

Using Revenue Management in Multiproduct Production/Inventory Systems: A Survey Study

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Abstract

The study aims at investigating how revenue management techniques can be applied in industries which offer multiple products. Most of the companies nowadays trend to produce multiproducts and they try to find the best method of selling. Therefore, revenue management can be considered as a new direction which should be developed for these firms. In this study, multi-product firms are mainly referred as firms offering a bundle of products or substitute products. In this regard, models and techniques applied in multiproduct firms are discussed and it is tried to provide basic models to better understand the problems, variables, customer choice models and constraints. The main methodology in this study is literature review. In order to carry out the research first revenue management applications and techniques are discussed to find a fit to this kind of industries.

The main findings of this study are (1) identifying and analyzing the most important factors affecting decision making regarding managing of bundling and substitute products and ultimately total revenue of multiproduct firms. (2) Summarizing the results and knowledge obtained from various studies within fields of bundling and substitute products. (3) Discussing the possibility of applying different revenue management techniques to these fields. (4) Identifying potentials and new directions for future study with respect to both revenue management techniques and multiproduct firms.

Keywords: Revenue Management, Multiproduct, Bundling Product, Substitute Product, Assortment Planning, Pricing, Inventory Management.

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Chapter 1

Introduction and Motivation

1 Introduction

1.1 Introduction and Motivation

Today's business and highly competitive markets force firms to coordinate their activities along with customers' desires and needs. Customers are more demanding than they used to be. They want more newly developed and customized products and seek for possible variety in their choice. For example, in a store shop one customer may be willing to buy an expensive prestigious brand of jeans, but another customer is only willing to pay for regular or low quality jeans. Consequently, firms are required to be highly flexible. Manufacturers should be able to develop their product lines to produce products with more variety. Moreover, they cannot produce as many units as they prefer to stock and sell in the market since products' life cycles are getting short and customers' preferences change rapidly. Retailers also have to anticipate customers' heterogeneous demands and carry not only different categories of products, but also different variety of one or similar products to satisfy diverse needs and preferences of customers. This diversity of needs has forced firms to change their marketing strategies from serving the whole market with the same product towards serving the market with several products, each for satisfying a specified portion of the market. Companies which offer different products or one product in several varieties are referred as multiproduct firms. Managing the business of multiproduct companies is a complex task. There exist many firms in each market, competing for customers, which lead to a high price competition. Most of firms, nowadays, produce and sell different types of products through their sale channels, therefore managers must manage their products as well as the inventories and prices of these products in a way that they avoid high amount of unsold goods, losing market shares and other difficulties. On the other hand, they should be able to satisfy their customers with proper products and services, reasonable price and availability of their products in order to survive in the market. New marketing and selling techniques have been developed to cope with these difficulties and to enable firms in making the maximum possible revenue from their production and sales activities. *Revenue management* is a contemporary concept which aims to improve the firms' revenue through understanding customers' behavior, forecasting their needs and demand in a real time, and adjusting firms' activities (e.g. production, marketing, sales) over time to respond the customers' behavior and the changing of market condition. Revenue management applications are well-known in service industries such as airline industries, however, its applications in many other industries

are not well studied in literature or utilized in practice e.g. manufacturing industries. Therefore, in this master thesis we are motivated to focus on identifying and analyzing managers' problem in regard to managing multiple products, and application of revenue management techniques in multiproduct firms.

1.2 Aim and Scope

The current master thesis aims to review the existing literature regarding problems associated with multiproduct firms and how revenue management is applied to solve these problems, in order to investigate advantages, and find new directions and potentials for future researches.

Through literature reviewed, we study the multiproduct problem from different perspectives, such as marketing, economics, operation management, operation research, etc. In each of these literature one or some aspects of the problem are taken into consideration; some literature try to analyze firm' costs, others focus on profit of the firms which can be gained by offering multiple products. In economics literatures many papers investigate the impact on customer's welfare. On the other hand in marketing and operation management, many literatures focus on products' attribute design and selection for different market segments. The following criteria show our view of multiproduct and multiproduct firms in this thesis. Products should meet all or some of these criteria:

- Products that use the same resource to be produced
- Products which can be used to fulfill the same needs of customers
- Products that can or must be used/consumed together

Having this view of multiproduct, now we define the scope of our study along with the following terms:

Bundling products: Products that are sold together as a bundle e.g. a package consisting of different cosmetic products. Generally the products within a bundle can be either complementary or substitute for each other.

Substitute products: Products which satisfy the same needs of customers and an increase in the sale of one product means a reduction in sale of other products e.g. selling a blue color shirt means less selling of a black color shirt from the same market.

Note that by complementary products we mean those products which increase in sale of one product usually increases the sales of other products e.g. PC and printers. Usually highly complement products are offered in bundles.

Figure 1.1 illustrates the way we study and organize this thesis with respect to aim and scope described above. We study revenue management, bundling, and substitute products in details to gain knowledge about the primary concepts and get acquainted with approaches and techniques used in recent literature regarding managing of bundling and substitute products. Ultimately, we integrate them in order to find fits, gaps and direction for future studies.

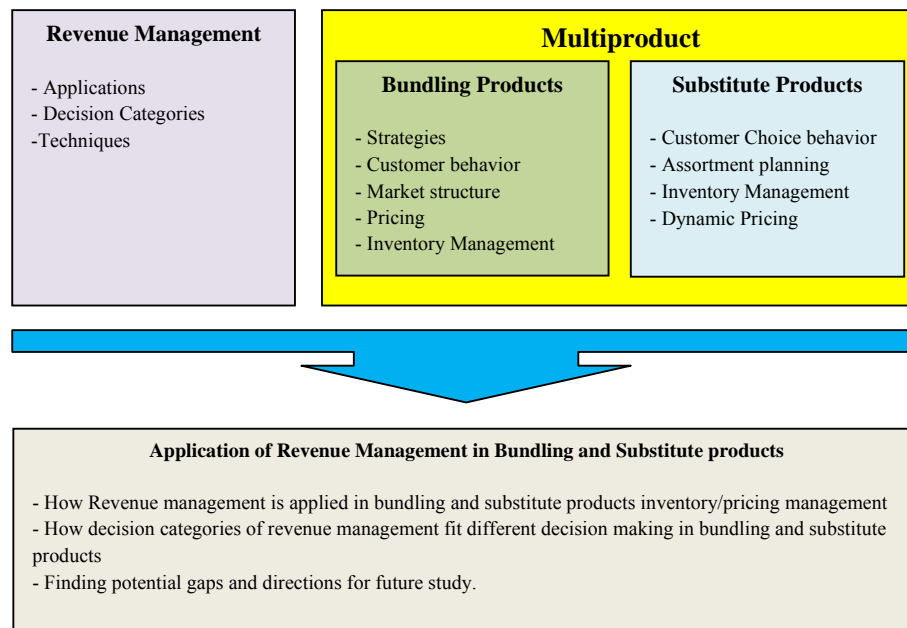


Figure 2.1 A concise map of content and objectives of thesis with respect to aim and scope

1.3 Limitation

In this thesis it is tried to cover most of significant works within the scope of our research, we tried to include the most significant and well known classical research as well as new and state of the art researches. Our main focus was on the researches considering mathematical models as well as economic and marketing perspectives. There are also some researches which are not aligned with our direction, even though they discussed multi products offering. Several journals and databases were searched in order to find relevant literature. We used

mostly keywords to search within these sources. In addition all relevant references given at the end of each paper were studied to ensure inclusion of significant studies. Among resources searched we can mention: European Journal of Operational Research, Operations Management, Journal of Price and Revenue Management, Manufacturing & Service Operations Management, Journal of Marketing Research, as well as Sciencedirect and Emerald Database. The limitation of our method for finding literature is that we might not find those researches which use different or unfamiliar keywords, yet relevant, for their researches.

1.4 Thesis outline

The thesis is organized as follows: Chapter 2 is dedicated to introduction and basic concepts of revenue management. Definitions, background and techniques of revenue management will be discussed as well. In chapter 3 we describe the concept of bundling products in detail. Motivations, different strategies of bundling, marketing and customer behavior towards bundling, economics perspective of bundling and application of revenue management in bundling are discussed and significant literatures in this field are reviewed. Chapter 4 discusses the concept of substitute products, the problems which arise in managing substitute products and their significance, customer behavior towards choosing substitute products, as well as general mathematical model. Finally, the related literatures will be reviewed and results will be compared. Chapter 6 concludes the report and discusses the possibility of applying new revenue management techniques which have not been used in multiproduct firms.

Chapter 2

Basics of Revenue Management

2 Basics of Revenue Management

In this chapter it is aimed to introduce the concept of revenue management (RM) and the most comprehensive definition of revenue management is discussed here as well. Moreover, the origin of revenue management practice, the conditions under which RM can be applied and different techniques of RM are described.

2.1 Introduction and Definition of Revenue Management

It is obvious that the most important objective of almost all firms is to increase the total profit of the firm through providing different services and products for customers in order to compete and survive in the market. On the other hand, in today's business there are numerous uncertainties in selling a product or service to customers and companies are acquiring whatever tools and techniques needed to cope with these uncertainties or even to take advantages of them. Competition forces firms to better and more carefully understand the importance of customer value for the products or services, which is diverse and plays a critical role in revenue and profit. The time at which a customer desire to purchase a product or service, where and at what place customers prefer the product to be available, the value customers assign to products and the prices they are willing to accept for different products, are decisions and issues which should be considered by firms who seek to obtain the highest possible profit from customers. These decisions are not easy to make since customers are not the same in their behaviors. Customers show different buying behaviors, they assign values in different ways for a particular products and the market for products are getting divided into much smaller and more distinguished segments where needs of customers within each segment differ. Selling products within each segment requires much more efforts than selling in an aggregate market. Perhaps it is required to design a product in different variants to respond each customer's unique need, products should be priced based on customers' valuation and willingness to pay and should capture the effects of time, place and other factors which affect customers' buying behavior. In addition, demand for products within each segment should be forecasted individually. Inventory levels of different product variants have to be determined in interaction with other products' demand; for example when products' demands are correlated. Other difficulties involve short life cycle of products and perishability of them which force firm to sell their products in a very short horizon but at the best possible time.

Revenue management is a very useful concept to make the decisions above easier and to provide firms the techniques and tools in order to maximize their expected profit in the competitive and diverse market. Revenue management can be implied as a system which aims to understand and anticipate consumers' behavior and effectively react to them in order to maximize profit of the firms. There are several synonymous names for revenue management; such as *yield management*, *price and revenue optimization* and *demand management*. Consequently there are several definitions for revenue management, but the recent and perhaps the best definition so far is as follow; Robert G. Cross author of the book "Revenue Management – Hard-Core Tactics for Market Domination" define revenue management as "*the art and science of predicting real-time customer demand at the micromarket level and optimizing the price and availability of products*" (Cross, 1997). It is a sophisticated form of supply and demand management that balances both pricing strategies and inventory management. Its primary purpose is managing customer demand through the use of variable pricing and capacity management to maximize profitability. It is essentially the process of allocating the right type of capacity to the right customer at the right time at the right price. It focuses the service organization on maximizing profitability by applying disciplined tactics to forecast consumer behavior at the micro-market level and control inventory availability at each price level at any one time. Revenue management practice can offer additional revenue from 3% to 8 % and it may result in possible profit increment of 50% to 100% in some cases (Cross, 1997).

2.2 Concepts and Techniques of Revenue Management

Practicing revenue management has introduced some new concepts and changed the conventional focus of marketing and sales activities in some ways. Revenue management practice requires attentions to be focused on prices rather than costs when balancing supply and demand. Price of products needs to be determined based on customers' valuation and their willingness to pay, unlike conventional approach which pricing was associated with total cost of products. Markets have to be segmented based on customers' type and their willingness to pay and products should be specifically designed and priced for each segment, therefore higher prices are charged for customers who assign higher value to the product and have higher willingness to pay, on the other hand lower prices are offered for customers with lower willingness to pay, perhaps at different place or time frame. Practicing of revenue management does not involve only pricing of products. Revenue management highlights distinguishing of the most valuable customers and serve them in a higher priority; e.g. if there

is one product left, a customer with higher willingness to pay is prioritized to get the product than a customer with lower willingness to pay. In another situation, higher amount of capacity is allocated for the most profitable product in prior to other less profitable products for other market segments. Product life cycle plays a significant role in practicing RM. Price of products changes dynamically over time and different life cycle of a product with a close interaction by inventory level of the product. For example, when a new cell phone is introduced to the market, first the customers who are interested in new products and pay much attention to be pioneer users of new technologies buy the cell phone and they do not care about the price as much as others do. Therefore the firm can charge them with a higher price. After a while, when the product gets mature regular customers begin buying the product, so the firm has to lower the price to increase the sale volume. When the end of product life cycle is approaching, the firm reduces the price again to capture demands from customers who are very sensitive to price. On the other hand, the firm prefers to sell all the remaining products before their life cycles end. Therefore, different prices can be charged without changing the product but by understanding the product life cycle and customers' behavior. From this discussion it can be implied that there are different strategies and decisions along with RM concept and each of them aims to maximize the expected profit of the firm. In general, Talluri and Van Ryzin (2005) distinguish three decision categories through which RM concept can bring higher profits. Different techniques and models of RM are also categorized within these decisions:

Structural decision: These decisions are mostly strategic decisions which do not tend to be frequently changed. How a firm segments the market for its products? Through which channel or mechanisms the firm aims to sell the product? Whether or not products are sold in bundle? How many and which products to include in a product line to be produced or in a store to be sold? These decisions are not easy to change when they are taken, but they have a significant effect on firm's profit. For example a firm sells complements products and the question is that whether selling products in a bundle can bring higher profit or not. After the decision is made and products are to be sold as bundles, it is very costly and difficult for the firm to change the decision as making bundles require its own packaging, advertisement, marketing efforts and so on. In another example, introducing a new product to be sold in a particular store has its own price of contracting, inventory management, and shelf space extension; therefore the decision should be made carefully in the first place to ensure maximum profit.

Quantity decisions: These decisions should be made usually in advance of selling periods and are easier to be changed during the selling horizon than structural decision. Decisions such as; how to allocate capacity to different product in a product line? How many products to order before the selling period begins to achieve highest possible profit? How many products to include in a bundle? How many seats should be reserved for first class passengers in an airline industry? And how to exploit demand by controlling availability of products over time to maximize revenue? These are examples of the decisions which are made under this category.

Price decisions: The price decision is perhaps the easiest one to be changed over time. The common decisions which firms have to make are: how to set prices of products over different market segments? How to forecast customers' willingness to pay? How to change price dynamically over time and product life cycle? How to price discriminate over product categories or different variants of one category?

Firms make their RM decisions sequentially and mostly based on above three stages. For example at higher level of RM a retailer decide about categories, assortment, styles and colors of products to stock in the store. This is a structural decision. In the next levels quantity and price of these products should be determined, however, the sequence of these decisions is not the same in all cases. A firm considers bundling of some of its complements products to be sold together. This decision accompanied with which products to be included in the bundle can be considered as a structural decision. Then the optimal order quantity of bundle and the associated price are determined in next stages.

2.3 Applications of Revenue Management

Revenue management first was accepted and implemented by the US airline industry. After that, many travel and hospitality companies have changed their focus towards revenue management. Since RM has its origin in service industries, conventionally RM concept was assumed to be suitable only for service industries where the time of production and consumption is simultaneous and the offered service cannot be stocked for future usage. As RM was adapted by different service firms and its techniques were developed, it has been discovered that RM can be implemented also in other industries. These developments call for models that have the potential to increase the other firms' revenue significantly as they did in airline and other service industries. Today, revenue management processes and

systems are implemented in number of industries, including manufacturing, advertising, energy, hi-tech, telecommunications, car rental, cruise line, railroad and retail.

Considering the aim of this paper, the important question which may arise is whether or not RM can be applied for industries which work with substitutable and bundling products. The answer is yes. To answer this question it is important to understand that substitutability or bundling of products is not limited to any industries and they can exist in most of services and production industries as well as retail industries. Even in some cases they are used as a practice of RM, for example a firm offers a bundle consisting two of its products at a discounted price to sell all the remaining products before they expire. Kimes (1990) described the characteristics of industries where RM can be applied, but her focus was on capacity restricted service industries. We introduce these characteristics and for each of them we give examples both from service industries and production industries in order to show the applicability of RM in different industries. Generally RM can be used to increase profitability of industries which their characteristics meet all or some of the following:

- ***Segmented market:*** The market for a service or product can be clearly segmented and price sensitivity of customers in each segment differs from other segments. Using RM techniques a firm can charge different prices or offer different quantities to different segments in order to maximize the profit e.g. in an airline industry, available seats can be divided into two class, business and economy, and priced differently. Also in a fashion industry, different market segments can be served with different quality where the associated prices and quantities produced for each segment is different.
- ***Fixed capacity:*** The firm is restricted to a fixed capacity for offering a service or production of products and it is significantly expensive or impossible to increase or decrease the capacity in the short run, but there may be the ability to shift it. A firm can apply RM to ration the capacity for the demand based on time (e.g. delivery or consumption time), market segment, popularity, etc. for example in an airline industry, from 200 available seats, 40 seats are allocated to the business segment and 160 seats to the economy segment. In a manufacturing industry with assemble to order (ATO) production, since the production capacity is restricted, the firm prioritizes orders so that charges higher prices for orders with short delivery time and

lower prices for those which have to wait for a free capacity and have a longer delivery time.

- ***Perishability:*** Sometimes the time dimension plays a significant role for the provision of the service or product. Once that time has passed, the inventory perishes and become valueless. In situation like this, RM concept implies that firm would better to sell all the products or service before it perishes even at a very low price. For instance, empty seats in an airplane after its departure has zero value and that is the reason sometimes a flight ticket may be bought at \$5 in low-demand seasons or days. On the other hand, a retailer has to sell all the bread in the store before their expiry date, otherwise they become valueless.
- ***Low marginal cost:*** The cost of providing additional units of capacity to sell is relatively low to the fixed cost of offering the service or the product. Applying RM techniques, a firm price the service or product based on the value it offers to customers not the cost. For example adding one passenger to a flight may cost \$10 but the firm can charge a customer for \$200 and another customer for \$5 based on the value the customers assign to the ticket, therefore the possible loss is negligible compared to the benefit.
- ***Advance booking/selling:*** The firm has the opportunity to evaluate and accept or reject order requests in advance of selling the products or the performance of the service, or the firm has the flexibility to adjust prices quickly to reflect variations in the balance of supply and demand. For example airline industries usually offer advance booking to assess demand and set the prices efficiently to reflect to the demand. Also online selling has helped MTO/ATO manufacturing companies to sell their products in advance and it provides them the ability to evaluate demand through time in order to adjust prices and capacity allocation.
- ***Stochastic and fluctuating demand:*** Demand fluctuates and shows definite peaks and valleys over time, which can be forecasted, but not with a high degree of certainty. Firms can manipulate demand by raising prices in high-demand periods and by lowering prices in low-demand to increase revenue and manage the capacity

utilization. For example flights on weekend usually are cheaper than flights during the week.

Different examples given above show that RM concept has the capability to be applied in many different industries and situation, using different techniques but with goal of increasing revenue. In the following we describe common techniques of revenue management.

2.4 Revenue Management Techniques

Here we highlight the most significant and common techniques of revenue management into two main categories of quantity and price based revenue management. However, in many cases these techniques are used together or they can be characterized under both categories.

2.4.1 Quantity Based Techniques

Booking limit:

In this technique it is assumed that there are different classes of customers. Booking limits control the amount of capacity that can be sold to each particular class at each point in time. Each segment or class of the customers is charged differently. Assume that we have 3 classes of customer for a product with a fixed quantity. Assume there is a quantity of 40 for a particular product, and the customers are classified in 3 classes. The first class of the customers are charged 30\$ for one unit of the product, the second class is charged 40\$ and the third class is charged 50\$. For example the number of the product which is sold to the first class is 15 and for the second class and the third class is 15 and 10 respectively. If the number of demand for one class exceeds the corresponding booking limit, the class would be closed and the product will not be available at that price. Therefore if the customer wants to buy the product, he should buy the product at another (higher) available price.

Bid pricing:

Bid pricing is simpler than booking limit in its nature. In this technique there is always a threshold price above which all customer bids are accepted. This threshold price or value can be updated based on time of sale or remaining capacity/inventory of product. For example at a specific point of time or specific remaining capacity of flight seats, firm determines \$50 as the threshold price. All customer bids are accepted and the product will be sold to customer as long as they are above \$50.

Overbooking:

In the businesses which advance booking is applicable, such as hotels and airlines, it is sometimes possible to consider a portion of capacity not to be used even after they are sold, for example no-show or cancellation in airline industries. Therefore using the concept of overbooking, firms can sell the capacity in advance, more than their actual capacity in order

to obtain higher revenues, since they expect a portion of customers to cancel their booking or not to show up at the time of consumption.

2.4.2 Price based techniques

Dynamic pricing:

Dynamic pricing is a technique, in which the price of a particular product changes over time according to the level of demand, the time length between purchase and consumption time, season of purchase, remaining available capacity and product life cycle. For example, in airline industries, the price of a seat is gradually increases as the time of departure is approaching. A flight seat in two month before departure time can be purchased at \$10, however, the same seat is sold at \$50 10 days before departure time. Moreover, If 24 hours before the departure time there are some unsold seats, the price may be dramatically reduced because free seats have no value after the departure time.

Usually in practicing dynamic pricing, prices can be increased, decreased, or remained the same interchangeably through time, but there are two special case of dynamic pricing where prices can only increase or decrease.

Mark down strategy:

The main concept is to lower the price over the time, especially for perishable product, fashion product, and high tech products which are very interesting for the customer in the beginning, but as the time keeps passing the products become less popular. Mark down technique is most suitable for products with relatively short life cycle time. In other words, we can consider markdown strategy as a mechanism of segmenting the market with respect to customers' different willingness to pay for a particular product. Therefore, customers with higher reservation price (willingness to pay) are buying the product earlier and at higher prices, and after a while when the price goes down customers with lower reservation price will buy that product. Other example can be perishable products and seasonal products when their expiration dates are approaching. In these situations, firms prefer to get rid of their products, otherwise they will be perished, and thus any amount of money is better than nothing.

Mark up pricing:

This technique is well deserved for companies with fix capacity on which booking should be made in advanced. In this technique the price of the service or the product is raising as the booking period is finishing. In other word, as the last day of the booking period is getting close and the amount of available capacity is getting lower, firms we can assign the free capacity to the customer with higher reservation price and lower price sensitivity. The best example could be rail ways and airlines industries.

Auction:

This technique can be considered as a practice of dynamic pricing but the difference is that customers suggest the price (bids) in auction and the firm has the power to accept or reject bids. Also it differs from bid pricing since in auctions there are no threshold prices above

which all bids are accepted. In fact the product will be sold to the highest bid possible and therefore there is going to be a near perfect price discrimination. This technique also provides a good mechanism to understand how different customers value a particular product. If the firm aims to sell N number of identical products in an auction, the firm announced the ascending price until the number of customers are willing to pay at that price are equal to N . This method is called Multiunit Auction.

In the next chapter we discuss bundling products and recent literature to the field will be reviewed.

Chapter 3

Bundling Products

3 Bundling Products

Bundling product as one of the common techniques in selling products is getting popular nowadays. Also, firms producing multi-product are trying to use bundling concept to take advantages of bundling from different perspectives which will be described here. Moreover, bundling can contribute to raising revenue if we can use this technique in proper ways which are going to be discussed in this chapter.

This chapter is divided into the six parts. In the first part we define bundling and discuss the very basic concepts of bundling. In the second part, we discuss the motivations for using bundling. Different bundling strategies will be described in the third part. The fourth part is dedicated to bundling from marketing perspective; in order to discuss different perceived value and different judgment of customers when we offer a bundle of products. A brief discussion and illustration regarding comparison of different strategies is discussed in part five. In the sixth part we will discuss revenue management features of bundling when we have monopoly or non-monopoly market, as well as comparison and reviewing of the existing literature in that field. Finally we have the summery and result of this chapter.

3.1 Bundling definitions

Here we define basics of bundling concepts. A classic definition of the bundling is “the practice of marketing two or more products and/or services in a single package for a special price” Guiltinan and Joseph (1987), however, the definition is very general. We can classify bundling techniques in terms of bundling pricing and bundling product as follows:

- 1- **Price bundling:** two or more products are sold together at a discount price than the sum of selling individual products
- 2- **Product bundling:** two or more products are sold together at any price. Therefore, the price of the bundle could be even higher than the sum of the prices of the products. This situation occurs when the integration of the products needs a special skill or adds value from customers’ point of view. The best example could be computer manufacturer that integrates different components and delivers as a PC at any price which can be even higher than the sum of the price of all components; but the customer pays because she knows that integration of the components needs special technology and this process made by the manufacturer has an added value for the customer. So the customer is willing to pay more because of this additional value.

