Network Planning of Single Frequency Broadcasting Networks

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Abstract

The Orthogonal Frequency Division Multiplexed (OFDM) scheme allows all transmitters in a radio network to transmit identical signals in the same frequency block, forming a so called Single Frequency Network (SFN). By avoiding frequency reuse, substantial improvements in bandwidth utilisation are possible. In this thesis we will focus on the coverage properties of an OFDM based SFNs, in particular the network planning of a such network. The simultaneous transmission creates severe artificial multipath propagation, which translates into intersymbol interference at the receiver. The network planning due to this kind of \textit{self-interference} is addressed in this thesis. An analysis on the receiver performance is presented, resulting in a method to perform coverage calculations. The receiver analysis show that the method that is extensively used in today’s SFN planning [6] can only be employed for some special OFDM signal constellations.

We show that the coverage properties depend on many different parameters, e.g. the OFDM constellation, transmitter power, antenna height. The length of the guard interval, which is employed to reduce the intersymbol interference between two consecutive OFDM frames, is a very important parameter. All transmitters that are situated such that they have a propagation delay shorter than the guard interval will contribute usefully at the receiver. It is thus essential to have high transmitter density to achieve good coverage. We show that for wide area (national) applications it is possible to use the existing FM infrastructure, provided that a large guard interval is employed to reduce the self-interference. In local SFN, the performance is shown to drop drastically mainly due to the low degree of transmitter diversity. Furthermore, if several closely local SFN are operating in the same frequency band it is preferable to use low transmitter antennas to reduce the interference among the networks.

Lastly, we show that an exact uniform spacing of the transmitters is not necessary. For wide area networks with an average distance of 70 km between transmitters, large displacement can be made without any significant losses in coverage. For local area networks, the maximum displacement is on the order of a couple of kilometres.
Acknowledgment

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Contents

1 Introduction ............................................................................. 1
  1.1 Background ........................................................................ 1
  1.2 Related Work .................................................................... 4
  1.3 Scope of the thesis ......................................................... 9
  1.4 Thesis outline ................................................................. 11

2 OFDM based Single Frequency Broadcasting Networks ......... 13
  2.1 The OFDM modulation scheme ........................................ 13
  2.2 Wide area coverage using SFN ........................................ 18
  2.3 Local area coverage using SFN ......................................... 19

3 Channel model ........................................................................ 23
  3.1 Introduction ..................................................................... 23
  3.2 Short term fading ............................................................. 24
  3.3 Expected local received power ......................................... 27
  3.4 Shadow fading ................................................................. 29

4 Receiver model ........................................................................ 33
  4.1 Artificial delay spread at the receiver ............................... 34
  4.2 Decorrelation on a single tap channel ............................... 36
  4.3 Expected local received power on a single tap channel ...... 39
  4.4 Receiver performance on a single tap channel ................. 43
  4.5 Decorrelation of received signals and its performance ...... 45
  4.6 The outage probability ..................................................... 48

5 Numerical results .................................................................... 51
  5.1 Assumptions ................................................................. 51
  5.2 Numerical results for wide area coverage using SFN ....... 52
  5.3 Numerical result for local coverage using SFN ............... 56
  5.4 Summary ....................................................................... 64

6 Transmitter locations and sensitivity analysis ...................... 67
  6.1 Transmitter location strategies ........................................ 67
  6.2 Sensitivity Analysis ......................................................... 72
  6.3 Summary ....................................................................... 77

7 Conclusions ............................................................................ 79
## References

83

### Appendix A.

91

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Derivation of the output from the correlation demodulator</td>
<td>91</td>
</tr>
<tr>
<td>Matrix representation of the demodulator output for all carriers</td>
<td>94</td>
</tr>
<tr>
<td>The interference distribution</td>
<td>96</td>
</tr>
<tr>
<td>Derivation of the normalised interfering phase, $\eta$</td>
<td>99</td>
</tr>
<tr>
<td>Derivation of the expected received power</td>
<td>100</td>
</tr>
<tr>
<td>Derivation of the weight function $R(\tau)$</td>
<td>102</td>
</tr>
</tbody>
</table>
Chapter 1
INTRODUCTION

1.1 Background

Radio broadcasting systems have the capacity to deliver an enormous amount of information to many receivers all over the world. Currently, the offered service could be an audio or a TV program. Despite the fact television has become an increasingly dominating medium in the world over the past 30 years, audio broadcasting still remains the major source of information and entertainment for the vast majority of the population. Moreover, audio is now experiencing a renaissance with its uniqueness to supply different radio programs to an increasing mobile audience.

In broadcasting, the operator wants to supply their programs to large regions, for instance, over a city or a country. The main difference among broadcasting systems is the size of the area where the transmitted information is to be received. The two most noted regional broadcasting services are the community radio and the local radio. The community radio sends out programs over a very small area of only a couple of kilometres. The equipment needed is cheap, and the transmitter antennas are often placed at a low elevation and transmit at very low power, on the order of 10 W. A more intense market in Sweden is the rapidly developing private local (commercial) FM radio. Today, there are twelve private radio stations in the Stockholm area. They use more sophisticated equipment to enhance the sound quality. The radio stations typically have a single transmitter site with a moderate radiated power on the order of 1kW, reaching areas located 30-40 km from the transmitter site.
For wide area services, the system becomes more complicated. To support large areas such as a country, the required infrastructure consists of a large number of transmitter sites regularly spread out. The adjacent transmitters are required to use different frequencies when broadcasting the same program to avoid interference at the receiver. To reduce the cost of the infrastructure it is preferable to have large distance between adjacent transmitters. However, the distance between the transmitters cannot be too large. A transmitter can only serve a limited area, due to the spherical structure of the earth, where receivers beyond the “radio horizon” experience a severe degradation in received powers resulting in reduced reception quality. By varying the height of the transmitter antenna, the distance to the “radio horizon” can be adjusted from a couple of kilometres up to 100 km. Typical antenna heights in the national FM broadcasting system lie between 100-300 meters.

The main limitation when providing radio communication services for a large number of programs is the shortage of radio frequency spectrum. Thus, it is essential to use the spectrum efficiently. A fundamental approach to achieve high spectrum utilisation in radio networks is to reuse the spectrum in geographically separated area. When reusing the spectrum, the transmitters must be sufficiently separated spatially such that they cannot interfere with each other. This is the external interference aspect of the network planning, which requires a frequency reuse planning strategy (see for instance [4] [34]). An illustration of a network with frequency reuse is shown in Figure 1.1.
A new type of broadcasting system is currently being developed, which is able to serve an arbitrary large area with the same program without reusing frequencies. This new type of network is often referred to as a **Single Frequency Network** (SFN). For broadcasting services where the amount of needed bandwidth is high, e.g. television broadcast, this offers enormous advantages. By avoiding frequency reuse, substantial improvements in bandwidth utilisation are possible. Figure 1.2 illustrates the differences between a conventional frequency reuse network and a Single Frequency Network.

In this thesis we will look at this new type of broadcasting system. An SFN can both be implemented for both wide area or local (small) area applications. Provided that the infrastructure consists of many transmitters spread out over a large area, the SFN can also have an arbitrary sub-area within this larger area as a local service area. The shape of this local sub-area can vary during operation depending on where the broadcaster wants to cover, which could be an attractive feature in future radio communication systems.
1.2 Related Work

Two new digital broadcasting systems are now under construction in Europe, one for audio [5] and one for TV [88]. The existing broadcasting systems are very costly from the operator’s point of view, and one of the strongest reason for developing new systems is to drastically reduces the operational and maintenance cost. Moreover, the lack of available frequencies is also a problem. For example, it is almost impossible to establish new radio channels in some European countries. The reception quality is an important requirement of the new systems. The introduction of digital techniques in sound production has brought the listeners very high sound quality, largely through the rapidly growing market for digital HI-FI equipment such as compact disc (CD) players. Unfortunately, the analog FM sound broadcasting system is not capable of providing this quality to its listeners. Two of the problems with FM technology is its sensitivity to multipath interference during mobile reception and the relatively poor frequency economy.

