Digital Dispersion Equalization and Carrier Phase Estimation in 112-Gbit/s Coherent Optical Fiber Transmission System

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Abstract

Coherent detection employing multilevel modulation format has become one of the most promising technologies for next generation high speed transmission system due to the high power and spectral efficiencies. With the powerful digital signal processing (DSP), coherent optical receivers allow the significant equalization of chromatic dispersion (CD), polarization mode dispersion (PMD), phase noise (PN) and nonlinear effects in the electrical domain. Recently, the realizations of these DSP algorithms for mitigating the channel distortions in the transmission system are the most attractive investigations.

The CD equalization can be performed by the digital filters developed in the time and the frequency domain, which can suppress the fiber dispersion effectively. The PMD compensation is usually performed in the time domain with the adaptive least mean square (LMS) and constant modulus algorithms (CMA) equalization. Feed-forward and feed-back carrier phase estimation algorithms are employed to mitigate the phase noise from the transmitter and local oscillator lasers. The fiber nonlinearities are compensated by using the digital backward propagation methods based on solving the nonlinear Schrodinger (NLS) equation and the Manakov equation.

In this dissertation, we present a comparative analysis of three digital filters for chromatic dispersion compensation, an analytical evaluation of carrier phase estimation with digital equalization enhanced phase noise and a brief discussion for PMD adaptive equalization. To implement these investigations, a 112-Gbit/s non-return-to-zero polarization division multiplexed quadrature phase shift keying (NRZ-PDM-QPSK) coherent transmission system is realized in the VPI simulation platform. With the coherent transmission system, these CD equalizers have been compared by evaluating their applicability for different fiber lengths, their usability for dispersion perturbations and their computational complexity. Meanwhile, the bit-error-rate (BER) floor in carrier phase estimation using a one-tap normalized LMS filter is evaluated analytically, and the numerical results are compared to a differential QPSK detection system.
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Chapter 1

Introduction

The performance of high speed optical fiber transmission systems is severely affected by chromatic dispersion (CD), polarization mode dispersion (PMD), phase noise (PN) and nonlinear effects. Coherent optical detection allows the significant equalization of transmission system impairments in the electrical domain, and has become one of the most promising techniques for the next generation communication networks. With the full optical wave information, the fiber dispersion, carrier phase noise and the nonlinear effects can be well compensated by the powerful digital signal processing (DSP).

In this chapter, we present an overview of the history and the state-of-the-art of the coherent transmission technologies. We will also make a discussion about the attractive techniques for mitigating the system distortions in the development of the high speed coherent communication systems. Furthermore, we will give a summary of our research work in digital coherent receivers and a description for the structure of the dissertation.

1.1 Structure of thesis

In this investigation, we present a detailed study in the DSP algorithms for mitigating the system impairments in the high speed coherent optical transmission system. Our research mainly focuses on the chromatic dispersion compensation and the carrier phase estimation in coherent systems. We give a comparative analysis on the different CD equalization methods, and describe an analytical evaluation on the phase estimation considering the equalization enhanced phase noise.

In chapter 1, we give an introduction of the history, the development and the rebirth of the coherent transmission technologies. The state-of-the-art of the coherent system and the attractive techniques are discussed in this part, meanwhile, the DSP algorithms for chromatic dispersion compensation, polarization mode dispersion equalization and phase noise mitigation are also briefly introduced.

In chapter 2, the influence of the fiber impairments and the system distortions on the high speed coherent communication systems are described relatively detailed. The impacts of the chromatic dispersion, the polarization mode dispersion, the phase noise and the nonlinear effects on the transmission system are analyzed and discussed respectively. This gives a brief overview of the basic knowledge for our research.

In chapter 3, we present the implementation of the 112-Gbit/s NRZ-PDM-QPSK coherent optical transmission system, which is realized in the VPI platform. Meanwhile, we describe a mathematical analysis of the 112-Gbit/s NRZ-PDM-QPSK coherent system. The theoretical modes consisting of the QPSK transmitter, the fiber
channel and the coherent receiver are established and analyzed in equations.

In chapter 4, the theoretical basic for our research work is described. The mitigation of the chromatic dispersion and the compensation of the carrier phase noise are significantly elaborated by analyzing the corresponding DSP algorithms, and the adaptive equalization of the polarization mode dispersion using the LMS and the CMA filters is also discussed briefly. Correspondingly, we also present the numerical simulation results in the 112-Gbit/s NRZ-PDM-QPSK coherent optical transmission system employing the DSP algorithms for chromatic dispersion compensation and carrier phase estimation.

In chapter 5, we make a summary about our research work in this dissertation, and present a brief overview of our publications. Moreover, we also give some suggestion and plans for our future investigations in coherent transmission technologies.

1.2 Historical background

Due to the high sensitivity of the receivers, coherent optical transmission systems were investigated extensively in the eighties of last century [1]. However, the development of the coherent technologies has been delayed for nearly 20 years because of the invention and the rapid progress of erbium-doped fiber amplifiers (EDFAs) in the high-capacity wavelength-division multiplexed (WDM) systems [2-4]. The coherent transmission techniques did not attracted the interests of investigation again until 2005 [5], since the efficient modulation formats such as M-ary phase shift keying (PSK) and quadrature amplitude modulation (QAM) were implemented by employing the digital coherent receivers. Meanwhile, full access to the optical wave information offers the possibility of electrical compensation of transmission impairments as powerful as traditional optical compensation techniques. Due to the two main merits, the reborn coherent detections brought us the enormous potential for higher transmission speed and spectral efficiency in the present optical fiber communication systems [2-4].

1. Coherent optical communications in last century

Employing coherent optical receivers with the heterodyne or the homodyne detection, the transmitted optical signals could be converted into the baseband electrical signals without any loss of information [1-4]. With an additional local oscillator (LO) source, the sensitivity of receiver was achieved to the limitation of the shot-noise. Furthermore, compared to the traditional intensity modulation direct detection (IMDD) system, the phase detection system can also improve the receiver sensitivity because the distance between symbols is extended by the use of the signal phasors on the complex plane [1]. The multi-level modulation formats such as quadrature phase-shift keying (QPSK) and QAM can be applied into optical fiber transmission systems by using the phase modulation modules, which can include more information bits in one transmitted symbols than before.

However, the advantages of the traditional coherent optical receivers grew fainter due
to the invention of erbium-doped fiber amplifiers [3]. The sensitivity of coherent receivers limited by the shot-noise become less significant, because the signal-to-noise ratio (SNR) in the WDM transmission channel using EDFAs is determined by the accumulated amplified spontaneous emission (ASE) noise, which is smaller than the shot noise [2-4]. Moreover, some technical difficulties in the realization of coherent optical receivers have also prevented the development of coherent detection. For example, the coherent optical receivers are rather difficult to implement due to the high complexity and cost in stable locking of the rapid carrier phase drift. For these above reasons, the investigations in the coherent optical communications have broken off for nearly twenty years [4]. At the same time, the EDFA-based fiber communication systems employing WDM techniques played the dominant roles in the optical transmission techniques during the nineties in the last century.

2. Rebirth of coherent optical communications

In recent years, there has been a renewed interest in the research community in coherent optical communication systems, due to the increment of the transmission-capacity in WDM systems [2]. With the demand of the ever-increasing bandwidth, the multi-level modulation formats based on the coherent detection need to be employed in the transmission systems to improve the spectral efficiency [6].

The first revival of the investigations in coherent optical communications comes from the differential QPSK (DQPSK) transmission experiment with the optical in-phase & quadrature (IQ) modulation and the optical delay detection [4,7]. We can duplicate the bit rate with keeping the same symbol rate because the optical signal can carry two or more bits in one transmitted symbol. The next step of coherent technologies rebirth arises from the high-speed digital signal processing [4,5]. With the rapid development of high-speed integrated circuits, treating the electrical signal in a digital signal processing core and retrieving the IQ components from the optical carrier become feasible. Using a phase-diversity homodyne receiver (intradyne receiver) followed by the DSP circuit, the demodulation of the 10-Gsample/s QPSK signal with the offline digital signal processing has been realized [8,9]. Meanwhile, more advanced and powerful DSP circuits are developed, and this can provide us with more efficient methods for carrier phase estimation to substitute the optical phase-locked loop (PLL) [2-4].

3. State-of-the-art coherent technologies

The main benefit of the digital coherent receivers is the post digital signal processing function [2-4]. The demodulation process is entirely linear in the coherent receivers, and all information of the transmitted optical signal including the state of polarization (SOP) is preserved. The signal processing techniques such as tight spectral filtering [5], chromatic dispersion compensation [6-10], polarization mode dispersion compensation and phase noise mitigation can be performed at the electrical domain after the coherent detection [11].
Once in the conventional coherent receiver, the polarization management turned out to be one of the main obstacles for the practical implementation [4,10]. For WDM systems, each channel requires a dedicated dynamic polarization controller, and this severely limits the practicality of the coherent receivers. In the digital coherent receivers, the polarization control can be solved by using the electrical adaptive polarization alignment, which is realized with much lower complexity and cost [11].

The next issue is the possibility and the applicability of the coherent transmission systems for any type of multi-level modulation formats. Besides the QPSK modulation format, the 8-PSK and the 16-QAM formats are also examined at 10-Gsymbol/s in the coherent systems [4,12-15]. Note that polarization multiplexing can always double the bit rate as mentioned before.

The most important technical issue is real-time operation of the digital coherent receivers, which depends on the computing speed of the analog-to-digital convertors (ADCs) and the DSP components. Now the novel components come out, for example the 50-GSample/s 8-bit ADCs (Tektronix DPO72004) appear in test equipment, and the module of gate CMOS-ASIC with 4 integrated ADCs over 20-GSample/s is also developed [16,17].

1.3 Brief summary of research field

The performance of high speed optical fiber transmission systems is severely affected by chromatic dispersion, polarization mode dispersion, phase noise and nonlinear effects [18-21], which can be well compensated in the coherent detection system employing the DSP circuit and the corresponding algorithms. Here we give a short overview about the development and the current status of the recently reported investigations related with our research work.

1. Chromatic dispersion equalization

Coherent optical receivers employing digital filters allow significant equalization of chromatic dispersion in the electrical domain, instead of compensation by dispersion compensating fibers (DCFs) or dispersion compensating modules (DCMs) in the optical domain [22-28]. Several digital filters have been applied to compensate the CD in the time and the frequency domain [25-31]. H. Bülow and A. Färbert et al. have reported their CD equalization work using the maximum likelihood sequence estimation (MLSE) method [23,26], which was the first DSP equalizer proposed. The MLSE electronic equalizer is implemented by using the Viterbi algorithm [26], where one is looking for the most likely bit sequence formed by a series of distorted signals. The MLSE is not tailored to a specific distortion but is optimum for any kind of optically distorted signal detected by the photodiode, provided the inter-symbol interference (ISI) does not exceed the equalized symbols each with a sampling period. S. J. Savory used a time-domain fiber dispersion finite impulse response (FD-FIR) filter to compensate the CD in the 1000 km and the 4000 km transmission fibers without using dispersion compensation fibers [28,29]. The realization of the FD-FIR filter arises from the digitalization of the inverse function of the time-domain impulse
response for the fiber channel [28]. The time window of the FD-FIR filter can be truncated by using the Nyquist frequency, which is determined to avoid the aliasing phenomenon in the digital system. M. Kuschnerov and F. N. Hauske et al. have used the frequency domain equalizers (FDEs) to compensate the CD in coherent communication systems [30,31], which are considered as the most efficient digital equalizers for chromatic dispersion compensation. The implementation of the frequency domain equalizers comes from the inverse impulse response of the fiber channel in the frequency domain. One of the most popular realization of the FDEs is the blind look-up (BLU) digital filter [30], which will be discussed in our thesis. It has been demonstrated in the investigation that the FDEs are more efficient than the FD-FIR filter and the adaptive digital filters, when the accumulated fiber dispersion is larger than 3000 ps/nm in the transmission system [32,33].

2. Carrier phase estimation

Phase noise from lasers is also a significant impairment in coherent optical transmission systems. The traditional method of demodulating coherent optical signals is to use an optical or electrical PLL to synchronize the frequency and phase of the local oscillator with the transmitter (TX) laser. Advances in high-speed very large-scale integration (VLSI) technology promise to change the paradigm of coherent optical receivers [34,35]. The frequency mismatch between the TX and the LO lasers can be tracked by DSP algorithms and compensated in the feed-forward and feed-back architectures. Recent reports have demonstrated that feed-forward carrier-recovery schemes can be more tolerant to the laser phase noise than the PLL-based receivers. Several feed-forward and feed-back carrier phase estimation (CPE) algorithms have been validated as effective methods for mitigating the phase fluctuation from the laser sources [34-40]. The feed-forward carrier phase estimation algorithms mainly arise from some basic principles. One is based on the maximum-likelihood detection to estimate the transmitted sequence. The receiver consists of a soft-decision phase estimation stage followed by hard-decision estimation of the carrier phase and the transmitted symbols [34,35]. The other popularly used algorithm is called the N-power carrier phase estimation method [37-39]. By applying this algorithm a common phase value is evaluated for a block of signal samples and subtracted from the received signal prior to making a decision on the data extracted from the signal. The carrier phase is estimated by raising the signal amplitude to the power of N in order to get rid of the phase modulation due to encoded data and by averaging contributions from a block of the signal samples. The feed-back carrier phase recovery is to employ a one-tap normalized LMS (NLMS) filter to implement the decision-direct phase estimation in the coherent system, where a parameter of step size can influence the performance of the NLMS filter [40].

However, the analysis of the phase noise in the transmitter and the local oscillator lasers is often lumped together in these algorithms, and the influence of the large chromatic dispersion on the phase noise in the system is not considered. Related work has been developed to deliberate the interplay between the digital chromatic dispersion equalization and the laser phase noise [41-47]. W. Shieh, K. P. Ho and A.
P. T. Lau et al. have provided the theoretical assessment and analysis to evaluate the equalization enhanced phase noise (EEPN) from the interaction between the LO phase fluctuation and the fiber dispersion in coherent transmission system [41-43]. C. Xie has investigated the impact of chromatic dispersion on both the LO phase noise to amplitude noise conversion and the fiber nonlinear effects [44,45]. I. Fatadin and S. J. Savory have also studied the influence of the equalization enhanced phase noise in QPSK, 16-level quadrature amplitude modulation (16-QAM) and 64-QAM coherent transmission systems by employing the time-domain CD equalization [28,46]. Due to the existence of EEPN, the requirement of laser linewidth can not be generally relaxed for the transmission system with higher symbol rate. It would be interesting to investigate the performance of the equalization enhanced phase noise in the coherent optical communication system employing different digital chromatic dispersion compensation methods, which will be also discussed in this dissertation.
Chapter 2

Channel impairments in transmission system

Chromatic dispersion, polarization mode dispersion, phase noise and fiber nonlinearities are the important distortions that affect the performance of optical transmission systems. We will present a brief introduction in this part for the above four types of impairments in the optical fiber communication system.

2.1 Chromatic dispersion

The chromatic dispersion of an optical medium is the phenomenon that the phase velocity and the group velocity of light propagation depend on the optical frequency. Group delay is defined as the first derivative of the optical phase with respect to the optical frequency, and chromatic dispersion is defined as the second derivative of the optical phase with respect to the optical frequency. Chromatic dispersion consists of the waveguide dispersion and the material dispersion [21,48-50]. The material dispersion occurs due to the changes in the refractive index of the medium with the changes in optical wavelength, which originates from the electromagnetic absorption. The waveguide dispersion occurs when the speed of a wave in a waveguide depends on its frequency for geometric reasons, independent of any frequency dependence of the materials. For a fiber, the waveguide dispersion arises from the dependence on the fiber parameters such as the core radius and index difference. The common evaluation of the chromatic dispersion (dispersion parameter D) is calculated by the time delay between the unitary wavelength difference after propagation through the unitary fiber length. The unit of D is normally expressed in ps/nm/km.

![Figure 1. Typical wavelength dependence of dispersion parameters in normal single-mode fibers.](image)
The chromatic dispersion for different wavelength in the standard single mode fiber (SSMF) is illustrated in Fig. 1 [51]. The example shows the characters of the single-mode fibers have zero dispersion at a wavelength of 1310 nm. We can also find that the chromatic dispersion value is around 16 ps/nm/km at 1550 nm, which is the operation wavelength for practical optical fiber transmission systems. Chromatic dispersion remains constant over the bandwidth of a transmission channel for long lengths of fiber. In traditional optical fiber communication systems, the chromatic dispersion is usually compensated by the dispersion compensation fibers. In coherent transmission systems, the chromatic dispersion can be equalized by using a digital filter, which will be discussed in Chapter 4.

2.2 Polarization mode dispersion

Polarization mode dispersion is a phenomenon of modal dispersion that two orthogonal polarizations of light propagate at different speeds due to the random imperfections and asymmetries in the waveguide, which cause random spreading of optical pulses. The ideal optical fiber core has a perfectly circular cross-section, where the two orthogonal fundamental modes travel at the same speed. However, in a realistic fiber, the random imperfections such as the circular asymmetries, can arouse the two polarizations to propagate at different speeds. The symmetry-breaking random imperfections consist of the geometric asymmetry (slightly elliptical cores) and the stress-induced material birefringences [48-51].

In the existence of PMD, the two polarization modes of the optical signal will separate slowly. Corresponding to the random imperfections, the pulse spreading effects is a random walk. Due to the characteristic of random variation, the evaluation of the polarization mode dispersion is calculated by the mean polarization-dependent time-differential, which is called the differential group delay (DGD), proportional to the square root of propagation distance. The unit of the polarization mode dispersion is in $\text{ps}/\sqrt{\text{km}}$ [52,53]. In practical single mode fibers, the value of PMD is from $0.1\text{ps}/\sqrt{\text{km}}$ to $1\text{ps}/\sqrt{\text{km}}$. The pulse spreading effects in the optical fiber is shown in Fig. 2 [54].

![Figure 2. The mode spreading due to the PMD in the optical fibers.](image)

The method for PMD compensation is to employ a polarization controller to compensate the differential group delay occurring in optical fibers. The PMD effects are random and time-dependent, therefore, an active feed-back device over time is
required. Such systems are therefore expensive and complex. In the digital coherent receivers employing DSP, the PMD can be compensated by the adaptive filters.

2.3 Laser phase noise

One of important sources for receiver sensitivity degradation in the coherent lightwave systems is the phase noise associated with the transmitter laser and the local oscillator laser [21]. Laser phase noise can be approximately regarded as a Wiener process caused by laser spontaneous emission, which can be modeled as the expression [34,55,56]:

\[ \phi(t) = \int_{-\infty}^{t} \delta \omega(\tau) d\tau \]  

(1)

where \( \phi(t) \) is the instantaneous optical phase, and \( \delta \omega(\tau) \) is the frequency noise with zero mean and autocorrelation \( R = 2\pi\Delta\nu \delta(\tau) \). It has been demonstrated that the laser output has a Lorentzian spectrum with a 3-dB linewidth of \( \Delta\nu \) [34].

![Figure 3](image-url)

(a) Without phase noise, (b) with phase noise laser linewidth TX=LO=150 kHz.

Phase noise is a significant impairment in the coherent transmission systems, since it impacts the optical carrier synchronization between the TX laser and the LO laser. In the non-coherent detection system (such as IMDD system), the carrier phase is not so important because the receiver only measures the energy of the optical signal. In the coherent systems, the information is encoded into the variation of carrier phase, therefore, the phase fluctuation over a symbol period has the significant influence on the signal demodulation in the receiver, as shown in Fig. 3. We can find in Fig. 3 that the QPSK constellation is distorted obviously by the phase noise. In the traditional coherent systems, the carrier phase noise is compensated by using the optical PLL in the receiver to track the phase changing with the time, which is rather difficult to realize the corresponding control circuits. In the modern digital coherent detection systems, the carrier phase noise can be well mitigated by using the DSP algorithms.
such as the feed-forward and the feed-back carrier phase estimation, which are relatively easy to implement.

### 2.4 Nonlinear effects

The transmission nonlinear impairments associated with long-distance high bit-rate optical fiber communication systems mainly include the fiber Kerr nonlinearities, the self-phase modulation (SPM), the cross-phase modulation (XPM) and the four-wave mixing (FWM) [57,58]. The signals transmitted through the optical fibers in presence of attenuation, chromatic dispersion and nonlinear effects follow the nonlinear Schrödinger equation (NLSE),

\[
\frac{\partial E(z,t)}{\partial z} + \frac{j}{2} \beta_2 \frac{\partial^2 E(z,t)}{\partial t^2} + \frac{\alpha}{2} E(z,t) = j \gamma |E(z,t)|^2 E(z,t)
\]

where \( E(z,t) \) is the electric field of the optical signal, \( \alpha \) is the attenuation coefficient, \( \beta_2 \) is the chromatic dispersion parameter, \( \gamma \) is the nonlinear coefficient, and \( z \) and \( t \) are the propagation direction and time, respectively. The nonlinear parameter \( \gamma \) depends inversely on the effective core area of the transmission fiber.

For WDM transmission systems, fiber nonlinear effects mainly consist of two aspects: inter-channel interference and intra-channel interference. Inter-channel nonlinear effects refer to the interference between different wavelength channels, which include the cross-phase modulation and the four-wave mixing. Intra-channel nonlinear effects indicate the interference between different modules in the same wavelength channel, which include self-phase modulation, intra-channel XPM and intra-channel FWM. Inter-channel nonlinearities are dominant for lower bit-rate transmission systems, and intra-channel nonlinearities are dominant for higher bit-rate transmission systems.

The fiber nonlinear effects are difficult to compensate in traditional high speed IMDD transmission systems. In digital coherent systems, the nonlinear effects can be mitigated by using the backward propagation methods based on solving the nonlinear Schrödinger equation and the Manakov equation [59,60].
Chapter 3

High speed PDM-QPSK coherent transmission system

In this chapter, we give a detailed analysis for the setup of the 112-Gbit/s NRZ-PDM-QPSK coherent optical transmission system realized in the VPI platform. Meanwhile, we describe the whole system by a mathematical model of the 112-Gbit/s NRZ-PDM-QPSK coherent transmission system. The theoretical modes consisting of the QPSK transmitter, the fiber channel and the coherent receiver are established and analyzed in equations.

3.1 Setup of 112-Gbit/s PDM-QPSK coherent transmission system

The setup of the 112-Gbit/s NRZ-PDM-QPSK coherent transmission system implemented in the VPI simulation platform is illustrated in Fig. 4 [61].

![Figure 4. Schematic of 112-Gbit/s NRZ-PDM-QPSK coherent optical transmission system.](image)


The data sequence output from the four 28-Gbit/s pseudo random bit sequence (PRBS) generators are modulated into two orthogonally polarized NRZ-QPSK optical signals by the two Mach-Zehnder modulators. Then the orthogonally polarized signals are integrated into one fiber channel by a polarization beam combiner (PBC) to form the 112-Gbit/s NRZ-PDM-QPSK optical signal. Using a local oscillator in the coherent receiver, the received optical signals are mixed with the LO laser to be transformed into four electrical signals by the photodiodes. Then they are digitalized by the 8-bit
analog-to-digital convertors (ADCs) at twice the symbol rate [62]. The sampled signals are processed by a series of digital equalizers, and the BER is then estimated from the data sequence of $2^{16}$ bits. The central wavelength of the TX laser and the LO laser are both 1553.6 nm. The standard single mode fibers with the CD coefficient equal to 16 ps/nm/km are employed in all the simulation work. Here we mainly concentrate our work on the CD compensation and the carrier phase noise mitigation methods in DSP techniques, and so we neglected the influences of fiber attenuation, polarization mode dispersion and nonlinear effects [59-63].

3.2 Theoretical analysis of system modules

Some mathematical expressions for analyzing the modules in coherent transmission system are presented in the following descriptions.

