Simulation of Trading Arrangements Impact on Wind Power Imbalance Costs

Mikael Amelin, Member, IEEE

Abstract—Uncertain wind power forecasts is a disadvantage in an electricity market where the majority of the trading is performed several hours before the actual delivery. This paper presents a model which can be used to study how changes in the trading arrangement—in particular changing the delay time between closure of the spot market and the delivery period or changing the imbalance pricing system—would affect different players in the electricity market. The model can be used in Monte Carlo simulation, which is demonstrated for an example system.

Index Terms—wind power generation, power generation economics, power system economics.

I. NOMENCLATURE

Variables

\( D \) demand
\( G \) generation
\( \delta \) imbalance
\( \lambda \) price

Functions

\( B_{\delta}(\diamond) \) value of \( \diamond \)
\( C_{\delta}(\diamond) \) cost of \( \diamond \)

Sets

\( C \) index set of consumers
\( G \) index set of producers

Specifiers

\( \diamond \) maximal value of \( \diamond \)
\( \diamond^{\epsilon} \) forecast error for \( \diamond \)
\( \diamond^{1} \) value of \( \diamond \) in the spot market
\( \diamond^{2} \) value of \( \diamond \) in the adjustment market
\( \diamond^{\uparrow} \) up-regulation of \( \diamond \)
\( \diamond^{\downarrow} \) down-regulation of \( \diamond \)
\( \diamond_{r} \) real-time period \( r \)
\( \diamond_{b} \) balance responsible player \( b \)

II. INTRODUCTION

THE concern of climate changes due to global warming have prompted many countries to set up ambitious plans to increase the amount of renewable electricity generation, and wind power is often an important part of these plans. Already today, about 15 % of the electricity generation in Denmark is from wind power [1]. The corresponding numbers for Spain is about 8% and for Germany about 5%. In total, the growth rate of the installed wind power capacity has been about 20% or higher the recent years. However, this fast development is to some extent been relying on subsidies. To decrease this dependency, it is important that electricity markets are designed in such a way that wind power can compete with other power sources on reasonable terms.

Wind power in a competitive electricity market has slightly different conditions compared to conventional thermal power or hydro power; the maximal wind power output depends on the wind speed, which can be difficult to predict more than a few hours in advance. Moreover, the energy contents of the wind is proportional to the cube of the wind speed; hence, a small forecast error in the wind speed may result in a large forecast error in available wind power capacity [2]. Accurate forecasts are important in competitive electricity markets, since most of the trading is settled before the actual time of delivery, and players who cannot fulfil their obligations will have to pay imbalance costs. The uncertainties in the wind power forecasts will therefore give wind power producers a disadvantage in comparison to thermal power and hydro power, which generally can follow the planned generation schedule, and in those cases when there is an outage, it is likely that there is enough time to start up a reserve unit to replace the lost capacity. The shorter the lead time from market gate closure to delivery, the lesser the disadvantage for wind power producers. However, shorter lead times also results in increased costs for the electricity trading, as more staff is needed by traders and in control centres.

There is a need for simulation tools that can be used to study the consequences of different trading arrangements. In [3] the consequences of different trading arrangement to the Danish wind power producers were analysed. This study used actual generation data and prices from 2003. In [4], the impact of the imbalance pricing was studied based on three years of historical data from the U.K.

The aim of this paper is to present a method to simulate electricity prices taking into account the trading arrangements of the electricity market. The method uses Monte Carlo simulation to estimate how the forecast errors affect generation, consumption and electricity prices. The paper is organised as follows: The general features of the trading arrangements in an electricity market are described in section III. A simple mathematical model is presented in section IV. Section V discusses Monte Carlo simulation and an example is given in section VI. Section VII summarises the conclusions of the paper. Finally, there is an appendix providing a further justification of the models used in section IV.
III. TRADING ARRANGEMENTS

A typical contract on a competitive electricity market is that the seller injects an amount of energy to the power system during a certain period, and the buyer extracts the same amount during the same period. The shortest trading period is generally one hour, but some electricity markets use shorter trading periods. The generation and the consumption should be in balance for each trading period, i.e., the seller should generate as much energy as consumed by the buyer, but does not have to be balanced momentarily. The responsibility to maintain the frequency (i.e., to keep the continuous balance between electricity generation and load) is given to a system operator. The result of this arrangement is that three phases of the electricity trading can be identified: an ahead market before each trading period, a real-time market during the trading period, and a post market after the trading period. The terminology and design of these phases vary from electricity market to electricity market. The objective of this section is to establish a general terminology that can be used to describe the main functions that may be found in any electricity market, and which can be used to compare different possible trading arrangements.

