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Forecast Performance Between SARIMA and SETAR Models: An Application to Ghana Inflation Rate

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**Forecast Performance Between SARIMA and SETAR Models:
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DEDICATION

To my beloved Parents.....Maxwell & Elizabeth

ABSTRACT

In recent years, many research works such as Tiao and Tsay (1994), Stock and Watson (1999), Chen et al. (2001), Clements and Jeremy (2001), Marcellino (2002), Laurini and Vieira (2005) and others have described the dynamic features of many macroeconomic variables as nonlinear. Using the approach of Keenan (1985) and Tsay (1989) this study shown that Ghana inflation rates from January 1980 to December 2009 follow a threshold nonlinear process. In order to take into account the nonlinearity in the inflation rates we then apply a two regime nonlinear SETAR model to the inflation rates and then study both in-sample and out-of-sample forecast performance of this model by comparing it with the linear SARIMA model.

Based on the in-sample forecast assessment from the linear SARIMA and the nonlinear SETAR models, the forecast measure MAE and RMSE suggest that the nonlinear SETAR model outperform the linear SARIMA model. Also using multi-step-ahead forecast method we predicted and compared the out-of-sample forecast of the linear SARIMA and the nonlinear SETAR models over the forecast horizon of 12 months during the period of 2010:1 to 2010:12. From the results as suggested by MAE and RMSE, the forecast performance of the nonlinear SETAR models is superior to that of the linear SARIMA model in forecasting Ghana inflation rates.

Though the nonlinear SETAR model is superior to the SARIMA model according to MAE and RMSE measure but using Diebold-Mariano test, we found no significant difference in their forecast accuracy for both in-sample and out-of-sample.

KEY WORDS: Ghana Inflation, SARIMA model, SETAR model, Forecast comparison, CH test, ZA test, KPSS test, HEGY test, Tsay test, Keenan test

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1 INTRODUCTION

In recent years there has been an increase in both applied and theoretical research in time series modelling and forecasting. The research in this area has contributed to the success of several economies in the world. One of the economic variables that have received much attention in time series modelling is inflation. This is because inflation is one of the macroeconomic variables that have great impact in every economy and its forecasting has great importance for policy makers, investors, firms, traders as well as consumers. For instance, forecasting future inflation will enable policy makers to foresee ahead of time the requirement needed to design economic strategies to combat any expected or unexpected change in inflation. It will also enable investors, firms and others governmental and nongovernmental organisation to develop and evaluate economic policies and business strategies and also to take good decisions on their financial planning. Inflation is the major focus of economic policy worldwide as described by [David \(2001\)](#). Inflation as defined by [Webster \(2000\)](#) is the persistent increase in the level of consumer prices or a persistent decline in the purchasing power of money. Inflation causes global concern because it can distort economic patterns when not anticipated. Inflation as described by [Aidoo \(2010\)](#) can cause uncertainty about the future price, interest rate, and exchange rate etc which as a result might increase the risk among potential traders and partners of a country.

In inflation modelling and forecasting, ARIMA class of models have been extensively used due to its ability in forecasting as compare to other linear time series models. The most commonly used model in the ARIMA class of models for inflation rates is the Seasonal Autoregressive Integrated Moving Average (SARIMA) model. For example, [Aidan et al \(1998\)](#) used SARIMA model to forecast Irish Inflation, [Junttila \(2001\)](#) applied SARIMA model approach in other to forecast finish inflation, and [Pufnik and Kunovac \(2006\)](#) applied SARIMA model to forecast short term inflation in Croatia, [Aidoo \(2010\)](#) applied SARIMA models to forecast Ghana inflation rates etc. The SARIMA model is an extension of the ordinary ARIMA model proposed by [Box and Jenkins \(1976\)](#). This model is use to analyzes time series data which contain seasonal and non-seasonal behaviors. The models are also known to be good in modeling and

forecasting other macroeconomic time series such as unemployment rate and GDP. Due to the effect from business cycles, the dynamic features of inflation and other macroeconomic variables have been described as nonlinear by other research work such as [Tiao and Tsay \(1994\)](#), [Stock and Watson \(1999\)](#), [Chen et al. \(2001\)](#), [Clements and Jeremy \(2001\)](#), [Marcellino \(2002\)](#) etc. [Laurini and Vieira \(2005\)](#) argue that Brazilian inflation rate follows a nonlinear process. This means that the inflation and other macroeconomic variables display different features during economic expansion and recession. Hence, these variables have asymmetric properties which can not be captured by the simple linear models and also the forecast values based on this model may not be reliable. The required model to capture this asymmetric fluctuations or behaviour is the nonlinear times series models. An example of the nonlinear type of models includes the Self Excited Threshold Autoregressive (SETAR) model which is a special type of the TAR proposed by [Tong \(1978, 1983\)](#) and further discussed by [Tong and Lim \(1980\)](#), the Smooth Transitional Autoregressive (STAR) models proposed by [Teräsvirta \(1994\)](#), and the Markov Switching Autoregressive (MS-AR) models introduced by [Hamilton \(1989\)](#). If the data generation follow a nonlinear process it is believed that a nonlinear model is suppose to perform better in terms of forecasting as compare to the linear model since it will be capable of handling the asymmetric features in the data. However as describe in some research the nonlinear models sometimes perform poor in forecasting as compare to the linear counterpart. In this we consider the Self Exited Threshold Autoregressive (SETAR) model.

In theoretical and applied research work of economic modelling the SETAR model have extensively been studied see [Tong \(1978, 1983\)](#), [Tong and Lim \(1980\)](#), [Tiao and Tsay \(1994\)](#), [Potter \(1995\)](#), [Clements and Smith \(1997\)](#), [Rothman \(1998\)](#), [Clements and Krolzig \(1998\)](#), [Clements et al \(1999\)](#), [Feng and Liu \(2002\)](#), [Ismail and Isa \(2006\)](#). The SETAR model is a set of different linear AR models, changing according to the value of the threshold variable(s) which is the past values of the series. The process is linear in each regime, but the movement from one regime to the other makes the entire process nonlinear. For some number of research work the model has proved to perform better as compare to other nonlinear models. For example, [Feng and Liu \(2002\)](#) compared the out-of-sample forecast performance between SETAR model and the linear

ARIMA model in forecasting the nonlinear Canadian real GDP data using two evaluation forecast techniques (multi-step and 1-step ahead forecast). They found out that the SETAR model performs better the ARIMA model in both in-sample and out-of-sample fit.

In this study, our main objective is to compare both in-sample and out-of-sample forecasting performance between linear SARIMA models and nonlinear SETAR model applied to monthly Ghana inflation rate, and to answer the question; Does Ghana inflation rates exhibit nonlinear behaviour? If so, do nonlinear models have superiority in forecasting Ghana inflation rates?

Also since there is limited amount of research concerning the application of SETAR model on inflation rate, we believed that this research will serve as a literature for other researchers who wish to embark on similar studies.

The study made use of monthly Ghana inflation rate from January 1980 to December 2010 which was obtained from the Statistical Service Department of Ghana. The study applied the SARMA and SETAR model following their modelling procedures in order to model the dynamics of Ghana inflation rates from 1980 to 2009. The remaining observations were used to assess the out-of sample forecast performance from both models. After obtaining the forecast from both models, root mean squared error (RMSE) and mean absolute error (MAE), was employed to measure the accuracy of the forecasting from both models. A model with a minimum of these statistics was considered to be the best in terms of forecasting. Also the Diebold-Mariano test of forecast accuracy was used to test the significant difference between the forecast from both models

The structure of the remaining part of the paper is as follows: Section 2 introduces the SARIMA and SETAR models and describes the modeling cycles in each model. Section 3 also describes source and features of Ghana inflation rates and also illustrates how the theoretical methodology of both models were applied to model and forecast the inflation rates and also how the forecast performance between the two models were measured. Section 4 presents the concluding remarks.

2 MODELS AND METHODS

In this section we discuss the competing models used in this research work. The discussion begins by introducing the linear SARIMA model and the modelling cycle associated with the model. Then we consider the nonlinear SETAR model as well. The section also discussed how the two models will be compared to each other using forecast accuracy measure.

2.1 SARIMA Model

Seasonal AutoRegressive Integrated Moving Average (SARIMA) model is the generalization of the well known Box-Jenkins ARIMA model to accommodate a data with both seasonal and non-seasonal feature. The ARIMA model which is known to be a combination of the AutoRegressive (AR) and Moving Average (MA) models utilize past information of a given series in other to predict the future. The AR part of the model deals with the past observation of the series whiles the MA part deals with the past error of the series (see [Hamilton, 1994](#); [Pankratz, 1983](#)). The ARIMA model is applied in the case where the series has no seasonal features and also differenced stationary. This means that an initial differencing is required for the data to be stationary. The ARIMA model with its order is usually presented as ARIMA (p,d,q) model where p , d , and q are integers greater than or equal to zero and refer to the order of the autoregressive, integrated, and moving average parts of the model respectively. The first parameter p refers to the number of autoregressive lags, the second parameter d refers to the order of integration that makes the data stationary, and the third parameter q gives the number of moving average lags (see [Pankratz, 1983](#); [Hurvich and Tsai, 1989](#); [Hamilton, 1994](#); [Kirchgässner and Wolters, 2007](#); [Kleiber and Zeileis, 2008](#); [Pfaff, 2008](#))

A process, $\{y_t\}$ is said to be ARIMA (p,d,q) if $\Delta^d y_t$ is described by a stationary ARMA(p,q) model. Δ means differencing of y_t in d order to achieve stationarity. In general, we will write the ARIMA model as

$$\phi(L)(1-L)^d y_t = \theta(L)\varepsilon_t; \quad \{\varepsilon_t\} \sim WN(0, \sigma^2) \quad (1)$$

where ε_t follows a white noise (WN) process. The autoregressive operator and moving average operator are defined as follows:

$$\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \dots - \phi_p L^p \quad (2)$$

$$\theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \dots + \theta_q L^q \quad (3)$$

$\phi(L) \neq 0$ for $|\phi| < 1$, the process $\{y_t\}$ is stationary if and only if $d=0$, in which case it reduces to an ARMA(p, q) process.

The generalization of ARIMA model to the SARIMA model occurs when the series contains both seasonal and non-seasonal behavior. This behavior of the series makes the ARIMA model inefficient to be applied to the series. This is because it may not be able to capture the behavior along the seasonal part of the series and therefore mislead to a wrong order selection for non-seasonal component. The SARIMA model is sometimes called the multiplicative seasonal autoregressive integrated moving average model and is denoted by ARIMA(p, d, q)(P, D, Q)_s. This can be written in its lag form as (Halim & Bisono, 2008):

$$\phi(L)\Phi(L^S)(1-L)^d(1-L^S)^D y_t = \theta(L)\Theta(L^S)\varepsilon_t \quad (4)$$

$$\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \dots - \phi_p L^p \quad (5)$$

$$\Phi(L^S) = 1 - \Phi_1 L^S - \Phi_2 L^{2S} - \dots - \Phi_P L^{PS} \quad (6)$$

$$\theta(L) = 1 - \theta_1 L - \theta_2 L^2 - \dots - \theta_q L^q \quad (7)$$

$$\Theta(L^S) = 1 - \Theta_1 L^S - \Theta_2 L^{2S} - \dots - \Theta_Q L^{QS} \quad (8)$$

where,

p, d and q are the order of non-seasonal AR, differencing and MA respectively.

P, D and Q is the order of seasonal AR, differencing and MA respectively.

y_t represent observable time series data at period t .

ε_t represent white noise¹ error (random shock) at period t .

L represent backward shift operator ($L^k y_t = y_{t-k}$)

S represent seasonal order (e.g. $s = 4$ for quarterly data and $s = 12$ for monthly data).

¹ The error term is said to be white noise if has the following characteristics:

$E(\varepsilon_t) = 0$, $E(\varepsilon_t^2) = \sigma^2$ and $E(\varepsilon_t \varepsilon_s) = 0$ for all $t \neq s$

For effective model selection, there is a need to follow the model building stages suggested by Box-Jenkins. These model building stages includes model identification, parameter estimation and evaluation and then forecasting stage.

2.1.1 Model Identification

In the identification stage of model building steps, we determine the possible SARIMA models that best fit the data under consideration. But before the search of the possible model for the data, the data under consideration must satisfy the condition of stationarity. This is because the SARIMA model is appropriate for stationary time series data (i.e. the mean, variance, and autocorrelation are constant through time). If a time series is stationary then the mean of any major subset of the series does not differ significantly from the mean of any other major subset of the series. Also if a data series is stationary then the variance of any major subset of the series will differ from the variance of any other major subset only by chance (see [Pankratz, 1983](#)). The stationarity condition ensures that the properties of the estimated parameters from the model are standard. That is the t statistic will asymptotically follow the usual t distribution.