3.2.1 Optical QPSK transmitter (Mach-Zehnder modulator)

The main structure of the QPSK transmitter is realized by using a nested Mach-Zehnder modulator [64-66]. The PRBS output sample pass into the non-return-to-zero signal generator to form the modulation wave, where the output bit sequence could be expressed as

$$x_i(t) = p \quad \text{and} \quad p = 0,1$$  \hspace{1cm} (3)

$$x_q(t) = q \quad \text{and} \quad q = 0,1$$  \hspace{1cm} (4)

The electric field transfer function $h_{MZM}(t)$ of the Mach-Zehnder modulator is given as the following equation,

$$h_{MZM}(t) = \alpha_{MZM} \cdot \exp(p \cdot j\pi)$$  \hspace{1cm} (5)

The I-channel electric field output from Mach-Zehnder modulator neglecting the attenuation of the Mach-Zehnder modulator is

$$E_i(t) = E_{CPW}(t) \cdot h_{MZM}(t)$$

$$= E_o \exp\left[j(\omega_{carrier}t + \phi_{carrier} + p \cdot \pi)\right] \quad p = 0,1$$  \hspace{1cm} (6)

$$E_{CPW}(t) = E_o \exp\left[j(\omega_{carrier}t + \phi_{carrier})\right]$$  \hspace{1cm} (7)

According to the same principle, we could obtain the Q-channel electric field as,

$$E_q(t) = E_{CPW}(t) \cdot h_{MZM}(t) \cdot \exp\left(j\frac{\pi}{2}\right)$$

$$= E_o \exp\left[j(\omega_{carrier}t + \phi_{carrier} + q \cdot \pi + \frac{\pi}{2})\right] \quad q = 0,1$$  \hspace{1cm} (8)
\[ E_{\text{QPSK}}(t) = E_r(t) + jE_q(t) \]  

### 3.2.2 Fiber propagation

The generalized nonlinear Schrödinger equation is used to describe the in-band effects for the fiber transmission, which is expressed as,

\[ \frac{\partial E(z,t)}{\partial z} = \left[ \hat{D} + \hat{N} \right] \cdot E(z,t) \]  

where \( E(z,t) \) denotes the slowly-varying complex-envelope of the electric field of the light wave, \( |E(z,t)|^2 \) characterizes its power, \( \hat{D} \) is the dispersion operator, and \( \hat{N} \) is the nonlinearity operator.

\[ \hat{D} = j\beta_2 \frac{\partial^2}{\partial t^2} + \frac{\beta_s}{6} \frac{\partial^3}{\partial t^3} - \frac{\alpha}{2} \]  

where \( \beta_2 \) describes the first order group-velocity dispersion, \( \beta_s \) is the second order GVD slope, and \( \alpha \) is the attenuation constant of the transmission fiber.

Nonlinear operator (with no Raman effect) is simply given by

\[ \hat{N} = -j\gamma |E(z,t)|^2 \]  

\[ \gamma = \frac{2m_2f_{\text{ref}}}{cA_{\text{eff}}} \]  

where \( \gamma \) depends on the nonlinear index \( n_2 \), the effective core area \( A_{\text{eff}} \), as well as the reference frequency of optical carrier wave \( f_{\text{ref}} \) and the velocity of light in vacuum \( c \).

The propagation of the optical signal in the fiber can be calculated by the split-step Fourier method. Assuming a propagation of optical signals in +z direction and an asymmetrical split-step algorithm, the mathematical formalism of the procedure can be described as the following description,

\[ E(z_0 + \Delta z, t) = \left[ \exp \left( \Delta z \hat{N} \right) E(z_0, t) \right] \exp \left( \Delta z \hat{D} \right) \]

### 3.2.3 Coherent Receiver

In the coherent receiver, the 2×4 90 degree hybrid structure is adopted to demodulate the received optical signal, which consists of four 3-dB 2×2 fiber couplers and a phase delay components of \( \pi/2 \) phase shift in one branch [67-70].

Assuming the electric field of the received optical signal is \( E_r(t) \), and the electric
field of the local oscillator laser is $E_{\text{LO}}(t)$, which is expressed as

$$E_{\text{LO}}(t) = E_{\text{LO}} \cdot \exp[j(\omega_{\text{LO}} t + \phi_{\text{LO}})]$$  \hspace{1cm} (15)

The output electric field components of the coherent receiver are calculated as follows,

$$
\begin{bmatrix}
    E_0^o \\
    E_{90}^o \\
    E_{180}^o \\
    E_{270}^o
\end{bmatrix} =
\frac{1}{2}
\begin{bmatrix}
    1 & 1 \\
    1 & j \\
    1 & -1 \\
    1 & -j
\end{bmatrix}
\begin{bmatrix}
    E_R^o \\
    E_{\text{LO}}^o
\end{bmatrix} =
\frac{1}{2}
\begin{bmatrix}
    E_R + E_{\text{LO}} \\
    E_R + E_{\text{LO}} \cdot \exp(j 90^\circ)
\end{bmatrix}
$$

(16)

where $E_0^o$, $E_{90}^o$, $E_{180}^o$, $E_{270}^o$ represent the four electric fields output from the 90 degree hybrid coherent receiver respectively. Due to the asymmetry of the 3-dB 2×2 fiber coupler, the two lower outputs are 90 degree phase shifted relative to the two upper outputs. With the additional 90 degree phase shift introduced, the output electric fields are revised as

$$
\begin{bmatrix}
    E_0^o \\
    E_{90}^o \\
    E_{180}^o \\
    E_{270}^o
\end{bmatrix} =
\frac{1}{2}
\begin{bmatrix}
    (-j) \cdot E_R + (-j) \cdot E_{\text{LO}} \\
    (-j) \cdot (-j) E_R + E_{\text{LO}} \cdot \exp(j \cdot 90^\circ) \\
    (-j) \cdot E_R - (j) E_{\text{LO}} \cdot \exp(j \cdot 90^\circ) \\
    (-j) \cdot (-j) E_R - j E_{\text{LO}} \cdot \exp(j \cdot 90^\circ)
\end{bmatrix}
$$

(17)
Chapter 4

Digital signal processing algorithms for coherent transmission system

In this chapter, the chromatic dispersion mitigation and the carrier phase noise compensation are implemented and analyzed with the corresponding DSP algorithms. The adaptive PMD equalization using the LMS and the CMA filters is also briefly discussed. Moreover, we also present the numerical simulation results for chromatic dispersion compensation and carrier phase estimation in the 112-Gbit/s NRZ-PDM-QPSK coherent optical transmission system.

4.1 Chromatic dispersion compensation

The popular digital filters involving the time-domain LMS adaptive filter and FD-FIR filter, as well as the frequency-domain filters are investigated for CD compensation. The characteristics of these filters are analyzed comparatively in the 112-Gbit/s NRZ-PDM-QPSK coherent transmission system.

4.1.1 Time domain equalizers

1. The LMS adaptive filter

The LMS filter employs an iterative algorithm that incorporates successive corrections to weights vector in the negative direction of the gradient vector which eventually leads to a minimum mean square error [71]. The principle of LMS filter is given by the following equations:

\[ y(n) = w^H(n)x(n) \]

(18)

\[ \bar{w}(n+1) = \bar{w}(n) + \mu \bar{x}(n)e^*(n) \]

(19)

\[ e(n) = d(n) - y(n) \]

(20)

where \( \bar{x}(n) \) is the digitalized complex magnitude vector of the received signal, \( y(n) \) is the complex magnitude of the equalized output signal, \( n \) represents the number of sample sequence, \( \bar{w}(n) \) is the complex tap weights vector, \( w^H(n) \) is the Hermitian transform of \( \bar{w}(n) \), \( d(n) \) is the desired symbol, which corresponds to one case of the vector \([1+i, 1-i, -1-i, -1+i]\) for the QPSK coherent transmission system, \( e(n) \) represents the estimation error between the output signal and the desired symbol, \( e^*(n) \) is the conjugation of \( e(n) \), and \( \mu \) is a key real coefficient called step size. In order to guarantee the convergence of tap weights vector \( \bar{w}(n) \), the step size \( \mu \) needs to satisfy the condition of \( 0 < \mu < 1/\lambda_{\text{max}} \), where \( \lambda_{\text{max}} \) is the largest eigenvalue of the correlation matrix \( R = \bar{x}(n)x^*(n) \).
The tap weights in LMS adaptive equalizer for 20 km fiber CD compensation is shown in Fig. 5. The convergence for 9 tap weights in the LMS filter with step size equal to 0.1 is shown in Fig. 5(a), and we can find the tap weights obtain their convergence after about 5000 iterations. The magnitudes of converged tap weights are shown in Fig. 5(b), and it can be found that the central tap weights take more dominant roles than the high-order tap weights.

2. The FD-FIR filter

Compared with the iteratively updated LMS filter, the tap weights in FD-FIR filter have a relatively simple specification [28,29], the tap weight in FD-FIR filter is given by the following equations:
where $D$ is the fiber chromatic dispersion coefficient, $\lambda$ is the central wavelength of the transmitted optical wave, $z$ is the fiber length in the transmission channel, $T$ is the sampling period, $N^A$ is the required maximum tap number for compensating the fiber dispersion, and $\lfloor x \rfloor$ denotes the nearest integer less than $x$.

Figure 6. Tap weights of FD- FIR filter.

The tap weights of FD-FIR filter according to Eq. (21) for 20 km fiber ($D=16$ ps/nm/km) are shown in Fig. 6 [72,73]. For a fixed fiber dispersion, the magnitudes of tap weights in FD-FIR filter are constant, whereas the real and the imaginary parts vary periodically.

4.1.2 Frequency domain equalizers

The frequency domain equalizers (FDEs) have become the more attractive digital filters for channel equalization in the coherent transmission systems due to the low computational complexity for large dispersion and the wide applicability for different fiber distance [28-33]. The fast Fourier transform (FFT) convolution algorithms involving the overlap-save (OLS) and the overlap-add zero-padding (OLA-ZP) methods are traditionally used for the equalization in the wireless communication systems [74-77]. In our research work, the OLS-FDE and the OLA-ZP-FDEs are applied to compensate the CD in the 112-Gbit/s NRZ-PDM-QPSK coherent optical transmission system [32,33,78,79].
1. **Overlap-save method**

The schematic of the FDE with overlap-save method is illustrated in Fig. 7 [74,75,78,79]. The received signals are divided into several blocks with a certain overlap, where the block length is called the FFT-size. The sequence in each block is transformed into the frequency domain data by the FFT operation, and afterwards multiplied by the transfer function of the FDE. Next, the data sequences are transformed into the time domain signals by the inverse FFT (IFFT) operation. Finally, the processed data blocks are combined together, and the bilateral overlap samples are symmetrically discarded. One of the most popular OLS-FDEs for chromatic dispersion equalization is the blind look-up filter we mentioned in our dissertation.

![Diagram of FDE with OLS method](image)

Figure 7. FDE with OLS method. The parts with slants are to be discarded.

2. **Overlap-add method**

The structure of the FDE with overlap-add one-side zero-padding (OLA-OSZP) method is shown in Fig. 8 [74-77]. The received data are divided into small blocks without any overlap, and then the data in each block are appended with zeros at one side. To be consistent with the OLS method, the total length of data block and zero padding is called the FFT-size, while the length of zero padding is called the overlap. The zero-padded sequence is transformed by the FFT operation, and multiplied by the transfer function of the FDE. Afterwards, the data are transformed by the IFFT operation. Finally the processed data sequences are combined by overlapping and adding. Note that half of the data stream in the first block is discarded.

The schematic of the FDE with overlap-add both-side zero-padding (OLA-BSZP) method is illustrated in Fig. 9 [74-77]. The received data are also divided into several blocks without any overlap, and then the data in each block are appended with equivalent zeros at both sides. The total length of data block and zero padding is called the FFT-size, and the length of the whole zero padding is called the overlap. The zero-padded sequence is transformed by the FFT operation, and multiplied by the
The influence of PMD and polarization fluctuation can be compensated adaptively by the decision-directed LMS filter [25,71], which is expressed as the following equations:

\begin{align*}
E_{12} + E_{21} & = E_{31} + E_{32} + E_{41} + E_{42} + E_{51} \\
E_{12} + E_{21} & = E_{31} + E_{32} + E_{41} + E_{42} + E_{51}
\end{align*}

Figure 8. FDE with OLA-OSZP method. The gray parts mean the appended zeros, and the parts with slants are to be discarded.

Figure 9. FDE with OLA-BSZP method. The gray parts mean the appended zeros, and the parts with slants are to be discarded.

4.2 Polarization mode dispersion and polarization rotation equalization

4.2.1 LMS adaptive PMD equalization
\[
\begin{bmatrix}
x_{\text{out}}(n) \\
y_{\text{out}}(n)
\end{bmatrix} =
\begin{bmatrix}
x_n^* \\
y_n^*
\end{bmatrix}
\begin{bmatrix}
x_{\text{in}}(n) \\
y_{\text{in}}(n)
\end{bmatrix}
\]
(23)

\[
\begin{align*}
\rightarrow w_{xs}(n+1) &= w_{xs}(n) + \mu_p \varepsilon_x(n) x_{\text{in}}(n) \\
\rightarrow w_{xs}(n+1) &= w_{xs}(n) + \mu_p \varepsilon_x(n) x_{\text{in}}(n) \\
\rightarrow w_{xs}(n+1) &= w_{xs}(n) + \mu_p \varepsilon_y(n) y_{\text{in}}(n) \\
\rightarrow w_{xs}(n+1) &= w_{xs}(n) + \mu_p \varepsilon_y(n) y_{\text{in}}(n)
\end{align*}
\]
(24)

\[
\begin{align*}
\varepsilon_x(n) &= d_x(n) - x_{\text{out}}(n) \\
\varepsilon_y(n) &= d_y(n) - y_{\text{out}}(n)
\end{align*}
\]
(25)

where \( x_{\text{in}}(n) \) and \( y_{\text{in}}(n) \) are the complex magnitude vectors of the input signals, \( x_{\text{out}}(n) \) and \( y_{\text{out}}(n) \) are the complex magnitudes of the equalized output signals respectively, \( w_{xs}(n) \), \( w_{xs}(n) \), \( w_{ys}(n) \) and \( w_{ys}(n) \) are the complex tap weights vectors, \( d_x(n) \) and \( d_y(n) \) are the desired symbols, \( \varepsilon_x(n) \) and \( \varepsilon_y(n) \) represent the estimation errors between the output signals and the desired symbols respectively, and \( \mu_p \) is the step size parameter. The polarization diversity equalizer can be implemented subsequent to the CD compensation.

### 4.2.2 CMA adaptive PMD equalization

The decision-directed CMA filter can also be employed for the adaptive compensation for the influence of the PMD and the polarization fluctuation [80,81], of which the tap weights can be expressed as:

\[
\begin{bmatrix}
x_{\text{out}}(n) \\
y_{\text{out}}(n)
\end{bmatrix} =
\begin{bmatrix}
x_n^* \\
y_n^*
\end{bmatrix}
\begin{bmatrix}
x_{\text{in}}(n) \\
y_{\text{in}}(n)
\end{bmatrix}
\]
(26)

\[
\begin{align*}
\rightarrow v_{xs}(n+1) &= v_{xs}(n) + \mu_q \eta_x(n) x_{\text{in}}(n) \\
\rightarrow v_{xs}(n+1) &= v_{xs}(n) + \mu_q \eta_x(n) x_{\text{in}}(n) \\
\rightarrow v_{ys}(n+1) &= v_{ys}(n) + \mu_q \eta_y(n) x_{\text{in}}(n) \\
\rightarrow v_{ys}(n+1) &= v_{ys}(n) + \mu_q \eta_y(n) x_{\text{in}}(n)
\end{align*}
\]
(27)

\[
\begin{align*}
\eta_x(n) &= 1 - |x_{\text{out}}(n)|^2 \\
\eta_y(n) &= 1 - |y_{\text{out}}(n)|^2
\end{align*}
\]
(28)

We can find that the CMA algorithm is based on the principle of minimizing the modulus variation of the output signal to update its weight vector.
4.3 Carrier phase recovery

4.3.1 The normalized LMS filter for phase estimation

The one-tap NLMS filter can be employed effectively for carrier phase estimation [40,71], of which the tap weight is expressed as

\[ w_{NLMS}(n+1) = w_{NLMS}(n) + \frac{\mu_{NLMS}}{|x_{PN}(n)|} x_{PN}^*(n) e_{NLMS}(n) \] (29)

\[ e_{NLMS}(n) = d_{PE}(n) - w_{NLMS}(n) x_{PN}(n) \] (30)

where \( w_{NLMS}(n) \) is the complex tap weight, \( x_{PN}(n) \) is the complex magnitude of the input signal, \( n \) represents the number of the symbol sequence, \( d_{PE}(n) \) is the desired symbol, \( e_{NLMS}(n) \) is the estimation error between the output signals and the desired symbols, and \( \mu_{NLMS} \) is the step size parameter.

The phase estimation using the one-tap NLMS filter resembles the performance of the ideal differential detection [38-40,82], of which the BER floor can be approximately described by an analytical expression,

\[ BER_{NLMS}^{floor} \approx \frac{1}{2} \text{erfc}\left(\frac{\pi}{4\sqrt{2}\sigma}\right) \] (31)

where \( 2\sigma^2 \) represents the total phase noise variance in the coherent transmission system.

4.3.2 Differential phase detection for phase estimation

It has been reported that the symbol delay detection can also be used for carrier phase estimation [38-40]. The coherent system can be operated in differential demodulation mode when the differential encoded data is recovered by a simple “delay and multiply algorithm” in the electrical domain. In such a case the encoded data is recovered from the received signal based on the phase difference between two consecutive symbols, i.e. the value of the complex decision variable \( \Psi = Z_k Z_{k+1}^* \exp[i\pi/4] \), where \( Z_k \) and \( Z_{k+1} \) are the consecutive k-th and (k+1)-th received symbols. The BER floor of the differential phase receiver can be evaluated using the principle of conditional probability [82], which is expressed as the following equation,

\[ BER_{DQPSK}^{floor} = \frac{1}{2} \text{erfc}\left(\frac{\pi}{4\sqrt{2}\sigma}\right). \] (32)

where \( 2\sigma^2 \) also represents the total phase noise variance in the coherent transmission system.
### 4.3.3 Principle of equalization enhanced phase noise

The scheme of the coherent optical communication system with digital CD equalization and carrier phase estimation is depicted in Fig. 10.

![Scheme of equalization enhanced phase noise in coherent transmission system.](image)

**Figure 10.** Scheme of equalization enhanced phase noise in coherent transmission system. 
$\Phi_{TX}$: phase fluctuation of the TX laser, $\Phi_{LO}$: phase fluctuation of the LO laser, $N(t)$: additive white Gaussian noise.

The transmitter laser phase noise passes through both transmission fibers and the digital CD equalization module, and so the net dispersion experienced by the transmitter PN is close to zero. However, the local oscillator phase noise only goes through the digital CD equalization module, which is heavily dispersed in a transmission system without dispersion compensation fibers. Therefore, the LO phase noise will significantly influence the performance of the high speed coherent system with only digital CD post-compensation. We note that the EEPN does not exist in a transmission system with entire optical dispersion compensation for instance using DCFs.

Theoretical analysis demonstrates that the EEPN scales linearly with the accumulated chromatic dispersion and the linewidth of LO laser [41-46], and the variance of the additional noise due to the EEPN can be expressed as

$$\sigma_{LO}^2 = \frac{2\pi}{c} \frac{D \cdot L \cdot \Delta f_{LO}}{T_s}$$  \hspace{1cm} (33)

where $\lambda$ is the central wavelength of the transmitted optical carrier wave, $c$ is the light speed in vacuum, $D$ is the chromatic dispersion coefficient of the transmission fiber, $L$ is the transmission fiber length, $\Delta f_{LO}$ is the 3-dB linewidth of the LO laser, and $T_s$ is the symbol period of the transmission system.

It is worth noting that the theoretical evaluation of the enhanced LO phase noise is only appropriate for the FD-FIR and the BLU dispersion equalization, which represent the inverse function of the fiber transmission channel without involving the phase noise mitigation.

Considering the effect of EEPN, the total phase noise variance can be revised as follows

$$\sigma^2 \approx \sigma_{TX}^2 + \sigma_{LO}^2 + \sigma_{EEP}^2$$  \hspace{1cm} (34)

$$\sigma_{TX}^2 = 2\pi \Delta f_{TX} \cdot T_s$$  \hspace{1cm} (35)
\[
\sigma^2_{LO} = 2\pi f_{LO} \cdot T_s
\]  

(36)

where \( \sigma^2 \) represents the total phase noise variance in the coherent transmission system, \( \sigma^2_{TX} \) and \( \sigma^2_{LO} \) are the original phase noise variance of the transmitter and the LO lasers respectively, and \( \Delta f_{TX} \) and \( \Delta f_{LO} \) are the 3-dB linewidth of the transmitter and the LO laser respectively.

**4.4 Simulation results**

In the following paragraph, numerical simulations are performed in the 112-Gbit/s NRZ-PDM-QPSK coherent transmission system to validate the effects of the digital chromatic dispersion compensation filters and the carrier phase estimation algorithms.

**4.4.1 CD compensation**

The CD compensation results using three digital filters are illustrated in Fig. 11. Figure 11(a) indicates the CD equalization with 9 taps for 20 km fiber and 243 taps for 600 km fiber using the LMS and the FD-FIR filters, as well as 16 FFT-size (8 overlap) for 20 km fiber and 512 FFT-size (256 overlap) for 600 km fiber using the BLU filter. Obviously, the FD-FIR filter is not able to compensate the CD in 20 km fiber entirely. About 3 dB optical signal-to-noise ratio (OSNR) penalty from the back-to-back result at BER equal to \( 10^{-3} \) can be observed. Then we investigate the CD compensation for different fiber lengths using the three filters, which are shown in Fig. 11(b). It can be found that the LMS filter and the BLU filter show the same acceptable performance for different fiber lengths, while the FD-FIR filter will not behave satisfactorily until the fiber length exceeds 320 km.
Figure 11. CD compensation using three digital filters neglecting fiber loss. (a) BER with OSNR. (b) BER with fiber length at OSNR 14.8 dB.

The CD equalization for 20 km and 600 km fibers using the LMS and the FD-FIR filters with different number of taps are shown in Fig. 12. Due to the optimum characteristic of LMS algorithm, the LMS filter has a slight improvement with the increment of tap number. However, the performance of the FD-FIR filter will degrade, when the tap number increases and exceeds the required tap number in Eq. (22). It is because the redundant taps will lead to the pass-band of the filter exceeding the Nyquist frequency, which will further result in the aliasing phenomenon. We also find in Fig. 12(a) that the FD-FIR filter does not achieve a satisfactory CD equalization performance for 20 km fiber even by using any other tap number.
Figure 12. CD compensation with different taps number using LMS filter and FD-FIR filter at OSNR 14.8 dB. (a) 20 km fiber. (b) 600 km fiber.

From the above description, the FD-FIR filter does not achieve an acceptable CD equalization performance for short distance fibers, but it can work well for long fibers. When we use a series of delayed taps to approximate the filter time window $A_{WT}$, the digitalized discrete time window $T_{DN} = N^4 \cdot T$ could not attain exactly the same value as the continuous time window $T_{ WN}$, which is illustrated in Fig. 13.

The malfunction of FD-FIR filter for short fibers arises from this reason, and now we provide a more detailed explanation. We calculate the relative error $p$ of time window to evaluate the precision of time window approximation, which is given by

$$ p = \frac{(T_{DN}^d - T_{WN}^d)}{T_{WN}^d}. $$

According to previous discussion, a short fiber will have a relative small time window to keep the signal bandwidth to be lower than Nyquist frequency to avoid the aliasing phenomenon. However, such a small time window is not easy to be digitalized accurately with a fixed sampling period $T$. In order to broaden the time window, we need to raise the Nyquist frequency correspondingly. The Nyquist frequency is defined as half of the sampling frequency of the system, and this means we need to increase the sampling rate in the ADC modules. With sampling rate being increased, the Nyquist frequency are also raised, meanwhile, the sample period $T$ is reduced,
which allows the broadened continuous time window to be digitalized more precisely. The relative errors of time window for different fiber length with different sampling rate are shown in Table 1, where the positive time error means the aliasing occurring. We could find the relative error of time window for 20 km is reduced obviously with the sampling rate changing from 2 samples per symbol (Sa/Sy) to 8 Sa/Sy. Furthermore, the time error for 20 km fiber with 8 Sa/Sy is equal to the time error for 320 km fiber with 2 Sa/Sy, which is the acceptable fiber length limitation shown in Fig. 11(b). So we consider this method could have significant improving effects on the FD-FIR filter equalization performance for short fibers.