A. The Ahead Market

The ahead market comprises all contracts which are signed before the time of delivery. This includes both contracts of physical delivery and financial derivatives. Physical contracts can be traded bilaterally or at a power exchange (spot market). Trading on a power exchange in generally performed on a daily basis. For example, bids to the Nordic power exchange Nord Pool Elspot should be submitted no later than at noon on the day before the hour of delivery [5], i.e., between 24 and 36 hour before the time of delivery.

After the closure of the spot market, there might be a possibility to trade in an adjustment market. An example of an adjustment market is the Nord Pool Elbas market, which is open up to one hour before the delivery hour [6]. This gives the players in the electricity a possibility to compensate for forecast errors or outages in power plants.

B. Real-time Market

During the delivery period, players are supposed to follow the resulting plans from the ahead market. However, deviations from the plans are inevitable. Smaller deviations are managed by automatic control systems (primary control and automatic generation control). The reserves of these systems are limited and the system operator may have to release them by procuring balancing resources from the players of the electricity market.

Up-regulation means that the system operator is buying energy, i.e., a producer activates an up-regulation bid by increasing the generation, whereas a consumer with controllable load can activate an up-regulation bid by decreasing the consumption. Similarly, down-regulation means that the system operator is selling energy, i.e., producers decrease the generation and consumers increase their load.

C. Post Market

After the trading period, measurements as well as trading in the ahead market and the real-time market are compiled by the system operator, and an imbalance can be calculated for each balance responsible player. A positive imbalance means that the player has generated or purchased more energy than the consumption or sales during that trading period. To settle this imbalance, the balance responsible player has to sell imbalance power to the system operator. Similarly, a player with a negative imbalance must buy imbalance power from the system operator.

There are many ways to set the prices of imbalance power. Usually, the prices are correlated to the prices in the real-time market. The two most common solutions are single imbalance pricing and dual imbalance pricing, which are illustrated in Fig. 1. With single imbalance pricing, the same price is applied to positive and negative imbalances; the price of imbalance power is equal to the up-regulation price if the net regulation during the trading period is upwards, and the down-regulation price is used for a net down-regulation trading period. Dual imbalance pricing means that a less favourable price is given to those players who are assumed to be the cause of the activation of regulating bids. During an up-regulation period those players who have not supplied enough energy, i.e., which have negative imbalances, must pay the up-regulation price (which is higher.

![Fig. 1. Examples of imbalance pricing systems. The up-regulation price can either be the highest price for an activated up-regulation bid (marginal pricing), or the mean cost of up-regulation (average pricing). The down-regulation price is defined in a similar manner. The system price can be obtained from the spot price at a power exchange. An alternative to using the system price in the dual imbalance pricing is to use the regulation price in the reverse direction, i.e., the down-regulation price during an up-regulation period and vice versa. Cf. [7].](image-url)
than the price in the ahead market), while the players having positive balance receive the same price as in the ahead market. During down-regulation on the other hand, those players having positive imbalance are assumed to have caused the need for regulation and therefore are getting paid the down-regulation price (which is less than the ahead market price), while the other players receive the same price as in the ahead market.

IV. MODELLING

The model used in this paper is quite simple; perfect competition is assumed and power system limitations (for example congestion management) are neglected. The players in the ahead market are assumed to base their decisions on forecasts; hence, the model does not presume perfect information, but the players at least have symmetrical information, i.e., they are aware of the competitors forecasts. Moreover, it is assumed that there are no links between the trading periods. The reason for these simplifications is to keep the focus of the modelling on the division of the trading in phases, and how forecast errors affect these simplifications is to keep the focus of the modelling on the congestion management). Are neglected. The players in the partition is assumed and power system limitations (for example consumption. However, if there are no direct regulation and down-regulation, i.e., deviations from the planned generation, the objective function is to maximise the value of consumption minus the total cost of generation (see Appendix). Hence, the model does not presume perfect information, but the players at least have symmetrical information, i.e., they are aware of the competitors forecasts.

