Bayesian Inference in Structural Second-Price Auctions

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Abstract

The aim of this thesis is to develop efficient and practically useful Bayesian methods for statistical inference in structural second-price auctions. The models are applied to a carefully collected coin auction dataset with bids and auction-specific characteristics from one thousand Internet auctions on eBay. Bidders are assumed to be risk-neutral and symmetric, and compete for a single object using the same game-theoretic strategy. A key contribution in the thesis is the derivation of very accurate approximations of the otherwise intractable equilibrium bid functions under different model assumptions. These easily computed and numerically stable approximations are shown to be crucial for statistical inference, where the inverse bid functions typically needs to be evaluated several million times.

In the first paper, the approximate bid is a linear function of a bidder's signal and a Gaussian common value model is estimated. We find that the publicly available book value and the condition of the auctioned object are important determinants of bidders' valuations, while eBay's detailed seller information is essentially ignored by the bidders. In the second paper, the Gaussian model in the first paper is contrasted to a Gamma model that allows intrinsically non-negative common values. The Gaussian model performs slightly better than the Gamma model on the eBay data, which we attribute to an almost normal or at least symmetrical distribution of valuations. The third paper compares the model in the first paper to a directly comparable model for private values. We find many interesting empirical regularities between the models, but no strong and consistent evidence in favor of one model over the other. In the last paper, we consider auctions with both private-value and common-value bidders. The equilibrium bid function is given as the solution to an ordinary differential equation, from which we derive an approximate inverse bid as an explicit function of a given bid. The paper proposes an elaborate model where the probability of being a common value bidder is a function of covariates at the auction level. The model is estimated by a Metropolis-within-Gibbs algorithm and the results point strongly to an active influx of both private-value and common-value bidders.

Keywords: Asymmetry, Bid function approximation, Common values, Gamma model, Gaussian model, Markov Chain Monte Carlo, Private values, Variable selection, Internet auctions.

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List of Included Papers

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Contents

1 Introduction

1.1 Historical background

Auctions have been used since antiquity for the sale of a variety of objects. Ancient Romans used auctions in commercial trade to liquidate property and estate goods. It is not known if the bidding process was increasing or decreasing. However, since the word "actus" in Latin means increasing, it is assumed that auctions were held in an increasing fashion. In 193 A.D. the Praetorian Guard sell off the entire Roman Empire by means of an auction.

Nowadays, auctions are widely used in many areas and account for a huge volume of economic transactions. Private firms sell products including real estates, fresh flowers, fish, houses and cars. Governments offer contracts through procurement auctions and every week sell foreign exchange, bills and bonds through auctions. During the last decade, Internet auctions have gained wide popularity where billions of dollars are turned over every day. Probably the biggest auction in the world is currently the keyword search auctions on Google.

An auction can be summarized as a bidding mechanism, described by a set of auction rules that specifies how the winner is determined and how much the bidder has to pay. There exists many different auction rules, but four basic types of auctions, where the object is awarded to the bidder with the highest bid, are particularly common and referred to as standard auctions. They are divided into open and sealed-bid auctions. The open auctions inlude the ascending-bid or English auction and the descending-bid or Dutch auction, while the first-price and second-price auctions are sealed-bid auctions. In oral auctions, like the English auction, bidders note each other's bids and can make counteroffers. In sealed-bid auctions the bidders submit only one bid simultaneously without revealing them to others.

The English auction is the oldest auction form and typically starts with low bids and increases in small predetermined portions until only one bidder is left. The Dutch auction is the counterpart to the English auction. Here the auctioneer begins at a usually high price and gradually lowers it until someone makes a sign to claim the item at the current price. In both first-price and second-price auctions the bidder with the highest bid wins, but in a second-price auction the winning bidder pays an amount equal to the next highest bid in contrast to the payment of the highest bid in first-price auctions.