Table 1. The relative error between continuous time window and discrete time window

<table>
<thead>
<tr>
<th>Fiber length (km)</th>
<th>20</th>
<th>20</th>
<th>20</th>
<th>320</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taps number</td>
<td>7</td>
<td>9*</td>
<td>33*</td>
<td>129*</td>
</tr>
<tr>
<td>Sampling rate (Sa/Sy)</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>T (ps)</td>
<td>17.9</td>
<td>17.9</td>
<td>8.9</td>
<td>4.5</td>
</tr>
<tr>
<td>T_W^A (ps)</td>
<td>144.2</td>
<td>144.2</td>
<td>288.4</td>
<td>576.7</td>
</tr>
<tr>
<td>T_N^A (ps)</td>
<td>125</td>
<td>160.7</td>
<td>294.6</td>
<td>575.9</td>
</tr>
<tr>
<td>(T_N^A-T_W^A)/T_W^A (%)</td>
<td>-13.3</td>
<td>11.46</td>
<td>2.18</td>
<td>-0.14</td>
</tr>
</tbody>
</table>

* means the limitation of the required tap number calculated in Eq. (22).

The improved method for 20 km fiber CD compensation using FD-FIR filter with different sampling rate is shown in Fig. 14. We find in Fig. 14(a) that the CD equalization performance shows an obvious improvement with the increment of the sampling rate, and the FD-FIR filter can equalize the CD in 20 km fiber entirely with 8 sampling points per symbol. The performance of BER with normalized time window (T_N^A/T_W^A) using FD-FIR filter is shown in Fig. 14(b), where a significant improvement can also be found. Meanwhile, we find that the FD-FIR filter performs better when the value of T_N^A/T_W^A is around 1.0, which is consistent with our preceding analysis.
Although this improved method increases the necessary tap number in the FD-FIR filter and the required sampling rate in the coherent transmission system, which may not be suitable for very long distance fibers and high speed communication systems, we could improve the FD-FIR filter to compensate the CD in short distance fibers significantly by increasing the ADC sampling rate. Meanwhile, we could also put an adaptive post-filter after the FD-FIR filter to compensate the penalty in CD equalization for short distance fibers. However, here we mainly concentrate on analyzing and comparing the inherent characteristics of the three digital filters in CD compensation. Therefore, we hope to find the reason and improvement method in terms of the intrinsic properties of the FD-FIR filter. The fiber lengths are usually no less than hundreds of kilometers in practical transmission systems, therefore, the FD-FIR filter can be applied reasonably for the CD equalization in the systems with 2 Sa/Sy ADC sampling rate.

The CD compensation results using different frequency domain equalization methods are illustrated in Fig. 15. The results refer to the CD equalization with 16 FFT-size for 20 km fiber and 512 FFT-size for 600 km fiber using OLS (BLU), OLA-BSZP and OLA-OSZP methods. The overlap size (or ZP) is all designated as half of the FFT-size. We can see that both of the OLA-ZP methods can provide the same acceptable performance as the OLS (BLU) method.
Figure 15. CD compensation results using OLS and OLA-ZP methods.

Figure 16 and Fig. 17 show the performance of CD compensation for 20 km and 40 km fibers using OLS (BLU) and OLA-ZP methods with different FFT-sizes and overlaps (or ZP), respectively. From Fig. 16 we can see that for a certain fiber length, the three FDEs can show stable and converged acceptable performance with the increment of the FFT-size. The critical FFT-size values (16 FFT-size for 20 km fiber and 32 FFT-size for 40 km fiber), actually indicate the required minimum overlap (or ZP) value which are 8 overlap (or ZP) samples for 20 km fiber and 16 overlap (or ZP) samples for 40 km fiber. The similar performance demonstrates that for a fixed overlap (or ZP) value, the maximum compensable dispersion in the OLS (BLU) method is the same in the OLA-ZP methods.

Figure 16. CD compensation using OLS and OLA-ZP methods with different FFT-sizes at OSNR 14.8 dB. The overlap is half of the FFT-size.
We have demonstrated that the overlap (or ZP) is the pivotal parameter in the FDE, and the FFT-size is not necessarily designated as double of the overlap (or ZP). Figure 17 illustrates that with a fixed FFT-size (4096 samples) the three FDEs are still able to work well for 4000 km fiber, provided the overlap (or ZP) is larger than 1152 samples (1152=4096×9/32), which indicates the required minimum overlap (or ZP) for 4000 km fiber.

![Figure 17. CD compensation for 4000 km fiber using OLS (BLU) and OLA-ZP methods with different overlaps at OSNR 14.8 dB. The FFT-size is 4096.](image)

### 4.4.2 Carrier phase estimation

1. Carrier phase estimation with three CD equalization methods

![Carrier phase estimation graph](image)
Figure 18. The one-tap NLMS phase estimation for different fiber length with inline DCF and digital CD compensation. (a) TX=4 MHz, LO=0 Hz, (b) TX=LO=2 MHz, (c) TX=0 Hz, LO=4 MHz.

Figure 18 shows the BER performance of the transmission system with different fiber length employing the optical and the digital dispersion compensation by further using a one-tap NLMS filter for phase noise compensation. Again the results are obtained under different combination of the transmitter laser and LO laser linewidths with the same summation. We can see clearly that influenced by the EEPN, the performance of FD-FIR equalization and BLU equalization reveals obvious fiber length dependence with the increment of LO laser linewidth. The OSNR penalty in phase noise compensation scales with the LO phase fluctuation and the accumulated dispersion.
This is in agreement with previous studies [41-46]. On the other hand, the dispersion equalization using the LMS filter shows almost the same behavior in the three cases. That is because the chromatic dispersion interplays with the phase noise of both TX and LO lasers simultaneously in the adaptive equalization. Moreover, Figure 18 also shows the LMS filter is less tolerant against the phase fluctuation than the other dispersion compensation methods when the one-tap NLMS carrier phase noise compensator is employed.

2. Evaluation of BER floor in the one-tap NLMS phase estimation with EEPN

![Figure 19. BER performance in NLMS-CPE for 2000 km fiber with FD-FIR dispersion equalization, T: theory, S: simulation. (a) different combination of TX and LO lasers linewidth with the same summation, (b) only TX laser phase noise.](image)
The performance of phase estimation using the one-tap NLMS filter with the FD-FIR dispersion equalization is compared with the theoretical evaluation using Eq. (31), as shown in Fig. 19. Figure 19(a) illustrates the numerical results for different combination of TX and LO lasers linewidths with the same summation. With the increment of OSNR value, the numerical simulation reveals the BER floor only influenced by the phase noise, which achieves a good agreement with the theoretical evaluation. Figure 19(b) denotes the results with only the analysis of TX laser phase noise, where a slight deviation is found between the simulation results and the theoretical analysis. It arises from the approximation in the analytical evaluation of the one-tap NLMS phase estimator in Eq. (31). It has been validated in our simulation work that the phase estimation with the BLU dispersion equalization performs closely the same behavior as the FD-FIR equalization.

3. Evaluation of BER floor in differential phase estimation with EEPN

The BER performance of the DQPSK coherent transmission system with the FD-FIR dispersion equalization is illustrated in Fig. 20. Figure 20(a) shows the simulation results for different combination of the TX and the LO lasers linewidths with the same summation, and Fig. 20(b) denotes the performance of the differential demodulation system with only the TX laser phase noise. It is found that the BER behavior in the DQPSK coherent system can achieve a good agreement with the theoretical evaluation in Eq. (32) for both Fig. 20(a) and Fig. 20(b). The consistence between simulation and theory in DQPSK demodulation is better than the one-tap NLMS phase estimation in the case of only TX laser phase noise.
In evaluating the BER floor of phase estimation, we have assumed that the intrinsic TX laser, LO laser phase noise and the EEPN are statistically independent. Obviously, the TX laser phase noise is independent from the LO laser phase noise and the EEPN. Here we mainly investigate the correlation between the intrinsic LO laser phase noise and the EEPN. The total phase noise variance in the coherent optical transmission system can be modified as

$$\sigma^2 = \sigma_{\text{TX}}^2 + \sigma_{\text{LO}}^2 + \sigma_{\text{EEP}}^2 + 2\rho \cdot \sigma_{\text{LO}} \sigma_{\text{EEP}}.$$  \hspace{1cm} (38)$$

where $\rho$ is the correlation coefficient between the intrinsic LO laser phase noise and the EEPN, and we have the absolute value $|\rho| \leq 1$.

We have implemented the numerical simulation in the DQPSK system for different combination of the intrinsic LO laser phase noise and the EEPN with the same summation, which is illustrated in Fig. 21(a). It can be found that the BER floor does not show tremendous variation due to the correlation between the LO laser phase noise and the EEPN. The BER floor reaches the lowest value at $\sigma_{\text{EEP}}^2 = 0.5\sigma_{\text{LO}}^2$, which corresponds to the maximum value of the term $|2\rho \cdot \sigma_{\text{LO}} \sigma_{\text{EEP}}|$. The cases for $\sigma_{\text{EEP}}^2 >> \sigma_{\text{LO}}^2$ and $\sigma_{\text{EEP}}^2 = 0$ correspond to the mutual term $|2\rho \cdot \sigma_{\text{LO}} \sigma_{\text{EEP}}| = 0$. From Fig. 21(a) we can find that $\rho$ is usually a negative value.

The absolute value of the correlation coefficient $|\rho|$ for different fiber length is illustrated in Fig. 21(b), in which we can see that the correlation coefficient $|\rho|$ determined from the numerical simulation achieves good agreement with the
theoretical approximation. With the increment of fiber length, the magnitude of correlation coefficient $|\rho|$ approaches zero rapidly. Consequently, we can neglect the correlation term in Eq. (38) when the fiber length is over 80 km. Therefore, the assumption in Eq. (34) is available when the fiber length exceeds 80 km in practical optical communication systems.

Figure 21. Phase noise correlation in DQPSK system with BLU dispersion equalization, T: theory, S: simulation. (a) BER performance in different combination of EEPN and LO phase noise with the same summation, (b) correlation coefficient for different fiber length.
Chapter 5

Conclusions

5.1 Summary of the dissertation work

In this dissertation, we present a comparative analysis of different digital filters for chromatic dispersion compensation and an analytical evaluation of carrier phase estimation with digital equalization enhanced phase noise. These investigations are performed in the 112-Gbit/s NRZ-PDM-QPSK coherent transmission system, which is realized in the VPI simulation platform.

In CD equalization, the LMS adaptive filter shows the best performance in terms of safety and stability. However, it requires slow iteration for guaranteed convergence, and also the tap weights update increases the computational complexity. The FD-FIR filter affords the simplest analytical tap weights specification with respect of equalizer specification. However, it does not show acceptable performance for short distance fibers. The blind look-up filter will be faster and much more computationally efficient from the aspect of speed and efficiency, especial for large fiber dispersion. However, its performance will degrade dramatically if the overlap in the equalizer does not reach the required minimum overlap size.

In the investigation of the EEPN in carrier phase recovery, the carrier phase estimation is implemented by using the one-tap normalized LMS filter. In the FD-FIR and the BLU dispersion equalization, the BER floor of the one-tap NLMS phase estimation with the enhanced phase noise is analytically evaluated, and the simulation results are compared to the differential phase detection system.

5.2 Summary of the appended papers

Paper I

A comparative analysis of three popular digital filters for chromatic dispersion compensation involving a time-domain least mean square adaptive filter, a time-domain fiber dispersion finite impulse response filter and a frequency-domain blind look-up filter, are applied to equalize the CD in a 112-Gbit/s NRZ-PDM-QPSK coherent transmission system in this paper. The characteristics of these filters are compared by evaluating their applicability for different fiber lengths, their usability for dispersion perturbations, and their computational complexity.

Paper II

We present a novel investigation on the enhancement of phase noise in coherent optical transmission system due to electronic chromatic dispersion compensation. Two types of equalizers, including the time domain FD-FIR filter and the frequency domain BLU filter are applied to mitigate the CD in the 112-Gbit/s PDM-QPSK transmission system. The BER floor in phase estimation using the optimized one-tap NLMS filter, and considering the equalization enhanced phase noise is evaluated
analytically including the correlation effects. The numerical simulations are implemented and compared with the performance of differential QPSK demodulation system.

**Paper III**

In this paper, an adaptive finite impulse response filter employing normalized LMS algorithm is developed for compensating the CD in a 112-Gbit/s PDM-QPSK coherent communication system. The principle of the adaptive normalized LMS algorithm for signal equalization is analyzed theoretically, and at the meanwhile, the taps number and the tap weights in the adaptive FIR filter for compensating a certain fiber dispersion are also investigated by numerical simulation. The CD compensation performance of the adaptive filter is analyzed by evaluating the behavior of the BER versus the OSNR, and the results are compared with other present digital filters.

**Paper IV**

The frequency domain equalizers employing two types of overlap-add zero-padding methods are applied to compensate the chromatic dispersion in the 112-Gbit/s NRZ-PDM-QPSK coherent transmission system. Simulation results demonstrate that the OLA-ZP methods can achieve the same acceptable performance as the overlap-save method. The required minimum overlap (or zero-padding) in the FDE is derived, and the optimum fast Fourier transform length to minimize the computational complexity is also analyzed.

**Paper V** (Not discussed in this thesis)

We evaluate the influence of laser phase noise for the coherent and the direct-detection orthogonal frequency division multiplexed (OFDM) systems employing radio frequency (RF) pilot tone phase noise cancellation. Novel analytical results for the common phase error and for the (modulation dependent) inter carrier interference are evaluated based upon Gaussian statistics for the laser phase noise. Numerical results are presented for OFDM systems with 4 and 16 PSK modulation, 200 OFDM bins and baud rate of 1 GS/s. It is found that about 225 km transmission is feasible for the coherent 4PSK-OFDM system over normal fiber.

**Paper VI**

Coherent optical receivers with digital filters can mitigate the impairments in optical transmission system. In this paper, an adaptive filter employing NLMS algorithm is developed for chromatic dispersion compensation in a 112-Gbit/s PDM-QPSK coherent communication system. The performance of the adaptive filter is analyzed by comparing with present digital filters.

**Paper VII**

In this paper, we demonstrate the chromatic dispersion equalization employing a time-domain FIR filter in a 112-Gbit/s PDM-QPSK coherent system. The required tap number of the filter is analyzed from anti-aliasing and pulse broadening. The dynamic
range of the filter is evaluated by using different number of taps.

5.3 Suggestions for our future work

In the CD compensation investigation, to demonstrate the fundamental features of the time-domain adaptive and fixed filters as well as the frequency-domain equalizers, different digital filters are applied to compensate the CD in the 112-Gbit/s NRZ-PDM-QPSK coherent optical transmission system. Besides the chromatic dispersion equalization, our analysis and comparison does not take into account the influences of PMD and fiber nonlinearities, only the carrier phase estimation is investigated. Future efforts should incorporate comparing the CD equalization performance of these methods in the return-to-zero (RZ) and the NRZ polarization division multiplexed QPSK transmission systems, as well as compensating the PMD and the fiber nonlinearities in such coherent systems. Furthermore, these CD compensation methods should be studied for the utilization in the QAM coherent systems.

In the carrier phase recovery investigation, three electronic dispersion equalizers are applied to compensate the CD in the coherent optical transmission system to investigate the impact of the dispersion equalization enhanced phase noise. The carrier phase estimation is implemented by using the one-tap normalized LMS filter, and the simulation results are compared to the differential phase detection system. Further investigation will involve the evaluation of the BER floor in phase estimation with LMS adaptive equalization, which is rather complicated due to the equal enhancement of both TX and LO lasers phase noise. Moreover, the effects of EEPN on different phase estimation algorithms in coherent transmission system will be also studied in the future investigations.
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**Acronyms**

ADCs: analog-to-digital convertors  
ASE: amplified spontaneous emission  
BER: bit-error-rate  
BLU: blind look-up  
CD: chromatic dispersion  
CMA: constant modulus algorithms  
CPE: carrier phase estimation  
DCFs: dispersion compensating fibers  
DCMs: dispersion compensating modules  
DGD: differential group delay  
DQPSK: differential quadrature phase shift keying  
DSP: digital signal processing  
EDFAs: erbium-doped fiber amplifiers  
EEPN: equalization enhanced phase noise  
FDEs: frequency domain equalizers  
FD-FIR: fiber dispersion finite impulse response  
FFT: fast Fourier transform  
FWM: four-wave mixing  
GVD: group velocity dispersion  
IF: intermediate frequency  
IFFT: inverse fast Fourier transform  
IMDD: intensity modulation direct detection  
IQ: in-phase and quadrature  
ISI: inter-symbol interference  
LMS: least mean square  
LO: local oscillator  
MLSE: maximum likelihood sequence estimation  
NLMS: normalized least mean square  
NLSE: nonlinear Schrödinger equation  
NRZ: non-return-to-zero  
OLA: overlap-add  
OLA-BSZP: overlap-add both-side zero-padding  
OLA-OSZP: overlap-add one-side zero-padding  
OLA-ZP: overlap-add zero-padding  
OLS: overlap-save and the methods  
OSNR: optical signal-to-noise ratio  
PBC: polarization beam combiner  
PDM: polarization division multiplexed  
PLL: phase-locked loop  
PMD: polarization mode dispersion  
PN: phase noise
PRBS: pseudo random bit sequence
PSK: phase-shift keying
QAM: quadrature amplitude modulation
QPSK: quadrature phase shift keying
SNR: signal-to-noise ratio
SOP: state of polarization
SPM: self-phase modulation
SSMF: standard single mode fiber
TX: transmitter
VLSI: very large-scale integration
WDM: wavelength-division multiplexed
XPM: cross-phase modulation
Published Papers
Chromatic dispersion compensation in coherent transmission system using digital filters

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Abstract: We present a comparative analysis of three popular digital filters for chromatic dispersion compensation: a time-domain least mean square adaptive filter, a time-domain fiber dispersion finite impulse response filter, and a frequency-domain blind look-up filter. The filters are applied to equalize the chromatic dispersion in a 112-Gbit/s non-return-to-zero polarization division multiplexed quadrature phase shift keying transmission system. The characteristics of these filters are compared by evaluating their applicability for different fiber lengths, their usability for dispersion perturbations, and their computational complexity. In addition, the phase noise tolerance of these filters is also analyzed.

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14. www.vpiphotonics.com
1. Introduction

The performance of high speed optical fiber transmission systems is severely affected by chromatic dispersion (CD) and polarization mode dispersion (PMD) as well as phase noise (PN) [1,2]. Coherent optical receivers employing digital filters allow significant equalization of chromatic dispersion in the electrical domain, instead of compensation by dispersion compensating fibers (DCF) or dispersion compensating modules (DCM) in the optical domain [3–10]. Several digital filters have been applied to compensate the CD in the time and the frequency domain [6–13]. H. Bülow and A. Färbert et al. have reported their CD equalization work using the maximum likelihood sequence estimation (MLSE) method [4,8]. S. J. Savory used a time-domain fiber dispersion finite impulse response (FD-FIR) filter to compensate the CD in 1000 km and 4000 km transmission fibers without using dispersion compensation fibers [10,11]. M. Kuschnerov and F. N. Haukse et al. have used the frequency domain equalizers (FDEs) to compensate the CD in coherent communication systems [12,13]. With the advent of these digital signal processing (DSP) techniques for CD compensation, the dream of optical transmission without any DCFs appears to become true. However, these reported digital filters have their own characteristics, and it is necessary to clearly understand the limitations of their utilizations.

In this paper, the time-domain least mean square (LMS) adaptive filter is demonstrated to compensate CD in a 112-Gbit/s non-return-to-zero polarization division multiplexed quadrature phase shift keying (NRZ-PDM-QPSK) coherent optical transmission system, which is realized in the VPI simulation platform [14]. Comparatively, the time-domain FD-FIR filter and the frequency-domain blind look-up (BLU) filter are also applied for CD equalization in this transmission system [10–13]. The principle of the LMS algorithm and the influence of the step size on the tap weights convergence in the adaptive filter are analyzed, and the structure of the FD-FIR filter and the BLU filter are also studied. Meanwhile, the effects of tap number in the LMS and the FD-FIR filters, as well as the FFT-size and overlap samples in the BLU filter are investigated for equalizing the fiber dispersion. We also present the detailed investigation in CD compensation using the three digital filters for different fiber lengths and illustrate their performance in the existence of dispersion perturbation due to the temperature variation. It is
found that the time-domain FD-FIR filter does not work well when the fiber length is shorter than a certain distance. The corresponding reason and an improvement method are explained and developed. The optimum selection of tap number in FD-FIR filter are analyzed and discussed. The computational complexity of the three algorithms is also compared. Furthermore, the phase noise compensation (PNC) after the CD equalization using the three filters is also analyzed. Based on this work, the characteristics of the three digital filters are analyzed and illustrated, and we can find the optimum approach and effective usage for each digital filter in CD equalization.

The characteristics of a normalized LMS (NLMS) adaptive filter were preliminarily analyzed by comparing with the FD-FIR filter and the blind look-up filter in our previous work [15]. However, the NLMS adaptive filter, acting as a band-limit filter, can meliorate the back-to-back result in the coherent transmission system, and make it converge to a back-to-back result with a 3-tap NLMS post-filter. In order to be comparable to the behavior of NLMS filter, the back-to-back result and the CD equalization performance of FD-FIR filter and BLU filter are processed by a post-added 3-tap NLMS equalizer. Hence, some peculiar characteristics of the time-domain adaptive filter and fixed FD-FIR filter as well as the frequency-domain BLU filter are concealed and obscured. While in this paper, the optical and electrical filters in the coherent transmission system have been designated with the optimal bandwidths. Meanwhile, the LMS adaptive filter which we employ here can guarantee the back-to-back results to be identical with and without a 3-tap LMS post-equalizer. Therefore, the true features of the LMS adaptive filter and FD-FIR filter as well as the blind look-up filter can be investigated and compared directly.

2. Digital signal processing modules in coherent receiver

The schematic of the DSP modules in digital coherent receiver for linear fiber impairments is illustrated in Fig. 1. The bit-error-rate (BER) is calculated through digital signal processing in five steps: CD compensation using FIR filters, adaptive PMD and polarization rotation (PR) equalization, carrier phase estimation (PE) for phase noise compensation, adaptive equalization for residual dispersion and symbol identification.

Fig. 1. Schematic of DSP modules in coherent receiver. The adaptive equalization is validated, when fixed CD filters are employed.

Generally, the LMS-based adaptive equalizer can compensate the carrier phase noise and the residual dispersion simultaneously. However, it only works satisfactorily when the phase fluctuation is small enough. The phase correlation between symbols with a long delay interval will fade out due to the large phase fluctuations occurring within the filter delay time. When we employ a single high-order adaptive filter for equalizing both PN and residual CD, the performance of the adaptive filter will degrade with the increment of the filter delay. Therefore, the function is realized by two steps, a one-tap NLMS filter for phase estimation and a multiple-tap LMS filter for residual dispersion equalization [16].
Our work focuses on analyzing and comparing the three DSP techniques for CD equalization. Meanwhile, the adaptive mitigation for PMD and polarization rotation is theoretically analyzed, and the phase noise compensation using an NLMS filter is numerically simulated. The necessity of the adaptive filter for residual dispersion is also discussed, when the fixed CD equalizer is used.

The current investigation does not consider the fiber nonlinear effects, since the DSP compensation for nonlinear effects represents a separate and highly involved study area which is beyond the scope of this paper.

3. Principle and Structures of Three Digital Filters for CD Equalization

3.1 Least Mean Square Adaptive Filter

The LMS filter employs an iterative algorithm that incorporates successive corrections to weights vector in the negative direction of the gradient vector which eventually leads to a minimum mean square error [17]. The principle of LMS filter is given by the following equations:

\[
y(n) = \mathbf{\hat{w}}^H(n) \mathbf{x}(n) \tag{1}
\]

\[
\mathbf{\hat{w}}(n+1) = \mathbf{\hat{w}}(n) + \mu \mathbf{x}(n) e^*(n) \tag{2}
\]

\[
e(n) = d(n) - y(n) \tag{3}
\]

where \( \mathbf{x}(n) \) is the digitalized complex magnitude vector of the received signal, \( y(n) \) is the complex magnitude of the equalized output signal, \( n \) represents the number of sample sequence, \( \mathbf{\hat{w}}(n) \) is the complex tap weights vector, \( \mathbf{\hat{w}}^H(n) \) is the Hermitian transform of \( \mathbf{\hat{w}}(n) \), \( d(n) \) is the desired symbol, which corresponds to one case of the vector \([1+i -1-i -1-i -1+i]\) for the QPSK coherent transmission system, \( e(n) \) represents the estimation error between the output signal and the desired symbol, \( e^*(n) \) is the conjugation of \( e(n) \), and \( \mu \) is a key real coefficient called step size. The tap weights vector \( \mathbf{\hat{w}}(n) \) is updated in a symbol-by-symbol iterative manner, and achieves their convergence when \( e(n) \) approaches zero.