According to the second definition price bundling is a practice of product bundling.

Generally we have the following situations for the price of the bundle. Assume that j indicates each single product, and p_j is the price of each single product. b Indicates bundles containing a number of single products and p_b is the price of the bundle.

Additive price means the situation in which the price of the bundle is exactly as same as the sum of the price of the products in the bundle.

$$p_b = \sum p_j$$

Sub-additive price means the situation in which the sum of the price of the products is higher than the price of the bundle. In other words, the bundle is sold at a discount price. This situation is normally used when the products have zero or negative demand correlation, the products in the bundle are not complementary for each other; in order to encourage the customers to buy more products. Also, this situation fits to the case of making bundle from only one kind of product.

$$p_b \leq \sum p_j$$

Super-additive price is the situation in which the price of the bundle is higher than the sum of the price of the products. This situation fits to the case of making bundle from complementary products where the customer would not prefer to buy each product separately. In other words since the customer would prefer to use the components together she is willing to pay more to have them as a package.

$$p_b \geq \sum p_j$$

Nowadays we can see a lot of products and services that are sold together through different techniques. A typical gym sells numbers of sports available at a time together at a single price; a tourist company offers a two-way flight and few nights staying at a hotel as well as a tour guide in one package; Microsoft offers a package of different software. These are examples of selling different services together. Also it is getting increasingly popular to sell a service with tangible products; for example automotive industries sell after-sale services with each car. Finally selling two or more products together is the most tangible kind of bundling. Fast foods offer a sandwich; French fries and a drink together; one can buy a cell phone and SIM card together with internet network connection subscription for a special period of time.

Also, another common example is to buy three shirts for price of two. The examples mentioned above are from different types of bundling. Although all of them are practices of bundling, each of them can be used for a particular goal. One important question is what motivates firms to practice bundling.

3.2 Motivations for bundling

The motivation for practicing bundling in conventional views is different from modern views, the conventional one is discussed here but the modern views are discussed in the following parts. Conventionally it was believed that if a firm has a monopoly market for product A and a competitive market for product B; if the firm sells both products together as a bundle, the firm can be better off. Consider product A is sold at the price p_A and B is sold as p_B and the optimal price of the bundle is p_O , according to the conventional argument the firm makes more revenue. This practice forces the customers to buy the bundle even if they are not willing to buy product B since A is not sold separately and the bundle is sold at the price $p_O = p_B + p_A$. This strategy aims to increase the demand for the competitive products. Surprisingly, in many cases not only the number of customers who want to pay for A does not increase, but also customers who are willing to buy A and not B do not buy the bundle any longer. Therefore, this strategy does not make sure an improvement in revenue. This conventional discussion is known as Chicago School Argument; for further discussion see Nalebuff (2003). By contrast, some companies in a monopoly market or near to monopoly market use the same strategy but in a different way. They add one or more products to their primary products and sell them as a bundle. The main reason would be improving the value of the product for customers rather than ensuring selling a competitive product. For example Microsoft is close to be considered as a monopoly operation system producer, and sells several software in a package called Microsoft Office which none of the software is sold individually. This discussion fits to the area of monopoly markets, but most of the firms that practice bundling to leverage their power in different markets are not in a monopoly market; therefore there should be other motivations that bundling can make a sense for them. These motivations are as follows.

3.2.1 Promotion of Sale

Customers can be motivated to buy more products or higher volume of a product if they are offered a discount for buying more. For example a customer is going to buy only one shirt but she is offered to buy three shirts for price of two. Therefore the customer gets motivated to

pay for two shirts instead of one. Assume in another situation where the product A is recommended to be used with product B, we can lower the price of A and increase the price of B depending on the situation. For example, Apple produces MacBook; which the operation system is not Windows, the operation system needs a compatible series of software which can be bought by the user several times. Here we can see the product of the company is implicitly a bundle consisting of one PC and compatible software, which the proportion of the consumption is not one-to-one. Thus if Apple reduces the price of MacBook, more potential customers will be motivated to buy. On the other hand, the company can raise the price of the compatible software. In a long run, the revenue from selling the package can be much higher than the reduction of the price of the PC since marginal cost for coping software is almost zero.

3.2.2 Reduction in customer value variation

In an example, Krishna (1990) shows that the customers' valuation, the value assigned to the product by a customer, for a bundle has lower variations compared with the case of selling each component individually. If the demand of two products A and B are correlated negatively, an increase in sale of one product leads to decrease in the sale of the other product, bundling of them makes the customers' value more homogeneous for the product. In other words, the standard deviation of the reservation price distribution for bundle is lower than the sum of the standard deviations of each product within the bundle especially when the correlation is negative. McAfee et al (1989), as a classic research in this field, showed that even if A and B are valued independently in a multiproduct monopolistic market, selling A and B in a bundle performs much better than selling them individually; because at this level demand are more aggregate than the situation in which the demand for each product is forecasted individually.

In case of symmetric reservation price distribution, selling the products only in a bundle effectively reduces the dispersion of buyers' valuation as long as the demands for both products are not perfectly correlated. Therefore, the standard deviation of the customer reservation price for the bundle is always less than the sum of standard deviation of the customer reservation price for each product (customer reservation price is discussed in more detail in 3.4.1). Definitely bundling should make sense for the products in other words if the products are totally irrelevant this argument does not make sense. Figure 3-1 shows two products which the demand for each of them is absolutely independent of the other demand.

As we can see in figure 3-1 the standard deviation of the bundle is much less than the sum of the standard deviation of the products.

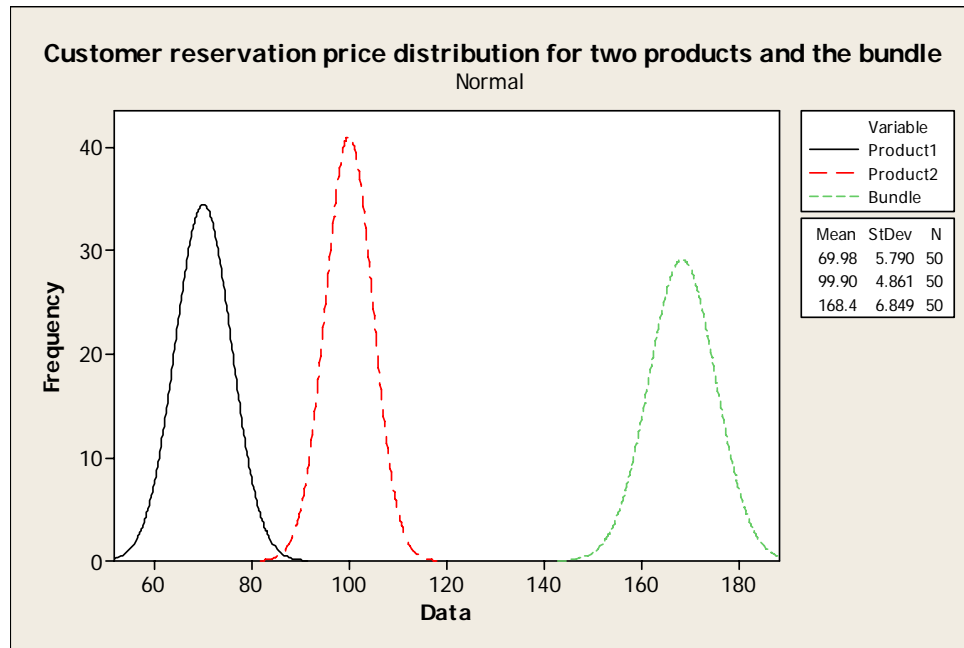


Figure 3-1 customer reservation price for two products and the bundle of them

Therefore, bundling of the products sometimes can help us to have more accurate forecast due to lower variation.

3.2.3 Cost saving

Selling two or more products together can have a lower marginal cost compared to selling them separately. For example selling some software in one CD has a lower marginal cost than selling them separately and consuming one CD for each. Moreover, Cost of packaging multiple products together is lower than packaging separately.

3.2.4 Increasing sale of products that are about to be expired

Often firms make a bundle of products to be sold at lower price to get rid of products that are about to expire. For example, a super market which sells regular meat for 3\$ per kg, make a bundle of packaged meat, which their date of the expiration is approaching, and sells 4kg for 7\$; therefore the customer is motivated to buy the bundle because of the discount. More bundling incentives can be found in Nalebuff (2003).

As we will discuss in this chapter, many research study try to find the optimal pricing policy for the firms which use bundling. Most of the papers have covered monopoly situation and a

few deal with a competitive market, we will discuss them as well in this chapter. The purpose of finding the optimal price is to gain more revenue and extracting the customer surplus as much as possible.

Bundling can be used under different kind of strategies which will be discussed below.

3.3 Bundling strategies

Here we will discuss the different strategy that can be used by a firm while practicing bundling. Nalebuff (2003) classifies three different bundling strategies as follows:

- 1- **Pure bundling:** two or more products are sold together, with the constant proportion of each in the final package. In other words, none of the products can be purchased individually and the proportion should be fixed.
- 2- **Mixed bundling:** two or more products are sold in a package. Also, products can be sold individually. Volume discount can be classified in mixed bundling category i.e. selling two or more units of a product at a discount price to motive the customers to purchase at higher volume.
- 3- **Pure component:** also known as unbundling means no matter how many products we have and what the relationship between them is, we always sell our product separately; even if our customer would like to buy two or more kinds of our products together, the customer has to buy them separately.

Different researches have been carried out to find the best bundling strategy which leads to more revenue. Surprisingly we cannot find a general strategy which always performs well. The performance of each strategy highly depends on the other factors like competitors, cost structure, cost of making bundle. We will discuss the comparison of the strategies later in this chapter.

The researches regarding bundling products can be classified in two categories. Some researches have been done to understand the behavior and perceive value of the customers toward bundling practice which we classified them as marketing perspective of bundling. Some research have been carried out to find the best bundling strategy under different conditions as well as finding the best price for the products. Moreover, some papers aim to find the optimal inventory as well as best price while practicing bundling. We classified these

literatures as revenue management practice of bundling. As follow first we discuss bundling from marketing perspective then we will discuss revenue management aspects of bundling.

3.4 Bundling from marketing perspective

In this part first describe the general customer behavior and illustrate bundling price and bundling products. Then we discussed the general value perceived by customers when we offer a bundle of products and psychological effects of bundling on the customer judgment.

3.4.1 Bundling and General Perceived value by Customers

First we describe some basic definition and concepts that will be used through the paper.

Reservation price: is the value assigned to the product by the customer. In other words the amount of money which a customer is willing to pay for the product is her reservation price. Customer reservation price depends on different factors including brands, quality, budget constraint of the customer, etc.

Consumer surplus: the difference between the customer reservation price and the actual price of a product is called consumer surplus.

Marginal Cost: marginal cost shows the cost of production for one unit of the product.

Transaction Cost: is the cost that a buyer should pay for making an economic exchange in addition to the price of the goods or the service. For example if a customer is going to buy a book online. In addition to the price of the book the customer is required to pay for shipment. The shipment cost is the transaction cost.

Sunk cost: the cost that has been already paid and it is not possible to recover it regardless of what happens in the future. In other words, the money that has been paid for a product but the product is not useable at the time of consumption. Also, the money is not recoverable. For example a customer buys a flight ticket and she misses the flight. Therefore, if the fee is not refundable the whole money is lost.

A typical customer buys a product if the reservation price is higher than the actual price; in other words whenever the consumer surplus is positive the customer buys the product.

When a customer is going to choose one product among several products she buys the one which offers the highest consumer surplus. Assume that i indicates the product and s_i is a consumer surplus associated with product i . Product i will be bought by the customer if:

$$(s_i \geq 0) \cap (s_i > s_j \text{ for all } j \neq i)$$

Assume that a firm offers two products called A and B. p_i, r_i and s_i show the price, the customer reservation price and the consumer surplus associated with product i respectively.

Consider figure 3-2 in which the vertical axes show the reservation price for A and the horizontal axes shows the reservation price associated with B. And We have four different area I,II,III and IV.

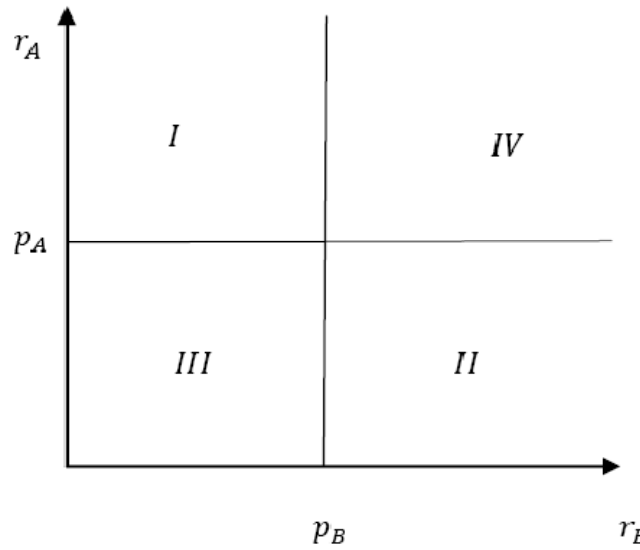


Figure 3-2 prices and reservation prices of products A and B

Customers in area III never buy either A or B because their reservation prices for none of the products are higher than the products' price. Customers who are located in I have higher reservation price than the price of product A, $r_A \geq p_A$, therefore they will buy A, likewise people in area II will buy product B because of the same reason. In addition, customers located in IV have the reservation price higher than the prices of both products and they buy both.

Assume that the firm sells a package consisting one of each at a price lower than the sum of the prices of both products, in other words, the company uses price bundling. Let p_{bundle} and r_{bundle} denote the price of the package and the customer reservation price for the package respectively. Therefore, $p_{bundle} \leq p_A + p_B$. See figure 3-3.

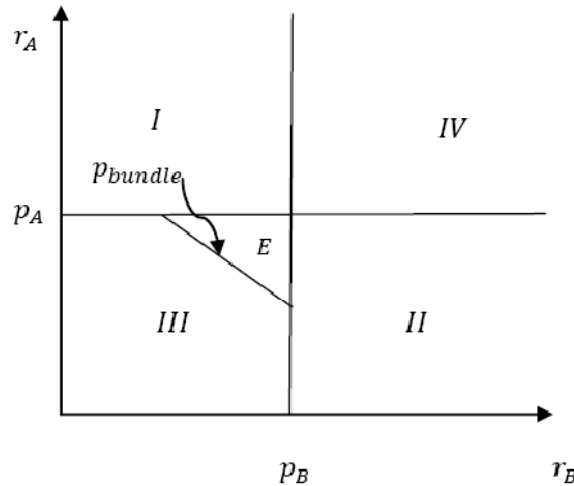


Figure3-3 price of the bundle

Now we expect the customers in triangle E to buy the bundle. Without offering the bundle at that price of the bundle customers in the triangle E would not buy anything. So we can see how bundling motivates customer to buy more. Also we can see a new segment of the market has been included. Also, bundling can change the customers' value and the customers' judgment which can lead to inclusion of new market segments. The following part discusses the effects of bundling on customers' judgment.

3.4.2 Bundling psychological effects

After describing the terms and the concepts needed from the marketing perspective, here we discuss the psychological effects of bundling on customer judgment regarding the price of the products. Psychological effect here means that the effects of bundling and different kind of discount on customer feeling and behavior. Not only bundling enables us to extract the consumer surplus, but also sometimes helps us to improve the revenue through changing the customer perceived value. In other words, sometimes bundling leads to different customer judgments compared to the case that products are offered separately. This property can be considered as a powerful leverage to understand the market and the customer's valuation. Here the most significant psychological effects are highlighted under the following categories in order to show how we can change the customer perceived value and customer judgment.

Paying for Bundles or Individuals:

If a firm can offer different product in a bundle even with no discount, additive bundle price, the customer pays less attention to the sunk cost compared to the case when the firm charges the customer for each product individually. But what cause these different feelings is the fact that a typical customer cannot easily identify the price of each individual products, in contrast to unbundle case which customer is easily able to find the price of each products. In other words, there is going to be an ambiguity regarding price for a customer when the products are offered as a bundle. This ambiguity makes the customer not to perceive clearly the cost and benefit of each product, consequently the customer pays less attention to the sunk cost if he cannot use one or a few products of the package. The following example is brought to clarify this discussion:

Consider a company offers traveling tours. In case of unbundling, customer A should pay 100\$ for a flight and 200\$ for two nights staying at a hotel and 50 \$ for a ticket of a classic music concert. In case of bundling customer B pays 350\$ for one flight, two nights staying at the hotel and the concert ticket together. Both customers attend to the flight as well as two nights staying at the hotel, but before the time of the concert, both are invited to a party. Which one would attend to the party and which one would go to the concert? According to Saman and Gourville(2001) customer A, who has paid the price for each item separately, is able to analyze the cost and the benefit of the concert clearly, consequently she perceives the sunk cost to be significant. Therefore it is less likely for customer A to refuse the concert and to attend the party instead than it is for the customer B who has paid the whole price as a bundle. Because of the ambiguity associated with price allocation to individual items, customer B would pay less attention to the cost and benefit of the concert individually. As a result, the first customer feels more sunk cost than the second one.

Generally when we have one-for-many payment i.e. customer pays one time for a number of products, there is a high degree of ambiguity to identify the price and transaction cost for each item individually; therefore less attention is going to be paid to the sunk cost. This attribute of the bundle pricing can be perfectly used when a company has a product which is not popular within a market segment. Normally a product cannot be sold individually, because the customers perceive the value of the consumption less than their willingness to pay or the customer may think that she would not use the product. In order to sell the product, or get rid of that, the company would choose to reduce the price as much as the customers'

willingness to pay; in this case there might be a problem associated with even marginal cost coverage. In contrast, if we use the property of the bundling price, which has been discussed before to make ambiguity in assigning the price to the product, the company can sell the product even at the desire price if the product is sold in a bundle together with a number of relevant products; the best situation is that the product is sold together with a number of products which are complementary for each other. Therefore, the customers perceive no sunk cost for the product and they are less likely to hesitate to buy the bundle only because of the unpopular product under condition that the other products are not available individually for sale.

Customer Valuation and Payment Decision:

Another psychological effect of bundling can be the customer decision regarding buying the products in a bundle or separately. If the customer is free to choose to buy the products either in a bundle or separately we can see that the customer decision is influenced by customer perceived value. Thaler (1985) suggest that customers make the decision regarding buying products as a bundle or as individual products based on their perceived value of the products. If the customer reservation price for a product is higher than the price of the product, buying the product has positive perceived value from the customer point of view, and if the reservation price is lower than the price of the product the product has a negative perceived value for the customer. Assume a firm which produces two complementary products x and y . X , Y and $(X + Y)$ represent the perceived value of the customer for product x and y and the overall perceived value respectively. If the products are going to be sold in a package the price is going to be additive. Here after we show positive perceived value for x by X and negative value by $-X$, the same for y . Thaler (1985) defined the situation for the overall perceived value as follows:

Multiple gains: The perceived values of both products are positive. So we have X , Y and (X, Y)

Mixed gains: When one of the products has a positive perceived value and one has a negative perceived value and the overall perceived value is positive; X , $-Y$, $(X, -Y)$ but $(X - Y \geq 0)$.

Mixed losses: When one of the products has positive perceived value and one negative value and the overall value is negative; X , $-Y$, $(X - Y < 0)$.

Multi losses: When both products are valued negatively and the overall is negative as well;
 $-X, -Y$ and $-(X+Y) < 0$.

Figure 3-4 shows different situation of overall perceived value by customers.

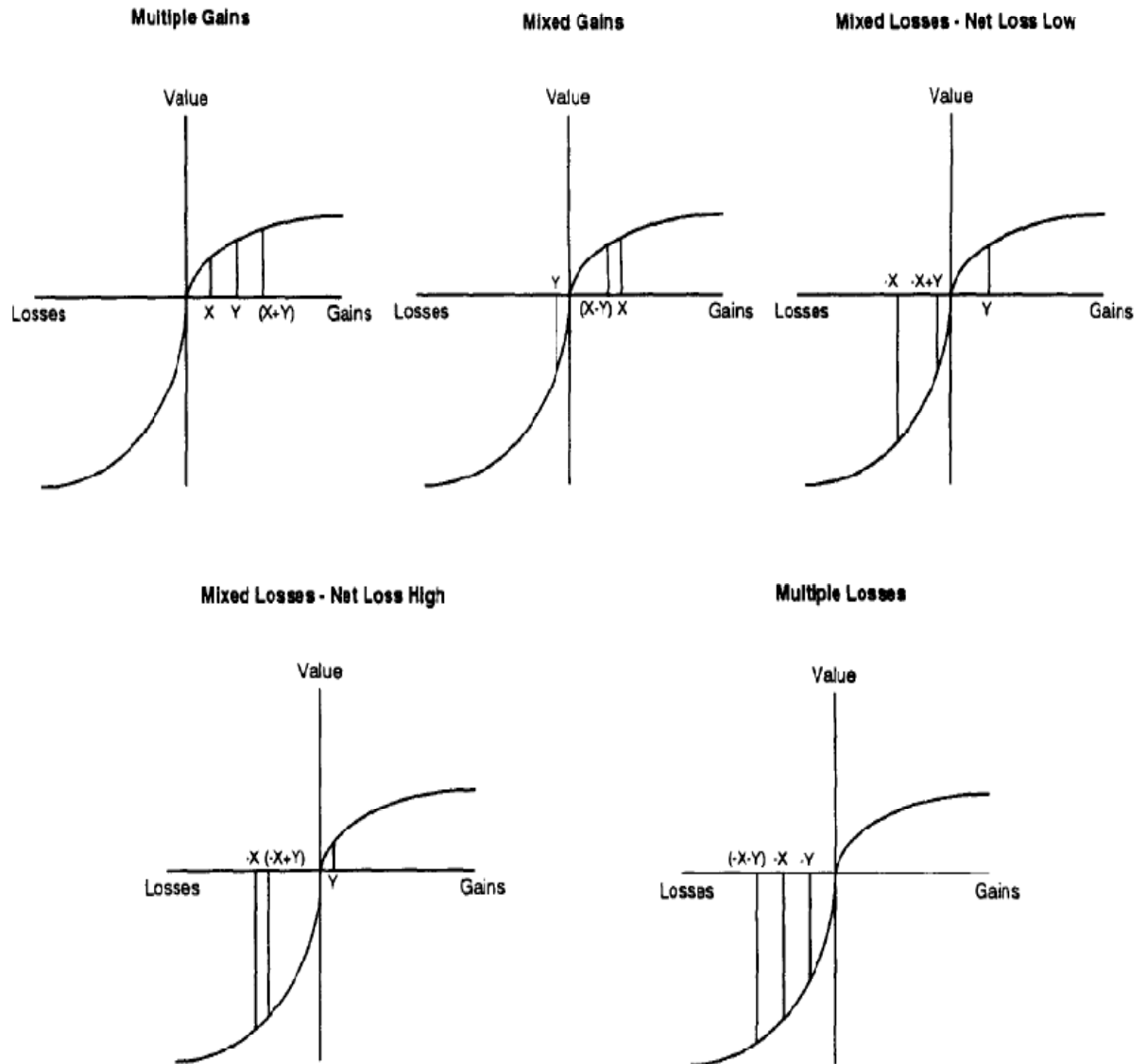


Figure 3-4 overall perceived value illustration

Source: Thaler (1985)

Kaicker et al (1995) examined the customer preference of payment under the different above condition to find out in each of the above situation customer would prefer to purchase the products in a bundle or pay separately for them. The result of the statistical research shows

for multiple gains, multiple losses and mixed gains customer would prefer to purchase the products in a bundle. In mixed losses situation if the customer does not feel a significant loss he also would prefer to purchase both products in the bundle but if he feels a significant loss he would prefer to buy the products separately.

Effects of different discount:

Another psychological effect on customers' perceived value can be created if there are different kinds of discount at a time. For instance, assumed a market with two kinds of products each product used to be sold at a regular price; but now they have another price in a seasonal sale, lower than the regular one. Also if the customer buys both products in a package the bundling price is even lower than the sum of the price of each product in the sale. In this situation what would be the saving value perceived by the customer because of buying in the sale period instead of buying at the regular time. There can be three different interpretations of the customer perceived saving value, since there are two kinds of discount during the seasonal sale period, one discount due to the sale period and one due to buying the bundle. A typical customer always thinks about the saving value. Therefore, saving value is the customer perceived value of a product based on the regular price and the current price of the product. There are three possible interpretations of saving value.

Interpretation 1: the total saving value is the differences between the sum of the prices of the individual products in the sale and the sum of the price of the individual product at regular price.