Since the pioneering work of Vickrey (1961) the theory of auctions as games of incomplete information has developed extensively, especially over the last decades. There has been a number of economic theorists that made a considerable amount of work in understanding the factors influencing auction prices for different types of goods. Klemperer (1999) outlines practical, empirical and theoretical reasons why auction theory is important. First, auctions constitute a market with a huge volume of economic transactions. This is especially true with the advent of Internet auctions in recent years where high-quality datasets are readibly available. Second, auctions are simple and can be explicitly modeled with well-defined game-theoretic forms that provide a very valuable platform for testing economic theory. Finally, through much fundamental theoretical work, auction theory has been important as a tool of understanding other methods of price formation in other competetive markets.

1.2 Valuations

Wolfstetter (1996) states that auctions are essentially used for rapid sales, to reveal the information about buyers' valuations and to prevent dishonest dealing between the seller's agent and the buyer. A key feature is the asymmetries in information. If the seller would knew the bidders' valuations, he could just offer the object to the bidder with the highest valuation at a price just below what the bidder is willing to pay. The valuations of the bidders are often classified into one of two standard paradigms: the independent private-value or the pure common-value paradigm. Within the private-value model each bidder knows his valuation (the value) and knowledge of other bidders' valuation would not affect a bidder's valuation. This is a reasonable model if the object for sale is used for consumption, e.g. a piece of furniture, a painting or private collectibles.

In the pure common-value model, bidders use their own private information to estimate the unknown value of the object that is the same for all bidders. Typical examples include objects that are derived from an unknown market price at the time of bidding, e.g. for the sale of oil contracts with an unknown amount of oil. The common-value model is a special case of a general specification called one of interdependent values. In this general setting, the bidders have only partial information regarding the value, which may be different for different bidders, and would be affected of knowing the information that other bidders posess. Each bidder uses his private information to estimate the value of the object.

1.3 Bid equilibrium

A bidder's strategy can be defined as a mapping from a conceived value of the object to a final bid. The strategic mapping from a bidder's value to a bid is the so called bid function. Nash (1951) proved for a general, finite non-cooperative game that there always exists at least one equilibrium point. In the setting of auctions this means that there always exist an equilibrium strategy that maps a bidder's value to an equilibrium bid. The consequence of the bid equilibrium is that no bidder (player) in the auction (game) can succeed with a better strategy given the strategy of the other players.

In auction theory, the equilibrium bidding strategies depend on the type of auction and the nature of the buyers and seller(s) in the auction. In this thesis, we typically consider second-price auctions with a symmetric equilibrium, in which all bidders follow the same strategy. We also assume that bidders are risk-neutral, which means that each bidder seeks to maximize his expected utility by maximizing his expected profit. The profit is defined as the difference between the bidder's value of the object and the bidder's payment. Risk-neutral bidders are commonly assumed in the literature and is a special case of risk-averse bidders, where the bidders seek to maximize their expected utility functions.

In second-price auctions with private-value bidders, Vickrey (1961) showed that it is a dominant strategy for a bidder to bid his value. The strategic problems for common-value bidders in second-price auctions are much harder, since their mapping from a value to a bid depends on the other bidders' distribution of values. The bidder faces a simple trade-off. Increasing the bid increases the probability of winning, while at the same time decreases a potential profit if the bidder wins. The expected profit for a bidder is calculated as the size of the profit from winning times the probability of winning. By maximizing the expected profit with respect to the bidder's bid, the equilibrium bid function can be derived.

In an influential article, Milgrom and Weber (1982) derive the equilibrium bid function for a symmetric second-price common value auction. In general, common value models are much more technically challenging than the models of private values. This makes it, in practice, difficult to specify distributional assumptions of valuations that yield closed-form solutions of the equilibrium bid function or at least neat implicit forms. A handful closed-form solutions have been derived, but mostly for highly specialized models, see e.g. Kagel and Levin (1986), Matthews (1984), and Levin and Smith (1991).