In order to guarantee the convergence of tap weights vector \( \mathbf{\hat{w}}(n) \), the step size \( \mu \) needs to satisfy the condition of \( 0 < \mu < 1/\lambda_{\text{max}} \), where \( \lambda_{\text{max}} \) is the largest eigenvalue of the correlation matrix \( R = \mathbf{x}(n) \mathbf{x}^H(n) \) [17]. If the step size \( \mu \) is chosen to be very small, then the algorithm converges very slowly. A large value of \( \mu \) could lead to a faster convergence, but the algorithm will be less stable and safe, because sometimes the step size \( \mu \) may be larger than \( 1/\lambda_{\text{max}} \). An efficient method to designate the step size \( \mu \) is to make it change with the time-dependent largest eigenvalue \( \lambda_{\text{max}} \), and this will lead to a fast and stable convergence, which refers to the variable-step-size LMS algorithm [17]. However, it is required to check the value of \( \lambda_{\text{max}} \) in each step, and so it will be cumbersome and computationally inefficient. In our simulation, we have chosen the step size to be a fixed small value, which can guarantee the convergence of the LMS filter.
The tap weights in LMS adaptive equalizer for 20 km fiber CD compensation is shown in Fig. 2. The convergence for 9 tap weights in the LMS filter with step size equal to 0.1 is shown in Fig. 2(a), and we can find the tap weights obtain their convergence after about 5000 iterations. Note that the convergence speed of the LMS algorithm will slightly slow down with the increment of the tap number for the same fiber dispersion. About 8000 iterations are required to obtain the guaranteed convergence when 21 taps are used in the LMS filter for equalizing 20 km fiber dispersion. The magnitudes of converged tap weights are shown in Fig. 2(b), and it can be found that the central tap weights take more dominant roles. For a fixed fiber dispersion, the tap weights in LMS adaptive filter approach zero, when the corresponding taps order exceeds a certain value, and this value indicates the least required taps number for compensating the CD effectively, which is 9 taps for 20 km fiber shown in this figure. This illustrates the optimization characteristic of the LMS adaptive algorithm.

3.2 Fiber dispersion FIR filter

Compared with the iteratively updated LMS filter, the tap weights in FD-FIR filter have a relatively simple specification [10,11], the tap weight $a_k$ in FD-FIR filter is given by the following equations:

$$a_k = \frac{je^{T^2}}{D\sqrt{\lambda^2 z}} \exp \left( -j \frac{\pi c T^2}{D\lambda^2 z} k \right) - \left\lceil \frac{N}{2} \right\rceil \leq k \leq \left\lfloor \frac{N}{2} \right\rfloor$$  \hspace{1cm} (4)

$$N^4 = 2 \times \left\lceil \frac{D\lambda^2 z}{2cT^2} \right\rceil + 1$$  \hspace{1cm} (5)

where $D$ is the fiber chromatic dispersion coefficient, $\lambda$ is the central wavelength of the transmitted optical wave, $z$ is the fiber length in the transmission channel, $T$ is the sampling period, $N^4$ is the required maximum tap number for compensating the fiber dispersion, and $\lfloor x \rfloor$ denotes the nearest integer less than $x$.

The tap weights of FD-FIR filter according to Eq. (4) for 20 km fiber ($D = 16 \text{ ps/nm/km}$) are shown in Fig. 3, which is the same with the one illustrated in our previous work [15]. For a fixed fiber dispersion, the magnitudes of tap weights in FD-FIR filter are constant, whereas the real and the imaginary parts vary periodically. The FD-FIR filter does not show the optimization characteristic, because excessive tap weights do not approach zero without window truncation. It is noted that the tap weights in Fig. 3 should be truncated properly for practical application. To avoid aliasing, the Nyquist frequency is chosen as the boundary of the frequency window in FD-FIR filter, namely the above tap weights are convolved with a sinc function in the time domain, and the redundant tap weights can be eliminated effectively.
To avoid the aliasing, the pass-band of the filter needs to be lower than the Nyquist frequency \([10,11]\), and so the continuous time window of the FD-FIR filter is limited by the following inequality:

\[
\frac{|P|e^{-\frac{\lambda^2 z}{2e^T}}}{2e^T} \leq T \leq \frac{|P|e^{-\frac{\lambda^2 z}{2e^T}}}{2e^T}
\] (6)

where the variables description is the same as the description in Eq. (4) and Eq. (5).

So the continuous time window \(T_w^a\) in the FD-FIR filter can be obtained from Eq. (6) as

\[
T_w^a = \frac{2|P|e^{-\frac{\lambda^2 z}{2e^T}}}{2e^T} = \frac{|P|e^{-\frac{\lambda^2 z}{2e^T}}}{e^T}.
\] (7)

The required tap weights number \(N^a\) is calculated from the continuous time window quantized by the sampling period \(T\) as

\[
N^a = 2 \times \left\lfloor \frac{T_w^a}{(2T)} \right\rfloor + 1.
\] (8)

Once the time window is determined, we can employ a Kaiser window to truncate the FD-FIR filter in the time domain for practical application \([10,11]\).

The above analysis of window width in FD-FIR is indeed rational and correct by using the anti-aliasing approach \([10,11]\). From a physical aspect, it will be also reasonable to determine the filter length based on the broadening of an pulse propagating in the dispersive fiber channel \([2,18]\). Assuming the signal is a Gaussian pulse, the width of the filter can be calculated according to the broadened pulse duration \([2,18]\), which is given by

\[
T_w^p = \frac{2}{\pi c^2 T} \sqrt{\pi c^2 T^2 + 4\lambda^2 D^2 z^2}
\] (9)

where the parameter descriptions are the same with Eq. (4) and Eq. (5). The corresponding tap number is obtained as

\[
N^p = 2 \times \left\lfloor \frac{T_w^p}{2T} \right\rfloor + 1 = 2 \times \left\lfloor \frac{1}{\pi c^2} \sqrt{\pi c^2 T^2 + 4\lambda^2 D^2 z^2} \right\rfloor + 1
\] (10)

where \(\left\lfloor x \right\rfloor\) denotes the nearest integer larger than \(x\).

The tap numbers for different fiber length calculated according to pulse broadening are illustrated in Table 1, and compared with the tap numbers calculated from the analysis of anti-aliasing.
Table 1. The tap number in FD-FIR filter calculated by pulse broadening and anti-aliasing

<table>
<thead>
<tr>
<th>Fiber length (km)</th>
<th>Tap number determined by pulse broadening (N_P)</th>
<th>Tap number determined by anti-aliasing (N_A)</th>
<th>N_P/N_A (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>7</td>
<td>9</td>
<td>77.8</td>
</tr>
<tr>
<td>100</td>
<td>27</td>
<td>41</td>
<td>65.9</td>
</tr>
<tr>
<td>600</td>
<td>157</td>
<td>243</td>
<td>64.6</td>
</tr>
<tr>
<td>1000</td>
<td>259</td>
<td>403</td>
<td>64.3</td>
</tr>
<tr>
<td>2000</td>
<td>517</td>
<td>807</td>
<td>64.1</td>
</tr>
<tr>
<td>5000</td>
<td>1287</td>
<td>2109</td>
<td>63.7</td>
</tr>
</tbody>
</table>

We find that most of the tap numbers calculated from pulse broadening are around 60% of the value calculated from the anti-aliasing analysis, which is consistent with the empirical factor in the reported results [10]. This number could be considered as a lower bound on the required number of taps for an effective CD compensation.

3.3 Blind look-up filter

The function of the blind look-up filter comprises two steps [12,13]. The first step is to utilize a blind adaptive FDE to estimate the accumulated chromatic dispersion in the transmission system, which has already been introduced and analyzed previously [15]. The second step is to achieve the CD equalization using an overlap-save fast Fourier transform (FFT) method [12,13,19–22], assuming the amount of dispersion has been estimated precisely. The structure of the overlap-save BLU equalizer is illustrated in Fig. 4. The time domain received digital signals are firstly divided into several sequence blocks with an overlap, where the block length is called the FFT-size. Then the sequence in each block are transformed into frequency domain data by the FFT operation, and afterwards multiplied by the inverse transfer function of the dispersive channel in the frequency domain. Furthermore, the data sequences are transformed into time domain signals by the inverse FFT (IFFT) operation, and finally the processed sequence blocks are combined together by the superposition with overlap samples. The recomposed data are the equalized output signal sequence. The zero-padded overlap-add method, which is another FFT convolution algorithm, can also be used for equalizing the chromatic dispersion in frequency domain [19,20].

![Fig. 4. Schematic of blind look-up adaptive filter.](image)

It should be noted that the value of overlap is the pivotal parameter determined by the dispersion to be equalized [22], and the FFT-size is configurable provided it is larger than the overlap. For a proper overlap value, a large FFT-size may be more efficient, however, it will cost more memory resource of the hardware. In our work, the FFT-size is designated as the double of the overlap, so that BLU filter can be applied conveniently for equalizing different fiber dispersion only by determining the required FFT-size.

The transfer function of the frequency domain equalizer is given by

\[ G(z, \omega) = \exp \left( -j \frac{D \lambda^2 z}{4 \pi c} \omega^2 \right) \] (11)
where the parameter definitions are also the same with Eq. (4) and Eq. (5).

The pass-band of the BLU filter is determined by the Nyquist frequency, which is also given by

$$-\omega_N \leq \omega_{BLU} \leq \omega_N$$  

(12)

where \( \omega_N \) is the Nyquist angular frequency of the system.

The tap weights specification in the blind look-up filter can be expressed as

$$b_k = \exp \left[ -j \frac{D \lambda^2 z}{\pi c} \left( \frac{k}{N_{FFT}} \omega_N \right)^2 \right] - \frac{N_{FFT}}{2} \leq k \leq \frac{N_{FFT}}{2} - 1$$  

(13)

where \( N_{FFT} \) is the FFT-size of the frequency domain equalizer.

In terms of the above analysis, the angular-frequency pass-band (AFPB) of BLU filter \( \omega_{BLU}^{PB} \) is a fixed value, whereas the AFPB of FD-FIR filter \( \omega_{FD-\text{FIR}}^{PB} = 2 \cdot \frac{\pi c}{D \lambda^2 z} \cdot N^A \cdot T \) 

varies with the tap number \( N^A \). This is the fundamental reason for the different CD equalization performance between the FD-FIR filter and the BLU filter.

4. Principle of equalization for polarization dependent impairments and phase noise

4.1 Adaptive PMD and polarization rotation equalization

The influence of PMD and polarization fluctuation can be compensated adaptively by the decision-directed LMS filter [17,23], which is expressed as the following equations:

$$\begin{bmatrix}
    x_{in}(n) \\
    y_{in}(n)
  \end{bmatrix} =
\begin{bmatrix}
    \rightarrow^H \omega_x(n) & \rightarrow^H \omega_y(n) \\
    \rightarrow^H \omega_x(n) & \rightarrow^H \omega_y(n)
  \end{bmatrix}
\begin{bmatrix}
    x_{out}(n) \\
    y_{out}(n)
  \end{bmatrix}$$  

(14)

$$\begin{align}
  \omega_x(n+1) &= \omega_x(n) + \mu_e \varepsilon_x(n) \omega_x(n) \\
  \omega_y(n+1) &= \omega_y(n) + \mu_e \varepsilon_y(n) \omega_y(n) \\
  \varepsilon_x(n) &= d_x(n) - x_{out}(n) \\
  \varepsilon_y(n) &= d_y(n) - y_{out}(n)
\end{align}$$  

(15)

where \( \omega_x(n) \) and \( \omega_y(n) \) are the complex magnitude vectors of the input signals, \( x_{in}(n) \) and \( y_{in}(n) \) are the complex magnitudes of the equalized output signals respectively, \( \rightarrow w_x \), \( \rightarrow w_y \) are the complex tap weights vectors, \( d_x(n) \) and \( d_y(n) \) are the desired symbols, \( \varepsilon_x(n) \) and \( \varepsilon_y(n) \) represent the estimation errors between the output signals and the desired symbols respectively, and \( \mu_e \) is the step size parameter. The polarization diversity equalizer can be implemented subsequent to the CD compensation.
4.2 Normalized LMS filter for phase estimation

The one-tap NLMS filter can be employed for phase estimation [16,17], of which the tap weight is expressed as

\[ w_{NLMS}(n+1) = w_{NLMS}(n) + \frac{\mu_{PN}}{|x_{PN}(n)|} x_{PN}^*(n) \zeta(n) \]  
\[ \zeta(n) = d_{PN}(n) - w_{NLMS}(n) \cdot x_{PN}(n) \]  

where \( w_{NLMS}(n) \) is the complex tap weight, \( x_{PN}(n) \) is the complex magnitude of the input signal, \( d_{PN}(n) \) is the desired symbol, \( \zeta(n) \) represents the error between the output signals and the desired symbols, and \( \mu_{PN} \) is the step size parameter.

5. Simulation investigation of PDM-QPSK transmission system

The setup of the 112-Gbit/s NRZ-PDM-QPSK coherent transmission system established in the VPI simulation platform is illustrated in Fig. 5. The data sequence output from the four 28-Gbit/s pseudo random bit sequence (PRBS) generators are modulated into two orthogonally polarized NRZ-QPSK optical signals by the two Mach-Zehnder modulators. Then the orthogonally polarized signals are integrated into one fiber channel by a polarization beam combiner (PBC) to form the 112-Gbit/s NRZ-PDM-QPSK optical signal. Using a local oscillator (LO) in the coherent receiver, the received optical signals are mixed with the LO laser to be transformed into four electrical signals by the photodiodes. Then they are digitalized by the 8-bit analog-to-digital convertors (ADCs) at twice the symbol rate [24]. The sampled signals are processed by the digital equalizer, and the BER is then estimated from the data sequence of 2^{16} bits. The central wavelength of the transmitter laser and the LO laser are both 1553.6 nm. The standard single mode fibers (SSMFs) with the CD coefficient equal to 16 ps/nm/km are employed in all the simulation work.

Fig. 5. Schematic of 112-Gbit/s NRZ-PDM-QPSK coherent optical transmission system. PBS: polarization beam splitter, MZI: Mach-Zehnder interferometer, OBPF: optical band-pass filter, PIN: PiN diode, LPF: low-pass filter.

Here we mainly concentrate our work on the CD compensation methods in DSP techniques, and so we neglected the influences of fiber attenuation, polarization mode dispersion and nonlinear effects in the comparison of the three filters. The PMD and polarization rotation
equalization could be realized by employing the adaptive LMS filter as we mentioned above. The carrier phase estimation can be implemented by using a one-tap NLMS adaptive filter [16], which will be analyzed in our simulations afterwards.

With the increment of launched optical power, the fiber nonlinearities such as self-phase modulation (SPM), cross-phase modulation (XPM) and four-wave mixing (FWM) need to be considered in the long-haul wavelength-division multiplexing (WDM) transmission systems [25]. The fiber nonlinear impairments can be mitigated and compensated by using the digital backward propagation methods based on solving the nonlinear Schrodinger (NLS) equation and the Manakov equation [26,27].

6. Simulation results

6.1 Static chromatic dispersion equalization

The CD compensation results using three digital filters are illustrated in Fig. 6. Figure 6(a) indicates the CD equalization with 9 taps for 20 km fiber and 243 taps for 600 km fiber using the LMS and the FD-FIR filters, as well as 16 FFT-size (8 overlap) for 20 km fiber and 512 FFT-size (256 overlap) for 600 km fiber using the BLU filter. Obviously, the FD-FIR filter is not able to compensate the CD in 20 km fiber entirely. About 3 dB optical signal-to-noise ratio (OSNR) penalty from the back-to-back result at BER equal to $10^{-3}$ can be observed. This phenomenon contradicts with our previous result [15], because the additional 3-taps NLMS equalizer in previous simulation has mitigated the CD equalization penalty of the FD-FIR filter for short fibers. Here we can observe and analyze the true features of the three digital filters without any interference. Then we investigate the CD compensation for different fiber lengths using the three filters, which are shown in Fig. 6(b). It can be found that the LMS filter and the BLU filter show the same acceptable performance for different fiber lengths, while the FD-FIR filter will not behave satisfactorily until the fiber length exceeds 320 km.

The CD equalization for 20 km and 600 km fibers using the LMS filter and the FD-FIR filter with different number of taps are shown in Fig. 7. Due to the optimum characteristic of LMS algorithm, the LMS filter has a slight improvement with the increment of tap number. However, the performance of the FD-FIR filter will degrade, when the tap number increases and exceeds the required tap number in Eq. (5). It is because the redundant taps will lead to the pass-band of the filter exceeding the Nyquist frequency, which will further result in the aliasing phenomenon. Here we should note that the redundant taps refers to no consideration of window truncation. We also find in Fig. 7(a) that the FD-FIR filter does not achieve a satisfactory CD equalization performance for 20 km fiber by using either the tap number ($N_A = 9$) analyzed by anti-aliasing or the tap number ($N_p = 7$) determined by pulse broadening.
From the above description, the FD-FIR filter does not achieve an acceptable CD equalization performance for short distance fibers, but it can work well for long fibers. When we use a series of delayed taps to approximate the filter time window $T_{W}^A$, the digitalized discrete time window $T_{N}^A$ could not attain exactly the same value as the continuous time window $T_{W}^A$, which is illustrated in Fig. 8. The malfunction of FD-FIR filter for short fibers arises from this reason, and now we provide a more detailed explanation.

We calculate the relative error $p$ of time window to evaluate the precision of time window approximation, which is given by

$$p = \frac{\left(T_{N}^A - T_{W}^A\right)}{T_{W}^A}.$$  \hspace{1cm} (19)

According to Eq. (7), a short fiber will have a relative small time window to keep the signal bandwidth to be lower than Nyquist frequency to avoid the aliasing phenomenon. However, such a small time window is not easy to be digitalized accurately with a fixed sampling period $T$. In order to broaden the time window, we need to raise the Nyquist frequency correspondingly. The Nyquist frequency is defined as half of the sampling frequency of the system, and this means we need to increase the sampling rate in the ADC modules. With sampling rate being increased, the Nyquist frequency are also raised, meanwhile, the sample period $T$ is reduced, which allows the broadened continuous time window to be digitalized more precisely.

The relative errors of time window for different fiber length with different sampling rate are shown in Table 2, where the positive time error means the aliasing occurring. We could find the relative error of time window for 20 km is reduced obviously with the sampling rate changing from 2 samples per symbol (Sa/Sy) to 8 Sa/Sy. Furthermore, the time error for 20 km fiber with 8 Sa/Sy is equal to the time error for 320 km fiber with 2 Sa/Sy, which is the acceptable fiber length limitation shown in Fig. 6(b). So we consider this method could have significant improving effects on the FD-FIR filter equalization performance for short fibers.

The improved method for 20 km fiber CD compensation using FD-FIR filter with different sampling rate is shown in Fig. 9. We find in Fig. 9(a) that the CD equalization performance shows an obvious improvement with the increment of the sampling rate, and the FD-FIR filter can equalize the CD in 20 km fiber entirely with 8 sampling points per symbol. The
performance of BER with normalized time window \( (T_N^/T_W^) \) using FD-FIR filter is shown in Fig. 9(b), where a significant improvement can also be found. Meanwhile, we find that the FD-FIR filter performs better when the value of \( T_N^/T_W^ \) is around 1.0, which is consistent with our preceding analysis.

<table>
<thead>
<tr>
<th>Fiber length (km)</th>
<th>20</th>
<th>20</th>
<th>20</th>
<th>20</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taps number</td>
<td>7</td>
<td>9*</td>
<td>33*</td>
<td>129*</td>
<td>129*</td>
</tr>
<tr>
<td>Sampling rate (Sa/Sy)</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>T (ps)</td>
<td>17.9</td>
<td>17.9</td>
<td>8.9</td>
<td>4.5</td>
<td>17.9</td>
</tr>
<tr>
<td>T_W (ps)</td>
<td>144.2</td>
<td>144.2</td>
<td>288.4</td>
<td>576.7</td>
<td>2306.8</td>
</tr>
<tr>
<td>T_N (ps)</td>
<td>125</td>
<td>160.7</td>
<td>294.6</td>
<td>575.9</td>
<td>2303.6</td>
</tr>
<tr>
<td>((T_N - T_W)/T_W) (%)</td>
<td>-13.3</td>
<td>11.46</td>
<td>2.18</td>
<td>-0.14</td>
<td>-0.14</td>
</tr>
</tbody>
</table>

* means the limitation of the required tap number calculated in Eq. (5).

It can also be found both in Fig. 7 and Fig. 9(b) that the FD-FIR filter does not necessarily obtain the best CD equalization performance when using the maximum tap number calculated in Eq. (5). In Fig. 7, the calculated tap numbers from Eq. (5) are 9 taps for 20 km fiber and 243 taps for 600 km fiber. For a fixed sampling rate, two main aspects need to be taken into consideration in selection of the practical tap number in FD-FIR filter. One is to keep the system bandwidth lower than Nyquist frequency to avoid aliasing, which is the smaller the better, and the other is to equalize the CD influence in the received signal, which is the larger the better. To obtain the best BER performance, we need to incorporate the two factors, and the optimum tap number is between the maximum number calculated from Eq. (5) and the minimum number calculated from Eq. (10).

Fig. 9. CD compensation using FD-FIR filter with different sampling rate. (a) BER with OSNR. (b) BER with normalized time window at OSNR 14.8 dB.

Although this improved method increases the necessary tap number in the FD-FIR filter and the required sampling rate in the coherent transmission system, which may not be suitable for very long distance fibers and high speed communication systems, we could improve the FD-FIR filter to compensate the CD in short distance fibers significantly by increasing the ADC sampling rate. Meanwhile, we could also put an adaptive post-filter after the FD-FIR filter to compensate the penalty in CD equalization for short distance fibers. However, here we mainly concentrate on analyzing and comparing the inherent characteristics of the three digital filters in CD compensation. Therefore, we hope to find the reason and improvement method in terms of the intrinsic properties of the FD-FIR filter. The fiber lengths are usually no less than hundreds of kilometers in practical transmission systems, therefore, the FD-FIR filter can be applied reasonably for the CD equalization in the systems with 2 Sa/Sy ADC sampling rate.
The performance of CD equalization using blind look-up filter with different FFT-size and overlap are illustrated in Fig. 10. With the increment of the FFT-size, the BLU filter can show stable and converged acceptable performance for a certain fiber length. Note that the FFT-size is designated as twice of the overlap in Fig. 10(a). The critical FFT-size values (16 FFT-size for 20 km fiber and 32 FFT-size for 40 km fiber), actually indicate the required minimum overlap value which are 8 overlap samples for 20 km fiber and 16 overlap samples for 40 km fiber. It is also found that the performance of the BLU filter reveals dramatic degradation as the overlap size decreases. We have demonstrated that the overlap is the pivotal parameter in the BLU filter, and the FFT-size is not necessarily designated as double of the overlap. Figure 10(b) illustrates that with a fixed FFT-size (4096 samples) the BLU filter could work well for 4000 km fiber, provided the overlap is larger than 1152 samples (1152 = 0.28125 × 4096), which indicates the required minimum overlap for 4000 km fiber.

Table 3. The minimum overlap size in BLU filter calculated according to pulse broadening

<table>
<thead>
<tr>
<th>Fiber length (km)</th>
<th>Overlap by pulse broadening</th>
<th>Overlap determined by CD equalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>40</td>
<td>14</td>
<td>16</td>
</tr>
<tr>
<td>600</td>
<td>158</td>
<td>176</td>
</tr>
<tr>
<td>4000</td>
<td>1032</td>
<td>1152</td>
</tr>
</tbody>
</table>

Actually, the minimum overlap in BLU filter can be calculated from the pulse width broadening [18,22], as illustrated in Table 3. We observe that the calculated overlap values are in accord with the values determined by the CD equalization performance in our work.