If this condition is assured then, the estimated model can be used for forecasting (see [Hamilton, 1994](#)). To check for stationarity, we sometimes test for the existence or nonexistence of what we called unit root. Unit root test is performed to determine whether a stochastic or a deterministic trend is present in the series. If the roots of the characteristic equation (such as Equation 2) lie outside the unit circle, then the series is considered stationary¹. This is equivalent to say that the coefficients of the estimated model are in absolute value is less than 1 (i.e. $|\phi_i| < 1$ for $i = 1, \dots, p$). In testing for unit root in a given series the features of the series must be known. When the series contains both seasonal and non-seasonal behaviour, the test of stationarity must be conducted on both components (seasonal and non-seasonal frequencies). In testing for stationarity under non-seasonal frequencies the most used approach is the one of [Kwiatkowski et. al. \(1992\)](#) and also [Zivot and Andrews \(1992\)](#).

¹ If a series is non-stationary, it means that the random shock in the series have permanent effect.

Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test proposed by Kwiatkowski et al. (1992) is an LM type test used to test the null hypothesis that a given observable series is level stationary and/or stationary¹ around a deterministic trend. As describe in Pfaff (2008) the test take the null hypothesis as a stationary process against the alternative hypothesis of unit root process. The model considered in the test is given by (see Pfaff, 2008):

$$y_t = \xi t + r_t + \varepsilon_t \quad (9)$$

where r_t is a random walk, i.e. $r_t = r_{t-1} + u_t$, and the error process u_t is assumed to be *i.i.d.*(0, σ_u^2); ξ_t is a deterministic trend; ε_t is also a stationary error. If $\xi = 0$, then this model is in terms of constant as deterministic regressor. The test statistics is constructed as either the series y_t is regress on only constant term (level) or constant term and deterministic trend (level and trend) depending on whether one wants to test level and/or trend stationary. Let the partial sum series of the residuals $\hat{\varepsilon}_t$ from the regression model be

$$S_t = \sum_{i=1}^t \hat{\varepsilon}_i, \quad t = 1, 2, \dots, T. \quad (10)$$

Then the KPSS test statistic for the null hypothesis of stationarity is given by:

$$LM = \frac{\sum_{t=1}^T S_t^2}{T^2 \hat{\sigma}_\varepsilon^2} \quad (11)$$

where $\hat{\sigma}_\varepsilon^2$ is an estimate of the error variance of ε_t from the regression model. The

optimal weighting function which correspond to the Bartlett window $w(s, l) = 1 - \frac{s}{l+1}$ is

used as suggested by the authors to estimate the long-run variance $\hat{\sigma}_\varepsilon^2$; that is

$$\hat{\sigma}_\varepsilon^2 = s^2(l) = T^{-1} \sum_{t=1}^T \hat{\varepsilon}_t^2 + 2T^{-1} \sum_{s=1}^l 1 - \frac{s}{l+1} \sum_{t=s+1}^T \hat{\varepsilon}_t \hat{\varepsilon}_{t-1} \quad (12)$$

where l is the lag truncation parameter. In this exercise $l = \text{integer}[4(T/100)^{1/4}]$.

The approximate upper tail critical values of the asymptotic distribution of the KPSS test are taken from Kwiatkowski et al. (1992).

¹ For a given series y_t the null hypothesis of the test is given by $H_0: y_t \sim I(0)$ and $H_1: y_t \sim I(1)$

As described by some research that some of the conventional unit root has low power against the null hypothesis and it is always advisable to use more than one test as discussed in [Cheung and Chin \(1997\)](#), [Maddala and Kim \(1998\)](#), [Gabriel \(2003\)](#) to obtain a robust conclusion about the properties of the underlying time series. For instance, [Perron \(1989\)](#) shown that the power of the ADF test has low power of rejecting the null hypothesis of unit root when there is a break in the underlying series. [Engel \(2000\)](#) warns about the use of the KPSS because of its lack of power. [Caner and Kilian \(2001\)](#), indicated that the KPSS tests show size distortions when the stochastic process is near to non-stationarity. [Chen \(2002\)](#) also investigated the behaviour of the KPSS test in the presence of breaks and found that the test has power to reject the null hypothesis of stationarity of the series in the presence of breaks. [Otero and Smith \(2003\)](#) also investigated the effect of the KPSS test in the presence of outliers and in their research they found that the power of the KPSS test to reject the null hypothesis of stationarity falls when the series has a unit root with outliers.

To avoid false conclusions the ZA test which is capable of handling data with breaks and also use a different approach from the KPSS test can be employed.

Zivot and Andrews (ZA) test proposed by [Zivot and Andrews \(1992\)](#) is usually applied to test for stationarity of an observable series which is believed to have been affected by breaks. The test is sometimes called a sequential break test. As discussed in [Perron \(1989\)](#), if there is a structural break in the observable series, the conventional unit root test such as ADF, KPSS, and PP test may reflect misspecification of the deterministic trend. So the ZA test which gives an alternative of the [Perron \(1989\)](#) test of unit root that assumes a known break point which is based on an exogenous phenomenon. With the ZA test the break points are endogenously determined within the model. The ZA test considers three different models in testing the null hypothesis of unit root against the alternative hypothesis of stationary with a one time break. The models considered in the test are given by (see [Narayan, 2005](#); [Harvie et. al, 2006](#); [Waheed et. al 2006](#)):

Model A

$$\Delta y_t = c + \alpha y_{t-1} + \beta t + \gamma DU_t + \sum_{j=1}^k d_j \Delta y_{t-j} + \varepsilon_t \quad (13)$$

Model B

$$\Delta y_t = c + \alpha y_{t-1} + \beta t + \theta DT_t + \sum_{j=1}^k d_j \Delta y_{t-j} + \varepsilon_t \quad (14)$$

Model C

$$\Delta y_t = c + \alpha y_{t-1} + \beta t + \gamma DU_t + \theta DT_t + \sum_{j=1}^k d_j \Delta y_{t-j} + \varepsilon_t \quad (15)$$

where y_t represent the observable series with $t = 1, 2, \dots, T$, Δ is the first difference operator, ε_t represent a white noise disturbance. Also DU_t represent an indicator dummy variable for a mean shift occurring at the break date (TB) while DT is an indicator dummy variable corresponding to the trend shift. The function of DU_t and DT is given by:

$$DU_t = \begin{cases} 1 & \text{if } t > TB \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad DT_t = \begin{cases} t - TB & \text{if } t > TB \\ 0 & \text{otherwise} \end{cases}$$

The Δy_{t-j} term in the model allows for serial correlation and ensures the disturbance term in the model is white noise. From above, Model A allows for a one-time change in the intercept, Model B allows for a one-time change in the trend, and Model C allows for one-time change in both the intercept and the trend.

According to [Zivot and Andrews \(1992\)](#), in the implementation of the ZA test the inclusion of the end points of the sample causes the asymptotic distribution of the statistics diverges to infinity. In this case some region must be chosen such that the end points of the sample are not included. The authors suggest that a trimming region be specified as $(0.15T, 0.85T)$. The test consider all points as a potential candidate of break point but the final break point suggested by each model is selected recursively by choosing the value of TB for which the absolute value of the one-sided t -statistic for α is minimized. The critical values of the ZA test can be obtain from [Zivot and Andrews \(1992\)](#).

The stationarity under the seasonal frequencies can also be test to determine if the seasonal behaviour in the data is deterministic or stochastic. The most common approach is the one of [Hylleberg et al \(1990\)](#), see also [Canova and Hansen \(1995\)](#).

Hylleberg-Engle-Granger-Yoo (HEGY) test proposed by [Hylleberg et al. \(1990\)](#) is used to test the presence of seasonal unit root in an observable series. The test was first developed to apply to quarterly time series by the authors. The approach was extended by [Franses \(1990\)](#) to be applied to monthly time series. As discussed in [Franses \(1991\)](#) the seasonal differencing operator Δ_{12} will have 12 roots on the unit circle which can be decomposed as:

$$\begin{aligned}
 1 - B^{12} = & (1 - B)(1 + B)(1 - iB)(1 + iB) \\
 & x \left[1 + (\sqrt{3} + i)B/2 \right] \left[1 + (\sqrt{3} - i)B/2 \right] \\
 & x \left[1 - (\sqrt{3} + i)B/2 \right] \left[1 - (\sqrt{3} - i)B/2 \right] \\
 & x \left[1 + (\sqrt{3} + i)B/2 \right] \left[1 - (\sqrt{3} - i)B/2 \right] \\
 & x \left[1 - (\sqrt{3} + i)B/2 \right] \left[1 + (\sqrt{3} - i)B/2 \right]
 \end{aligned} \tag{16}$$

where all the terms other than $(1 - B)$ correspond to seasonal unit roots. Testing for unit roots in monthly time series is equivalent to testing for the significance of the parameters in the auxiliary regression presented below:

$$\begin{aligned}
 \varphi^*(B)y_{8,t} = & \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-1} + \pi_4 y_{3,t-2} + \pi_5 y_{4,t-1} \\
 & + \pi_6 y_{4,t-2} + \pi_7 y_{5,t-1} + \pi_8 y_{5,t-2} + \pi_9 y_{6,t-1} + \pi_{10} y_{6,t-2} \\
 & + \pi_{11} y_{7,t-1} + \pi_{12} y_{7,t-2} + \mu_t + \varepsilon_t,
 \end{aligned} \tag{17}$$

where μ_t represent the deterministic part in the regression model consisting of a constant, 11 seasonal dummy variables or a trend. $\varphi^*(B)$ is a polynomial function of B for which the usual assumption applies and where

$$\begin{aligned}
 y_{1,t} &= (1 + B)(1 + B^2)(1 + B^4 + B^8)y_t, \\
 y_{2,t} &= -(1 - B)(1 + B^2)(1 + B^4 + B^8)y_t, \\
 y_{3,t} &= -(1 - B^2)(1 + B^4 + B^8)y_t, \\
 y_{4,t} &= -(1 - B^4)(1 - \sqrt{3}B + B^2)(1 + B^2 + B^4)y_t, \\
 y_{5,t} &= -(1 - B^4)(1 + \sqrt{3}B + B^2)(1 + B^2 + B^4)y_t, \\
 y_{6,t} &= -(1 - B^4)(1 - B^2 + B^4)(1 - B + B^2)y_t, \\
 y_{7,t} &= -(1 - B^4)(1 - B^2 + B^4)(1 + B + B^2)y_t, \\
 y_{8,t} &= (1 - B^{12})y_t.
 \end{aligned}$$

The estimates of the π_i can be obtained by applying the ordinary least squares method. Testing for the significance of the π_i terms implies testing for both seasonal and non-seasonal unit roots. The null hypothesis of unit roots is tested by t -test of the separate π_i 's. The test involves the use of one-sided t -test in testing for the null hypothesis of $\pi_1 = 0$ and the null hypothesis of $\pi_2 = 0$. The two-sided t -test are used in testing for the null hypothesis of $\pi_i = 0, i = 3 \dots 12$. The F -test is used in testing the null hypothesis that pairs of π_i 's are equal to zero simultaneously (e.g. $\pi_3 = \pi_4 = 0$) as well as the joint test of π_i 's ($\pi_3 = \dots = \pi_{12} = 0$). There is no seasonal unit root if π_2 through π_{12} are significantly different from zero. If $\pi_1 = 0$, then the presence of non-seasonal unit root 1 can not be rejected. According to Franses (1991), pairs of the complex unit roots are conjugates, so roots are only present when pairs of π_i 's are equal to zero simultaneously and also in the case of all $\pi_i, i = 1, 2 \dots 12$ are equal to zero, it is appropriate to apply the Δ_{12} filter. The critical values for t -tests of the separate π_i 's, and for F -tests of pairs of π_i 's, as well as for a joint F -test of $\pi_3 = \dots = \pi_{12}$ can be taken from Franses (1990).