The lack of closed form solutions has two major drawbacks. First, it is hard to see how the bid function depends on various distributional components of the model, which makes it more difficult to bring out model characteristics. Second, to evaluate the bid function one has to make use of numerical integration, which is very time demanding. This is a crucial step for econometric analysis of auction data (e.g. likelihood/Bayesian estimation) where the equilibrium bid function has to be evaluated over and over again. Bajari and Hortacsu (2003) reduce the computational complexity significantly in their model by exploiting a linearization property, but the inverse bid function in the very complicated likelihood function still needs to be evaluated by time-consuming numerical integration.

1.4 Revenue equivalence principle

At the late seventies the major contributions came in the mechanism-design field of auction theory. Roughly during the same time, independent of each other, Myerson (1981) and Riley and Samuelson (1981) generalized Vickrey's results about the equivalence in expected seller revenue for many different auctions, including the four standard auctions. As Klemperer (1999) mention, in his broad survey of the literature in auction theory, the theorem is so fundamental that any reader who is unfamiliar with the result is strongly urged to learn it.

In short, the theorem of *revenue equivalence principle* can be described as follows. Assume that risk-neutral, private-value bidders draw their values independently from the same distribution. Then, any symmetric and increasing equilibrium of any standard auction yields the same expected revenue to the seller. The counterpart of this principle to interdependent values and affiliated (correlated) signals is called the *revenue ranking (linking) principle*, where the highest expected seller revenue is obtained in English auctions, followed by the second-price and first-price auctions.

1.5 The winner's curse

The winner's curse is by far the most highlighted phenomenon in common value auctions. The concept origins from the *curse* of winning the auction when the price exceeds the unknown market value at the time of bidding. Wilson (1969) introduced the common-value model and developed the first closed-form equilibrium analysis of the winner's curse. However, it was the three Atlantic Richfield engineers, Capen, Clapp, and Campell (1971) that introduced the name of the concept. They found out that oil firms suffered from the winner's curse during the oil era in the Gulf of Mexico after the 1950s, when business had paid off less than expected. Later, Thaler (1988) views the empirical results of the winner's curse as an anomaly in his comprehensive discussion of the concept.

The winner's curse can be clarified by the following countervailing trade-offs. Bidders in common-value auctions face effects from both competition and information perspectives. More bidders introduce more competition that gives a bidder incentives to submit a higher bid (*competition effect*). However, a bidder must also account for the risk of overestimating the value of the object if he wins, since his signal is then the highest signal among bidders. This implies that a bidder should also lower his bid when facing more bidders (*overestimation effect*). In equilibrium, the overestimation effect is always larger than the competition effect and bidders correct for the winner's curse by lowering their bids as the number of bidders increases (Krishna, 2002).

1.6 Asymmetric bidders

In recent years, auctions with asymmetric bidders have been actively studied. Asymmetry is present when at least one of the main assumptions in the modeling of auctions is dropped, e.g. assuming risk-averse bidders instead of risk-neutral, relaxing the assumption of independently drawn valuations, and allowing for both private-value and common-value components. In the mechanism-design literature of auction theory, Maskin and Riley (1985) bring out many key ideas by weakening the main assumptions on the nature of bidders and focusing on only two bidders with private values. In addition, Maskin and Riley (2000) analyze asymmetric auctions by distinguishing between *weak* and *strong* private-value bidders. Apart from private values, Goeree and Offerman (2002,2003) and Jackson (2009) analyze auctions with both private-value and common-value components, and Reny and Zamir (2004) prove the existence of equilibria in general asymmetric first-price auctions with interdependent values. In second-price auctions with both private-value and common-value bidders, Tan and Xing (2011) prove the existence of a monotone pure-strategy equilibrium.

1.7 Structural econometric auction models

Reiss and Wolak (2007) give a broad introduction to the logic of structural econometric models, including models for auctions, and compare them to other types of econometric models. Over the last decades the structural estimations of auction data have become increasingly popular. Laffont and Vuong (1996) came with major contributions in this field and emphasize that auction models are particularly suited for structural estimation, where many datasets are readily available and well-defined game forms exist.