6.2 Dynamic chromatic dispersion equalization

The performance of the three filters for 2000 km fiber CD compensation in a dynamic dispersive system with an approximate ±0.2% chromatic dispersion variation (due to the change of temperature) are shown in Fig. 11. We can see that except the LMS adaptive filter, the fixed FD-FIR and BLU filters will degrade due to the existence of the small dispersion variation. So it is necessary to add a post LMS adaptive filter (with 5 ~7 taps) after the fixed CD equalizer to compensate the residual dispersion in the practical coherent optical transmission systems. We have verified that the equalization penalty of FD-FIR and BLU filters in this 2000 km fiber transmission system with ±0.2% CD perturbation can be entirely compensated by a 5-tap LMS adaptive filter.
6.3 Computational complexity of the three filters

We have also investigated the complexity of the three digital filters, and we evaluated the computational complexity in terms of complex multiplications per transmitted bit (Mults/bit) [28,29], which could be calculated as the following equations:

\[
C_{\text{LMS}} = \frac{L_{\text{CD}}(1 + n_{\text{SC}})}{\log_2(M)}
\]

(20)

\[
C_{\text{FD-FIR}} = \frac{L_{\text{CD}} \cdot n_{\text{SC}}}{2\log_2(M)}
\]

(21)

\[
C_{\text{BLU}} = \frac{1 + \log_2\left(\frac{N_{\text{FFT}}}{M}ight)}{\log_2(M) \cdot (1 - L_{\text{CD}}/N_{\text{FFT}})}
\]

(22)

where \(C_{\text{LMS}}, C_{\text{FD-FIR}}\) and \(C_{\text{BLU}}\) are the computational complexity of the three filters respectively, \(L_{\text{CD}}\) is the filter length for CD equalization, \(M\) is the number of points in signal constellation, \(n_{\text{SC}}\) is the oversampling ratio in samples per symbol, \(N_{\text{FFT}}\) is the length of FFT operation in the BLU filter. In the 112-Gbit/s NRZ-PDM-QPSK transmission system with a 2 Sa/Sy sampling rate at the ADC modules, we have \(M = 4\) and \(n_{\text{SC}} = 2\) in the above formulas.

<table>
<thead>
<tr>
<th>Fiber dispersion (ps/nm)</th>
<th>320</th>
<th>3200</th>
<th>9600</th>
<th>32000</th>
<th>96000</th>
</tr>
</thead>
<tbody>
<tr>
<td>LMS filter (Mults/bit)</td>
<td>27</td>
<td>243</td>
<td>729</td>
<td>2421</td>
<td>7269</td>
</tr>
<tr>
<td>FD-FIR filter (Mults/bit)</td>
<td>9</td>
<td>81</td>
<td>243</td>
<td>807</td>
<td>2423</td>
</tr>
<tr>
<td>Blind look-up filter (Mults/bit)</td>
<td>15</td>
<td>24</td>
<td>30</td>
<td>33</td>
<td>39</td>
</tr>
</tbody>
</table>

The complexity of the three filters for different fiber dispersion is illustrated in Table 4. We find that frequency-domain BLU filter is much more efficient than the time-domain LMS and FD-FIR filters when the accumulated chromatic dispersion is large (usually larger than 3000 ps/nm). Moreover, the LMS filter needs higher computational complexity than the FD-FIR filter because it consumes the additional multiplications operation to update the tap weights per symbol duration.

6.4 Phase noise compensation

An example of phase noise compensation using the one-tap NLMS filter is illustrated in Fig. 12. The simulation setup is the same as Fig. 5, where the linewidths of transmitter and LO lasers are both set to be 500 kHz. The figure illustrates the BER characteristics in phase estimation after the 600 km fiber CD equalization. It can be found that the LMS filter is more tolerant of phase noise than the FD-FIR and the BLU filters. Without phase noise compensation, the BER floor
of LMS filter is around $10^{-3}$, which is much better than FD-FIR and BLU filters (above $10^{-2}$). Meanwhile, we find the phase noise compensation using the one-tap NLMS equalizer can achieve a satisfactory performance with a small penalty from the back-to-back result for all the three filters. More detailed investigation of the phase noise influence is in progress and will be presented in a separate publication.

![Phase noise compensation using the NLMS filter after dispersion equalization. w/o means without.](image)

**Fig. 12.** Phase noise compensation using the NLMS filter after dispersion equalization. w/o means without.

## 7. Conclusions

To demonstrate the fundamental features of the time-domain adaptive and fixed filters as well as the frequency-domain equalizer, the LMS filter, the FD-FIR filter and the blind look-up filter are applied to compensate the CD in a 112-Gbit/s NRZ-PDM-QPSK coherent optical transmission system. Compared to our previous work, they are analyzed by evaluating their dynamic range and applicability in CD equalization without using an adaptive 3-tap post-equalizer, and some peculiar characteristics of these filters are consequently revealed. In terms of safety and stability, the LMS adaptive filter shows the best performance in CD equalization. However, it requires slow iteration for guaranteed convergence, and also the tap weights update increases the computational complexity of this algorithm. The LMS filter is especially appropriate for the practical communication systems in which the undefined factors and sudden perturbations probably exist. With respect of equalizer specification, the FD-FIR filter affords the simplest analytical tap weights specification. However, it does not show acceptable performance for short distance fibers. This can be rectified and improved by multiplying the ADC sampling rate or adding an adaptive LMS post-filter. In a practical system, the FD-FIR filter concatenate with an LMS filter (few taps) is usually used due to the lower complexity than a single LMS dispersion equalizer. From the aspect of speed and efficiency, the blind look-up filter will be faster and much more computationally efficient, especial for large fiber dispersion. However, its performance will degrade dramatically if the overlap in the equalizer does not reach the required minimum overlap size. It has been also demonstrated in our investigation that the LMS filter is more tolerant to the chromatic dispersion perturbation and the carrier phase noise than the other two filters.

Besides the chromatic dispersion equalization, our analysis and comparison does not take into account the influences of PMD and fiber nonlinearities, only the carrier phase estimation is investigated. Future efforts should incorporate comparing the CD equalization performance of the three methods in the return-to-zero (RZ) and the NRZ polarization division multiplexed QPSK transmission systems, as well as compensating the PMD, phase noise and the fiber nonlinearities in such coherent systems. Furthermore, these three methods should be studied for the utilization in the quadrature amplitude modulation (QAM) coherent systems.
Paper II
Analytical estimation of phase noise influence in coherent transmission system with digital dispersion equalization

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Abstract: We present a novel investigation on the enhancement of phase noise in coherent optical transmission system due to electronic chromatic dispersion compensation. Two types of equalizers, including a time domain fiber dispersion finite impulse response (FD-FIR) filter and a frequency domain blind look-up (BLU) filter are applied to mitigate the chromatic dispersion in a 112-Gbit/s polarization division multiplexed quadrature phase shift keying (PDM-QPSK) transmission system. The bit-error-rate (BER) floor in phase estimation using an optimized one-tap normalized least-mean-square (NLMS) filter, and considering the equalization enhanced phase noise (EEPN) is evaluated analytically including the correlation effects. The numerical simulations are implemented and compared with the performance of differential QPSK demodulation system.

OCIS codes: (060.1660) Coherent communications; (060.2330) Fiber optics communications.

References and links

The performance of high speed optical fiber transmission systems is severely affected by chromatic dispersion (CD), polarization mode dispersion (PMD), phase noise (PN) and nonlinear effects [1–4]. Coherent optical detection allows the significant equalization of transmission system impairments in the electrical domain, and has become one of the most promising techniques for the next generation communication networks [4–10]. With the full optical field information, the fiber dispersion and carrier phase noise can be well compensated by the efficient digital signal processing (DSP). Several feed-forward and feed-back carrier phase estimation (CPE) algorithms have been validated as effective methods for mitigating the phase fluctuation from the laser sources [9–13]. However, the analysis of the phase noise in the transmitter (TX) and the local oscillator (LO) lasers is often lumped together in these algorithms, and the influence of the large chromatic dispersion on the phase noise in the system is not considered.

Related work has been developed to deliberate the interplay between the digital chromatic dispersion equalization and the laser phase noise [14–21]. W. Shieh, K. P. Ho and A. P. T. Lau et al. have provided the theoretical assessment and analysis to evaluate the equalization enhanced phase noise (EEP) from the interaction between the LO phase fluctuation and the
fiber dispersion in coherent transmission system [14–17]. C. Xie has investigated the impact of chromatic dispersion on both the LO phase noise to amplitude noise conversion and the fiber nonlinear effects [18,19]. I. Fatadin and S. J. Savory have also studied the influence of the equalization enhanced phase noise in QPSK, 16-level quadrature amplitude modulation (16-QAM) and 64-QAM coherent transmission systems by employing the time-domain CD equalization [20]. Due to the existence of EEPN, the requirement of laser linewidth cannot be generally relaxed for the transmission system with higher symbol rate. It would be of interest to investigate in more detail the bit-error-rate (BER) performance of the equalization enhanced phase noise in the coherent optical communication system employing electronic chromatic dispersion compensation.

In this paper, we present a detailed analysis on the impact of the dispersion equalization enhanced phase noise in the coherent transmission system using two electronic dispersion equalizers: a time domain fiber dispersion finite impulse response (FD-FIR) filter and a frequency domain blind look-up (BLU) filter [22–25]. The investigation is performed in a 112-Gbit/s non-return-to-zero polarization division multiplexed quadrature phase shift keying (NRZ-PDM-QPSK) transmission system, which is implemented in the VPI simulation platform [26]. The carrier phase estimation is realized by using a one-tap normalized least mean square (NLMS) filter [13,27], of which the leading order performance is analytically described. Using the FD-FIR and the BLU equalization, the bit-error-rate floor in the NLMS-CPE considering the EEPN is analytically evaluated. Simulation results are used to validate the error-rate floor prediction based upon the theoretical analysis. As an important reference case, a 28-Gsymbol/s differential QPSK (DQPSK) transmission system is also implemented for differential phase detection, of which the basic theory is already reported [28–30]. The CPE performance of the one-tap NLMS digital filter and the differential phase demodulation are comparatively analyzed.

2. Principle of equalization enhanced phase noise

The scheme of the coherent optical communication system with digital CD equalization and carrier phase estimation is depicted in Fig. 1. The transmitter laser signal including the phase noise passes through both transmission fibers and the digital CD equalization module, and so the net dispersion experienced by the transmitter PN is close to zero. However, the local oscillator phase noise only goes through the digital CD equalization module, which is heavily dispersed in a transmission system without dispersion compensation fibers (DCFs). Therefore, the LO phase noise will significantly influence the performance of the high speed coherent system with only digital CD post-compensation. We note that the EEPN does not exist in a transmission system with entire optical dispersion compensation for instance using DCFs.

Theoretical analysis demonstrates that the equalization enhanced phase noise scales linearly with the accumulated chromatic dispersion and the linewidth of the LO laser [14–21], and the variance of the additional noise due to the EEPN can be expressed as follows - see e.g [14]:

![Fig. 1. Scheme of equalization enhanced phase noise in coherent transmission system. MZI: Mach-Zehnder interferometer, $\Phi_{\text{TX}}$: phase fluctuation of the TX laser, $\Phi_{\text{LO}}$: phase fluctuation of the LO laser, $N(t)$: additive white Gaussian noise, ADC: analog-to-digital converter.](image-url)
\[ \sigma_{\text{EEPN}}^2 = \frac{\pi \lambda^2}{2c} \cdot \frac{D \cdot L \cdot \Delta f_{\text{LO}}}{T_s} \] (1)

where \( \lambda \) is the central wavelength of the transmitted optical carrier wave, \( c \) is the light speed in vacuum, \( D \) is the chromatic dispersion coefficient of the transmission fiber, \( L \) is the transmission fiber length, \( \Delta f_{\text{LO}} \) is the 3-dB linewidth of the LO laser, and \( T_s \) is the symbol period of the transmission system.

It is worth noting that the theoretical evaluation of the enhanced LO phase noise is only appropriate for the FD-FIR and the BLU dispersion equalization, which represent the inverse function of the fiber transmission channel without involving the phase noise mitigation. The analysis of the phase noise enhancement due to the least-mean-square (LMS) adaptive dispersion equalization will be further discussed in a separate publication, and a preliminary example is presented in Section 5.1.

3. Principle of normalized LMS filter for carrier phase estimation

3.1 Principle of normalized LMS filter

The one-tap NLMS filter can be employed effectively for carrier phase estimation [13,27], of which the tap weight is expressed as

\[ w_{\text{NLMS}}(n+1) = w_{\text{NLMS}}(n) + \frac{\mu_{\text{NLMS}}}{\|x_{\text{PN}}(n)\|^2} x_{\text{PN}}^*(n) e_{\text{NLMS}}(n) \] (2)

\[ e_{\text{NLMS}}(n) = d_{\text{FE}}(n) - w_{\text{NLMS}}(n) \cdot x_{\text{PN}}(n) \] (3)

where \( w_{\text{NLMS}}(n) \) is the complex tap weight, \( x_{\text{PN}}(n) \) is the complex magnitude of the input signal, \( n \) represents the number of the symbol sequence, \( d_{\text{FE}}(n) \) is the desired symbol, \( e_{\text{NLMS}}(n) \) is the estimation error between the output signals and the desired symbols, and \( \mu_{\text{NLMS}} \) is the step size parameter.

It has been demonstrated that the one-tap NLMS carrier phase estimation can be implemented by using the feed-forward control scheme [13]. Therefore, it is not difficult to implement the NLMS-CPE in a parallel-processing circuit for the real-time QPSK coherent transmission system.

3.2 BER floor of phase estimation using NLMS filter with EEPN

The phase estimation using the one-tap NLMS filter resembles the performance of the ideal differential detection [13,28–30], of which the BER floor can be approximately described by an analytical expression (see Appendix):

\[ BER_{\text{NLMS}} = \frac{1}{2} \text{erfc} \left( \frac{\pi}{4\sqrt{2}\sigma} \right) \] (4)

\[ \sigma^2 \approx \sigma_{\text{TX}}^2 + \sigma_{\text{LO}}^2 + \sigma_{\text{EEPN}}^2 \] (5)

\[ \sigma_{\text{TX}}^2 = 2\pi f_{\text{TX}} \cdot T_s \] (6)

\[ \sigma_{\text{LO}}^2 = 2\pi f_{\text{LO}} \cdot T_s \] (7)
where \( \sigma^2 \) represents the total phase noise variance in the coherent transmission system, \( \sigma_{TX}^2 \) and \( \sigma_{LO}^2 \) are the original phase noise variance of the transmitter and the LO lasers respectively, and \( \Delta f_{TX} \) is the 3-dB linewidth of the transmitter laser.

It should be noted that the variance of the original TX and LO phase noise are also considered in our derivation in addition to the enhanced LO phase noise included in the total phase noise. The deduction of the EEPN does not consider the intrinsic phase fluctuation from the TX and the LO lasers (see e.g [14]). We assume that the intrinsic TX laser, LO laser phase noise and the EEPN are statistically independent. This assumption is reasonable for practical communication systems with the transmission fiber length over 80 km, which will be discussed in Section 6.2. Therefore, the total phase noise variance can be calculated as the summation of the above three items. The BER floor of phase estimation using the one-tap NLMS filter with EEPN is shown in Fig. 2, which is obtained from Eq. (4). In Fig. 2(a), we could see that the BER floor rises with the increment of the fiber length and the LO laser linewidth, which demonstrates that the EEPN influences the performance of the high speed coherent transmission system significantly. Figure 2(b) indicates the BER floor in phase estimation for different combination of TX and LO lasers linewidth while keeping the sum of linewidths \( \Delta f_{TX} + \Delta f_{LO} \) constant. To make the BER floor induced by only the TX phase noise be above \( 10^{-9} \), we need to select a large value (76 MHz and 122 MHz) of the laser linewidth in Fig. 2(b), which may not be used in the practical case. It can be found clearly that the EEPN arises from the LO phase noise, because the BER floor does not change with the increment of fiber length when there is only phase fluctuation from the TX laser. Theoretical investigation demonstrates that the enhanced LO phase noise plays the dominant role in total phase fluctuation, when the accumulated fiber dispersion is larger than 700 ps/nm (about 45 km normal transmission fiber) [14,16].

3.3 Optimization of the step size in NLMS filter

According to the reported investigation, the step size \( \mu_{NLMS} \) has an optimal value to provide the best performance of the one-tap NLMS phase estimator for a certain laser phase noise [13]. Roughly speaking, a smaller step size will deteriorate the BER floor induced by the laser phase noise due to the fast phase changing occurring in the long effective symbol average-span. By contrast, a larger step size will degrade the NLMS phase estimator on the sensitivity of optical signal-to-noise ratio (OSNR), but influence the BER floor induced by the phase fluctuation little. The performance of the one-tap NLMS-CPE using different step size is shown in Fig. 3, where both of the TX and LO lasers linewidths are 5 MHz, and the fiber length is 2000 km. We
can see that the one-tap NLMS-CPE shows the best performance when using the optimum step size ($\mu=0.25$), and the BER floor in NLMS-CPE is deteriorated obviously when the smaller step size ($\mu=0.025$) is used. Meanwhile, we find that only the OSNR sensitivity is degraded while the BER floor has no significant variation, when the larger step size ($\mu=1$) is employed in the one-tap NLMS-CPE. Note that the OSNR value is all defined in 0.1 nm and the penalty between the back-to-back result and the theoretical limit (at BER=10^{-3}) is around 1.8 dB.

![Fig. 3. Phase estimation using the one-tap NLMS filter with different value of step size, $\mu$ is the step size. (a) NLMS-CPE with FD-FIR dispersion equalization, (b) NLMS-CPE with BLU dispersion equalization.](image)

From the above analysis, it is important to determine the optimum step size in the application of the one-tap NLMS phase estimation. Corresponding to the definition of the original phase noise from TX and LO lasers, we employ an effective linewidth $\Delta f_{\text{eff}}$ to describe the total phase noise in the coherent system with EEPN, which can be defined as the following expression:

$$\Delta f_{\text{eff}} = \frac{\sigma_{\text{TX}}^2 + \sigma_{\text{LO}}^2 + \sigma_{\text{EEP}}^2}{2\pi T_i}$$

(8)

In Fig. 4(a), we studied the optimum step size for different effective linewidth in the 112-Gbit/s NRZ-PDM-QPSK coherent optical transmission system, which is applicable for both the FD-FIR filter and the BLU filter. It is found that the optimum step size increases with the effective laser linewidth. Note that the one-tap NLMS filter is employed with the optimum step size value for phase estimation in our simulation work.

![Fig. 4. The optimum step size and the OSNR penalty in NLMS-CPE. (a) optimum step size for different effective linewidth, (b) OSNR penalty in NLMS phase estimation with the optimum step size for FD-FIR and BLU equalization.](image)
Using the FD-FIR and the BLU dispersion equalization, the OSNR penalty from back-to-back result at BER=$10^{-3}$ in the one-tap NLMS-CPE with the optimum step size are illustrated in Fig. 4(b). It is found that the OSNR penalty scales exponentially with the increment of the effective laser linewidth.

It can be found in Fig. 3 and Fig. 4(b) that the coherent system using the FD-FIR and the BLU dispersion equalization has closely the same performance in the one-tap NLMS carrier phase estimation. Therefore, we will only analyze one of the two digital filters in our later discussion.

4. Simulation investigation of PDM-QPSK transmission system

The setup of the 112-Gbit/s NRZ-PDM-QPSK coherent transmission system implemented in the VPI simulation platform is illustrated in Fig. 5. The data sequence output from the four 28-Gbit/s pseudo random bit sequence (PRBS) generators are modulated into two orthogonally polarized NRZ-QPSK optical signals by the two Mach-Zehnder modulators. Then the orthogonally polarized signals are integrated into one fiber channel by a polarization beam combiner (PBC) to form the 112-Gbit/s NRZ-PDM-QPSK optical signal. Using a local oscillator in the coherent receiver, the received optical signals are mixed with the LO laser to be transformed into four electrical signals by the photodiodes. The four electrical signals are processed by further using the Bessel low-pass filters (LPFs) with a 3-dB bandwidth of 19.6 GHz. Then they are digitalized by the 8-bit analog-to-digital convertors (ADCs) at twice the symbol rate [31]. The sampled signals are processed by the digital equalizer, and the BER is then estimated from the data sequence of $2^{16}$ bits. The central wavelength of the transmitter laser and the LO laser are both 1553.6 nm. The standard single mode fibers (SSMFs) with the CD coefficient equal to 16 ps/nm/km are employed in all the simulation work.

Here we neglected the influences of fiber attenuation, polarization mode dispersion and nonlinear effects in our simulation. The PMD and polarization rotation equalization could be realized by employing the adaptive LMS and constant modulus algorithm (CMA) equalizers [32]. The chromatic dispersion compensation is implemented by using the digital filters with appropriate parameters, which has been analyzed in our previous work [33].

Fig. 5. Schematic of 112-Gbit/s NRZ-PDM-QPSK coherent optical transmission system. PBS: polarization beam splitter, OBPF: optical band-pass filter, PIN: PiN diode.

With the increment of launched optical power, the fiber nonlinearities such as self-phase modulation (SPM), cross-phase modulation (XPM) and four-wave mixing (FWM) need to be considered in the long-haul wavelength-division multiplexing (WDM) transmission systems.
The fiber nonlinear impairments can be mitigated and compensated by using the digital backward propagation methods based on solving the nonlinear Schrödinger (NLS) equation and the Manakov equation [34,35].

5. Simulation results

5.1 Phase estimation considering EEPN with different CD equalization

In Fig. 6, the FD-FIR equalization performance in the coherent transmission system with different fiber length is compared with the adaptive LMS dispersion equalization, where the results are processed by further using a one-tap NLMS filter for carrier phase estimation. The results are obtained under different combination of TX and LO lasers linewidth while keeping the sum of linewidths $\Delta f_{TX} + \Delta f_{LO}$ constant. We can clearly see that influenced by the EEPN, the performance of FD-FIR equalization (the same in BLU equalization) reveals obvious fiber length dependence with the increment of LO laser linewidth. The OSNR penalty in phase estimation scales with the LO phase fluctuation and the accumulated dispersion. On the other hand, the dispersion equalization using the LMS filter shows almost the same behavior in the three cases [27]. That is because the dispersion interplays with the phase noise of both TX and LO lasers simultaneously in the adaptive equalization. Moreover, Fig. 6 also shows that the LMS filter is less tolerant against the phase noise than the other two dispersion equalization methods when carrier phase estimator is employed.

![Fig. 6. Carrier phase estimation for various fiber length with different CD compensation methods, where the linewidth of the TX and the LO lasers are in different combination while keeping the sum of linewidths constant. (a) FD-FIR filter, (b) LMS filter.](image)

5.2 Evaluation of BER floor in phase estimation with EEPN

In Fig. 7, the performance of carrier phase estimation using the one-tap NLMS filter with the FD-FIR dispersion equalization is compared with the theoretical evaluation using Eq. (4).
Figure 7 illustrates the numerical results for different combination of TX and LO lasers linewidth while keeping the sum of linewidths $\Delta f_{\text{TX}} + \Delta f_{\text{LO}}$ constant. With the increment of OSNR value, the numerical simulation reveals the BER floor only influenced by the phase noise, which achieves a good agreement with the theoretical evaluation. Figure 7(b) denotes the results with only the analysis of TX laser phase noise, where a slight deviation is found between the simulation results and the theoretical analysis. It arises from the approximation in the analytical evaluation of the one-tap NLMS phase estimator in Eq. (4), of which only the leading order is considered. It has been validated in our simulation work that the phase estimation with the BLU dispersion equalization performs closely the same behavior as the FD-FIR equalization.