To meet these new demands, digital audio broadcasting has been proposed. One such proposed system is the European DAB system which has been developed within the EUREKA-147 group supported by the EBU, the European Broadcasting Union [5]. This consortium has participants from most of the public European broadcasting companies. The standardization of the European DAB system has now been finished by the European Telecommunication Standardization Institute (ETSI) [5], and a good description on the DAB system can be found in a collection of papers [37] published by EBU. The DAB system uses sophisticated signal processing to compress and transmit digital signals. The radio channel causes multipath interference at the receiver which generates two effects, frequency selective fading and intersymbol interference. To counter the multipath interference, a method called Orthogonal Frequency Division Multiplexing (OFDM) has been proposed, [7], [36], [48]-[72]. In this modulation scheme, a high rate data stream is modulated onto a large number of orthogonal narrow band frequency multiplexed signals. The slow bit rate on each carrier and the insertion of a guard interval between consecutive OFDM signals make the symbols sufficiently long to overcome the problem with intersymbol interference. The information transmitted on the carriers that are affected by the frequency selective fading can be retrieved by means of powerful error correcting codes together with time and frequency interleaving [35]. The main drawback with
OFDM schemes is caused by Doppler shift due to receiver’s moments. This causes losses in subcarrier orthogonality, also termed interchannel interference (ICI), and results in an error floor [52] [72]. By using diversity schemes, e.g. antenna diversity and error correcting codes, the performance can be improved [72]. The concept of OFDM has shown to be of great interest not just for sound broadcasting services, but also for terrestrial digital TV systems. All of the digital television broadcasting systems proposed for Europe use OFDM [94], and one of these system, the HD-DIVINE [96] modem, has also been a candidate to be a U.S standard [95].

When distributing a program over a number of transmitters, the COFDM scheme does not require the transmitters to use different frequencies. In contrast to the analog systems, all transmitters in the radio network broadcast the same information in the same frequency block simultaneously [20]. The receiver in such a network experiences a severe artificial multipath propagation as illustrated in Figure 1.3. Provided that the receiver is able to overcome this problem, substantial improvements in frequency economy can be made. In particular for wide area applications, this is very attractive, since no extra frequencies are needed for frequency reuse as in a conventional network. In both [78] and [85] national network planning is addressed showing that the frequency economy is improved by an order of magnitude compared to the existing FM-network. The networks in [78] and [85], are ideal with no interference from other closely located networks. When planning a real national network, rules and regulations need to be considered such that other networks can co-exist. Interference problems, in particular, at the borders between countries must be considered and as a result reduction in spectral efficiency is made.

![Figure 1.3](image_url)

**Figure 1.3** An illustration of self-interference in national SFN with one transmitter in the centre of each hexagon.
Analog systems have a smooth transition from good to poor quality and can thus output a poor quality without any sudden outages. Digital systems, such as DAB, suffer from abrupt failures of the system unless high requirements are put on the coverage properties. In [74], they claim that a coverage probability higher than 90% is needed for the DAB system. In terrestrial digital-TV broadcasting, an hierarchical digital transmission scheme has been proposed [72] [91] [92] to avoid these sudden outages. The basic idea is to employ multiresolution modulation schemes that allow reception of various levels of video quality depending on channel conditions. This allows for graceful degradation and increases the transmission region over which video of some quality can be received.

The coverage properties in an SFN is improved due to the diversity of receiving signals from many transmitters [12]. Thus in SFN, it is now possible to attain better coverage properties by inserting more transmitters into the service area. The performance in all diversity system depends on how many signals it receives and if the signals are correlated. Often in such systems, the performance increases as a function of the number of diversity branches, i.e. more received signals yield better performance. However, if the received signals are correlated, then the performance often decreases compared to the uncorrelated situation. In SFNs, the coverage properties are therefore expected to change due to these two aspects. There are also other limitations in SFNs, signals received from transmitters very far away can cause intersymbol interference instead of providing an useful contribution. This kind of self-interference is particularly noticeable in wide area networks with sparse transmitter density. The planning process for wide area networks is now concerned with avoiding excessive propagation delays rather than avoiding frequency reuse interference. The effects of receiving several delayed OFDM signals have been studied in [6] [7] [9], where the expected useful and interfering components are derived. The guard interval between two consecutive OFDM signals is a very important parameter in “self-interference” planning. All signals that arrive at the receiver within the time span corresponding to the duration of the guard interval are treated as useful. All other received signals partly or totally interfere [6] [7] [9]. The drawback with all these papers is that no explanation is given on how the expected useful and interfering components relate to the performance of the receiver. A reason why no results have been produced could be that by the receiver manufactures consider this information as confidential. Nevertheless, the expected useful and interfering components are extensively used in the SFN planning.
1.2 RELATED WORK

The network planning for local single frequency networks is a mixture of frequency reuse planning and “self-interference” planning [25]. Figure 1.4 illustrates several local SFNs. Each network generates its own interference environment while the other nearby SFNs generate the external interference. Recently, the Swedish government has decided to introduce local networks in Sweden [97]. The intention is to have local networks in Stockholm, Gothenburg and Malmö. Since these cities are somewhat separated, the external interference can be neglected and only self-interference planning is necessary. To achieve good coverage in such networks is a challenging task. The number of transmitters required to serve an area must be determined, as well as the configuration of the transmitter sites, such as antenna height, radiated power, direction.

How to design local SFN with several other nearby SFNs is an even more difficult task. To achieve high spectrum efficiency, it is essential to be able to reuse the spectrum within a short spatial distance. How close can the networks be located? Does there exist a minimum reuse distance? Where should the transmitters be located to minimise interference among networks and still maintain good coverage in each service area? As we see all these questions relate to how the configuration of the network should appear to achieve a short reuse distance. To limit the interference between networks, a limit on the maximum allowed interfering field strength has been applied. This is a good way to avoid interference between countries and has also been applied in the planning of the FM-system. Today, the digital systems are not dependent on the actual magnitude of the interfering field-strength. The limiting factor in these systems is instead the ratio between the useful and interfering field-strength. It would be nearly impossible to support dense local networks if a restriction was put on the field-strength. To utilise the spectrum efficiently, network planning should focus on increasing the ratio between the useful and interfering field-strength.

Figure 1.4 Several local SFNs. Same frequencies are reused in each of the gray areas.
Most of the work done so far on SFN planning has been performed within the EUREKA-147 together with the EBU (R1-DIG) [5]-[33]. Three modes of operation have been defined for the European DAB standard. Mode-I and II are supposed to be used for terrestrial networks for national and local area SFNs. Mode-III is intended to be used for satellite and hybrid satellite-terrestrial broadcasting. Although the EBU (R1-DIG) project has developed a method to calculate the coverage probability, no real method to design an SFN has been produced. The main reason that trial and error is often used is due to the large complexity of the problem. To reach a more structured network planning, some general planning rules are needed. By applying these rules a good initial network configurations could be obtained. To find even better solutions, sophisticated optimization schemes can be used where the initial state is given from the rules of thumb [98] [99].

In the literature, coverage results have been presented in two ways. A continuous coverage plot over the service area, or calculation of the outage probability. An exact treatment of the relevant statistics for continuous coverage plots over the whole service area makes excessive demands on the computing time. Approximation methods [8], [75]-[77] have therefore been used when summarising the useful and interfering field strengths on a power sum basis, where the distribution of the resulting powers are assumed to be lognormal. The coverage probability for a fixed location is then derived from the probability that the useful field strength is larger than a minimum field strength threshold [8] [20]. The threshold is derived using the noise power, the self-interference and protection ratios.

In [78], a first study on the outage probability was performed, where both national and local area networks were considered. The outage probability is defined as the probability that a randomly selected receiver within the service area cannot receive the transmitted program. The results in [78] show that very good coverage (>99%) can be achieved in national area networks. At modest transmitter powers (typically 100 W/program) the system becomes limited by its own self-interference and no further improvements is obtained by increasing the power level. For local networks with only a few transmitters in each region the performance is shown to drop drastically mainly due to the low degree of diversity.

The spectral efficiency of local SFNs has often been questioned, especially since
the natural frequency gain as in a wide area SFN cannot be obtained. In spite of this, we expect to be able to reuse an OFDM frequency block within a short distance. This requires careful network planning to avoid propagating interference to other nearby local SFNs. By using directional antennas pointed towards the central parts of the service area, it is possible to reduce the external interference generated among the local SFNs [22] [103]. Other design methods can also be applied. In particular, this thesis investigates how the height of the transmitter antenna influences the coverage. By changing the antenna height of the transmitter, we can use the increased propagation loss beyond the radio horizon as a beneficial feature in the design of the network. Previous results on conventional FM-planning [101] have indicated that it is necessary to locate the interfering transmitters beyond the radio horizon to obtain good coverage. Furthermore, for local networks with smaller service areas, very lower radiated transmitter power is expected to be used. Although the power does not have the same influence on the system performance as in a conventional FM network, it is advantageous to reduce it as much as possible.

The planning parameters for single frequency networks are the subject of several field trials [42]-[47]. A comparison of measured and predicted field strength is presented in [45]. The predicted field strength is the median received path loss according to the CCIR (the International Radio Consultative Committee) recommendation for broadcasting over land [1]. The results show that the predicted field strength needs to be adjusted by a factor 10 due to a change in the receiver antenna height from 10 meters to 2 meters. The network performance has been measured in [46], and the results so far indicate significant improvements in coverage due the diversity of receiving several signals. The variation of the signal level is also shown to decrease even though there are relatively large differences in the signal level from the different transmitters.

