Master of Science Thesis

Phenomenology of Dark Matter from Non-Minimal Universal Extra Dimensions

Johan Bonnevier

Typeset in LaTeX

Abstract

We give a general introduction to particle physics, and in particular to particle physics in extra dimensions. Furthermore, we introduce the concept of dark matter (DM) and discuss some suggestions for what it originates from. Next, we calculate the cross-section for DM annihilation in the framework of non-minimal universal extra dimensions. This process gives mono-energetic gamma-ray lines with energy close to the DM particle mass. The calculations are performed for two different DM candidates, the $U(1)_Y$ gauge boson B^1 and the electrically neutral $SU(2)_L$ gauge boson Z^1. The DM candidate is always the lightest particle of the theory (LKP). When the Z^1 is the LKP, we get a larger cross-section, even though it is heavier that the B^1 is when it is the LKP. The reason why the cross-section is larger for the Z^1 is that many more Feynman diagrams contribute to this process, since Z^1 has non-negligible self-interactions.

Keywords: Kaluza–Klein dark matter, non-minimal universal extra dimensions.

Sammanfattning

En allmän introduktion till partikelfysik ges. I synnerhet introduceras partikelfysik inom ramarna för extra dimensioner. Vidare introduceras begreppet mörk materia (MM) och några möjliga förklaringar till vad den består av. Efter detta beräknas tvärsnittet för annihilation av MM inom modellen icke-minimala universella extra dimensioner. Denna process skapar gammastrålar med en energi som i princip är lika med MM-partikelns massa. Beräkningarna har utförts för två olika MM-kandidater, $U(1)_Y$-gaugebosonen B^1 och den laddningsneutrala $SU(2)_L$-gaugebosonen Z^1. MM-kandidaten är alltid den lättaste partikeln inom den betraktade teorin. Störst tvärsnitt får man när Z^1 tas som den lättaste partikeln, och alltså som MM-kandidat, trots att den är tyngre än vad B^1 är när den är tagen som teorins lättaste partikel. Anledningen till att tvärsnittet blir större för Z^1 är att den har icke försumbara självväxelverkningar, vilket gör att många fler Feynmandiagram bidrar.

Nyckelord: Kaluza–Klein-mörk materia, icke-minimala universella extra dimensioner.

iv

Preface

This master of science thesis is the result of my work at the Department of Theoretical Physics at the Royal Institute of Technology (KTH) from June 2010 to March 2011.

This thesis is on the subject extra dimensions, and its possibility to describe what dark matter (DM) is.

Overview of the thesis

The thesis is structured as follows: In ch. 1, an introduction to the subject will be given. In ch. 2, the standard model of particle physics is introduced together with the concept of extra dimensions. In ch. 3, DM will be discussed. In ch. 4, two specific DM annihilation processes will be studied. The two processes have different DM particles, but both of them describe DM that annihilates into two photons. Finally, in ch. 5, we summarize and draw some conclusions.

Notation and conventions

In this thesis, the Einstein summation convention will be used. Also, natural units will be used, *i.e.* $c = \hbar = 1$.

Ordinary four-dimensional indices will be denoted by lower-case Greek letters. Indices of the full higher-dimensional spacetime will be denoted by upper-case Roman letters. KK-numbers will be denoted by lower-case Roman letters.

The following sign convention for the Minkowski metric will be used:

$$(g_{\mu\nu}) = \text{diag}(1, -1, -1, -1).$$

Acknowledgments

I would like to thank my supervisor Prof. Tommy Ohlsson for giving me the opportunity to do my master thesis here at the Department of Theoretical Physics. Furthermore, I thank Prof. Ohlsson for proof reading this thesis and useful guidance. I would also like to thank Henrik Melbéus, who has acted as a second supervisor to me, for introducing me to the subject and for good comments on this thesis. Thanks to Dr. Alexander Merle for many useful and interesting discussions. Thanks to everyone else here at the department that have made my time here so nice. Finally, thanks to my family and to my girlfriend Sara for all of your support.

Contents

To my family
Thank you for everything

Chapter 1

Introduction

The word physics comes from the ancient Greek word $\varphi\acute{v}\sigma\iota\varsigma$ ("physis"), which means Nature. In physics, we try to *describe* phenomena that we observe in Nature as accurately as possible in terms of mathematical formulas. Another important role of physics is to predict phenomena not yet seen in Nature. Note that we did not use the word *explain* above. To explain why things are like they are is a question that should be left for the philosophers to answer.

Mankind has for a very long time now tried to answer fundamental questions like "where do we come from" and "what is the smallest building block in Nature". Today, we have enough knowledge to describe much of what we have seen here at Earth and in space. However, there is still very much that we do not have a complete understanding of. Some of those things we hope to get the full picture about in the coming years, thanks to some new and powerful experiments, like the Large Hadron Collider (LHC) [1] or the IceCube [2].

Work performed in physics is usually in the direction of *either* theory *or* experiment. These different areas work together in symbiosis. The role of the theoreticians is to describe phenomena discovered by the experimentalists, and also to suggest new things for the experimentalists to look for.

Theoretical physics can be divided into several subclasses, one of which is phenomenology of particle physics. What is meant by *phenomenology* in this context is the task of relating theory with experiments.

Elementary particle physics is the branch of physics which concerns itself with the smallest known building-blocks of Nature, the elementary particles. What defines an elementary particle is that it does not consist of any smaller, more fundamental parts. What is to be considered as elementary is not written in stone, but has changed over time due to better and better experiments.

Perhaps the best and most famous example of this is the fact that the ancient Greeks named the *atom* with the word $\alpha\tau\acute{o}\mu o\varsigma$. This actually means indivisible, and hence, an atom was considered to be elementary in ancient Greece. Actually this was the general belief for a very long period of time. Today, nevertheless, it is

known that atoms actually consist of protons and neutrons, which in turn consist of quarks.

Nature, at the sub-atomic scale, is well described by quantum theory. This is a rather strange theory, based on probabilities, but has nevertheless proven to give an excellent description of observed phenomena. Michio Kaku wrote in 1994 [3]:

> "In fact, it is often stated that of all the theories proposed in this century, the silliest is quantum theory. Some say that the only thing that quantum theory has going for it, in fact, is that it is unquestionably correct."

There are four known forces in Nature. Namely, the electromagnetic, weak, strong, and gravitational forces. Actually, the electromagnetic and the weak forces combine to form the electroweak force at energies around 250 GeV. At even higher energies, around 10^{15} GeV, the electroweak and the strong forces are also expected to unify. Theories where this happens are usually called Grand Unified Theories [4]. Of the four forces, gravity is significantly weaker than all the rest. This is one of the things that still puzzles people in the scientific community. Gravity also differs from the rest of the forces in that it is described by its own theory. This theory is called General Relativity and was invented by Albert Einstein [5]. All the other forces are contained within the same theory, which will be described below.

A theory that unifies all forces in Nature is generally referred to as a Theory of Everything, but at the time of writing there is no good candidate for such a theory. The best description we have of Nature at the smallest scale, to this date, is the so-called Standard Model of particle physics (SM) [4]. The name was chosen by one of the inventors of this theory, namely Steven Weinberg. What makes the SM so incredible is that one has been able to predict new particles before they where found in experiments using this theory. In fact, measurements in quantum electrodynamics, which is a subclass of the SM, have shown agreement between theory and experiment to the precision of 10^{-9}, which is the best acceptance ever accomplished by man.

Although the SM in its full glory describes so much of what we observe, it actually fails at several points, and hence, one can conclude that the SM is not the most fundamental theory of Nature. Firstly, the SM does not make any reference to gravity. This might seem to be a small problem, since gravity is negligible at small length-scales. However, a fundamental theory of Nature should be able to incorporate this. Secondly, the SM offers no description to what the so-called dark matter (DM) consist of. A third thing is that the neutrinos are taken to be massless within the framework of the SM. This has turned out to be wrong, since the phenomenon known as neutrino oscillations demands that neutrinos are massive.

Many models have been suggested with the aim to solve one or more of the problems with the SM. Examples of such theories are supersymmetry [6] and string theory [7]. Another branch that has attracted some attention lately are theories with extra spatial dimensions (ED). Extra-dimensional theories are usually

referred to as Kaluza–Klein theories after the inventors of these theories, Theodore Kaluza [8] and Oskar Klein [9]. The starting point of their ideas was to try to unify electromagnetism with gravity during the 1920's.

One has to remember when constructing new theories, such as the ones mentioned above, that they should still be able to describe the phenomena that our current theories describe well. This is actually a strong constraint, which helps one to set bounds on new theories. Furthermore, one does not want to end up with a new description of Nature that suggests phenomena which are not there. The general belief is that the description of Nature should be as simple as possible.

The focus in this thesis will lay on EDs. One of the motivations to introduce EDs is that by doing this, it turns out that a possibility to solve the DM problem arises.

A common reaction to the suggestion of EDs is, "If there are extra dimensions, where are they, and why can we not see them?". The answer to this was first given by Oskar Klein [9]. He proposed that the reason why we cannot see the EDs is that they could be small and closed, *e.g.* in the form of a circle with a tiny radius.

An example of an extra-dimensional theory is the Randall–Sundrum model [10], which aims to solve the hierarchy problem. Another theory which has got some attention lately is universal extra dimensions (UED), proposed about ten years ago by Appelquist *et al* [11].

One interesting aspect about UED is that it naturally offers DM candidates. These DM particles could annihilate into gamma rays with energies close to the DM particle's mass, and hence give a clear signature to look for in experiments. One satellite that currently looks for gamma ray signals is the Fermi Large Area Telescope [12]. There are also ground-based experiments, *e.g.* MAGIC [13], that search for these signals.

There has been some work done in the minimal version of UED. The authors of refs. [14, 15] have investigated the gamma ray signals in this theory. However, similar work has not yet been done in the more general non-minimal case. The main goal of this thesis will be to investigate some parts of the non-minimal UED.

If it will be possible in the future to show that additional spatial dimensions actually exist, the way most people view Nature will be changed. What one has to remember is that not much at all would in fact change, since, then the extra dimensions would already be here. Just like when it was discovered that the Earth is not flat and when it was first observed that Earth is orbiting the Sun and not the other way around, this is something that will become the new reality and will not be considered strange by coming generations.

The scientific area where EDs would play the most important role is without doubt elementary particle physics. The reason is simply that particles are small enough to "feel" the effect of the EDs, in contrast to macroscopic objects like ourselves. This is the reason why it is interesting to study EDs in the language of elementary particles.

Chapter 2

The Standard Model and its extension to extra dimensions

In this chapter, the basics of the so-called Standard Model (SM) will be discussed. We will also mention some problems that are related to the SM. Continuing, it will be described how the SM could be extended to extra dimensions. In particular, a model known as universal extra dimensions (UED) will be studied. We will investigate both the minimal and non-minimal versions of UED, and, in connection to this, some important differences that arises between the two versions.