In the real-time trading, the system operator is procuring up- and down-regulation, i.e., deviations from the planned generation and consumption. However, if there are no direct regulation costs then maximisation of income from activated down-regulation bids minus the costs of activated up-regulation bids is equivalent to maximisation of the total value of consumption minus the total cost of generation (see Appendix). Hence, the same optimisation problem can be used to simulate all phases of the trading. The objective function is to maximise the value of consumption minus the cost of production and the constraint is that load balance must be maintained.

maximise \[ \sum_{c \in \tilde{C}} B_{D_c}(D_c) - \sum_{g \in \tilde{G}} C_{G g}(G_g) \]  
subject to \[ \sum_{c \in \tilde{C}} D_c + \sum_{c \in \tilde{C}} D_c = \sum_{g \in \tilde{G}} G_g + \sum_{g \in \tilde{G}} G_g, \]  
\[ 0 \leq D_c \leq D_c \quad \forall c \in \tilde{C}, \]  
\[ 0 \leq G_g \leq G_g \quad \forall g \in \tilde{G}. \]

A. Spot Market

The model assumes that the ahead market can be divided in a spot market and an adjustment market. Most of the traders in the spot market would be using a power exchange; any bilateral contracts can be assumed to be following the price of the power exchange. The players have the choice to not participate in the spot market, and only trade at the adjustment market, but that seems like an unlikely strategy, and it is therefore reasonable to assume that all players participate in the spot market, i.e.,

\[ \hat{C} = C, \quad \hat{C} = \emptyset, \quad \hat{G} = G, \quad \hat{G} = \emptyset. \]

The maximal demand of the consumers and the maximal generation of the producers are given by the initial forecasts

\[ D_c = D_c^* \forall c \in \hat{C}, \]  
\[ G_g = G_g^* \forall g \in \hat{G}. \]

The output of the spot market problem is the planned consumption and generation, i.e.,

\[ D_c = D_c \forall c \in \hat{C}, \]  
\[ G_g = G_g \forall g \in \hat{G}. \]

Finally, the spot market price, \( \lambda^1 \), is given by the dual variable of the constraint (2).

B. Adjustment Market

Not all players are interested in trading in the adjustment market. There might be technical reasons to avoid changing production plans, or the player may not have the resources to update forecasts and reschedule their operations accordingly. Hence, the sets \( \hat{C}, \hat{G}, \hat{C} \) and \( \hat{G} \) must be defined to match the trading strategies of the players. The maximal demand and the maximal generation of players participating in the adjustment market are given by the forecasts:

\[ \tilde{D}_c = \tilde{D}_c^* \forall c \in \tilde{C}, \]  
\[ \tilde{G}_g = \tilde{G}_g^* \forall g \in \tilde{G}. \]

The players who do not participate in the adjustment market will stand by their plans from the spot market, i.e.,

\[ D_c = D_c^* \forall c \in \hat{C}, \]  
\[ G_g = G_g^* \forall g \in \hat{G}. \]

The output of the adjustment market problem is updated plans for consumption and generation, i.e.,

\[ D_c^2 = D_c^2 \forall c \in \hat{C}, \]  
\[ G_g^2 = G_g^2 \forall g \in \hat{G}. \]

Finally, the adjustment market price, \( \lambda^2 \), is given by the dual variable of the constraint (2).

C. Real-time Market

As described in section III.B, the real-time trading includes both automatically activated reserves and slower reserves procured by the system operator during the trading period. The automatic control systems are neglected in this model; it is assumed that the variable limits used in the real-time problem corresponds to the mean generation or demand during the period, and that the continuous variations around the mean are not larger than what can be managed by the automatic control systems. Moreover, it is assumed that the mean frequency corresponds to the nominal value so that the automatic control systems neither generate more nor less than planned; hence, the energy difference between the planned and actual generation and consumption is covered only by up- or down-regulation activated by the system operator.

These assumptions are reasonable if the time periods considered in the real-time market are short. Hence, it is possible to divide the simulation of the real-time market is divided in a
number of real-time periods, which are indexed by \( r = 1, \ldots, R \). Another advantage of this division is that it enables the model to detect trading periods during which both up- and down-regulation occurs. (However, such periods are probably quite rare—less than 1% of the time in the for example the Nordic electricity market [8]—and the loss of accuracy when choosing \( R = 1 \) should not be too large.)