1.3 Scope of the thesis

In this thesis we will focus on the coverage (outage) properties of OFDM based SFNs. The symbol error rate in an OFDM receiver is investigated for a channel with an artificial delay. This is to verify the proposed performance measure in [6]. The influence of correlated signals and the diversity gain of receiving signals from several transmitters will be evaluated for both national (self-interference
only) and local (both self and external interference) single frequency networks.

We also investigate different methods to reach high efficient local SFN. Results on conventional FM-planning [101] have indicated that it is necessary to locate the interfering transmitters beyond the radio horizon to achieve good coverage results. By changing the antenna height, the increased propagation loss beyond the radio horizon can be used in the design of the network. The impact of different antennas heights is therefore also addressed in the thesis. Moreover, in today’s FM system the power is a very important parameter. To cover a wide area the transmitters emit on the order of 10-100 kW per program. In national SFN, it has been shown that a very good coverage can be obtained by using modest power (100 W). In local networks we expect to use even lower power levels.

The length of the guard interval can affect a local network more than a national network, in particular at the borders due to the low degree of diversity. The impact of the guard interval length and the density of the transmitters which has been shown to be very important, will be analysed for local networks. For local networks we attempt to provide solutions to the following questions:

- How does the guard interval length affect the system?
- How many transmitters do we need to cover a given area?
- Do we need to use high radiated powers to yield good coverage?
- How will the height of the antennas affect the coverage?

Lastly, we study sensitivity/robustness for both wide area and local single frequency networks. One main issue is where the transmitter should be located to reach good coverage. How sensitive/robust is the coverage property for different transmitter locations, i.e. is it important to spread the transmitter regularly over the service area or can a randomly selected transmitter position also establish the same performance? If only a few number of transmitters are available, a careful planning strategy is perhaps necessary. On the other hand, a random spread of transmitters could yield enough coverage if the number of available transmitters is large. By providing the network designer with a rough planning strategy, this problem can be solved.

Throughout our investigation, we use the ETSI DAB [5] standard as a numerical example. However, most results are not confined to this application and can be generalised to other SFN applications.
1.4 Thesis outline

In the next chapter, the OFDM modulation scheme is described together with some basic models that are used for wide area and local area single frequency networks. The most important parameters for the system are also defined. In chapter 3, the broadcasting channel model is described in detail using models that consider site parameters such as antenna height, power and transmitter location.

The performance of a receiver subjected to an artificial delay spread channel is evaluated in chapter 4. It includes detailed descriptions of how different carriers in the OFDM signal cause self-interference within the network. The analysis concludes by deriving a weighting function that assumes a crucial role in the subsequent SFN planning. Our coverage performance measure (the outage probability) is also defined. Numerical results are presented in chapter 5 for both wide area and local area single frequency networks.

A sensitivity/robustness study is performed in chapter 6 to evaluate if an exact transmitter location planning strategy is necessary in SFNs. Chapter 7 summarizes the conclusions.
Chapter 2
OFDM BASED SINGLE FREQUENCY BROADCASTING NETWORKS

In this chapter, we describe the OFDM modulation scheme and the models for wide and local area networks using Single Frequency Networks (SFNs). The chapter starts with a description of the modulation scheme and some of its important parameters, and proceeds with some basic definitions of the network models.

2.1 The OFDM modulation scheme

A communication link in any digital radio communication system can be described by Figure 2.1. A data sequence, denoted \( \{d\} \), is mapped to a signal constellation \( x(t) \) in the modulator. The signal is sent from the transmitter, distorted by the time varying radio channel, \( h(t) \). At the receiver, noise, \( z(t) \), is added before the receiver makes an estimate, \( \{\hat{d}\} \), of the transmitted data sequence.

The modulation scheme for DAB is called OFDM (Orthogonal Frequency Division Multiplexing). This novel modulation scheme has been considered as a candidate for many different applications such as digital transmission at high rates over twisted-pair telephone subscriber loops [67], mobile radio systems [50], and broadcasting systems [5] [88]. The attraction to OFDM is mainly due to its nice properties. It has been proven to be robust against multipath interference, and combined with error correcting codes (COFDM), it can handle very severe interference environments such as the mobile radio channel.
OFDM is based on the theory of Fourier transform, where the signal is modulated by means of an inverse Fourier transform which yields the following multiplexed signal

\[
x_r(t) = Re \left[ x(t) \right] \\
= Re \left[ \sum_{q=-\infty}^{\infty} \sum_{l=0}^{N-1} D_{q,l} p(t - qT_{tot}) e^{j2\pi f_l(t - qT_{tot})} \right] \\
= \sum_{q=-\infty}^{\infty} \sum_{l=0}^{N-1} I_{q,l} \cos \left( 2\pi f_l(t - qT_{tot}) \right) p(t - qT_{tot}) \\
- \sum_{q=-\infty}^{\infty} \sum_{l=0}^{N-1} Q_{q,l} \sin \left( 2\pi f_l(t - qT_{tot}) \right) p(t - qT_{tot})
\]

where \(N\) is the number of carriers, \(T_{tot}\) is the total duration of a symbol and \(D_{q,l}\) is the information symbol on carrier \(l\) in frame \(q\), e.g. in QPSK \(D_{q,l} = \pm \left( 1 / \sqrt{2} \right) \pm j \left( 1 / \sqrt{2} \right)\). The definition of the inphase component, \(I_{q,l}\), and the quadrature component, \(Q_{q,l}\), can be read out directly from (2.1). The OFDM modulator is shown in Figure 2.2. The equivalent complex representation of the OFDM signal is denoted by \(x(t)\), which will be used throughout in the forthcoming analysis. The transmitted signal \(x(t)\) consists of several OFDM frames. Each frame, denoted by \(q\), is transmitted under \(T_{tot}\) seconds, and the information symbols are minimally separated in frequency, i.e. \(f_l = f_0 + l/T\). The signal \(p(t)\) is assumed to be a rectangular pulse of length \(T_{tot}\)

\[
p(t) = \begin{cases} 
1 & -T_g \leq t \leq T \\
0 & \text{otherwise}
\end{cases}
\]

where \(T_{tot} = T_g + T\). \(T_g\) represents the duration of the useful symbol and \(T_g\) is a guard interval between two consecutive OFDM frames. The guard interval is a cyclic extension in time of the OFDM signal.
The shape of $p(t)$ is rectangular in DAB, but other pulse shapes such as the raised cosine can be employed. In this thesis, we consider only the rectangular pulse shape. At the receiver side, the inverse procedure is applied as illustrated in Figure 2.3. The correlation demodulator maps the received signal into a complex symbol representation, and the receiver then estimates the transmitted sequence. The
receiver is analysed in detail in Chapter 4.

A receiver in a radio network experiences multipath propagation caused by reflection in the vicinity of the receiver. The reflected signals arrive to the receiver with slightly different delays. As a result, intersymbol interference (ISI) is introduced into the system, mainly between consecutive OFDM frames. The length of the guard interval then becomes important and must be chosen with some care to prevent ISI. It has been shown that ISI can be neglected in many cases if the length of the guard interval is chosen to be longer than the mean delay spread of the channel. There are also drawbacks when introducing a guard interval. A fraction of the available bandwidth, \( \frac{T_g}{T + T_g} \), is lost provided that it is not utilised for other purposes. In [71] for instance, the guard interval is decisive in obtaining a low complexity frame synchronisation. The power efficiency also decreases by the same factor. The OFDM spectrum density, when the guard interval is omitted, can be expressed as

\[
S(f) = T \sum_{k=0}^{N-1} \sin^2 (f - f_k) \tag{2.3}
\]

yielding a spectrum where the carriers are overlapping in the frequency domain as depicted in Figure 2.4. However, it can easily be shown that all signals are orthogonal in the time domain. Due to the minimal carrier separation, the OFDM scheme becomes sensitive to frequency errors and the Doppler spread [68] [72].

![Figure 2.4 Signal spectrum for different carriers](image-url)
The capability to overcome multipath interference has made it possible to form a Single Frequency Network, where all transmitters use identical signals that occupy the exact same frequency block. The simulcasting creates a severe artificial delay spread at the receiver side, as graphically illustrated in Figure 2.5. The delay spread of these signals depends on the distance to the transmitters. By applying a large guard interval, the OFDM receiver is capable of handling to some extent the interference generated by the network. The losses in bandwidth by using a large guard interval is negligible compared to the substantial improvements in frequency economy from not requiring the transmitters to use different frequencies as with conventional systems.

Due to the artificial propagation delays between the transmitters and the receiver, the interval in which the receiver may receive energy corresponding to a certain transmitted symbol is usually much larger than the used detection time (the receiver window) [78], see Figure 2.6. The receiver needs to choose a time (frame) synchronisation position for this window to obtain as much energy as possible from the received signals. A study in this problem has been performed in [102], where several schemes are evaluated.

