



An application of DOE in the evaluation of optimization functions in a statistical software

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Abstract

The aim of this master thesis is to map the performance of an optimization function used in a statistical software. There exists several different methods to solve optimization problems. One diligently used method is the desirability approach which is the one used in this software. By applying methods in design of experiment, DOE, the behavior and the precision of a desirability approach for optimization problems is investigated. The thesis begins with an investigation and a description of the desirability approach followed by a discussion with proposed improvements.

Sammanfattning

Syftet med detta examensarbete är att kartlägga och beskriva en optimeringsfunktion i en statistisk programvara. När det gäller att lösa optimeringsproblem existerar det ett flertal metoder. En flitigt använd metod är målfunktionsmetoden vilken är den som används i programvaran i detta arbete. Genom att tillämpa metoder inom försöksplanering har beteendet och precisionen för denna målfunktionsoptimering karakteriserats. Uppsatsen inleds med en utredning och en beskrivning av målfunktionsmetoden, detta följs utav en diskussion om eventuella förbättringar.

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1 Introduction

1.1 Objective

The main objective of this thesis is to investigate the performance and the precision of an optimization function using a certain desirability function approach. Another objective is to open a discussion for possible improvements of this desirability function. In order to perform requested tasks the founders suspect that the desirability function of their use lacks of precision regarding other important tasks. The investigation part is about how to detect when and why these defects occur and the latter part is about how to avoid them and to propose further improvements.

1.2 Methods and references

Methods in design of experiment, from now on written as DOE, are widely used in e.g. manufacturing, social studies and agriculture studies to find models that describe certain situations and processes of interest. Lately DOE has been increasingly used in software evaluation problems (e.g. Wakeland, Martin and Raffo [6]). By using methods in DOE to evaluate a software performance one will be able to characterize the impact of different parameters, all adjustable in the software.

1.3 Structure

This thesis presents the complete process for the investigation. To make it easier for the reader, the thesis begins with some necessary definitions. It also gives a short introduction to what kind of problems that can occur when using this optimization software. The reader will before entering the section that treats the investigation also be given an understanding about how an optimizing process with a desirability approach works. Then the investigation is presented with the preface, the process and the analysis. The reader can then follow a discussion for possible improvements to this desirability approach.

2 Background and definitions

2.1 Design of experiments, DOE

This Section is an introduction to methods in DOE and a description how to apply it on this investigation. DOE was first developed to reduce cost in terms of money and time when evaluating different processes. Before DOE was introduced the COST technique was used where one variable was changed at a time while the others were held fixed. With DOE it was possible to vary several variables simultaneously and the number of experiments required to obtain the same size of information could be significantly reduced. Hence, the cost could be reduced.

DOE is mainly used for three objectives.

- **Screening.** When using DOE for screening purposes the aim is to identify which important factors that have influence on the process of interest. This is often the opening step in a process evaluation.
- **Optimization.** For optimization purposes the number of experiments required are more than for screening but DOE is still much more effective than other approaches. By using DOE in optimization problems the objective is to predict the response value for all possible combinations of factors within the experimental region and then be able to locate an optimal experimental point.
- **Robustness testing.** The reason for robustness testing is to ascertain that the method is robust to small fluctuations in the factor levels. This is often the last part of an investigation when the screening and the optimization is done.

In the investigation of this thesis the main objective is screening. The aim is to identify important factors that are affecting the optimizing precision. For a more extensive description about areas where DOE is used and the benefits of DOE see e.g. Eriksson et al. [4].

The three objectives mentioned are the base for the design created for an investigation. When a design is set up and the experiments have been carried out and all the results are gathered, the next step is to fit a model to the data. Due to different objectives, different methods are used. A common used method that takes into account linear, interaction and quadratic terms is Multiple Linear Regression, MLR. The three simplest model complexities are expressed due to the following equations.

Let y be the response variable and let it be dependent of k factors. Then y can be modeled in e.g. the three following complexities

- single term linear model

$$y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \epsilon, \quad (1)$$

- interaction model

$$y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_i^k \sum_{j>i}^k \beta_{ij} x_i x_j + \epsilon,$$

- quadratic model

$$y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_i^k \sum_{j>i}^k \beta_{ij} x_i x_j + \sum_{i=1}^k \beta_{ii} x_i^2 + \epsilon,$$

where the error term ϵ is assumed to be a random variable with $E[\epsilon] = 0$ and $V[\epsilon] = \sigma^2$, β_i and β_{ij} are regression coefficients and x_i is the level of the i :th factor.

In order to estimate the regression coefficients in the MLR model the method of least squares is used. Then if we write (1) in terms of each observation among the n experiments as

$$y_j = \beta_0 + \sum_{i=1}^k \beta_i x_{ij} + \epsilon_j, \quad j = 1, \dots, n, \quad (2)$$

then the least squares function is

$$L = \sum_{j=1}^n \epsilon_j^2.$$

The method of least squares chooses the β 's in (2) for which L is minimized.

The next step in creating a good model to a response variable is normally to test for significance of the regression coefficients. Since this thesis is investigating an algorithm instead of a chemical process there is no variation in the experiments and hence, no need to test for significance. For the interested reader an excellent and extensive introduction to design and analysis of experiments can be found in Montgomery [5].

2.2 The desirability approach

The desirability approach for an optimization problem strives to find the most desired solution. For an optimization problem containing several response variables, each response variable have an individual desirability function assigned to it. The desirability function

describes how desired a given solution is. To find the most desired solution the optimizer minimizes the desirability function. In the case of several response variables an overall desirability function is used. The overall desirability function is a weighted average of the individual desirability functions.