The Canova-Hansen (CH) test proposed by Canova and Hansen (1995) is one of the well known tests which are used to test for whether seasonality in observable time series is stochastic or deterministic. The test is usually considered as an extension of the KPSS test proposed by Kwiatkowski et al (1992) to test for null hypothesis of stationary seasonal against the alternative of seasonal unit root (non-stationary due to seasonal unit root). As discussed in Caner (1998), the CH test statistics is a Lagrange Multiplier tests which include serially correlated and heteroscedastic processes. The autocorrelation in the process is handled by using a nonparametric adjustment. Given a regression model as in Banik and Silvapulle (1999):

$$y_t = x_t' \beta + d_t' \alpha + e_t \quad t = 1, 2, \dots, n \quad (18)$$

where y_t is the dependent variable, x_t is set of fixed regressors which includes intercept and/or linear trend, d_t is a set of deterministic seasonal component and e_t is a white noise process. The CH test consider trigonometric representation of (18) as

$$y_t = \mu + x_t' \beta + f_t' \gamma + e_t \quad (19)$$

where $f_t = \begin{pmatrix} f_{1t} \\ \vdots \\ f_{qt} \end{pmatrix}$, $\gamma = \begin{pmatrix} \gamma_1 \\ \vdots \\ \gamma_q \end{pmatrix}$, $q = s/2$ ($s = 12$ for monthly data),

$$f_{jt}^1 = \begin{bmatrix} \cos\left(\frac{j\pi t}{q}\right), \sin\left(\frac{j\pi t}{q}\right) \end{bmatrix} \text{ for } j < q \text{ and } f_{qt}^1 = \cos(\pi t).$$

In the test, in order to distinguish between non-stationarity at a seasonal frequency and at the zero frequency, it is required that y_t does not have a unit root at the zero frequency. If y_t has a unit root at zero frequency then $\Delta y_t = y_t - y_{t-1}$ is considered as dependent variable (see [Banik and Silvapulle, 1999](#)).

In testing for unit root at a specific frequency, we rewrite (19) in such a way for individual seasonal frequency as:

$$y_t = \mu + x_t^1 \beta + \sum_{j=1}^q f_{jt}^1 \gamma_j + e_t \quad (20)$$

where γ_j represent the seasonal cycle for the frequency $(j\pi/q)$. Hence test for a seasonal unit root at frequency $(j\pi/q)$ reduces to testing for unit root in γ_j . Letting $\widehat{\Omega}_{jj}^f$ denote the j th block diagonal of $\widehat{\Omega}^f$, the test statistic which is an LM test under the null hypothesis of stationary at the seasonal frequency $(j\pi/q)$ is given as:

$$L_{(\pi j/q)} = \frac{1}{n^2} \sum_{t=1}^n \widehat{F}_{jt}^1 (\widehat{\Omega}_{jj}^f)^{-1} \widehat{F}_{jt}, \quad (21)$$

for $j = 1, 2, \dots, q$ where $\widehat{F}_{jt} = \sum_{i=1}^t f_{ji} \widehat{e}_i$ is the sub-vector of \widehat{F}_t partitioned conformably with γ . When the null hypothesis is satisfied, the distribution of $L_{(\pi j/q)}$ is non-standard and the critical values are given in [Canova and Hansen \(1995\)](#).

According to [Hylleberg \(1995\)](#), the CH and the HEGY test complement each other.

When the stationarity condition of the data is satisfied, the possible models suitable for the data can now be determined. The order of the model which AR, MA, SAR and SMA terms can be determined with the help of the ACF and the PACF plot of the stationary series. The ACF and PACF give more information about the behavior of the time series. The ACF gives information about the internal correlation between observations in a time series at different distances apart, usually expressed as a function

of the time lag between observations. These two plots suggest the model we should build. Checking the ACF and PACF plots, we should both look at the seasonal and non-seasonal lags. Usually the ACF and the PACF has spikes at lag k and cuts off after lag k at the non-seasonal level. Also the ACF and the PACF has spikes at lag ks and cuts off after lag ks at the seasonal level. The number of significant spikes suggests the order of the model. Table 2.1 and 2.2 below describes the behaviour of the ACF and PACF for both seasonal and the non-seasonal series (see [Shumway and Stoffer, 2006](#)).

Table 2.1: Behavior of ACF and PACF for Non-seasonal ARMA(p, q)

	AR(p)	MA(q)	ARMA(p, q)
ACF	Tails off at lag k $k=1,2,3,\dots$	Cuts off after lag q	Tails off
PACF	Cuts off after lag p	Tails off at lags k $k=1,2,3,\dots$	Tails off

Table 2.2: Behavior of ACF and PACF for Pure Seasonal ARMA(P, Q)_s

	AR(P) _s	MA(Q) _s	ARMA(P, Q) _s
ACF	Tails off at lag ks $k=1,2,3,\dots$	Cuts off after lag Qs	Tails off at lag ks
PACF	Cuts off after lag Ps	Tails off at lags ks $k=1,2,3,\dots$	Tails off at lag ks

The ACF and PACF plot suggest the possible models that can be obtained for the data but it does not give the final model for the data. This means that for a given series, several possible models can be obtained. In order to select the best model among the possible models, the penalty function statistics such as Akaike Information Criterion (AIC or AICc) or Bayesian Information Criterion (BIC) can be used (see [Sakamoto et. al., 1986](#); [Akaike, 1974](#); and [Schwarz 1978](#)). The AIC, AICc and BIC are a measure of the goodness of fit of an estimated statistical model. Given a data set, several competing

models may be ranked according to their AIC, AICc or BIC with the one having the lowest information criterion value being the best. These information criterion judges a model by how close its fitted values tend to be to the true values, in terms of a certain expected value. The information criterion value assigned to a model is only meant to *rank* competing models¹ and tell you which one is the best among the given alternatives. The criterion attempts to find the model that best explains the data with a minimum of free parameters but also includes a penalty that is an increasing function of the number of estimated parameters. This penalty discourages over fitting. In the general case, the AIC, AICc and BIC take the form as shown below:

$$AIC = 2k - 2\log(L) \quad \text{or} \quad 2k + n\log\left(\frac{RSS}{n}\right) \quad (22)$$

$$AICc = AIC + \frac{2k(k+1)}{n-k-1} \quad (23)$$

$$BIC = -2\log(L) + k\log(n) \quad \text{or} \quad \log(\sigma_e^2) + \frac{k}{n}\log(n) \quad (24)$$

where

k = the number of parameters in the statistical model, (p+q+P+Q+1)

L = the maximized value of the likelihood function for the estimated model.

RSS = the residual sum of squares of the estimated model.

n = the number of observation, or equivalently, the sample size

σ_e^2 = the error variance

The AICc is a modification of the AIC by [Hurvich and Tsai \(1989\)](#) and it is AIC with a second order correction for small sample sizes. [Burnham & Anderson \(1998\)](#) insist that since AICc converges to AIC as n gets large, AICc should be employed regardless of the sample size.

¹ If two or more different models have the same or similar AIC or BIC values then the principles of parsimony can also be applied in order to select a good model. This principle states that a model with fewer parameters is usually better as compared to a complex model. Also some forecast accuracy test between the competing models can also help in making a decision on which model is the best.

2.1.2 Parameter Estimation and Evaluation

After identifying a possible model for the data, the next step in the model building procedure is to estimate the parameters of the selected model. The parameters are estimated using method of maximum likelihood estimation (MLE). At this stage we get precise estimates of the coefficients of the chosen model. That is we fit the chosen model to our time series data to get estimates of the coefficients. This stage provides some warning signals about the adequacy of our model. In particular, if the estimated coefficients do not satisfy certain mathematical inequality conditions¹ that model is rejected.

After estimating the parameters of the chosen model, we then check the adequacy of that model which is usually called model diagnostics or model evaluation. Ideally, a model should extract all systematic information from the data. The part of the data unexplained by the model (i.e., the residuals) should be small as possible. The diagnostic check is used to determine the adequacy of the chosen model. These checks are usually based on the residuals of the model. One assumption of the SARIMA model is that, the residuals of the model should be white noise. If the assumption of are not fulfilled then different model for the series must be search for. A statistical tool such as Ljung-Box Q statistic, ARCH–LM test and *t*-test can be used to test the hypothesis of independence, constant variance and zero mean of the residuals respectively.

Ljung-Box statistic proposed by [Ljung and Box \(1978\)](#) is used to check if a given observable series is linearly independent. The test is usually used to check if there is higher-order serial correlation in the residuals of a given model. The null hypothesis of linearly independence of the series is examined by the test. The Ljung-Box test statistic is given by:

$$Q(h) = T(T+2) \sum_{k=1}^h \frac{\hat{\rho}_k^2}{T-k} \quad (25)$$

where

¹ After the estimation of the parameters of the model, usually the assumptions based on the residuals of the fitted model are critically checked. The residuals are the difference between the observed value or the original observation and the estimate produced by the model. For the case of SARIMA model the assumption or the condition is that the residuals must follow a white noise process. If this assumption is not met, then necessary action must be taking.

$\hat{\rho}_k$ = the sample autocorrelation at lag k

T = the sample size

h = the number of time lags included in the test

When the null hypothesis is satisfied, $Q(h)$ is asymptotically χ^2 distributed with h degrees of freedom. The null hypothesis of linear independence is rejected if the p -value associated with $Q(h)$ is small (p -value $< \alpha$) or when the value of $Q(h)$ is greater than the selected critical value of the chi-square distribution with h degrees of freedom.

ARCH-LM test of Engle (1982) is used to check for conditional heteroscedasticity of the squared residuals a_t^2 of a given model. Suppose a linear regression model given by;

$$a_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \dots + \alpha_m a_{t-m}^2 + e_t, \quad t = m+1, \dots, T, \quad (26)$$

where e_t denotes the error term, m is a prespecified positive integer, and T is the sample size. According to Tsay (2005), the test for conditional heteroscedasticity which is also known as Arch effect is the Lagrange Multiplier test and is equivalent to the usual F statistic for testing $\alpha_i = 0$ ($i = 1, \dots, m$) in the above Equation (26). The null hypothesis of no Arch effect in the squared residuals (*i.e.* $\alpha_1 = \dots = \alpha_m = 0$) is examined by the test.

Let $SSR_0 = \sum_{t=m+1}^T (a_t^2 - \bar{a})^2$ where $\bar{a} = \left(\frac{1}{T}\right) \sum_{t=1}^T a_t^2$ is the sample mean of a_t^2 , and

$SSR_1 = \sum_{t=m+1}^T \hat{e}_t^2$ where \hat{e}_t is the least squares residual of the prior linear regression. The F

statistic as in Tsay (2005) is given by:

$$F = \frac{(SSR_0 - SSR_1) / m}{SSR_1 / (T - 2m - 1)}, \quad (27)$$

When the null hypothesis is satisfied, F is asymptotically $\chi^2(m)$ distributed with m degrees of freedom. The null hypothesis of no Arch effect is rejected if the p -value associated with F is small (p -value $< \alpha$) or when the value of F is greater than the selected critical value of the chi-square distribution with m degrees of freedom

2.1.3 Forecasting From Seasonal ARIMA Models

The last step in Box-Jenkins model building approach is Forecasting. After a model has passed the entire diagnostic test, it becomes adequate for forecasting. For example Given Seasonal ARIMA (0,1,1)(1,0,1)₁₂ model we can forecast the next step which is given by (see [Cryer & Chan, 2008](#))

$$y_t - y_{t-1} = \Phi(y_{t-12} - y_{t-13}) + \varepsilon_t - \theta\varepsilon_{t-1} - \Theta\varepsilon_{t-12} + \theta\Theta\varepsilon_{t-13} \quad (28)$$

$$y_t = y_{t-1} + \Phi y_{t-12} - \Phi y_{t-13} + \varepsilon_t - \theta\varepsilon_{t-1} - \Theta\varepsilon_{t-12} + \theta\Theta\varepsilon_{t-13} \quad (29)$$

The one step ahead forecast from the origin t is given by

$$\hat{y}_{t+1} = y_t + \Phi y_{t-11} - \Phi y_{t-12} - \theta\varepsilon_t - \Theta\varepsilon_{t-11} + \theta\Theta\varepsilon_{t-12} \quad (30)$$

The next step is

$$\hat{y}_{t+2} = \hat{y}_{t+1} + \Phi y_{t-10} - \Phi y_{t-11} - \Theta\varepsilon_{t-10} + \theta\Theta\varepsilon_{t-11} \quad (31)$$

and so forth. The noise terms $\varepsilon_{13}, \varepsilon_{12}, \varepsilon_{11}, \varepsilon_{10}, \dots, \varepsilon_1$ (as residuals) will enter into the forecasts for lead times $l = 1, 2, \dots, 13$, but for $l > 13$ the autoregressive part of the model takes over and we have

$$\hat{y}_{t+l} = \hat{y}_{t+l-1} + \Phi y_{t+l-12} - \Phi y_{t+l-13} \text{ for } l > 13 \quad (32)$$