**Figure 2.5** Multipath propagation due to simulcasting.

**Figure 2.6** a) The transmitted symbol. The symbol consists of the basic symbol (T) and the guard interval (T_g). b) Propagation delayed signals from different transmitters and the receiver window of length T.
Three European DAB modes have been defined by the European Telecommunication Standardization Institute (ETSI) [5]. Mode-I is supposed to be used for national terrestrial single frequency networks in bands I-III with a maximum distance of 100 km between adjacent transmitters. Mode-II is intended for local SFN in bands I-V and in the L-band. However, recent studies suggest using the L-band also for national terrestrial networks [41]. For Mode-II, the distances among the transmitters can be as much as 25 km. Mode-III is intended to be used for satellite and hybrid satellite-terrestrial broadcasting below 3 GHz with a maximum terrestrial inter-transmitter distance of 12.5 km. The different DAB mode parameters are shown in Table 1.

### Table 1. DAB parameters

<table>
<thead>
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<th>mode-II</th>
<th>mode-III</th>
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<td>1536</td>
<td>384</td>
<td>192</td>
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<tr>
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<td>246 µs</td>
<td>62 µs</td>
<td>31 µs</td>
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<tr>
<td>T</td>
<td>1000 µs</td>
<td>250 µs</td>
<td>125 µs</td>
</tr>
<tr>
<td>T&lt;sub&gt;tot&lt;/sub&gt;</td>
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<td>312 µs</td>
<td>156 µs</td>
</tr>
<tr>
<td>f&lt;sub&gt;o&lt;/sub&gt;</td>
<td>230 MHz</td>
<td>230 MHz or 1.5 GHz</td>
<td>1.5 GHz</td>
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<tr>
<td>Total bandwidth</td>
<td>1.5 MHz</td>
<td>1.5 MHz</td>
<td>1.5 MHz</td>
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</table>

**2.2 Wide area coverage using SFN**

To reduce the cost for a wide area terrestrial network, large spatial distances between transmitters are preferable. Currently the national FM system has an average distance of 60 km between any two transmitters. When introducing a new terrestrial wide area broadcasting system, such as intended for DAB, it is of great economical interest to be able to utilise the existing infrastructure. Therefore, the DAB system has a certain mode which is designed for national coverage, mode-I.

The interference environment for a wide area SFN differs from the interference in conventional radio systems. There is no frequency reuse in wide area SFN, thereby allowing the external interference to be neglected. We emphasis that this only holds for very large countries where no constraints are made within the country. For small countries, problems with external interference occur at areas
2.3 Local area coverage using SFN

SFN techniques provide very high frequency economy in wide area coverage applications. The majority of radio broadcasters must also distribute programs destined for only one region. Such networks cover limited areas and use only a few transmitters. A small region cannot retain the same diversity gain as in a wide area network. In contrast to wide area coverage, local networks are subjected to external interference which requires a separation among local networks. In our evaluation, both external interference and self-interference must be considered.

Figure 2.7 A model for a national network built with hexagonal services areas. $R$ is the radius of the cell and $D$ is the adjacent transmitter distance.
The service area in local networks can assume many shapes, and the transmitters location may not be chosen at will. However, we assume that the service area can be modelled as a set of local networks of roughly hexagonal shape. A local network consists of $M$ hexagons each with a transmitter with an omnidirectional antenna located in the centre. Each local network uses one OFDM frequency block to broadcast its programs. A total of $K$ OFDM frequency blocks are assumed to be available. $K$ is therefore also the frequency reuse factor of the network. Since the radio spectrum is a limited resource, it is essential to have a low frequency reuse factor, particularly in broadcasting systems where the applications uses a large bandwidth, e.g. Television broadcast. In FM-broadcasting, the minimum published frequency reuse factor is seven [73], whereas in reality this factor is close to 9. By introducing local SFN designed to handle severe external interference, gains in frequency economy could be made. The local networks are further assumed to be none overlapping as depicted in Figure 2.8. The networks can of course be overlapping, which would result in an even more denser network configuration. Figure 2.8 shows some different configurations for local networks for various combinations of $M$. A local network is also characterised by the following two parameters:

![Figure 2.8](image)

**Figure 2.8** A model for local area network with frequency reuse factor 3 and $M=\{1,3,7,19\}$. 
• The *radius of the local service area*, $R_{\text{service}}$, defined as the largest distance from the centre of the local network to a point at the border (Figure 2.8).

• The *reuse distance*, $R_{\text{reuse}}$, defined as the minimum possible distance between a transmitter within the local service area and the border of an external interfering SFN (Figure 2.8). The reuse distance can directly be determined from the radius of the local service area, $R_{\text{service}}$, and the frequency reuse factor, $K$. 
Chapter 3
CHANNEL MODEL

In this chapter, we describe our model for the broadcasting channel. The chapter starts with a short introduction on the different parameters that affect the channel characteristics and then continue with a more detailed description in the three subsequent sections.

3.1 Introduction

The radio communication channel is characterised by many different parameters. Propagation measurements characterise the radio communication environment where the signal fading can be decomposed into a path loss component together with a slow varying long-term (shadowing) component and a fast (short-term) varying component. The path loss component is modelled as a deterministic function describing the distance dependent losses whereas the other two components vary depending on the receiver location. For broadcasting systems with large spatial transmitter distances, the spherical structure of the earth needs to be considered. For instance, receivers beyond the “radio horizon” experience a degradation in received power. The distance to the “radio horizon” can be adjusted by the height of the transmitter antenna. This type of phenomenon is thus included in the path loss component. Other important transmitter site parameters are the directivity (antenna pattern) and the radiated power.

Radio signals are not only subjected to distance dependent propagation loss, but also to path loss affected by the topography of the terrain. The shadow fading is caused by large obstacles in the terrain between the receiver and the transmitter. Moreover, reflections of various types of signal scatters, both mobile and stationary, cause a multipath propagation environment at the receiver. The received radio signal thus consists of a “sum” of many reflected signals. This type
of fading is often referred to as multipath fading or short term fading. How these different parameters are related is shown in the forthcoming sections.

3.2 Short term fading

The short term fading is generated by scattering in the vicinity of the receiver. The multipath fading is often modelled as a wide sense stationary uncorrelated scattering (WSSUS) channel [4]. The channel for transmitter $i$ is modelled as an $M$-tap delay line, where each channel tap represents reflection of the transmitted signal in the vicinity of the receiver. The impulse response of the channel is given by

$$h(t) = \sum_{c=1}^{M} h_{i(c)} \delta\left(t - \tau_{i(c)}\right),\quad (3.1)$$

where $\tau_{i(c)}$ is the delay of tap $i^{(c)}$. The delays are assumed to be independent random variables uniformly distributed between $[\tau_i, \tau_i + T_a]$, where $\tau_i$ is a constant, defined as the propagation delay between transmitter $i$ and the receiver. $T_a$ is the mean delay spread of the channel [70]. The envelope of channel tap $h_{i(c)}$ is assumed to Rayleigh distributed, i.e. $h_{i(c)}$ has following probability density function

$$Pr\left(h_{i(c)} = a\right) = \frac{a}{\sigma_m^2} e^{-a^2/2\sigma_m^2},\quad (3.2)$$

where $\sigma_m^2$ is a function of the expected local received power $\overline{P_{rx}}$ (see section 3.3). We assume that the expected received power from each tap is a constant, i.e. $\sigma_m^2 = \sigma^2 \forall m$. A typical impulse response in a single frequency networks is depicted in Figure 3.1.

The coherence time is a measure of how fast the channel changes, e.g. the channel characteristics change when a receiver is moving. The channel can be modelled as constant during one symbol time if the coherence time is much larger than the time it takes to send a symbol. The time dependency (the autocorrelation) of the channel is described by [3].

\[ \rho_i(\Delta T) = \left[ J_0\left(\frac{2\pi v_f}{c} \Delta T\right) \right]^2, \]  

(3.3)

where \( J_0(x) \) is the zero order Bessel function, \( v \) the velocity of the mobile receiver and \( c \) the speed of light. The correlation is close to zero for \( J_0(2.45) \) which yields the coherence time, \( (\Delta T)_c \). The following relation must be fulfilled to avoid problems with a channel that changes too rapidly

\[ (\Delta T)_c = \frac{2.45 c}{2\pi v_f} > T_{tot}, \]  

(3.4)

where \( T_{tot} \) is the total symbol duration of an OFDM-signal. In DAB (Digital Audio Broadcasting) it is desirable that the channel has the same characteristics between two consecutive OFDM-frames. The different DAB mode parameters together with the coherence time for the channel are shown in Table 2. The results of Table 2 indicate that it is appropriate to assume a constant channel over two consecutive symbols.
Another important parameter characterising the fading channel is the coherence bandwidth, \( (\Delta F)_c \). The coherence bandwidth gives insight into how much of the transmitted signal bandwidth is exposed to a similar channel. A channel is said to be frequency selective if the signal bandwidth is wider than the coherence bandwidth, whereas the channel can be considered as flat or frequency non-selective if the signal bandwidth is narrower than the coherence bandwidth. The coherence bandwidth can be determined using \[ 3 \]

\[
\rho_f (\Delta F) = \frac{1}{1 + (2\pi FT_0)^2},
\]

where \( T_0 \) is the mean delay spread of the channel. To reach a low correlation the frequency separation must abide by,

\[
1 + (2\pi FT_0)^2 \ll 1.
\] (3.6)