2.1   The Standard Model

The SM of particle physics is our most fundamental theory at the moment. It describes the world in terms of elementary particles that interact with each other. There is much experimental evidence to support this theory. However, the SM makes no reference to gravity and is also unable to describe things like dark matter and the hierarchy problem. All of this indicates that the SM is the low-energy limit of some more fundamental theory.

The particles in the SM can be divided into three groups. Those are *fermions*, *vector/gauge bosons*, and *scalars*, and will all be described below.

In the SM, there are three kinds of forces, the electromagnetic (EM), the weak, and the strong. They are mediated by particles known as bosons. Bosons are particles with integer spin. The force carrier of the EM-field is called the photon, which is simply a light-particle. The strong force is carried by gluons, of which there are eight different ones. Both the photon and the gluons are massless. The weak force is mediated by the W and Z bosons, which differ from the photon and

gluons, since they are massive. They gain their masses because of a spontaneously broken symmetry.

The force carriers mediate forces between the so-called matter particles. The matter particles are not bosons as the force carriers, but fermions. A fermion is a particle with half-integer spin. Within the SM, all fermions are spin-1/2 particles.

The matter particles can be grouped together to form one group called leptons, and one called quarks. The structure of these groups can be visualized as follows:

Leptons:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}, \tag{2.1}$$

Quarks:

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}. \tag{2.2}$$

What is written inside the parentheses is usually referred to as flavors or generations.

Up to now, we have discussed bosons and fermions. What about the scalars? In the SM, there is only one scalar particle, called the *Higgs* boson. This particle has not yet been observed experimentally, but will be found (if it exists) in the coming years in the LHC at CERN. One usually refers to the Higgs boson as the particle responsible for the masses of all other massive particles.

The gauge group of the SM is $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, which is spontaneously broken by the Higgs mechanism to $SU(3)_C \otimes U(1)_Q$. Of interest for this thesis is only the $SU(2)$ and $U(1)$ part of the total gauge group.

2.1.1   Quantum Field Theory

In particle physics, the framework one uses is Quantum Field Theory (QFT). As the name indicates, this is a theory of quantized fields. To easier describe what QFT is, we start by discussing some classical field theory, and then relate this to the quantized version.

In classical particle mechanics, the fundamental object one works with is the *action*, S, the time integral of the *Lagrangian*, L, *i.e.* $S = \int L \mathrm{d}t$. However, when going to a local field theory, the Lagrangian becomes the spatial integral of the Lagrangian density[1], $\mathcal{L}$. This object could be a function of both the fields ϕ, and their derivatives $\partial_\mu \phi$. We thus get the action for a local field theory

$$S = \int \mathcal{L}(\phi, \partial_\mu \phi) \mathrm{d}^4 x. \tag{2.3}$$

The only thing that changes, when going from classical fields to quantized ones, is that the fields become operators, with certain commutation relations.

---

[1]Normally when working with field theory, one refers to the Lagrangian density when one says Lagrangian and this convention will also be adopted here.

2.1.2 Ghosts

In the case of non-Abelian gauge fields, one has to introduce additional fields called ghost fields, denoted by c^a. These fields are non-physical, and serve to cancel some degrees of freedom that come from the longitudinal and time-like polarizations, which happen to cancel each other exactly in the Abelian case.

The ghost fields are complex, anti-commuting scalars. Of importance later on, when deriving Feynman rules for the extra-dimensional ghosts, will be that they have an even Kaluza–Klein expansion. What is meant by this will be explained later on in this chapter.

One thing that one has to pay attention to when including ghosts in one's calculations is that $c^\pm \equiv \frac{1}{\sqrt{2}} \left(c^1 \mp c^2 \right)$. This has the profound effect that c^+ and c^- are not each other's Hermitian conjugates.

2.1.3 Problems with the Standard Model

The first thing one notices when studying the SM is that it makes no reference to gravity. To describe gravity, we usually use Einstein's theory of General Relativity. The problem is that it has so far been impossible to combine this with the SM, since it has proven difficult to describe gravity in the language of QFT.

Secondly, there is no suitable DM particle within the SM. The DM is thought to have a particle origin, so in order to have a complete theory of particle physics, this is something that needs to be contained within the theory.

Thirdly, the SM fails to describe the phenomenon called neutrino oscillations. In order for the neutrinos to oscillate, it is a necessity that they have masses. This is however not the case within the SM framework, where they are taken to be massless. Although neutrino oscillations is an interesting research area, it will not be discussed further in this thesis. For more information on this subject the reader could consult *e.g.* ref. [16].

Furthermore, in order to solve the hierarchy problem, *i.e.* the problem to describe why the Higgs mass differs so much from the Planck mass, one has to consider extensions of the SM. The most common choice to solve this is to make supersymmetric extensions of the SM. More on this topic may be found in ref. [17].

There are, as seen above, a number of problems with the SM that need to be addressed. Nevertheless, the SM is very successful in describing all observed particle interactions up to energies of order 10^2 GeV.

We shall now continue by discussing one possible extension of the SM, namely that of extra spatial dimensions. This will give us the opportunity to address some of the problems mentioned above.

2.2   Extra dimensions

Common sense tells us that our world is four-dimensional. It consists of three spatial dimensions and one time dimension. Nevertheless, it is possible that there could be more dimensions at higher energy scales, or if one likes, smaller length scales.

It is not very hard to picture extra dimensions even in our four-dimensional world: Consider for example the game of billiards. The balls are confined to move in a two-dimensional plane. However, the sound waves are able to escape the table and enter into the third dimension. That is why we can hear the balls colliding. Furthermore, if you are not very good at billiards, it can happen that you make one of the balls jump off the table, and thus probe the extra dimension.

Today, there are several different models that suggest one or more extra dimensions, which could have flat or curved geometry. One of the most popular ones is *string theory*, which in some cases has seven extra dimensions.

One thing that could differ between different extra-dimensional theories is that some suggest that only certain of the particles in our four-dimensional world are allowed to probe the extra dimension, and others indicate that all of the SM particles are allowed to move in the higher-dimensional space-time.

The model that will be focused on in this thesis is one called *universal extra dimensions* (UED). This is a theory with small extra spatial dimensions of flat geometry. Since the extra dimensions are small, all the SM fields are free to propagate there.

2.2.1   Kaluza–Klein theories

Extra-dimensional theories are usually referred to as Kaluza–Klein (KK) theories [8, 9]. The name comes from the two pioneers in this area, Theodore Kaluza and Oskar Klein. They came up with these ideas during the 1920's, while trying to unify the EM-force with gravity.

In KK theory, one makes a so-called KK decomposition of the higher-dimensional fields, to get a four-dimensional view of them. How this works is most easily explained by the means of an example.

Consider performing a KK decomposition on a five-dimensional scalar field. In five dimensions, the action of a complex scalar field ϕ with mass m is given by

$$S = \int \mathrm{d}^4 x \int_0^{2\pi R} \mathrm{d}y \left((\partial_M \phi)^* \left(\partial^M \phi \right) - m^2 \phi^* \phi \right). \tag{2.4}$$

This is a field defined on a circle, and hence, it will be periodic with period $2\pi R$. This enables us to make a Fourier expansion of the five-dimensional field

$$\phi(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \phi^{(n)}(x^\mu) \exp\left(\frac{\mathrm{i}ny}{R}\right). \tag{2.5}$$

Now, putting the expanded fields into eq. (2.4) we can easily integrate out the fifth dimension, using the orthonormality of the basis functions. The resulting four-dimensional effective action then reads

$$S = \sum_{n=-\infty}^{\infty} \int \mathrm{d}^4x \left[\partial_\mu \phi^{(n)*} \partial^\mu \phi^{(n)} - \left(m^2 + \frac{n^2}{R^2}\right) \phi^{(n)*} \phi^{(n)} \right]. \tag{2.6}$$

By studying this action, we can immediately see that we get four-dimensional complex scalar fields with masses $m^{(n)} = \sqrt{m^2 + n^2/R^2}$. The infinite number of fields in this sum is called a KK-tower. Notice that the only difference between two arbitrary modes is their mass.

2.2.2 Universal extra dimensions

The model of UED was proposed in 2000 by Appelquist, Cheng, and Dobrescu [11]. In this model, one adds one or more extra dimension(s), perpendicular to our ordinary four space-time dimensions. The added dimensions are flat and compact.

The model that will be considered here is one where a single extra dimension is added with the topology of a circle S^1 with a small radius R. However, as it will turn out, the topology of a circle will introduce unwanted degrees of freedom (d.o.f.). This is not such a serious problem as it may appear at first sight. As it turns out, there is an elegant solution that will remove these extra d.o.f.

By considering the UED model, the problem introduced by the unwanted d.o.f. is that fermions in a higher-dimensional space are non-chiral. Chiral fermions are something one has to demand if one wants to get back the SM in the low-energy limit. However, there is a way around this problem. By identifying the opposite points $-y$ with y, one introduces an $S^1/\mathbb{Z}_2$ orbifold (fig. 2.1), and the unwanted extra d.o.f. are removed, allowing for chiral fermions [18].

Thus, we have introduced a parity transformation $-y \to y$. The five-dimensional fields have to be either even or odd under this transformation. If we now rewrite the expression in eq. (2.5) as a sum of an odd and an even part, we get

$$\phi(x^\mu, y) = \phi_{\mathrm{even}}(x^\mu, y) + \mathrm{i}\phi_{\mathrm{odd}}(x^\mu, y), \tag{2.7}$$

where

$$\phi_{\mathrm{even}}(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \phi^{(0)}(x^\mu) + \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} \phi^{(n)}(x^\mu) \cos\left(\frac{ny}{R}\right) \tag{2.8}$$

and

$$\phi_{\mathrm{odd}}(x^\mu, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} \phi^{(n)}(x^\mu) \sin\left(\frac{ny}{R}\right). \tag{2.9}$$
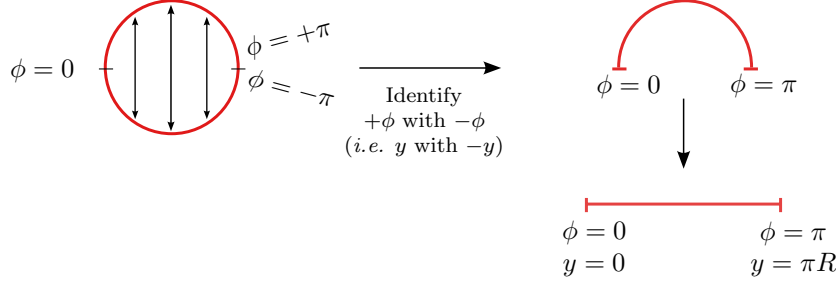
Figure 2.1. Orbifolding a circle to an interval. The picture has been adopted from ref. [19].

One can now see from eqs. (2.8) and (2.9) that the even part has a zero mode, in contrast to the odd one, which does not have a zero mode. This is an important observation, since the parity should not be affecting our "normal" four-dimensional fields. Another important consequence of the UED model is that when considering the Fourier expansion of the gauge fields one gets additional scalars. These scalars have odd expansions and will hence not show up in the zero-mode part of the theory, as they should not.