Interpretation 2: the total saving value is the differences between the price of the bundle in sale and the sum of the individual price in the sale.

Interpretation 3: the total saving of value is the difference between the price of the bundle and the sum of the prices of the individual products at the regular price.

Yadav and Monroe (1993) show that second interpretation is the most likely way in which a consumer perceives value. If we accept this discussion we can argue that even when we offer a discount for our product, bundling can change the customer judgment.

As it was discussed above customer behavior, perceived value, decisions and judgment can be changed in some cases when a bundle of product is offered. Therefore, if bundling is used in a proper way, it can improve the profit through changing customer willingness to pay, judgment and perceived value.

After discussing the psychological effects of bundling we continue with comparison of different bundling strategies.

3.5 Comparing different bundling strategies

A lot of scientific researches have been carried out to compare the three bundling strategies in different conditions. The results of the researches show that each strategy has advantages and disadvantages depending on the situation. Therefore, the best strategy is determined based on different factors. Adams and Yellen (1976) as one of the early researches in this field made a good and well-known discussion regarding comparison of these strategies. Each strategy is evaluated in terms of the following criteria. These criteria can be best achieved through perfect price discrimination. But price discrimination would not be possible entirely due to legal restrictions or insufficient information of the market.

- A) ***Complete Extraction:*** no consumer feels any surplus.
- B) ***Exclusion:*** no consumer consumes the product if his reservation price is less than the cost of production.
- C) ***Inclusion:*** anyone whose reservation price is more than the cost of the product consumes the product.

According to above criteria Adams and Yellen (1976) compared three different bundling strategies to find out which of them is more close to perfect price discrimination, because perfect price discrimination makes the highest possible revenue.

Definitely pure component strategy never violates exclusion criterion; because we never set the price lower than the production cost in this strategy. On the other hand, extraction criterion is violated by pure bundling strategy since there would be some customers who are willing to pay for one or a number of products in the bundle more than the price. Mixed bundling facilitates inclusion. In general none of the strategies ensure us to have complete extraction. Also there are few circumstances under which we can choose the best strategy among the three strategies; these circumstances highly depend on the cost structure of our product and distribution of customer reservation price. On the other hand the competitive

situation, monopoly or non-monopoly, makes different condition of evaluation of the strategies.

Schmalensee (1984) has followed the research of Adams and Yellen, (1976) in order to find the situation in which each bundling strategy performs best. The criteria for this research are not as same as the ones for Adams and Yellen, (1976). Assume that there is a monopoly market in which a firm produces two products. The customer reservation price for the product pair follows bivariate normal distribution. The main result of Schmalensee (1984) can be summarized as follows.

Let μ as the mean value of customer reservation price. Let c production cost. And α is defined as

$$\alpha = \mu - c$$

The numerical studies show that if α value is high enough in a symmetric case it makes higher welfare for the customer. Also in case of symmetric case pure bundling performs better than unbundling for any value of the correlation and α .

As the figure shows the shadow area is the consumer surplus (see figure 3-5).

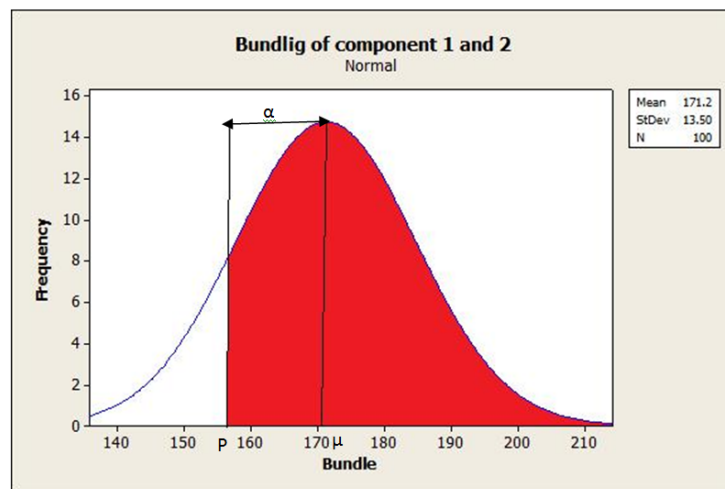


Figure 3-5 consumer surpluses

Mixed bundling is more profitable than unbundling when the products demands are negatively correlated. In other word, mixed bundling is the best if the products are relatively

substitute for each other. For positive imperfect correlation, not equal to 1, and for most of positive values of α , mixed bundling is better than unbundling.

For conclusion, based on the results of the researches the decision regarding the best bundling strategies is highly depends on the cost of the product, the price and the mean value of the reservation price distribution and the dependency of the products as well as competitive situation in the market.

After a general discussion about the performance of each bundling strategy the next part discusses different mathematical models of strategies and the optimal price and inventory under each strategy to achieve the highest revenue.

3.6 Bundling from Revenue Management Perspective

As it was mentioned before, bundling concept and strategies can be observed from revenue management perspective as well. In this part the main focus is on applying different quantitative techniques in bundling.

We divide the discussion in two categories. The first category is dedicated to the models and discussions where the market is monopoly. In the second part discussions under non-monopoly market will be presented.

3.6.1 Bundling in Monopoly Market

In this part first the basic model of bundling is discussed and then different assumptions and variables are presented. Finally the existing literature will be discussed.

3.6.1.1 General problem statement:

Assume that there is a company, in monopoly market, produces different number of products. The company is going to find out which bundling strategy among pure component, pure bundling or mixed bundling is the best strategy, also what is the optimal price of each component under each strategy. A typical customer buys the bundle because of the following reasons:

- 1- The customer feels more surplus from buying the bundle than buying one product.
- 2- Buying one product is not possible due to stockout of the product but there is a bundle containing the product.
- 3- Bundle consists of complementary products which customer would prefer to use the products together, then buying each product separately does not make a sense as long

as the bundle makes greater or equal surplus than buying each product separately does.

However there are some other reasons which make sense for buying bundle which have been discussed earlier in this chapter. But the above three reasons are common in modeling in the existing literature.

The general model for pure bundling is as follows:

$$Max Z = \sum_i Pr(r_{bi} \geq p_{bi}) \cdot N \cdot (p_{bi} - c_{bi}) \quad (1)$$

Where:

- $Pr(r_{bi} \geq p_{bi}) = \int_{p_i}^{\infty} f(r_{bi}) dR$
- Z is the total revenue.
- Pr shows probability.
- i is the bundle indicator because bundles can be different in the number of items.
- r_{bi} indicates customer reservation price for bundle i .
- $f(r_{bi})$ is continues probability density function of customer reservation price for bundle i .
- p_{bi} is optimal price of the bundle i .
- c_{bi} is the cost of making the bundle, production and forming.
- N shows the number of the people in the market.

$Pr(r_{bi} \geq p_i) \cdot N$ Shows the number of customer in the market who buy the bundle i .

In the model the optimal price is the variable of the model which we are going optimized in order to obtain the maximum revenue. The result of the model highly depends on $f(r)$; therefore, finding a suitable probability distribution function is important. Different kind of probability distribution functions have been considered in different researches which will be discussed later.

If the company applies mix bundling strategy the model should be made in such a way to find the optimal price of each single product as well. Assume that there is a firm producing couple

of products A and B which can be sold either as a bundle, one of each, or separately. p_A, p_B and p_b are the optimal price of A, B and the bundle respectively. Depending on the situation the price of the bundle could be additive, sub-additive or super-additive, which should be mentioned as a constraint in the model. A customer buys either one of the products or the bundle to maximize the consumer surplus. Also, customers leave without purchasing if the customer reservation price for the bundle and each single product are less than the corresponding price of the bundle and the products.

The general model for mixed bundling is as follows:

$$\begin{aligned} \text{Max } z = & Pr(r_A \geq p_A, r_A - p_A > r_B - p_B, r_A - p_A > r_b - p_b) \cdot (p_A - c_A) \cdot N + Pr(r_B \geq \\ & p_B, r_B - p_B > r_A - p_A, r_B - p_B > r_b - p_b) \cdot (p_B - c_B) \cdot N + Pr(r_b \geq p_b, r_b - p_b > r_A - \\ & p_A, r_b - p_b > r_B - p_B) \cdot (p_b - c_b) \cdot N \end{aligned} \quad (2)$$

Model 2 is a typical mix bundling model which can be modified based on the situation.

The general model for pure component:

$$\text{Max } z = Pr(r_A \geq p_A, r_A - p_A > r_B - p_B) \cdot (p_A - c_A) \cdot N + Pr(r_B \geq p_B, r_B - p_B > r_A - p_A) \cdot (p_B - c_B) \cdot N \quad (3)$$

Model 3 considers no bundling. Model 2 and 3 are the same as long as the price of the bundle consider additive.

After discussing the basic model of bundling pricing, in next part we review literatures of bundling products within monopoly market. We also distinguish different models based on following attributes:

Products: Products can be mentioned as goods or services. Depending on the situation the number of products in a bundle can be changed. It is a common assumption to have one unit of each product in the bundle, but this assumption does not have to be made always. Also, the number of item in a bundle can be either fixed or varying, mostly when we discuss the model for a company offering service as product we can considers variable number of products offered in the bundle.

Price of the bundle: Sometimes price of the bundle is considered to be variable and should be determined. Sometimes the price is restricted by the assumption of sub-additive or super-

additive. Sometimes the price of the bundle is additive so the price of the single products make the price of the bundle.

Bundling Strategies: As it was mentioned before, the model can be made in such a way to find the best price of the each product and/or the bundle under the assumption of one of the bundling strategy; or can be to find the best strategy over the three strategies as well. Table 3-2 shows a summary of the literature.

Customer arrival rate: In order to determine the demand for a product or a bundle of products over a particular time period it is common to define customer arrival rate. Customer arrival shows how customers come to our system, store. Arrival rate shows the average number of people coming to our system per unit of time. Arriving to our system does not mean that the customers buy something. Customers make their decision whether to buy based on their behavior, reservation price and consumer surplus.

Customer arrival rate usually is considered to follow Poisson distribution. Poisson distribution is not time-dependent, and it is memory less, so that the number of future customer arriving to the system does not depend on the number of pervious customer arrivals.

Customer reservation price: Jedidi and Zhan(2002) , make a good discussion for finding consumer reservation price by combining the economic theory of consumer choice and preference estimation of traditional conjoint. In the existing model and literature customer reservation price is assumed to be a probability distribution. Different kinds of probability distributions are assumed as follow:

- **Uniform distribution:**

This assumption indicates that the customer reservation price is distributed uniformly over a specific price area. Although assuming Uniform distribution of reservation price is computationally convenient, the assumption is not much realistic.

- **Normal distribution:**

Normal distribution is a continues distribution for reservation price, but the problem regarding normal distribution is the fact that normal distribution performs for negative numbers as well as positive numbers. But in reality nobody has a negative reservation price. This problem can be solved by considering the appropriate mean value and standard deviation for the distribution. On the other, hand the shape of normal distribution is symmetric but we know that often the probability of the reservation price for higher

prices is not the same as the probability of the reservation price for lower prices. However, when we are discussing a bundle of product, in fact we consider a number of product which each of them is valued differently by different customers. Therefore if we assume that the reservation price for each individual product follows normal distribution, we can conclude that the reservation price for the bundle follows normal distribution as well. In some cases it is better to use a multivariate normal distribution because each customer makes the decision for the bundle depending on the other individual product in the bundle. If we assumed the reservation price of each individual product as normal distribution the reservation price of the bundle of two products follows bivariate normal distribution.

- **Weibull distribution and Gamma distribution:**

Weibull distribution and Gamma distribution are also considered as customer reservation price. These two distributions have the advantage of asymmetric shape which normal distribution does not. Therefore, by assuming Weibull distribution or Gamma distribution as customer reservation price we can get rid of the problem regarding symmetric shape. Also Weibull distribution and Gamma distribution do not perform over negative numbers as normal distribution does.

- **Bivariate Random Distribution:**

Whenever we have two variables distributed over the same space, the joint probability function of the variables is called bivariate distribution. Papers which consider two products and bundle of them usually use bivariate distribution .

Switch of a customer: Whenever the firm offers more than one product or bundle customers may switch from one bundle to the other, depending on the available time, price, quality etc. For instance, if the firm offers more than two products, customers can choose any combination of the products as the bundle. Therefore, there are several number of bundles which the customer can choose, either to stay with the bundle that he used to buy or to choose a new one.

Customer Available time: Another factor that can influence customer decision is available time for consumption of the product. If we offer our product as service, bundle of service, customer reservation price is an important factor for making decision. On the other hand, since the times of production and consumption time are simultaneous for services, even if the

customer reservation price is higher than the price of the service, the customer is not going to buy the bundle if she does not have available time for consumption.

Market size: Market size is the number of potential people who might be interested in the products and the bundle. Market size can be considered differently. One way can be to assume that the number of customers in each segment is well known, in other words the number of people willing to pay is well known (as Hanson and Martin, 1990). Also it is common to assume that the market size is fixed. So the expected number of people is the probability that the customer reservation price is higher than the price times the market size.

In the following part we will review the literatures in terms of bundling. We present each paper and its main results.

3.6.1.2 Literature Review of Monopoly Market

Hanson and Martin (1990) discuss pricing strategy for all possible bundles in a single firm; it is assumed that customer reservation price is well known for each segment of the market, and customer are segmented based on clustering method, clustering is a method of data reduction by categorizing them in such a way to maximize the similarities and minimize differences. Inputs to the model are customer reservation price of each segment of the market, number of people in each segment and cost of supplying a customer of particular segment for specific bundle. Price of the bundle is additive.

Bulut et al (2009) discuss the optimal pricing under different bundling strategies; pure bundling, mixed bundling or pure component. In addition authors consider the effects of other factors including the degree of contingency (complementarily or substitutability), the level of the initial stocks and the shape of the reservation price distributions on the revenue, price and selling of the bundle. The main results demonstrate that when the initial inventory level is high and the reservation price is negatively correlated, pure bundling is the best. Also, if the initial inventory level of both products are equal and high enough, the best strategy is pure bundling, but numerical results show that if the initial inventory of the products is not equal, mixed bundling strategy performs better, because if the company follows pure bundling strategy and initial inventories are not equal, the product with higher initial inventory level will remain unsold when the other product stockouts and there are not enough items for making a bundle. Also it is shown that bundling of complementary products makes much revenue than making a bundle from substitute products.

Gürler et al (2008) tried to maximize the profit through finding the optimal number of the bundle that should be made for a finite horizon and be kept as initial stock. But the initial inventory level is fixed for both individual products. The manager should decide that how many bundles should be made from this initial inventory. The authors consider the case which there is a cost associated with bundling of two individual products. The result of Gürler et al (2008) expresses that for lower demand correlation the bundling cost has a significant effect on the number of bundles, but for highly positive correlation the bundling cost is negligible. Also, the result indicates that when the optimal price of the product are set lower than the mean value of the reservation price distribution, the company can increase the price of the bundle and take mix bundling as the main strategy. Also, the numerical study shows that as the variation in customer reservation price increases the optimal number of bundles and the total profit decrease.

Ernest and Kouvelis (1999) tried to optimize the profit through finding the best order quantity for a company offering two individual products as well as bundle of them, in case of shortage of each the bundle would be used as alternative or in case of shortage of the bundle both product can be purchased. The result shows that generally profitability are highly depends on several factors, the probability distribution and mean value as well as the price of the product, inventory holding cost etc. The result shows that there is higher profitability when there is correlation for demand both positive and negative. Positive correlation leads to increase in stocking level of products and the package as well. Another result of the paper is optimal inventory control of multi product firms sometimes increase the inventor holding costs but the total profit is always higher.

MacCardle. FKevin et al (2004) tried to find out the effect of product bundling on retailer promotion of sale. Two different classes of products have been considered fashion and basic. The best price for the bundle is determined as well as the best order quantity.

Chang and Tayi (2009) assume a situation in which a firm produces different kinds of products. The firm has some customers which have purchased different kinds of bundles in their previous shopping. Now there is possibility for them to either purchase the same bundle as before or switch to another bundle. On the other hand there are always new customers who can choose a bundle freely. Authors aim to find how this transition can affect the profit. They consider a firm offering different kinds of services. Each bundle consists of none zero number of services. Therefore, even one service is assumed to be a bundle. The results

showed that for the higher retention rate of the customers, the firm performs better. Therefore the firm should reduce the switching rate of the customers. In order to increase the retention rate of customers for a particular bundle, the company can reduce the price of that bundle.

Venkatesh and Mahajan (1993) discuss an optimal pricing model to maximize the profit; but the research mainly discusses the product in terms of service not goods. The research argues that when we offer services in a bundle apart from the reservation price, the time availability of the customer is another factor of customer behavior because even if the reservation price of the customer is higher than the price of the bundle of the service, the customer will not buy the bundle when he does not have enough time to consume the bundle. Another result from the research is in this kind of situation mixed bundling performs better than the other strategies.

Ansari et al (1996) discuss the number of items that should be included in a bundle to maximize the profit and show that the number of items included in a bundle does affect the profit. The research discusses a firm producing services. First the firm is going to determine the optimal number of items in the selling bundle in such a way to maximize profit. In the second part the research considers a non-profit firm which is going to maximize the resource utilization. Conclusion from this discussion is the fact that not only type of making the bundle affects the maximum profit but also the number of items in the bundle makes a significant effect on the final profit. The result from the research for the non-profit organization the optimal number of the item in the bundle is higher than the profit organization situation.

Ferrer J.C et al (2010) tried to maximize the profit as well but the bundle is a combination of a service and a good; the good is the same in all bundles but the service is an identical service with different level of quality so that the higher is the quality level, the higher is the cost. The service associated to each product is subscription based and offered to the customer by an entering fee for the first period and the subscription fee for other period. Authors considered the possibility of the switching of the customer by charging them higher or lower periodic fee for the service. They conclude that the optimal price in dynamic pricing only depends on the number of the customer; the firm has to change properly the admission fee and the periodic fee by the number of the customer. Periodic price is increasing over the number of the customer and the admission fee is decreasing over the number of the customer. Also Ferrer J.C et al (2010) show that by reducing the flexibility through increasing the switching cost, the customer number for each bundle grows, and it provides a higher profit for the firm.

Chang and Tayi (2009) and Ferrer J.C et al (2010) have found the same conclusion which is the fact that the less the switch of the customer the more the revenue, but they have suggested different ways to reduce the switch of the customer.

Bakos and Brynjolfsson, (1999) discuss the selling of Information material through internet as a bundle. As it was mentioned before, selling service as a bundle makes the bundle more flexible than selling tangible goods. Also, as a result Bakos and Brynjolfsson(1999) indicate that bundling of information good is profitable because mostly in these industries marginal cost is zero. Bakos and Brynjolfsson(1999) also considered the case when the marginal cost is not zero and high enough, then since bundling reduces the dispersion of the reservation price, bundling would result to reduction in the profit. Therefore based on the result when the marginal cost is high enough bundling of the products is not recommendable in these industries.

Venkatesh and Kamakura, (2003) consider a monopoly case in which the firm produces two products. The research tries to find out which bundling strategy performs better if the products are complimentary or substitute for each other. The research considers the reservation price distribution as Uniform distribution and the cost of production for both products are equal. The optimal price of the products and the bundle under different strategies are entirely depending on the maximum reservation price, the cost of production and the degree of contingency. Degree of contingency, complementarily, is defined as follows

$$\theta = \frac{r_b - (r_A + r_B)}{r_A + r_B}$$

Where r_b , r_A and r_B are reservation price of the bundle and products A and B respectively.

In addition, authors showed that highly substitutable product, and moderate substitutable products should be offered separately. If the cost of the product is high and close to maximum reservation price pure bundling is the appropriate option for complimentary product. Otherwise, if the costs of the products are moderate, complimentary products are recommended to be offered as pure bundle, also mixed bundling could be a good strategy as well. Mixed bundling is recommended for independent or weak complimentary and mixed product. Table 3-1 summarizes the characteristics of literatures reviewed.

Reference	Product	Number of Products	The Goal of the paper	Bundling strategy	Price of the bundle
<i>Hanson and Martin (1990)</i>	Goods	Many	Finding the best bundling strategy	Purebundling/compon-ent Mixed bundling	Additive
<i>Bulut et al,(2006)</i>	Goods	2 components	Pricing of the components and the bundle	Pure bundling/component	Additive/Sub additive
<i>Chang and Tayi(2009)</i>	Service	From one to infinity	Finding the optimal price of the bundle	Individual sell is considered as a bundle so the strategy is pure bundling	Variable
<i>Gurleret al,(2009)</i>	Goods	2 Components	Optimal Number of Bundle that should be formed for a finite Horizon	Mixed Bundling	additive
Venkatesh and Mahajan (1993)	Service	Many	Optimal Pricing	Pure/Component/bundle And Mixed bundling	Variable
Ernest and Kouvelis(1996)	Goods	2componens	Finding the best order quality for each component to maximize the price	Mixed strategy	Fix
Ansari et al(1996)	Service	Many	Optimal Price to maximixe Profit / utilization	Pure/Component/bundle And Mixed bundling	Variable
Ferrer., et al.(2010)	Goods and Service	Many	Optimal Pricing	Pure bundling, of a good and a service. Bundles are differ in the quality of the service	Variable
Bakos and Brynjolfsson(1999)	Service	Many	Optimal Price of the bundles	Pure Bundling	Variable/ Subadditive/supper additive
Venkatesh and Kamakura, 2003	Goods	2 Components	the best strategy for different contingency level	Pure/Component/bundle And Mixed bundling	Variable
MacCardle. Fkevin, et al, (2004)	Goods	2 components	Finding the best strategy, the best price the best order quantity	Pure component /Pure bundling	Variable

Table 3-1 summery of basic characteristics of literatures

Table 3-2 classified the literatures in terms of contingency, demand pattern, arrival pattern and reservation price.

Paper	Degree of Contingency	Demand Pattern	Arrival Pattern	Reservation Price
<i>Hanson and Martin (1990)</i>	-	Maximum Surplus	Static Model	Well known for each market segment
<i>Bulut et al,(2006)</i>	Yes	Maximum Surplus	Poission	Bivariat Normal Distribution
<i>Chang and Tayi(2009)</i>	-	Maximum Surplus	Static Model	Uniform distribution
<i>Gurleret al,(2009)</i>	Yes	Maximum Surplus	Poission	Bivariat Normal Distribution
Venkatesh and Mahajan (1993)	-	Maximum Surplus and available time	Static Model	Bivariat Normal/weibulle Distribution
Ferrer., et al.(2010)	Only Positive	Maximum Surplus	Static Model	Multi Nominal Logit Model
Bakos and Brynjolfsson(1999)	Yes	Maximum Surplus	Static Model	Uniform distribution
Venkatesh and Kamakura, 2003	Yes	Maximum Surplus	Static Model	Uniform distribution
MacCardle. Fkevin, et al, (2004)	Positive, negative, independent	Maximize surplus	Static Model	Uniform distribution

Table 3-2 comparison of the papers in terms of demand model

3.6.2 Bundling in Non-monopoly Market

The main focus under the assumption of monopoly market is determining the price in such a way to maximize revenue. In non-monopoly market each firm not only must consider customers' behavior but also should consider the competitors actions and reactions. In a non-monopoly market customers can buy different products from different firms instead of buying a all products from one firm. Firms in non-monopoly situation should make their decision in three steps. The first step is compatibility which means whether or not a firm produces its products compatible with the competitors' products. Note that we are discussing the products that are complimentary for each other i.e. they should be used together. For example in

computer industries a firm can produce an operational system compatible with all software available in the market or can make an operation system which is only compatible with the firm's software. The second decision that should be made is to define which bundling strategy should be used. And the third decision is to find the optimal price according to the strategy and the competitors' behavior.

The models and the discussions that have been carried out in this category are different from each other in terms of market structure, customer choice variability, compatibility, bundling strategy and marginal costs of products. As follow we review these factors and then we will present the significant researches and their results as well.

Market structure: The market structure for non-monopoly markets can be addressed as duopoly or oligopoly. In duopoly markets there are only two competitors but in oligopoly market more than two competitors play in the market.

Customer perceived value: Different customers can perceive the same value for a product or they can assign different values to the product.

Compatibility: It means whether or not the firms produce their products compatible with the complementary products from the other firms. For example if company A produces one computer device and company B produces another device, if the two devices are compatible, the companies are assumed to hold compatibility assumption. Compatibility was mentioned as one of the decision making stage, however, sometime it is assumed that compatibility is not a concern of decision making and the company already produces compatible products.

Bundling strategy: This stage is influenced by the compatibility decision, because if the companies would prefer not to make a compatible products, all strategies are dominated by pure bundling because no customers is going to buy components from different firms to make the bundle herself .

Marginal cost: Marginal cost can be considered as zero, equal or unequal for all the products.