A good estimate of the coherence bandwidth is obtained when the denominator in (3.6) is equal to 10, that is, a correlation coefficient close to 0.1 yielding

\[
(\Delta F)_c = \frac{3}{2\pi T_0} = \frac{1}{2T_0}.
\] (3.7)

In a single frequency network, the variation in \( T_0 \) is dependent on the location of the receiver. In some parts of the service area, the artificial delay spread determines the length of the delay spread, e.g. when there is large variations in the propagation delay between the receiver and any transmitter. In such cases, \( T_0 \) could be on the order of \( ms \) yielding a very narrow coherence bandwidth. In other
parts of the service area, all signals arrive at almost the same time to the receiver, and the “natural” delay spread determines the mean delay spread. In suburban and urban areas, the delay spread for a single transmitter is between 0.1-10 μs. The coherence bandwidth is then in the range 0.05-5 MHz. The maximum “natural” delay spread of the channel is assumed to be 5 μs in this thesis. With this simple analysis of the channel we can conclude that the channel is slow varying and could be either frequency selective or flat faded.

3.3 Expected local received power

The expected local received power from transmitter \( i \) is given by

\[
P_{rx} = E \left[ \left( \sum_{c=1}^{M} h_{i,c} \right)^2 \right] = \sum_{c=1}^{M} E \left[ (h_{i,c})^2 \right] = \sum_{c=1}^{M} \sigma_c^2 = 2M\sigma^2 = \frac{P_{erp}Y_i}{L(r_i)} = P_i
\]

(3.8)

where \( P_i \) is known as the expected local received signal power, and is generated by transmitter \( i \) at a distance \( r_i \) from the receiver. \( L(r_i) \) is the corresponding average (median) received path loss according to the CCIR propagation model for broadcasting over land [1]. \( P_{erp} \) is the effective radiated power and \( Y_i \) is a random variable describing the shadow fading effect.

The current CCIR propagation model are established for a receiving antenna height of 10 meters. This model comprises sets of curves relating field strength to distance with transmitter antenna height as an parameter and was derived to predict the path loss when designing FM-systems for stationary receivers. Since we are interested in mobile receivers, the model requires adjustments to compensate for a receiving antenna height of roughly two meters. At this low height, the shadowing is more evident, and field trials [45] have indicated a need to decrease the received power level in the CCIR model by 10 dB to accommodate the new antenna height. The relationship between field strength and power for an omnidirectional receiver antenna with an antenna gain of 1 is [34]

\[
P = \left( \frac{3 \times 10^8 E}{2\pi f} \right)^2 \frac{1}{120} \quad [W],
\]

(3.9)

where \( E \ [V/m] \) is the field strength and \( f \) is the used frequency in \([Hz]\). The average
(median) received path loss can now be obtained as

\[
L(\xi)_{\text{dB}} = E_{\text{ccir}}(\xi)_{\text{dB (\mu V/m)}} + 20\log(\frac{3\times10^8}{2\pi f}) - 120 - 30 - 2.15 - 10 - 10\log(120)
\]

(3.10)

where \(E_{\text{ccir}}(\xi)_{\text{dB (\mu V/m)}}\) is given in the CCIR curves (Figure 3.2). We also compensate for the specific transmitter configuration employed in the CCIR curves, i.e. a power corresponding 1 kW (-30 dB) from a half-wave dipole (-2.15 dB).

An interesting propagation phenomenon in broadcasting applications is the effect of the radio horizon. This effect is included in the CCIR propagation model where the received power beyond the radio horizon drastically decreases. The distance to the radio horizon, \(D_{\text{RH}}\), can be approximated by [34]

\[
D_{\text{RH}} = 4.1 \left( \sqrt{h_t} + \sqrt{h_r} \right) \quad [\text{km}]
\]

(3.11)

\[\text{Figure 3.2} \quad \text{Field strength (dB(\mu V/m)) for 1 kW using half-wave dipole.} \quad h_t=\{37.5, 150, 300\} \text{ meters.} \quad \text{VHF} = 30-250 \text{ MHz, UHF} = 450-1000 \text{ MHz.} \]
where \( h_t \) and \( h_r \) are the antenna heights for the transmitter and the receiver (meters), respectively. For higher frequencies the effect of the radio horizon is more pronounced, as illustrated in Figure 3.2. The external interference generated onto other nearby SFN can be reduced by using the increased propagation loss beyond the radio horizon. A design goal for a local SFN is to cover as much as possible of its service area. We can easily achieve high coverage by using high antenna positions. However, choosing the antenna height trades-off the coverage in the SFN and the generated external interference to other nearby SFNs. Thus, if the antenna height is chosen such that the distance to the radio horizon is larger than the reuse distance \( D_{RH} > R_{reuse} \), see Figure 2.8), a substantial portion of the transmitted signal power will reach another region using the same frequency block. Another approach to reduce the generated interference is to decrease the other transmitters’ radiated power. Note that the shape of the path loss is maintained, although the level is decreased.

### 3.4 Shadow fading

The shadow fading has been characterised in the literature by an one-dimensional distribution which is well approximated by a log-normal function [4]. The shadowing gain \( Y_i \) for transmitter \( i \) can thus be expressed as,

\[
Y_i = 10^{\vartheta_i/10},
\]

where \( \vartheta_i \) is normally distributed with zero mean and a standard deviation \( \sigma \) of 8.3 dB [34].

The coverage in a single frequency network is dependent on receiving signals from several transmitters [78], yielding a diversity gain. The performance in a diversity system increases as a function of the number of diversity branches. Furthermore, the correlation properties between the received signals are essential, where high correlation often yields a degradation compared to an uncorrelated situation. In SFN, a similar shadow fading gain could be obtained if a large obstacle shadows the signal from several transmitters as depicted in Figure 3.3. The correlation properties of the shadow fading between the receiver and the transmitters are therefore of great interest, since it can change the performance.
Figure 3.3 Shadow fading environment

Let us assume that these shadow fading effects between the receiver and two transmitters are correlated with respects to the difference in the azimuth angle. Figure 3.4 shows the angle between the receiver and two transmitters in a hexagonal grid with the transmitters located in the centre of each cell. The vector between the receiver and transmitter \( i \) is denoted as \( r_i \). The following correlation function is assumed:

\[
R_0 (r_i, r_{i-1}) = E [ \vartheta (r_i) \vartheta (r_{i-1}) ] = R_0 (\Delta_i) .
\]

(3.13)

where \( \Delta_i \) is the azimuth angle between vector \( r_i \) and \( r_{i-1} \), as illustrated in Figure 3.4.

The correlation depends on the roughness of the terrain and differs for different types of terrains. In a smooth terrain one would expect to have high correlation, whereas in hilly terrain with large height variations the correlation is rather low. To model the angular correlation, we use a simple decreasing correlation function,

\[
R_\theta (r_i, r_{i-1}) = \sigma^2 a |\Delta_i| 0 \leq a \leq 1 ,
\]

(3.14)

where \( a \) is the normalised autocorrelation parameter. This rather simple model yields the following stochastic process generating the shadow fading for the different transmitters
Figure 3.4 Angle difference between two transmitters and the mobile.

\[ \vartheta (r_i) = a |\Delta_i| \vartheta (r_{i-1}) + \sqrt{1 - a^2} \sigma X_i, \]

where \( X_i \) is modelled as an independent Gaussian random process with zero mean and standard deviation 1. Figure 3.5 shows the normalised correlation function for different values of \( a \). In the literature, the shadow gains for the transmitters are often assumed to be independent, which correspond here to the case where \( a \) is equal to zero.

In reality, this correlation function is periodic with period \( 2\pi \), but the exponential decaying correlation function proposed here yields a reasonable approximation. The model above is also slightly pessimistic, since all transmitters lying in the linear transmission path beyond the closest transmitter (i.e. \( \tilde{r}_i = k \tilde{r}_{i-1} \) where \( k \) is a positive constant) have the same shadowing gains. In that case, the model does not describe any changes in the shadow fading, i.e. it is not distance dependent. Consider for example the case when a nearby transmitter has line-of-sight to the receiver and a faraway transmitter has a large obstacle in between these sites as illustrated in Figure 3.6. Since the model is not distance dependent, this means that the shadowing gain for both these two transmitters could be either line-of-sight or shadowed, which is of course not true. We have indicated that the model has several drawbacks and needs to be refined. However, as an initial assumption, this model leads to reasonably realistic shadow fading environment. A natural extension is to use real measured data to find a more appropriate correlation function.
Figure 3.5 Normalised angle correlation function for different values of $a$.