2.2.3   Minimal universal extra dimensions

The orbifold $S^1/\mathbb{Z}_2$ has two fixed points under the $\mathbb{Z}_2$ transformation, $y = 0$ and $y = \pi R$, *i.e.* the endpoints of the interval. At these points, we could have localized terms in the Lagrangian, so-called boundary localized terms (BLTs). One often makes the ansatz that the BLTs vanish at the cut-off scale of the higher-dimensional theory. By doing this, one obtains what is called the minimal UED (MUED). This is the model that will be used in the calculations of $B^1 B^1 \to \gamma\gamma$ to come in ch. 4.

2.2.4   Non-minimal universal extra dimensions

By not removing the BLTs, one gets non-minimal UEDs. This has been discussed in ref. [20]. This change turned out to affect the spectrum quite much in some regions of parameter space. For this thesis, the most interesting aspects of this non-minimal case is that it opens up possibilities to have more possible dark matter candidates than in the minimal version, which only offers one candidate, B^1. The calculation of $Z^1 Z^1 \to \gamma\gamma$ in ch. 4 heavily depends on this new opportunity. In fact, adding the BLTs also makes the H^1 a viable DM candidate. This will however not be investigated further in this thesis.

Chapter 3

Dark matter

The motion of planets and stars in the Universe is mostly well-described by Newton's theory of gravity. There are nevertheless exceptions. For example, the planet Mercury does not fully obey Newton's theory. At first, it was believed that this was due to another, so far unseen, planet. As it turned out, this was not what was causing the deviation from the expected motion. Instead the solution could be found when Einstein pointed out that Newton's theory was not completely correct, but that gravity should rather be described using General Relativity (GR).

The dark matter problem is quite analogous to the problem with Mercury's precession. Either it could be that there is still something wrong in our field equations describing gravity, or it could be that there is some invisible stuff around to give rise to the deviations from what one would expect. The general belief today is that there is something unnoticed that makes up for the deviations, namely what we call *dark matter* (DM).

What DM really consists of is still unknown. It is anyway believed to have a particle origin. The reason why it is called dark is because we cannot directly observe it, since it does not reflect electromagnetic radiation.

To figure out what DM really is, we need to consider physics beyond the SM. This is because there is no particle in the SM that has the right properties to be the DM particle. Well, since it today is thought that neutrinos are massive, they could in principle be a DM candidate. However, the neutrinos are ruled out, since they are simply too light to make up more than a small portion of the DM. Also, they are highly relativistic, which has turned out not to be preferred.

Even though it is unknown what DM actually is, it is a well-established concept. There is considerable experimental support from astrophysics and cosmology that DM exists. This will be discussed in some detail below.

3.1   The need for dark matter

A strong indication that DM exists comes from observations of how galaxies rotate. This was first pointed out in the 1980's by Rubin *et al* [21]. However, the existence of DM was suggested already back in 1933 by Zwicky [22]. He noticed that there was something strange with the mass-to-light ratio from the galaxy cluster Coma.

Another piece of evidence for the existence of DM, comes from strong gravitational lensing. What strong gravitational lensing means is that light passing by a galaxy is affected more by gravity than one would naively expect from just considering the visible mass of the galaxy. This clearly indicates that there should be more mass around than what we are able to directly detect.

3.1.1   Different types of dark matter

In order for a particle to be a good DM candidate, there are some properties that need to be fulfilled. Namely, 1) it cannot take part in the electromagnetic interaction, since we cannot see it, 2) it is not allowed to decay[1].

Furthermore, it looks from simulations of large scale structure formations like *cold* DM (CDM) is preferred over *hot* DM [23,24]. What cold means in this context is that the particles move slowly, and hence, they are non-relativistic. In fact, one often makes the approximation that two DM particles annihilate at rest. What is meant by hot above is ultra-relativistic *i.e.* $v_{\mathrm{DM}} > 0.95c$, where c is the speed of light.

Except for the types of DM mentioned above, one could also consider something called *warm* DM which could originate from *e.g.* massive sterile neutrinos [25]. Warm DM means neither cold nor hot, but has a velocity around $0.1c - 0.95c$. This will not be discussed further here.

3.2   Detecting dark matter

To detect DM, there are mainly two alternatives. The first one is to study when normal matter nuclei get scattered by DM. This is called *direct detection*. It is thought that the Earth is moving through a dark matter halo that is confined to our galaxy. Thus, it sometimes happens that the DM interacts with our ordinary matter. The DAMA/LIBRA [26] and CDMS [27] collaborations are two experimental groups that are looking for traces of this interaction. The CDMS upper limit on the scattering cross-section for a 70 GeV weakly interacting massive particle (WIMP) is $3.8 \cdot 10^{-8}$ pb at 90 % confidence level. The DAMA/LIBRA collaboration has claimed direct detection of DM. However, this is generally not considered as solid evidence for DM. Both the experiments mentioned above are placed deep underground in order to reduce the background noise.

---

[1]It should anyway have a long enough life-time to still be around in the right amount.

The second way is to look for final state particles from DM annihilation. This is called *indirect detection*. There are two possible indirect detection methods. One could either look for particles created when DM annihilates, or one could look for missing energy in accelerator experiments such as the LHC which would indicate that DM has been produced in the collision. The calculations performed in this thesis will result in a mono-energetic gamma ray signal, which means that the signal should be searched for using indirect detection techniques.

3.2.1 Mono-energetic gamma rays

Using the technique of looking for gamma rays, the DM particles are obviously not observed themselves. However, DM annihilating into gamma rays would give a clear signature, since the energy would be close to the DM particle mass. An example of an experiment which is looking for such gamma rays is the space based satellite Fermi Large Area Telescope (Fermi LAT) [12]. Other examples are the ground based MAGIC telescopes [13], placed on the Canary Island La Palma. Since the mirror of a ground based telescope can be made much larger, these types of experiments can look for higher energies, and hence, more massive DM candidates.

Yet another possibility could be to look for neutrinos created from DM instead of gamma rays. An example of an experiment searching for neutrinos from DM is IceCube [28].

3.3 Dark matter hidden in extra dimensions

As already mentioned, we need to turn to physics beyond the SM in order to have a DM candidate. If we consider the UED model, opportunities to have a DM particle open up. The particle that could be the DM particle is the lightest KK particle (LKP). This is a possibility, since the conservation of KK parity makes the LKP stable. The kind of DM one is talking about in this context is Kaluza–Klein DM (KKDM). This is the type of DM of interest for the calculations to be performed later on in this thesis.

The usual suggestion for the LKP is the first KK mode of the hyper-charge gauge boson B^1, a.k.a. the KK-photon. The reason why we can consider the B^1 to be the KK-photon is that the first level Weinberg angle is bounded as $\sin(\theta_W^{(1)}) \lesssim 0.05$ for $R^{-1} \geq 300$ GeV. We will use the value of $R^{-1} \geq 500$ GeV for the calculation of B^1 annihilation, and hence, this approximation is valid.

There are other possibilities for the LKP, *e.g.* the KK-graviton. The phenomenology of this DM will be that of a super-WIMP [29, 30]. This will however not be discussed further in this thesis.

3.3.1   Dark matter in non-minimal UED

As mentioned in section 2.2.4, when one takes into account the BLTs one could have the situation that the first KK-mode of B^1 is not the LKP [20]. Keeping the BLTs gives rise to the following boundary kinetic terms and mass terms:

$$\mathcal{L}_{BLT} = -\frac{r_B}{4g_Y^2}B_{\mu\nu}B^{\mu\nu} - \frac{r_W}{4g_Y^2}W^a_{\mu\nu}W^{a\mu\nu} + r_H(D^\mu H)^\dagger D_\mu H + \mu_b^2 H^\dagger H - \lambda_b(H^\dagger H)^2.$$
(3.1)

The LKP in the allowed regions is either the B^1 or the $W^{3,1}$. For $r_W \gtrsim r_B$, $W^{3,1}$ is the LKP, whereas for $r_W \lesssim r_B$, B^1 is the LKP.

Since we have chosen to call the KK-photon B^1 because of the small Weinberg angle, we should in principle stick with the notation $W^{3,1}$ for the excited $SU(2)$ gauge boson. However, we will from now on call this Z^1 for simplicity even though this is in another basis.
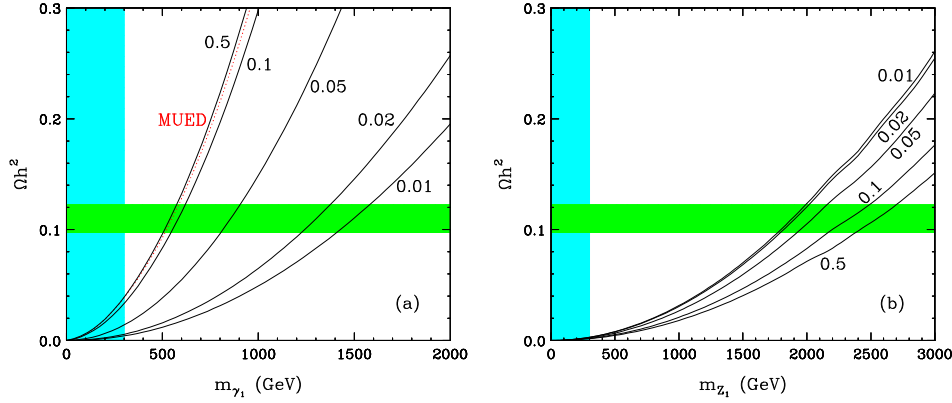

Figure 3.1. Relic density of the LKP (left panel: B^1, right panel: Z^1) as a function of the LKP mass. The (black) solid curves show the LKP relic density for several choices of the mass splitting between the LKP and the KK quarks. It is assumed that singlet and doublet KK quarks are degenerate. The green horizontal band denotes the preferred WMAP [31] region for the relic density $0.1037 < \Omega_{CDM}h^2 < 0.1161$. The cyan vertical band delineates values of m_{LKP} disfavored by precision data. The (red) dotted curve is the result from the full calculation in MUED, including all co-annihilation processes, with the proper MUED choice for all masses. Both figures are adopted from ref. [32].

The situation where the B^1 is taken to be the LKP has been thoroughly investigated in refs. [14,15]. However, to have the Z^1 as the LKP is not very well studied in the literature. This is the reason why we have chosen to focus on this later on in this thesis.

If the Z^1 turns out to be the LKP, it has been shown in ref. [32], from a relic density calculation, that its mass should be in the range $1800 \text{ GeV} - 2700 \text{ GeV}$, depending on the relative mass difference between the LKP and the heavier particles. This is summarized in fig. 3.1, where also the results for the B^1 are presented.