Only players with controllable generation and load are participating in the real-time market. These players are included in the sets \( \mathcal{G} \) and \( \mathcal{C} \) respectively. The remaining players must then belong to the sets \( \mathcal{G} \) and \( \mathcal{C} \). The maximal demand and the maximal generation of players participating in the real-time market are given by the available capacities during each real-time period:

\[
D_c = \overline{D}_{c,r} \quad \forall c \in \mathcal{C}, 
\]

\[
\overline{G}_g = \overline{G}_{g,r} \quad \forall g \in \mathcal{G}. 
\]

The demand and the generation of the other players is price insensitive during the real-time phase, and is therefore treated as parameters equal to their varying real-time values, i.e.,

\[
D_c = D_{c,r} \quad \forall c \in \mathcal{C}, 
\]

\[
G_g = G_{g,r} \quad \forall g \in \mathcal{G}. 
\]

The output of solving the real-time market problem for each real-time period is the actual generation and load of the players participating in the real-time trading. The activated up- and down-regulation can then be calculated by comparing these values to the planned values:

\[
D_{c,r} = \begin{cases} 
D_{c}^2 - D_{c,r} & \text{if } D_{c}^2 > D_{c,r}, \\
0 & \text{if } D_{c}^2 \leq D_{c,r},
\end{cases} 
\]

\[
D_{c,r} = \begin{cases} 
D_{c,r} - D_{c}^2 & \text{if } D_{c,r} > D_{c}^2, \\
0 & \text{if } D_{c,r} \leq D_{c}^2,
\end{cases} 
\]

\[
G_{g,r} = \begin{cases} 
G_{g} - G_{g,r}^2 & \text{if } G_{g,r} > G_{g}^2, \\
0 & \text{if } G_{g,r} \leq G_{g}^2,
\end{cases} 
\]

\[
G_{g,r} = \begin{cases} 
G_{g,r}^2 - G_{g,r} & \text{if } G_{g,r}^2 > G_{g,r}, \\
0 & \text{if } G_{g,r}^2 \leq G_{g,r}.
\end{cases} 
\]

Notice in (20) and (21) that consumers up-regulate by decreasing the load and down-regulate by increasing the load.

If marginal pricing is applied, the up- and down-regulation prices are given by

\[
\lambda^+ = \begin{cases} 
\max_{r} \lambda_{r} & \text{if } \sum_{r} \left( \sum_{c} D_{c,r}^2 + \sum_{g} G_{g,r}^2 \right) > 0, \\
\lambda^1 & \text{otherwise},
\end{cases} 
\]

\[
\lambda^+ = \begin{cases} 
\min_{r} \lambda_{r} & \text{if } \sum_{r} \left( \sum_{c} D_{c,r}^2 + \sum_{g} G_{g,r}^2 \right) > 0, \\
\lambda^1 & \text{otherwise},
\end{cases} 
\]

D. Post Market

The imbalance of a player who is balance responsible for the consumers \( \mathcal{C}_b \) and the producers \( \mathcal{G}_b \), is calculated as generation + purchase – consumption – sales, i.e., as

\[
\delta_b = \sum_{g \in \mathcal{G}_b} \left\{ \frac{1}{R} \sum_{r=1}^{R} \left( G_{g,r}^+ - G_{g,r}^- + G_{g,r}^- G_{g,r}^- G_{g,r}^- G_{g,r}^- \right) - G_{g,r}^2 \right\} + \sum_{c \in \mathcal{C}_b} D_{c,r}^2 - \frac{1}{R} \sum_{r=1}^{R} \left( D_{c,r}^+ + D_{c,r}^- - D_{c,r}^- D_{c,r}^- D_{c,r}^- \right). 
\]

The income or cost of the imbalance trading is then calculated as \( \lambda_b \delta_b \), where \( \lambda_b \) is the imbalance price which is applicable for the balance responsible player \( b \) (cf. Fig. 1).