2.3 The optimizer

This subsection is a short introduction to how to use the optimizing function in the software. When a user is about to optimize a process there are a few pre-settings to consider. First the user has to define whether the individual response variables are about to be maximized, minimized or if a target value solution is desired. Then, depending on the type of optimization, specification limits have to be defined. If the response variable is about to be maximized, a lower limit and a target value is required. If the response variable is about to be minimized, an upper limit and a target value is required. When a response variable is of the third alternative and a target solution is desired, a lower value, an upper value and a target value have to be defined.

- **Lower limit.** The lower specification limit is the lowest acceptable value for the response variable. Used in maximization and target solution optimizing situations.
- **Upper limit.** The upper specification limit is the highest acceptable value for the response variable. Used in minimization and target solution optimizing situations.
- **Target value.** The target value is the most desired solution for the response variable. For maximization/minimization problems, improvements above/under this point would have no appreciable value. When a target solution is desired, this is the value the optimizer strives to reach.

Another setting the user can change is the weight setting. By changing the weight setting the user can rank the importance between the response variables. The default value for the weight is 1 and it varies between 0.1 and 1. When all response variables are of the same importance, the optimization process will focus on finding a solution for all response variables to be inside the specification limits. By changing the weight the shape of the desirability function will change and it will have a great importance for how the optimization process will proceed. More discussion about that will follow further on in the text.

When all the settings are done the optimizer starts searching for a solution. The optimizer uses the Nelder-Mead simplex method [1] to find the most desired solution. In the sections below, all the definitions necessary for a more detailed description of the optimization process, are given.

2.4 The desirability function

In an optimization problem with several response variables one has to take into account several conditions for the response variables. Since it is not always possible to reach the optimum value for all responses, one has to compromise. To be able to compare different compromises, the desirability function was developed to provide a mathematical solution to this problem. This approach was developed by Edwin C. Harrington [2] in 1965. Derringer and Suich [3] in 1980 developed a modification of Harrington's approach in order to illustrate how several response variables can be transformed into one desirability function, which can be optimized. To fulfill the interests required for this software yet another, unpublished, modification was developed and it is this version of the desirability function that has been evaluated in this thesis. Figure 1 shows a graph with a visualization of this desirability function.

Definition 1 *Let y be a response variable to be optimized. The individual desirability function for the i :th response variable is*

$$d_i(g(y_i)) = 100 * (e^{\lambda_i * g(y_i)} - 1) \quad (3)$$

where

$$\lambda_i = \frac{-\ln \frac{100}{(100 - Limit_i)}}{\frac{100(L_i - P_i)}{(T_i - P_i)}}, \quad Limit_i = 90 + 80 * \log_{10}(w_i) \quad (4)$$

w_i = "the user defined weight" and $g(y_i) = 100 * ((y_i - P_i)/(T_i - P_i))$. T , L and P are defined as:

- T_i = User desired target value for the i :th response variable.
- L_i = User defined worst acceptable response value for the i :th response variable. L = Upper limit if $y > T$ and L = Lower limit if $y < T$.
- P_i = Worst value for the i :th response variable computed from the starting simplex. P is never closer to the Target than L .

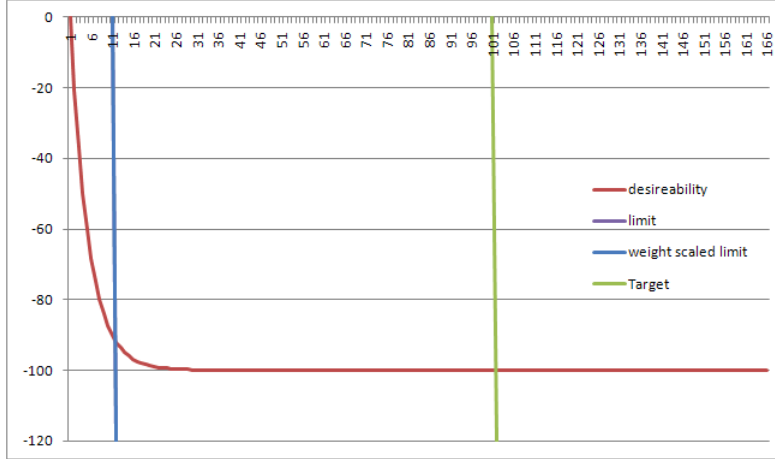


Figure 1: The graph shows the desirability function when the weight is set to $w = 1$. On the horizontal axis is the scaled response function, Y , that is about to be optimized. Y is scaled to the range 0 to 100 where 0 is the value for the worst starting point among the set of starting simplexes and 100 for the user specified target value. On the vertical axis is the range for the desirability function between the starting value 0 and the target value solution at $-100 * w$.

The appearance of the desirability function is constructed in a way that it will take value

- 0 at the border. The worst point among the set of starting simplexes is set to the border value.
- $-90 * w$ at the weight scaled specification limit.
- $-100 * w$ for a solution at the target value, T , where w is the user specified weight.

2.5 The overall desirability function

In the case where the optimizer intends to find a solution where several response variables are included an overall desirability function is used. The overall desirability function is a weighted average of the individual desirability functions, weighted by the user specified weights, w .

Definition 2 *The overall desirability function $f(d(g))$ is calculated from*

$$f(d(g)) = \frac{\sum_{i=1}^M w_i d_i}{\sum_{i=1}^M w_i}$$

where M is the number of response variables and w is the user specified weight.

2.6 Log(D)

The value that the optimizer returns to the software user as a measure of how good the solution is, is called $\text{Log}(D)$, known as "the overall distance to target".

Definition 3 Let y_i be the i :th response variable with its specification values T_i and L_i defined in definition 1. $\text{Log}(D)$ is then defined by

$$\text{Log}(D) = \log_{10} \left(\frac{\sum_{i=1}^M \left(\frac{y_i - T_i}{T_i - L_i} \right)^2}{M} \right)$$

where M is the number of responses.

The overall distance to target gives a positive value for a non-acceptable solution and a negative value when the solution satisfies or at least nearly satisfies all the response variables specifications.