18

Chapter 4

Dark matter annihilation

In this chapter, we consider Kaluza–Klein (KK) dark matter (DM) annihilation. We can have annihilation into photons at one-loop level. However, there will be no tree-level processes, since there is no such diagram that would respect KK number conservation. Throughout our calculations we will only consider the zeroth and first KK modes, since higher modes are increasingly heavier and hence make a smaller contribution. We will also use the approximation that the zero-mode particles are massless. The last thing we assume is that the contribution from the Higgs sector is unimportant for an order of magnitude estimate, since the couplings are so much weaker in that sector.

The produced gamma-rays from the kind of processes we are considering are mono-energetic with a clear signature, and could be observed by future gamma-ray experiments. The operational experiment today, Fermi LAT [12], does not reach high enough in energy to observe the mono-chromatic signals from the processes considered here. However, detection could perhaps be made via the continuous gamma-ray spectrum from other decay channels. This spectrum has been calculated for the B^1 in ref. [33], but has not yet been calculated for the Z^1. An experiment that goes high enough in energy to cover the energy of the produced gamma-rays is MAGIC [13]. However, it is at the moment not clear if the energy resolution of around 10 % [34] is good enough to resolve the peak from the continuous spectra produced by indirect annihilation to photons.

The cross-section for the annihilation process $B^1 B^1 \to \gamma\gamma$ was first calculated in ref. [14]. In ref. [15], the authors verified this result and also calculated the contributions from the γZ and γH final states, which not surprisingly turned out to give just a small enhancement of the signal coming from the $\gamma\gamma$ final state alone.

We will here also discuss the process $B^1 B^1 \to \gamma\gamma$. After this, we will turn our attention to the process $Z^1 Z^1 \to \gamma\gamma$, which offers a much more involved calculation.

4.1 $B^1 B^1 \to \gamma\gamma$

We have chosen to calculate this rather well-studied process in order to check that our software implementation behaves in a reasonable manner. Most steps in this calculation are the same as the ones performed in section 4.2. However, there are some differences worth pointing out.

First of all, we have an Abelian theory in this case, which means that there will be no self-interactions. Neglecting the Higgs sector, the only type of diagram in this process will be the top-left one in fig. 4.2. Since we furthermore have a $U(1)$ theory instead of $SU(2)$, we get diagrams with both singlet and doublet fermions running in the loop.

In this process, all the divergences canceled, even though we did not consider more than the first KK level. This will turn out not to be true in general. However, it is in fact quite expected to be true in this case, since one in principle has two times photon-scattering in QED. This is of course a process without divergences, since all physical processes should be free from divergences.

The result of $B^1 B^1 \to \gamma\gamma$ is presented in fig. 4.1 as a function of the mass-splitting between the LKP and the heavier particles at the first KK level. The zero mode particles are taken to be massless, since they are so much lighter than the first exited ones.

4.2 $Z^1 Z^1 \to \gamma\gamma$

In this section, we will focus on another DM candidate, namely Z^1. As mentioned in earlier chapters, it has been shown in ref. [20] that this could possibly be the LKP and hence, a DM candidate. The main contribution to monochromatic gamma ray lines will most probably also in this process, as in B^1 annihilation, come from the final state with $\gamma\gamma$.

4.2.1   The diagrams

The diagrams that contribute to this process have the structures shown in fig. 4.2. Actually there is a total number of 42 diagrams plus crossings for this process, which is why we have chosen to only display some representative diagrams here. Also, we have neglected the Higgs sector of the theory, since the couplings appearing are so weak anyway compared to the ones included, and we are most interested in an order of magnitude estimation as we anyway have to make so many assumptions.

4.2.2   The amplitude

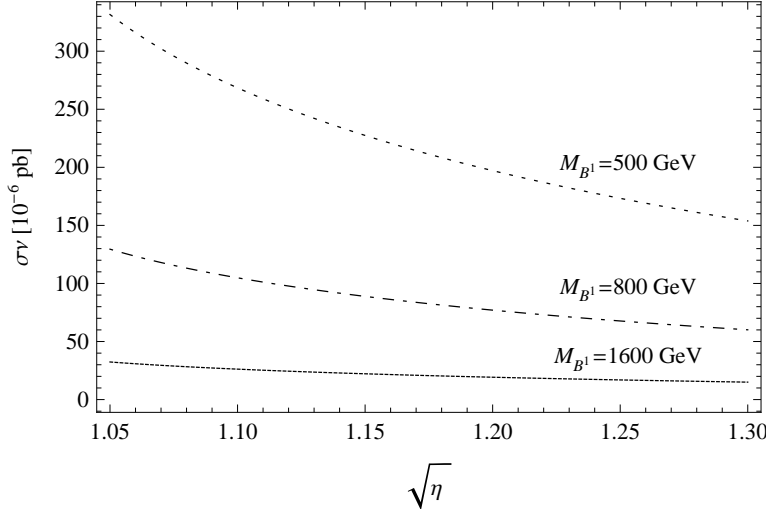The amplitude for this scattering process can be written as

Figure 4.1. The annihilation cross-section for $m_{B^1} = 500, 800, 1600$ GeV as a function of the mass splitting between the LKP and the heavier particles at the first KK-mode, *i.e.* $\eta = \left(\frac{m_{f1}}{m_{B^1}}\right)^2$. This result is in agreement with the earlier ones calculated in refs. [14, 15]. We see that the cross-section depends quite heavily on the mass of the LKP, as there is roughly one order of magnitude between the lightest and heaviest possibility. It is however not surprising that the difference is so large, since the cross-section is proportional to the inverse of the mass squared.

$$\mathcal{M} = \epsilon_1^{\mu_1}(p_1)\epsilon_2^{\mu_2}(p_2)\epsilon_3^{\mu_3}(p_3)\epsilon_4^{\mu_4}(p_4)\, \mathcal{M}_{\mu_1\mu_2\mu_3\mu_4}(p_1, p_2, p_3, p_4)\,, \qquad (4.1)$$

where all momenta are taken to be ingoing and ϵ_i are polarization vectors.

Since the Z^1 is a WIMP, it has the typical velocity $v \sim 10^{-3}$, and we can use the approximation that the Z^1s annihilate at rest. This means that we may put the ingoing momenta equal, *i.e.* $p := p_1 = p_2 = (m_{Z^1}, \vec{0})$. The relation for total momentum conservation can thus be written as

$$2\, p^\mu + p_3^\mu + p_4^\mu = 0. \qquad (4.2)$$

If we also take into account that contraction with a polarization vector and the corresponding momenta should give zero, the most general form of the polarization tensor is

$$
\begin{aligned}
\mathcal{M}^{\mu_1\mu_2\mu_3\mu_4} &= g_2^2 e^2 \Big(\frac{A}{m^4}\, p_3^{\mu_1} p_4^{\mu_2} p^{\mu_3} p^{\mu_4} + \frac{B_1}{m^2}\, g^{\mu_1\mu_2} p^{\mu_3} p^{\mu_4} + \frac{B_2}{m^2}\, g^{\mu_1\mu_3} p_4^{\mu_2} p^{\mu_4} \\
&\quad + \frac{B_3}{m^2}\, g^{\mu_1\mu_4} p_4^{\mu_2} p^{\mu_3} + \frac{B_4}{m^2}\, g^{\mu_2\mu_3} p_3^{\mu_1} p^{\mu_4} + \frac{B_5}{m^2}\, g^{\mu_2\mu_4} p_3^{\mu_1} p^{\mu_3}
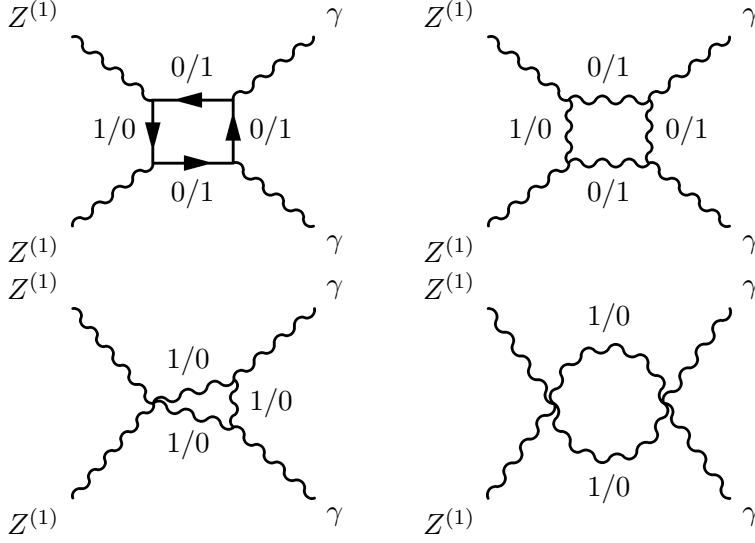\end{aligned}
$$

Figure 4.2. Representative diagrams that contribute to the process $Z^1 Z^1 \to \gamma\gamma$. The internal particles are $f_d^{(0/1)}$, $W^{\pm(0/1)}$, and $W_5^{\pm(1)}$. The d is to indicate $SU(2)$ doublets. In total, the process has 42 diagrams plus necessary crossings. The notation 1/0 refers to first or zeroth KK-mode. Note that there are no triangular or bubble diagrams in the fermion case, since fermions do not have self-interactions. Some vertex rules are given in Appendix B and some are taken from ref. [39].

$$
\begin{aligned}
&+ \quad \frac{B_6}{m^2} \; g^{\mu_3\mu_4} p_3^{\mu_1} p_4^{\mu_2} + C_1 \; g^{\mu_1\mu_2} g^{\mu_3\mu_4} + C_2 \; g^{\mu_1\mu_3} g^{\mu_2\mu_4} \\
&+ \quad C_3 \; g^{\mu_1\mu_4} g^{\mu_2\mu_3} \Big),
\end{aligned}
\tag{4.3}
$$

where A, B_i, and C_i are dimensionless coefficients that depend on the external momenta and the masses of the particles running in the loops. If we now take Bose symmetry into account it turns out that not all the A, B_i, and C_i are independent.

We know from ref. [35] that the cross-section of two particles $\mathcal{A}$ and $\mathcal{B}$, with a final state consisting of two particles, is given by

$$
\sigma = \int \frac{1}{2E_\mathcal{A} 2E_\mathcal{B} |v_\mathcal{A} - v_\mathcal{B}|} \frac{\mathrm{d}\Omega_{\mathrm{CM}}}{4\pi} \frac{1}{8\pi} \left(\frac{2|\vec{p}|}{E_{\mathrm{CM}}} \right) |\mathcal{M}|^2,
\tag{4.4}
$$

where CM stands for center-of-mass and $|\vec{p}|$ is the magnitude of the 3-momentum of either particle in the CM frame.