V. MONTE CARLO SIMULATION

A generic computer simulation is based on a mathematical model, which describes the relation between a set of random inputs, \( \mathbf{Y} \), and a set of outputs to be studied, \( \mathbf{X} \). The model is represented by a function \( g \), i.e., \( \mathbf{X} = g(\mathbf{Y}) \). In this case the function \( g \) is defined indirectly from the solutions of the optimisation problem described in section IV. The inputs are the forecasted values (both for the spot market and the adjustment market) of the available capacity of each generating unit and the maximal demand of each consumer, as well as the real outcome of these variables for each real-time period. The inputs will obviously be correlated to each other; determining suitable probability distributions for the inputs is a major challenge, which remains to be studied further in the future. In this paper, it is sufficient to notice that the probability distribution of all inputs must be known, whereas the probability distribution of the outputs, \( F_X \), is unknown. In fact, the objective of the simulation is to determine the statistical properties of \( F_X \).

Although it is possible to estimate the entire probability distribution, it is generally sufficient to estimate the expectation values of the most interesting outputs. This is done by randomising a series of outcomes of \( Y \). Given \( n \) random scenarios, \( y_1, \ldots, y_n \), the expectation values of the outputs can be estimated as the mean of observed output values, i.e.,

\[
m_X = \frac{1}{n} \sum_{i=1}^{n} x_i = \frac{1}{n} \sum_{i=1}^{n} g(y_i). 
\]

When comparing different systems with the same inputs, it is generally efficient to apply correlated sampling to get good estimates of the difference between the two systems, \( g_1 \) and \( g_2 \) [9]. The estimated difference is calculated as

\[
m_{X_1 - X_2} = \frac{1}{n} \sum_{i=1}^{n} (x_{1,i} - x_{2,i}) = \frac{1}{n} \sum_{i=1}^{n} (g_1(y_i) - g_2(y_i)). 
\]

Using correlated sampling is quite natural when studying the impact of trading arrangements. The inputs, forecasts and real-time values, are the same regardless of the delay time between
the spot market and the delivery hour, the imbalance pricing system etc.

In addition to the prices, it is interesting to study how the surplus of different players are affected. The total surplus, is a measure of the overall economic efficiency of the system, and is defined as the total value of consumption minus the total production cost, i.e.,

$$TS = \frac{1}{R} \sum_{r=1}^{R} \left( \sum_{c \in C} B_{Dc}(D_{c,r}) - \sum_{g \in G} C_{Gg}(G_{g,r}) \right). \quad (29)$$

The total surplus of a balance responsible player is equal to the total value of the player’s consumption minus the total generation cost of the player plus the sum of the results from trading in the spot market, adjustment market, real-time market and post market, which may be stated as

$$BRPS = \sum_{c \in C_b} \left( \frac{1}{R} \sum_{r=1}^{R} (B_{Dc}(D_{c,r}) + \lambda^1 D^1_{c,r} - \lambda^2 D^2_{c,r}) ight)$$

$$- \lambda^1 D^1_{r} - \lambda^2 (D^2_{g} - D^1_{g})$$

$$+ \sum_{g \in G_b} \left( \lambda^1 G^1_{g,r} + \lambda^2 (G^2_{g} - G^1_{g}) \right)$$

$$+ \sum_{r=1}^{R} \left( (\lambda^1 G^1_{g,r} - G^1_{g,r} - C_{Gg}(G_{g,r})) \right)$$

$$+ \lambda b \delta b. \quad (30)$$

VI. EXAMPLE

This section will provide an example which demonstrates how inputs can be created and which outputs that can be obtained. The system is completely fictitious, and no conclusions should be drawn from the numerical values of this simulation. The problem of identifying suitable probability distributions for forecasts and the real-time values must be studied further; the objective of this example is just to give an impression of what is required of the inputs.

A. Trading Arrangements

The trading period in this example system is one hour, and there is only one real-time period in the simulation. The system has been simulated using four different trading arrangements:

- **Perfect information.** All players are assumed to have perfect forecasts, which means that the results of the spot market trading will be equal to the real-time operation, and there will be no need for up- or down-regulation. This model is used as a benchmark for the other trading arrangements.
- **Six hour delay, dual imbalance pricing.** In this model the spot market is assumed to be closed six hours before the delivery hour, whereas the adjustment market is closed one hour before the delivery hour. Dual imbalance pricing is applied in the post market.
- **Six hour delay, single imbalance pricing.** This model is the same as the previous, except that single imbalance pricing is applied in the post market.
- **One hour delay, dual imbalance pricing.** Here the spot market is closed just one hour before the delivery hour and there is no separate adjustment market. Dual imbalance pricing is applied in the post market.