6. Differential phase detection

6.1 Differential QPSK demodulation system

The coherent optical transmission system can be operated in differential demodulation mode when the differential encoded data is recovered by a simple “delay and multiply algorithm” in the electrical domain. In such a case the encoded data is recovered from the received signal based on the phase difference between two consecutive symbols, i.e. the value of the complex decision variable $\Psi = Z_k Z_{k+1}^* \exp\{i \pi/4\}$, where $Z_k$ and $Z_{k+1}$ are the consecutive k-th and (k+1)-th received symbols. The BER floor of the differential phase receiver can be evaluated using the principle of conditional probability [28], which is expressed as the following equation:

$$\text{BER}^{\text{DQPSK}}_{\text{floor}} = \frac{1}{2} \text{erf}c\left(\frac{\pi}{4\sqrt{2}\sigma}\right).$$

The BER performance of the 28-Gsymol/s DQPSK coherent transmission system with the FD-FIR dispersion equalization is illustrated in Fig. 8. Figure 8(a) shows the simulation results for different combination of TX and LO lasers linewidth while keeping the sum of linewidths $\Delta f_{\text{TX}} + \Delta f_{\text{LO}}$ constant, and Fig. 8(b) denotes the performance of the differential demodulation system with only the TX laser phase noise. It is found that the BER behavior in the DQPSK coherent system can achieve a good agreement with the theoretical evaluation in Eq. (9) for both Fig. 8(a) and Fig. 8(b). The consistence between simulation and theory in DQPSK demodulation is better than the one-tap NLMS phase estimation in the case of only TX laser phase noise.
6.2 Correlation between EEPN and intrinsic LO phase noise

In the evaluation for the BER floor of the one-tap NLMS phase estimation in Section 3.2, we have assumed that the intrinsic TX laser, LO laser phase noise and the equalization enhanced phase noise are statistically independent. Obviously, the TX laser phase noise is independent from the LO laser phase noise and the EEPN. Here we mainly investigate the correlation between the intrinsic LO laser phase noise and the EEPN. The total phase noise variance in the coherent optical transmission system can be modified as

$$\sigma^2 = \sigma_{TX}^2 + \sigma_{LO}^2 + \sigma_{\text{EEPN}}^2 + 2\rho \cdot \sigma_{LO} \cdot \sigma_{\text{EEPN}}$$  \hspace{1cm} (10)

where $\rho$ is the correlation coefficient between the intrinsic LO laser phase noise and the EEPN, and we have the absolute value $|\rho| \leq 1$.

We have implemented the numerical simulation in the DQPSK system for different combination of the intrinsic LO laser phase noise and the EEPN while keeping a constant sum, which is illustrated in Fig. 9(a). It can be found that the BER floor does not show tremendous variation due to the correlation between the LO laser phase noise and the EEPN. The BER floor reaches the lowest value at $\sigma_{\text{EEPN}}^2 = 0.5 \sigma_{LO}^2$, which corresponds to the maximum value of the term $|2\rho \cdot \sigma_{LO} \cdot \sigma_{\text{EEPN}}|$. The cases for $\sigma_{\text{EEPN}}^2 \gg \sigma_{LO}^2$ and $\sigma_{\text{EEPN}}^2 = 0$ correspond to the mutual term $|2\rho \cdot \sigma_{LO} \cdot \sigma_{\text{EEPN}}| = 0$. From Fig. 9(a) we can find that $\rho$ is usually a negative value.
The EEPN arises from the electronic dispersion compensation where the phase of the equalized symbol fluctuates during the time window of the digital filter, while the intrinsic LO laser phase fluctuation comes from the integration during the consecutive symbol period. Therefore, we could give an approximate theoretical evaluation of the correlation coefficient as

$$|\rho| \approx \frac{T_s}{N \cdot T}. \quad (11)$$

where \(N\) is the required tap number (or the necessary overlap) in the chromatic dispersion compensation filter, and \(T\) is the sampling period in the transmission system. The tap number (or the necessary overlap) can be calculated by the fiber dispersion to be compensated [22,25], and the sampling period \(T = \frac{1}{2}T_s\) when the sampling rate in the ADC modules is selected as twice the symbol rate.

The absolute value of the correlation coefficient \(|\rho|\) for different fiber length is illustrated in Fig. 9(b), in which we can see that the correlation coefficient \(|\rho|\) determined from the numerical simulation achieves good agreement with the theoretical approximation. With the increment of fiber length, the magnitude of correlation coefficient \(|\rho|\) approaches zero rapidly. Consequently, we can neglect the correlation term in Eq. (10) when the fiber length is over 80 km. Therefore, the assumption in Eq. (5) is valid when the fiber length exceeds 80 km in practical optical communication systems. A more accurate analytical expression for correlation coefficient could be derived from the mathematical definition of correlation between two random variables by using the probability density functions (PDFs) of the LO phase noise and the EEPN [9,16]. This will be studied in our future work.

Note that in Fig. 9 we use the BLU dispersion equalization rather than the FD-FIR equalization, because we need to employ various fiber length for different combination of the LO laser phase noise and the EEPN, while the FD-FIR filter shows the malfunctional performance for short distance fibers [33].

7. Conclusions

To evaluate the impact of the dispersion equalization enhanced phase noise, two electronic dispersion equalizers involving the FD-FIR filter and the BLU filter are applied to compensate the CD in the 112-Gbit/s NRZ-PDM-QPSK coherent optical transmission system. The carrier phase estimation is implemented by using the one-tap normalized LMS filter, of which the performance is analyzed in detail. The BER floor of the one-tap NLMS phase estimation with the enhanced phase noise is analytically evaluated, and the simulation results are compared to the differential phase detection system. As a novel result, it is found that the FD-FIR and the BLU dispersion equalization with the one-tap NLMS carrier phase estimation have very similar phase noise statistic performance as using differential phase detection. The small deviation of significance for shorter transmission distance (when the EEPN is small) is tentatively attributed to the high order analysis in the NLMS phase estimation. We have for the first time evaluated the correlation between the LO phase noise and the EEPN in detail, and have given a novel quantitative explanation for the size of the correlation. It is verified that the correlation effect is only of practical significance for the transmission distance less than 80 km normal transmission fiber.

The evaluation of the BER floor in phase estimation with the LMS adaptive dispersion equalization is rather complicated due to the equal enhancement of both TX and LO lasers phase noise as shown in the simulation example. This will be investigated in our future work. Moreover, the effects of EEPN on different phase estimation algorithms in coherent transmission system will be also studied in the future investigations.
Appendix: BER floor of phase estimation using one-tap NLMS filter

Assuming that the one-tap NLMS filter has converged through numbers of iterations [27], the k-th output symbol after the carrier phase estimation can be expressed as

\[
y_{PE}(k) = w_{NLMS}(k) \cdot x_{PN}(k)
= b_k E_k \exp\left[j(\phi_k - \Phi_k)\right]
\]  \hspace{1cm} (A1)

\[
x_{PN}(k) = E_k \exp\left(j\phi_k\right)
\]  \hspace{1cm} (A2)

\[
w_{NLMS}(k) = b_k \exp\left(-j\Phi_k\right)
\]  \hspace{1cm} (A3)

\[
e_{NLMS}(k) = d_{PE}(k) - y_{PE}(k)
\]  \hspace{1cm} (A4)

where \(x_{PN}(k)\) is the k-th received symbol, \(y_{PE}(k)\) is the k-th output symbol after the carrier phase estimation, \(w_{NLMS}(k)\) is the k-th tap weight in the one-tap NLMS filter, \(E_k\) is the complex envelope of the electrical field of the received symbol, \(\phi_k\) is the total phase fluctuation in the received symbol, \(b_k\) is a positive real coefficient of tap weight, \(\Phi_k\) is the estimated phase in the one-tap NLMS filter.

To achieve a satisfactory effect of phase noise compensation, we have the estimated error:

\[
\left|e_{NLMS}(k)\right| \ll 1.
\]  \hspace{1cm} (A5)

Namely, we have the estimated phase as

\[
\Phi_k \approx \phi_k.
\]  \hspace{1cm} (A6)

Therefore, for the (k+1)-th received symbol, the (k+1)-th output symbol after the CPE can be describes as following equations:

\[
y_{PE}(k+1) = w_{NLMS}(k+1) x_{PN}(k+1)
\approx \left[b_k E_{k+1} + \frac{\mu_{NLMS}}{E_k} e_{PE}(k) E_{k+1} E_{k+1}^* \right] \exp\left[j(\phi_{k+1} - \phi_k)\right],
\]  \hspace{1cm} (A7)

\[
x_{PN}(k+1) = E_{k+1} \exp\left(j\phi_{k+1}\right),
\]  \hspace{1cm} (A8)

\[
w_{NLMS}(k+1) = w_{NLMS}(k) + \frac{\mu_{NLMS}}{x_{PN}(k)} e_{PE}(k) x_{PN}^*(k).
\]  \hspace{1cm} (A9)

It is found that the function of the one-tap NLMS filter resembles the ideal differential detection. The BER floor in the one-tap NLMS-CPE can be calculated correspondingly [28–30], which is described as the following expression:

\[
BER_{NLMS}^{floor} \approx \frac{1}{2} \text{erfc}\left(\frac{\pi}{4\sqrt{2}\sigma}\right).
\]  \hspace{1cm} (A10)

The BER floor in phase estimation using the one-tap NLMS filter with different phase noise variance is illustrated in Fig. 10.
Fig. 10. BER floor of phase estimation using one-tap NLMS filter.
Paper III
Normalized LMS digital filter for chromatic dispersion equalization in 112-Gbit/s PDM-QPSK coherent optical transmission system

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Abstract

High bit rates optical communication systems pose the challenge of their tolerance to linear and nonlinear fiber impairments. Coherent optical receivers using digital signal processing techniques can mitigate the fiber impairments in the optical transmission system, including the chromatic dispersion equalization with digital filters. In this paper, an adaptive finite impulse response filter employing normalized least mean square algorithm is developed for compensating the chromatic dispersion in a 112-Gbit/s polarization division multiplexed quadrature phase shift keying (PDM-QPSK) coherent optical communication system, which is established in the VPI simulation platform. The principle of the adaptive normalized least mean square algorithm for signal equalization is analyzed theoretically, and at the meanwhile, the taps number and the tap weights in the adaptive finite impulse response filter for compensating a certain fiber chromatic dispersion are also investigated by numerical simulation. The chromatic dispersion compensation performance of the adaptive filter is analyzed by evaluating the behavior of the bit-error-rate versus the optical signal-to-noise ratio, and the compensation results are also compared with other present digital filters.

1. Introduction

Fiber impairments such as chromatic dispersion (CD) severely impact the performance of high speed optical fiber transmission systems [1,2]. Although current systems use dispersion compensation fibers (DCFs) to compensate the chromatic dispersion distortion, this increases the complexity and cost of the transmission systems. Digital coherent receivers allow equalization for linear transmission impairments in the electrical domain [3,4], and have become the most promising alternative approach to dispersion compensation fibers. While coherent detection was experimentally demonstrated as early as 1979, its use in commercial systems has been hindered by the additional complexity, due to the need to track the phase and polarization of the incoming signal [5]. In a digital coherent receiver these functions are implemented in the electrical domain leading to a dramatic reduction in complexity. Furthermore since coherent detection maps the entire optical field within the receiver bandwidth into the electrical domain it maximizes the efficacy of the signal processing. This allows fiber impairments which have traditionally limited high bit rate systems to be overcome adaptively [6–15].

Several digital filters have been applied to compensate the chromatic dispersion in the time domain and the frequency domain [14,15]. In this paper an adaptive finite impulse response (FIR) filter employing normalized least mean square (NLMS) algorithm is developed to compensate CD in a 112-Gbit/s polarization division multiplexed quadrature phase shift keying (PDM-QPSK) coherent optical transmission system. The principle of the NLMS algorithm and the structure of the adaptive filter are analyzed and investigated, and the influence of the step size on the convergence of the tap weights in the adaptive filter is also discussed and illustrated. The performance of the NLMS filter is characterized by evaluating the behavior of the bit-error-rate (BER) versus optical signal-to-noise ratio (OSNR) in the PDM-QPSK system using VPI simulation platform [16], and compared with a fiber dispersion FIR (FD-FIR) filter and a blind look-up adaptive filter [14,15].

2. Principle of normalized least mean square adaptive filter

The normalized least mean square algorithm is an iterative adaptive algorithm that can be used in the highly time varying sig-
nal environment. The NLMS algorithm uses the estimates of the gradient vector from the available data. The NLMS algorithm incorporates an iterative procedure that makes successive corrections to the weights vector in the direction of the negative of the gradient vector which eventually leads to the minimum mean square error. The transfer function $h(n)$ of the NLMS adaptive filter is given by:

$$h(n + 1) = h(n) + \mu e(n)x'(n)/||x(n)||^2$$ (1)

$$e(n) = d(n) - h(n)x(n)$$ (2)

where $x(n)$ is the input signal, $d(n)$ is the desired symbol, $e(n)$ represents the estimation error between the output signal and the desired symbol, $x'(n)$ is the Hermitian of the input signal $x(n)$, and $\mu$ is a coefficient called step size parameter. The transfer function $h(n)$ is updated in a symbol-by-symbol iterative manner, and achieves convergence when $e(n)$ approaches to zero.

In order to guarantee the convergence of the transfer function $h(n)$, the step size $\mu$ needs to satisfy the condition of $0 < \mu < 1/\lambda_{\text{max}}$, where $\lambda_{\text{max}}$ is the largest eigenvalue of the correlation matrix $R = x(n)x'(n)$. The convergence speed of the algorithm is inversely proportional to the eigenvalue spread of the correlation matrix $R$. When the eigenvalues of $R$ are spread, convergence may be slow. The eigenvalue spread of the correlation matrix is estimated by computing the ratio of the largest eigenvalue to the smallest eigenvalue of the matrix. If the step size $\mu$ is chosen to be very small, then algorithm converges very slowly. A large value of $\mu$ may lead to a faster convergence, but may be less stable around the minimum value. An appropriate step size is usually selected as $\mu = 0.1$ in the NLMS adaptive filter.

The schematic of the linear adaptive normalized least mean square equalizer with $N$ taps is shown in Fig. 1, where $T$ is the symbol period, and coefficients $W_i$ are the tap weights corresponding to the NLMS transfer function vector $h(n)$. The linear adaptive NLMS equalizer consists of a tapped delay line that stores data samples from the input signal. Once per symbol period, the equalizer outputs a weighted sum of the values in the delay line and updates the tap weights to prepare for the next symbol period. The tap weights vector are usually initialized with zero value, and updated according to the estimation error between the output signal and the desired signal.

3. Simulation investigation of PDM-QPSK transmission system

The installation of the 112-Gbit/s PDM-QPSK coherent optical transmission system established in the VPI simulation platform is illustrated in Fig. 2. The electrical data from four 28-Gbit/s pseudo random bit sequence (PRBS) generators are modulated into two orthogonally polarized QPSK optical signals by two Mach–Zehnder modulators, which are then integrated into one fiber transmission channel by a polarization beam combiner to form the 112-Gbit/s PDM-QPSK optical signal. Using an optical local oscillator (LO) in the coherent receiver, the received optical signals are mixed with the LO laser to be transformed into four electrical signals by the photodiodes and then digitalized by the analog-to-digital converters (ADCs) at double sampling rates. Thus the impairments of chromatic dispersion in the transmission channel could be equalized with diverse digital filters. In this simulation installation, the intermediate frequency between the transmitter and the LO laser is set to be zero, and the linewidth of both laser diodes are set to be 0 Hz.

4. Simulation results of chromatic dispersion compensation

To illustrate the features of the NLMS filter, the compensation of CD from a standard single mode fiber with dispersion coefficient $D = 16\text{ps}/(\text{nm km})$ are investigated and analyzed by comparing with the FD-FIR filter. Compared with the iteratively updated tap weights in the NLMS adaptive filter, the tap weights in the FD-FIR filter have a relatively simple specification, the tap weights $a_k$ in the FD-FIR filter is given by [14].

$$a_k = \sqrt{\frac{j\pi D^2}{C_1^2}} \exp \left(-\frac{j\pi D^2}{C_1^2} z_k^2 \right) - \frac{N}{2} \leq k \leq \frac{N}{2}$$ (3)

$$N = 2 \times \left[ \frac{|D| z_k^2}{2\pi C_1^2} \right] + 1$$ (4)

where $T$ is the symbol period, $D$ is the fiber chromatic dispersion coefficient, $\lambda$ is the central wavelength of the transmission optical wave, $z$ is the fiber length in the transmission channel, $k$ is the taps order and $N$ is the required upper bound of taps number for compensating the fiber dispersion.

The distribution of converged tap weights in the NLMS filter for 20 km fiber are shown in Fig. 3(a)–(c), respectively, and the tap weights of the FD-FIR filter for 20 km fiber are shown in Fig. 3(d). We can see that in the NLMS filter, the central tap weights take more dominant roles in the chromatic dispersion equalization, while in the FD-FIR filter, the tap weights magnitudes are constant for a certain length fiber, whereas the real parts and the imaginary parts of the tap weights vary periodically with the tap orders increasing. For a fixed fiber dispersion, the tap weights in NLMS adaptive filter approach to zero, when the corresponding taps order exceeds a
certain value, and this value indicates the least required taps number for compensating the chromatic dispersion effectively. This also illustrates the optimization characteristic of the NLMS adaptive algorithm. While in the FD-FIR filter, the excessive tap weights do not approach to zero when the chromatic dispersion in the transmission fiber channel has been equalized effectively.

The chromatic dispersion compensation results are shown in Fig. 4, where Fig. 4(a) indicates the CD equalization for 20 and 6000 km fibers using the NLMS filter and the FD-FIR filter, respectively, both digital filters using 9 taps for 20 km fiber and 2411 taps for 6000 km fiber, and Fig. 4(b) shows the influence of taps number increment on CD compensation effects for 20 km fiber with the two digital filters. It could be seen in Fig. 4 that the NLMS adaptive filter could achieve the same CD compensation performance as the FD-FIR filter both using the appropriate number of taps. The NLMS filter has a better improvement with the number of taps increasing. However, the FD-FIR filter behaves worse than its effective CD compensation performance when the number of taps increases exceeding the required number of taps, and this is because the FD-FIR filter is not an optimal adaptive equalizer. In order to improve this FD-FIR filter performance in practical system, an adaptive equalizer could be posted after the FD-FIR filter, and thus the polarization dependent effect and the residual chromatic dispersion could be compensated.

To investigate the required number of taps in NLMS adaptive filter and FD-FIR filter for a certain value of fiber dispersion, the required numbers of taps in the filters for different fiber lengths with the same CD coefficient are shown in Table 1. We could see that the NLMS adaptive needs fewer taps than FD-FIR filter to achieve an acceptable CD compensation performance (BER better than $10^{-3}$) with the fiber length increasing.

In the previous description, although the NLMS filter shows a slightly better performance than the FD-FIR filter, the normalized LMS equalizer requires a higher computational complexity than the FD-FIR equalizer. For an N length filter, the NLMS adaptive equalizer needs $2N$ complex multiplications per signal symbol, while the FD-FIR equalizer needs N complex multiplications per signal symbol.

The structure of the frequency domain blind look-up adaptive equalizer is shown in Fig. 5(a). In the blind look-up filter, the dig-
italized electrical signals are firstly transformed by the fast Fourier transform (FFT) operation and then multiplied by the inverse transfer function of the dispersive channel in frequency domain, and then the processed signals are transformed into time domain signals by the inverse fast Fourier transform (IFFT) operation [15]. Starting at the initial value 280 ps/nm, the dispersion applied to the adaptive filter is increased in steps of 2 ps/nm up to the negative of maximum possible dispersion in the 112-Gbit/s PDM-QPSK coherent optical transmission system. The simulation results of CD compensation for 20 km fiber using the blind look-up filter with various FFT-sizes N are illustrated in Fig. 5(b), the dotted line in Fig. 5(b) is the back-to-back measurement result. The overlap- add FFT method is employed in the blind look-up adaptive equalizer for CD compensation, and the overlap sample number is selected as the half of the FFT-size [17]. The required FFT-size values in this frequency domain adaptive filter for different fiber length are also illustrated in Table 1.

In the above simulation results, we could see that the CD equalization of the NLMS adaptive filter could reach nearly the same performance as the FD-FIR filter with necessary taps and the blind look-up adaptive filter with 16 FFT-size and eight overlap samples in the frequency domain equalization operation, when the appropriate parameters are set in the NLMS filter.

5. Discussion

The NLMS adaptive filter could also be used for phase estimation in carrier recovery in PDM-QPSK coherent transmission system, when the linewidth of semiconductor lasers for the transmitter and local oscillator is in the range of several hundred kilohertz [18,19]. Using a one-tap NLMS adaptive filter, the phase noise compensation using a one-tap NLMS adaptive filter, (a) semiconductor linewidth is 500 kHz, (b) semiconductor linewidth is 1000 kHz, (c) semiconductor linewidth is 1500 kHz.

Table 1

<table>
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<th>Fiber length (km)</th>
<th>Taps number in NLMS filter</th>
<th>Taps number in FD-FIR filter (upper bound)</th>
<th>FFT-size in blind look-up filter (with less than 0.5 dB penalty)</th>
<th>FFT-sizes in blind look-up filter</th>
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*a Necessary number of taps in NLMS and FD-FIR filters and FFT-sizes in blind look-up filter for a certain length fiber.

Fig. 4. Performances of CD compensation using NLMS and FD-FIR digital filters neglecting fiber loss, (a) BER with OSNR using NLMS and FD-FIR filters, (b) BER with taps number using NLMS and FD-FIR filters at OSNR 14.6 dB.

Fig. 5. Blind look-up adaptive equalizer, (a) schematic of blind look-up adaptive filter, (b) CD equalization using blind look-up filter at OSNR 14.6 dB.

Fig. 6. Phase noise compensation using a one-tap NLMS adaptive filter, (a) semiconductor linewidth is 500 kHz, (b) semiconductor linewidth is 1000 kHz, (c) semiconductor linewidth is 1500 kHz.
noise of coherent fiber optical communication system could be eliminated after the group-velocity dispersion (GVD) is compensated by digital filters, such as NLMS adaptive filter, FD-FIR filter and blind look-up adaptive filter. We put an 803-tap NLMS filter for 2000 km fiber CD equalization in the first step, and then use a one-tap NLMS filter to compensate the phase noise in the second step. The BER performance for carrier-phase estimation using the one-tap NLMS adaptive filter with different step size is illustrated in Fig. 6, when the semiconductor linewidth of transmitter and LO lasers is 500, 1000 and 1500 kHz, respectively. We could see in Fig. 6 that too small step size degrades the BER performance of the phase noise compensation, this is due to the longer averaging-span length induces the larger phase error between the signal and the reference through the non-negligible phase noise. The value of step size to obtain the best performance is 0.1, and the power penalty from back-to-back measurement is about 2 dB at BER = 10^{-3} in such a case. As the laser linewidth is further increased, the optimum value of the step size also increases [18,19].

6. Conclusions

In this paper an adaptive finite impulse response digital filter employing normalized least mean square algorithm is developed to compensate the chromatic dispersion in a 112-Gbit/s polarization division multiplexed quadrature phase shift keying coherent optical transmission system. The performance of the NLMS adaptive filter for CD equalization is compared with the FD-FIR filter and the blind look-up adaptive filter. The NLMS adaptive filter shows the best performance in CD compensation, whereas it requires slow iteration for guaranteed convergence. The FD-FIR filter affords simple analytical tap weights specification, whereas giving slightly poor performance with taps number increasing. The blind look-up filter shows the same performance as the NLMS filter, and the frequency domain filter will be more efficient when the equalizer length is large [20]. The one-tap NLMS adaptive filter could also be used in the carrier-phase estimation in the high speed coherent transmission system, and the BER performance have about 2 dB penalty from the back-to-back measurement after phase noise compensation.

References

Paper IV
Frequency-domain Chromatic Dispersion Equalization using Overlap-add Methods in Coherent Optical System

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Summary

The frequency domain equalizers (FDEs) employing two types of overlap-add zero-padding (OLA-ZP) methods are applied to compensate the chromatic dispersion in a 112-Gbit/s non-return-to-zero polarization division multiplexed quadrature phase shift keying (NRZ-PDM-QPSK) coherent optical transmission system. Simulation results demonstrate that the OLA-ZP methods can achieve the same acceptable performance as the overlap-save method. The required minimum overlap (or zero-padding) in the FDE is derived, and the optimum fast Fourier transform length to minimize the computational complexity is also analyzed.

1 Introduction

Fiber impairment such as the chromatic dispersion (CD) severely impacts the performance of the high speed optical fiber transmission systems \cite{1,2}. Current systems usually use dispersion compensation fibers (DCFs) to suppress the CD distortion in the optical domain, which increases the cost and deteriorates the nonlinear tolerance of the transmission systems. Coherent detection allows dispersion equalization in the electrical domain, and has become a promising alternative approach to optical dispersion compensation (ODC) \cite{3,4}.

