical particles, but not by ellipsoids, which instead tend to be closer to the single phase measurements. This behavior is consistent with the hypothesis that spheres do not contribute as much additional TKE production as the ellipsoids do.
Large nearly neutrally buoyant particles in turbulence

Figure 11. Probability density functions for the three cases
5. Rotational Dynamics

Results of the particle rotation rates measurements are reported here. Although this data have to be considered preliminary results, due to their limited statistical significance, it has to be pointed out that to the authors knowledge, until now, such data were only available in DNS. The statistics reported here are computed from 89 particle measurements for Case B, and 107 for Case C. For each measurement we determined the instantaneous rotation rate vector $\Omega = (\Omega_x, \Omega_y, \Omega_z)$ as explained in paragraph 2.4. Given the isotropy of the flow, it is reasonable to assume that also the three components of the rotation vector are isotropic, therefore, to increase the accuracy of our statistical estimates we combine the measurements of the three components in a single array called $\Omega$. Another quantity of interest is the magnitude of the rotation vector $|\Omega|$.

At first mean and standard deviation of $\Omega$ and $|\Omega|$ are computed. The results are shown in table 5, with their respective 95% confidence intervals. The results show that mean is centered at zero in both cases, which is expected, given that the mean flow in the tank is nearly zero. The standard deviation is slightly larger for Case C, as confirmed by the p.d.f.’s shown in figure 12(a). Here a Gaussian distribution with the same mean and standard deviation as the data sets is used and compared to the actual histograms. The relatively large confidence intervals do not allow to make strong conclusions.

Instead, the mean of $|\Omega|$ has to be obviously larger than zero, and it seems that ellipsoids rotate faster in average than sphere. This result can be explained by the fact that the longer axis of the ellipsoidal particles provide act as a longer ‘lever arm’ for the fluid to act on the particle, thus the available torque is larger than for spheres. Similar to figure 12 a, in fig. 12b a gaussian p.d.f. is used to fit the data.

The next stage of this research program will take advantage of the fact that our measurements resolve both fluid and particle phases simultaneously, and thus will evaluate the coupling between the fluid and particle phases by examining conditional statistics near the particle boundaries.

<table>
<thead>
<tr>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_\Omega$ [°s$^{-1}$]</td>
<td>$4.9 \leq -0.1 \leq 4.7$</td>
</tr>
<tr>
<td>$\sigma_\Omega$ [°s$^{-1}$]</td>
<td>$24.5 \leq 26.2 \leq 27.9$</td>
</tr>
<tr>
<td>$\mu_{</td>
<td>\Omega</td>
</tr>
<tr>
<td>$\sigma_{</td>
<td>\Omega</td>
</tr>
</tbody>
</table>

Table 5. Summary of rotation statistics.
Large nearly neutrally buoyant particles in turbulence

6. Discussion

The fact that ellipsoidal particles provide a lower reduction in TKE than spherical ones is somewhat surprising if we consider that the total surface area for the case of ellipsoidal particles is about 1.5 times larger than for the spheres. Thus, we have to admit that the amount of additional dissipation by ellipsoidal particles is smaller than for spherical ones, or ellipsoidal particles introduce additional mechanisms of turbulence production. The first hypothesis could be explained if ellipsoids tend to align with the flow gradients in such a way that dissipation is minimized. In the second case we could speculate that due to their rotation, elongated particles tend to sustain vortices of a size larger or equal to their major axis.

Spectral analysis has shown that indeed ellipsoidal particles sustain the large scales more than spherical ones. This observation is also confirmed by the inspections of the tails of the p.d.f’s of the velocity components, and this seem to support the hypothesis that elongated particles sustain large structures due to their rotational dynamics. However, in order to verify both hypothesis, a detailed analysis of flow gradients around the particles and more converged
statistics on rotation rates would be needed. This would require a spatial resolution much higher than what presented here, which instead has been optimized to measure global effects on turbulence modulation, hence, the analysis of velocity gradients is left as future work.

7. Conclusions

We have reported an experimental technique with which we can simultaneously measure rotational rates of arbitrarily shaped particles and fluid velocity by combining standard Stereoscopic Particle Image Velocimetry and Index-of-Refracion matching. This technique has applied to study the interaction of Taylor-scale spherical and non-spherical particles in Homogeneous Isotropic Turbulence (HIT), and has provided the capability of obtaining experimental data so far only available from DNS.

Single-phase measurements have shown that the flow is indeed homogeneous and isotropic over the entire measurement area, and an inertial range with power-law decay rate of -5/3 is established. These measurements have been used as a reference to compare with the particle-laden cases. The aim was to investigate the effect of particle shape on turbulence modulations. To do this we tested spherical and ellipsoidal particles of 8 mm diameter, which is on the order of the estimated Taylor length scale. The ellipsoids had aspect ratio 2. In order to test the effect of particle shapes only and to avoid concentration gradients, particles were nearly neutrally buoyant. We found that spherical particles provide a 15% turbulence reduction, about five times more than what provided by ellipsoidal particles at the same volume fraction ($\phi_v \approx 0.1\%$), and with less particle surface area available. These results suggest that there might be a turbulence production mechanism for ellipsoidal particles that is not present for spheres. This hypothesis is supported by the analysis of the spectra. Pivoting effect is observed for both spherical and ellipsoidal particles, but for the latter, the reduction in the small wavenumber region is less evident. Preliminary results on statistics of rotational rates show that ellipsoidal particles tend to have an enhanced rotational velocity as compared to spheres.


References