B. System Data

The system has three balance responsible players: One for wind power, one for thermal power plants and one for the consumers. All players participate in the spot markets and adjustments markets when applicable. The wind power producers and consumers do not participate in the real-time market.

The installed wind power capacity is 4 000 MW, and the available generation capacity and its forecasts are described by a multivariate normal distribution with mean

$$\mu = [1, 320, 1, 320, 1, 320]^T$$

and the covariance matrix

$$\Sigma = \begin{bmatrix} 80 000 & 16 000 & 1 600 \\ 16 000 & 80 000 & 64 000 \\ 1 600 & 64 000 & 80 000 \end{bmatrix}. $$

The first element in this distribution represents the forecast at the time of the spot market trading, the second element is the forecast at the time of the adjustment market trading, and finally the third element is the actual wind power generation during the trading period. The mean relative forecast errors from this distribution is about 26% for the first forecast and about 11% for the second.

The thermal units are divided in two groups. The base load group consists of 21 units with a generation cost between 10–20 €/MWh. The peak load units group consists of 15 units in the price range 21–35 €/MWh. Each unit has a capacity 500 MW, and the Mean-Time-To-Failure is 1 900 h and the Mean-Time-To-Repair is 100 h. The available capacity of the thermal units is based on a persistence forecast, i.e., if the unit is operational at the time of the forecast then it is assumed to be operational during the delivery period and vice versa. This results in eight possible combinations of forecasts for the spot market and adjustment market as well as the real-time value of the delivery period (see table I).

The demand curve of the system is shown in Fig. 2. Only the base load part of the demand curve is random. The base load is normally distributed with mean 10 000 MWh/h and standard deviation 1 000 MWh/h. The base load forecasts for the spot and adjustments markets is generated by multiplying a relative forecast error to the true value. The relative forecast error is normally distributed with mean

$$\mu = [1, 1]^T$$

and the covariance matrix

$$\Sigma = \begin{bmatrix} 0.004 & 0.003 \\ 0.003 & 0.003 \end{bmatrix}. $$

The mean relative forecast errors from this distribution is about 1.6% for the first forecast and about 1.4% for the second.
C. Results

The four systems described above were each simulated for the same set of 10,000 scenarios. The results are presented in Table II. The results show how different trading arrangements affect the different balance responsible players.

<table>
<thead>
<tr>
<th>Forecast</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability [%]</td>
<td>4.71</td>
<td>0.05</td>
<td>&lt;0.01</td>
<td>0.24</td>
<td>0.25</td>
<td>&lt;0.01</td>
<td>0.05</td>
<td>94.70</td>
</tr>
<tr>
<td>Forecast 1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>Forecast 2</td>
<td>0</td>
<td>0</td>
<td>500</td>
<td>500</td>
<td>0</td>
<td>0</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>Real value</td>
<td>0</td>
<td>500</td>
<td>0</td>
<td>500</td>
<td>0</td>
<td>500</td>
<td>0</td>
<td>500</td>
</tr>
</tbody>
</table>

**TABLE II**

**ESTIMATED EXPECTATION VALUES FOR THE EXAMPLE SYSTEM**

<table>
<thead>
<tr>
<th>Output</th>
<th>Perfect information</th>
<th>Six hour delay, dual imbalance pricing</th>
<th>Six hour delay, single imbalance pricing</th>
<th>One hour delay, dual imbalance pricing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total surplus [€/h]</td>
<td>410 229</td>
<td>410 229</td>
<td>410 229</td>
<td>410 229</td>
</tr>
<tr>
<td>Wind power surplus [€/h]</td>
<td>30 254</td>
<td>30 057</td>
<td>30 061</td>
<td>30 269</td>
</tr>
<tr>
<td>Thermal power surplus [€/h]</td>
<td>81 995</td>
<td>81 739</td>
<td>81 739</td>
<td>81 850</td>
</tr>
<tr>
<td>Consumer surplus [€/h]</td>
<td>297 980</td>
<td>298 425</td>
<td>298 429</td>
<td>298 155</td>
</tr>
</tbody>
</table>

VII. CONCLUSIONS

The trading arrangements in an electricity market will have different consequences for different players. A long delay time between the closure of the spot market and the delivery period will be a larger problem for small players with intermittent power generation compared to large players with more easily forecasted generating units. This paper has presented a simulation method which can be used to investigate the impact of different trading arrangements.