3.6.2.1 Literature Review of Bundling in Non-monopoly Market

Matutes and Regibeau (1992), discusses a case in which there are two firms producing two parts of one system, Software and hardware, if the firms produce the products compatible to the products of each other then the customers can make the bundle from both firm. For example if the first company make the bundle at a lower price than the sum of each

individual product price and raise the price of both product for a individual selling, the other firm will reduce the price of the both products and tries to sell them individually; as you can see here we have a typical game theory to deal with, finally the paper tries to define which bundling strategy is the best in that situation; for this typical case of duopoly authors discuss that , if both firm produce compatible products pure bundling strategy is dominated by pure component strategy and if they produce incompatible products all bundling strategies are equivalent to pure bundling, to see the complete discussion and the proof see Matutes and Regibeau (1998) .

Another research made by Matutes and Regibeau (1992) discussed and compared the performance of mixed bundling and pure component strategies. The results indicate that each strategy affects both firms depending on the customer reservation price.

Economides (1993) also discussed the duopoly situation to clarify if using mixed bundling is the appropriate strategy. The research proofs that if one firm uses mixed bundling the reaction of the other firm is to use mixed bundling as well but finally the profit gained by both firms is lower than the situation if they both use pure component strategy.

Farrell et al (1998) discussed profitability of competing on pure bundling versus competing on component. In this research it is assumed that there are two kinds of products that are absolutely complimentary for each other and both should be used together as a system. Also there are a number of competitors in the market. The research question is which of the following strategy makes the best performance for the company. The first strategy is to produce the products in such a way that either they are not compatible with the other products produced by other firms in the market or only to use pure bundling strategy even if products are compatible. The second strategy is to produce compatible products and sell them as a single product. The research shows that the cost of production and the price of the products are important factors in determining the strategy. Also the number of competitors in the market is important as well, duopoly market or oligopoly. Also the research considers a few firms as a sample in different markets and industries and discusses these strategies for the firms and the best strategy was suggested to the company.

Liao and Tauman (2000) continued the research and discussed that even if the products from one company are compatible with products from competitors, bundling increase the consumer surplus, or at least the surplus made by bundling is not lower than the surplus made by pure component strategy. The comparison has been summarized in the table 3-3.

Reference	Market structure	Customer taste	Marginal Cost	Compatibility
Matutes and Regibeau (1992)	Duopoly	Vary	Zero	Decision to make during the game
Anderson, Leruth (1993)	Duopoly	Vary	Vary and equal for both products	Compatible
Economides (1993)	Duopoly	Vary	Zero	Compatible
Farrell et al (1998)	Oligopoly	Constant	Constant and Stochastic for different scenarios	Decision to make during the game
Liao and Tauman(2000)	Oligopoly	Vary	Vary firm to firm as well as product to product	Compatible

Table 3-3 Comparison of non-monopoly markets

3.7 Summery

In this chapter we tried to discuss the customer behavior and customer preference when we can use one of the bundling strategies. It is discussed the judgment and the customer perceived value in different situations and we show that if we can understand the customer perceived value and the customer judgment we can offer our product in such a way to increase our revenue.

Also we discuss three different bundling strategies that can be applied by a multi product firm. We also show that each strategy has advantages and disadvantages. Decision regarding the best strategy among the three strategies is a decision that should be made based on the situation in which the firm operates. The most important factors influencing the decision making are as follows:

Initial inventory level of the products: If the initial inventory is high pure bundling is the best strategy. If the initial inventory level is not very high and the initial inventory levels are not equal mixed bundling performs even better than pure bundling.

Degree of complementary or substitution: the higher complementarity leads to the higher inventory level.

Demand correlation: either positive or negative has positive impact of inventory management compare with the correlation is zero. In other words, usually there is a correlation between our products, either negative or positive, we have a lower cost of inventory compare with zero correlation which leads to a higher profit.

Customer reservation price: customer reservation price distribution and its variation have a significant effect on the total profit as well as on strategies; different reservation price distribution would result to different bundling strategies. Generally it can be concluded that as the variation in reservation price increases total profit reduces. Especially if there is a high demand correlation the impact of customer reservation price is more significant.

The other important factors are marginal cost of production, cost of forming the bundle, demand of the products, planning for a single finite horizon of infinitive horizon, customer arrival model, monopoly or non-monopoly structure of the market etc, which we cannot conclude any general rule for the mentioned item and we have to consider them case by case. Since firms differ in the mentioned elements the optimal strategy would be different.

If we consider a firm producing different kind of products and if the firm allows the customer to switch from one bundle to the other bundle then we have to think about the effect of switching rate on the overall performance. As the researches show the more is the switching rate the less is the profitability. Therefore the company facing this situation should reduce the switching rate. One of the best treatments could be charging customer with switching fee.

In chapter 4 we continue the discussion of application of RM considering substitute products.

Chapter 4

Substitute Products

4 Substitute Products

As we discussed in chapter 1, substitute products are another type of multiproduct which we study in the current thesis besides bundling products. The aim of this chapter is to provide an overview one may need to know about substitute products to conduct a research in relevant fields, or to apply RM techniques in managing substitutable products. This chapter is organized as follow; first the concept of substitute products is introduced and required terms are defined. Next, the problems associated with managing substitute products are described from a supply chain perspective as well as the significance of the problems and effects they have on profitability of firms. In consequence, a basic mathematical model will be presented accompanied with detailed demand models to capture substitutability and substitution behavior of customers. Finally the most significant and relevant literatures will be reviewed and results will be discussed.

4.1 Introduction to substitute products

It is a common experience that a customer goes to a store and expects to buy a particular product but she ends with another similar product instead. This behavior of customers is called “*substitution*”. Consequently, *substitute products* are defined as: “*goods that can be used to satisfy the same needs, one in the place of another. The buyer carries out an actual and conscious process of choice about them, which leads the buyer to prefer one to another*” (Piana. V., 2005). A few examples can make the definition more clear; a customer wants to buy a specific brand of milk, but since she cannot find it, she buys from another brand. A white shirt can be bought by a customer in case that her preferred red shirt is sold out. The different brands of milk and the white vs. red shirts in examples above are substitutes for each other. In addition, in some cases, a product can be a substitute for itself; e.g. a new developed version of software may be purchased at a higher price instead of the old version. Fresh doughnuts and old doughnuts can be substitute goods if a seller offers a discount for old doughnuts to get rid of them. The common aspect of the mentioned examples is that these substitute products satisfy the similar needs of consumers and consumers have the power to decide which product to prefer and purchase.

The definition of substitute products described above is different from classical economic view about them, to some extent. In classical textbook of microeconomics these goods can be of any types as long as consumers accept one product instead of the other. In other words, substitute products are not limited to satisfy the same needs of customers. For example a car

can be a substitute for a house for a particular customer in case of budget constraint; he/she cannot afford buying a house. To be more specific about our view of substitute products and their characteristics, it is worth to describe why consumers prefer some products to others, or why they are indifferent about some of them and willing to accept a substitute product. The fundamental concept of substitute products is derived from theory of consumer preferences which will be expressed in following section.

4.1.1 Consumer Preferences and Economics view of Substitute products

Consumers make choices every day; what car to buy, what kind of food to eat and cloths to purchase. Further, when it comes to making decision about a product from many options, it becomes even more difficult for the consumer to decide which one to purchase. Consumer's choice is also restricted by limited resources such as budget. Theory of consumer choice focuses on how consumers choose goods and services with limited resources. As we study consumer decision facing a set of different products, it is necessary first to understand how a consumer compares the desirability of different sets of goods and form the preferences. Preference is defined as *“the power or ability to choose one thing over another with the anticipation that the choice will result in greater satisfaction, greater capability or improved performance. In economics and other social sciences, preference refers to the set of assumptions relating to a real or imagined "choice" between alternatives and the possibility of rank ordering of these alternatives, based on the degree of happiness, satisfaction, gratification, enjoyment, or utility they provide”* (Kreps, 1990).

Let us consider product A and product B as two consumption goods. We will use symbol “ $>$ ” to show preferences, for example $A > B$ means a consumer prefers goods A to B. Now we can suppose that given any two consumption goods, A and B, the consumer can rank them as to their desirability. That is, the consumer can determine that one of the consumption goods is strictly better than the other e.g. $A > B$ implies that the consumer strictly prefers A to B. Consumers also can be indifferent about two goods. $A \sim B$ indicates that the consumer's satisfaction by consuming A is the same as she/he consumes B. In addition, $A \succsim B$ implies that customers see product A as preferred or indifferent to B. Using these symbols above, economists usually make some assumptions about preferences, referred as axiom of consumer preferences:

Axiom 1: preferences are Complete. We assume that any two goods can be compared. That is, given any A and B goods, we assume that $A \succ B$, or $B \succ A$, or $A \sim B$ when consumer is indifferent between the two goods.

Axiom 2: preferences are Reflexive. We assume that any good is at least as good as itself: $A \succcurlyeq A$

Axiom 3: preferences are Transitive. If $A \succcurlyeq B$ and $B \succcurlyeq C$, then we assume that $A \succcurlyeq C$. In other words, if the consumer thinks that A is at least as good as B and that B is at least as good as C, then the consumer thinks that A is at least as good as C.

Axiom 4: preferences are continuous. If A is preferred to B, then goods “sufficiently close” to A must also be preferred to B.

These axioms consequently are used to understand how a particular product is preferred to other products. Economists show preferences in terms of *utility*. The phrase product A is preferred to B means that A has higher utility than B. A utility function is a way of assigning a number to every possible consumption goods such that more-preferred goods get assigned larger numbers than less-preferred goods. That is, A is preferred to B if and only if the utility of A is larger than the utility of B. in symbols, $A \succ B$ if and only if $U(A) > U(B)$ where U denotes the utility of goods. The only property of a utility assignment that is important is how it orders goods. The utility is important only to enable us to *rank* the different consumption goods and form an ordered preference over goods. The size of the utility difference between any two consumption goods doesn't matter.

Based on classical economics, substitute products are seen as everything that can be substituted by anything as long as they provide the same utility for customers. Providing the same utility implies the concept of *perfect substitutes* which means products can be substitutes if and only if they offer the same utility for customers. This view cannot define the concept of *imperfect substitutes* which will be defined later in this section. By contrast, our view of substitution is fundamentally the same but different in some aspects. Now it is the time to explain our constrained view of substitute products and define some important concepts which will be used repeatedly through this paper. To be clear about the definition of substitute products we again define them as at the beginning of this chapter: “Substitute products are referred to goods that can be used to satisfy the same needs, one in the place of another. The buyer carries out an actual and conscious process of choice about them, which

leads the buyer to prefer one to another”. From this definition some characteristics of these substitute products can be implied as: a small set of goods which are almost similar in their functions and are to satisfy the same needs of consumers. These products are carefully compared in terms of prices and quantities, more importantly in terms of quality. Having this view, the imperfect substitutes can be products which satisfy the same needs but differ in some functionality or attributes to the others which can bring different utility to consumers e.g. a developed version of software compared to the old version.

4.1.2 Substitution Definitions

The following definitions will be used in the entire chapter repeatedly corresponding to characteristics of substitute products.

Substitution: product A is a substitute for product B when a consumer which prefers product B, buys product A instead to satisfy her needs (the reasons will be discussed in 4.2). This behavior is called substitution.

Substitution probability: the probability that a customer accepts a substitute product instead of her preferred product.

Substitute demand: The term of substitute demand is used to describe demand for a product which has arisen because that product is in some way a substitute for some other products (McGillivray and Silver, 1978).

Full substitution: full substitution is referred to the case when substitution probability is equal to 1.

Partial substitution: it is referred to the case when substitution probability is between 0 and 1.

Downward/Upward substitution: it is referred to the case when demands for a substitutable product can be satisfied only by a higher/lower class of the same product.

4.2 Problem Description

Having substitution and substitute products defined, one important question is what problems and difficulties may arise due to presence of substitution in managing a supply chain of multiple products when the goal is maximizing the expected profit through RM techniques. Optimizing expected profit of a supply chain involving different entities (producers,

distributors, retailers, etc) which works with multiple products is a bothersome task. Introducing demand correlation and substitution behavior among these products make this task even more difficult. This section sheds light on the nature of substitution and effects it may have on different levels of a supply chain optimization. Let us consider a supply chain consisting of one producer and one retailer both dealing with substitute products. For simplicity we assume a decentralized supply chain in which producer and retailer make decision independently. Each of them tries to maximize their expected revenue. Figure 4.1 shows the problem from a broad view. There are three sequential stages of decision making both for the retailer and producer.

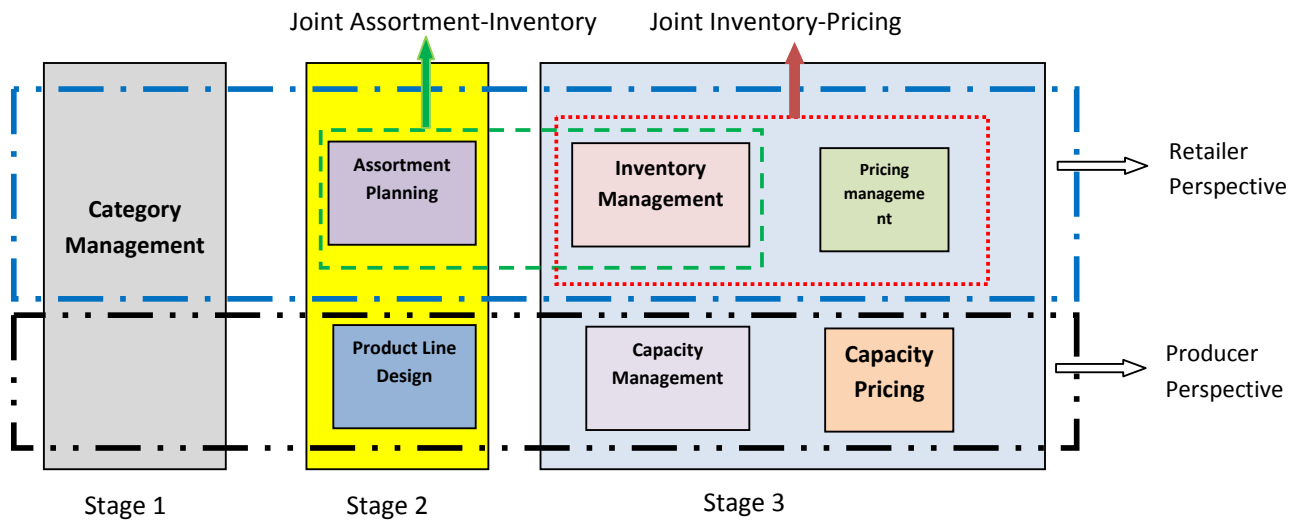


Figure 4.1 A three-stage decision making framework for substitute products

At stage 1, before producing and starting a product line a producer has to decide in which category of products she tends to produce, such as home appliance, food and beverage, apparel, etc. it may be required to go even further in one category, for example producing TV series and digital cameras under category of digital electronic devices. On the other hand, a retailer also has to make a clear decision about the different categories of products she needs to stock and sell in her store. We call this level of decision making as “*product category management*”. There is a large stream of research in managing categories of products and effects of category management on firm’s profit and consumer satisfaction in marketing literatures, for example see Chong et al (2001), but as products from different categories are usually different in their nature and usages and therefore substitution effect has no influence

at this stage, at least by our definition of substitution. We do not go through this problem more in detail, but focus on stage 2 and 3. To be more specific, we describe the problem at stage 2 and 3 from producer and retailer's perspective individually.

Producer's perspective: at stage 2 and after making the decision about the product category or family to be produced, the producer has to focus on the variety of the selected category in her product line. For example she will decide to produce shirts with the same design but five different colors in four sizes. Someone may think about producing product in different classes, but fundamentally the same; e.g. different classes of printers. Usually a product line has the capability of producing several similar products within a product family. The question is to select products which ensure highest revenue from the product line. This problem is commonly called "*product line design (PLD)*" problem in literatures.

Next, at stage 3, since a product line usually has a restricted amount of capacity, producer has to allocate her product line capacity to demand for different product which arrives over time. This is called "*capacity-sizing decision (capacity management)*". Usually products with higher profit margin and market share or products with higher priority are allocated with more capacity. Many companies try to increase their market shares, therefore they expand their product lines by introducing improved versions of their existing products, as mentioned a new improved version of a cell phone or software. These new products have the potential to be substituted for sales of the existing products. So it is important to control the capacity allocated to new and old products and their price. If new product is priced close to the old version then customers switch to new version and remaining inventory of the old version is salvaged. Thus, at this stage *capacity pricing* also plays a significant role in capacity allocation and lead time management. Firms usually charge higher price for high priority demands and lower price for the demands which have to wait for capacity and those which have long lead times. If a customer's demand cannot be satisfied by her preferred product, usually firms serve customers with the same products from higher or lower class.

Effect of substitutability here concerns more the supply side than the demand side as the power and control for substitution behavior and substitute products to be produced are under decision of a firm, this kind of substitution is referred as "*supplier driven substitution*" and sometimes the term "*cannibalization effects*" is used instead of substitution effects. These products are also referred as "*substitutes in production*".

Retailer's perspective: products stocked in a store, Stock Keeping Units (SKUs), are usually from different categories of products, such as meats, breads, dairy, etc in a grocery store or digital cameras, and TV series in a home appliance store. These products can be divided into subcategories such as different types of digital cameras differing in the quality, brand, usage, etc. Even these subcategories can be decomposed into more detail, for example to different colors. These products within subcategories are mutually similar but differ in one or some attributes as mentioned, like quality, size, brand, color, etc. These products are referred as “*substitutes in consumption*”. After deciding about different categories of products to stock in the store, at stage 2, a retailer has to decide on selecting the optimal set of products within subcategories which maximize the total profit. In fact the retailer should determine the optimal assortment of product variants to stock for each category of products. This problem is commonly called “*assortment planning*” in literatures. In making assortment decision of which products to stock, retailers must compare the expected increase in revenue that an additional item can generate with its additional costs of presentation, logistics and inventory investment.

Further, at stage 3, the retailer should incorporate inventory management and pricing of these substitute products to gain highest expected profit. Inventories of substitute products have to be decided jointly since demand for one product depends on inventory levels of other products within category. We refer to this level as “*inventory management*” of substitute products shown in figure 4.1. As the inventory levels of substitute products are determined, now it is the time to price them to control substitution behavior and to influence inventory of stocked products through fixed or dynamic pricing and to ascertain maximum expected profit which we call it “*pricing management*”.

From figure 4.1 and discussion above, following results can be extracted:

First, we can divide substitute products in two categories. Products can be substitutes in terms of consumption or production.

- ***Substitutes-in-Consumption:*** Two or more goods that satisfy the same needs of consumers and Consuming one good means that buyers have no need to consume another.

- ***Substitutes-in-Production:*** Two or more goods that are produced using the same resources and partially satisfy the same needs for consumers. Producing one good prevents sellers from using resources to produce another (cannibalization).

Second, we distinguish three levels of decision in managing substitute products. Now based on these levels we can introduce main sources of substitution. Three sources of substitution can be identified: assortment-based substitution, stockout-based (inventory based) substitution, and price-based substitution. We define them as follows:

- 1) ***Assortment-based substitution:*** a customer identifies a favorite product based on advertisements or past shops, but she cannot find it in a particular store because they do not carry it, thus she buys another product.
- 2) ***Stock-out based substitution:*** it occurs in a situation where a customer is willing to buy a certain product but she finds it is stocked out, and then she buys another product as a substitute.
- 3) ***Price-based substitution:*** it occurs in a situation a retailer uses differential pricing to make certain products to be substitutable. The customer has a favorite choice but because of price switch to another product.

Moreover, we mentioned the term “*supplier driven substitution*”, which on the other hand is “*customer driven substitution*”. In customer driven substitution, customers have the ability to choose among substitute products or in some case not to purchase. Focus of this chapter is on substitution behavior driven by customers rather than firms and therefore products which are substitute in consumption. We made a clear distinction between different levels of decision making dealing with substitute products, however, in many cases and literatures some of these decisions are made in a joint fashion. For example, most of literatures consider joint assortment-inventory problem or joint product line-pricing problem. In some literatures also the problem of joint inventory-pricing decision is considered. Later we discuss these literatures further in detail.

To finish this part it is worth to mention that many literatures categorize their study into two streams of work with regard to substitution; “*static substitution*”, where only assortment-based substitution can occur, and “*dynamic substitution*”, where both assortment and stock-out based substitution are considered.

4.3 Significance of Substitute products problems

In this section significance and difficulties of managing substitute products will be discussed. Here also we take a level by level approach and argue the task in hand based on three decision making levels explained in previous section. These decision making levels exactly correspond to three decisions we addressed in chapter 2 in applying RM.

4.3.1 Assortment planning problem with substitute products

An assortment is a set of products that a retailer offers to be sold in each of her stores at each point in time. It is one of the most basic and strategic decisions each retailer has to make. Retailers usually face a tradeoff in planning for their assortment. This tradeoff is between three elements. They have to make a balance among how many different categories they need to carry (variety or retailer's breadth), how many stock-keeping-units within these categories (depth) and how much inventory they have to stock for each (service level), see K  k et al (2006) for a recent review of assortment planning with both substitute and without substitute products.

Assortment planning has a great impact on sales and gross margin of a retailer. A broad product variety has two advantages for retailers: first, customers have different preferences and therefore a wider range of variants in an assortment appeals a larger portion of customers. Second, a wider product variation allows a retailer to price discriminate among different customers. While customers with higher willingness to pay can be served by products with additional features or improved quality and performance, and retailer can extract more revenue from them, a retailer also does not lose customers who have a lower willingness to pay by providing lower quality products. On the other hand, this broad product variety creates additional problems for forecasting and inventory management. When a variant is added to the assortment, it increases the probability that a customer purchase from the assortment, but including more choice alternatives reduces the volume of demand for each individual variant. This fragmenting of total demand increase the demand variability of each variant, which in turn tends to increase inventory cost, also adding a variant to the assortment may involve a fixed cost.

Assortment planning under substitution forms one of the core problem domains in revenue management, and many variants of these problems have been studied extensively in the literature. Effect of substitutability on assortment planning is studied by Stassen et al (2002) and is illustrated in figures 4.2 to 4.5.