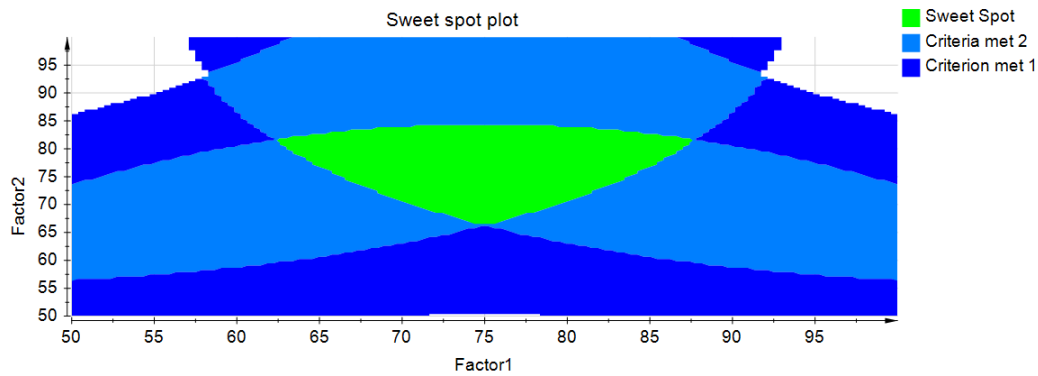


Figure 2: The plot shows an illustration of the sweet spot in the two-factor example for three response variables. The two factors vary on the x-axis and the y-axis from minimum value of 50 and a maximum value of 100. For three response variables either none, one, two or all three criteria can be satisfied. This is illustrated by the white area (none), dark blue area (one), light blue area (two) and the green area also known as the sweet spot (all three).

2.7 The sweet spot

A main objective of using the optimizer is to find a region in experimental space where to execute experiments within the specification limits for all the response variables. This region is sometimes called the sweet spot, from the frequent use of a sweet spot plot to locate and visualize such a good region. An example of a sweet spot is illustrated in Figure 2.

Definition 4 *The sweet spot is the experimental space where the factor settings result in response values satisfying all the required specifications.*

2.8 Acknowledged defects

When studying the optimizer some defects and limitations have been detected. There are mainly two types of defects. Type 1 occurs when the optimizer chooses a solution outside the sweet spot, when it is indeed possible to reach a solution inside. Type 2 occurs when the optimizer chooses a solution inside the sweet spot but not the best one, even when it is possible to reach a better solution. The two types of problems are illustrated in Figure 3.

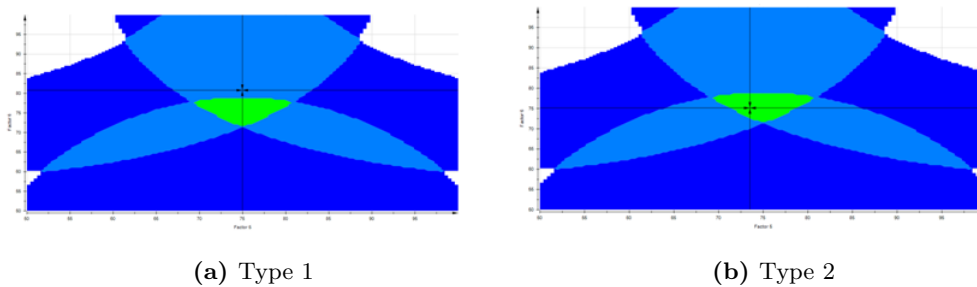


Figure 3: The different types of defects are illustrated in the two plots above. The solution found by the optimizer is shown by the black cross. In the left plot the solution lies outside the sweet spot and illustrates the type 1 defect. In the right plot the solution lies inside the sweet spot but not centered where the best solution is.

3 Test protocol

3.1 Choice of parameters

To create a design for the investigation one must have knowledge of which adjustable parameters that might have influence on the behavior of the optimizer. We have already discussed some of the adjustable parameters such as the weight and the specification limits when we introduced how to use the optimizer in Section 2.3. The parameters chosen are listed below with justifications and their respective level settings.

- Weight, w . The weight is directly affecting the desirability function, see Eq (3) and Eq (4). The weight has been used as a setting of importance when the optimization concerns several response variables. This investigation uses the fact that the weight affects the shape of the desirability function and the same weight is being used for all response variables. The range for the weight is $w \in [0.1, 1]$ with default value $w = 1$. Since the weight has a logarithmic impact on the desirability function (Eq (4)) the factor levels for the weight is set to non-equidistant values, i.e. $w = 0.2, 0.5, 1$.
- Target Position, $T - pos$. The Target Position parameter aims to answer whether the position of the target relative to the optimal solution and the sweet spot affects the precision of the parameter. This parameter has the following level settings.
 - 0 when the target value is far outside the sweet spot.
 - 20 when the target value is just outside the sweet spot.
 - 40 when the target value is on the border of the sweet spot.
 - 60 when the target value is just inside the sweet spot.
 - 80 when the target value is far inside and close to the optimal solution.
 - 100 when the target value is centered in the sweet spot and there exists a target solution for all response variables.

Remark. The data is constructed in a way that the optimal solution due to the overall desirability function is always centered in the sweet spot.

- Size of the sweet spot, $Size$. This parameter is chosen to answer if the precision of the optimizer changes when the area for acceptable solutions differs in size. Three levels were chosen and simply named as: Small, Medium and Large. The sizes were chosen as proportions of the total search area.
- Number of response variables, $\#Resp$. This factor was chosen to investigate if the uncertainty of the optimizer grows with increasing number of responses to take into account. In order to have control over the test protocol, the factor levels were limited to a maximum value of four. The chosen factor levels are 2, 3 and 4.