Figure 3.6 A simple illustration when our proposed correlated shadow fading model does not work.
Chapter 4
RECEIVER MODEL

In this chapter, a performance measure for an OFDM-based single frequency network is derived. The analysis is based on the simple performance analysis presented in [6], where the expected useful power and interference were derived for an artificial delay spread channel. The channel for each transmitter was assumed to consist of only one channel tap. The drawback in [6] is that no explanation was given for how these derived expected powers relate to the receiver performance. We provide a refined analysis of the receiver performance for the same channel model as in [6] and also for a more realistic environment. The Symbol Error Rate (SER) is used as our performance measure of the received signal. The receiver uses a correlation demodulator combined with a symbol detector. In the derivation of the performance measure, we show that it is possible to determine the contribution from each single channel tap. By adding all contributions, an estimate of the sent symbols can then be made.

As we shall see, the expected useful and interfering powers for delayed OFDM signals can be obtained by using a weighting function combined with the expected local received power. The weighting function is crucial in SFN planning and we show that the special weighting function derived in [6] is only valid for an OFDM modulation scheme that have the same structure as previously described in Chapter 2.

Finally, the coverage performance measure (the outage probability), which is used in the evaluation of the networks is described.
4.1 Artificial delay spread at the receiver

The transmitters in an SFN (wide area or local area coverage networks) are fed with identical synchronous data and broadcast identical signals that occupy the same frequency block. The signals arrive at the antenna of the receiver at different delays. The receiver experiences two types of time dispersion: the “natural” time dispersion caused by reflections in the vicinity of the receiver, and the artificial delay spread caused by the reception of signals from several transmitters at slightly different delays. The sum of all these signals at the receiver can be expressed as

$$s(t) = \sum_{i=1}^{n} h_i(t) * x(t) + z(t),$$

(4.1)

where * denotes convolution, $x(t)$ is the transmitted OFDM signal, $h_i(t)$ the channel impulse response from transmitter $i$, and $z(t)$ the Additive White Gaussian Noise (AWGN). The artificial delay spread is graphically illustrated in Figure 4.1.

It is assumed that the receiver uses a correlation demodulator on every carrier as shown in Figure 4.2. The complex output on carrier $k$ is denoted $\hat{D}_{q,k}$ and can be obtained as follows (for the sake of simplicity $q$ is assumed zero),

$$\hat{D}_{0,k} = \frac{1}{T} \int_{0}^{T} \left( \sum_{i=1}^{n} h_i(t) * x(t) \right) g_k(t) * dt + z_k,$$

(4.2)

where $g_k(t) = e^{j2\pi f_k t}$, $0 \leq t \leq T$, and $z_k$ is the noise component on carrier $k$. The channel for transmitter $i$ is modelled as a wide sense stationary uncorrelated scattering Rayleigh fading channel, described by its impulse response as

![Figure 4.1 Multipath propagation due to simulcasting.](image-url)
Figure 4.2 Illustration of an OFDM demodulator

\[ h_i(t) = \sum_{c=1}^{M} h_{j_{(c)}} \delta(t - \tau_{j_{(c)}}) \quad (4.3) \]

where the tap gains, \( \{ h_{j_{(c)}}, c = 1, 2, \ldots, M \} \), are i.i.d Rayleigh processes. Furthermore, the channel is assumed to be slow varying compared to the symbol rate. It has been shown by Russel et. al.[72] that time variations lead to severe interchannel interference (ICI), and that the system performance is limited by these effects. In DAB, these effects can be omitted as shown in Chapter 3. Using the linearity property in (4.2), we can rewrite the output from the correlation demodulator as

\[ \hat{D}_{0,k} = \sum_{i=1}^{n} \left( \frac{1}{T} \int_{0}^{T} (h_i(t) x(t)) g_k(t) \, dt \right) + z_k \]

\[ = \sum_{i=1}^{n} \sum_{c=1}^{M} \frac{1}{T} \int_{0}^{T} h_{j_{(c)}}(t-\tau_{j_{(c)}}) g_k(t) \, dt + z_k \quad (4.4) \]

\[ = \sum_{i=1}^{n} \sum_{c=1}^{M} \hat{D}_{0,k}(\tau_{j_{(c)}}) + z_k \]
where $\hat{D}_{0,k}(\tau)$ is defined as the output from the correlation demodulator from the channel tap with the delay of $\tau_i$. Equation (4.4) shows that it is possible to analyse the received contribution from each single tap separately. The exact output for a single tap after correlation is derived in the next section.

### 4.2 Decorrelation on a single tap channel

If we assume that the receiver is synchronized on the first received reflection, i.e. the first channel tap, then all other delayed signals arrive either within the time span of the guard interval or later. Two disjunct delay situations must be analysed separately in OFDM. In Figure 4.3 these two different situations are illustrated. In the first scenario a delayed signal arrives within the receiver window and thus the signal (symbol) does not overlap into the next receiver window. This holds for transmitters located close to the receiver. In the second scenario, the transmitters are further away from the receiver and the transmitted symbol may arrive such that some parts of the signal is within the receiver window and the other part of the signal overlaps into the next receiver window position. The received OFDM signal is then no longer orthogonal and some intersymbol interference (ISI) is generated. Below we give an expression of the output from a correlation demodulator on a noise free channel for these two scenarios.

In what follows, it is assumed that the OFDM signal is delayed with $\tau$ and its tap gain is denoted $h$. The detailed derivation of the results presented in this section can be found in appendix A.

**Scenario 1:** $0 \leq \tau \leq T_g$

In this case, the output from the correlation demodulator can be expressed as

$$D_{0,k}(\tau) = he^{-j2\pi f_k \tau}D_{0,k},$$

(4.5)

where $D_{0,k}$ is the symbol transmitted on frequency $f_k$. There are no orthogonality losses in this scenario.
Scenario 2: $T_g \leq \tau \leq T_{tot}$

The output from the correlation demodulator now depends on both the previous transmitted OFDM frame ($q=-1$) and the present one ($q=0$), according to

$$
\hat{D}_{0,k}(\tau) = h \left[ \mu_{kk}(\tau) D_{0,k} + \sum_{l=0, l \neq k}^{N-1} \mu_{lk}(\tau) D_{0,l} + \sum_{l=0}^{N-1} \lambda_{lk}(\tau) D_{-1,l} \right] \quad (4.6)
$$

where the first term within the parentheses corresponds to the wanted (useful) part of the incoming signal. The second term corresponds to the losses in frequency orthogonality from the present OFDM frame, while the third term is interference from the previous OFDM frame. Replacing the parameters $\lambda_{kk}(\tau)$ and $\mu_{kk}(\tau)$ in (4.6) by their expressions given in appendix A yields

$$
\hat{D}_{0,k}(\tau) = h \left[ (1 - \alpha) e^{-j2\pi f_k \tau} D_{0,k} + \alpha e^{-j2\pi f_k (\tau - T_g)} D_{-1,k} + I_{0,k}(\tau) \right] \quad (4.7)
$$

where $\alpha = (\tau - T_g) / T$ is a normalised delay and $I_{0,k}(\tau)$ is the interference caused by the frequency orthogonality losses.

The interference on a certain carrier described by (4.6) consists of a large sum of complex stochastic variables (the data) with almost the same variance. Usually when $N$ is large, the central limit theorem is applicable, and the interference can be approximated by a complex Gaussian distribution, yielding an interference
with a Rayleigh distributed envelope and an uniformly distributed phase between 
$[-\pi, \pi]$. This assumption may not be valid here, since the interfering components have different variance. Simulation results presented in Appendix A indicate that the interference envelope (normalised by the useful envelope) follows a Rice distribution [2], i.e.

$$ Pr\left[a\right] = I_0\left(\frac{aa_0}{\sigma^2}\right) \frac{a}{\sigma^2} \exp\left(-\frac{a^2 + a_0^2}{2\sigma^2}\right) $$

(4.8)

where $I_0$ is the zero order modified Bessel function of first kind and

$$ a = \frac{\alpha e^{-j2\pi f_k(T_{\text{tot}} - \tau)}}{(1 - \alpha) e^{-j2\pi f_k\tau} D_{-1,k}} \left[D_{-1,k} + l_{0, k}(\tau)\right] $$

$$ a_0 = \frac{\alpha e^{-j2\pi f_k(T_{\text{tot}} - \tau)}}{(1 - \alpha) e^{-j2\pi f_k\tau} D_{0,k}} = \frac{\alpha}{(1 - \alpha)} $$

(4.9)

$$ \sigma^2 = \frac{E\left[l_{0,k}(\tau)^2\right]}{(1 - \alpha) e^{-j2\pi f_k\tau} D_{0,k}^2} = \frac{2\alpha (1 - \alpha)}{(1 - \alpha)^2} = \frac{2\alpha}{(1 - \alpha)} $$

The Rice distribution is sometimes used to model the statistics of signals transmitted through some radio channels. The received signal then consists of a strong direct path plus AWGN. In our case, the previous sent symbol on the same carrier, which is the second term in (4.7), may sometimes (depending on $\alpha$) be the dominating interference. The frequency orthogonality losses, $I_{0,k}(\tau)$, is well approximated as AWGN. Note that the Rayleigh distribution is a special case of the Rice distribution for $a_0 = 0$.