2.2 SETAR Model

Self Excited Threshold Autoregressive (SETAR) model is a class of the Threshold Autoregressive (TAR) model proposed by [Tong \(1978\)](#) and further discussed in [Tong and Lim \(1980\)](#), [Tong \(1983, 1990\)](#). The SETAR model is a set of different linear AR models, changing according to the value of the threshold variable(s) which is the lagged values of the series. The process is linear in each regime, but the movement from one regime to the other makes the entire process nonlinear. The two regime version of the SETAR model of order p is given by (see [Boero and Marrocu, 2004](#)):

$$y_t = \begin{cases} \phi_0^{(1)} + \sum_{i=1}^{p^{(1)}} \phi_i^{(1)} y_{t-i} + \varepsilon_t^{(1)} & \text{if } y_{t-d} \leq \tau \\ \phi_0^{(2)} + \sum_{i=1}^{p^{(2)}} \phi_i^{(2)} y_{t-i} + \varepsilon_t^{(2)} & \text{if } y_{t-d} > \tau \end{cases} \quad (33)$$

where $\phi_i^{(1)}$ and $\phi_i^{(2)}$ are the coefficient in lower and higher regime respectively which needs to be estimated; τ is the threshold value; $p^{(1)}$ and $p^{(2)}$ are the order of the linear AR model in low and high regime respectively. In this work the order of the AR model in both regimes are equal. y_{t-d} is the threshold variable that governs the transition between the two regimes with d being the delay parameter which is a positive integer ($d < p$); $\{\varepsilon_t^{(1)}\}$ and $\{\varepsilon_t^{(2)}\}$ are sequence of independently and identically distributed random variables with zero mean and constant variance (i.e. *i.i.d.* $(0, \sigma_\varepsilon^2)$). The two regime SETAR model in its simplest form can be written as SETAR (2, p , d). As discussed in [Tsay \(2005\)](#), the properties of the general SETAR model are hard to obtain. Also from the discussion of [Franses and van Dijk \(2000\)](#), little is known about the condition under which the SETAR models generate time series that are stationary. Such condition has only been established for first-order SETAR model. For effective model selection, we follow the procedure discussed in [Franses and van Dijk \(2000\)](#). The approach of SETAR modelling start with AR(p) model specification and linearity against SETAR model, SEATR model identification, estimation and evaluation of the selected model and then forecasting.

2.2.1 AR Specification and Linearity Test

In order to apply the SETAR model to an observable time series, the series must first be nonlinear in nature. That is the existence of nonlinear behaviour in the series must first be checked. To test for nonlinearity in the series we first have specifies an appropriate linear $AR(p)$ model for the series under consideration. As discuss in Franses and van Dijk (2000), the choice of the maximum lag order is based on the autoregressive lag order that minimize the AIC value. After determine the linear $AR(p)$ model we then test for linearity using a well known linearity test such Keenan test and Tsay F -test.

Keenan test was introduced by Keenan (1985) to detect nonlinearity in an observable time series. The test is considered as a special case of the RESET test proposed by Ramsey (1969). It is a special case in the sense that it avoids multicollinearity. As describe in Cryer and Chan (2008), the Keenan test for nonlinearity analogous to Tukey's one degree of freedom for nonadditivity test. As in Cryer and Chan, the Keenan test is motivated by approximating a nonlinear stationary time series by a second-order Volterra expansion which is give by:

$$y_t = u + \sum_{u=-\infty}^{\infty} \theta_u \varepsilon_{t-u} + \sum_{v=-\infty}^{\infty} \sum_{u=-\infty}^{\infty} \theta_{uv} \varepsilon_{t-u} \varepsilon_{t-v} \quad (34)$$

where $\{\varepsilon_t, -\infty < t < \infty\}$ is a sequence of independent and identically distributed with zero-mean random variable. The process $\{y_t\}$ is linear if the double sum of the right-hand side of (34) does not exist. Thus we can test the linearity of the time series by testing whether or not the double sum of (34) does not exist. That is, the test requires that one distinguish between linearity versus a second-order Volterra expansion, by examining $\theta_{uv} = 0$ as well as the coefficients on higher orders.

It is shown in Cryer and Chan (2008) that the Keenan's test is equivalent to testing if $\eta = 0$ in the multiple regression model (with the constant 1 being absorb into θ_0):

$$y_t = \theta_0 + \phi_1 y_{t-1} + \dots + \phi_m y_{t-m} + \eta \hat{y}_t^2 + \varepsilon_t \quad (35)$$

The Keenan's test statistic for the null hypothesis of linearity ($H_0 : \eta = 0$) is given as:

$$\hat{F} = \frac{\eta^2 (n - 2m - 2)}{RSS - \eta^2} \quad (36)$$

where

m = lag order of the linear autoregressive process

n = same size considered

RSS = the residual sum of squares from the AR(m) process

When the null hypothesis is satisfied, \hat{F} is approximately F -distributed¹ with 1 and $n - 2m - 2$ degrees of freedom. The null hypothesis of linearity is rejected if the p -value associated with \hat{F} is small (p -value $< \alpha$) or when the value of \hat{F} is greater than the selected critical value of the F -distribution with 1 and $n - 2m - 2$ degrees of freedom.

Tsay's F -test introduced by Tsay (1989) is a test for detecting nonlinearity in an observable time series. The test considers a more general nonlinear alternative and is a combined version of the nonlinear test of Keenan (1985), Tsay (1986), and Petrucci and Davies (1986). According to the author the test is based on arranged autoregression and predictive residuals. In the Tsay's arranged regression approach, the linear AR(p) model is considered in the null against the alternative hypothesis of nonlinear threshold model. For an AR(p) regression with n observation as $y_t = (1, y_{t-1}, \dots, y_{t-p})\beta + a_t$ for $t = p+1, \dots, n$ where β is a $(p+1)$ dimensional vector of coefficients and a_t is the noise. The author refers to $(y_t, 1, y_{t-1}, \dots, y_{t-p})$ as a case of data for the AR(p) model. Then an arranged autoregression is an autoregression with cases rearranged based on the values of a particular regressor. Consider a two regime TAR(2; p, d) model with n observations, the threshold variable y_{t-d} may assume values $\{y_h, \dots, y_{n-d}\}$, where $h = \max\{1, p+1-d\}$. Let π_i be the time index of the i th smallest observation of $\{y_h, \dots, y_{n-d}\}$. Then the arranged autoregression with the first s cases in the first regime and the rest in the second regime is given by:

$$y_{\pi_i+d} = \begin{cases} \Phi_0^{(1)} + \sum_{v=1}^p \Phi_v^{(1)} y_{\pi_i+d-v} + a_{\pi_i+d}^{(1)} & \text{if } i \leq s \\ \Phi_0^{(2)} + \sum_{v=1}^p \Phi_v^{(2)} y_{\pi_i+d-v} + a_{\pi_i+d}^{(2)} & \text{if } i > s \end{cases} \quad (37)$$

¹ $F(1, n - 2m - 2)$

where s satisfies $y_{\pi_s} < \tau_1 \leq y_{\pi_{s+1}}$. The arranged autoregression provides a means by which the observations are separated into two groups such that if the true model is indeed TAR(2;p,d) process, the observations in a group follow the same linear autoregressive model. According to the author the separation of the observation does not require knowing the precise value of τ_1 and only the number of observation in each group depends on τ_1 . But since the threshold value is unknown, however the sequential least square estimates $\hat{\Phi}_v^{(1)}$ are consistent for $\Phi_v^{(1)}$ if there is sufficiently large number of observations in the first regime.

For the arranged autoregression, let $\hat{\beta}_m$ be the vector of least squares estimates based on the first m cases, P_m the associated $X'X$ inverse matrix, and x_{m+1} the vector regressor of the next observation to enter the autoregression $y_{d+\pi_{m+1}}$. Then the recursive least squares estimates can be computed efficiently by

$$\hat{\beta}_{m+1} = \hat{\beta}_m + K_{m+1}[y_{d+\pi_{m+1}} - x_{m+1}'\hat{\beta}_m], \quad (38)$$

$$D_{m+1} = 1.0 + x_{m+1}'P_mx_{m+1}, \quad (39)$$

$$K_{m+1} = P_mx_{m+1} / D_{m+1}, \text{ and} \quad (40)$$

$$P_{m+1} = \left(I - P_m \frac{x_{m+1}x_{m+1}'}{D_{m+1}} \right) P_m \quad (41)$$

and the predictive and standardized predictive residuals is given by:

$$\hat{a}_{d+\pi_{m+1}} = y_{d+\pi_{m+1}} - x_{m+1}'\hat{\beta}_m \quad (42)$$

and

$$\hat{e}_{d+\pi_{m+1}} = \hat{a}_{d+\pi_{m+1}} / \sqrt{D_{m+1}} \quad (43)$$

For fixed p and d , the effective number of observations in arranged autoregression is $n - d - h + 1$. Assuming the recursive autoregressions begin with b observation so that there are $n - d - b - p - h$ predictive residuals available. We do the least squares regression

$$\hat{e}_{\pi_i+d} = \omega_0 + \sum_{v=1}^p \omega_v y_{\pi_i+d-v} + \varepsilon_{\pi_i+d} \quad (44)$$

for $i = b + 1, \dots, n - d - h + 1$, and compute the associated F statistic under the null hypothesis of linear AR(p)

$$\hat{F}(p, d) = \frac{(\sum \hat{e}_t^2 - \sum \hat{\varepsilon}_t^2) / (p + 1)}{\sum \hat{\varepsilon}_t^2 / (n - d - b - p - h)}, \quad (45)$$

where the $\hat{\varepsilon}_t$ is the square residual of (44) and the argument (p, d) of \hat{F} is used to signify the dependence of the F -ratio on p and d . Suppose that y_t is a linear stationary autoregressive process of order p , then for large n the statistic $\hat{F}(p, d)$ follows an asymptotic F distribution with $p + 1$ and $n - d - b - p - h$ degrees of freedom.

The null hypothesis of linearity is rejected if the p -value associated with $\hat{F}(p, d)$ is small (p -value $< \alpha$) or when the value of $\hat{F}(p, d)$ is greater than the selected critical value of the F -distribution with $p + 1$ and $n - d - b - p - h$ degrees of freedom

2.2.2 Model Identification

After the null hypothesis of linearity has been rejected we then select appropriate SETAR model that best fit the data. In this research we consider two regime SETAR model where the order p of AR model in both regimes are equal, that is SETAR(2; p, d). For a given nonlinear time series, different SETAR models with different delay parameter d and threshold value τ can be identified. The value of delay parameter is defined as the value for which the Tsay F statistic is significant and maximum. Also according to [Tsay \(1989\)](#) the predictive residual can be used to locate the threshold values once the need for a threshold model is detected, see details from [Franses and van Dijk \(2000\)](#), [Zivot and Wang \(2006\)](#). From all the possible models by a grid search the choice of the best model can be selected based on the minimum of the usual information criterion which are the AIC and BIC. The AIC and BIC for the AR model in the two regimes as defined by [Tong \(1990\)](#) and presented in [Franses and van Dijk \(2000\)](#) is given by:

$$AIC(p_1, p_2) = n_1 \ln \hat{\sigma}_1^2 + n_2 \ln \hat{\sigma}_2^2 + 2(p_1 + 1) + 2(p_2 + 1) \quad (46)$$

$$BIC(p_1, p_2) = n_1 \ln \hat{\sigma}_1^2 + n_2 \ln \hat{\sigma}_2^2 + (p_1 + 1) \ln n_1 + (p_2 + 1) \ln n_2 \quad (47)$$

where

$n_j, j = 1, 2$ is the number of observations in the j th regimes and

$\hat{\sigma}_j^2, j = 1, 2$ is the variance of the residuals in the j th regimes

p_1 and p_2 are the selected lag order in regime 1 and 2 respectively for which the information criterion is minimized.

2.2.3 Parameter Estimation and Evaluation

After the desired model has been selected, the next step is to estimate the parameters of the selected model. The parameters can be estimated using a sequential conditional least square method. According to Franses and van Dijk, by using this method the resulting estimates are equivalent to maximum likelihood estimates (MLE) under the additional assumption that the residuals are normally distributed.

After the parameters of the selected model have been estimated, we then evaluate the adequacy of the selected model by accessing the residuals from the model which is usually called model diagnostics. The approach of access the adequacy of SETAR model may follow the same way as describe in section 2.1. As describe by Franses and van Dijk (2000), the usual ARCH-LM test and the t -test can be used to test the hypothesis of constant variance and zero mean of the residuals respectively. But for the test of serial correlation the authors argue that the Ljung-Box statistic is invalid for the residuals from the nonlinear SETAR model and hence suggested the LM-type test proposed by Breusch-Godfrey.