In order to reach our final expression for the cross-section, we need to use the following relation for massive vector bosons from ref. [35]

$$\sum_{\epsilon^\mu p_\mu = 0} \epsilon^\mu(p)\epsilon^{\nu\star}(p) = -\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{m^2}\right). \tag{4.5}$$

Finally, summing over final states and average over initial states, we get the cross-section

$$
\begin{aligned}
\sigma v \quad &= \frac{\alpha_{\mathrm{SU(2)}}^2 \alpha_{\mathrm{em}}^2}{144\pi m_{Z^1}^2} \Big\{ |A|^2 + 3|B_1|^2 - 16|B_2|^2 + 4|B_6|^2 + 12|C_1|^2 + 24|C_2|^2 \\
&\quad + 2\Re\left[A\left(B_1^\star + B_6^\star + C_1^\star\right) + B_1 B_6^\star + 3B_1 C_1^\star + 4B_6 C_1^\star + 2B_6 C_2^\star + 6C_1 C_2^\star\right] \Big\},
\end{aligned}
\tag{4.6}
$$

where the scalars A, B_i, and C_i are given in appendix C. Also, the relation $g_2^2 e^2 = 16\pi^2 \alpha_{\mathrm{em}} \alpha_{\mathrm{SU(2)}}$ has been used to reach this final expression. Furthermore, notice that we have called σv the cross-section. The reason for this is that σv is what determines the observable flux.
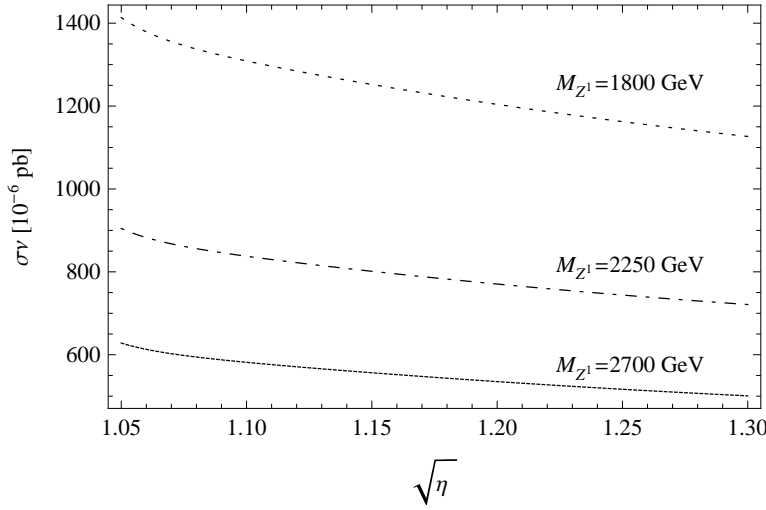

Figure 4.3. The annihilation cross-section for $m_{Z^1} = 1800, 2250, 2700$ GeV as a function of the mass splitting between the LKP and the heavier particles at the first KK-mode.

4.2.3 Result

The coupling constants could be taken from ordinary running to be $\frac{1}{\alpha_{\mathrm{em}}}(1 \text{ TeV}) \approx 123$ and $\frac{1}{\alpha_{\mathrm{SU(2)}}}(1 \text{ TeV}) \approx 95$. The result may then be summarized in fig. 4.3. The

first KK mode fermions and bosons are taken to have the same mass, so we only have one mass splitting parameter η. We see that the difference between the lightest and the heaviest allowed mass of the LKP is around a factor of two. Recall that this difference was an order of magnitude in the B^1 case. We also see that the cross-section decreases with larger mass splitting and larger LKP-mass.

4.2.4   Divergence non-cancellation

It turns out the the ultra-violet divergences do not cancel for Z^1 annihilation with our approximations. However, since the divergences do not necessarily cancel at each KK-level, this is not a severe problem. To analyze the divergences of our process, the divergent part of two additional processes that both resemble the one here in some way have been studied. They were $\gamma\gamma$-scattering in the SM and in scalar QED. In those models the divergences canceled (as they must). The results of these calculations could then be compared to what was obtained in our calculation. From a divergence point of view, Z^1 annihilation is actually almost two times photon-scattering in the SM. The thing that in the end differs from two times the SM are contributions from a few diagrams containing scalars, which is the reason to treat photon-scattering in scalar QED. Doing all these steps, we can feel confident that the remaining divergences do not come from some mistake in our calculations.
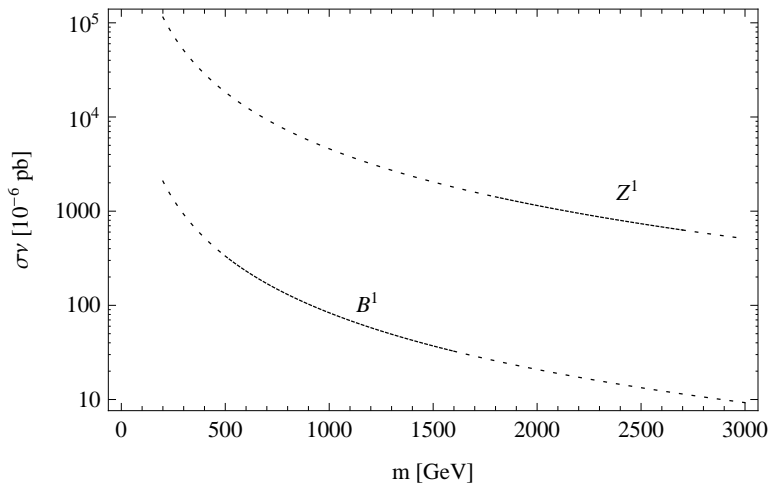

Figure 4.4. The annihilation cross-section for KKDM as a function of the mass, for mass splitting $\frac{m^{(1)}}{m_{\text{LKP}}} = 1.1$. The indicate parts of the curves show allowed mass regions from the relic abundance point of view. It is interesting to note that the LKP candidate Z^1, which arose from the non-minimal set up of UED, turned out to give a cross-section, which is around one order of magnitude larger than the one from B^1.

If one would consider all the KK-levels, the divergences should of course cancel in total for our process. However, the contributions to the finite part of the result are still much smaller from each higher level, and so it is anyway enough to consider the zeroth and first KK-levels and then simply remove the remaining divergences by hand.

4.3 Summarizing the results

We compare the results from the two calculations in fig. 4.4. It is interesting to notice that the cross-section for Z^1 as the LKP is roughly one order of magnitude larger than for B^1. This is quite surprising since $\sigma v \sim 1/m_{\mathrm{LKP}}^2$ and $m_{Z^1} > m_{B^1}$. However, in order to know how good the observational prospect are, one would have to calculate the continuous spectrum of gamma-rays to see if the peak from the direct annihilation to gamma-rays is possible to distinguish from the background. If this would not be the case, one has to use some other experimental technique in order to look for this DM candidate. The authors of ref. [14] actually calculated this background and showed that the peak, in the case of B^1, could be resolved from the background given that some new experiment that reaches high enough in energy and with good enough resolution will be performed.

Chapter 5

Summary and conclusions

This thesis has treated the subject of extra-dimensional physics. In particular, we have considered the phenomenology of Kaluza–Klein dark matter (DM) annihilation by introducing one extra spatial dimension. This has been considered from the point of view of a model called universal extra dimensions (UED). Both the minimal and non-minimal versions of this model have been studied. In the minimal version, only one possible DM candidate exists, namely B^1. This is however not the case in non-minimal UED, where also Z^1 and H^1 are possible DM candidates.

In ch. 1, we gave a short introduction to theoretical physics, particle physics, extra dimensions, and DM. In ch. 2, we discussed the Standard Model of particle physics, and some phenomena that this model fails to describe. After that, we treated how this could be adjusted in order to allow for extra dimensions. We introduced the concept of Kaluza–Klein decomposition and the UED model. We handled both the minimal and non-minimal versions of the UED model. In ch. 3, we described the need for DM and discussed some suggestions for what it could consist of. In particular, we treated the possibility that DM could originate from extra dimensions, as the lightest Kaluza–Klein particle. Of great interest was the possibility to have different DM candidates if one considered the non-minimal UED model. Finally, in ch. 4, we calculated the annihilation cross-section for two different DM candidates. The results are summarized in fig. 4.4, where one can see that the annihilation cross-section is larger for the heavier particle Z^1 than for B^1. Both particles have a decreasing cross-section with mass and scale as $1/m_{\mathrm{LKP}}^2$, which is why it is a bit surprising that the cross-section is larger for Z^1. This originates from the fact that Z^1 obeys a non-Abelian gauge theory, and hence, has self-interactions. This has the profound effect that more Feynman diagrams will contribute to its annihilation cross-section, which turned out to compensate for the heavier mass.

There are experiments that cover the energy range of these candidates, *e.g.* the ground-based MAGIC telescopes [13]. However, the energy resolution is probably not good enough as it is today. The first telescope has an energy resolution of around 20 % for the energy scale of interest, while the second telescope has a

resolution of about 10 % [34]. If future improvements of these telescopes, or similar ones, can make the energy resolution even better, detection or exclusion of our DM candidates could be made. However, not only the energy resolution is of importance. One also has to investigate if the peak in the DM flux from direct annihilation to photons could be resolved from the continuous background of indirect annihilation to photons.

It would be interesting to expand fig. 4.4 to incorporate H^1 as the LKP as well. Moreover, it would be interesting to calculate the background signal from Z^1 annihilation and H^1 annihilation. In the case that the H^1 is taken as the LKP, the relic abundance is furthermore not calculated. Hence, this would be yet another fascinating investigation to perform in connection to the topic treated in this thesis.

Appendix A

Passarino–Veltman functions

In this appendix, Passarino–Veltman (PV) functions [36] will discussed.

The main reason to use PV functions is that one easily can remove all tensor-structure from integrals, and one then only has to calculate scalar integrals in the end. Moreover, the solutions to such scalar integrals exist in the literature [37]. Another advantage is that the divergences of these functions are known, which means that one can check algebraically what happens when summing a lot of Feynman diagrams.

We will use the two-point PV functions in the discussion below, but the principle is the same for three- and four-point functions. For all the definitions in these cases the reader could consult ref. [36]. However, one should notice that they use a different signature for the metric.

A.1   The two-point functions B

The definition of the two-point PV function is given by

$$B_0; B_\mu; B_{\mu\nu}(p; m_1, m_2) = \int \frac{\mathrm{d}^n q}{(2\pi)^n} \frac{1; q_\mu; q_\mu q_\nu}{[q^2 - m_1^2]\,[(q+p)^2 - m_2^2]}, \qquad \text{(A.1)}$$

where p is the external momentum and m_i are masses of the particles running in the loop.