The interference phase is also analysed in appendix A, and for $\alpha = 0$, the probability density function of the interference phase relative to the phase of the wanted signal is approximately uniformly distributed between $[-\pi, \pi]$.

However, for large normalised delays, i.e. $\alpha = 1$, the density function assumes a periodic shape caused by the symbol constellation of the dominating interference from the previous symbol. The expected interfering power from the previous symbol, and the frequency orthogonality losses are derived in the next section together with the expected useful power.
4.3 Expected local received power on a single tap channel

Scenario 1: $0 \leq \tau \leq T_g$
Using the result from (4.5), the expected received power on carrier $k$ for a single channel tap is

$$E\left[|\hat{D}_{0,k}(\tau)|^2\right] = E\left[|he^{-j2\pi/\tau}D_{0,k}|^2\right] = E[|h|^2]$$  \hspace{1cm} (4.10)

This last equation is the crucial part in SFN planning. The equation states that all signals arriving within a time span of the guard interval are expected to contribute with useful power. By having a short spatial distance among transmitters, more useful contributions can be obtained yielding improved coverage properties. SFN can thus be seen as a transmitter diversity system.

Scenario 2: $T_g \leq \tau \leq T_{tot}$
The transmitters far away from the receiver contribute both an useful and an interfering part as described earlier. These transmitter generate self-interference in the network. The expected local received power on carrier $k$ can be directly obtained from (4.6) combined with Appendix A as

$$E\left[|\hat{D}_{0,k}(\tau)|^2\right] = E[|h|^2] \left( (1 - \alpha)^2 + \alpha^2 + 4\alpha^2 \sum_{m = 1}^{K} \sin^2(m\alpha) \right)$$

$$+ 2\alpha^2 \sum_{m = K + 1}^{L} \sin^2(m\alpha)$$

$$= E[|h|^2] (a + b + c + d)$$  \hspace{1cm} (4.11)

where $K = \min(N-k-1,k)$ and $L = \max(N-k-1,k)-K$.

$$a = (1 - \alpha)^2$$
$$b = \alpha^2$$
$$c = 4\alpha^2 \sum_{m = 1}^{K} \sin^2(m\alpha)$$
$$d = 2\alpha^2 \sum_{m = K + 1}^{L} \sin^2(m\alpha)$$  \hspace{1cm} (4.12)

The terms $a$, $b$, $c$ and $d$ can be seen as weighting functions of the received power. The first term, $a$, in (4.11) is the useful amount, and the second term, $b$, is the interference from the previous symbol. The interference power caused by losses in frequency orthogonality is described by the term $c+d$. The total expected
interference power, \( b+c+d \), is a function of the number of carriers. Figure 4.4 plots the terms \( a, b, c, d \) and the sum \( b+c+d \) for \( K = 10 \) (dashed-dotted) and 100 (solid) with \( L = 1000 \) as a function of the normalised delay \( \alpha \). It shows that for modest values of \( K \), the third term, \( c \), can be approximated by \( 2 (\alpha - \alpha^2) \) while the fourth term, \( d \), is negligible. Hence, the weighting function for the received interfering power can be approximated as

\[
 b + c + d = \alpha^2 + 2 (\alpha - \alpha^2) = 1 - (1 - \alpha)^2
\]  

(4.13)

Using the approximation in (4.13) leads to the following relation between the useful and interfering power

\[
P_{use} = E [|h|^2] (1 - \alpha)^2
\]  

(4.14)

\[
P_{int} = E [|h|^2] (1 - (1 - \alpha)^2)
\]  

(4.15)

Both functions in (4.14) and (4.15) can be seen as the expected total power multiplied with the weighting function, which only includes the normalised delay, \( \alpha \).

Figure 4.4 Different weighting functions as a function of \( \alpha \).
A summary of the analysis: (The expected received power for any delay, $\tau$)

The expected local useful power can now be expressed using a weighting function, $Q(\tau)$. For delays within the guard interval the weighting function is one, whereas for larger delays it can be described using (4.14). The weighting function, $Q(\tau)$, can thus be expressed as,

$$Q(\tau) = \begin{cases} (1 + \beta)^2 & -T \leq \tau \leq 0 \\ 1 & 0 \leq \tau \leq T_g \\ (1 - \alpha)^2 & T_g \leq \tau \leq T + T_g \\ 0 & \text{otherwise} \end{cases}$$

where $\beta = \tau/T$ for negative delays, often referred to as pre-echoes. Essentially, this is the same result as in [6]. The weighting function is illustrated in Figure 4.5.

The relation to the expected interference power is given by following weighting function $(1 - Q(\tau))$. Similar to equations (4.14) and (4.15), we get

$$P_{use} = E[|h|^2] Q(\tau)$$

$$P_{int} = E[|h|^2] (1 - Q(\tau))$$

The weighting functions $Q(\tau)$ and $(1 - Q(\tau))$ depend on the structure of the OFDM signal described in Chapter 2. Other weighting functions can however be obtained. For example, if the carrier separation is $2/T$, and not $1/T$ as we assumed in the earlier derivations, the interfering weighting function changes its shape entirely, whereas the useful weighting function maintains the same shape. We denote the interfering weighting function by $R(\tau)$ and is illustrated in Figure 4.6. By only utilising half of the available spectrum, the interference from the subcarriers is reduced resulting in $R(\tau)$ (see appendix A for details). Besides the carrier separation, the pulse shape of $p(t)$ given in (2.2) and frequency offsets also lead to different weighting functions.

Another type of weighting function has been proposed in [78], denoted by $L(\tau)$, and is shown in Figure 4.7. This function is quite optimistic since the useful power is assumed to be directly proportional to the degree of overlap into the following symbol. This function has no realistic OFDM structure (it is beyond the author’s knowledge). From the network planners’ point of view, it is desirable to use the same design rules independent of which function is used. We therefore compare the coverage properties using different weighting functions. Note that all the presented weighting functions have the same shape in the interval $[0, T_g]$, i.e.
all signals arriving within this interval are treated as useful. This is the crucial property that makes the construction of SFNs possible. It should also be emphasised here that this is not always true. A time varying channel for which the Doppler spread cannot be omitted would lead to entirely different weighting functions.

In the next two sections, the weighting function \( Q(\tau) \) is used in the receiver performance investigation. Before we proceed in evaluating the performance of the receiver for a single channel tap, we define the expected Signal-to-Interference Ratio (SIR) as

\[
\Gamma = \frac{P_{use}}{P_{int} + N_0} = \frac{E[|h|^2]Q(\tau)}{E[|h|^2](1 - Q(\tau)) + N_0}
\]  

(4.19)

where the noise component, \( N_0 \), is included.

**Figure 4.5** The weighting function, \( Q(\tau) \).

**Figure 4.6** The weighting function, \( R(\tau) \).
4.4 Receiver performance on a single tap channel

The main objective in this section is to show the relationship between the receiver performance and the expected SIR given in (4.19). The Symbol Error Rate (SER) is used as the receiver performance, and the output from the correlation demodulator is used in the symbol estimator to make a decision on what symbol has been sent. The estimator suppresses the phase shift $2\pi f_k \tau$ and then the detector selects the symbol that minimises

$$
\min_{\hat{D}} \left\{ e^{2\pi f_k \tau} [\hat{D}_{0,k} (\tau) + z_k] - D \right\}^2
$$

(4.20)

where $z_k$ is the noise component on carrier $k$. For OFDM signals with a delay shorter than the guard interval, this is an optimal detection [2]. However, it is not an optimal detection for a signal with a delay larger than the guard interval. For instance, it is possible to make a joint detection over all carriers utilising the structure in the interference as shown in the matrix representation in Appendix A.