The **Breusch–Godfrey (BG) test** proposed by Breusch (1979) and Godfrey (1978) is a Lagrange Multiplier test used to test for higher-order serial correlation in the residuals from a given regression model. Suppose a regression model given by:

$$y_t = \beta_0 + \beta_1 x_t + u_t \quad (48)$$

where u_t is the OLS residuals from the regression model which might follow an autoregressive process of order p which is given by:

$$u_t = \alpha_1 u_{t-1} + \alpha_2 u_{t-2} + \cdots + \alpha_p u_{t-p} + \varepsilon_t \quad (49)$$

The BG test uses the OLS estimation procedure to solve the auxiliary regression model given by:

$$\hat{u}_t = \beta_0 + \beta_1 x_t + \alpha_1 \hat{u}_{t-1} + \alpha_2 \hat{u}_{t-2} + \cdots + \alpha_p \hat{u}_{t-p} + \varepsilon_t \quad (50)$$

The test statistic for the null hypothesis ($H_o : \alpha_1 = \alpha_2 = \cdots = \alpha_p = 0$) is given by:

$$LM = TR^2 \quad (51)$$

where

T = the sample size

R^2 = the usual coefficient of determination calculated from the model

When the null hypothesis is satisfied, LM is asymptotically $\chi^2(p)$ distributed with p degrees of freedom. The null hypothesis of no serial correlation of any order up to p is rejected if the p -value associated with LM is small (p -value $< \alpha$) or when the value of LM is greater than the selected critical value of the chi-square distribution with p degrees of freedom.

2.2.4 Forecasting From SETAR Model

The important aim of considering nonlinear type of model such as SETAR as compare to the linear counterpart is to adequately describe the dynamic behaviour of the observable series under consideration and also to produce adequate forecast values that are far better than the one produced by the simple linear models. SETAR models have been successful been used to model and forecast a number of economic and financial data.

The optimal one step-ahead forecast from the origin t is given by (see, [Franses and van Dijk, 2000](#)):

$$\hat{y}_{t+1|t} = E[y_{t+1} | \Omega_t] = F(x_t; \phi) \quad (52)$$

where \hat{y}_{t+1} is the forecast value for the time $(t+1)$, and Ω_t is the history of the time series up to and including the observation at time t . $F(x_t; \phi)$ is the nonlinear function that represent the SETAR model. The next optimal step-ahead forecast is given by:

$$\hat{y}_{t+2|t} = E[y_{t+2} | \Omega_t] = E[F(x_{t+1}; \phi) | \Omega_t] \quad (53)$$

In general, the linear conditional expectation operator E can not be interchanged with the nonlinear operator F , that is

$$E[F(\cdot)] \neq F(E[\cdot]) \quad (54)$$

Put differently, the expected value of a nonlinear function is not equal to the function evaluated at the expected value of its argument. Hence,

$$E[F(y_{t+1}; \phi) | \Omega_t] \neq F(E[y_{t+1} | \Omega_t]; \phi) = F(\hat{y}_{t+1|t}; \phi) \quad (55)$$

The optimal h-step-ahead forecast can be obtained as

$$\hat{y}_{t+h|t} = E[y_{t+h} | \Omega_t] = F(x_{t+h-1}; \phi) \quad (56)$$

2.3 FORECAST COMPARISON

In applied economic and financial modelling, the core point for estimating an econometric or time series model is so that the estimated model can be used to predict future value for decision making and policy evaluation. Forecasting is the process of making statements about events whose actual outcomes have not yet been observed. It is an important application of time series. If a suitable model for the data generation process (DGP) of a given time series has been found, it can be used for forecasting the future development of the variable under consideration. A good model for forecasting can be described as a model that produces minimum forecast errors as compare to other competing models. And to choose a final model for forecasting the accuracy of the model must be higher than that of all the competing models. The accuracy for each model can be checked to determine how the model performed in terms of both in-sample and out-of-sample forecast. Usually the model producing fewer out-of-sample forecast errors is preferred than a model producing fewer in-sample forecast errors. The forecast errors are the difference between the actual observations and the observations predicted by the estimated model. Usually in time series modelling some of the observations are left out during the model building process in order to assess accuracy of the out-of-sample forecast of the estimated models. The accuracy of the models can be compared using forecast measure such as Mean Absolute Error (MAE), Root Mean Square Error

(RMSE). A model with a minimum of MAE or RMSE is considered to be the best for forecasting. In mathematical notation the MAE and RMSE are define as:

$$MAE = \frac{1}{T} \sum_{t=1}^T |\hat{y}_t - y_t| = \frac{1}{T} \sum_{t=1}^T |e_t| \quad (57)$$

$$RMSE = \sqrt{\frac{1}{T} \sum_{t=1}^T (\hat{y}_t - y_t)^2} = \sqrt{\frac{1}{T} \sum_{t=1}^T (e_t)^2} \quad (58)$$

where y_t is the actual observation , \hat{y}_t is the forecasted value and T is the sample size.

The DM test proposed by Diebold-Mariano (1995) can also be used to check if there exist significant differences between the forecast accuracy of the two competing models. The null hypothesis of DM test is that there is no difference between the forecast accuracy from the two models against the alternative hypothesis that there is difference between the forecast accuracy from the two models. One sided test can also be performed.

3 DATA AND EMPIRICAL RESULTS

This section starts by describing the source and properties of the inflation series and how both models under consideration were applied. The section also describes how the forecast results from both were compared to each other. The entire analysis made use of automatic codes and functions produced by [R Development Core Team \(2010\)](#).

3.1 Descriptive Statistics

The data employed in this research is inflation rate of Ghana from January 1980 to December 2010, which is made up of 372 observations. The data was obtained from the Statistical Service Department of Ghana. The series is seasonally unadjusted monthly observations. The full sample is divided into two portions, that is, the first part which starts from 1980:1 and ends in 2009:12 (360 observations) is used for model construction purposes while the 12 months observations are retained to assess the out-of-sample forecast performance of the derived models, and hence are not used in modelling.

Figures 3.1 and 3.2 below describe the dynamic behaviour of Ghana inflation. The right panel of Figure 3.1 display the time series plot of inflation rates in its original form which is denoted by R_t while the left panel display the density plot of the inflation rates. Table 3.1 also provides summary statistics of the inflation rates series.

From Figure 3.1, it can be confirmed that the inflation of Ghana exhibit volatility starting from somewhere around 1993. The volatility in Ghana inflation series can be attributed to several economic factors. Some of those factors are partly transmitted internationally. Examples of these factors include increases in monetary aggregates (money supply), exchange rate depreciation, petroleum price increases, and poor agricultural production (see [Aidoo, 2010](#)) etc. [Ocran \(2007\)](#) describe inflation in Ghana as monetary phenomena. The summary statistics shown in Table 3.1 implies that the dynamic structure of the inflation rates series contains an asymmetric pattern with a high variation among the observations. The sample moments suggest that the right tail of the distribution is fatter than the left tail and also has a higher peak (leptokurtic) which is confirmed by the density plot. The p-value of the Jarque-Bera normality test also

confirms the asymmetric nature of the distribution of the inflation rates series at 5% level of significance.

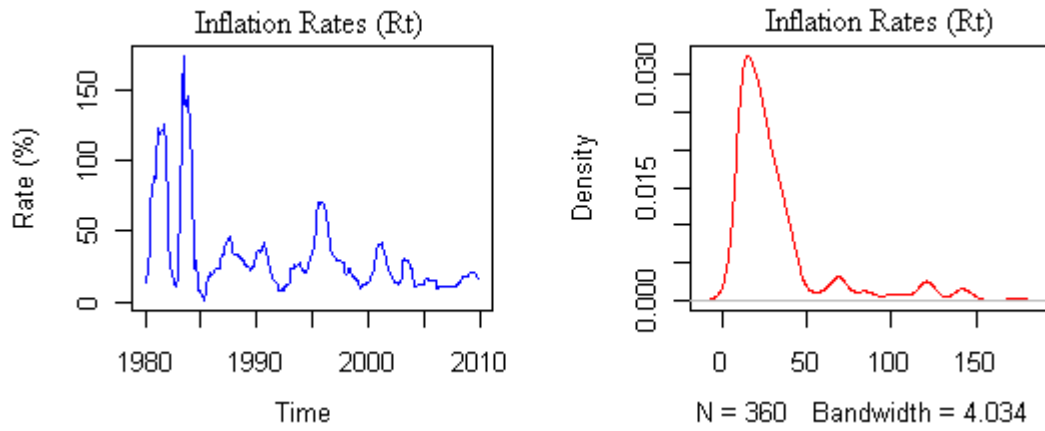


FIG 3.1: Monthly Inflation Rates of Ghana (1980:1–2009:12)

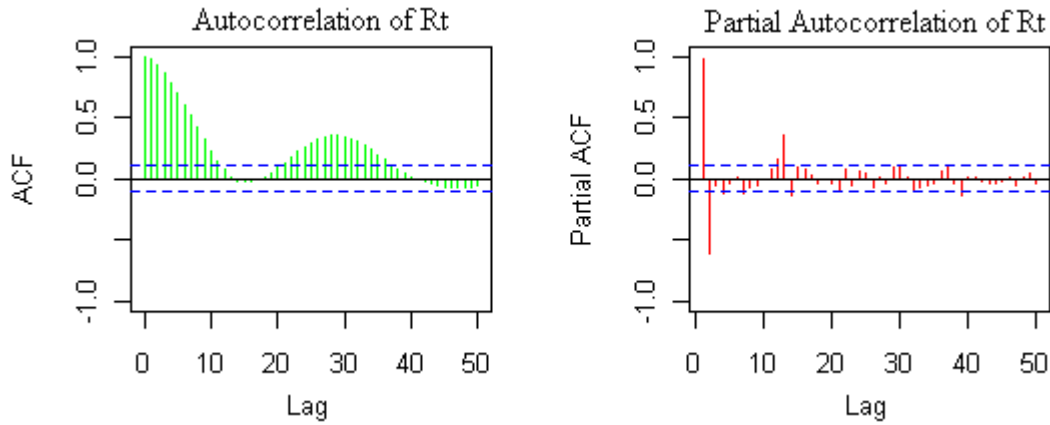


FIG 3.2: ACF and PACF of Inflation Rates (1980:1–2009:12)

Table 3.1: Descriptive Statistics of R_t

Statistic	Sub-sample (N=360)
Minimum	1.14
Maximum	174.14
Mean	32.09
Median	22.93
Standard Deviation	29.78
Skewness	2.36
Kurtosis	5.39
Jarque-Bera (p-value)	2.2e-16

3.2 SARIMA Modelling

In this section we use Seasonal ARIMA model approach to model and forecast Ghana inflation rates. In the modeling cycle, we follow the Box and Jenkins procedure as described in section 2.2.

3.2.1 Stationarity Test and Model Identification

As described in Section 2, the approach of SARIMA modelling requires stationary series hence we have to test for unit root in the inflation series. Using the method of KPSS in testing for stationarity, we test the null hypothesis that the inflation rate (R_t) is stationary. Table 3.2 below presents the results from the unit root test with which the critical values at 5% level are placed in the parenthesis. In the test, we considered a model with a constant term as well as a model with constant term and a trend. From the test results we reject the null hypothesis of stationarity for a model with only constant term. But for a model with constant term and trend we fail to reject null hypothesis of stationarity. Looking at the situation, making a decision for stationarity or unit root becomes very difficult. As described by several research works, most conventional unit root test has problems when there is chock within the series. [Caner and Kilian \(2001\)](#) show by simulation that the KPSS test is subject to immense size distortions when the null is close to the alternative of a unit root. From the left panel of Figure 3.1 above, it is clear that Ghana inflation rates have experienced series of chocks within the period of this study. In this situation we employ the ZA unit root test approach to enable us to draw conclusion on stationarity or unit root in the inflation series. This method is capable of handling time series with chocks. From the test results as shown in Table 3.2, we conclude that the inflation series has unit root. This is confirmed in both cases, that is using a model with only constant term and also a model with both constant term and trend. Now since the series is non-stationary we consider first differencing to render it stationary and it is denoted by Y_t . After considering the first differencing, we use both test to check again if the series is now stationary or has second unit root. From the test results as shown in Table 3.3 below, we conclude at 5% level of significance that the first differenced series is now stationary.