Now, the Lorentz structure of B_μ and $B_{\mu\nu}$ can be moved from the internal to the external momenta according to the following relations:

$$B_\mu = p_\mu B_1, \qquad \text{(A.2)}$$

$$B_{\mu\nu} \quad = \quad p_\mu p_\nu B_{21} - g_{\mu\nu} B_{22}. \tag{A.3}$$

The functions B_1, B_{21}, and B_{22} are related to A and B_0, where the one-point function A is defined as

$$A(m) = \int \frac{\mathrm{d}^n q}{(2\pi)^n} \frac{1}{q^2 - m^2}. \tag{A.4}$$

The set of equations linking B_1, B_{21}, and B_{22} to A and B_0, which have known solutions, are

$$p^2 B_1 = \frac{1}{2} \left[A(m_1) - A(m_2) - (p^2 + m_2^2 - m_1^2) B_0 \right],$$

$$p^2 B_{21} + B_{22} = \frac{1}{2} \left[A(m_2) + (m_1^2 - m_2^2 - p^2) B_1 \right], \tag{A.5}$$

$$A(m_2) - m_1^2 B_0 = p^2 B_{21} + 4 B_{22} + \frac{1}{2}(m_1^2 + m_2^2 + \frac{1}{3} p^2).$$

When working out traces in n dimensions, one could get factors of n appearing in the amplitudes [35]. One has to be careful when taking the limit $n \to 4$. One should use the following relations for the two-point functions

$$nB_0 \quad = \quad 4B_0 - 2, \tag{A.6}$$

$$nB_1 \quad = \quad 4B_1 + 1. \tag{A.7}$$

The last thing one wants to know about the PV functions is how the divergences look like. For the two-point functions, they are

$$\mathrm{div}(B_0) \quad = \quad \frac{2}{\epsilon}, \tag{A.8}$$

$$\mathrm{div}(B_1) \quad = \quad -\frac{1}{\epsilon}. $$

A.2   Solutions to scalar integrals B_0 and C_0

Here we present the results for the scalar integrals needed in ch. 4, calculated using ref. [37]. We have chosen to present the expressions without divergences, since we have handled them separately, *viz.*

$$C_0(4, 0, 0; \eta, \eta, \eta) \quad = \quad -\frac{1}{2} \arctan^2 \left(\frac{1}{\sqrt{\eta - 1}} \right), \tag{A.9}$$

$$C_0(1, 0, -1; 0, \eta, \eta) \quad = \quad \frac{1}{2} \left\{ \mathrm{Li}_2 \left(-\frac{1}{\eta} \right) - \mathrm{Li}_2 \left(\frac{1}{\eta} \right) \right\}, \tag{A.10}$$

$$C_0(1, 0, -1; \eta, 0, 0) \quad = \quad \frac{1}{2} \left\{ \mathrm{Li}_2 \left(-\frac{\eta}{\eta - 1} \right) - \mathrm{Li}_2 \left(\frac{\eta}{\eta + 1} \right) \right\}, \tag{A.11}$$

$$B_0(4; 0, 0) \quad = \quad 2 - 2 \log 2 + \mathrm{i}\pi, \tag{A.12}$$

$$
\begin{aligned}
B_0(4;\eta,\eta) &= 2 - \log\eta - 2\sqrt{\eta-1}\,\arctan\left(\frac{1}{\sqrt{\eta-1}}\right), & \text{(A.13)}\\
B_0(0;\eta,\eta) &= -\ln(\eta), & \text{(A.14)}\\
B_0(1;0,\eta) &= 2 - \eta\log\eta + (\eta-1)\log(\eta-1), & \text{(A.15)}\\
B_0(-1;0,\eta) &= 2 + \eta\log\eta - (\eta+1)\log(\eta+1), & \text{(A.16)}
\end{aligned}
$$

where $\eta \equiv \left(\frac{m_{\text{internal}}}{m_{\text{initial}}}\right)^2$ and the Spence function or dilogarithm is defined by

$$
\text{Li}_2(z) \equiv -\int_0^1 dt\,\frac{\log(1-zt)}{t} = \sum_{k=1}^\infty \frac{z^k}{k^2}\,. \tag{A.17}
$$

Also, note that we have used a different normalization in the above expressions than in the definition (A.1).

A.3 LERG-I

There is a Mathematica software package that performs the PV reduction, called LERG [38]. The main feature of this program is to solve systems of equations like the ones in eq. (A.5). One then gets a linear combination of scalar integrals, which are solvable.

Another feature of LERG is that it can handle the situation when the momenta of the ingoing particles are the same, which is the case for WIMP DM annihilation. In this situation, the normal PV reduction scheme does actually not work.

One should notice that LERG is implemented with a different signature for the metric than the one used in this thesis.

Appendix B

Feynman rules

In this appendix, the method of deriving interaction Feynman rules from a Lagrangian using the path integral formalism will be discussed. This method has been used in order to derive the new rules needed to complete the calculation of $Z^1 Z^1$ annihilation.

B.1 An example

As an illustrative example, the Feynman rule of $A_\mu^{(0)} A_\nu^{(0)} W_5^{(1)\pm} W_5^{(1)\mp}$ will be derived carefully. This will introduce the necessary concepts, which then easily could be generalized.

For this example we are interested in the non-Abelian part of the kinetic Lagrangian. It looks like

$$\mathcal{L} = -\frac{1}{4} F_{MN}^i F^{iMN}, \tag{B.1}$$

where $M, N = 0, 1, 2, 3, 5$. The definition of F_{MN}^i is

$$F_{MN}^i = \partial_M A_N^i - \partial_N A_M^i + g f^{ijk} A_M^j A_N^k. \tag{B.2}$$

Equation (B.1) can be rewritten as

$$\mathcal{L} = -\frac{1}{4} (F_{\mu\nu}^i F^{i\mu\nu} + 2 F_{5\mu}^i F^{i5\mu} + F_{55}^i F^{i55}). \tag{B.3}$$

Looking at the three parts obtained, we see that the first term is simply the vector-vector interactions. The second term is the vector-scalar interactions, and the third term is zero due to the antisymmetry of F_{MN}^i.

Some vertex rules are already existing in the literature, see *e.g.* ref. [39]. If we now focus on the four-point interaction between vectors and scalars, it is enough to

keep the second term in eq. (B.3). Since we are interested in a four-point vertex, we do not keep terms with partial derivatives. Thus, we obtain the Lagrangian

$$\mathcal{L}' = -\frac{1}{2}g_2^2\epsilon^{ijk}\epsilon^{ilm}A_5^j A_\mu^k A^{l5}A^{m\mu}, \tag{B.4}$$

where we have used the structure constants ϵ^{ijk}, since we have an $SU(2)$ gauge group. The Fourier expansion of the gauge fields are

$$A_\mu^i = \frac{1}{\sqrt{\pi R}}A_\mu^{i(0)} + \sqrt{\frac{2}{\pi R}}\sum_{n=1}^\infty A_\mu^{i(n)}\cos\left(\frac{ny}{R}\right), \tag{B.5}$$

$$A_5^i = \sqrt{\frac{2}{\pi R}}\sum_{n=1}^\infty A_5^{i(n)}\sin\left(\frac{ny}{R}\right). \tag{B.6}$$

Since we are interested in the interaction between two zero-modes and two scalars from the first KK-mode, we get by explicitly writing out the integral over the extra dimension

$$\mathcal{L}'' = \frac{1}{2}g_2^2\epsilon^{ijk}\epsilon^{ilm}g^{\mu\nu}\frac{1}{\pi R}\frac{2}{\pi R}A_5^{j(1)}A_5^{l(1)}A_\mu^{k(0)}A_\nu^{m(0)}\int_0^{\pi R}\mathrm{d}y\sin^2\left(\frac{y}{R}\right). \tag{B.7}$$

This can now be simplified if we redefine the coupling constant $\frac{1}{\sqrt{\pi R}}g_2 \to g_2$. Equation (B.7) then reads

$$\mathcal{L}'' = \frac{1}{2}g_2^2\epsilon^{ijk}\epsilon^{ilm}g^{\mu\nu}A_5^{j(1)}A_5^{l(1)}A_\mu^{k(0)}A_\nu^{m(0)}. \tag{B.8}$$

We want to have a final state with two photons in our process. With that in mind, we put the gauge index of the zero-mode particles equal to 3. Using the properties of the Levi-Civita tensor, the non-vanishing terms then are

$$\begin{aligned}
\mathcal{L}''' &= \frac{1}{2}\sum_{i=1,2}g_2^2 g^{\mu\nu}\epsilon^{ij3}\epsilon^{il3}A_5^{j(1)}A_5^{l(1)}A_\mu^{3(0)}A_\nu^{3(0)} \\
&= \frac{1}{2}g_2^2 g^{\mu\nu}A_\mu^{3(0)}A_\nu^{3(0)}\left(A_5^{1(1)}A_5^{1(1)} + A_5^{2(1)}A_5^{2(1)}\right).
\end{aligned}$$

$$\tag{B.10}$$

Now, we can use the definition of $W^\pm$, namely

$$W_M^\pm = \frac{1}{\sqrt{2}}\left(A_M^1 \mp \mathrm{i}A_M^2\right). \tag{B.11}$$

Also $A_\mu^{3(0)}$ can be rewritten using

$$A_M^3 = s_w A_M + c_w Z_M. \tag{B.12}$$

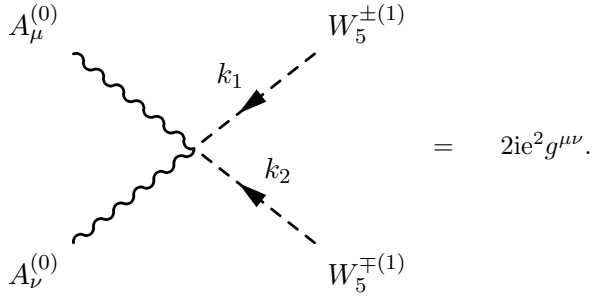Putting in these two relations into eq. (B.9), one gets for the photon

$$\mathcal{L}^{(\text{iv})} = e^2 g^{\mu\nu} A_\mu^{(0)} A_\nu^{(0)} W_5^{(1)+} W_5^{(1)-}, \qquad (\text{B.13})$$

where also the relation $s_w g_2 = e$ has been used.

The last step is to take the functional derivative of this expression w.r.t. all the fields involved. This gives an extra factor of two, since we in principle have $(A^{(0)})^2$. After taking the functional derivative we introduce an additional i, since the functional integral is defined as

$$\int \mathcal{D}A e^{\text{i}S}. \qquad (\text{B.14})$$

Finally, we arrive at the following Feynman rule:
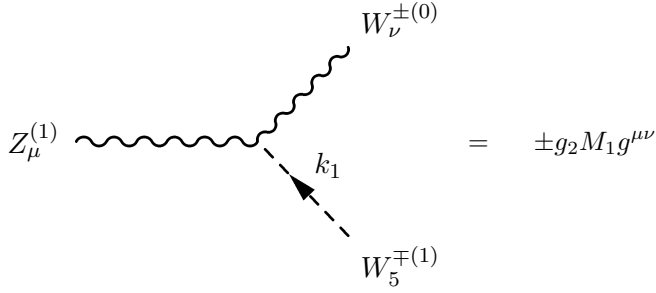
This completes our calculation of the Feynman rule for the four-point function with two photons and two ED scalars. All other rules needed are given in section B.2. The derivations of those rules are similar to the one just performed above.