Several digital filters have been applied to compensate the CD in the time and the frequency domain \cite{4-6}. Compared to the time-domain fiber dispersion finite impulse response (FD-FIR) and adaptive least mean square (LMS) filters, the frequency domain equalizers (FDEs) have become the more attractive digital filters for channel equalization in the coherent transmission systems due to the low computational complexity for large dispersion and the wide applicability for different fiber distance \cite{4-8}. The fast Fourier transform (FFT) convolution algorithms involving the overlap-save (OLS) and the overlap-add zero-padding (OLA-ZP) methods are traditionally used for the frequency domain equalization in the wireless communication systems \cite{9-12}. Recently, the OLS-FDE employed in the coherent optical communication system was reported, where the received data sequence is divided into small blocks with a certain overlap before they are equalized \cite{13,14}.

In this paper, two types of FDEs employing the OLA-ZP FFT convolution methods are investigated to compensate the CD in a 112-Gbit/s non-return-to-zero polarization division multiplexed quadrature phase shift keying (NRZ-PDM-QPSK) coherent optical transmission system \cite{9-12}. In the OLA-ZP equalization, the received data sequence is divided into small blocks without any overlap, but appended with zero-padding. The CD compensation results using the two OLA-ZP methods are compared with the OLS method by evaluating the behavior of the bit-error-rate (BER) versus the optical signal-to-noise ratio (OSNR) as well as the FFT-sizes and the overlap sizes. The minimum value of the overlap (or zero-padding), which is the pivotal parameter in FDE, is evaluated according to the equalized dispersion. Moreover, the optimum FFT-size in FDE is also analyzed to minimize the computational complexity.

2 Principle of OLS and OLA-ZP methods

2.1 Overlap-save method

The schematic of the FDE with overlap-save method is illustrated in Fig. 1 \cite{9,10,13,14}. The received signals are divided into several blocks with a certain overlap, where the block length is called the FFT-size. The sequence in each block is transformed into the frequency domain data by the FFT operation, and afterwards multiplied by the transfer function of the FDE. Next, the data sequences are transformed into the time domain.
signals by the inverse FFT (IFFT) operation. Finally, the processed data blocks are combined together, and the bilateral zero-padding samples are symmetrically discarded.

Fig. 1: FDE with OLS method. The parts with slants are to be discarded

The transfer function of FDE is expressed as follows [1],

\[
G_c(z, \omega) = \exp\left(-jD\lambda z/4\pi c\right)
\]

(1)

where \( D \) is the CD coefficient, \( \lambda \) is the operation wavelength of the laser, \( c \) is the light speed in vacuum, \( \omega \) is the angular frequency, and \( z \) is the fiber length.

2.2 Overlap-add one-side zero-padding method

The structure of the FDE with overlap-add one-side zero-padding (OLA-OSZP) method is shown in Fig. 2 [9-12]. The received data are divided into small blocks without any overlap, and then the data in each block are appended with zeros at one side. To be consistent with the OLS method, the total length of data block and zero padding is called the FFT-size, while the length of zero padding is called the overlap. The zero-padded sequence is transformed by the FFT operation, and multiplied by the transfer function of the FDE. Afterwards, the data are transformed by the IFFT operation. Finally the processed data sequences are combined by overlapping and adding. Note that half of the data stream in the first block is discarded.

Fig. 2: FDE with OLA-OSZP method; the gray parts mean the appended zeros, and the parts with slants are to be discarded

2.3 Overlap-add both-side zero-padding method

The schematic of the FDE with overlap-add both-side zero-padding (OLA-BSZP) method is illustrated in Fig. 3 [9-12]. The received data are also divided into several blocks with no overlap, and then the data in each block are appended with equivalent zeros at both sides. The total length of data block and zero padding is called the FFT-size, and the length of the whole zero padding is called the overlap. The zero-padded sequence is transformed by the FFT operation, and multiplied by the transfer function of the FDE, and then transformed by the IFFT operation. The processed data blocks are also combined together by overlapping and adding. Note that half of the data stream in the first block is discarded.

Fig. 3: FDE with OLA-BSZP method; the gray parts mean the appended zeros, and the parts with slants are to be discarded

2.4 Minimum overlap in FDE

Actually, the value of the overlap in the OLS method or the zero-padding in the OLA-ZP methods is the pivotal parameter in the FDE determined by the dispersion to be equalized. The FFT-size can be configurable provided it is larger than the overlap (or zero-padding). The required minimum overlap (or ZP) in the OLS or the OLA-ZP method can be calculated from the pulse width broadening (PWB) [1,15],

\[
N_p = 2x \left[ \frac{T_p}{2T} \right] + 2
\]

(2)

\[
T_p = \frac{2}{\pi T} \sqrt{\pi^2 c^2 T^4 + 4\lambda^2 D^2 z^2}
\]

(3)

where \( T_p \) is the duration width of a broadened Gaussian pulse, \( T \) is the sampling period, and \( \left[ x \right] \) denotes the nearest integer larger than \( x \).

Table 1: The minimum overlap in FDE; \( N_c \): minimum overlap determined by PWB, \( N_S \): minimum overlap determined by CD equalization simulation

<table>
<thead>
<tr>
<th>Fiber length (km)</th>
<th>( N_c )</th>
<th>( N_S )</th>
<th>((N_c-N_S)/N_p )%</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>8</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>40</td>
<td>14</td>
<td>16</td>
<td>14.29</td>
</tr>
<tr>
<td>600</td>
<td>158</td>
<td>176</td>
<td>11.39</td>
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<td>1000</td>
<td>260</td>
<td>288</td>
<td>10.77</td>
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<td>2000</td>
<td>518</td>
<td>576</td>
<td>11.2</td>
</tr>
<tr>
<td>4000</td>
<td>1032</td>
<td>1152</td>
<td>11.63</td>
</tr>
<tr>
<td>6000</td>
<td>1546</td>
<td>1674</td>
<td>8.28</td>
</tr>
</tbody>
</table>

The minimum overlap (or ZP) in the FDE for different fiber length is illustrated in Table 1. The CD coefficient of the fiber is 16 ps/nm/km. The value \( N_p \) represents the minimum overlap (or ZP) calculated from Eq. (2), and the value \( N_S \) represents necessary overlap (or ZP) determined in the dispersion equalization simulation, which will be discussed in the Section 4. We can find that the simulation results achieve a good agreement with the theoretical analysis. It will be demonstrated in our simulation that the performance of CD equalization will degrade drastically, when the overlap (or ZP) in FDE is less than \( N_c \). Therefore, the column 4 in Table 1
provides the significantly meaningful information that the theoretical overlap (or ZP) \( N_{\text{overlap}} \) plus about 15% additional supplement can cover the necessary overlap (or ZP) \( N_{\text{overlap}} \) determined in the numerical simulation, which could achieve the satisfactory CD equalization in practical work for the fiber length up to 6000 km.

For a proper overlap (or ZP) value, a large FFT length may be more efficient. However, it will cost more computational complexity and hardware memory resources [16]. The efficient selection of the FFT-size will be discussed later. In our simulation work, the FFT-size is designated as the double of the overlap (or ZP) unless otherwise stated. Therefore, the FDE algorithms can be applied conveniently for equalizing different fiber dispersion only by determining the required FFT-size.

### 2.5 Optimization of FFT-size

From the above analysis, the required overlap (or ZP) depends on the fiber dispersion to be equalized, and any integer, provided larger than the overlap, can be theoretically selected as the FFT-size. However, an optimal FFT-size can be selected to obtain the minimum complexity for frequency domain equalization [16]. The complexity in FDE for different FFT-size using several classical FFT algorithms (such as radix-2 and radix-4 FFT operation) is evaluated by the number of multiplications per symbol (Mul/Sym), which can be calculated as [16,17],

\[
N_{\text{Mul}} = \frac{N_{\text{FFT}} \cdot \left[ 6C \cdot \log_2(N_{\text{FFT}}) + 3 \right]}{N_{\text{FFT}} - N_{\text{Overlap}} + 1} \tag{4}
\]

where \( N_{\text{FFT}} \) is the FFT-size in FDE, \( N_{\text{overlap}} \) is the required overlap (or ZP) derived from the fiber dispersion, and \( C \) is a constant positive varying for different FFT algorithms. In classical FFT algorithms, \( C = \frac{1}{2} \) corresponds to the radix-2 FFT algorithm (FFT-size equal to power of two), and \( C = \frac{3}{8} \) corresponds to the radix-4 FFT algorithm (FFT-size equal to power of four) [17].

![Fig. 4: The complexity for different FFT-size in FDE](image)

The complexity (defined as multiplications per symbol) for 600 km and 2000 km fibers CD equalization using different FFT-size in FDE is shown in Fig. 4. We can see that the optimum FFT-size values to minimize the complexity for 600 km and 2000 km fibers dispersion equalization are 1024 and 4096 respectively in both of the radix-2 and the radix-4 FFT algorithms.

The optimum FFT-size in FDE for different fiber length to minimize the complexity is illustrated in Table 2. We can find that the radix-4 FFT algorithm can achieve a lower complexity than the radix-2 FFT algorithm. Note that the more sophisticated split-radix FFT algorithms can achieve the lowest complexity for FDE with \( C \leq 1/3 \) in Eq. (4) [16,17].

### Table 2: The optimum FFT-size in FDE for different fiber length

<table>
<thead>
<tr>
<th>Length (km)</th>
<th>Radix-2 FFT ( N_{\text{FFT}} )</th>
<th>Radix-4 FFT ( N_{\text{FFT}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>32 23.04 64</td>
<td>18.53</td>
</tr>
<tr>
<td>40</td>
<td>64 26.35 64</td>
<td>20.71</td>
</tr>
<tr>
<td>600</td>
<td>1024 38.98 4096</td>
<td>30.12</td>
</tr>
<tr>
<td>1000</td>
<td>2048 41.21 4096</td>
<td>32.03</td>
</tr>
<tr>
<td>2000</td>
<td>4096 44.63 4096</td>
<td>34.33</td>
</tr>
<tr>
<td>4000</td>
<td>16384 48.02 16384</td>
<td>36.82</td>
</tr>
<tr>
<td>6000</td>
<td>16384 49.69 16384</td>
<td>38.09</td>
</tr>
</tbody>
</table>

### 3 NRZ-PDM-QPSK coherent system

The setup of the 112-Gbit/s NRZ-PDM-QPSK coherent system implementation in VPI simulation platform is illustrated in Fig. 5 [18]. The electrical data output from the four 28-Gbit/s pseudo random bit sequence (PRBS) generators are modulated into two orthogonally polarized NRZ-QPSK optical signals by two Mach-Zehnder modulators, and then integrated into one fiber channel by a polarization beam combiner (PBC) to form the 112-Gbit/s PDM-QPSK optical signals. Using a local oscillator (LO) in the coherent receiver, the received optical signals are mixed with the LO laser and transformed into four electrical signals after the photodiodes, which are then digitalized by the analog-to-digital converters (ADCs) at twice the symbol rate. The CD coefficient in the transmission fiber is 16 ps/nm/km, and the central wavelengths of the transmitter and the LO lasers are both 1553.6 nm. Here the influences of fiber channel attenuation, polarization mode dispersion, phase noise and nonlinear effects are neglected.

![Fig. 5: The 112-Gbit/s NRZ-PDM-QPSK transmission system](image)
size. We can see that both of the OLA-ZP methods can provide the same acceptable performance as the OLS method.

From Fig. 7 we can see that for a certain fiber length, the three methods can show stable and converged acceptable performance with the increment of the FFT-size. The critical FFT-size values (16 FFT-size for 20 km fiber and 32 FFT-size for 40 km fiber), actually indicate the required minimum overlap (or ZP) value which are 8 overlap (or ZP) samples for 20 km fiber and 16 overlap (or ZP) samples for 40 km fiber. The similar performance demonstrates that for a fixed overlap (or ZP) value, the maximum compensable dispersion in the OLS method is the same in the OLA-ZP methods.

We have demonstrated that the overlap (or ZP) is the pivotal parameter in the FDE, and the FFT-size is not necessarily designated as double of the overlap (or ZP). Figure 8 illustrates that with a fixed FFT-size (4096 samples) the three filters are still able to work well for 4000 km fiber, provided the overlap (or ZP) is larger than 1152 samples (1152=4096×9/32), which indicates the required minimum overlap (or ZP) for 4000 km fiber. The minimum overlap (or ZP) in FDE determined from simulation is illustrated in Table 1.

5 Conclusions

For the first time to our knowledge, we present the detailed comparative analysis for three types of frequency domain equalization, including overlap-save, overlap-add one-side zero-padding and overlap-add both-side zero-padding methods. They are applied to compensate the CD in the 112-Gbit/s NRZ-PDM-QPSK coherent optical transmission system. Our analysis demonstrates that both the two overlap-add zero-padding methods and the overlap-save method can achieve the same performance in frequency domain CD equalization. The required minimum overlap (or zero-padding) is given out by the analytical expression, and the simulation results show the theoretical overlap plus a 15% additional supplement can provide the acceptable equalization performance. The optimum FFT-size in FDE is also analyzed to obtain the minimum computational complexity. The radix-4 FFT algorithm can be employed to achieve nearly the same lowest complexity as the sophisticated split-radix FFT method.

References

[18] www.vaniphotonics.com
Paper V
Phase noise influence in optical OFDM systems employing RF pilot tone for phase noise cancellation

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Summary
For coherent and direct-detection Orthogonal Frequency Division Multiplexed (OFDM) systems employing radio frequency (RF) pilot tone phase noise cancellation the influence of laser phase noise is evaluated. Novel analytical results for the common phase error and for the (modulation dependent) inter carrier interference are evaluated based upon Gaussian statistics for the laser phase noise. In the evaluation it is accounted for that the laser phase noise is filtered in the correlation signal detection. Numerical results are presented for OFDM systems with 4 and 16 PSK modulation, 200 OFDM bins and baud rate of 1 GS/s. It is found that about 225 km transmission is feasible for the coherent 4PSK-OFDM system over normal (G.652) fiber.

1 Introduction
Current coherent optical communications research has focus on achieving high capacity system bit-rates (100 Gb/s – 1 Tb/s) with the possibility of efficient optical multiplexing (MUX) and demultiplexing (DEMUX) on sub-band level (the order of 1 Gb/s). An essential part of the optical system design is the use of Digital Signal Processing (DSP) techniques in both transmitter and receiver in this way eliminating costly hardware implementations of MUX/DEMUX, dispersion compensation, polarization tracking and control, clock extraction etc.

In the core part of the network emphasis has been on long-range (high sensitivity) where coherent (homodyne) system implementations of \(n\)-level Phase-Shift-Keying (nPSK) and Quadrature Amplitude Modulation (nQAM) have proven superior performance. When it comes to efficient high-capacity low granularity optical MUX/DEMUX Orthogonal Frequency Division Multiplexing (OFDM) technology becomes an interesting alternative. The efficient MUX/DEMUX capability of OFDM systems using Inverse Fast Fourier Transformation (IFFT) algorithms in the channel MUX stage (and Fast Fourier Transformation (FFT) algorithms in the channel DEMUX stage) is of special interest in the Metro-/Access parts of the optical network where high system sensitivity is not a prime factor. OFDM systems can be viewed as a sub-carrier multiplexed optical system and – due to the need of a strong “DC” optical carrier wave (in order to avoid clipping distortion effects) – these systems should be expected to have lower sensitivity (shorter reach) than nPSK or nQAM systems with equivalent capacity. However, OFDM systems have other advantages due to the distributed capacity in many tightly spaced signal channels in the frequency domain. These advantages include highly efficient optical reconfigurable optical networks (efficient optical MUX/DEMUX), easy upgrade of transmission capacity using digital software (Digital Inverse Fast-Fourier-Transform (DIFFT) can be used for channel MUX and DFFT for channel DEMUX) and adaptive data provisioning on optical per OFDM-channel basis (i.e. optical ADSL implementation to make transmission agnostic to underlying physical link).

Optical coherent systems can be seen as a parallel technology to currently implemented systems in the radio (mobile) domain. It is important to understand the differences between radio and optical implementations and these implementations and these are mainly that the optical implementations operates at significantly higher transmission speeds than their radio counterparts and that they use signal sources (transmitter and local oscillator lasers) that are significantly less coherent than their radio counterparts. For nPSK and nQAM systems DSP technology in the optical domain is entirely focused on high speed implementation of simple functions such as AD/DA currently operating at 56 Gbaud or below. The use of high constellation transmission schemes is a way of lowering the DSP speed relative to the total capacity.

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Using OFDM as MUX/DEMUX technology and implementing hundreds or thousands channels is an alternative way of very effectively lowering the DSP speed (per channel) and still maintaining 100 Gb/s (or more) system throughput. Both Direct Detection and Coherent (heterodyne) detection is considered for OFDM implementations (DD-OFDM and CO-OFDM systems) and the low channel baud-rate leads to a significant influence of the laser phase noise. Especially for CO-OFDM systems the influence is severe. The theory basis for dealing with the phase noise influence has been presented for radio OFDM systems in [1-4] and several accounts for optical systems can be found in [5-9].

Using nPSK or nQAM systems with DSP based dispersion compensation leads to strong influence of laser phase noise which is further enhanced by equalization enhanced influence of the local oscillator phase noise [10-12]. OFDM systems may use wrapping of the signal in the time domain (cyclic prefix) to account for dispersion effects in this way eliminating the need for DSP based compensation. Using an RF carrier which is adjacent to or part of the OFDM channel grid is an effective way of eliminating the phase noise effect [5] but it has to be noted that the dispersion influenced delay of OFDM channels will make the elimination non-complete and this leads to a transmission length dependent (dispersion enhanced) phase noise effect [7-9]. The purpose of this paper is to investigate this in detail for both DD-OFDM systems and CO-OFDM systems for nPSK and nQAM OFDM channel constellations using an accurate (analytical) model framework which allows direct physical insight into the problem. We will use this to derive important practical OFDM design guidelines.

2 Theoretical analysis

2.1 Instantaneous power representations

The theoretical analysis follows [1-4]. Optical channel plans for the DD-OFDM and CO-OFDM systems with N channels are shown in Fig. 1. It appers that the radio frequency (RF) pilot carrier is transmitted separately from the OFDM band in DD-OFDM (and self-heterodyning techniques are used to extract the OFDM signal in the receiver (Rx)). In CO-OFDM the RF channel is in the center of the OFDM band. The purpose of our analysis is to find the limiting influence of laser phase noise (i.e. the resulting BER-floor which needs to be below the order of 10^-10 in order to make practical use of Forward-Error-Correction (FEC) techniques possible).

In the following we will present the derivation for CO-OFDM systems explicitly whereas for DD-OFDM systems only main results will be given. During a symbol period T the complex envelope (constellation position) of the transmitted OFDM signal (defined as a microwave signal with a frequency relative to the center of the OFDM band) is [1]:

\[ s(t) = e^{j \varphi(t)} \sum_{k=-\infty}^{\infty} a_k e^{j 2 \pi k \nu t} \]  

We note that this is the analogue output after digital inverse fast Fourier transformation (DIFT) of the digitized input sampled with N samples separated by T/N, and each sample specifying one OFDM channel constellation \( a_k \).

The RF carrier is injected into the analogue signal at grid position \( k=0 \) prior to optical modulation that brings \( s(t) \) onto the optical carrier wave [5] – see Fig. 1. Thus, this grid position is not used for data transmission. \( \psi(t) \) denotes the laser phase noise. After coherent detection with an LO laser with the same frequency as the RF the (anologue) signal at the DFFT output of the receiver – including correlation detection - is for bin \( k \) (DFFT coefficient \( f_k \)) [1]:

\[ r_k = \frac{1}{T} \int_{-\infty}^{\infty} s(t) e^{-j 2 \pi k \nu t} dt \]  

This assumes that the dispersion dependent frequency offset between signal bins has been completely compensated (for instance using frequency estimation). In the case of no frequency offset and no phase noise influence orthogonality between the channels means that \( r_k = a_k \).

Taylor expansion is employed to identify the leading order phase noise influence in (2). The resulting Common Phase Error (CPE) for channel \( k \) is (assuming that any resulting constant phase error is ideally corrected [1,4]):

\[ \frac{j}{T} \int_{-\infty}^{\infty} (\psi(t) - \psi(t + k \tau)) dt \]  

where \( \tau = DL^2 \Delta f / c \) \( (D \) is the fiber dispersion coefficient, \( L \) the fiber length, \( \lambda \) the laser transmission wavelength, \( \Delta f \) the frequency separation between OFDM channels and \( c \) is the velocity of light) specifies the dispersion influence (between adjacent OFDM channels). The Inter-Carrier Interference (ICI) is:

\[ \frac{j}{T} \sum_{k=-\infty}^{\infty} a_k \int_{-\infty}^{\infty} (\psi(t) - \psi(t + k \tau)) e^{-j 2 \pi k \nu t} dt \]  

The use of a common RF pilot tone in the system [5, 8] - which is complex conjugated and multiplied with the OFDM signal channels - is modeled as providing a common phase reference of \( \psi(t) \) thus eliminating the phase noise influence which is not due to dispersion for the CPE and the ICI. The filtering by the correlation receiver must be accounted for – see (3). The effect of filtering is to reduce the phase difference variance by a factor of 2/3 – see [13, 14]. Thus for CPE-influence for bin \( k \) the phase noise variance is \( \sigma^2_{\psi} = 4 \pi a k \nu_k \) \( (1 \leq k \leq N/2) \). \( \Delta \nu_k \) denotes the Intermediate Frequency (IF) signal laser linewidth which is the sum of linewidths from the transmitter (Tx) and local oscillator (LO).

For DD-OFDM systems a similar derivation as above specifies the CPE phase noise variance \( \sigma^2_{\psi} = 4 \pi a k (N + k) \nu / 3 \) \( (1 \leq k \leq N) \) where \( \nu \) denotes the Tx linewidth.
For the ICI influence we note that we have an OFDM symbol dependence – see (4). We will follow [1] in assuming that the ICI contribution from the symbol $r$ ($r \neq k$) is an independent Gaussian distributed contribution. This is a reasonable assumption in the case of many OFDM channels. Using (4) the ICI disturbance of the constellation phase - described through the in-phase contribution from the integrand - can be approximated to have zero mean value and a variance of $\sigma_{\phi,r} = a_r / a_i \sqrt{4 \pi \nu \Delta f / (3 \sqrt{2})}$ for CO-OFDM systems (and $\sigma_{\phi,r} = a_r / a_i \sqrt{4 \pi \nu \Delta f / (3 \sqrt{2})}$ for DD-OFDM systems). This consideration allows a specification of the worst case ICI influence which happens in the case of a $k$-constellation close to the origin and an $r$-constellation far away from the origin. It is probably reasonable to describe the total ICI influence averaging over all possible constellation points of the interfering OFDM symbol $r$. Denoting this average by $\langle \sigma_{\phi,r} \rangle$ we have a total (worst case) ICI influence given by the variance $\sigma_{\phi} = \sum_k \langle \sigma_{\phi,r} \rangle$. The total phase noise variance that affects the constellation phase detection for channel $k$ is now $\sigma_{\phi} = \sigma_{\phi_{\text{const}}} + \sigma_{\phi_{\text{CPE}}}$. Following [11, 12] the Bit-Error-Ratio for nPSK channel modulation (and for 2nQAM) is approximately:

$$BE_{r} = \frac{1}{2 \log_e n} \text{erfc} \left( \frac{\pi}{n \sqrt{2 \sigma_{r}}} \right)$$

(5)

With $n = n$ for nPSK, $n = 2n$ for 2nQAM. The total OFDM system BER considers contributions from $N$ OFDM symbols (see [8])

$$BE_{r} = \frac{1}{N} \sum_{r=0}^{N} BE_{r}$$

(6)

for CO-OFDM systems, and

$$BE_{r} = \frac{1}{N} \sum_{r=0}^{N} BE_{r}$$

(7)

for DD-OFDM systems.