Bajari and Hortacsu (2005) mention three conditions that must apply for structural estimation of auction data. First, the bidders' goal is to maximize their expected utility. This is basicly an assumption of rational bidders. If the bidders are risk-neutral they maximize their expected profits. Second, bidders are able to compute the relationship between their bid and the probability of winning the auction. That is, they are able to compute the optimal combination of the probability of winning and the amount of the profit if they win. Third, given their beliefs, bidders are able to correctly maximize their expected utility.

These assumptions of rationally are quite strong, but there exists a number of papers that test for necessary conditions. Guerre, Perrigne, and Vuong (2000) point out that a necessary condition for rationality in private value auction models is, in principal, to the test if the bid function is increasing in values. Paarsch and Hong (2006) survey the field of structural econometrics of auction data.

1.8 Internet auctions

Recently, over the last decade, Internet auctions have gained wide popularity. Bajari and Hortacsu (2004) argue in their survey of online auctions that auctions on the Internet grow at an impressive pace and are one of the most successful forms of electronic commerce. Lucking-Reiley (2000) survey 142 online auctions and estimate eBay as the world's largest auction site by far. At eBay, millions of items are sold every day in thousands of categories from which high-quality datasets become available to buyers and sellers through completed auction listings.

To explore the determinants of bidder and seller behaviour, Bajari and Hortacsu (2003) examine a dataset of coin auctions from eBay. According to several empirical findings for auctions with a fixed end time, e.g. Wilcox (2000) and Ockenfels and Roth (2006), bids tend to arrive very late in these auctions. In the spirit of Wilson (1977), Bajari and Hortacsu (2003) show that late bidding in their independent symmetric common value model of eBay auctions is a symmetric Nash equilibrium. In this environment each bidder is assumed to place only one bid in the very last minute of the auction, so that no other bidders have time to revise their bids. As a consequence, they estimate eBay auctions as independent second-price common value auctions.

2 Summary of the papers

2.1 Paper I: Bayesian Inference in Structural Second-Price Common Value Auctions

Structural econometric modeling of auction data has become increasingly popular in recent years, especially with the advent of high-quality datasets from Internet auctions. Bajari and Hortacsu (2003) made a number of important contributions to the field by simplifying the analysis of common value auction models with a stochastic number of bidders. They proved that a symmetric Nash equilibrium exists in their study of eBay auctions, which allowed them to model eBay coin auctions as independent second-price common value auctions. The analysis of common value models is well-known for being technically challenging under realistic model assumptions.

In this paper, we refine and extend the analysis in Bajari and Hortacsu (2003). An important obstacle in their model is the need for numerical integration to solve the equilibrium bid function. This is very time-consuming since the likelihood function of bids needs to be evaluated many times for inference. A key contribution in our approach is a very accurate approximation of the equilibrium bid as a linear function of the bidder's signal, giving fast and numerically stable evaluations of the likelihood function.

We use both simulated and real data to analyze our model. The real data was carefully collected by human inspections and contains bids and auction-specific characteristics from 1050 eBay coin auctions. To estimate the data we use an efficient Bayesian framework for variable selection that brings out the posterior probability of including a given covariate in the model. We find the publicly available book value and the condition of the auctioned object as the main determinants for bidders' valuations, whereas the eBay's detailed seller information is essentially ignored by the bidders. We also show that our approximate bid function does not distort inference for a number of economic implications, such as the bidders' correction for the winner's curse by lowering their bids. Finally, we document good out-of-sample predictions of auction prices.