B.2 List of Feynman rules
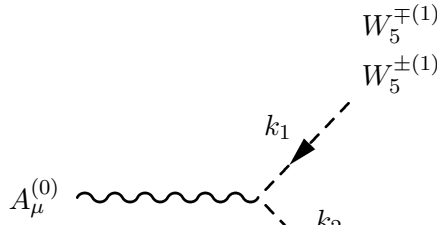
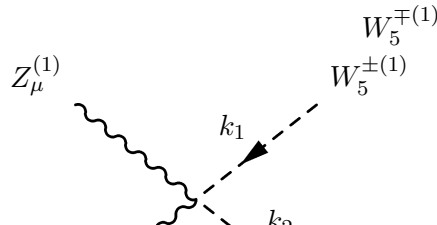Here we present a list of all derived Feynman rules:
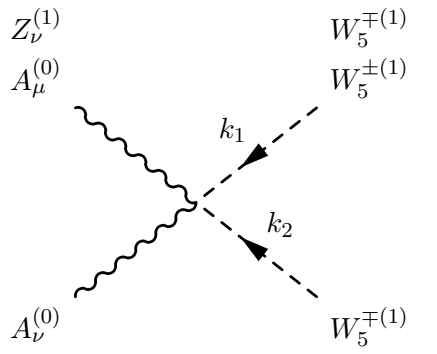
$$= \quad \mp e M_1 g^{\mu\nu}$$

$$= \quad \pm \mathrm{i} e (k_2 - k_1)^{\mu}$$

$$= \quad \mathrm{i} g_2^2 g^{\mu\nu}$$

$$= \quad 2\mathrm{i} e^2 g^{\mu\nu}$$

$$
c^{\mp(1/0)}
$$

$$
A_\mu^{(0)} \qquad\qquad k_1 \qquad \bar{c}^{\mp(1/0)} \qquad\qquad = \qquad \mp e k_1^\mu
$$

$$
c^{\mp(1/0)}
$$

$$
Z_\mu^{(1)} \qquad\qquad k_1 \qquad \bar{c}^{\mp(0/1)} \qquad\qquad = \qquad \mp g_2 k_1^\mu
$$

In the above rules the parameter $M_1 = \frac{1}{R}$, where R is the size of the extra dimension.

Appendix C

Amplitude coefficients

In this appendix, we present the explicit expressions for the coefficients A, B_i, and C_i used in ch. 4, *viz.*

$$
\begin{aligned}
A &= -12g_{\text{eff}}^2 B_0(-1,0,\eta) + \frac{12(g_{\text{eff}}^2 - 3g_{\text{eff}}^2\eta)B_0(1,0,\eta)}{1-\eta} \\
&\quad - \frac{24g_{\text{eff}}^2\eta B_0(4,\eta,\eta)}{\eta-1} + 24B_0(-1,0,\omega) \\
&\quad - \frac{24(1-3\omega)B_0(1,0,\omega)}{1-\omega} + \frac{48\omega B_0(4,\omega,\omega)}{\omega-1} \\
&\quad + \frac{16\left(g_{\text{eff}}^2\eta^2 + 2g_{\text{eff}}^2\eta\right)C_0(0,0,4,\eta,\eta,\eta)}{1-\eta} \\
&\quad + \frac{8\left(-5g_{\text{eff}}^2\eta^2 - 2g_{\text{eff}}^2\eta + g_{\text{eff}}^2\right)C_0(1,0,-1,0,\eta,\eta)}{1-\eta} \\
&\quad - \frac{32\left(\omega^2 + 2\omega\right)C_0(0,0,4,\omega,\omega,\omega)}{1-\omega} \\
&\quad - \frac{16\left(-5\omega^2 - 2\omega + 1\right)C_0(1,0,-1,0,\omega,\omega)}{1-\omega} + 8(g_{\text{eff}}^2 - 2), \quad\quad\quad (C.1) \\
B_1 &= -\frac{2\left(-5g_{\text{eff}}^2\eta^2 - 2g_{\text{eff}}^2\eta + 19g_{\text{eff}}^2\right)B_0(1,0,\eta)}{3(1-\eta)(\eta+1)} \\
&\quad + \frac{4B_0(4,0,0)(4g_{\text{eff}}^2\omega + 4g_{\text{eff}}^2 + 3\eta\omega - 47\eta + 3\omega - 47)}{3(\eta+1)(\omega+1)} \\
&\quad + 2g_{\text{eff}}^2 B_0(-1,0,\eta) + \frac{4(4g_{\text{eff}}^2 - g_{\text{eff}}^2\eta)B_0(4,\eta,\eta)}{3(1-\eta)} \\
&\quad + \frac{4\left(3\omega^3 - 14\omega^2 - 23\omega + 94\right)B_0(1,0,\omega)}{3(1-\omega)(\omega+1)}
\end{aligned}
$$

$$+4\omega B_0(-1,0,\omega) - 2B_0(0,\omega,\omega) - \frac{2(91-31\omega)B_0(4,\omega,\omega)}{3(1-\omega)}$$

$$-\frac{8g_{\text{eff}}^2\eta^2 C_0(0,0,4,\eta,\eta,\eta)}{\eta-1} - \frac{8g_{\text{eff}}^2 C_0(1,0,-1,0,\eta,\eta)}{1-\eta}$$

$$-\frac{2\left(3\omega^2+37\omega+24\right)C_0(0,0,4,\omega,\omega,\omega)}{1-\omega}$$

$$+\frac{4\left(-6\omega^2-5\omega+43\right)C_0(1,0,-1,0,\omega,\omega)}{1-\omega} - 4(9-\omega)C_0(0,1,-1,0,0,\omega)$$

$$-32C_0(0,0,4,0,0,0) + \frac{1}{3}(31-8g_{\text{eff}}^2), \tag{C.2}$$

$$\begin{aligned}
B_2 &= \frac{\left(-13g_{\text{eff}}^2\eta^2 - 10g_{\text{eff}}^2\eta - 13g_{\text{eff}}^2\right)B_0(1,0,\eta)}{3(1-\eta)(\eta+1)}\\[2mm]
&+\frac{B_0(4,0,0)(16g_{\text{eff}}^2\omega + 16g_{\text{eff}}^2 + 15\eta\omega - 191\eta + 15\omega - 191)}{6(\eta+1)(\omega+1)}\\[2mm]
&-g_{\text{eff}}^2 B_0(-1,0,\eta) + \frac{2(5g_{\text{eff}}^2\eta + 4g_{\text{eff}}^2)B_0(4,\eta,\eta)}{3(1-\eta)}\\[2mm]
&+\frac{\left(55\omega^2 + 4\omega + 361\right)B_0(1,0,\omega)}{6(1-\omega)(\omega+1)} + \frac{5}{2}B_0(-1,0,\omega)\\[2mm]
&+\frac{1}{2}(3\omega+1)B_0(0,0,0) + \frac{1}{8}(-12\omega-1)B_0(0,\omega,\omega)\\[2mm]
&-\frac{7(13\omega+107)B_0(4,\omega,\omega)}{24(1-\omega)} + \frac{4\left(g_{\text{eff}}^2\eta^2 + 2g_{\text{eff}}^2\eta\right)C_0(0,0,4,\eta,\eta,\eta)}{1-\eta}\\[2mm]
&+\frac{4\left(2g_{\text{eff}}^2\eta^2 + g_{\text{eff}}^2\eta\right)C_0(1,0,-1,0,\eta,\eta)}{\eta-1}\\[2mm]
&-\frac{\left(33\omega^2 + 143\omega + 232\right)C_0(0,0,4,\omega,\omega,\omega)}{4(1-\omega)}\\[2mm]
&-\frac{3\left(-\omega^4 - 3\omega^3 + 25\omega^2 + 7\omega + 12\right)C_0(0,1,-1,0,0,\omega)}{2\omega(\omega+1)}\\[2mm]
&-\frac{\left(3\omega^4 - 27\omega^3 + 11\omega^2 - 155\omega - 36\right)C_0(1,0,-1,0,\omega,\omega)}{2(1-\omega)\omega}\\[2mm]
&-\frac{8(\omega+7)C_0(0,0,4,0,0,0)}{\omega+1} + \frac{1}{24}(64g_{\text{eff}}^2 - 137), \tag{C.3}
\end{aligned}$$

$$\begin{aligned}
B_6 &= \frac{\left(41g_{\text{eff}}^2\eta^2 + 26g_{\text{eff}}^2\eta - 31g_{\text{eff}}^2\right)B_0(1,0,\eta)}{3(1-\eta)(\eta+1)}\\[2mm]
&+\frac{B_0(4,0,0)(8g_{\text{eff}}^2\omega + 8g_{\text{eff}}^2 + 15\eta\omega - 103\eta + 15\omega - 103)}{3(\eta+1)(\omega+1)}\\[2mm]
&+5g_{\text{eff}}^2 B_0(-1,0,\eta) + \frac{2(4g_{\text{eff}}^2 - 13g_{\text{eff}}^2\eta)B_0(4,\eta,\eta)}{3(1-\eta)}
\end{aligned}$$

$$+\frac{\left(-9\omega^3 - 73\omega^2 - 73\omega + 227\right)B_0(1,0,\omega)}{3(1-\omega)(\omega+1)} + (-\omega-9)B_0(-1,0,\omega)$$

$$+(1-\omega)B_0(0,0,0) + \frac{1}{4}(-4\omega-1)B_0(0,\omega,\omega) + \frac{(253\omega-397)B_0(4,\omega,\omega)}{12(1-\omega)}$$

$$-\frac{4\left(g_{\text{eff}}^2\eta^2 + 2g_{\text{eff}}^2\eta\right)C_0(0,0,4,\eta,\eta,\eta)}{1-\eta}$$

$$-\frac{4\left(-3g_{\text{eff}}^2\eta^2 - g_{\text{eff}}^2\eta + g_{\text{eff}}^2\right)C_0(1,0,-1,0,\eta,\eta)}{1-\eta}$$

$$-\frac{\left(-13\omega^2 + 3\omega + 122\right)C_0(0,0,4,\omega,\omega,\omega)}{2(1-\omega)}$$

$$-\frac{\left(\omega^4 - 5\omega^3 + 41\omega^2 + 9\omega + 18\right)C_0(0,1,-1,0,0,\omega)}{\omega(\omega+1)}$$

$$-\frac{\left(\omega^4 + 29\omega^3 + 19\omega^2 - 87\omega - 18\right)C_0(1,0,-1,0,\omega,\omega)}{(1-\omega)\omega}$$

$$-\frac{4(\omega+15)C_0(0,0,4,0,0,0)}{\omega+1} + \frac{1}{12}(29 - 16g_{\text{eff}}^2), \tag{C.4}$$

$$C_1 = \frac{\left(-5g_{\text{eff}}^2\eta^2 - 2g_{\text{eff}}^2\eta + 19g_{\text{eff}}^2\right)B_0(1,0,\eta)}{3(1-\eta)(\eta+1)}$$

$$-\frac{8B_0(4,0,0)(g_{\text{eff}}^2\omega + g_{\text{eff}}^2 - 11\eta - 11)}{3(\eta+1)(\omega+1)} - g_{\text{eff}}^2 B_0(-1,0,\eta)$$