Target Position	Weight	Sweet spot size	#Resp
0	1	Small	2
20	1	Small	2
40	1	Small	2
60	1	Small	2
80	1	Small	2
100	1	Small	2
0	0,2	Medium	2
20	0,2	Medium	2
40	0,2	Medium	2
60	0,2	Medium	2
80	0,2	Medium	2
100	0,2	Medium	2
0	0,5	Medium	2
20	0,5	Medium	2
40	0,5	Medium	2
60	0,5	Medium	2
80	0,5	Medium	2
100	0,5	Medium	2
0	1	Medium	2
20	1	Medium	2
40	1	Medium	2
60	1	Medium	2
80	1	Medium	2
100	1	Medium	2
0	0,2	Large	2
20	0,2	Large	2
40	0,2	Large	2
60	0,2	Large	2
80	0,2	Large	2
100	0,2	Large	2

Figure 4: A subset of the full factorial design in 162 runs.

Due to comfortable and easy computer tools there is no need to reduce the number of experiments. Therefore a full factorial design was made. The design contains a total of $3 * 6 * 3 * 3 = 162$ runs. A subset of the design can be seen in Figure 4.

3.2 Choice of measurements

To chose adequate measurements for the analysis one has to make clear which questions there are to be answered. In this thesis two types of known defects have been introduced. The questions that follow from those descriptions (Section 2.8) would be, why and when do these defects occur and how large are they? To measure and describe those defects in order to answer the questions, the following measurements have been developed.

- Distance from optimal solution, *Dist*. *Dist* is an average distance from the optimal solution expressed in the test data response values. The knowledge of the test data allows such a comparison.

$$Dist = \frac{\sum_{i=1}^M |y_{i-observed} - y_{i-optimal}|}{M} \tag{5}$$

where M is the number of response variables.

Before moving on, some reflexions over the selected parameters are necessary. Beginning with the weight, one has already noticed that the weight setting directly affects the shape and properties of the desirability function. It is therefore of great interest to be able to describe how this change of shape of the desirability function affects the optimizer’s precision. To answer the questions whether the target position and/or the size of the sweet spot matters, the Target Position parameter and the Size parameter were constructed. The combination of those two will include experiments where the size of the sweet spot changes while the target moves from far outside sweet spot ($T - pos = 0$), to just outside/inside the sweet spot ($T - pos = 20, 40$ and 60), to close to the optimal solution ($T - pos = 80$) and, finally to exactly at the optimal solution ($T - pos = 100$). The last parameter chosen is the Number of response variables. These response variables are further discussed and explained in Section 3.3.

With all the design parameters defined with their respective levels, the actual design protocol can be constructed. Since there is no variation in the measurements there is no need for replicates. Due to comfortable and

- The number of rejected response variables, $\#Rejects$, is a discrete measurement of how many response variables for which the given solution not lies inside the respective specification limits. This measurement will show when type 1 defects occur.

3.3 Test data

The test protocol discussed above with the chosen parameters requires a set of test data that meet certain demands. Because the investigation requires that the parameters can be controlled pursuant to the design the test data has to be constructed in a way that allows that. To meet those criteria a two-factor test data was constructed. The response functions are conical with peaks at the corners of the factor space. The investigation aims to test up to four response variables and their contour plots are illustrated in Figure 5.

Remark. When the Number of Response variables parameter is set to three, the test data is adjusted so that the distances between the conical peaks are equal for all three of them.

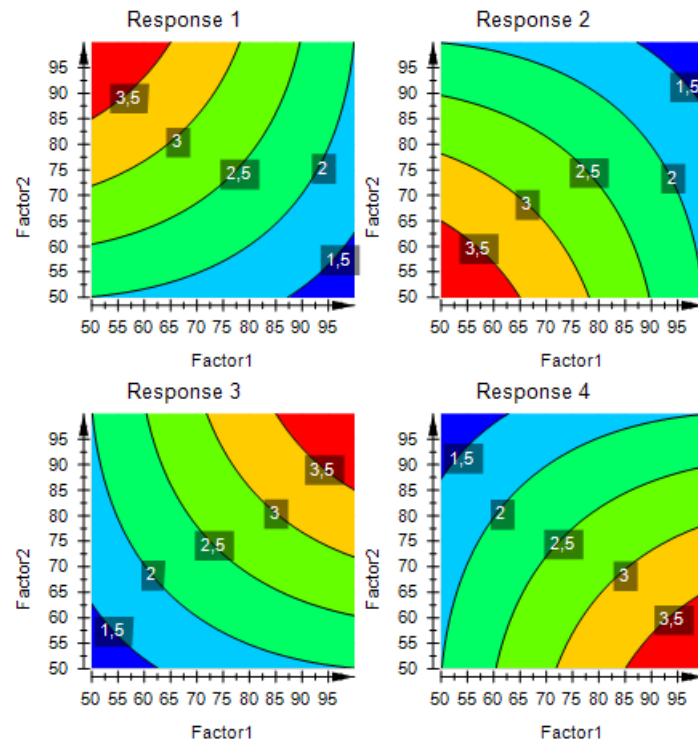


Figure 5: Contour plots of the four response surfaces in the test design. The response variables are conical with their peaks in the corners of the experimental space.

4 Results

4.1 Execution of the experiments

Imagine one experiment out of the 162 in the investigation design. The factor settings for that certain experiment can be inserted manually in the software and then one can run the optimizer and observe the results. This is a possible way to carry on with the investigation, although a rather time-consuming and laborious choice. Not just the initiation of the parameter settings for 162 experiments but also the necessary calculations needed to obtain the measurements of interest, would be massive. Therefore a computer tool that communicates with the software was used to implement the investigation.