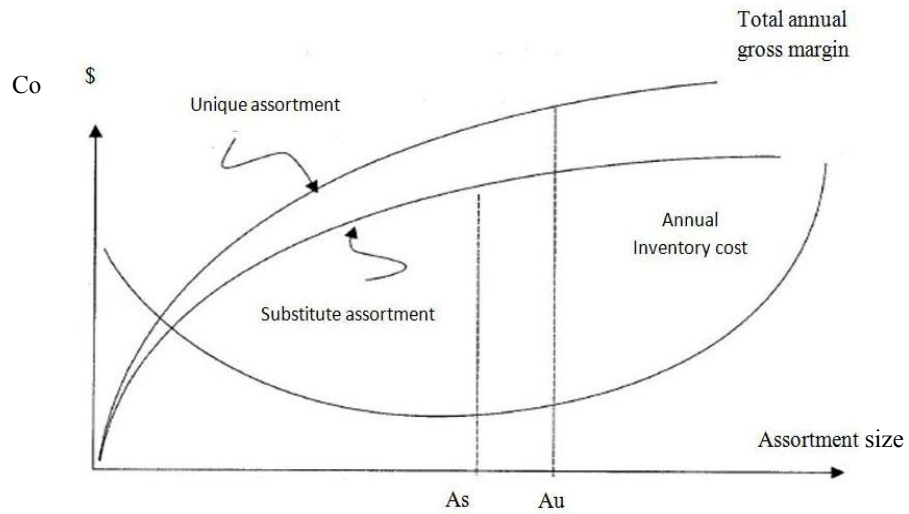


Figure 4.2 Optimal depth of unique Vs substitute assortments

Source: Stassen et al (2002)

Note:

*Au: Optimal assortment size with unique products

*As: Optimal assortment size with substitute products

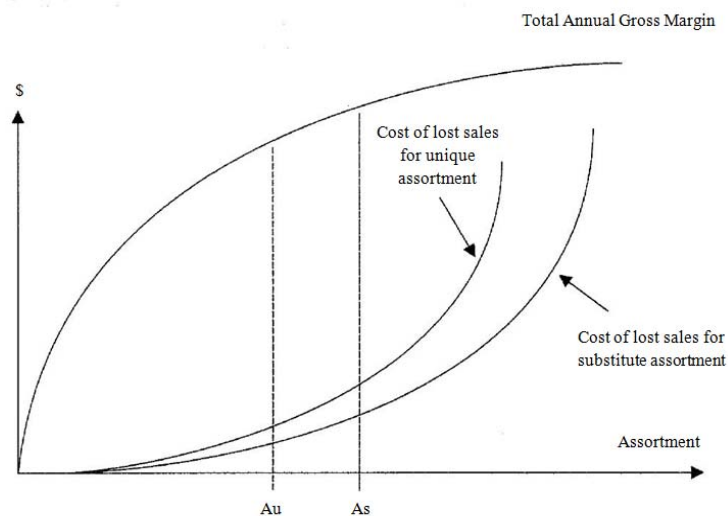


Figure 4.3 Cost of lost sales due to stockouts

Source: Stassen et al (2002)

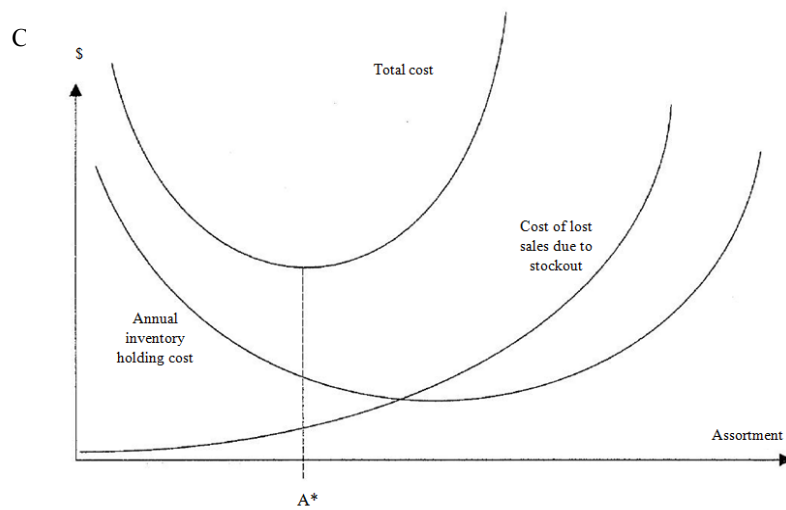


Figure 4.4 Cost of lost sales due to stockouts and Inventory cost

Source: Stassen et al (2002)

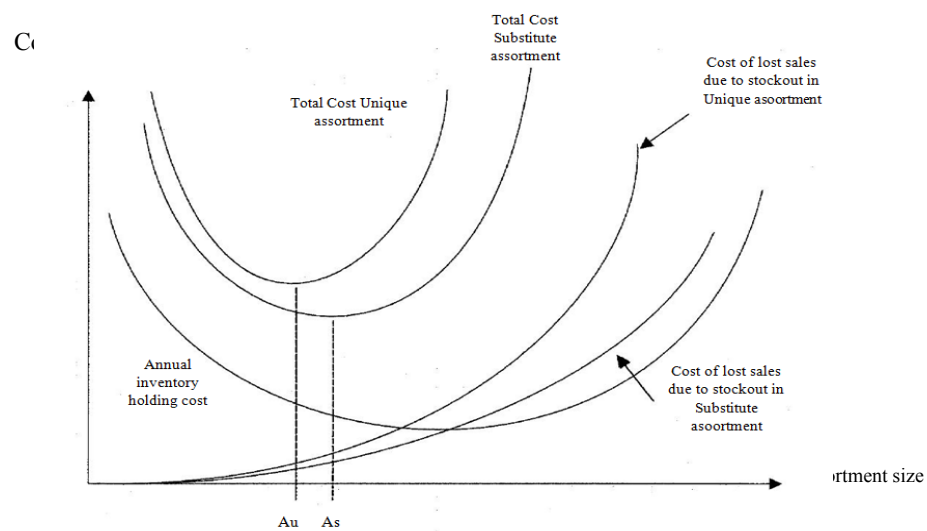


Figure 4.5 Unique assortment VS Substitute assortment

Source: Stassen et al (2002)

Figure 4.2 shows the relationship between assortment size, inventory holding cost and total gross margin for both unique and substitute assortment. The total gross margin is the sum of each variant's gross margin contribution. As Figure 4.2 illustrates, there are two functions of gross margin, one for products with low degree of substitutability or unique products, the other for products with high degree of substitutability or substitute products. Unique items can bring more margins either through new or current customers but substitute products bring lower gross margin because additional unit volume reduces the sales and margins of existing items. Gross margins have a positive slope and are concave. The holding cost function has the same effect on unit volume for both substitute and unique products. As assortment increases, the holding cost decrease because it reduces the holding capacity per item and the ability to increase unit sales. However, if the assortment continues to increase, the holding cost begin to increase as well because more slow movers are added to the category and total category sales increases at a decreasing rate (Stassen et al, 2002). It is clear that optimal assortment with substitute products carry less items than unique product assortment. Figure 4.3 shows cost of lost sales due to stockout for both unique and substitute products. Given a fixed category space, increasing the assortment decreases the capacity for existing items which results in increasing probability of stockouts for all items and lost sales. This figure illustrates that the optimal level of assortment with unique products is lower than for substitute products. Figure 4.4 shows total cost of an assortment, the sum of inventory holding cost and lost sales. In figure 4.5 we combine the total cost and gross margin of assortments for both unique and substitute products. It implies that if the lower expected cost of lost sales in the assortment with substitute products is greater than the reduction in total gross margin that will be received as a result of substitutability, then the optimal level of assortment will increase; otherwise it will decrease (Stassen et al, 2002).

The common approach in academic researches regarding assortment planning is to formulate an optimization problem and to decide jointly multiproduct assortment and inventory decision, which usually give rise to complex optimization models that are computationally challenging. However, retailers mostly carry different assortment at each stores, academic literatures have focused mostly on single assortment in a single store. Within this single assortment, products are only quality-differentiated and from the same generic type. They satisfy essentially the same customer need, and usually differ from each other due to existence or non-existence of a feature, or the performance of the products with respect to some measure.

4.3.2 Inventory management problem of substitute products

When a retailer sells similar products, she has to be aware of substitution effects among them. In presence of substitution, demand for a particular product is not only influenced by its own characteristics, but also the inventory level of products with similar characteristics (Rajaram and Tang, 2001). It implies that the observed sale of an item is no longer equal to its core consumer demand (the case where all items are available). Sales of items which are stockout can be underestimated because they only can be met to the extent of available stock; on the other hand sales of other items can be overestimated since a portion of demand comes from consumer substitution. “Assume there are 50 units of product A and 30 units of product B at a store. Assume further, B is sold out soon and A is sold out after a while since A is more in number. Then the result you get is that 50 of A and 30 of B are sold. However the product that really sells is B”, stated by Mr. Toshifumi Suzuki, chief executive of Seven & Eleven of Japan to demonstrated a possible biased interpretation if substitution is ignored (Anupindi et al 1998). As a result, forecasting the accurate demand for each product and determining its right order quantity to assure maximum profit is difficult for retailers. Retailers should incorporate inventory decisions of all products when selling substitute products. For example, one may expect to reduce the safety stock of a particular item since she can count on other items to be sold as substitute in case of high demand or by adjusting the price she make some products substitute for others to avoid stock-outs. It may lead to lower stocking costs. Reality shows retailers frequently buy too little volume of some products, which results in lost sales and profit margins, and too much of other products whose prices then are marked down at the end of season (Karakul and Chan, 2008). Only these inventory related costs is count for \$25 billion a year in the U.S retail industry (Frazier, 1986).

4.3.3 Pricing problem of substitute products

Pricing in a multi-product competitive market environment is a difficult task. Adding substitution among products makes it even more complex. In this environment, price competition exists between a firm and its competitor's products. Also this pricing challenge exists between different product variants belonging to the same company. When different product variants are substitutable, demand for certain products does not only depend on their own price, but also depends on the price of other products. This occurs, for example, when customers are willing to substitute their favorite quality and style product for a cheaper one they can afford. Customers buy their preferred product as long as the product price charged by the company reflects their perceived value of the product. Otherwise, customers may

switch to a lower-priced substitute product offered by the same company or a competitor. Therefore the demand of different variants can be changed or controlled via differential pricing. An increase in the price of one substitute good causes an increase in demand for the other. A decrease in the price of one substitute good causes a decrease in demand for the other. Economists call this the cross-price effect. Cross price elasticity for two substitutes will be positive. Firms, instead of charging the same price for all variants, charge different prices to manipulate the product demand to maximize their profit. Thus firms should estimate the cross-price elasticity of product variants to predict the effect on the quantity demanded and total revenue of them. Moreover pricing of substitute products must interact to the inventory levels of them. When a firm faces inventory scarcity for a particular product, intuition suggests increasing its price and reducing the price of substitutes. Joint optimizing of price and inventory attracted many scholars in recent years, see Chan et al (2004) for a comprehensive review. As research by AMR (2001) suggests that if pricing decision is integrated into capacity and inventory decision, it can increase the expected profit by \$90 billion in U.S manufacturing industry alone.

4.4 Problem Modeling

In this part we aim to model the problem of involving substitution in both inventory-pricing management and assortment planning. We make the model with basic assumption usually made in literatures. The common way of modeling perishable products or one period inventory management is to use newsvendor model to maximize the expected profit or minimize total cost over a finite horizon. As our focus is on RM based models, the model is a profit maximization model. Many scholars use newsvendor models to find optimal inventory. On the other hand several papers consider pricing decision using newsvendor model for single product, see Petruzzi and Dada (1999) for a review of models. Models with additional complexities and assumptions will be studied in literature review.

Model assumptions:

- N is a set of possible products to be selected in a assortment and $N = \{1, 2, \dots, n\}$.
- S is the set of products stocked in the store and $S \subset N$.
- π_i is the expected profit from product i
- P is the vector of prices and $P = (p_1, p_2, \dots, p_n)$.

- C is the vector of costs so that $C = (c_1, c_2, \dots, c_n)$
- d_i is the demand for product i . In other words, number of customers choosing product i and $D = (d_1, d_2, \dots, d_n)$
- x_i denote the number of units of products i stocked by the store and $X = (x_1, x_2, \dots, x_n)$
- There is a zero lead time for products and no restriction exists in supply process.

The retailer's problem is threefold:

1. To find optimal order quantities for substitute products (x_i^*)
2. To set optimal prices for substitute products (p_i^*)
3. To determine the optimal set of products to include in the assortment (S^*)

The retailer's problem can be modeled as follow:

A retailer must maximize the total profit obtained from each product variants:

$$\text{Max} \sum_{i=1, \dots, n} \pi_i$$

$$\pi_i = E[p_i \min\{x_i, d_i\} - c_i x_i]$$

Hence, for a given assortment S including a set of products, the expected profit $\pi(S)$ is:

$$\pi(S) = \sum_{i \in S} E[p_i \min\{x_i, d_i\} - c_i x_i]$$

Further to determine the optimal assortment, we have to seek to find the optimal S yielding the maximum total expected profit:

$$\pi^*(S) = \pi(S^*) = \text{Max}_{S \subset N} \text{Max} \pi(S)$$

The model can be seen as a multi-item newsvendor model; however, pricing and assortment decision bring additional complexity to the model. It can be seen from the model that it requires two levels of problem solving. First the optimal stock level of a given assortment S should be determined as well as the products' prices if prices are to be determined endogenously. The expected profit that S yields is determined as well. Then in the next level the retailer must select the best S (S^*) among different set of assortments which yields the highest profit. Solving these three problems simultaneously is very difficult and in many

cases impossible. Therefore in most literatures, it is aimed to solve one or two of these problems and assume the other remaining problem to be exogenously determined. As mentioned before this model describe a very simple model, however there are many possible extensions to this model which make it more complex but realistic. Multiple periods, dynamic assortment, dynamic pricing, etc are other desirable extensions.

So far we have modeled the problems in a very basic form, but the most important question yet to be answered is how substitution is integrated into the model described above. The most significant part of the above model is the underlying demand function (D) . It is the demand function which defines how substitution occurs in the model and how solution approach should be selected. The following section discusses the demand processes which are commonly used in substitute products literatures.

4.4.1 Demand models

The most important function of economics and operations management models is the demand function. This importance is increased when we model a multi-product problem which there is correlation among them as well as other effects such as substitutability or complementarity.

Here we first explain general demand model which describes consumer's choice among different substitute products $N = \{1, 2, \dots, n\}$. These products are differentiated in their attributes. We defined q_i as the probability that an arriving customer choose product i from set N . Probability of no purchase is then $q_0 = 1 - \sum_{i \in N} q_i$. q_i is also referred as the market share of product i and $\sum_{i=1}^n q_i \leq 1$.

The demand for a particular product i is:

$$d_i = q_i \Lambda + \varepsilon_i$$

where Λ is a random variable with finite mean and variance, representing the volume of potential customers. ε is the vector of random error terms which has zero mean and finite variance which reflects the demand uncertainty for products i . This demand model consists of two components: a controllable part q_i which is usually sensitive to price, attribute or other controllable variables (will be described in 4.4.2) and a stochastic part (Λ and ε) which are independent. The expected demand for product i is proportional to its market share q_i . In the literature it is common to model the demand randomness in two ways:

1- Additive randomness model:

$$d_i = q_i\Lambda + \varepsilon_i$$

where ε_i is a zero-mean value which does not depend on price, or other attributes of products and only shifts the demand randomly about its mean. The potential problem of this demand model is demand could be negative if the variance of ε_i is large and $q_i\Lambda$ is small relatively, therefore cautions should be made when using it for products that coefficient of variation of their demand uncertainty is high.

2- Multiplicative randomness model:

$$d_i = q_i\Lambda\varepsilon_i$$

where ε is nonnegative value, usually is set to 1, and it only scales the mean demand by a random value.

Using either of the above models, the main difference among demand functions is the way they find q_i , the market share of products, in an appropriate way to incorporate different aspects of consumer choice, such as price sensitivity, substitution behavior, etc. therefore it is useful to understand the structure of consumer choice and components of choice probabilities.

4.4.2 Consumer choice

Understanding the nature of consumer behavior facing discrete choice has a great significance in marketing and operation research specially literatures in multiproduct revenue management which is the focus of this study. Discrete choice models are statistical procedures that model choices made by people among a finite set of alternatives. Discrete choice models statistically relate the choice made by each person to the attributes of the person and the attributes of the alternatives available to the person. For example, the choice of which car a person buys is statistically related to the person's income and age as well as to price, fuel efficiency, size, and other attributes of each available car. The models estimate the probability that a person chooses a particular alternative.

Choice Set:

The choice set is the set of alternatives that are available to the person. For a discrete choice model, the choice set must meet three requirements:

1. The set of alternatives must be *exhaustive*, meaning that the set includes all possible alternatives. This requirement implies that the person necessarily does choose an alternative from the set.
2. The alternatives must be *mutually exclusive*, meaning that choosing one alternative means not choosing any other alternatives. This requirement implies that the person chooses only one alternative from the set.
3. The set must contain a *finite* number of alternatives, meaning that there are a countable number of alternatives in the set. This third requirement distinguishes discrete choice analysis from regression analysis in which the dependent variable can (theoretically) take an infinite number of values.

Consumer Utility:

Here we show that the discrete choice models can be derived from utility theory. Random-utility models are based on a probabilistic model of individual customer's utility.

This derivation is useful for four reasons:

1. It gives a precise meaning to the probabilities q_{ni}
2. Heterogeneity of customers' preferences can be accounted by probabilistic models.
3. A firm can model unobserved variables which affect the customer's choice and make it uncertain for the firm (alternatives such as customer's budget).
4. It can model variety-seeking behavior of customers and their preference changes over time.

U_{ni} is the utility that person n obtains from choosing alternative i . The behavior of the person is utility-maximizing: person n chooses the alternative that provides the highest utility. Dummy variable y_{ni} shows the choice of each customer for each alternative as:

$$y_{ni} = \begin{cases} 1, & \text{if } U_{ni} > U_{nj} \forall j \neq i \\ 0, & \text{otherwise} \end{cases}$$

The person's choice depends on many factors, some of which can be observed and some of which cannot. The utility that the person obtains from choosing an alternative is decomposed into a part that depends on variables that can be observed and a part that depends on variables that cannot be observed. In a linear form, this decomposition is expressed as:

$$U_{ni} = \beta z_{ni} + \varepsilon_{ni}$$

Where:

z_{ni} indicates observed variables relating to alternative i for person n that depends on attributes of the alternative, such as price, quality, brand.

β is a corresponding coefficients for observed variables, and

ε_{ni} captures the impact of all unobserved factors that affect the person's choice, such as customer's budget.

Therefore the probability of choosing alternative i by person n is :

$$q_{ni} = Pr(y_{ni} = 1) = Pr(U_{ni} > U_{nj}, \quad \forall j \neq i)$$

Different random utility models arise from different distributions of ε_{ni} for all i and different treatments of β models take many forms, including: Binary Logit, Binary Probit, Multinomial Logit, Conditional Logit, Multinomial Probit, Nested Logit, Generalized Extreme Value Models, Mixed Logit, and Exploded Logit. For more information see Talluri and van Ryzin (2004). In section 4.4.3 we will explain Multinomial Logit model which is frequently used in consumer choice modeling. MNL is useful mostly in case of static substitution. In section 4.4.4 another type of demand function, exogenous demand, which is common in substitute demand modeling will be described. In this type of modeling the choice probability of consumers is given and only substitution behavior of consumers should be incorporated in the function. This type of modeling is common in stockout based substitution.

4.4.3 Multinomial Logit demand model

The Multinomial Logit (MNL) model is a form of random-utility model which is commonly used in economics and marketing literatures. In MNL each customer associates a utility U_j

with each alternative in the set, $j \in S$. This utility is decomposed into two parts, the deterministic component of the utility u_j and a random component ε_j .

$$U_j = u_j + \varepsilon_j$$

It is assumed that the component ε_j is i.i.d random variables with a Gumbel random variable, also known as Double Exponential. Therefore while the expected utility of an alternative is the same for all customers, the actual utility is different for them as a result of ε_j . The cumulative density function of ε_j is:

$$F\{\varepsilon_j \leq X\} = e^{-e^{-(\frac{x}{\mu} + \gamma)}}$$

where γ is Euler's constant (0.57722). The mean value of ε_j is zero, and the variance is $\mu^2\pi^2/6$. μ is the scale parameter. The higher is μ the higher is degree of heterogeneity among the customers. An individual chooses the product with the highest utility among the set of available options. The Gumbel distribution provides some useful analytical properties. The Gumbel distribution is closed under maximization, meaning that the distribution of the maximum of independent Gumbel random variables with the same scale parameter μ is also a Gumbel random variable.

The probability that an individual chooses product j from S is

$$q_j(S) = \Pr\{U_j = \max_{i \in S \cup \{0\}} (U_i)\}$$

It can be shown that:

$$q_j(S) = \frac{e^{u_j/\mu}}{\sum_{i \in S \cup \{0\}} e^{u_i/\mu}}$$

See Anderson et al. (1992) for a proof, and Van Ryzin and Mahajan (1999) for more details on its relation to other choice models.

The restrictive property of the MNL model is known as its Independence of Irrelevant Alternatives (IIA) property. This property holds if the ratio of choice probabilities of two

alternatives is independent of the other alternatives in the choice process. Formally, this property is:

For all $S \subset N$, $T \subset N$, and two alternative i and j :

$$\frac{q_i(S)}{q_j(S)} = \frac{q_i(T)}{q_j(T)}$$

IIA property is not realistic in cases where the choice set is decomposed to some subgroups and products within a subgroup are more similar with each other than with products outside the subgroup. It says that the relative likelihood of choosing i and j is independent of the choice set containing these alternatives. The famous example of “blue bus/red bus paradox” from K  k et al (2006) illustrates this property. Consider an individual going to work and has the same probability of using his or her car or of taking the bus: $\Pr\{car\} = \Pr\{bus\} = 1/2$. Suppose now that there are two buses available that are identical except for their color, red or blue. Assume that the individual is indifferent about the color of the bus he or she takes: The choice set is $\{car, red\ bus, blue\ bus\}$. One would intuitively expect that $\Pr\{car\} = 1/2$ and $\Pr\{red\ bus\} = \Pr\{blue\ bus\} = 1/4$. However, the MNL model implies that $\Pr\{car\} = \Pr\{red\ bus\} = \Pr\{blue\ bus\} = 1/3$.

The Nested Logit Model is one way to deal with the IIA property. It is introduced by Ben-Akiva (1985). Choice process involves two step; first choice among subgroups i.e. brands, second choice within a subgroup i.e. color or taste. The choice set N is partitioned into subsets $N_l, l = 1, \dots, m$ such that $\bigcup_{l=1}^m N_l = N$ and $N_l \cap N_k = \emptyset$ for any l and k . The utility from the choice within subset N_l is also Gumbel distributed with mean $\mu \ln \sum_{j \in N_l} e^{u_j/\mu}$ and the scale parameter μ . As a result, the choice process between the subsets follows the MNL model as well and the probability that a consumer chooses variant j in subset N_l is $q_j(N) = q_{nl}(N) \times q_j(N_l)$.

An important shortcoming of MNL model is related to substitution between different products. MNL model in its simplest form is unable to capture an important characteristic of the substitution behavior, stockout based substitution. In MNL model utility of the no-purchase option with respect to the utility of the products in S determines the rate of substitution. As an example consider a set of two products 1 and 2. $\mu = 1$ and $u_0 = u_1 = u_2$.

Probability of choosing option i is $\exp(u_i) / (\exp(u_0) + \exp(u_1) + \exp(u_2)) = 1/3$ for $i = 0, 1, 2$. Thus, two thirds of the customers are willing to make a purchase from the category. If the second product is unavailable, the probability of her choosing the first product is $\exp(u_1) / (\exp(u_0) + \exp(u_1)) = 1/2$. That is, half of the consumers whose favorite is stockout will switch to the other product as a substitute and the other half will prefer no-purchase alternative to the other product. As we see, the average substitution rate is $1/2$. This rate depends on products' u_i . One way to control the substitution rate is using varying u_0 .

Locational Choice Model:

The Locational Choice model was introduced by Hotelling (1929) to study the pricing and location decisions of competing firms. Lancaster (1966, 1975) extended the Locational model of consumer choice behavior. In his model, products are viewed as a bundle of their characteristics (attributes) and each product can be represented as a vector in the characteristics space. The components of each product indicate amount of each characteristic that product involves. For example, a camera can be defined as its image quality, speed, size and reliability. Each individual choose a point in the characteristics space, which corresponds to his or her most preferred combination of characteristics.

The utility of variant j to the consumer is:

$$u_j = k - p_j - g(y, z_j)$$

where z_j denote the location of variant j in attribute space R . y is the customer's ideal product. k is a positive constant, p_j is the price, and $g: R \rightarrow R$ is a distance function, representing the disutility associated with the distance from the consumer's ideal point. The consumer chooses the variant that gives him or her maximum utility.

4.4.4 Exogenous Demand Model

In exogenous demand models the demand for each product is specified and customer's substitution behavior is determined. Therefore the model does not require a random utility model to generate demand for products. This is the most commonly used demand model in the literature of inventory management for substitutable products. The following assumptions show the characteristics of the choice behavior of customers, see K  k et al (2006).

In exogenous demand model it is assumed that every customer chooses her favorite variant j from the set N with probability q_j and $\sum_{j \in N \cup \{0\}} q_j = 1$.

q_j is deterministic and therefore demand for product j in case of no substitution is:

$$D_j = q_j \cdot D, \text{ where } D \text{ is the total demand for all products.}$$

In case of substitution, if the favorite product is not available for any reason, with probability δ customer chooses a second favorite and with probability $1 - \delta$ she decide not to purchase. The probability of substituting product j for k is α_{kj} . Demand for product j considering both initial deterministic demand and demand from substitution is:

$$D_j = q_j \cdot D + \alpha_{kj} [q_k \cdot D - x_k]^+, \quad \text{and } [a]^+ = \max \{0, a\}$$

x_k is the order quantity of product k .