$$-\frac{2(4g_{\text{eff}}^2 - g_{\text{eff}}^2\eta)B_0(4,\eta,\eta)}{3(1-\eta)}$$

$$-\frac{16\left(-\omega^2 - \omega + 11\right)B_0(1,0,\omega)}{3(1-\omega)(\omega+1)} - B_0(0,\omega,\omega)$$

$$+\frac{(349 - 61\omega)B_0(4,\omega,\omega)}{12(1-\omega)} + \frac{4g_{\text{eff}}^2\eta^2 C_0(0,0,4,\eta,\eta,\eta)}{\eta-1}$$

$$+\frac{4g_{\text{eff}}^2 C_0(1,0,-1,0,\eta,\eta)}{1-\eta} - \frac{\left(3\omega^2 - 87\omega - 44\right)C_0(0,0,4,\omega,\omega,\omega)}{2(1-\omega)}$$

$$+\frac{2\left(\omega^2 + 6\omega + 9\right)C_0(0,1,-1,0,0,\omega)}{\omega+1}$$

$$-\frac{2\left(-3\omega^2 - 6\omega + 41\right)C_0(1,0,-1,0,\omega,\omega)}{1-\omega}$$

$$+\frac{4(5\omega+3)C_0(0,0,4,0,0,0)}{\omega+1} + \frac{1}{12}(16g_{\text{eff}}^2 - 59), \tag{C.5}$$

$$C_2 = \frac{\left(-13g_{\text{eff}}^2\eta^2 - 10g_{\text{eff}}^2\eta - 13g_{\text{eff}}^2\right)B_0(1,0,\eta)}{3(1-\eta)(\eta+1)}$$

$$+\frac{B_0(4,0,0)(8g_{\text{eff}}^2\omega + 8g_{\text{eff}}^2 - 9\eta\omega - 79\eta - 9\omega - 79)}{3(\eta+1)(\omega+1)}$$

$$-g_{\text{eff}}^2 B_0(-1,0,\eta) + \frac{2(5g_{\text{eff}}^2\eta + 4g_{\text{eff}}^2)B_0(4,\eta,\eta)}{3(1-\eta)}$$

$$+\frac{\left(23\omega^2 + 32\omega + 149\right)B_0(1,0,\omega)}{3(1-\omega)(\omega+1)} + 3B_0(-1,0,\omega) + B_0(0,\omega,\omega)$$

$$+\frac{(-83\omega - 325)B_0(4,\omega,\omega)}{12(1-\omega)} + \frac{4\left(g_{\text{eff}}^2\eta^2 + 2g_{\text{eff}}^2\eta\right)C_0(0,0,4,\eta,\eta,\eta)}{1-\eta}$$

$$-\frac{4\left(2g_{\text{eff}}^2\eta^2 + g_{\text{eff}}^2\eta\right)C_0(1,0,-1,0,\eta,\eta)}{1-\eta}$$

$$+\frac{2\left(7\omega^3 + 34\omega + 9\right)C_0(1,0,-1,0,\omega,\omega)}{(1-\omega)\omega}$$

$$-\frac{\left(17\omega^2 + 79\omega + 104\right)C_0(0,0,4,\omega,\omega,\omega)}{2(1-\omega)}$$

$$-\frac{2\left(2\omega^3 + 15\omega^2 + 6\omega + 9\right)C_0(0,1,-1,0,0,\omega)}{\omega(\omega+1)}$$

$$-\frac{16(\omega+3)C_0(0,0,4,0,0,0)}{\omega+1} + \frac{1}{12}(32g_{\text{eff}}^2 - 73), \tag{C.6}$$

where $\eta = \left(\frac{m_{\text{fermion}1}}{m_{Z^1}}\right)^2$, $\omega = \left(\frac{m_{\text{boson}1}}{m_{Z^1}}\right)^2$, and g_{eff}^2 is defined in eq. (C.7). The above given coefficients have been calculated using the software LERG-I [38], which is shortly described in section A.3. The solutions to the scalar integrals $B_0(...)$ and $C_0(...)$ are given in section A.2. Notice that some of the scalar integrals become infra-red divergent for some particular argument. The reason is that we have treated the zero mode W bosons as massless, since they are so much lighter than the first exited ones. In order to remove these divergences, we have introduced a small parameter $\epsilon = 0.01$ instead of zero in in the numerical calculations, at places where this was a problem.

The definition of g_{eff} is

$$g_{\text{eff}}^2 \equiv \sum Q^2 T_3^2 = \frac{1}{4}\sum Q^2, \tag{C.7}$$

where the sum is over all $SU(2)$ doublet fermions.

Bibliography

[1] LHC/LC Study Group, G. Weiglein *et al.*, *Physics interplay of the LHC and the ILC*, Phys. Rept. **426**, 47 (2006), `hep-ph/0410364`.

[2] IceCube Collaboration, M. Danninger and K. Han.

[3] M. Kaku, *Hyperspace; a scientific odyssey through parallel universes, time warps, and the tenth dimension* (Anchor Books, New York, NY, 1995).

[4] Particle Data Group, K. Nakamura *et al.*, *Review of particle physics*, J.Phys.G **G37**, 075021 (2010).

[5] A. Einstein, *The Foundation of the General Theory of Relativity*, Annalen Phys. **49**, 769 (1916).

[6] S. Dimopoulos and D. W. Sutter, *The Supersymmetric flavor problem*, Nucl. Phys. **B452**, 496 (1995), `hep-ph/9504415`.

[7] P. Horava and E. Witten, *Eleven-Dimensional Supergravity on a Manifold with Boundary*, Nucl. Phys. **B475**, 94 (1996), `hep-th/9603142`.

[8] T. Kaluza, *On the unification problem in physics*, (1984), English translation of Kaluza's paper which appeared in Sitzungsberichte of the Prussian Acad. of Sci., p.966, 1921.

[9] O. Klein, *Quantum Theory and Five-Dimensional Theory of Relativity. (In German and English)*, Z. Phys. **37**, 895 (1926).

[10] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, Phys. Rev. Lett. **83**, 3370 (1999), `hep-ph/9905221`.

[11] T. Appelquist, H. C. Cheng and B. A. Dobrescu, *Bounds on universal extra dimensions*, Phys. Rev. **D64**, 035002 (2001), `hep-ph/0012100`.

[12] Fermi LAT Collaboration, A. Cuoco *et al.*, *Anisotropies in the Diffuse Gamma-Ray Background from Dark Matter with Fermi LAT: a closer look*, (2010), `1005.0843`.

[13] MAGIC collaboration, C. Baixeras *et al.*, *Design studies for ECO, the European gamma-ray observatory*, (2004), `astro-ph/0403180`.

[14] L. Bergstrom *et al.*, *Two photon annihilation of Kaluza-Klein dark matter*, JCAP **0504**, 004 (2005), `hep-ph/0412001`.

[15] G. Bertone *et al.*, *Gamma Ray Lines from a Universal Extra Dimension*, (2010), `1009.5107`.

[16] E. K. Akhmedov, *Neutrino physics*, (1999), `hep-ph/0001264`.

[17] S. P. Martin, *A Supersymmetry Primer*, (1997), `hep-ph/9709356`.

[18] D. Hooper and S. Profumo, *Dark matter and collider phenomenology of universal extra dimensions*, Phys. Rept. **453**, 29 (2007), `hep-ph/0701197`.

[19] R. Sundrum, *To the fifth dimension and back. (TASI 2004)*, (2005), `hep-th/0508134`.

[20] T. Flacke, A. Menon and D. J. Phalen, *Non-minimal universal extra dimensions*, Phys. Rev. **D79**, 056009 (2009), `0811.1598`.

[21] V. C. Rubin, N. Thonnard and W. K. Ford, Jr., *Rotational properties of 21 SC galaxies with a large range of luminosities and radii, from NGC 4605 (R = 4kpc) to UGC 2885 (R = 122 kpc)*, Astrophys. J. **238**, 471 (1980).

[22] F. Zwicky, *Spectral displacement of extra galactic nebulae*, Helv. Phys. Acta **6**, 110 (1933).

[23] J. Diemand, M. Kuhlen and P. Madau, *Dark matter substructure and gamma-ray annihilation in the Milky Way halo*, Astrophys. J. **657**, 262 (2007), `astro-ph/0611370`.

[24] V. Springel *et al.*, *Prospects for detecting supersymmetric dark matter in the Galactic halo*, Nature **456N7218**, 73 (2008).

[25] A. D. Dolgov and S. H. Hansen, *Massive sterile neutrinos as warm dark matter*, Astropart. Phys. **16**, 339 (2002), `hep-ph/0009083`.

[26] R. Bernabei *et al.*, *New results from DAMA/LIBRA*, Eur. Phys. J. **C67**, 39 (2010), `1002.1028`.

[27] CDMS, D. S. Akerib *et al.*, *Low-threshold analysis of CDMS shallow-site data*, Phys. Rev. **D82**, 122004 (2010), `1010.4290`.

[28] F. Halzen and S. R. Klein, *IceCube: An Instrument for Neutrino Astronomy*, Rev.Sci.Instrum. **81**, 081101 (2010), `1007.1247`.

[29] J. L. Feng, A. Rajaraman and F. Takayama, *SuperWIMP Dark Matter Signals from the Early Universe*, Phys. Rev. **D68**, 063504 (2003), `hep-ph/0306024`.

[30] J. L. Feng, A. Rajaraman and F. Takayama, *Superweakly-interacting massive particles*, Phys. Rev. Lett. **91**, 011302 (2003), `hep-ph/0302215`.

[31] N. Jarosik *et al.*, *Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Sky Maps, Systematic Errors, and Basic Results*, Astrophys. J. Suppl. **192**, 14 (2011), `1001.4744`.

[32] S. Arrenberg *et al.*, *Kaluza-Klein Dark Matter: Direct Detection vis-a-vis LHC*, Phys. Rev. **D78**, 056002 (2008), `0805.4210`.

[33] L. Bergstrom *et al.*, *Gamma rays from Kaluza-Klein dark matter*, Phys. Rev. Lett. **94**, 131301 (2005), `astro-ph/0410359`.

[34] MAGIC collaboration, M. M. Hendrik Bartko, Search for dark matter with the magic telescope, conferance talk, the hunt for dark matter, FNAL, 2007.

[35] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Perseus Books, Cambridge, Massachusetts, 1995).

[36] G. Passarino and M. Veltman, *One Loop Corrections for e+ e- Annihilation Into mu+ mu- in the Weinberg Model*, Nucl.Phys. **B160**, 151 (1979).

[37] G. 't Hooft and M. Veltman, *Scalar One Loop Integrals*, Nucl.Phys. **B153**, 365 (1979).

[38] R. G. Stuart, *Algebraic reduction of one loop Feynman diagrams to scalar integrals*, Comput. Phys. Commun. **48**, 367 (1988).

[39] M. Gustafsson, *Light from Dark Matter*, PhD thesis, Stockholm University, 2008.