There exists a COM-interface¹ that via Visual Basic Application(VBA) programming communicates with the software from a Microsoft Excel worksheet. By creating a Microsoft Excel worksheet containing the investigation design and a program that will send the settings to the software one can easily and swiftly return the results of interest. The programming was done so that the calculated results of interest were returned immediately, an illustration of the process can be seen in Figure 6. The program registers the parameter settings for the i :th experiment, sends it to the software and then activates the optimizer and runs it. Thereafter it returns the information of interest to the worksheet.

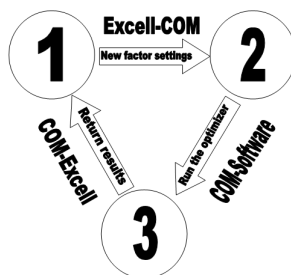


Figure 6: In step 1 the COM-interface registers the new parameter settings from the excel worksheet. In step 2 it assigns the new settings to the optimizer in the software, runs the optimizer and registers the information of interest. In step 3 the program calculates the measurements in the requested form and returns it to the worksheet.

After all results of interest are gathered, the analysis of the investigation can begin. The analysis is performed in the software. The analysis of the full factorial design was initiated in the software. The result is a mathematical model that describes the relationship between the selected parameters (factors) and the measurements (responses).

¹COM-interface was constructed by the software company to communicate with the program from an outside source. Mainly for de-bugging and quality testing purposes.

4.2 Introductory analysis

The full factorial analysis aims to describe the performance of the optimizer by the parameter settings defined in Section 3.1. A regression model was fitted by Multiple Least Squares regression, MLR. The full analysis with describing figures are shown below.

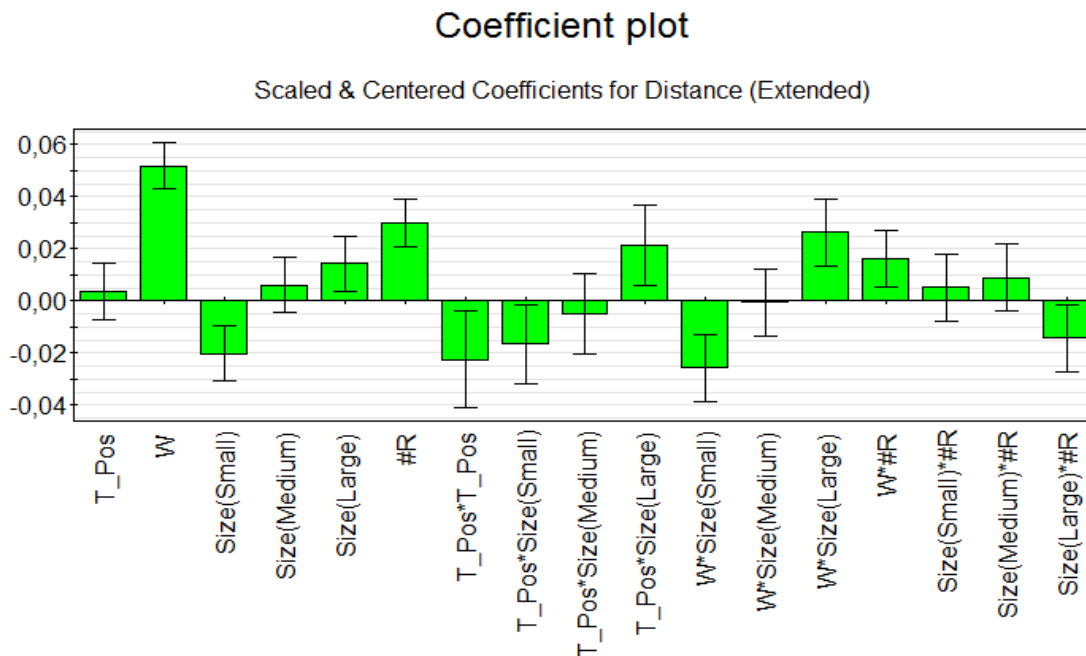


Figure 7: Coefficient plot for the distance measurement.

A first insight of the individual parameter effects on the optimization precision can be obtained by taking a look at a coefficient plot. The coefficient plot in Figure 7 shows the estimated parameter effects on the distance measurement. The first parameter is the Target Position and it shows no major effect on the distance. Although, after a deeper analysis of the effect one can notice that the quadratic term of the Target Position parameter seems of great importance. Therefore one must study how the quadratic dependents reflects on the distance. In Figure 8 one can see that the quadratic dependents of the Target Position peaks for the mid-values while it seems to have better precision for the range values. A first attempt to analyze the Target Position effect on the distance measurement is to notice that for the values where the target is positioned close to the border of the sweet spot ($T - pos = 20, 40$ and 60) the precision is worse than for values far outside or close to the optimal solution. An explanation to why this behavior occurs is discussed in details in Section 5.

Main Effect for Target Position, resp. Distance

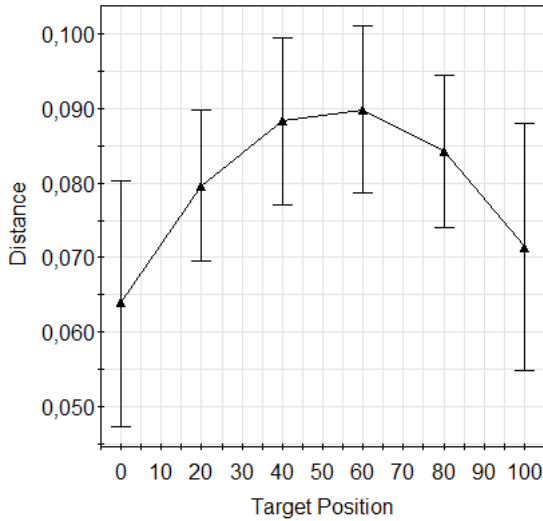


Figure 8: Illustration of how the quadratic term for the Target Position parameter affects the distance.