α_{kj} is specified by a substitution probability matrix. The substitution matrix may take different forms to account for different probabilistic mechanisms. For example the matrix can have the same value for all substitution probabilities to account for random substitution behavior, or values can be such that only substitution with the most similar products is allowed (other values are zero) which is called adjacent substitution matrix. Examples below show some different structure of substitution matrix for a four-product case:

Random substitution matrix:

$$\begin{bmatrix} 0 & \frac{\delta}{3} & \frac{\delta}{3} & \frac{\delta}{3} \\ \frac{\delta}{3} & 0 & \frac{\delta}{3} & \frac{\delta}{3} \\ \frac{\delta}{3} & \frac{\delta}{3} & 0 & \frac{\delta}{3} \\ \frac{\delta}{3} & \frac{\delta}{3} & \frac{\delta}{3} & 0 \end{bmatrix}$$

Adjacent substitution matrix:

$$\begin{bmatrix} 0 & \delta & 0 & 0 \\ \frac{\delta}{2} & 0 & \frac{\delta}{2} & 0 \\ 0 & \frac{\delta}{2} & 0 & \frac{\delta}{2} \\ 0 & 0 & \delta & 0 \end{bmatrix}$$

Proportional substitution matrix:

Note: $\lambda = d_1 + d_2 + d_3 + d_4$

$$\begin{bmatrix} 0 & \frac{\delta d_2}{(\lambda - d_1)} & \frac{\delta d_3}{(\lambda - d_1)} & \frac{\delta d_4}{(\lambda - d_1)} \\ \frac{\delta d_1}{(\lambda - d_2)} & 0 & \frac{\delta d_3}{(\lambda - d_2)} & \frac{\delta d_4}{(\lambda - d_2)} \\ \frac{\delta d_1}{(\lambda - d_3)} & \frac{\delta d_2}{(\lambda - d_3)} & 0 & \frac{\delta d_4}{(\lambda - d_3)} \\ \frac{\delta d_1}{(\lambda - d_4)} & \frac{\delta d_2}{(\lambda - d_4)} & \frac{\delta d_3}{(\lambda - d_4)} & 0 \end{bmatrix}$$

In the proportional substitution matrix, the general expression for α_{kj} is:

$$\alpha_{kj} = \delta \frac{d_j}{\sum_{l \in N \setminus \{k\}} d_l}$$

It implies that the probability of choosing more popular products is higher than unpopular products. It is interesting since the proportional substitution matrix has a very similar property with utility-based models such as the MNL model; $\alpha_{kj} > \alpha_{kl}$ if $d_j > d_l$. If a store doesn't carry the whole assortment, then realized substitution rate from variant k to other products is:

$$\sum_{j \in S} \alpha_{kj} = \delta \sum_{j \in S} d_j / \sum_{l \in N \setminus \{k\}} d_l$$

which is increasing in the set S . This means that consumer's willingness to substitute increases as the set of potential substitutes increase.

What if the customer's first substitute choice is also not available? If several substitution attempts are allowed, consumers may repeat the same procedure for choosing another substitute or may stop buying. The lost sales probability $1 - \delta$ and the substitution probabilities can be the same for each substitution attempt or may be defined differently for each substitution attempt.

As it is obvious exogenous model can better capture the different substitution behavior. MNL model cannot account for adjacent substitution, one-product substitution, or within subgroup substitution but it has the capability to show how consumers choose from a set. Exogenous models on the other hand can differ among products within a subgroup and across subgroups by different substitution rates. Therefore MNL model is better for assortment based and static substitution and exogenous models are better for dynamic substitution problems.

4.5 Literature Review:

The aim of this section is to review recent and the most significant literatures concerning managing substitute products. To review papers we group literatures to four groups to be easy to follow. In the first group we study literatures concerning inventory management of substitute products, either general inventory management or newsvendor models. In the second group first we briefly review product line design (PLD) problem literatures and consequently literatures involving assortment planning problem will be discussed. In the third group papers concerning dynamic pricing of substitute product will be reviewed, and in the forth group we review papers which study joint inventory-pricing of substitute products. There may be some subgroups within each group, if appropriate, to further classify literatures. While reviewing each paper we describe the problem, the assumptions made, the structure of the model and solving approach. Important parts of models, particularly demand functions, may be discussed more in detail and finally results will be highlighted.

At the end of this part we will make a classification table for papers based on characteristics below:

Length of time horizon: Problems are usually modeled using a single period (newsvendor) problem, however some papers assume multiple periods. We distinguish the length of horizon while reviewing papers.

Price: Price of products can be categorized based on two terms. Prices can be exogenously given, for example determined by the competitive market, or to be endogenously determined through optimization. Price also can be remain fixed through selling horizon or dynamically changed as a function of inventory, demand, etc. Adding pricing problem to the models makes them more complex.

Costs: Commonly there is a fixed procurement cost for products. This cost may be equal for all products variants or not. Also some papers consider holding and shortage cost in their models, but many of them ignore holding and shortage costs as well as salvage value for overstocked products. We will distinguish these characteristics of models as well.

Supply process: Most of papers assume no restriction on the supply process. Such restriction may involve limiting the number of variants included in the assortment or shelf space restriction since in long run they can be relaxed at some costs. It is common to have zero lead

time for inventory replenishment if inventory replenishment and dynamic assortment is allowed. When appropriate, we note these assumptions in literatures.

Substitution: The importance of substitution type is remarkable in modeling since it usually determines the primary demand function and solving approach. We note substitution type and all assumption made in substitution behavior of consumers.

Demand function: The most important distinction of demand type is whether it is deterministic or stochastic. In deterministic model, demand is a known function of some parameters like price. But in stochastic models it consists of a known part as well as a stochastic or random part. Consumer behavior, such as arrival process and particularly preference are important parts of a demand functions.

4.5.1 Inventory management with product substitution

This section reviews the literatures concerning inventory management of substitutable products. We first review general inventory models then we discuss inventory management of these products under newsvendor setting and models which aim to maximize the expected profit. Focus of early papers regarding substitute products are to identify the effects which accounting for substitution may have on inventory policies. Usually total costs and order up-to-levels of these products are examined. Intuition suggests accounting for substitutability and risk pooling leads to a reduction in total costs and inventory levels.

4.5.1.1 General Inventory management with substitute products

McGillivray and Silver (1978) was first to study inventory management of substitutable products. They consider an inventory model of N similar items with a (R, S_i) system of control with substitutable, stochastic demand. Stockout based substitution occurs between products with some probabilities and demand follows normal distribution. Authors assume normal distribution for the sake of convenience, but claim that the results are closely the same with other distribution, particularly when the mean value of demand is sufficiently large. The same unit cost and shortage penalty cost is assumed for products since products are mutually similar. Without this assumption analysis becomes far more complex for authors. An analytical solution is developed for optimal order up-to-level in case where substitution factor is equal to 1, meaning that consumers with probability of 100 % substitute facing out-of-stock products. With above assumption they show that the expected total costs are lower if substitution is considered in the model. However they state that an exact solution is not possible for the case of partial substitution (substitution factor is between 0 and 1), they

developed a heuristics to determine order up-to-level and show its performance. Authors show that the saving can be substantial where substitution probability is high and increase as the number of substitute possibilities increases.

Gerchak and Mossman (1992) explore the effects of demand randomness and risk pooling on optimal inventory levels and associated costs. In contrast with intuition, they show that there are some specific conditions where the optimal inventory levels may increase due to risk pooling and substitution effects.

Drezner et al (1995) develop an economic order quantity (EOQ) model in presence of two substitutable products. In their model one product can be substitute for the other product, at a fixed cost. Authors' intuition was that if accounting for substitution is profitable, then the rate of substitution does not matter and either no substitution is optimal or complete substitution is optimal. Surprisingly results show that full substitution is never optimal and either no substitution or partial substitution is optimal. They prove when the difference in holding costs of two products are sufficiently large, then the total cost of partial substitution is marginally lower than cost of full substitution.

Gurnani and Drezner (2000) extend the Drezner et al (1995) to find optimal order quantity and substitution quantity. In their model there are N products which only downward substitution can occur among them, if products stockout, at a fixed cost. They assume different unit costs, selling prices and holding costs for products. Annual demand also is known for all products. To account for substitutable demand, authors define a time variable that is the time at which a particular product is exhausted and assume products with higher inventory holding costs to exhaust faster than products with lower holding costs. Using this variable they expressed order quantity of products in terms of the products demand and transfer demand of stockout products, at the time they exhausted. The objective function therefore is to find optimal stockout time for each product and order quantity respectively. It is shown that the total cost function is not convex and intractable. By formulating a new cost function with new variables, they managed to make the problem convex and optimally solve it.

Deniz et al (2004) analyze inventory policy of one perishable product which has a life time of two periods. After one period the product becomes old and after two periods, it perishes. The old and new product can be substituted for each other. Authors model an infinite period problem to find optimal order up-to-level at the beginning of each period. The goal is to

minimize total cost. They consider four cases of full, downward, upward and no substitution. They show that it is not guaranteed that substitution will always be beneficial. Moreover they investigated situations in which each of four substitution types mentioned above are suitable based on different operational costs. For example they claim that downward substitution policies are always beneficial with respect to average freshness of products. On the other hand blood services can benefit from full substitution since there is a high penalty costs for leaving customers unsatisfied.

In contrary with conventional opinion for inventory pooling and that if the demand of various sources are pooled it will increase the profit and lead to lower inventory levels and cost. Yang and Schrage (2009) study the situation where risk pooling particularly full and partial substitution increase inventory levels. They indicate that in case of full substitution and right skewed demand distribution, optimal inventory levels increase, also in case of partial substitution and symmetric demand distribution such as normal and uniform, they show conditions when inventory levels increase. The papers reviewed above where general inventory models trying to investigate effects of substitution on total costs and order quantity of substitutable products. Another stream of research concerning substitutable products is inventory management of substitutable products under newsvendor setting, which is more relevant to purpose of this thesis and nature of revenue management models.

4.5.1.2 Substitutable products under newsvendor model setting and profit maximization:

Newsvendor problem is a special case of inventory management where a retailer makes a procurement decision for her products to be sold over a finite horizon. Newsvendor models are classified as single or multiple periods. In single period model, ordering decision is made for products that have no or low salvage value. In multiple periods model the excess inventory of one period can be carried over multiple period before end of same review period.

Two-Product Problems:

Parlar and Goyal (1984) study a two substitutable products problem in a newsvendor model. They demonstrate that the optimal order quantities of each product can be found by maximizing an expected profit function, which is strictly concave for a wide range of problem parameter values.

Parlar (1985) study the optimal ordering policy of two perishable and substitutable products as a stochastic control problem. There is one product which perishes in two periods. An old product is one-period old and a new product is the fresh product (as Deniz, 2004). An

example could be a bakery shop. The manager should decide how many units of fresh goods to bake or to order each morning considering inventory level of old goods remaining in the store. These two products can be substitute for each other as stockout occurs. Authors model an infinite horizon problem and objective function is to find optimal order quantity in each period to maximize expected profit. They assume an identical unit cost for products and the old product is cheaper than the fresh one. It is proved that for one period problem, the profit function is a strictly concave function of order quantity of the new product. To solve the problem for multiple periods, they formulate problem as a Markov decision problem (MDP) and solve it by linear programming. They analyzed sensitivity of optimal policy to problem parameters and particularly show the high sensitivity of optimal order quantity to changes in substitution probabilities.

Pasternack and Drezner (1991) model a stochastic, single period newsvendor problem with two full substitutable products. They managed to develop an expression for optimal order up-to-level and show that profit increases by accounting by substitution, however, order up-to-level may increase or decrease.

Khonja et al (1996) extend the model of Pasternack and Drezner(1991) for two substitutable products, but they allow the substitution factor to be other than one. They consider unique price and cost as well as salvage value for two products. In the single period newsvendor problem with stochastic demand, authors cannot show the concavity of the problem, therefore optimal solution for order quantities through optimization method is difficult to obtain. Monte Carlo simulation provides a tool to obtain lower and upper bound for the problem. The numerical study shows the benefit of accounting for substitution.

Ernst and Kouvelis (1999) consider a case of two products which are not direct substitutes. There is a bundle of these products which can be substitute for either of them in case of stockout. If customer needs the bundle and it is not available, then she buys one unit of each product. Price of bundle is cheaper than sum of prices of individual items. In single period problem and presence of unique prices, costs and salvage value for products, and stochastic random demand, authors prove that if the joint cumulative distribution of demand is continuous, the expected profit function is concave. With numerical study it is shown that demand correlation among products and bundle, either positive or negative, results in higher profit compared to case of uncorrelated demand. As demand become more negatively

correlated, profitability increases. Also as the substitution probabilities increases the inventory level of bundle increases.

Roychowdhury (2009) models a two-product problem where only upward substitution is allowed. Model is based on a multiple finite and also infinite periods. Author proves the concavity of expected profit for both cases.

Multiple Products Problem:

Bassok et al (1999) study a multiple inventory problem and extend the previous works to N products with full downward substitution (excess demand for class i can be satisfied using product j for $(i \geq j)$). Each product can have different demand properties which are stochastic. They assume order, holding, penalty a salvage costs are proportional and there is a substitution cost, proportional to quantity substituted. Excess inventory has a salvage value and there is a cost for shortage. The model is a single period model and where retailer observes the inventory on hand at the beginning of each period and place orders for each products. Then after the demand is realized she has to allocate the product to satisfy the demand. They show that the profit function is concave and submodular and demonstrated that a greedy allocation policy is optimal for the case of downward substitution. They particularly indicate that most gain from substitution is obtained in problems with high salvage value of products high demand variability, low substitution cost, low profit margins and similarity of products in terms of prices and costs.

Rajaram and Tang (2001) analyze a substitute products problem by proposing an effective demand function to approximate demand under substitution. First they study a two product case where stockout based substitution can occur during the period. Assuming equal unit cost c , selling price p , and salvage value r , and normal distribution for demand with mean μ_i and standard deviation σ_i :

$$D_i^e = D_i + \alpha_{ji}[D_j - Q_i]^+ \quad j \neq i, (a)^+ = \max\{0, a\}$$

$$D_i^e = D_i + \alpha_{ji}[D_j - Q_i]^+ = D_i + \alpha_{ji}(D_j - \min\{D_j, Q_j\})$$

$$= D_i + \alpha_{ji} D_j \left(1 - \frac{\min\{D_j, Q_j\}}{D_j}\right)$$

D_i^e is the effective demand consisting of primary demand of product i and demand derived from product substitution. α_{ji} represents portion of unmet demand of product j which

substitute to product i . Due to complexity of profit function in the newsvendor model with demand substitution and demand correlation, authors approximate the effective demand by a more tractable variable \widehat{D}_i^e . To do so first they define a service rate for each product $\gamma_i(Q_i)$ as:

$$\gamma_i(Q_i) = \frac{\min\{D_j, Q_j\}}{D_j} = \frac{E(\min\{D_j, Q_j\})}{E(D_j)}$$

Therefore:

$$\widehat{D}_i^e = D_i + \alpha_{ji}D_j(1 - \gamma_i(Q_i))$$

Authors test the validity of this approximation through simulation and show that the error is negligible when Q_i is sufficiently large (more than one standard deviation above the mean of demand). Even with the new demand function, the profit function is difficult to maximize. Thus a heuristic is developed to find optimal service rate for products using mean value and standard deviation of proposed effective demand. The order quantities resulted from heuristic provide a lower bound for the problem. An upper bound is derived by using a lagrangian relaxation. Finding optimal service rate for products leads to finding optimal order quantities. Through numerical study they extend their model to multiproduct case. Their investigation suggests that demand substitution always lead to higher profit and they highlight the significance of designing products to be substitutable. In analyzing sensitivity of order quantities to model parameters, they show that total order quantities is increasing in the level of demand correlation, since the variance of demand under substitution increase as demand correlation increases. This leads to a lower profit as well. In agreement with Pasternack and Drezner (1991), it is proved that total order quantity with substitution may increase or decrease compared to without substitution, depending on the underlying parameters such as substitution rate, level of demand uncertainty and correlation.

Nagarajan and Rajagopalan (2008) investigate the nature of optimal inventory policies with substitute products. First they consider a two-product problem and show that profit function is concave for single period, finite and infinite multiple periods model. It is claimed that optimal inventory of a product can be partially decoupled from inventory of other products in some conditions like if the inventory level of an item is sufficiently high or the substitution fractions are low, demand variability is not too high and products' service levels are high.

Through numerical analysis they extend the model to case of more than two products and it is shown that the results above are hold in case of multiple substitute products.

Myers (2009) studies perishable and substitute products problem within multiple product variants. There are N products which have fixed shelf lives (equal or different to each other). Substitution can take place between product variants and between different ages of a particular product. Stockout based substitution is allowed and products are ranked for each customer based on her preference. For example if a customer prefer a fresh unit of product i and it is not available, there are four possibilities: 1. substitute with older item but the same product if available, 2. substitute with fresh item of other product based on ranked preferences if available, 3. substitute with old item of other products based on ranked preferences if available, 4. Decide to not to purchase. Author models the problem according to (R, S_i) system of control, also newsvendor as a special case. Products have different purchasing cost, holding cost and selling price. Demand for products are stochastic, but with known distribution and parameters. Using simulation-optimization approach and the results obtained, author develops a heuristic to find optimal ordering policies for both cases of newsvendor and (R, S_i) . First the aggregate amount of order quantity for all products is determined and in the next step, this aggregate quantity is allocated to different products. Through analysis the effects of different parameters on expected profit are investigated and it is shown that profit increases as the level of substitution increases.

Extensions:

Netessine and Rudi (2003) consider modeling of substitutable products to analyze optimal inventory in two case of centralized and competitive inventory management. They allow stockout-based substitution to occur but one substitution attempt can be made. In a single period model, authors prove that the problem of more than two products and substitution is not concave or even quasi-concave. In case of centralized management, assuming a multivariate normal distribution for demand, they obtain analytically tractable solution. They show that under multivariate normal distribution, the retailer's profit is decreasing function of any coefficient of correlation if order quantities are either held fixed or adjusted optimally as correlation changes. For the case of competitive management, it is shown that a Nash equilibrium exist which can be found from first order condition. Comparison of two cases revealed that competition leads to higher inventory levels. However, through counterexample it is shown that in asymmetric case there are situations where the inventory level of at least

one product is higher in noncompetitive model than in competitive. Finally authors demonstrate substitution is more beneficial when demand correlation of products is low.

Kraiselburd et al (2004) study the issue of contracting between manufacturer and retailer where substitution exists among products in a single period. In their model a manufacturer face lost sale in case a customer substitute to another product but the retailer gain a profit from selling substitute products. They demonstrate that when the manufacturer has high influence on demand, Vendor Managed Inventory (VMI) outperforms Retail Managed Inventory (RMI). Also they show that there are situations where RMI performs better than VMI particularly when the level of substitution is high.

Gurler and Yilmaz (2010) extended the two product problem to the case of channel coordination. They assume a supply chain considering one manufacturer and one retailer. In a newsvendor setting and stockout based substitution, a retailer has an agreement with manufacturer to return unsold products at some credits. Authors prove the concavity of channel profit function as well as manufacturer's profit function. But due to complexity of retailer's profit function, they only consider special case of one-way full substitution and show its concavity. Study shows that channel coordination is not achieved for the case of full return with full credits, and in no return case as well.

Huang et al (2010) model a competitive newsvendor for substitute products in a single period. Each product's inventory policy is managed by one decision maker. Decision makers have perfect information about their competitor's policies. It is shown that expected profit of each product is concave in its order quantity and authors characterize a unique Nash equilibrium for the competition model. To find the effective demand under substitution, authors use the service rate heuristic (as Rajaram and Tang, 2001). Through numerical analysis it is demonstrated that with symmetric cost parameters and identically distributed demand, competitive models always lead to higher inventory levels. The higher degree of substitution results in higher inventory levels and higher expected profits. Moreover, as demand variation increases, inventory levels inclined and it enhances the impacts of product substitution and demand correlation.

4.5.2 Assortment planning Problem

4.5.2.1 Product Line Design Problem

Here we discuss the problem of selecting products to be produced in a product line. These literatures are related to assortment planning problem. The increasing need of offering a selection of products which satisfy customers in a variety of segments rather than offering a single product leads product line design and pricing to be among the most critical decision each firm has to make. A product can be defined as a set of attributes where each attribute can have different levels; therefore a product line is a set of different attribute level configurations (Fruchter et al, 2006). There are two type of product differentiation. In a horizontally differentiated, products cannot be ordered based on their features. Products are ranked based on customers e.g. milk with different tastes. In a vertically differentiated, products can be ordered based on their attributes such as quality. For example the higher quality product outranks the lower quality product in customer's view. See Lanchester (1990). Firms have to balance the potential gains from increasing product variety against the costs and possible cannibalization effects on existing products in the line. The main difficulties in product line design are to simultaneously determine how many products to offer, how to differentiate them along their key attributes and how to price them, considering products demand cross effects and shared resources (Schon, 2010).

The literature on product line design and economies of product variety has long history which are reviewed by Lancaster (1990), Eliashberg and Steinberg (1993), Krishnan and Ulrich (2001) and Ramadas (2003), each from different points of view. Literatures in product line design consider products that are differentiated usually by a single attribute (e.g. quality along with price) and allow customers to self-select from variants offered. Two streams of research can be identified: in the first category, literatures seek analytical results to understand the structure of solution and many restrictive assumptions are imposed to the model such as no fixed costs, constant unit cost and a single attribute differentiated products. In the second stream which models are less restricted, optimal solution through optimization is the objective even at the expense of making analytical solution impossible. These stream of researches were pioneered by Mussa and Rosen (1978), considering a monopolist's product line design problem. Moorthy (1984) developed a theory of market segmentation based on consumer self-selection and finite number of market segment and a general utility function. In their model the segmentation involves choosing a subset of vertically differentiated products which differ in a single attribute and determining their prices. They stated that there

are two types of competition regarding this problem. Competition across firms and competition within a firm's own product line ("cannibalization"). Considering a monopolist firm they show that (i) consumer self-selection has significant implications for how products and prices are chosen by firms and (ii) product line selection (market segmentation) problem is not solved by looking at each market separately, even if there are no economies of scale in production. This is resulted from cannibalization effects. They do not consider operational issues such as fixed cost or inventory costs. Dobson and Kalish (1993) consider the designing and pricing a line of substitute products in presence of fixed cost and variable cost for introducing each product profile and their cannibalization effects on other products. The objective of the firm is either maximizing profit or customers' total welfare. Having the complexity of NP-complete problem, they proposed a greedy heuristic for efficiently solving the problem, but they do not address any analytical result of optimal assortment. Netessine and Tylor (2005) analyze the same problem as Moorthy (1984) with taking into account the classical EOQ production cost. They show that the number of products, products' quality, inventory cost and production cost are affected by each other interactively and suggest that all these effects with heterogeneity of customer tastes and size of market segments should be considered in product line decision.

De Groote (1994) shows that considering a concave production costs and horizontally differentiated products in context of Locational choice models with deterministic demand and consumer preferences, firm chooses a product line to satisfy the whole market and product locations are equally spaced. Alptekingolu et al (2004) consider two multi-products firms compete on the basis of product variety and price; one is mass-customizer and the other mass-producer. They show that a mass-producer reduces the product variety in case of competition with mass-customizer to reduce the intensity of price competition, compared to monopolist mass-producer. Chen et al (1998) study optimal product positioning and pricing, extending Lancaster's model to incorporate varying prices and quality levels in the attribute space, as well as varying reservation prices of customers. They show that the optimal solution can be found by using dynamic programming under stochastic demand and static substitution. Hopp and Xu (2005) address strategic impact is modular design on the optimal length of product line and price levels under MNL demand model. They do not consider economies of scale but a high product design cost leading to economies of scope. They show that if a firm reduces the cost of introducing new product via modularity, it should offer more variety in product line and because it can serve larger market share in a multiple segment market, it can

charge a price premium. Also they find a risk neutral firm can increase variety by reducing variable cost and charge a premium price, while a risk-averse firm may reduce the variety to reduce variable costs. In a single segment market, the more risk averse the firm is, the less variety it should offer. They show the monotonicity of product line length by introducing a product assortment order called “*monotonic weak majorization*”. Fruchter et al (2006) proposed a genetic algorithm approach to obtain optimal product line design. Schon (2010) find an optimization solution for profit maximization product line problem with continuous prices under MNL consumer choice.

These literatures discussed the product assortment from a manufacturer’s perspective. Product assortment from a retailer’s perspective requires some different approaches, such as taking into account inventory decision. In the next section we review some of recent paper for assortment planning problem. These papers will be reviewed based on their difference in modeling the customer’s demand.