Table 3.2 Unit Root Test for Inflation Rates

Test type	Test Statistic	
	<i>Constant</i>	<i>Constant + trend</i>
KPSS	1.5638 (0.463)	0.1035 (0.146)
ZA	-4.6685 (-4.8)	-4.7929 (-5.08)

Table 3.3 Unit Root Test for difference Inflation Rates

Test type	Test Statistic	
	<i>Constant</i>	<i>Constant + trend</i>
KPSS	0.0269 (0.463)	0.0243 (0.146)
ZA	-9.8368 (-4.8)	-10.438 (-5.08)

From the acf and pacf plot as display by in Fig 3.2, we can see that the inflation series slowly decline and portray a sin wave pattern which describe seasonal and non-seasonal component of the series. And since the series have both features it becomes necessary to test for the behaviour of the seasonality (deterministic or stochastic) in the data. In this case we also need to test for seasonal unit root. This will enable us to be sure that the data is now stationary (i.e. at both seasonal and non-seasonal frequencies) for modelling.

Using HEGY approach as described in Section 2, we test the null hypothesis that the first non-seasonal differenced inflation rates (Y_t) has seasonal unit root. From the HEGY test results as presented in Table 3.4, we reject the null hypothesis of unit root at the seasonal frequencies. The test also confirms that the non-seasonal part of the series is also stationary. As described by some researchers, some times most of the test has low power of rejecting the hypothesis and in such case it usually becomes advisable to employ more than one test before drawing conclusion. In other to make good conclusion we also employ the seasonal unit root test approach by Canova-Hansen. From the results as presented in Table 3.5 we fail to reject the null hypothesis of no seasonal unit root at all the seasonal frequency in our first differenced series (Y_t). Hence we conclude that the first differenced series is now stationary at both seasonal and non-seasonal frequency.

Table 3.4: HEGY Seasonal Unit Root Test for Y_t

<i>t</i> -statistics (one sided)	Variable	
	Constant	Constant + Seasonal Dummies
π_1	-4.569*	-4.344*
π_2	-4.130*	-4.640*
<i>t</i> -statistics (two sided)		
π_3	-6.192*	-6.456*
π_4	-2.238*	-1.964 *
π_5	-7.748*	-7.925 *
π_6	2.807	3.429
π_7	-9.383*	-9.281 *
π_8	-1.423*	-1.829 *
π_9	-12.350*	-12.596*
π_{10}	6.011	5.839
π_{11}	-7.429*	-7.861*
π_{12}	-0.099	-0.068
<i>F</i> -statistics		
π_3, π_4	21.704*	22.315*
π_5, π_6	33.610*	37.090*
π_7, π_8	46.098*	44.978*
π_9, π_{10}	76.267*	79.370*
π_{11}, π_{12}	28.368*	31.693*

* null hypothesis of seasonal unit root is rejected at 5% significant level

Table 3.5: CH Seasonal Unit Root Test of Y_t

Frequency	L-statistic
$\pi/6$	0.328**
$\pi/3$	0.069 **
$\pi/2$	0.336 **
$2\pi/3$	0.085 **
$5\pi/6$	0.059 **
π	0.105 **

** not significant at 5% level

Following the Box-Jenkins procedure, since our data is now stationary and has no unit root at both seasonal and non-seasonal frequencies, the next step in the model building procedure is to determine the order of the AR and MA for both seasonal and non-seasonal component. This can be determined by using the acf and pacf plot of the series as suggested by Box-Jenkins and also described in section 2. The Figure 3.3 below displays the acf and pacf plot of the first differenced series and ninety-five percent (95%) confidence bands ($\pm 2/\sqrt{T}$) are plotted on both panels. As described in section 2, the non-seasonal order are denoted by p and q while the seasonal order are denoted by P and Q . From the figure the acf has an exponential decay starting from nonseasonal lag 1 and seasonal lag 12 and the pacf also has a spike at lag 1 and seasonal lag 12. After comparing about 16 different models using their information criterion the most appropriate models were selected. The models are presented in Table 3.6 below with their AIC and BIC values. The choice of the models is based on the model with a minimum AIC and BIC value.

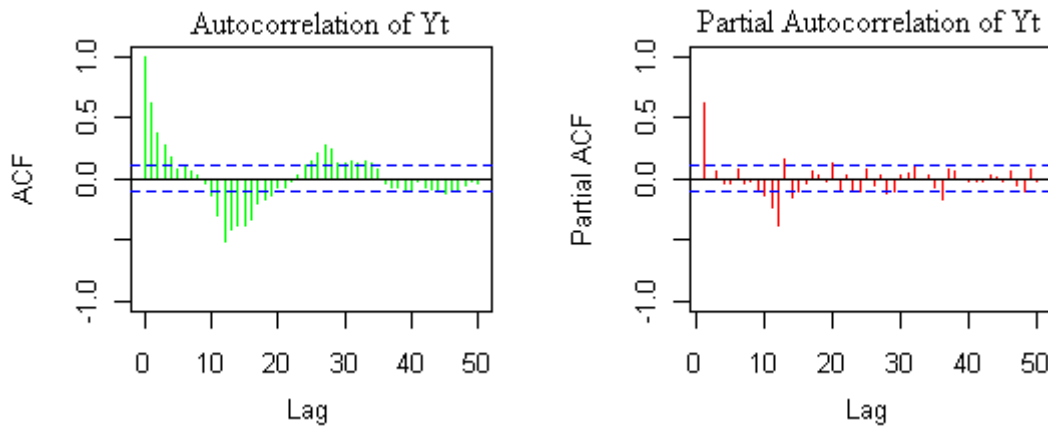


FIG 3.3: ACF and PACF of first non-seasonal differenced series

Table 3.6: AIC and BIC for the Suggested ARIMA Models

Model	AIC	BIC
ARIMA(1,1,0)(2,0,1) ₁₂	2052.99	2072.40
ARIMA(1,1,0)(1,0,1)₁₂	2052.34	2067.87
ARIMA(1,1,0)(0,0,2) ₁₂	2052.37	2067.87

3.2.2 Parameter Estimation and Evaluation

Looking at Table 3.6, $\text{ARIMA}(1,1,0)(1,0,1)_{12}$ could be judge as the best model that fit the data well since it has the minimum value for both AIC and BIC. As described by some research work (e.g. [Geunts and Ibrahim, 1975](#)), the selected model is not necessary the model that provides best forecasting results and since we are interested in a model that will give the best out of sample forecast results, hence we decided to maintain the three models for further assessment and accuracy test. From our derived models, using the method maximum likelihood the estimated parameters of the models with their corresponding standard error is shown in Tables 3.7, 3.8 and 3.9 below.

After the parameters of the models have been estimated the models needs to be checked to determine if they satisfies all the assumptions of seasonal ARIMA model. That is the residuals of the model must follow a white noise process. This is to say that residuals should have zero mean, constant variance and also uncorrelated. Figure 3.4 below display the acf of the residuals of the selected SARIMA models. From the plot we can see that the autocorrelation of the residuals from the three models are all zero, hence we can conclude that the residuals are uncorrelated. Using ARCH-LM and t test, we can test for constant variance and zero mean assumption respectively of the residuals of the selected models. Table 3.10 below provides the test results for ARCH-LM, t and Ljung-Box test. From the table, since the p-value of the ARCH-LM and t test is greater than 5% significant level, we fail to reject the null hypothesis of no ARCH effect and the null hypothesis of approximately zero mean respectively in the residuals of the selected models. Hence we conclude that there is a constant variance among residuals of the selected models and the true mean of the residuals is approximately equal to zero. Also since the p-values for the Ljung-Box test exceed 5%, indicating that there is no significant departure from white noise for the residuals.

Thus, since the selected models satisfy all the necessary assumptions, now we can say that the models can provide an adequate representation of the data.

Table 3.7: Estimates of Parameters for ARIMA (1,1,0)(2,0,1)₁₂

Variable	Estimate	Standard Error	95% Confidence Interval	
			Lower Limit	Upper Limit
AR(1)	0.5520	0.0448	0.464	0.640
SAR(1)	0.1132	0.0996	-0.082	0.308
SAR(2)	0.0928	0.0781	-0.060	0.246
SMA(1)	-0.7894	0.0834	-0.953	-0.626
$\hat{\sigma}^2$	16.93			

Table 3.8: Estimates of Parameters for ARIMA (1,1,0)(1,0,1)₁₂

Variable	Estimate	Standard Error	95% Confidence Interval	
			Lower Limit	Upper Limit
AR(1)	0.5506	0.0449	0.463	0.639
SAR(1)	0.0366	0.0947	-0.149	0.222
SMA(1)	-0.7112	0.0790	-0.866	-0.556
$\hat{\sigma}^2$	17.01			

Table 3.9: Estimates of Parameters for ARIMA (1,1,0)(0,0,2)₁₂

Variable	Estimate	Standard Error	95% Confidence Interval	
			Lower Limit	Upper Limit
AR(1)	0.5501	0.0448	0.462	0.638
SMA(1)	-0.6770	0.0548	-0.784	-0.570
SMA(2)	-0.0200	0.0581	-0.134	0.094
$\hat{\sigma}^2$	17.01			

Table 3.10: Residuals Diagnostics Test for SARIMA model

Model	P-value		
	<i>t test</i>	<i>ARCH-LM test</i>	<i>Ljung-Box test</i>
ARIMA(1,1,0)(2,0,1) ₁₂	0.5971	0.6594	0.07149
ARIMA(1,1,0)(1,0,1) ₁₂	0.6369	0.5483	0.0711
ARIMA(1,1,0)(0,0,2) ₁₂	0.6411	0.5402	0.06929

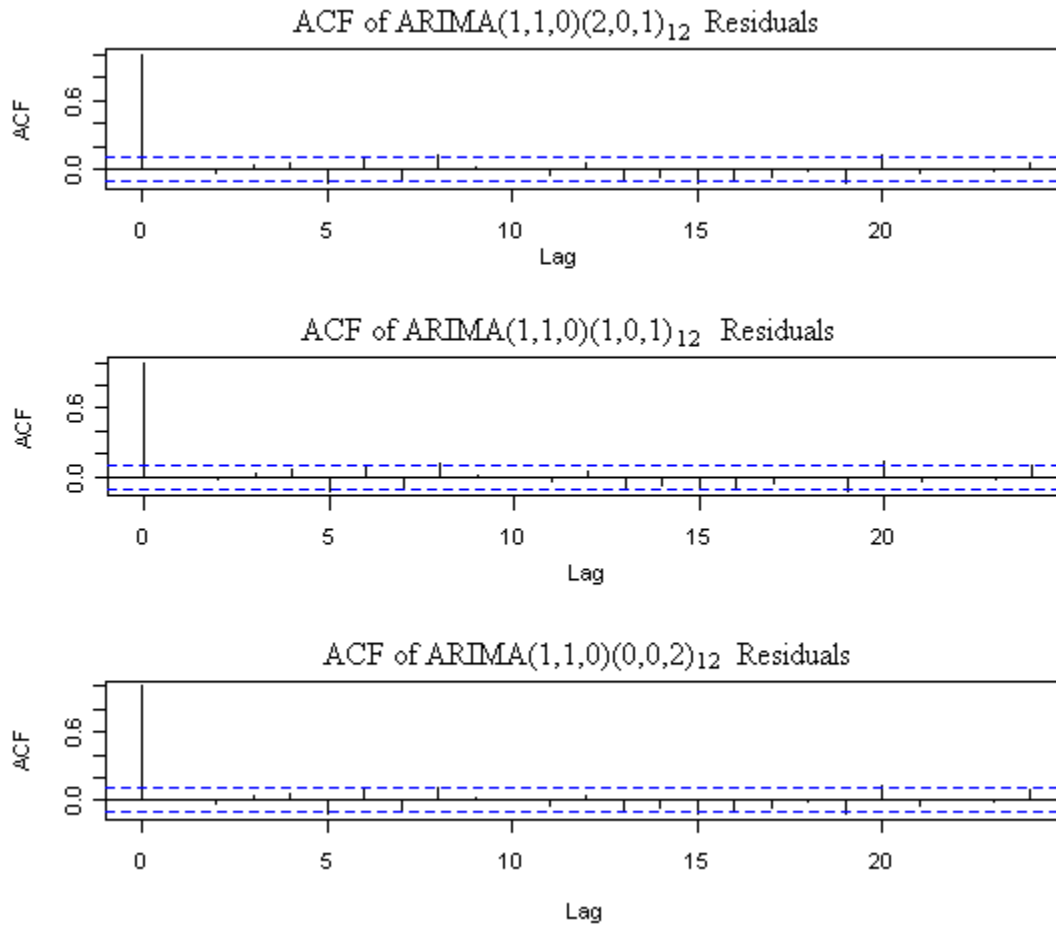


FIG 3.4: ACF Plot of the Residuals of the Selected Seasonal ARIMA Models

3.3 SETAR Modelling

In this section we use 2 regime Self Excited Threshold Autoregressive (SETAR) model approach to model and forecast Ghana inflation rates. In the SETAR modelling cycle we follow the approach presented in [Franses and van Dijk \(2000\)](#).