2.2 Paper II: Bayesian Inference in Structural Second-Price Auctions with Gamma Distributed Common Values

The valuations in auction models are intrinsically non-negative. Nevertheless, distributional assumptions are often used in the literature that allow for negative values. In our paper, we explore this issue by proposing an extension of the Gamma model in Gordy (1998) as an alternative to the Gaussian modeling of valuations in Wegmann and Villani (2011, henceforth WV). Similar to the Gaussian model in WV, a key contribution for the Gamma model is a very accurate approximation of the equilibrium bid function. The approximate bid function is non-linear, but since the approximate inverse bid (the approximate signal) is an explicit function of a given bid, we also obtain fast and numerically stable likelihood evaluations for the Gamma model.

The Bayesian framework with variable selection and the eBay coin auction dataset in WV are used to compare the performances between the Gaussian and Gamma models. The posterior results are quite similar between the models. The Gaussian model fits the data and predicts auction prices slightly better than the Gamma model. We find evidence that this is probably due to an almost normal or at least symmetrical distribution of valuations, where the density only attains small probabilities of negative values. Finally, we document for simulated datasets with different degrees of skewness that the superiority of the Gamma model for highly skewed data diminish when the value distribution becomes more symmetrical. These are findings that agree with the results for the eBay dataset.

2.3 Paper III: Bayesian Comparison of Private and Common Values in Structural Second-Price Auctions

Most of the literature in auction theory focus on either the private value or common value paradigm. Attempts to distinguish between the paradigms have been extensive during the last couple of decades, but mostly within first-price auctions. We compare the Gaussian model for common values in WV to a directly comparable model for private values in second-price auctions. The comparison is performed in numerous ways by using the Bayesian framework on the eBay coin auction data in WV.

Both models fit the data well with a slight edge for the more robust common-value model. The private-value model is better in predicting auction prices, but the more complex common value model is more robust in predicting some auctions. We find many interesting empirical regularities between the models. First, the winner's curse effect in common values explains the differences in the estimates for the expected values. Second, the optimal minimum bids in the common value model are much closer to the actual minimum bids than the optimal choice of zero or close to zero for the private value model. Third, we find no evidence of the winner's curse in the data, since the average bids do not decrease for an increasing number of bidders. Fourth, the models seem to capture the correlation between bids equally well.

Our results indicate that the value of the object probably includes both a private and a common value component, since we find evidence for both private and common values in different ways. However, auction models with a combination of private and common values have not yet seen the light in the literature. It is indeed a promising area of future research.

2.4 Paper IV: Bayesian Inference in Structural Second-Price Auctions with both Private-Value and Common-Value Bidders

Auction models with asymmetric bidders have received much attention in recent years. Tan and Xing (2011) show the existence of a montone pure-strategy equilibrium for symmetric second-price auctions with both private-value and common-value bidders. In their setting, the dominant strategy for a private-value bidder is to bid his value, while the bid equilibrium for a common-value bidder is the solution to an ordinary differential equation (ODE) that depends on the parameters in the private-value distribution. To solve the highly complicated ODE one needs to resort to numerical integration methods which are too time-consuming to be used for statistical inference.

We assume the model in Tan and Xing (2011) and derive a very accurate approximation of the equilibrium bid function. The approximate inverse bid is an explicit function of a given bid, which virtually takes no time to evaluate, giving fast and numerically stable evaluations of the likelihood function. We use Bayesian methods to evaluate the model on the eBay coin auction data in WV. Since we do not model auctions with a minimum bid, we use data from 464 auctions where the minimum bids have a negligable effect on the bidding process.

We propose a model where the probability of being a common-value bidder is a function of auction-specific covariates. An interesting feature of this modeling is the possibility to make inference, through Bayesian variable selection, on the probability of being a private-value or a common-value bidder in a given auction. We use a Metropolis-within-Gibbs algorithm to sample from the posterior in our Bayesian inference. Our main findings are that the empirical results for the common-value distribution are essentially the same as the results for a model with only common-value bidders, whereas the estimates of the parameters in the private-value distribution are more affected by the influx of common-value bidders. Finally, there is a slightly larger probability of being a common-value bidder, but this probability does not seem to depend on the covariates at the auction level.

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