4.5.2.2 Assortment planning Problem with MNL model:

Van Ryzin and Mahajan (1999) formulated a model to find inventory policy and optimal assortment for a multiple product problem. Each customer considers the subset S from N potential variant to be offered, then customers choose to purchase a variant in S or choose not to purchase, based on MNL model. The choice set S increase the probability of customer purchase because a customer chooses to purchase only if $\text{Max}_{j \in S} \{U_j\} \geq U_0$. On the other hand because of substitutability customer may switch from variant i to j if $U_i > U_j$, thus the selection of S affects the choice decision of each customer and the total demand for each variant. Also they assumed that customers choose based on the knowledge of set S without any knowledge about the inventory status of them. If a chosen variant is out of stock customer leaves the store without considering a stock-out based substitution. However their model can be unrealistic, they expressed some of situation where their assumption can be approximately appropriate. They assume $p_j = p$ and $c_j = c$ for all j (unique price and cost for all variants), also ordering variants based on their popularity such that $U_1 \geq U_2 \geq \dots \geq U_n$. Customers arrival visiting the store is Poisson with mean λ and number of customers selecting variant j is normally distributed with mean $\lambda \times q_j(S)$ and standard deviation $\sigma(\lambda q_j(S))^\beta$. Authors consider two models of aggregate demand. In independent population model, each customer assigns a unique utility to a variant. $U_{ij} = U_j + \varepsilon_{ij}$ where U_{ij} Is the utility customer i assign to variant j . In the trend following population all customer assign an

identical utility for variants, which results each customer to make the same choice. But this choice is not observable for retailer prior to making assortment decision. Since there is not any stockout based substitution, optimal inventory levels of both cases are determined easily from newsvendor model. When expected profit of each variant is determined, in the next step retailer should determine the optimal set of these variants to carry in the store. The objective is to maximize the total assortment profit:

$$\text{Max}_{S \in N} \sum_{j \in S} \pi_j(S)$$

Considering the net profit of adding a variant j to assortment S the goal is to find whether it is profitable to add a new variant to assortment or not. If the net profit is more than the profit losses of the products in S then adding j is profitable. They showed that the net profit function is quasi-concave in v_j (preference of product j) in the interval $[0, \infty]$. This leads to result that the optimal assortment always consist of the most popular variants when environment fit the assumption of homogeneous group of products with identical price and costs. They also demonstrated that high margins products provide incentive for retailers to broaden the range of variants in the set. Furthermore, when the no-purchase preference is sufficiently low, the retailer has the interest to decrease the breadth of the assortment.

Aydin and Ryan (2000) studied the retailer's problem in product line selection and pricing of variants to determine the structure of optimal assortment. A product line consists of quality-differentiated products of the same type. Their models differ from Van Ryzin et al (1999) from supply perspective. They assumed there is an infinite supply for all models which means all demand is met. Customers arrive at the store based on a Poisson process and their arrival is not function of variety offered by the retailer. The customer chooses a model based on MNL model. Since all demand is met, thus stock-out based substitution effect is ignored. In their model utility of each product is defined as the reservation price of a customer for that product. $U_i = \alpha_i + \varepsilon_i$ where α_i is the mean reservation price and ε_i is the error term with Gumbel distribution. They studied three cases. In the first case they considered a product line with fixed prices and the problem is whether or not to add a new model and at what price. They showed that the profit function of adding this new model to the assortment is unimodal in its price which means if there is no fixed cost for introducing a new model, there is a minimum price beyond which it is always profitable to introduce the new model. If there is a fixed cost for introducing a new product, then either there is a range of prices for which it is

profitable to introduce the new model or else it is not profitable to introduce this new model. In the second case, they assumed the assortment is predetermined and the problem now is to jointly determine prices of these models. They found that the expected profit in their model is not jointly quasi-concave in the prices of the assortment, but the optimal prices are unique solutions to the first-order optimality condition. They proved that the necessary condition for optimality is that the profit margins of all models $p_j - c_j$ are equal while infinite supply condition is hold. The joint pricing problem can be converted to an optimization problem over a single variable, defined as profit margin. Finally in the last case and to determine the optimal assortment, they showed when we cannot apply Van Ryzin assumption to order variants such that $U_1 \geq U_2 \geq \dots \geq U_n$, considering the infinite supply and ordering variants like $\alpha_1 - c_1 \geq \alpha_2 - c_2 \geq \dots \geq \alpha_n - c_n$ (average margin) the retailer should select K items from N models which have highest average margins and found that the optimal price is strictly concave function of the number of models in the product line. It implies that by increasing variety a retailer can increase her profit but this increase become quite small as the number of models increases.

Maddah and Bish (2007) extended assortment planning to jointly determine the set of variants together with their price and inventory levels. They assumed a single period and newsvendor type inventory model with no salvage value and shortage cost (as Aydin and Van Ryzin). They considered customer choice as a MNL model with Poisson customer arrivals. Also they ignored stockout based substitution effects. Their model differ in the way that unlike common assumption in the literature that demand is either additive with demand standard deviation being independent of price or multiplicative with demand coefficient of variation not depending on the price, they modeled demand as “*mixed multiplicative/additive*”. Both demand standard deviation and the coefficient of variation are function of price vector of product line. They demonstrated that their model can be seen as a multi-item newsvendor model with items having normal demand and under additive complexity of pricing and assortment decision. On the structure of optimal assortment they proved that if prices of items in S are fixed at some price vector P , then the expected profit from S And the expected contribution of item j are both strictly pseudo-convex in α_j The mean reservation price of item j . It implies that at fixed prices, either (i) there is minimum values of α_j beyond which it is optimal to add item to assortment or (ii) it is not optimal to add j to the assortment at any value of α_j this result extend the previous result from Van Ryzin (1999) when all items have the same cost and price. The optimal assortment consists of

items with the largest value of mean reservation price α_j , but in the special case where items can be ordered such that $\alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_n$ And $c_1 \leq c_2 \leq \dots \leq c_n$. If this condition cannot be applied the optimal assortment has items with largest average margin ($\alpha_j - c_j$) as Aydin (2000). In the numerical study and counter examples they showed that this results cannot be applied where there is a finite inventory, but they observed assortment having this structure yield expected profit that are close to optimal. Furthermore, in the investigating the characteristics of expected profit as a function of price they observed that the first-order optimally condition are not sufficient in their case and second-order optimally condition appears to be sufficient. They found that “equal profit margin” property of infinite supply case proved by Aydin et al (2000) is no longer hold for the case of infinite inventory but these profit margins would be approximately equal at optimality and based on these result they proposed simple heuristic to find optimal solution of assortment decision.

Cachon et al (2005) extended Van Ryzin model with presence of consumer search. They considered three models. First, the no-search model which is the same as Van Ryzin’s model with the same result. Then they studied two consumer search model. In both search models a customer does not purchase if no-purchase variant has the highest realized utility. But customer may not purchase even when there is an acceptable product in retailer’s assortment, because customer is uncertain about products outside the store’s assortment and may prefer to continue the search and explore other alternatives within another store. This search decision depends on several factors such as: the realized utility of the best variant, the cost of search and the customer’s expectation on the value of search outside the retailer’s assortment. The first search model is independent assortment search model which a consumer believe the retailer’s assortment is unique and thus utility of search is independent of the assortment e.g. jewelry stores and antique dealers. In the overlapping assortment search model, there are a limited number of products available in the market e.g. digital camera. Therefore expanding the retailer’s assortment reduces the value of search because search leads to fewer new variants. They found in no-search model and independent search model the profit function is quasi-convex in v_j (preference of product) and therefore Van Ryzin results is hold. But in the overlapping search model the profit function is not quasi-convex in v_j and hence the optimal assortment may not be in the set of popular variants and may consist of an unprofitable product to reduce search probability of customers. It implies that not considering consumer search may result in narrower assortment and lower profit.

Mahajan and Van Ryzin (2001a) extended the study to case of dynamic substitution. They used a sample path analysis to analyze structural properties of expected profit and they showed that the problem is not even quasi-concave. Proposing a stochastic gradient algorithm and two newsvendor heuristics and comparing the results, it is shown that a retailer's stock levels for more popular products are higher than for less popular products.

Gaur and Honhon (2006) study optimal product assortment and inventory levels under a locational choice model. The decision includes the number of products to offer, their locations in the attribute space and their inventory levels. Both product selection and product design. The goal is to maximize the profit. They consider a category with horizontally differentiated products, thus all products have the same price and costs (as Mahajan and Van Ryzin, 1999) and there is a fixed cost for including a product in the assortment. The assortment carried by the retailer is presented by a vector of product specifications $b = (b_1, \dots, b_n)$ where n is the number of products in the assortment and b_j is the location of product j on the preference spectrum. Each customer is characterized by specification of his most preferred goods in the preference spectrum, defined as the good that represent the optimal transfer of characteristics to him. A consumer with most preferred good x_i associates an utility U_{ij} with a product j in the assortment given by:

$$U_{ij} = Z - p - g(|x_i - b_j|)$$

Z is a positive constant representing the surplus associated with the product category, p is the price of product and g is a strictly increasing function representing the disutility associated with the distance between customer's most preferred product and product's specification. The utility of no purchase $U_{i0} = 0$. The coverage interval of product j is the interval which contains the most preferred goods of all customers who obtain a non-negative utility from product j . thus the first choice interval of product j in the assortment b is the sub-interval that contains the most preferred goods of all customers who choose product j as a first choice. They extend Lanchester's model to stochastic demand and random distribution of customers. They assume that the specification of the most preferred goods of all customers are independent and identically distributed with continuous probability distribution. They consider only unimodal distribution meaning that there is a unique most popular product and that as a product has more distance from the most popular product, it has less popularity. Customer's arrival is Poisson process with rate λ and demand for each product has normal. They investigate the characteristics of optimal assortment in both static and dynamic substitution case. Under

static substitution they showed that the optimal assortment consists of products equally spaced out (as De Groote, 1994) such that there is no substitution among them regardless of the distribution of the assortment and such that the most popular product is not offered. Under dynamic substitution they obtain bounds on profit where static substitution for the lower bound and a retailer-controlled substitution for the upper bound (as Van Ryzin and Mahajan, 1999) then proposed two heuristics to solve the problem. They show that a retailer provides higher variety under dynamic substitution than under static substitution and locate products closer to each other on the attribute space, resulting in more variety specially increasing of higher consumer density, so that consumers can derive positive utility from more than one product and the retailer can benefit from substitution. Furthermore, the optimal assortment may not include the most popular product because of fragmentation effect. Finally they demonstrate that the static solution can be a good approximation for the dynamic substitution problem when consumers are more homogenous in the preferences, or when the coefficient of variation of demand is low. In other cases dynamic substitution has a large impact on profits.

4.5.2.3 Assortment planning Problem with exogenous demand model

Smith and Agrawal (2000) develop a stochastic demand model to incorporate substitution effects in the problem where the retailer has to determine order quantity and products that should be included in the assortment. They consider both assortment and stockout-based substitution, but only one substitution attempt is allowed. The total demand for all products is known and the number of customers who prefers purchasing item i follows a negative binominal distribution. They show some appealing properties of negative binominal distribution for retail applications. Authors use a random substitution matrix to incorporate substitution probabilities. Due to complexity of the problem they assume order quantity is set to achieve a fixed service level.

$$\Pr\{\bar{D}_i(x) \leq q_i\} \geq \gamma_i(Q_i)$$

and

$$\Pr\{\bar{D}_i(x) \leq q_i - 1\} \leq \gamma_i(Q_i)$$

$\gamma_i(Q_i)$ is the product i 's service level.

Authors outline that the probability that a customer choose a certain product depends on previous arrivals, and substitution attempted.

$$g_i(x, m) = f_i + \sum_{i \neq j} f_i [1 - x_j A_j(x, m)] \alpha_{ji}$$

$g_i(x, m)$: Pr {item i is demanded by m -th arriving customer, either as initial preference or as substitute}

f_i : Pr {an arriving customer initially prefers item i }

$A_j(x, m)$: Pr {item j is still in stock for m -th arriving customer}

α_{ji} : Portion of unsatisfied demand j to be met by product i

Even with one substitution attempt, determining the demand is difficult and authors again use service level to develop upper and lower bound for the demand function.

$$\gamma_i(Q_i) \leq A_i(x, m) \leq 1$$

Using the lower bound as demand probability, under newsvendor model they solve the problem by a nonlinear integer programming. Through numerical study they find some insight into the problem, particularly they indicate that ignoring substitution may lead to less profitable assortment and inventory levels are significantly influenced by substitution effects. Even at zero fixed cost, assortment size tends to be lower when substitution is accounted.

Kok and Fisher (2007) provided a methodology for both selecting product carried in the assortment and inventory levels for them and applied this methodology at Albert Heijin, a supermarket chain in Netherlands. The objective function is as follow:

$$\text{Max}_{f_j, j \in N} \sum_j G_j(f_j, D_j(f, d))$$

$$\text{S. t: } \sum_j f_j w_j \leq \text{shelf space}$$

The decision process is to allocate a discrete number of facing to each product in order to maximize the total expected profit subject to a shelf space constraint. f_j is the number of facing allocated to product j and w_j is the width of a facing of product j . G_j is the average gross profit from product j . the effective demand rate of product j , $D_j(f, d)$ depends on facing allocation and demand rate of all products in the assortment.

$$D_j(f, d) = d_j + \sum_{k: f_k=0} d_k \alpha_{kj} + \sum_{k: f_k>0} \alpha_{kj} L_k(f_k, d_k) \quad ; \text{ similar to Smith and Agrawal}$$

L_k is the lost sales of product k . they estimate $L_k(f_k, d_k)$ via simulation. They showed that the problem is a knapsack problem with a non-linear and non-separable objective function, whose coefficients need to be calculated for every combination of the decision variables. Thus, they proposed an iterative heuristic that solves a series of separable problems. Using their optimization algorithm the retailer can add or delete products from her assortment or delete products from assortment if she carry full assortment. They also investigated some structural properties of the optimal assortment from heuristic algorithm. They showed product with higher demand, higher margin or smaller physical size may be included first in the assortment also with more inventory assigned to them. Furthermore, products with lower demand variability and smaller size should be included first in the assortment, but more inventories can be assigned to products with higher demand variance and larger case size if the available space is sufficiently high.

In the very recent paper by Honhon et al (2009), they modeled a single-period newsvendor setting consisting of N substitute products with prices and cost exogenously given. In their model each customer is defined by a customer type which a customer type is a sequence of products that a customer of that type is willing to purchase, arranged in decreasing order of preference. As an example a customer of type (1,2,4) first choose product 1, product 2 as the second choice and 4 as the third choice. Then the number of possible customer's type with N product is $|\tau| = \sum_{j=0}^n C_j^n j! = \sum_{j=0}^n \frac{n!}{(n-j)!} \cdot \alpha_t$ is the proportion of customer type t in the customer population and is exogenously given. This type of customer behavior results from a utility maximization procedure where each customer assign a utility U_j to products and then select the greatest values of utility vector which are also greater than U_0 as choice candidates. This customer choice model is very general and other models like MNL and Locational choice model can be presented as special case by efficiently restriction on the set of possible types. The total number of customers D is stochastic and continuous variable with known distribution from the past. There is a constant proportion of each customer type at all times. They considered customer come sequentially and inventory levels are updated after each arrival. Also customer can dynamically substitute when their choices are not available. They formulated the problem as a dynamic program where the value function measures the maximum expected profit that can be obtained from remaining customers given the current assortment. They showed the value function is a convex non-increasing function of demand

and has a piecewise structure. However the profit function had the properties of multiple local optimum, they managed to determine the optimal solution by providing an algorithm with a pseudo-polynomial complexity of $O(8^n)$ in dynamic programming. On the structure of optimal assortment they demonstrated given the profit margin of a product as the return on the product (selling price – purchase cost) and the risk of product (selling price – salvage value) they proved that products stocked in the optimal solution are such that the return and risk parameters of the subsets of products with positive inventory seen by subsequent customers are ordered in a specific way. E.g. if optimal assortment contains product 1 and 3 and the optimal quantities are such that product 1 is the first one to run out, then the return of subset $S = \{1, 3\}$ is greater or equal to set $\{3\}$, also the risk of $\{1, 3\}$ is greater or equal to subset $\{3\}$. Subsets that do not follow such an order cannot be included in optimal solution. To the best of the authors' knowledge this is the first, guaranteed running time, optimal assortment and inventory decision under dynamic substitution. From the structure of optimal assortment they proposed an efficient heuristic for the general case where total demand is discrete random variable and the proportion of customers of each type is random.

Yucel et al (2009) formulate a multi-product inventory and assortment problem combining with supplier selection under exogenous demand. They consider a single period stochastic demand case by a scenario-based approach for representing demand randomness. They do not make any specific assumption on the demand distribution of individual products or assortment itself. They model random demand by a joint demand distribution for all products in the category which is represented by a collection of demand scenario with given probabilities. This distribution may be obtained by market share of products. These assumptions give them the advantages of simplicity and flexibility to integrate several constraints and costs item in their model such as supplier selection, shelf space limitation and multi-level substitution behavior. They model a dynamic substitution where each customer is allowed for two level of substitution. Fixed ordering cost, fixed cost of supplier selection, inventory holding cost and customer substitution cost are integrated to the problem. They solve the problem with a proposed mixed-integer programming and find some managerial insight. Varying levels of substitution costs affect purchasing, ordering, inventory management and supplier selection decision. High substitution and lost sales cost result in extended assortment and increased service level, but reduce profit due to increase in operational costs.

Extensions of assortment planning:

So far, all papers reviewed regarding assortment planning isolated the retailer from supply chain consideration. Villas-Boas (1998) study effects of decentralized decision making on product line selection to show how optimal assortment in case of centralized and decentralized supply chain differs, but does not address how a decentralized supply chain can be coordinated. The centralized model is similar to Moorthy (1984) to determine product line and pricing of them, in the decentralized case, the manufacturer determine her optimal product line and the retailer decide which of these products to include in the assortment and pays a transfer fee for each product, but she is free to set the price to end consumers. The author finds the equilibrium solution for the optimal product line, optimal transfer and retailer prices. There is no inventory cost and all demand is met.

Aydin and Hasnman (2003) continue this study and model a problem with inventory cost and focus on how to achieve coordination and ignore the pricing problem. Their model is similar to Van Ryzin and Mahajan (1999) with MNL consumer choice. In the centralized case where the firm is both manufacturer and retailer, they extend the result of Van Ryzin and Mahajan (1999) to this case. In the decentralized case, the retailer keeps inventory of the end products and chooses her assortment. The manufacturer decides separable but has the capability to practice delayed differentiation. They find that the centralized chain offers at least as many as the decentralized one, and delayed differentiation provide more profit for both parties. Moreover, if the manufacturer pays a per-product fee to the retailer, it will induce the retailer to offer supply-chain-optimal number of end products and it makes both parties better off.

Singh et al (2005) study the model of Van Ryzin and Mahajan (1999) in a situation where there is wholesaler own the inventory and stocks them, when a customer order a particular product to the retailer, her order is shipped directly from wholesaler. This model can fit the common practice in internet retailing. Retailer has to pay for each drop-shipping. In this case the variety is higher than the traditional channel. They also consider a vertically integrated firm with multiple retailers and show that the retailer stock more popular products and less popular products are stocked at warehouse and shipped to customers directly. The assortment size at retailer decrease when number of retailers increase or drop-shipping cost decrease.

Customers make purchase decision in complex ways. As some of customer demand can be met by substitute products, some customers may be willing to purchase a certain set of products simultaneously i.e. tops and skirts is women wear. These products are called

complement products. It seems important to incorporate both substitution and complementarity behavior of purchase decision in making assortment decision. Agrawal and Smith (2003) formulate this problem where customers wish to purchase in set and also may accept to substitute if the set is not available. Each customer has an initial preferred purchase set. If the set is not available there are three options. 1. Substitute a smaller set that does not contain the missing item 2. Substitute a completely different purchase set 3. Not purchase anything. Authors ignore stockout base substitution and only assortment based substitution may happen. The substitution pattern is captured by a transition matrix substitution (as Smith and Agrawal, 2000). A set can be purchased only when all the items in the set are available. As their previous work, authors model the customer arriving by negative binominal distribution. The solving approach is similar to their previous work. They develop a mixed integer, linear programming formulation to optimize the assortment and through numerical examples they demonstrate that; customers' preferences to purchase items in set or their willingness to substitute may lead to larger or smaller assortment depending on the fixed cost. Ignoring substitution and complementarity can have substantial impacts on profit. Also, while increasing customers' willingness to substitute may increase the profit; increasing complementarity among items may decrease the profit. As the substitution possibilities become more for customers, profit tends to decrease since there is no small subset of items which can satisfy most of customers.

4.5.3 Dynamic Pricing for substitute products:

Li et al (2003) consider dynamic pricing model for two substitutable products. Consumer choice follows a MNL model and customers choose a product based on quality and the corresponding price. Therefore a price based substitution may occur. Retailer set the prices for each arriving customer. Authors assume different distributions of number of customers such as deterministic, geometric, or bounded. They find some structural properties of optimal policy and develop an algorithm to compute it and show the monotonicity of optimal expected revenue to prices.

Bitran et al (2004) study the dynamic pricing for a family of substitute products. In contrast with most of papers which consider substitute products within a certain subcategory of product family such as different colors, quality, etc of the same product using a MNL model to incorporate substitution behavior, authors in this paper aim to model substitution in different customer segments but in a family of products, which differ in price and quality and customers have the same taste for the quality but differ in their budget. They claim that MNL

model cannot rank the products based on the attributes above in a single way for each customers. Thus they propose a different consumer choice model called Walrasian choice model (WAL) because of its derivation based on a budget constraint. The model is as follow: products are ranked based on non-price attributes such that $u_1 > u_2 > \dots > u_n$, where u represent the utility of each product. Each customer prefer product i to j if $i < j$. When a customer goes to the store, she has a private pair (ω, u_0) where ω is the budget constraint and u_0 is non-purchase utility. To choose a particular product, customer solves the following utility maximization problem:

$$\begin{aligned}
 & \text{Max } u_0 x_0 + \sum u_i x_i \\
 & \text{subject to: } \sum p_i x_i \leq \omega \\
 & x_0 + \sum x_i = 1 \\
 & x_0, x_i \in \{0,1\}
 \end{aligned}$$

The model can be solved with a greedy algorithm. A customer search from product 1 and as soon as she finds a product which $p_i < \omega$ and $u_0 < u_i$, she buys the product. If for a product she finds $u_0 > u_i$ or the list is exhausted, she does not buy. Then probability of choosing a product depends on price of that product and the next alternative. Authors allow for price based substitution, but they relax stockout based substitution. When a product is stocked out, its price become the same as its next better alternative, therefore its choosing probability becomes zero. A closed form derivation to the problem is not available, hence two approximated solution are proposed. In an unlimited supply case, they consider an infinite inventory and optimal prices obtained with a line-search procedure. The second approximation is to consider deterministic demand and they show optimality condition under KKT conditions. Authors find that as inventory of a product declines, its price increases and as time elapses, the price of products decrease. Also as we have more products in the set, price of the cheapest product decreases and the opposite effect for the most expensive, revenue increases but the slope of this increase tends to zero rapidly.

Li and Chen (2009) model a dynamic pricing problem for multiple substitute products with MNL consumer choice model. Only price based substitution is allowed and consumer's

arrival rate is time-dependent. Assuming one arrival in each time interval, retailer set the prices based on available inventory levels. Selling time is finite and there is no salvage value and replenishment. Retailer should maximize the expected profit. Authors show that the expected profit is not quasi-concave in the price; therefore they transform price space to sales probabilities space. Since they cannot find a close form solution to the problem, a numerical algorithm is provided to compute optimal solutions. Authors develop a real-time dynamic pricing model for the case where estimating customer arrival distribution is an intractable problem, based on cumulative customer arrival observed to the date and inventory available.

4.5.4 Joint Optimization of Inventory and Price for substitute products:

Tang and Yin (2007) develop a model to investigate joint inventory-pricing of two substitute products under fixed and variable pricing strategy. They assume a deterministic demand function and products are price based substitutes. Authors model two cases, where in the first case the prices of products are fixed and equal. In the second case there is a varying price for products. It is shown the optimal price under variable pricing strategy is equal to optimal retail price with fixed pricing strategy plus or minus an adjusted term where this adjustment term is decreasing in the product substitutability factor. As the substitutability factor inclines, the gap between optimal prices decreases. It is shown that retailer always gains higher profit considering variable pricing, but this additional profit decreases as products become more substitutable. They extend the model to a limited capacity and competitive model and demonstrate the structure above still preserves. When capacity become restricted, retailer may are to charge higher price and reduce the order quantity. In further restriction, she has to order only most popular product for the whole capacity.

Karakul and Chan (2008) in their model consider two products, an existing and a new developed product, which new one can be substituted for old one in case of stockout (one-way full substitution). Also they allow for price based substitution. Price of the existing product is determined by market share and firm should determine order quantities of both products and price of new product. They show that problem is not concave in prices, but find some properties of optimal solution. Authors analytically demonstrate that the expected profit, optimal order quantity and price of new product are higher when substitution is allowed and order quantity of existing product becomes lower. Using these properties, they prove that objective function is unimodal by showing that first order conditions have a unique solution for the problem. Through numerical analysis, some important results are obtained. Considering substitution in pricing and procurement policy tends to increase total profit (by

30 % in their case) and reduce the total inventory levels. Also results show that joint optimization of price and order quantity seems to have a small effect as compared to the effect that substitution may have on expected profit, hence one may solve the pricing and procurement problem of substitute products separately without a big loss in profit.

Aydin and Porteus (2008) study the inventory-pricing problem of multiple products in a given assortment where products are only price based substitutes. They model a single period newsvendor problem with stochastic demand. They assume a multiplicative demand function where expected demand for each product is proportional to its attractiveness. Authors consider three special cases under attraction model. The first is the MNL model which is the most common model in economic literature. The second model is MCI model. Models are as follows:

MCI model:

$$v_i(p_i) = \alpha_i p_i^{-\beta_i}$$

$v_i(p_i)$ is the attractiveness of product i .