The next parameter in the coefficient plot is the Weight. It is clearly important and it has a positive effect on the distance. This means that when we change the weight level from the smallest ($w = 0.2$) and up, the precision deteriorates. Compared with the other parameters it has a large impact on the distance.

After noticing the weight influence the reader might have seen the next factor which is of a qualitative type, the Size of the sweet spot. When the Size parameter was defined it was constructed so that the investigation would include different sizes of the sweet spot for each position of the target ($T - Pos$). A quick look at the coefficient plot in Figure 7 shows how a smaller area leads to a better precision while a larger area seems to deteriorate the precision. The last main parameter is the Number of response variables. It shows a positive effect on the distance when the number of responses increases. Since the optimal solution is known one can now see that the optimizer causes a solution that increases the average distance as it has to take into account a larger number of response variables in its calculations. The first look at the coefficient plot has improved the understanding of the individual parameters effect on the distance measurement. The improved understanding concerns in which direction the different parameter settings affect the distance but also the mutual rank of dependence between the parameters. Leaving the single parameters behind for a while, the investigation can now proceed with the more complex combinations. The quadratic term of the Target Position parameter has already been discussed.

The interaction terms of interest is the Size*Target Position and Size*Weight terms and somehow also the Weight*Number of response variables. Let us begin with the interaction

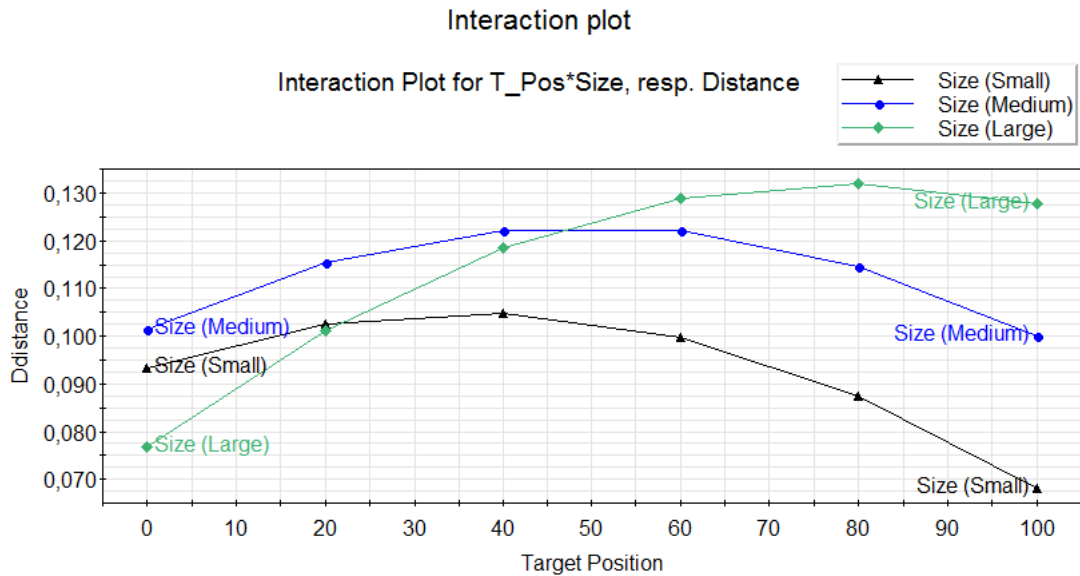


Figure 9: The graph visualize how the interaction between the Target Position parameter and the Size parameter affect the distance.

terms including the Size parameter. Figure 9 shows a graph that describes the Size*Target Position interaction and Figure 10 the Size*Weight interaction.

Another factor introduced in Section 3.2 was the number of rejected responses. This factor counts the number of response variables for which the solution did not satisfy the specification limits. To fit a model for these situations is not of a great interest, since there were quite few occurrences. Instead each situation where such a defect occurred was analyzed individually. Mainly, situations where the optimizer chooses a solution outside the sweet spot occurred for the following parameter settings: $Weight = 0.2$, $Size = Small$, $T - pos = 0$ or 20 . Thus, for a small sweet spot area, a target value outside the sweet spot and with a low weight, there is a risk that the optimizer chooses a solution outside the sweet spot. For a discussion why these situations might occur, see Section 5.

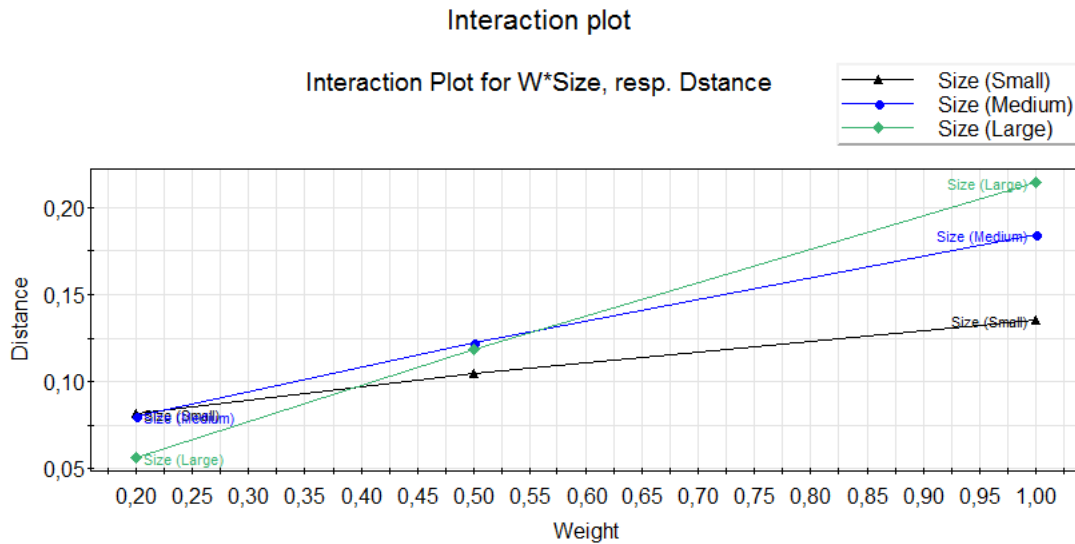


Figure 10: The graph visualize how the interaction between the Weight parameter and the Size parameter affect the distance.