3.3.1 Linearity Test

As described in section 2, in order to model a time series with SETAR model, the series must be nonlinear, hence we have to test for the existence of nonlinearity in the inflation rates. To test for nonlinearity in the series we first specifies linear $AR(p)$ model. Using akaike information criterion, we found $AR(16)$ model for the series. The choice of the maximum lag order is based on the autoregressive lag order that gives the minimum AIC value. After determine the linear AR model we employ Tsay F -test and the Keenan 1-degree test to test for linearity against the alternative of nonlinearity for the Keenan test. The F -test of Tsay has the alternative of threshold-type nonlinearity. Both linearity tests depend on the linear $AR(16)$ model selected. Table 3.11 below summarizes the results from the Tsay and Keenan 1-degree test. From the results, in the Keenan 1-degree test we reject the null hypothesis of linearity since the p-value of 0 is less than the 5% significant level. Also in the Tsay test, we reject the null hypothesis of no threshold nonlinearity since the p-value is less the 5% significant level. Hence we conclude that the data follows a threshold nonlinear. From both test about the nonlinearity of our data we conclude that the inflation rates of Ghana is nonlinear and it can be well explained by the regime switching model as compare to the simple linear model.

Table 3.11: Linearity test

Test	Test statistic	P-value	Decision
Keenan 1-degree	22.187	< 0.001	Linearity rejected
Tsay	14.28	< 0.001	No threshold nonlinearity rejected

3.3.2 Model Identification

After confirming that the inflation data is nonlinear, we then identify the specific SETAR model that best fit the data. We do this by determine the autoregressive lag order p in each regime and the threshold variable y_{t-d} where d represent the delay parameter. We choose the model with p lag order for both regimes and y_{t-d} threshold variable that minimize the AIC value. After performing a grid search on all possible combination of SETAR models that can be fitted to the data, SETAR (2;16,8) model with a threshold variable y_{t-8} and SETAR (2;16,9) model with a threshold variable y_{t-9} could be appropriate to explain the nonlinearity in the inflation data. These two models of course have closed minimum AIC value. The AIC values of these competing SETAR models are presented in Table 3.12 below.

3.3.3 Parameter Estimation and Evaluation

From the model identification stage we found that SETAR(2;16,8) and SETAR(2;16,9) models with threshold variable y_{t-8} and y_{t-9} respectively could be judge as the best model that fit the data well since it has the minimum value for both AIC. As done in section 3.2.1, we prefer to maintain the two models for further assessment on their forecast ability. From our derived models, following the approach of [Franses and van Dijk \(2000\)](#), we use the method of conditional least squares to estimate the parameters of the models. Table 3.13 and 3.14 below present the estimated parameters of the selected SETAR models with their corresponding threshold value.

After the parameters of the two SETAR models have been estimated we check the residuals of the models for best fit. That is we check for nonexistence of serial autocorrelation, zero mean and constant variance of the residuals. As done in section 3.2 we use the ARCH-LM and t test to check for constant variance and zero mean of the residuals respective. Lagrange Multiplier BG test was also used to check for higher-order serial correlation. From the results as shown in Table 3.15, we fail to reject the null hypothesis of all the three test for SETAR (2;16,9) model. On the hand, we reject the hypothesis of no ARCH effect and no serial correlation up to order 6 for SETAR (2;16,8) model.

Table 3.12: AIC for the Suggested SETAR Models

Model	AIC	BIC
SETAR(2;16,8)	807.7925	943.806
SETAR(2;16,9)	807.2694	943.283

Table 3.13: Estimates of Parameters for SETAR (2;16,8)

Coefficient	Low Regime			High Regime		
	<i>Estimate</i>	<i>Std Error</i>	<i>t-value</i>	<i>Estimate</i>	<i>Std Error</i>	<i>t-value</i>
Constant	1.5763	2.4127	0.6533	0.1622	0.3399	0.4773
ϕ_1	0.5466	0.1605	3.4064	1.6874	0.0545	30.9872
ϕ_2	0.2186	0.3737	0.5850	-0.5780	0.0976	-5.9216
ϕ_3	0.2125	0.4610	0.4608	-0.1528	0.0773	-1.9756
ϕ_4	-0.1686	0.4568	-0.3690	0.0060	0.0716	0.0832
ϕ_5	-0.0106	0.4325	-0.0246	0.0962	0.0677	1.4201
ϕ_6	0.1378	0.3854	0.3575	-0.0092	0.0679	-0.1360
ϕ_7	0.0124	0.4223	0.0295	-0.1523	0.0671	-2.2675
ϕ_8	-0.0794	0.4670	-0.1699	0.1705	0.0674	2.5300
ϕ_9	0.2847	0.4959	0.5742	-0.0176	0.0678	-0.2588
ϕ_{10}	-0.0345	0.3308	-0.1041	-0.0737	0.0673	-1.0946
ϕ_{11}	0.0145	0.3230	0.0450	-0.0645	0.0674	-0.9560
ϕ_{12}	-0.3895	0.3315	-1.1748	-0.2657	0.0674	-3.9434
ϕ_{13}	0.0923	0.3110	0.2968	0.5675	0.0703	8.0722
ϕ_{14}	-0.0316	0.2641	-0.1197	-0.1603	0.0785	-2.0419
ϕ_{15}	-0.0439	0.2892	-0.1517	-0.2044	0.0731	-2.7951
ϕ_{16}	0.2011	0.1396	1.4408	0.1490	0.0414	3.6012
Threshold value	12					
proportion	15.99%			84.01%		

Table 3.14: Estimates of Parameters for SETAR (2;16,9)

Coefficient	Low Regime			High Regime		
	<i>Estimate</i>	<i>Std Error</i>	<i>t-value</i>	<i>Estimate</i>	<i>Std Error</i>	<i>t-value</i>
Constant	2.8430	2.0642	1.3773	0.2277	0.3599	0.6328
ϕ_1	0.5688	0.1095	5.1968	1.6615	0.0555	29.9616
ϕ_2	0.1639	0.1723	0.9516	-0.4646	0.1131	-4.1094
ϕ_3	0.2864	0.2801	1.0225	-0.2820	0.1049	-2.6877
ϕ_4	-0.1298	0.3230	-0.4019	0.0230	0.0729	0.3153
ϕ_5	-0.0754	0.2569	-0.2935	0.1173	0.0700	1.6748
ϕ_6	0.0625	0.2550	0.2450	0.0022	0.0694	0.0312
ϕ_7	0.0468	0.3075	0.1522	-0.1797	0.0686	-2.6210
ϕ_8	0.0744	0.3546	0.2099	0.1813	0.0680	2.6645
ϕ_9	-0.0162	0.3824	-0.0425	-0.0024	0.0685	-0.0346
ϕ_{10}	0.0145	0.3784	0.0382	-0.0780	0.0673	-1.1592
ϕ_{11}	-0.0159	0.3061	-0.0519	-0.0693	0.0673	-1.0288
ϕ_{12}	-0.3130	0.3046	-1.0274	-0.2644	0.0675	-3.9186
ϕ_{13}	0.0820	0.3206	0.2559	0.5631	0.0695	8.1025
ϕ_{14}	0.0883	0.2954	0.2987	-0.1253	0.0792	-1.5807
ϕ_{15}	-0.0294	0.2181	-0.1350	-0.2468	0.0782	-3.1535
ϕ_{16}	0.0435	0.1401	0.3106	0.1596	0.0411	3.8821
Threshold value	13.22					
proportion	21.22%			78.78%		

Table 3.15: Residuals Diagnostics Test SETAR models

Model	P-value		
	<i>t-test</i>	<i>ARCH-LM test</i>	<i>BG test</i>
SETAR(2;16,8)	1	0.01536	0.0022
SETAR(2;16,9)	1	0.06523*	0.0640**

Note: * no ARCH effect null hypothesis was not rejected

** no serial correlation up to order 6 null hypothesis was not rejected

3.4 Forecast Comparison between SARIMA and SETAR Models

The main task of this research work is to compare the forecast ability between the linear SARIMA model and the non-linear SETAR model. Forecast values are of importance for decision making and policy formulation. As described by [Box and Jenkins \(1976\)](#), forecasting provide basis for economic and business planning, inventory and production control and control and optimization of industrial processes. Obtaining a good model that produce best forecast is the core point of every policy maker/planner.

Once the selected models from both approaches have been shown to satisfy all the model assumptions, we can conclude that the models are adequately and can be used to predict the inflation rates. Hence, we compare the forecast performance between the selected models using MAE and RMSE. The preferred model is based on the model with minimum value of MAE and RMSE. Table 3.16 and 3.17 below summarizes the results from both in-sample and out-of-sample forecast accuracy measure of SARIMA and SETAR model respectively. According to the results shown in Table 3.16 below, since seasonal ARIMA(1,1,0)(0,0,2)₁₂ have the minimum value MAE and RMSE for both in-sample and out-of sample forecast measure as compare to other SARIMA models, hence we conclude that it is the best linear models to compete with the nonlinear model. Similarly between the two selected SEATR models we use the same approach of MAE and RMSE to assess their predictive ability in other to choose the best one between them. The comparison is also done for both in-sample and out-of-sample forecast. According to the results shown in Table 3.17 below, though SETAR (2;16,8) model have the minimum value of MAE and RMSE for both in-sample and out-of sample forecast measure as compare to SETAR(2;16,9) model, but since it does not meet some of the model assumption hence we conclude that SETAR(2;16,9) model is the best non-linear models to compete with the linear model.

In comparison of different models within the same type, we also verified as discussed in [Geunts and Ibrahim \(1975\)](#) that the model that produced minimum AIC values does not necessary means that it is a final model that will give best forecast as compare to the other models. For instance all the SARIMA models selected have the same number of parameters but the model that gave the minimum information criterion did not produce minimum forecast errors as compare to other models. This was also the

same case with the SETAR models selected. Based on the out-of-sample forecast assessment from the linear ARIMA(1,1,0)(0,0,2)₁₂ and the nonlinear SETAR(2,16,9) models over the forecast horizon of 12 months during the period of 2010:1 to 2010:12, the forecast measure MAE and RMSE suggest that the nonlinear SETAR model outperform the linear SARIMA model. This nonlinear SETAR model also produced minimum in-sample forecast errors as compare to linear SARIMA model.

Though the nonlinear SETAR model outperform the linear SARIMA model as suggested by the forecast measure MAE and RMSE, but it is interesting to know whether there is significant difference in forecast from the two models. Using the approach of Diebold and Mariano (1995), we can test the null hypothesis that there is no difference between the forecast accuracy from the two models against the alternative hypothesis that the selected SETAR provide better forecast accuracy as compare to the selected seasonal ARIMA model. The results from the test are presented in Table 3.18. From the test results, we fail to reject the null of equal forecast accuracy at 5% level of significance and conclude that the forecast results from both models are almost the same.

Table 3.16: Forecast Comparison among SARMA models

Model	In-sample		Out-of-sample	
	MAE	RMSE	MAE	RMSE
ARIMA(1,1,0)(2,0,1) ₁₂	2.063	4.108	4.258	5.249
ARIMA(1,1,0)(1,0,1) ₁₂	2.046	4.119	4.054	4.765
ARIMA(1,1,0)(0,0,2) ₁₂	2.045	4.119	3.707	4.571

Table 3.17: Forecast Comparison among SETAR models

Model	In-sample		Out-of-sample	
	MAE	RMSE	MAE	RMSE
SETAR(2;16,8)	1.647	2.850	3.657	4.298
SETAR(2;16,9)	1.665	2.850	3.679	4.322

Table 3.18: Forecast Accuracy Test Results

Forecast	DM statistic	P-value
In-sample	-1.4416	0.9253
Out-of-sample	-0.8236	0.0679

4 CONCLUSION

In this paper we have shown that Ghana inflation rates follow a nonlinear process and the behaviour of this process can be modeled by a nonlinear threshold model. By using a two regime nonlinear SETAR model we study both in-sample and out-of-sample forecast performance of this model by comparing it with the linear SARIMA model.

After modelling the inflation series by the two models we comparing the forecast performance between the two models by using the forecast measure mean absolute error (MAE) and root means square error (RMSE).