α_i is the quality of product and β_i is the consumer population's price sensitivity for product i .

The third case is the demand model with multiple customer segments:

$$v_i(p_i) = \sum_{k=1}^m \beta_k \exp(\alpha_{ik} - p_i)$$

$$\sum_{k=1}^m \beta_k = 1$$

m is the number of customer segments where a customer belongs to segment k with probability β_k .

Since the original problem is not concave in price they work with profit expression written in terms of inverse demand function. Under the assumption of price based substitution and assuming that product attractiveness is strictly decreasing in price of the product, Authors demonstrate that there is a unique solution to the problem which satisfy first order condition and that their problem is well behaved under attraction model and where the first order condition is necessary and sufficient for optimality. Therefore they propose that a hill

climbing algorithm could be enough to find the optimal solution for price vector. It is indicated the effect of changes in cost and quality of one product on the other product is not obvious, but intuition is that an increase in cost of one product leads to a decrease in optimal prices of other products. Also if the quality of one product, with assumption of how attractiveness function depends on quality, increases then the optimal prices of all products will increase.

Song and Xue (2007) study the joint inventory and pricing decision of a multiple substitute products over a finite planning horizon. They consider a multiple periods model under newsvendor setting where products are only price based substitutes. Authors formulate a general stochastic demand model and develop MNL and Locational choice model as special cases to their model. At the beginning of each period, the current inventory is reviewed and replenishment orders and prices are set. Demand for current period depends on newly set prices. Since the problem is not jointly concave and tractable, authors use the inverse of market share vector and express the price vector as a function of market share vector. They show that the new revenue function is jointly concave in order up-to-level and product market share. To solve the problem as a dynamic programming model, they propose a branching algorithm to find the optimal policy. If the inventory of a product is below its optimal order up-to-level, then retailer places an order. If it is over the optimal order up-to-level, retailer does not order. Therefore the optimal policy involves three components. A not-to-order list for products overstocked. The base stock levels of products and the target market shares of them where the base stock levels and market shares depend on the initial inventory of no-to-order list.

Dong et al (2009) develop a model to study dynamic pricing and inventory policy of substitutable products in a given assortment where customers choose their preferred product using a MNL model. Authors model the problem as a stochastic dynamic programming under multiple periods and newsvendor model. To overcome the problem of not quasi-concavity of model in price vector, authors define an inverse function which maps the sales probability space into the price space. They derive optimal solution to the problem and further to show the value of dynamic pricing authors compare three pricing schemes. Static pricing where firm apply fixed prices for the selling season. Unified dynamic pricing where all products have identical prices and these identical prices can change as time passes. Mixed dynamic pricing which is combination of static and dynamic pricing. It is shown that the optimal prices of products in stock are affected not only by the inventory scarcity of the system (total

inventory) and individual items' inventory levels, but also the quality level of all in stock products. Due to complex relationship they cannot derive general properties for the dynamic pricing policies. However, through performance comparison it is shown that all pricing policies perform well when the procurement cost of products are low, but dynamic pricing has a significant value when there is high procurement cost and scarce inventory. Unified pricing well suits situation where product's quality levels are quite close, such as cloth with different colors but same style. Dynamic pricing is most effective to products with potentially high profit. To obtain optimal initial inventory levels, authors stated that the profit function is not quasi-concave in inventory levels, but the profit function of single product dynamic pricing problem is concave in inventory (see Galloego and Van Rayzin, 1994). Hence they use this property to find inventory of each single product with dynamic pricing obtained from previous problem, and use a pooling inventory approach to obtain a close to optimal inventory policy.

Table 4.2 Summarize significant studies with respects to their characteristics and assumptions made in modeling substitute products. Table 4.1 introduce notation for our classification system.

Elements	Code	Description	Codes
Decision variables	DV	Price/Inventory/Assortment	P,I,A
Planning Horizon	PH	Single period/Multiple periods	S,M
Demand Type (consumer choice)	DT	Attraction models/ Exogenous	A,E
Products	P	Two products/Multiple products	T,M
Substitution Type	ST	Static/Dynamic	S,D
Substitutability	S	Full/Partial/Downward/Upward/Neither(in static substitution)	F,P,D,U,N
Prices	(\$)	Fixed/Dynamic	F,D
Other	O	Competition/Supply chain/Neither	C,S,N

Table 4.1: Notations for classification system

	Codes							
Reference	DV	PH	DT	P	ST	S	(\$)	O
McGillivray and Silver (1978)	I	M	E	T,M	D	F,P	F	N
Parlar (1985)	I	M	E	T	D	F,P	F	N
Pasternack and Drezner (1991)	I	S	E	T	D	F	F	N
Khouja et al (1996)	I	S	E	T	D	F	F	
Ernst and Kouvelis (1999)	I	S	E	M	D	F,P	F	N
Bassok et al (1999)	I	S	E	M	D	F,D	F	N
Rajaram and Tang (2001)	I	S	E	T,M	D	F,P	F	N
Netessine and Rudi (2003)	I	S	E	M	D	F,P	F	C
Kraiselburd et al (2004)	I	S	E	M	D	F,P	F	S
Nagarajan and Rajagopalan (2008)	I	S,M	E	T,M	D	F,P	F	N
Myers (2009)	I	S,M	E	M	D	F,P	F	N
Gurler and Yilmaz (2010)	I	S	E	T	D	F,D	F	S
Huang et al (2010)	I	S	E	M	D	F,D	F	C
Li et al (2003)	P	S	A	T	S	N	D	N
Bitran et al (2004)	P	S	A	M	S	N	D	N
Li and Chen (2009)	P	S	A	M	S	N	D	N
Van Ryzin and Mahajan (1999)	I,A	S	A	M	S	N	F	N
Aydin and Ryan (2000)	P,A	S	A	M	S	N	F	N
Aydin and Hasnman (2003)	I,A	S	A	M	S	N	F	S
Maddah and Bish (2007)	P,I,A	S	A	M	S	N	F	N
Agrawal and Smith (2003)	A	S	A	M	D	F,P	F	N
Cachon et al (2005)	A	S	A	M	S	N	F	N
Mahajan and Van Ryzin (2001a)	I,A	S	A	M	D	F,P	F	N
Gaur and Honhon (2006)	I,A	S	A	M	S	N	F	N
Smith and Agrawal (2000)	I,A	S	E	M	D	F,P	F	N
Kok and Fisher (2007)	I,A	S	E	M	D	F,P	F	N
Honhon et al (2009)	I,A	S	E	M	D	F,P	F	N
Yucel et al (2009)	I,A	S	E	M	D	F,P	F	S
Tang and Yin (2007)	I,P	S	E	T	S	N	F,D	C
Karakul and Chan (2008)	I,P	S	E	T	D	F,D	F	N
Aydin and Porteus (2007)	I,P	S	A	M	S	N	F	N
Song and Xue (2007)	I,P	M	A	M	S	N	D	N
Dong et al (2009)	I,P	M	A	M	S	N	F,D	N

Table 4.2 Classification of papers

4.6 Summary and results

In this section we compare papers concerning substitution in regard to their approaches and model characteristics and we try to summarize the results. The fundamental difference for papers we reviewed is the goal of modeling. A few papers which model to minimize the total cost and investigate total inventory levels seem to have different results in some cases compared to models that their goal is to maximize the expected profit. For example Deniz et al (2004), Yang and Schrage (2009) show that accounting for substitution may or may not be beneficial with respect to model parameters and special conditions, but all profit maximization models claim that a higher profit is obtained by accounting for substitution.

Literatures consider different characteristics in their modeling to extend their previous studies and to shed light on uninvestigated effects and problems. As we see models begin with two products cases with restrictive assumptions on substitution behavior and the other operational characteristics such as identical supply costs or selling prices, and in the later papers models become more general and realistic. It is tried to incorporate all the substitution behaviors and to consider multiple product cases. The reason for this trend in literatures seem to be due to difficulties in solving the models, but results obtained from most of papers are the same in most cases. Studies prove that a firm or a retailer can always increase the expected profit by accounting for substitute products and customers' substitution behaviors compared to policies which ignore these effects. This increase in profit depends on some parameters in a model.

The substitution probability has a positive effect on expected profit. As substitution probability increases, a retailer may expect to gain a higher profit. Also, when the number of possible substitutes for customers increases, profit tends to be higher. On the other hand, the effect of substitution probability and in general substitution behavior on optimal inventory level is not that clear. As most of papers indicated, inventory levels of products may increase or decrease when a retailer incorporate substitution behavior. But intuition suggests that as a retailer aim to maximize the expected profit, the total inventory level of substitute products is higher than when substitution is ignored, but this result cannot be extended to individual item level. Inventory of some products may increase and some decrease. Mahajan and Van Ryzin (2001) show that retailers stock more of more popular products and less of less popular products. From these results it can be implied that accounting for substitution is most profitable when substitution probabilities are high and there is scarcity in inventories. Nagarajan and Rajagopalan (2008) claim that when the inventory levels for products are

sufficiently high and customer desire a very high service level, and substitution probabilities are low, inventory policy of a product can be decoupled from other products without a big loss in profit.

Demand variability and correlation also have a significant impact on both profit and inventory levels of substitute products. Results show that higher demand variation enhance the impact of substitution and also demand correlations among products. When demand for products becomes more negatively correlated, accounting for substitution become more significant and profit tends to increase. Ernst and Kouvelis (1999) demonstrate that when substitute products can be purchased in bundles, positively correlated demands also tend to increase profit compared to non-correlated demands.

When it comes to pricing, results are almost intuitive. Studies prove as inventories become scarce, prices go higher, but at a constant level of inventories, time has an inverse effect on prices. As time elapses, prices decline. Substitution probabilities affect price difference among products. The higher the substitution probability, the lower the price difference among different products. The reason probably is that products are much similar to their substitute in high substitution probability cases. On the other hand, as the number of substitutes inclines, the prices range increase. It implies that the prices of the cheapest products decrease and prices of the most expensive products increase and it leads to a higher profit. Impact of variable and dynamic pricing is highlighted in literatures for substitute products particularly when inventories are scarce and procurement costs are high.

On the structure of optimal assortment for substitute products, generally speaking, an optimal assortment consists of the most popular products and products which have high average margins. Also with respect to some conditions, the optimal assortment may involve some unprofitable products, for example to reduce consumer search probability or lost sales (Cachon et al, 2005), or optimal assortment does not involve the most popular products, for example to avoid fragmentation effect (Gaur and Honhon, 2006). As a retailer increases the variety of products, profit increases but the slope of this increase become close to zero rapidly and in some conditions with high fixed and procurement costs, profit may decreases. Accounting for substitute products results in a smaller assortment size compared to assortment without substitution. Moreover, higher substitution rates leads to a smaller assortment size.

In the next chapter we provide the interesting areas which have the potentials to be studied in future research, in both bundling and substitute products fields.

Chapter 5

Conclusion

5 Conclusion

5.1 Introduction

It can be observed that different techniques of RM have been increasingly applied to manufacturing industries. An important reason may be the opportunities that operating in an e-commerce environment has offered to industries. Consequently, application of RM in the area of manufacturing and supply chain management has gained the attention of many recent academic researches. In the current thesis we reviewed most of the significant literatures which were within the scope of bundling and substitute products. Many literatures had their focus on developing insights towards different problems which exist in managing these kinds of products and the strategies which may be applicable. On the other hand, several literatures concerned the ways different problems can be modeled mathematically and solved to the optimality in order to offer firms the highest possible profit. But still there is a huge potential for further research in the field. The gap between challenges in reality and research done to the date is still significant. Some of the challenges are due to complex nature of customers' behavior in a multiproduct environment, and some others are due to the fact that even these complex behaviors can be modeled, solving these models is in most of cases complicated and are restricted to the current technologies and knowledge.

In this chapter we first discuss the RM three decision categories defined in chapter 2, as well as RM techniques which have been applied to bundling and substitute products, and the possibility for further applications. Moreover, we argue the limitations, potentials and problems which have not been addressed in literatures, individually for both bundling and substitute products.

5.2 Decision Categories of bundling and substitute problems

Here we analyze the problems discussed in chapter 3 and 4 based on three decisions below to understand how different problems in bundling and substitute products can be characterized under these decision categories.

Structural decision: as mentioned in chapter 2 these decision are mostly strategic and based on the nature and type of problems different techniques are applied to solve them and find the optimal policy. We categorize the structural decisions that are relevant to concept of bundling and substitute products as follows:

- **Bundling products:**

- Finding the best bundling strategy over the three strategies; pure bundling, mixed bundling and pure component
- Strategic decision regarding whether or not making a compatible product with the competitors' products in a non-monopoly market.
- How to change the bundling strategies over life cycle of the products or the bundle to ensure maximum profit.

In the literatures which we have reviewed in chapter 3, finding the best bundling strategy has been considered in many literatures. Market segmentation has been mentioned as one of the most important factor in applying RM techniques, however, it has not been considered clearly in the literatures which aimed to find best bundling strategies. The other important issue which is yet to be considered in literatures is the significance of products' life cycles and how selecting optimal bundling strategies would change over products' life cycles.

- **Substitute products:**

- Finding the optimal assortment of substitute products to stock
- Determining the products to be introduced in a product line of substitute products

All the literatures we reviewed regarding these decision were about finding the structure of optimal assortment or products within a product line, however, issues such as how this optimal structure may change over time and life cycle of products or how introduction of a new product affect this structure are yet be investigated.

Quantity decision:

- **Bundling products:**

- The optimal number of products to be included in a bundle
- The optimal number of bundles to be formed

Although a few researches consider the number of services that should be included in a bundle, there is still a great potential for further researches in this field, especially when products are tangible goods. Regarding inventory management, most of the researches do not consider possibility of replenishment and they consider effects the initial level of inventory on bundling strategy decision. Also the optimal number of bundles that should be formed has the potential to be studied further.

- **Substitute products:**

- Finding the optimal inventory levels for a given assortment of substitute products
- How to allocate a restricted capacity for producing substitutable products or allocating shelf space to different substitute products.

Finding the optimal inventory of substitutable products has been the main focus of most of the papers we reviewed in chapter 4, but how restrictions such as shelf space may change the optimal inventory or even optimal assortment is not investigated much in literatures.

Price decision:

- **Bundling products:**

- Finding the optimal price of bundles and single products under different bundling strategies
- Dynamic pricing of the bundles

Most of researches in the field of bundling have been studying the optimal price of the bundles and the optimal price of each single product, however, pricing techniques such as dynamic pricing have not been considered significantly in literatures to account for the effect of life cycle and inventory levels of bundles.

- **Substitute products:**

- Finding the optimal price of substitute products in presence of substitutability effects among them
- Dynamic pricing and joint price-inventory management of substitute products

Besides many research that have been done to find optimal inventory policy for substitute products, pricing and dynamic pricing of these products were focus of a few papers which we reviewed in chapter 4. On the other hand the problem of joint inventory-price decision for substitute products was studied in few papers but the analysis were highly restricted due to complexity of models.

5.3 Revenue Management Techniques for Bundling and substitute products

In this part we argue different RM techniques that have been used for bundling and substitute products and highlights techniques that are applicable to these products but have not been utilized very much to the date.

Booking limit:

This technique can be applied when the firm has the possibility of serving the customer in both bundle and pure component, however, it has not been considered in literature significantly. Using this technique a firm can use different bundling strategies depending on the remaining available inventory. For example if a company offers three complementary products called A, B and C, and this products can be sold either in a bundle or as pure components. But the firm can achieve higher revenue through selling bundle of products. If the products' initial inventory levels are not equal (50, 60 and 75 are initial inventory levels for A, B and C respectively), the company can sell the products as a pure bundle; each bundle consists of one of each products. The bundle is sold at a super-additive price for the first 45 inventory units; in other word 45 inventory units is the booking limit. After selling the 45 bundles, the number of remaining inventory are 5, 15 and 30 for products A, B and C respectively. Now the company can use pure component strategy. Therefore, higher revenue can be achieved due to super-additive price of the bundle at the beginning of the selling period.

On the other hand, practice of booking limit has been used in only few papers for managing inventory/pricing of substitute products. The reason may be the nature of these products and the ability of replenishing inventory through selling horizon for many of these products. However, there are such cases that can fit the requirement for applying the booking limit technique. For example, in selling seasonal goods or products which have to be sold before a particular point in time, like Christmas, and presence of substitute products, booking limit can be applied in order to make a balance between demands for different substitute products by controlling the availability of products. A good example can be a firm offering a number of gifts to be sold before valentine; these gifts are substitute for each other. During the selling horizon the firm notices that one of the products is more popular and selling much faster than the others. Thus the firm limits the availability of the popular product as its inventory gets scarce, in order to push demand toward two other products. Then after a while the popular product will be available at a higher price to make sure obtaining the highest possible revenue.

Another case can be situations where a firm sells a mature existing product and a newly developed version of that product that is a substitute for the existing product. Therefore a firm can control availability of the developed product to make sure all existing are sold.

Bid pricing:

Bid pricing has not been applied for substitute or bundling products since it is not common to sell consumption goods through customers' bid. But in some cases there is possibility to apply this technique. Consider the case a few items of some substitute products are available after a selling period. These products now have a very low demand and but they have been produced at a high cost, therefore the company desire to sell these products at any price as long as it is higher than their production cost. An example can be a series of laptops which are similar in their functionalities and features but they are outdated. Since these laptops are substitutes, a customer will be cautious in bidding process, because he knows that there will be other alternatives if he cannot purchase his first choice.

Over booking:

This technique does not seem to be suitable for firms offering multiproduct since advance selling and possibility of no show-up and cancelation are the prerequisites for applying this technique which is not common in selling consumption goods.

Dynamic pricing:

Literature study showed that there is a lack for practicing dynamic pricing for bundling products, but since there is not a significance different between practicing dynamic pricing for a single product or a package of products, the term of dynamic pricing for bundling products seems not to be different from single products. However, it might be interesting to investigate how existence of single products may affect dynamic pricing of bundles. On the other hand dynamic pricing techniques have been used for substitute products in a few papers, but still there are plenty of rooms for further study.

In the following part we present some possible areas for further research based on above discussion and other important factors which may further the existing knowledge and develop the insights in the area of bundling and substitute product.

5.4 Suggestion for future work:

Potentially interesting and important research areas for future study will be discussed here based on literatures we reviewed and results obtained. Since problems in bundling products and substitute products in many cases are different in their characteristics we discuss the further research area for each of them individually.

Bundling products:

There is a great potential for research in the field of bundling by relaxing the monopoly market assumption. Indeed the research in non-monopoly market for practicing bundling strategies and pricing is quite limited. A few number of studied in this regard mainly discuss firms producing two kind of tangible products and the literature are lacking the problem regarding multi service companies i.e. considering a number of competitors in a market. Each one produces several services the model can be to determine whether to have a service compatible with other services offered by the competitors as well as determining the optimal bundling strategy and the price of the bundle.

Another direction for further research can be evaluating the effect of introducing a new bundle to the market and discussing if the new bundle affects of the old bundles and old strategies and pricing. This discussion also is recommended to be made under the assumption of competitive market structure.

Also, to investigate whether it is beneficial to include a newly developed product into a bundle i.e. consider a company which have been producing a bundle and now the company has recently developed a new product the question is whether or not it is recommendable to include the new product to the bundle or in which situation it is beneficial. Products within the bundle might be either complementary or substitute.

In the research that have considered customer arrival time, the customer arrival time is modeled as a homogeneous Poission Process. Although Poission process is the most suitable one, since the rate is homogeneous it is not close to reality so modeling the customer arrival process as a non-homogeneous process can close the gap between the models and reality.

Market segmentation which is an important factor in marketing studies has not been considered significantly in researches in the field of bundling. A new direction in the field of bundling can be segmenting the market and make the bundle customize to each segment of the market in such a way to fulfill the requirement of each segment properly.

Substitute products:

In field of substitute products, apparently academic literatures in most cases have not offered an exact solution for either inventory-pricing or assortment planning due to complexity of problems. Most of papers use approximation to model underlying demand model for substitute products and heuristics are proposed to optimize the model. Therefore the most important research area could be to find better approximations or even exact solution for the problems.

Moreover, there are some contradictory results in different studies. For example, effects of substitution on inventory levels, particularly on inventory of individual items, or assortment size. Yucel et al (2009), Agrawal and Smith (2003), show that substitution may increase assortment size and decrease profit. Cachon et al (2005) also show that an unprofitable product may be introduced to the assortment. It seems that these problems have to be further investigated to shed light on characteristics of situations that these contradictory results are hold.

Researchers ignore market segments with vertically differentiated products and focus on one segment with horizontally differentiated products. In contrast with marketing literature in which market segmentation and product positioning and price discrimination among them seem to attract a great attention, this consideration of multiple consumer segments and variety of vertically differentiated products and even groups of products are taken into account in very few literatures (Bitran et al, 2006 and Aydin and Porteus, 2007). An interesting case for future study can be modeling a consumer choice with a two-step choosing probability. In the first step a customer chooses a proper segment among vertically differentiated products, and in the second step she choose among horizontally differentiated substitute products. As discussed in section 4.4.3 MNL model cannot capture this behavior but a nested MNL model may be appropriate. None of papers we reviewed consider this realistic behavior. This consideration may lead to a higher profit.

A strategic point to be considered would be market position of the firm or retailer. The assortment a retailer carries defines its image and perception that consumers have about the retailer. For example a specialty retailer offer deep assortment in a narrow variety of product categories but a warehouse offer shallow product assortment with broad variety of product categories (Mantrala, et al, 2009). One retailer may carry fashionable and expensive brands to attract prestigious customers, on the other hand a retailer may carry an unprofitable product

or brand only to maintain customers or reduce consumer's search probability. This issue also has not been considered and integrated into assortment planning except (Cachon et al, 2005). Also some retailers are known for their cheap prices (Always On Sale). Managing inventory and prices of substitute products for these stores seems to be different with respect to their customers and unique characteristics. When taking this view, other aspect of a retailer position in market and the type and mix of products she sells should be taken into account. While a broad range of variety is the main driver of consumer in one store e.g. consumer goods, price promotion or past shopping experience and services are drivers for other stores e.g. home appliance. It seems interesting to integrate the market position and type of store into models.

Customers are assumed to purchase from a store in an isolate manner and disregard with other categories of products and customers' basket. Consumers often purchase from different category of products at a particular shopping trip because of complementarities or similar purchase cycle (Manchanda et al, 1999; Russel et al, 1997). Also there are situations where a customer purchase two or more complements products from two separate categories of substitute products. An example may be cell phones and communication options. Another example could be customers who desire to make a set, like a set of sport bag and snickers from the same brand or color which probability of selecting a product depends on probability of selecting the other one. This phenomenon has been addressed by Agrawal (2003) but there is still huge potential for further study.

Considering operational and technical points of view, it is found that demand function is the key difference between literatures. While most of papers assume an exogenous demand with given distribution, some others use a MNL model to model consumer choice. Customer arrivals and their choice behaviors are mostly assumed to be Poisson process with a constant rate which does not change along the time, except Li and Chen (2009). This assumption is not realistic where consumers arrival rate change along the time e.g. in weekends. Another factor which is commonly assumed to be stable is consumer preferences. An item that represents the first choice of a consumer at a specific point in time may not be her first choice in another point in time under a different situation. Therefore no single most preferred item exist for all the time and it is a function of decision circumstances at the time of purchasing (Bettman, Luce and Payne, 1998). Incorporating time of purchase and also other aspects of consumer choice such as store choice may be a possible research direction for obtaining more profitable assortment and inventory-pricing policy. Even when a customer finds her acceptable product,

she may be willing to search for other stores and explore other alternatives (Cachon et al, 2005; Mantrala et al, 2008).

The other common assumption is that consumers buy only a single unit at each shopping trip which may not be the case in reality. In none of papers we reviewed they consider a multiple purchase behavior. The difficulty gets bigger when the MNL model cannot support this kind of behavior and maybe there is a need for more advanced consumer choice models.

Dynamic assortment planning is the other issue which has not been considered in literature. Revising the assortment is necessary since many new products are introduced. Products life cycle also intensify the necessity of dynamic assortment. Products in their late steps of life cycle require less variety than mature or new products. Demand learning through time also may help to revise the assortment and making it more profitable.

Supply chain consideration also has the potential to be integrated more in planning for price, inventory and assortment. Shelf space restriction also rarely has been considered in literatures for substitute products. McIntyre and Miller (1999) argue that for a fixed shelf space, the process of selecting and pricing the assortment become inseparable when retailer allows for cross product effects such as substitutability or complementarities. Few literatures modeled joint assortment planning and pricing, which integrated with other elements of marketing such as advertising and promotion may be another avenue for further research.

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