4.3 Target solution analysis

This part of the analysis is for a specific situation of interest. It is an investigation of the precision when there exists a solution where all response variables is able to reach the target value. This is the situation where the Target Position parameter is set to $T - pos = 100$. A subset of the complete design is used for this part. The subset consists of the experiments when the Target Position factor is set to $T - pos = 100$ which includes a number of 27 runs. After fitting a model to describe the distance for this situation a coefficient plot as above can give a first insight of the parameter dependents. The coefficient plot for the distance measurement for this target solution situation is show in figure 11.

Overall this reduced investigation shows the same parameter effects as for the complete investigation. The only difference is the Number of Response variables parameter that seems to have much lesser effect for the target solution analysis. More advanced studies of this investigation shows that when a target solution is possible to reach for all response variables, level settings of the parameters can lead to a solution very close to the optimal solution. Those level settings can be obtained from this analysis. From the coefficient plot in Figure 11 those settings would be, low weight and a small search area.

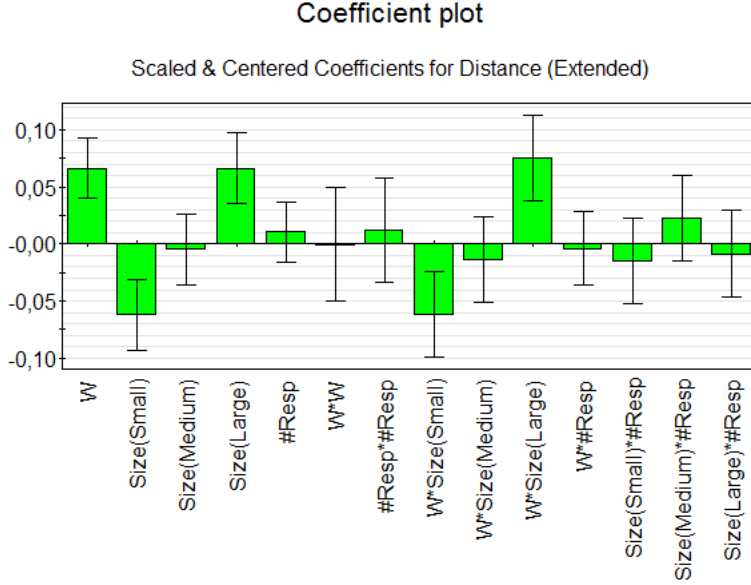


Figure 11: Coefficient plot for "Target situation"-model.

5 Conclusions and discussion

In this chapter the results received and presented in the previous chapters is analyzed and explained. In the introduction of this paper some questions were raised. Those questions concerned why the defects occur, how large they were and what parameter settings that caused them. The investigation has so far characterized the behavior and precision of the optimizer and with the knowledge of the desirability approach, in this section it is explained how the results can be described by the choice of the desirability approach.

We start with the weight parameter. The investigation has shown that a smaller value of the weight parameter results in a better precision by the optimizer. This can be explained by the method the optimizer uses. A smaller weight changes the shape of the desirability function in a way that makes it reach its minimum value, $d_{min} = -100 * w$, at a response value closer to its scaled target value, 100. Since the optimizer minimizes the overall desirability function in order to find the best solution, the optimizer can strive for a solution closer to the target value for a lower weight value. Figure 12a & 12b give a good visualization of this explanation. The weight parameter's original function was to set the mutual rank of the response variables in an optimization process. Since a response variable with weight $w = 0.2$ has a desirability function with a minimum value at, $d_{min} = -100 * 0.2 = -20$, while one with weight $w = 1$ has a minimum at, $d_{min} = -100 * 1 = -100$ the optimizer gains more by minimizing the latter one. Therefore,

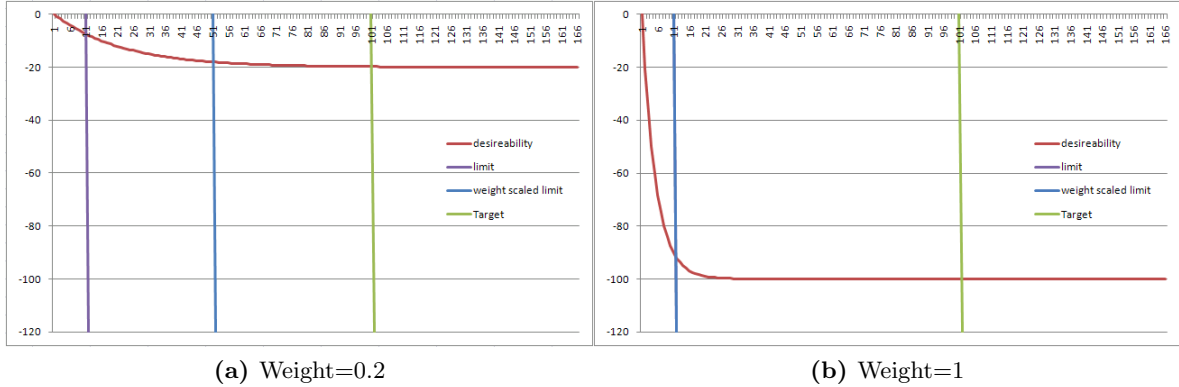


Figure 12: The two graphs shows how the shape and the properties of the desirability function changes as the weight goes from $w = 0.2$ to $w = 1$

by setting all responses at the same, lesser weight value, the precision is improved.