3 Results and discussions

We consider a normal transmission fiber ($D=16$ psec/nm/km) transmission distances up to 500 km, transmission wavelength $\lambda = 1.55 \mu m$, $c = 3 \times 10^{8}$ m/sec, OFDM channel separation $\Delta f = 1$ GHz i.e. baud rate 1 GS/s (symbol time $T = 1$ nsec), channel modulation as 4 and 16 PSK, number of channels $N = 200$. We select an IF linewidth of $\Delta \nu = 4$ MHz in the CO-OFDM system and a Tx linewidth of $\Delta \nu = 4$ MHz in the DD-OFDM system.

A sketch of the CO-OFDM and DD-OFDM channel plan is shown in Fig. 1 with the position of the RF phase compensating carrier indicated. The phase noise influence depends on the frequency difference between RF carrier and the OFDM signal bins. It is obvious that in CO-OFDM systems a lower influence of phase noise is expected than in DD-OFDM systems due to the relative RF channel position.

Fig. 2 shows the phase noise variance for different bin-positions in the OFDM channel grid and it is seen that the further away from the RF carrier the more phase noise influence results. As expected the phase noise influence is mostly pronounced for DD-OFDM.

The results of Fig. 2 are transformed into resulting BER values in Fig. 3 (using (5)) and using (6) and (7) the averaged BER is evaluated in Fig. 4. It is apparent that the phase noise influence is mostly pronounced for DD-OFDM systems and most severely for higher constellations (16 PSK in this example). In order to allow practical use of Forward-Error-Correction (FEC) a phase noise error-rate floor should be below the order of $10^{-4}$. It is seen that in order to realize a BER floor below the order of $10^{-4}$ a transmission distance of about 225 km (40 km) can be realized for CO-OFDM-4PSK (DD-OFDM-4PSK) systems whereas much shorter
distances of about 10-20 km is allowed for the 16PSK systems.

![Graph](image1)

(a)

![Graph](image2)

(b)

Fig. 3: \(BER_f\) (equation (5)) versus fiber length for different channels (k-values as indicated) for 8 and 16 PSK based (a) CO-OFDM and (b) DD-OFDM systems with 1 GS/s OFDM channels.

![Graph](image3)

Fig. 4: Total averaged BER (equations (6-7)) for 4 and 16 PSK CO-OFDM and DD-OFDM with 200 channels each with a baud rate of 1GS/s.

4 Conclusion

For coherent and direct-detection orthogonal frequency division multiplexed (CO-OFDM and DD-OFDM) systems employing Radio Frequency (RF) phase cancellation the influence of signal laser phase noise is evaluated in the ideal case where the dispersion induced frequency offset as well as any constant phase error are ideally compensated. Novel analytical results for the common phase error (CPE) and for the (modulation dependent) inter carrier interference (ICI) are derived based upon Gaussian statistics for the laser phase noise. (The Gaussian assumption for the ICI influence must be further investigated in future research.) In the derivation it is accounted for that the laser phase noise is filtered in the correlation OFDM signal channel detection. Numerical results are presented for OFDM systems with 4 and 16 PSK modulation, 200 OFDM signal bins and baud rate of 1 GS/s and using normal (G.652) fiber for transmission. An Intermediate Frequency linewidth of 4 MHz is considered for CO-OFDM and a transmitter laser linewidth of 4 MHz is considered for DD-OFDM systems. It is found that in order to realize a phase noise induced BER floor below the order of \(10^{-6}\) a transmission distance of 225 km is feasible for CO-OFDM with 4PSK modulation whereas a distance of about 40 km is obtained for the equivalent DD-OFDM system. Much shorter distances of about 10-20 km is allowed for system using 16PSK modulation. Thus, it is possible to use normal DFB lasers (with linewidths of 1-10 MHz) in OFDM systems intended for shorter-range (access/metro) use.

An alternative system implementation must be used for long range (in the order of 1000 km transmission distance) OFDM applications with DFB-type transmitter and local oscillator lasers. Such long range systems should use RF pilot tone phase noise cancellation, coherent detection and a dispersion compensating fiber at the entrance of the coherent receiver to balance (at least roughly) the fiber dispersion. Longer range systems can also be implemented using lasers with sub-MHz linewidths and without the need for dispersion compensation fiber in the transmission path; but it has to be noted that sub-MHz (external cavity based) lasers are more expensive and have life-time problems compared to DFB lasers.

The current model is simplified in considering only dispersion induced phase noise and not accounting for the dispersion induced frequency offset. To model the practical ICI influence of the frequency offset is an important future research task which should also consider the combined influence of additive noise and phase noise. Also modeling should be performed including the practical implementation of MUX/DEMUX using discrete Fourier Transformation techniques and the implementation of the conversion between digital and analogue signal representations in the Tx and Rx.

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Paper VI
Digital Compensation of Chromatic Dispersion in 112-Gbit/s PDM-QPSK System

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ABSTRACT

High bit rates optical communication systems pose the challenge of their tolerance to linear and nonlinear fiber impairments. Coherent optical receivers using digital signal processing techniques can mitigate the fiber impairments in the optical transmission system, including the chromatic dispersion equalization with digital filters. In this paper, an adaptive finite impulse response filter employing normalized least mean square algorithm is developed for compensating the chromatic dispersion in a 112-Gbit/s polarization division multiplexed quadrature phase shift keying coherent communication system, which is established in the VPI simulation platform. The principle of the adaptive normalized least mean square algorithm for signal equalization is analyzed theoretically, and at the meanwhile, the taps number and the tap weights in the adaptive finite impulse response filter for compensating a certain fiber chromatic dispersion are also investigated by numerical simulation. The chromatic dispersion compensation performance of the adaptive filter is analyzed by evaluating the behavior of the bit-error-rate versus the optical signal-to-noise ratio, and the compensation results are also compared with other present digital filters.

Key Words: Coherent optical receivers, digital signal processing, chromatic dispersion equalization, polarization division multiplexed quadrature phase shift keying, normalized least mean square algorithm, adaptive finite impulse response filter

1. INTRODUCTION

Fiber impairments such as chromatic dispersion (CD) severely impact the performance of high speed optical fiber transmission systems1,2. Although current systems use dispersion compensation fibers (DCFs) to compensate the chromatic dispersion distortion, this increases the cost of the transmission systems. Digital coherent receivers allow equalization for linear transmission impairments in the electrical domain3,4, and have become a promising alternative approach to dispersion compensation fibers. While coherent detection was experimentally demonstrated as early as 1979, its use in commercial systems has been hindered by the additional complexity, due to the need to track the phase and polarization of the incoming signal5. In a digital coherent receiver these functions are implemented in the electrical domain leading to a dramatic reduction in complexity. Furthermore since coherent detection maps the entire optical field within the receiver bandwidth into the electrical domain it maximizes the efficacy of the signal processing. This allows fiber impairments which have traditionally limited high bit rate systems to be overcome adaptively6-15.

It is possible to completely compensate chromatic dispersion with zero penalty in coherent detection receivers by means of electronic equalization techniques13-15. Several digital filters have been applied to compensate the chromatic dispersion in the time domain and the frequency domain14,15. In this paper an adaptive finite impulse response (FIR) filter employing normalized least square (NLMS) algorithm is developed to compensate CD in a 112-Gbit/s polarization division multiplexed quadrature phase shift keying (PDM-QPSK) coherent optical transmission system. The principle of the NLMS algorithm and the structure of the adaptive filter are analyzed and investigated, and the influence of the step size on the convergence of the tap weights in the adaptive filter is also discussed and illustrated. The performance of the NLMS filter is characterized by evaluating the behavior of the bit-error-rate (BER) versus optical signal-to-noise ratio (OSNR) in the PDM-QPSK system using VPI simulation platform16, and compared with a fiber dispersion FIR (FD-FIR) filter and a blind look-up adaptive filter14,15. The tap weights distribution in the NLMS adaptive filter and the FD-FIR

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filter are investigated and compared in the CD equalization numerical simulation. The required taps number in the NLMS filter and FD-FIR filter and the fast Fourier transform (FFT) size in the blind look-up filter are also studied for compensating different fiber chromatic dispersion. The characteristics of the three digital filters are gradually analyzed and illustrated by comparing their CD compensation simulation results.

2. PRINCIPLE OF NORMALIZED LEAST MEAN SQUARE ADAPTIVE FILTER

The normalized least mean square algorithm is an iterative adaptive algorithm that can be used in the highly time varying signal environment. The NLMS algorithm uses the estimates of the gradient vector from the available data. The NLMS algorithm incorporates an iterative procedure that makes successive corrections to the weights vector in the direction of the negative of the gradient vector which eventually leads to the minimum mean square error. The transfer function $\tilde{h}(n)$ of the NLMS adaptive filter is given by

$$\tilde{h}(n+1) = \tilde{h}(n) + \mu \frac{e(n)}{\|\hat{x}(n)\|^2} \hat{x}(n)$$

(1)

$$e(n) = d(n) - \tilde{h}(n)\hat{x}(n)$$

(2)

where $\hat{x}(n)$ is the input signal vector, $\hat{x}(n)$ is the Hermitian of $\hat{x}(n)$, $d(n)$ is the desired symbol, $e(n)$ represents the estimation error between the output signal and the desired symbol, $e'(n)$ is the conjugation of estimation error $e(n)$, and $\mu$ is a coefficient called step size parameter. The transfer function vector $\tilde{h}(n)$ is updated in a symbol-by-symbol iterative manner, and achieves convergence when $e(n)$ approaches to zero.

In order to guarantee the convergence of the transfer function $\tilde{h}(n)$, the step size $\mu$ needs to satisfy the condition of $0 < \mu < 1/\lambda_{\text{max}}$, where $\lambda_{\text{max}}$ is the largest eigenvalue of the correlation matrix $R = \hat{x}(n)\hat{x}^H(n)$. The convergence speed of the algorithm is inversely proportional to the eigenvalue spread of the correlation matrix $R$. When the eigenvalues of $R$ are spread, convergence may be slow. The eigenvalue spread of the correlation matrix is estimated by computing the ratio of the largest eigenvalue to the smallest eigenvalue of the matrix. If the step size $\mu$ is chosen to be very small, then algorithm converges very slowly. A large value of $\mu$ may lead to a faster convergence, but may be less stable around the minimum value, an optimal value of step size is usually selected as 0.1.

![Figure 1. Schematic of the adaptive normalized least mean square filter.](image)

The schematic of the linear adaptive normalized least mean square equalizer with $N$ weights is shown in Fig. 1, where $T$ is the sampling period, and coefficient $W_i$ is the tap weights corresponding to the NLMS transfer function vector.
The linear adaptive NLMS equalizer consists of a tapped delay line that stores data samples from the input signal. Once per symbol period, the equalizer outputs a weighted sum of the values in the delay line and updates the tap weights to prepare for the next symbol period. The tap weights value are updated according to the estimation error between the output signal and the desired signal.

3. SIMULATION INVESTIGATION OF PDM-QPSK TRANSMISSION SYSTEM

The installation of the 112-Gbit/s PDM-QPSK coherent optical transmission system established in the VPI simulation platform is illustrated in Fig. 2. The electrical data from four 28-Gbit/s pseudo random bit sequence (PRBS) generators are modulated into two orthogonally polarized QPSK optical signals by two Mach-Zehnder modulators, which are then integrated into one fiber transmission channel by a polarization beam combiner to form the 112-Gbit/s PDM-QPSK optical signal. Using an optical local oscillator (LO) in the coherent receiver, the received optical signals are mixed with the LO laser to be transformed into four electrical signals by the photodiodes and then digitalized by the analog-to-digital converters (ADCs) at double sampling rates. Thus the impairments of chromatic dispersion in the transmission channel could be equalized with diverse digital filters.

4. SIMULATION RESULTS OF CHROMATIC DISPERSION COMPENSATION

To illustrate the features of the NLMS filter, the compensation of CD from a standard single mode fiber with dispersion coefficient \( D = 16 \, \text{ps/(nm\cdot km)} \) are investigated and analyzed by comparing with the FD-FIR filter. Compared with the iteratively updated tap weights in the NLMS adaptive filter, the tap weights in the FD-FIR filter have a relatively simple specification, the tap weights \( a_k \) in the FD-FIR filter is given by

\[
a_k = \frac{f e T^2}{D \lambda z} \exp\left(-j \frac{\pi e T^2}{D \lambda z} k^2\right) - \frac{N}{2} \leq k \leq \frac{N}{2}
\]

\[
N = 2 \times \left[ \frac{|\log_{10}z|}{2e T^2} \right] + 1
\]

where \( D \) is the fiber chromatic dispersion coefficient, \( \lambda \) is the central wavelength of the transmission optical wave, \( z \) is the fiber length in the transmission channel, and \( N \) is the required minimum taps number for compensating the fiber dispersion.

The distribution of converged tap weights in the NLMS filter for 20 km fiber are shown in Fig. 3(a), (b) and (c), respectively, and the tap weights of the FD-FIR filter for 20 km fiber are shown in Fig. 3(d). We can see that in the NLMS filter, the central tap weights take more dominant roles in the chromatic dispersion equalization, while in the
FD-FIR filter, the tap weights magnitudes are constant for a certain length fiber, whereas the real parts and the imaginary parts of the tap weights vary periodically with the tap orders increasing. For a fixed fiber dispersion, the tap weights in NLMS adaptive filter approach to zero, when the corresponding taps order exceeds a certain value, and this value indicates the least required taps number for compensating the chromatic dispersion effectively. This also illustrates the optimization characteristic of the NLMS adaptive algorithm. While in the FD-FIR filter, the excessive tap weights do not approach to zero when the chromatic dispersion in the transmission fiber channel is equalized effectively.

Figure 3. Tap weights of NLMS and FD-FIR filters (Tap orders are centralized), (a) Magnitudes of NLMS filter weights, (b) Real parts of NLMS filter weights, (c) Imaginary parts of NLMS filter weights, (d) FD-FIR filter tap weights.

The chromatic dispersion compensation results are shown in Fig. 4, where Fig. 4(a) indicates the CD equalization for 20 km and 6000 km fibers using the NLMS filter and the FD-FIR filter respectively, both digital filters using 9 taps for 20 km fiber and 2411 taps for 6000 km fiber, and Fig. 4(b) shows the influence of taps number increment on CD compensation effects for 20 km fiber with the two digital filters.

Figure 4. Performances of CD compensation using three digital filters neglecting fiber loss, (a) BER with OSNR using NLMS and FD-FIR filters, (b) BER with taps number using NLMS and FD-FIR filters at OSNR 14.6 dB, (c) CD equalization using blind look-up filter at OSNR 14.6 dB.

It could be seen in Fig. 4 that the NLMS adaptive filter could achieve the same CD compensation performance as the FD-FIR filter, and the NLMS filter have a better improvement with the increment of taps number. For an acceptable CD compensation performance (BER better than $10^{-3}$), the required minimum numbers of taps are 9 taps in the NLMS filter and 9 taps in the FD-FIR filter for 20 km fiber, and 2305 taps in the NLMS filter and 2411 taps in the FD-FIR filter for 6000 km fiber, and the NLMS filter needs fewer taps than the FD-FIR filter for an acceptable compensation effect with the fiber length increasing.
Table 1. Necessary taps number in NLMS and FD-FIR filters and FFT-sizes in blind look-up filter for a certain length fiber with CD coefficient $D = 16 \text{ps/(nm} \cdot \text{km)}$

<table>
<thead>
<tr>
<th>Fiber length (km)</th>
<th>Taps number in NLMS filter</th>
<th>Taps number in FD-FIR filter</th>
<th>FFT-sizes in blind look-up filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>9</td>
<td>9</td>
<td>32</td>
</tr>
<tr>
<td>40</td>
<td>15</td>
<td>17</td>
<td>32</td>
</tr>
<tr>
<td>400</td>
<td>115</td>
<td>161</td>
<td>256</td>
</tr>
<tr>
<td>4000</td>
<td>1495</td>
<td>1607</td>
<td>2048</td>
</tr>
<tr>
<td>6000</td>
<td>2305</td>
<td>2411</td>
<td>2048</td>
</tr>
</tbody>
</table>

To investigate the required taps number in NLMS adaptive filter and FD-FIR filter for a fixed fiber dispersion, the required minimum taps number in the filters for different fiber length with the same CD coefficient are shown in Table 1. We could see that the NLMS adaptive could use fewer taps than FD-FIR filter to compensate the CD with the fiber length increasing.

The structure of the frequency domain blind look-up adaptive equalizer is shown in Fig. 5. In the blind look-up filter, the digitalized electrical signals are firstly transformed by the fast Fourier transform (FFT) operation and then multiplied by the inverse transfer function of the dispersive channel in frequency-domain, and then the processed signals are transformed into time domain signals by the inverse fast Fourier transform (IFFT) operation. Starting at the initial value of 240 ps/nm, the dispersion applied to the adaptive filter is increased in steps of 2 ps/nm up to the negative of maximum possible dispersion in the 112-Gbit/s PDM-QPSK coherent optical transmission system. The simulation results of CD compensation for 20 km fiber using the blind look-up filter with various FFT-sizes $N$ are illustrated in Fig. 4(c), the dotted line in Fig. 4(c) is the back-to-back measurement result.

Figure 5. Schematic of blind look-up adaptive filter.

In the above simulation results, we could see that the CD equalization of the NLMS adaptive filter could reach the same performance as the FD-FIR filter and the blind look-up adaptive filter with 32 symbols in the FFT operation.

5. CONCLUSIONS

In this paper an adaptive finite impulse response digital filter employing normalized least mean square algorithm is developed to compensate the chromatic dispersion in a 112-Gbit/s polarization division multiplexed quadrature phase shift keying coherent optical transmission system. The performance of the NLMS adaptive filter for CD equalization is compared with the FD-FIR filter and the blind look-up adaptive filter. The NLMS adaptive filter shows the best performance in CD compensation, whereas it requires slow iteration for guaranteed convergence. The FD-FIR filter affords simple analytical tap weights specification, whereas giving slightly poor performance with taps number increasing. The blind look-up filter shows the same performance as the NLMS filter, furthermore, it will decrease the computational complexity when the filter length is large.
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Paper VII
Digital Chromatic Dispersion Compensation in Coherent Transmission System Using a Time-Domain Filter

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Abstract: We demonstrate the chromatic dispersion equalization employing a time-domain filter in a 112-Gbit/s polarization division multiplexed quadrature phase shift keying coherent system. The required tap number of the filter is analyzed from anti-aliasing and pulse broadening. The dynamic range of the filter is evaluated by using different number of taps.

OCIS codes: (060.1660) Coherent communications; (060.2330) Fiber optics communications

1. Introduction

Fiber impairment such as chromatic dispersion (CD) severely impacts the performance of high speed optical transmission systems [1,2]. Digital coherent receivers allow equalization of fiber dispersion in the electrical domain [3], and have become the most promising alternative approach to dispersion compensation fibers (DCFs). In this paper, a time-domain fiber dispersion finite impulse response (FD-FIR) filter is developed to compensate the CD in a 112-Gbit/s non-return-to-zero polarization division multiplexed quadrature phase shift keying (NRZ-PDM-QPSK) coherent transmission system. The principle of the FD-FIR filter is analyzed by numerical simulations. The lower limit of the required tap number is determined for the first time by the dispersion induced pulse broadening, and the upper limit is calculated based on the analysis of anti-aliasing. The dynamic range of the filter is investigated by the variation of tap number in CD equalization.

2. Principle of time-domain FD-FIR filter

The FD-FIR filter is a time-domain feed-forward digital filter, of which the tap weight \(a_k\) can be described as the following expressions [3],

\[
a_k = \sqrt{\frac{j\pi c T^2}{D \lambda z}} \exp \left(-j \frac{\pi c T^2}{D \lambda z} k^2\right) \quad \text{for} \quad -N^d \leq k \leq N^d
\]

where \(D\) is the CD coefficient, \(\lambda\) is the central wavelength of optical wave, \(z\) is the fiber length, \(T\) is the sampling period, \(\left[ x \right]\) denotes the nearest integer less than \(x\), and \(N^d\) is the required tap number determined by the anti-aliasing [3], which means the pass-band of FD-FIR filter needs to be lower than the Nyquist frequency.

Meanwhile, we can also determine the filter length based on the broadening of a pulse propagating in the dispersive fiber channel [2,4]. For the Gaussian pulse, the tap number of the FD-FIR filter can be calculated according to the broadened pulse duration [2,4],

\[
N^p = 2 \sqrt{\frac{1}{\pi T^2} \left( \pi^2 c^2 T^4 + 4 \pi^2 D^2 z^2 \right)} + 1
\]

where \(\left[ x \right]\) denotes the nearest integer larger than \(x\).

Table 1. The tap number calculated by pulse broadening and anti-aliasing (D=16 ps/nm/km)

<table>
<thead>
<tr>
<th>Fiber length (km)</th>
<th>Tap number (N^p) by pulse broadening</th>
<th>Tap number (N^a) by anti-aliasing</th>
<th>(N^p/N^a) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>7</td>
<td>9</td>
<td>77.8</td>
</tr>
<tr>
<td>600</td>
<td>157</td>
<td>243</td>
<td>64.6</td>
</tr>
<tr>
<td>1000</td>
<td>259</td>
<td>403</td>
<td>64.3</td>
</tr>
<tr>
<td>2000</td>
<td>517</td>
<td>807</td>
<td>64.1</td>
</tr>
<tr>
<td>4000</td>
<td>1031</td>
<td>1615</td>
<td>63.8</td>
</tr>
</tbody>
</table>

The tap numbers for different fiber lengths analyzed from pulse broadening and anti-aliasing are illustrated in Table 1. We find that most of the tap numbers obtained from pulse broadening are around 60% of the value calculated from the anti-aliasing analysis, which is consistent with the reported empirical factor of 0.6 [3]. The tap numbers derived from pulse broadening and anti-aliasing are considered as the lower and upper bounds on the required number of taps for an effective CD compensation, respectively.
3. Simulation investigation of PDM-QPSK transmission system

The setup of the 112-Gbit/s NRZ-PDM-QPSK coherent transmission system established in the VPI simulation platform is illustrated in Fig. 1 [5]. The data output from four 28-Gbit/s pseudo random bit sequence (PRBS) generators are modulated into two orthogonally polarized NRZ-QPSK optical signals by two Mach-Zehnder modulators, which are then integrated into one fiber channel by a polarization beam combiner to form the 112-Gbit/s NRZ-PDM-QPSK optical signal. In the coherent receiver, the received optical signals are mixed with the local oscillator (LO) laser to be transformed into four electrical signals by the photodiodes, which are then digitalized by the 8-bit analog-to-digital convertors (ADCs) at twice the symbol rate. The central wavelength of the transmitter and the LO lasers are both 1553.6 nm, and the transmission fibers are all with the CD coefficient D=16 ps/nm/km.

![Fig. 1. Schematic of 112-Gbit/s NRZ-PDM-QPSK coherent optical transmission system](image)

4. Simulation results

The CD equalization for 20 km and 600 km fibers using the FD-FIR filter with different number of taps are illustrated in Fig. 2. The results indicate that the filter achieves the best performance when employing the tap number between the values derived from pulse broadening and anti-aliasing. The performance of the filter will degrade, if the tap number is less than the number N^o or exceeds the number N^A. This is because the inadequate taps can not generate sufficient delay for dispersion compensation, and the redundant taps will lead to the pass-band of the filter exceeding the Nyquist frequency to result in the aliasing phenomenon. Figure 2(a) also shows that the FD-FIR filter does not achieve a satisfactory CD equalization performance for 20 km fiber by using either the tap number (N^o=9) obtained from anti-aliasing or the tap number (N^A=7) derived from pulse broadening. This penalty can be compensated by using a post-added 3-tap least mean square (LMS) adaptive filter or by increasing the sampling rate from 2 samples per symbol (Sa/Sy) to 8 Sa/Sy in the ADCs modules, as shown in Fig. 2(a). In the x-axis of 8 Sa/Sy, the T value in Eq. (2) and Eq. (3) is decreased by a factor of 4.

![Fig. 2. CD compensation with different tap number at OSNR 14.8 dB, (a) 20 km fiber, (b) 600 km fiber](image)

5. Conclusions

In this paper, the time-domain FD-FIR filter is developed to equalize the CD in a 112-Gbit/s NRZ-PDM-QPSK coherent system. For the first time to our knowledge, we demonstrate the analytical expression of the lower bound on the required tap number base on the dispersion induced pulse broadening, which is consistent with the empirical factor [3]. Simulation results indicate that the filter shows the best performance when employing the tap number between the values calculated from the analysis of pulse broadening and anti-aliasing.

6. References