The next step will be to develop suitable probabilistic models concerning wind power and load forecasts, and how they are correlated to the actual outcome. Given such models, it is easy to further develop and adopt the model presented here, in order to consider transmission bottlenecks, energy storage, average pricing or any other aspect of the electricity trading that can be of interest.

APPENDIX

This appendix will show that maximizing the value minus the costs of changing schedules is equivalent to maximizing the value of the actual consumption minus the cost of the actual generation, provided that there are no additional costs to change planned generation or consumption.

Consider an optimisation problem where the objective function is to maximise the value of deviating from an earlier plan. The values and costs are defined as follows:

\[ B^+_{Dc}(\Delta D_c^+ - \Delta D_c^-) - B^-_{Dc}(D_c - \Delta D_c^-), \]

\[ B^+_{Gg}(\Delta G_g^- - \Delta G_g^+) - C^-_{Gg}(G_g - \Delta G_g^-), \]

\[ C^+_{Dc}(\Delta D_c^- - \Delta D_c^+) - B^-_{Dc}(D_c - \Delta D_c^+), \]

\[ C^+_{Gg}(\Delta G_g^- - \Delta G_g^+) - C^-_{Gg}(G_g - \Delta G_g^-), \]

where \( D_c \) and \( G_g \) are the planned consumption and generation respectively, and \( \Delta D_c \) and \( \Delta G_g \) represent changes upwards and downwards.

The same player cannot simultaneously up-regulate and down-regulate; hence, the producers can be divided in two subsets \( G_\uparrow \subseteq G \), \( G_\downarrow \subseteq G \), \( G_\uparrow \cap G_\downarrow = \emptyset \), such that \( \Delta G_g^+=0 \ \forall \ g \in G_\uparrow \) and \( \Delta G_g^- = 0 \ \forall \ g \in G_\downarrow \). A similar division can be applied to the consumers. This means that the objective function can be rewritten:

\[ \sum_{c \in C} (B^+_{Dc}(\Delta D_c^+ - \Delta D_c^-) - C^-_{Dc}(\Delta D_c^-)) \]

\[ \sum_{g \in G} (B^+_{Gg}(\Delta G_g^- - \Delta G_g^+) - C^-_{Gg}(\Delta G_g^-)) \]

because the benefit and cost functions (16)–(19) are equal to zero when the corresponding variable is zero. Substitution of (31)–(34) into (36) yields

\[ \sum_{c \in C} B^+_{Dc}(D_c + \Delta D_c^+) - B^-_{Dc}(D_c) \]

\[ - \sum_{c \in C} B^+_{Dc}(D_c - \Delta D_c^-) \]
However, the previous plans, $D_c$ and $G_g$ cannot be changed at this point, and are therefore considered as constants in the objective function. Moreover, the planned values plus the change is equal to the new plan, which is here denoted $D_c^*$ and $G_g^*$ respectively. Substituting the new plan into (37) and removing the constant values results in the following objective function:

$$
\begin{align*}
\min \sum_{c \in C^+} B_{Dc}(D_c^*) - \sum_{c \in C^-} B_{Dc}(D_c^*) + \\
- \sum_{g \in G^-} C_{Gg}(G_g^*) - \sum_{g \in G^+} C_{Gg}(G_g^*) = \\
\{C = C^+ \cup C^-, G = G^- \cup G^+ \} = \\
\sum_{c \in C} B_{Dc}(D_c^*) - \sum_{g \in G} C_{Gg}(G_g^*)
\end{align*}
$$

which concludes the proof.

REFERENCES


BIOGRAPHY

Mikael Amelin is a research associate in the Electric Power Systems Lab at the School of Electrical Engineering of the Royal Institute of Technology (KTH) in Stockholm. He received his masters, licentiate and doctors degrees from the same institute in 1997, 2000 and 2004 respectively.

His research interests include Monte Carlo techniques, analysis and modeling of electricity markets, as well as rural electrification in developing countries.