We proceed with the Target Position parameter. From the definition in Section 3.1 we remember the meaning of the different level settings. Earlier we discussed the quadratic effect of this parameter and noticed that the worst precision occurred for level settings $T - pos = 40$ and $T - pos = 60$, and a pretty good precision for $T - pos = 0$ and $T - pos = 100$. For situations where the target is positioned just outside/inside specification limits, the optimizer tends to stop too early and return a rather poor solution. This pattern or common behavior of the optimizer could be explained by the fact that when the target is positioned close to the sweet spot border, a solution close to this point solves the target value for at least one response variable and satisfies the specification limits for the other response variables. Let us remind ourselves from the discussion above about the weight dependence where we concluded that the optimizers lack of incitement to find a solution closer to the target when a solution inside specification limits is found. Again remember the weight discussion, where the shape of the desirability function caused the optimization process to stop searching for a better solution. Therefore a situation like this when the target value is positioned at or close to the border of the sweet spot, the incitements to proceed the search is even lesser then for the situation just discussed.

For the same reasons as for the previous analyzed parameters the Size parameter's effected on the distance is caused by the shape of the desirability function. A larger area for acceptable solutions where the optimal solution is centered in this area leads to a solution further from the optimal then a smaller area does.

The last main parameter investigated is the Number of Response variables. We saw an evident trend where the precision of the optimizer deteriorated as the number of responses increased. There is no obvious reason for this trend. One possible explanation could be developed from the same argument as for the Target Position parameter. We have shown

that the optimization process stopped iterating for better solutions pretty much as soon as a solution inside sweet spot was found. The chosen solution is therefore often a solution where a target value is found for one response variable while the other remains close to the border. Hence, as the number of responses increases, the number of responses with a solution close to the border and far from their target value increases as well and the average distance gets larger.

We now have a good characterization of the parameters' effect on the optimizer precision and we can begin a discussion about eventual improvements. The analysis has so far made it clear that a great part of the lack of precision of the optimizer is based on the shape of the desirability function. A fair question to ask in this moment would be, why to chose this certain desirability function?

During development of this software the desirability approach was chosen as optimization strategy. The desirability approach is widely used for optimization problems, specifically when it concerns several response variables. What separates different desirability approaches is the selection of desirability function. For this software the developers constructed a new function. It has similar properties as other desirability functions but also has some significant differences. Due to deficient documentation, arguments for the chosen desirability function is not available. When this desirability function was developed, the main objective, the optimizer would satisfy, was clearly to find a solution inside the specification limits. The shape of the desirability function gives the optimization process incitements to proceed iterating for a better solution while it still operates outside the sweet spot. As soon as the optimization process iterates a simplex inside the sweet spot it has clearly not the same incitement to proceed. This has simply to do with the shape of the function. This investigation has shown that one way to improve the precision is to set a lower weight to the response variables. But that is only an option if there exists a target solution inside the sweet spot. This investigation has also shown that elsewhere a conflict can occur and the optimizer chooses a solution outside the sweet spot instead of improving the search. One must therefore be careful and have a great knowledge of the data in the investigation. Hence, the weight changing option is therefore not the perfect solution to this problem. Another option to improve the precision due to this investigation would be to reduce the size of the sweet spot. This option demands a certain knowledge of the data. By changing the specification limits in order to reduce the sweet spot size, the optimizer will find a solution closer to the optimal value. The user must before changing the limits have control over the data in order to not exclude the optimal solution. Let us now proceed and instead of changing the desirability function in use we shall now consider other opportunities.

The desirability function in this software today has the certain properties previously discussed. By changing different parameters such as the weight parameter one can only affect those properties to some level. If it is desired to improve the use of this desirability approach and offer the software users more opportunities it could be worth to consider different desirability functions for different response variables depending on the desired

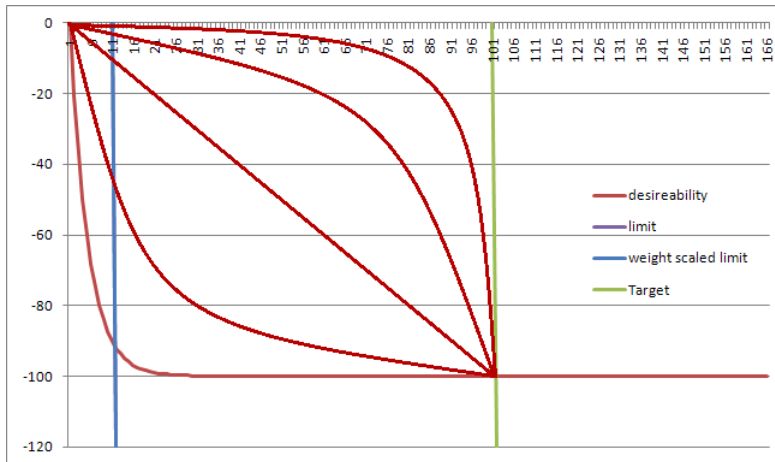


Figure 13: The graph shows five different desirability function for five different objectives. Depending on the objective the gain in minimizing the desirability function differs. For a response variable which main objective is to find a solution inside specification limits the original one (downwards convex) is to prefer while the upper one (downwards concave) is to prefer when a target solution is desired.

solution for this certain response variable. When Derringer and Suich [3] introduced their modification of Harrington's [2] desirability approach, they argued for different shapes and properties of the desirability function for different desired objectives for the response variables. If the main objective is to find a solution inside the specification limits the desirability function gains much for values just inside the limits and then it flattens out, like the one used in this software. If the objective on the other hand was to obtain a solution close to the target value, the desirability function was constructed in a way that it gained very little for all values not so close to the target and gained much of values close to the target. Figure 13 shows how such a desirability approach would look like compared to the one used here. By assigning different desirability functions to the response variables, due to their individually desired solution, a more satisfying result might be available.

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