Based on the in-sample forecast assessment from the linear SARIMA and the nonlinear SETAR models the forecast measure MAE and RMSE suggest that the nonlinear SETAR model outperform the linear SARIMA model. Also using multi-step-ahead forecast method we predicted and compared the out-of-sample forecast of the linear SARIMA and the nonlinear SETAR models over the forecast horizon of 12 months during the period of 2010:1 to 2010:12. From the results as suggested by MAE and RMSE, the forecast performance of the nonlinear SETAR models is superior to that of the linear SARIMA model in forecasting Ghana inflation rates.

Though the nonlinear SETAR model is superior to the SARIMA model according to MAE and RMSE measure but using Diebold-Mariano test, we found no significant difference in their forecast accuracy for both in-sample and out-of-sample.

REFERENCES

- Aidan, M., Geoff, K. and Terry, Q. (1998). Forecasting Irish Inflation Using ARIMA Models. CBI Technical Papers, 3/RT/98:1-48, Central Bank and Financial Services Authority of Ireland.
- Aidoo, E. (2010). Modelling and Forecasting Inflation Rates in Ghana: An Application of SARIMA Models. Master Thesis, Hogskolan Dalarna.
- Akaike, H. (1974). A New Look at the Statistical Model Identification. IEEE Transactions on Automatic Control 19 (6): 716–723.
- Banik, S. and Silvapulle, P. (1999). Testing for Seasonal Stability in Unemployment Series: International Evidence. *Empirica* 26: 123–139.
- Boero, G. and Marrocu, E. (2004). The Performance of SETAR Models: A Regime Conditional Evaluation of Point, Interval and Density Forecasts. *International Journal of Forecasting* 20: 305-320.
- Box, G.E.P. and Jenkins, G.M. (1976) Time Series Analysis: Forecasting and Control. Holden-Day, San Francisco.
- Breusch, T.S. (1979). Testing for Autocorrelation in Dynamic Linear Models. *Australian Economic Papers* 17: 334-355.
- Burnham, K.P., and Anderson, D.R. (1998). Model Selection and Inference. Springer-Verlag, New York.
- Caner, M. (1998). A Locally Optimal Seasonal Unit-Root Test. *Journal of Business & Economic Statistics* 16: 349–356.
- Caner, M. and Kilian, L. (2001). Size Distortions of the Null Hypothesis of Stationarity: Evidence and Implications for the PPP debate. *Journal of International Money and Finance* 20: 639–657.
- Canova, F. and Hansen, B.E (1995), Are Seasonal Patterns Constant Over Time? A Test for Seasonal Stability. *Journal of Business and Economic Statistics* 13, 237-252.
- Chen M.Y. (2002). Testing Stationarity against Unit Roots and Structural Changes. *Applied Economics Letter* 9: 459–464.
- Chen, X., Racine, J. and Swanson, N.R. (2001). Semiparametric ARX Neural Network Models with an Application to Forecasting Inflation. Working Paper, Economics Department, Rutgers University.
- Cheung, Y.W., Chinn, M.D. (1997). Further investigation of the uncertain Unit Root in GNP. *Journal of Business & Economic Statistics* 15: 68–73.
- Clements, M.P. and Jeremy, S. (2001). Evaluating Forecasts from SETAR Models of Exchange Rates. *Journal of Money and Finance*, 20:133-148.

- Clements, M.P. and Krolzig, H.M. (1998). A Comparison of the Forecast Performance of Markov-Switching and Threshold Autoregressive Models of U.S. GNP. *Econometrics Journal* 1, C47–75.
- Clemens, M.P. and Smith, J. (1997). The Performance of Alternative Forecasting Method for SETAR models. *International Journal of Forecasting* 13: 463–475.
- Clements, M.P., Franses, P.H. and Smith, J. (1999). On SETAR non-linearity and Forecasting. *Econometric Institute Report* 141, Erasmus University Rotterdam, Econometric Institute.
- Cryer J.D. and Chan, K.S. (2008) *Time Series Analysis with Applications in R*. 2Ed. Springer Science +Business Media, LLC, NY, USA.
- David, F.H. (2001). Modelling UK Inflation, 1875-1991. *Journal of Applied Econometrics*, 16(3): 255–275.
- Diebold, F.X. and Mariano, R.S. (1995) Comparing Predictive Accuracy. *Journal of Business & Economic Statistics*, 13, 253-263.
- Engel, C. (2000). Long-run PPP may not hold after all. *International Journal of Economics* 57: 243–273.
- Engle, R.F. (1982). Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica* 50: 987-1007.
- Feng, H. and Liu, J. (2002). A SETAR Model for Canadian GDP: Non-linearities and Forecast Comparisons. Working Paper EWP 0206, University of Victoria.
- Franses, P.H. (1990). Testing for Seasonal Unit Root in Monthly Data. *Econometric Institute Report* No.9032/A, Erasmus University Rotterdam.
- Franses, P.H. (1991). Seasonality, Non-stationarity and the Forecast of Monthly Time Series. *International Journal of Forecasting* 7: 199-208.
- Franses, P.H. and van Dijk, D. (2000). *Nonlinear Time Series Models in Empirical Finance*. Cambridge University Press (Virtual Publishing).
- Gabriel, V.J. (2003). Cointegration and the Joint Confirmation Hypothesis. *Economics Letter* 78:17–25.
- Geunts, M. and Ibrahim, I. (1975). Comparing the Box-Jenkins Approach with the Exponentially Smoothed forecasting Model Approach to Hawaii Tourist. *Journal of market Research* 12, 182-188.
- Godfrey, L.G. (1978). Testing Against General Autoregressive and Moving Average Error Models when the Regressors Include Lagged Dependent Variables. *Econometrica* 46: 1293-1302.
- Halim, S. and Bisono, I.N. (2008). Automatic Seasonal Autoregressive Moving Average Models and Unit Root Test Detection. *International Journal of Management Science and Engineering Management*, 3(4): 266-274.

- Hamilton, J.D. (1994). *Time Series Analysis*. Princeton University Press, Princeton, NJ.
- Harvie, C., Pahlavani, C. and Saleh, A.S. (2006). Identifying Structural Breaks in the Lebanese Economy 1970-2003: An Application of the Zivot and Andrews Test. WP 06-02, University of Wollongong Economics Working Paper Series.
- Hurvich, C.M., and Tsai, C.L. (1989). Regression and Time Series Model Selection in Small Sample. *Biometrika* 76: 297-307.
- Hylleberg, S., Engle, R., Granger, C. & Yoo, B. (1990), Seasonal Integration and Cointegration. *Journal of Econometrics* 44: 215-238.
- Ismail, M.T. and Isa, D. (2006). Modelling Exchange Rates Using Regime Switching Models. *Sains Malaysiana* 35: 55–62.
- Junttila, J. (2001). Structural Breaks, ARIMA Model and Finnish Inflation Forecasts *International Journal of Forecasting* 17: 203–230.
- Keenan, D. M. (1985), A Tukey nonadditivity-type test for Time Series Nonlinearity, *Biometrika* 72: 39-44.
- Kirchgässner, G. and Wolters, J. (2007) *Introduction to Modern Time Series Analysis*. Springer-Verlag Berlin Heidelberg.
- Kleiber, C. and Zeileis, A. (2008). *Applied Econometrics with R*. Springer Science +Business Media, LLC, NY, USA.
- Kwiatkowski, D., Phillips, P. C. B., Schmidt, P. and Shin, Y. (1992). Testing the Null Hypothesis of Stationarity against the Alternative of a Unit Root. *Journal of Econometrics* 54: 159–178.
- Laurini, M. and Vieira, H. (2005). A Dynamic econometric Model for Inflationary inertia in Brazil. IBMEC, forthcoming.
- Ljung, G.M. and Box, G.E.P. (1978). On a Measure of Lack of Fit in Time Series Models. *Biometrika* 65: 297–303.
- Maddala G.S., Kim, I.M. (1998) *Unit roots, Cointegration and Structural Change*. Cambridge University Press.
- Marcellino, M. (2002). Instability and Non-Linearity in the EMU. Working Paper 211, Bocconi University, IGIER.
- Narayan, P.K. (2005). The Structure of Tourist Expenditure in Fiji: Evidence from Unit Root Structural Break Tests. *Applied Economics* 37: 1157–1161.
- Ocran, M.K. (2007). A Modelling of Ghana's Inflation Experience: 1960–2003. African Economic Research Consortium, RP_169.
- Otero, J. and Smith, J. (2003). The KPSS Test with Outliers. Warwick Economic Research Papers No. 690, Department of Economics, Warwick University.

- Pankratz, A. (1983). *Forecasting with Univariate Box-Jenkins Models: Concepts and Cases*. John Wiley & Sons. Inc. USA.
- Perron, P. (1989). The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis. *Econometrica* 57, 1361 – 1401.
- Petrucelli, J., and Davies, N. (1986). A Portmanteau Test for Self-Exciting Threshold Autoregressive-Type Nonlinearity in Time Series. *Biometrika* 73: 687-694.
- Pfaff, B. (2008). *Analysis of Integrated and Cointegrated Time Series with R*. 2Ed, Springer Science +Business Media, LLC, NY, USA.
- Potter, S. (1995). A nonlinear Approach to U.S. GNP. *Journal of Applied Econometrics* 10: 109-125.
- Pufnik, A. and Kunovac, D. (2006). Short-Term Forecasting of Inflation in Croatia with Seasonal ARIMA Processes. Working Paper W-16, Croatia National Bank.
- R Development Core Team (2010). *R: A language and environment for statistical computing*. R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0, URL <http://www.R-project.org>.
- Ramsey, J. B. (1969). Tests for Specification Errors in Classical Linear Least Squares Regression Analysis. *Journal of the Royal Statistical Society Series B* 31: 350–371.
- Rothman, P. (1998). Forecasting Asymmetric Unemployment Rates. *The Review of Economics and Statistics* 80: 164-168.
- Sakamoto, Y., Ishiguro, M., and Kitagawa G. (1986). *Akaike Information Criterion Statistics*. D. Reidel Publishing Company.
- Schwarz, G. E. (1978). Estimating the Dimension of a Model. *Annals of Statistics* 6 (2): 461–464.
- Shumway, R.H. and Stoffer, D.S. (2006). *Time Series Analysis and Its Applications With R Examples* 2Ed, Springer Science +Business Media, LLC, NY, USA.
- Stock, J.H. and Watson, M.W. (1999). Forecasting Inflation. *Journal of Monetary Economics* 44: 293-335.
- Teräsvirta, T. (1994). Specification, Estimation, and Evaluation of Smooth Transition Autoregressive Models. *Journal of the American Statistical Association* 89(425): 208-218.
- Tiao, G.C. and Tsay, R.S. (1994). Some Advances in Non-linear and Adaptive Modelling in Time Series. *Journal of forecasting*, 13:109-131.
- Tong, H., (1978). On a threshold model, in C.H. Chen (ed.), *Pattern Recognition and Signal Processing*, Amsterdam: Sijthoff & Noordhoff, 101–41.
- Tong, H. (1983). *Threshold Models in Non-linear Time Series Analysis*, Springer: New York.

- Tong, H., (1990). *Non-Linear Time Series: A Dynamical Systems Approach*, Oxford: Oxford University Press.
- Tong, H. and Lim, K.S. (1980). Threshold Autoregressions, Limit Cycles, and Data, *Journal of the Royal Statistical Society B* 42: 245–92 (with discussion).
- Tsay, R.S. (1986). Nonlinearity Test for Time Series. *Biometrika* 73: 461-466.
- Tsay, R.S. (1989). Testing and Modeling Threshold Autoregressive Processes. *Journal of the American Statistical Association* 84: 231–240.
- Tsay, R.S. (2005). *Analysis of Financial Time Series* 2ED. John Wiley & Sons, Inc., Hoboken, New Jersey.
- Waheed, M., Alam, T. and Ghauri, S.P. (2006). Structural Breaks and Unit Root: Evidence from Pakistani Macroeconomic Time Series. MPRA Paper No. 1797. Munich Personal RePEc Archive.
- Webster, D. (2000). *Webster's New Universal Unabridged Dictionary*. Barnes & Noble Books, New York.
- Zivot, E., and Andrews, D.W.K. (1992). Further Evidence on the Great Crash, the Oil Price Shock, and the Unit Root Hypothesis, *Journal of Business and Economic Statistics* 10: 251-70.
- Zivot, E. and Wang, J. (2006). *Modeling Financial Time Series with S-Plus*. Springer Science+Business Media, Inc., New York.