The receiver performance is studied in an interference limited environment, where the noise component in (4.20) can be neglected. The SIR in an interference limited systems is

$$
\Gamma \to \frac{(1 - \alpha)^2}{1 - (1 - \alpha)^2} = \frac{Q(\tau)}{1 - Q(\tau)} \quad \text{when } E[|h|^2] \to \infty
$$

(4.21)

The SER is shown in Figure 4.8 for a full scale simulation using a coherent QPSK. The graph also includes the analytical expression for the symbol error probability (SER) of coherent QPSK modulation over an AWGN channel [2].
Figure 4.8 SER in an interference limited system when receiving only one signal.

\[ P_e = 2Q(\sqrt{\Gamma}) [1 - 0.5Q(\sqrt{\Gamma})] , \]  
where in this case \( \Gamma \) is given in (4.21) and \( Q(x) \) is the modified complementary error function, defined as,

\[ Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-z^2/2} dz \]  

(4.23)

The result shows that the analytical expression for the SER on an AWGN channel fits with the simulation result. Besides this, the main result is that the error rate decreases as function of the expected SIR given in (4.21). Note that this result is valid only for a single tap channel. In the next section we investigate the SER for a more complicated channel, including artificial delay spread.
4.5 Decorrelation of received signals and its performance

In this section we combine the output from the correlation demodulator according to (4.4) yielding an expression when several signals are received. The total output from the correlation demodulator can be expressed as,

$$
\hat{D}_{0,k} = \sum_{i=1}^{n} \sum_{c=1}^{M} \hat{D}_{0,k}(\tau_{j_{i,c}}) + z_k = \sum_{m=1}^{nM} \hat{D}_{0,k}(\Delta_m) + z_k
$$

(4.24)

where \( n \) is the number of transmitters in the SFN and \( M \) is the number of channel taps per transmitter. By ordering the delays as \( 0 < \Delta_0 < \Delta_1 < ... < \Delta_{nM} < T_g < \Delta_H < ... < \Delta_{nM} \) and defining \( \alpha_i = (\Delta_i - T_g)/T \) we can express the output after the correlation demodulator as

$$
\hat{D}_{0,k} = \left( \sum_{i=1}^{H-1} \hat{h}_i e^{-j2\pi f_0 \Delta_i} + \sum_{i=H}^{nM} \hat{h}_i e^{-j2\pi f_0 \Delta_i} (1 - \alpha_i) \right) D_{0,k} + 
\sum_{i=H}^{nM} \hat{h}_i \left( \alpha_i e^{-j2\pi f_0 (\Delta_i - T_g)} D_{-1,k} + I_{0,k}(\Delta_i) \right) + z_k
$$

(4.25)

where \( I_{0,k}(\Delta_j) \) is frequency orthogonality losses given in (4.7). A more compact description is given by

$$
\hat{D}_{0,k} = a e^{j\Theta} D_{0,k} + I_{tot} + z_k,
$$

(4.26)

where \( I_{tot} \) is the total interference. The envelope \( a \) is Rayleigh distributed from our definition of the channel, and \( \Theta \) is the received phase. If the total interference is independent of \( a \) and \( \Theta \), and its distribution is approximated by a complex Gaussian distribution, then the SER becomes similar to any modulation scheme over a flat Rayleigh fading channel. The receiver performance is then easily analysed, and the SER for this type of channel is well defined in the literature [2].

Based on the analysis done for a single tap channel, an expression for the expected local received SIR is obtained by combining (4.19), (4.16) and (4.24),

$$
\Gamma = \frac{\sum_{i=1}^{H-1} E[|h_i|^2] + \sum_{i=H}^{nM} E[|h_i|^2] (1 - \alpha_i)^2}{\sum_{i=H}^{nM} E[|h_i|^2]} \left( 1 - (1 - \alpha_i)^2 \right) + N_0
$$

(4.27)

or as
where we have used the expression (4.6) for the weighting function \( Q(\Delta_i) \). Since the “natural” time dispersion is much shorter than the symbol duration equation (4.28) can be approximated to

\[
\Gamma = \sum_{i=1}^{nM} \frac{E[h_i^2] Q(\Delta_i)}{\sum_{i=1}^{nM} E[h_i^2] (1 - Q(\Delta_i)) + N_0}
\]

The SER is investigated in an interference limited system, where the expected SIR is given by

\[
\Gamma = \frac{\sum_{i=1}^{n} \left( \frac{P_{erp}}{L(r_i)} \right) 10 \log_{10} \left( \frac{1}{1 - Q(\tau_i)} \right)}{\sum_{i=1}^{n} \left( \frac{P_{erp}}{L(r_i)} \right) (1 - Q(\tau_i)) + N_0}
\]

Figures 4.9 and 4.10 show simulation results from a number of typical (randomly picked) SFN channels using QPSK modulation. The network consists of 7 transmitters with an adjacent transmitter distance of 7 km and can be considered as interference limited. Two different types of channel are used, both with a maximum “natural” delay spread of 5 µs. The circles in Figure 4.9 represent a channel with one channel tap per transmitter, i.e. \( M = 1 \), and the channel in Figure 4.10 has \( M = 4 \) taps. The graphs also include the expected SER over an AWGN channel and a flat (non-frequency selective) Rayleigh fading channel [2]. The simulation results show that SER decreases as a function of increased SIR. Utilising methods to increase the locally received SIR leads to a decreased SER, yielding better coverage properties.

Another interesting observation is that the same result is obtained for the different channels. The symbol duration \( T \) used in SFN is chosen to be much larger than the natural time dispersion which causes this effect, i.e. the intersymbol interference due to reflections in the close area to the receiver can thus be neglected, making the approximation in (4.29) valid. In the rest of the thesis, we shall use this approximation.
Figure 4.9 SER in an interference limited system with artificial delay spread. Number of channel taps per transmitter $M=1$.

Figure 4.10 SER in an interference limited system with artificial delay spread. Number of channel taps per transmitter $M=4$. 
4.6 The outage probability

The receiver performance is a complicated function of many of the variables characterising the detailed propagation conditions as described in the previous sections. We have seen that the Signal-to-Interference Ratio (SIR), defined as

$$\Gamma = \frac{P_{use}}{P_{self} + P_{ext} + N}$$  \hspace{1cm} (4.31)

is a reasonable measure of the received signal quality. $P_{use}$ is proportional to the part of the total signal power that falls within the receiver window (useful) and $P_{self}$ is proportional to the signal power that overlaps into the following symbol (self-interference). $P_{ext}$ is the interfering power from external transmitters broadcasting other programs, and $N$ denotes the thermal noise power in the receiver.

The external interference was excluded in the previous analysis. However, for a randomly selected receiver position and time unit, this interference can be assumed as a complex Gaussian interference, i.e. under this snapshot, the interference can be seen as regular noise. Using the weighting function $Q(\tau_i)$ given in (4.16), a relation between the useful and the interfering power can be determined, and the expected SIR at the receiver can be expressed as

$$\Gamma = \frac{\sum_{i=1}^{n} P_i Q(\tau_i)}{\sum_{j \in ext} P_j + N}$$  \hspace{1cm} (4.32)

The quality on the receiver end not only depends on the expected SIR but also on whether error correcting codes are employed. In DAB and the digital TV systems for example, very powerful coding schemes are used. Employing error correcting coding schemes results in entirely different SER curves than in Figure 4.9 and Figure 4.10. Often the reception is either of very high quality or extremely poor. Therefore, it is more appropriate to consider the outage probability as our performance measure, defined as

$$Pr[\Gamma < \gamma_0],$$ \hspace{1cm} (4.33)

where $\Gamma$ is the signal-to-interference ratio at a randomly selected receiver within the service area, and $\gamma_0$ is the protection ratio, i.e. the minimum required SIR to provide the required reception quality. To ensure a good quality service, the requirements on the outage probability are quite high. In [74] they claim that a
coverage probability higher than 90% is needed for the DAB system.

The coverage properties differ considerably within a local service area. In particular, the central parts of the service area have better coverage compared to the area close to the border. Earlier estimations on the outage probability ([78] table 1) have shown that the outage probability for the whole regional service area is determined mainly by the worst areas, i.e. near the borders of the service area.
Chapter 5
NUMERICAL RESULTS

In this chapter we present numerical results for both wide area and local Single Frequency Networks (SFNs).

5.1 Assumptions

All the results presented in this chapter are obtained using the parameters specified in the ETSI DAB standard [5], see Table 1. However the results are not restricted to this application. To estimate the outage probability for an SFN we have used Monte-Carlo simulation, by sampling a large number of randomly selected receiver positions in the service area. We assume that our modulation scheme requires $\gamma_o = 10$ dB which is the value suggested in [22]. The bandwidth of a DAB OFDM signal is 1.5 MHz and can contain 4-6 radio programs. We have assumed that a stereo audio program requires $B = 250$ kHz which gives a total number of 6 programs per OFDM frequency block. The thermal noise power is given by $N_0 = FkT_0B$, where $k = 1.38 \times 10^{-23} (J/K)$ and $T_0 = 290 (K)$. A value between 6 dB and 10 dB for the receiver noise factor, $F$, has been suggested in the literature [32] [41]. In our numerical examples we assume a receiver noise factor of 10 dB, yielding a thermal noise power of $N_0 = -140$ dBW per program. The outage probability is considered as sufficient if it drops below 1%, i.e. we require at least 99% coverage.

As earlier mentioned in Chapter 2, the receiver needs to choose a time (frame) synchronisation position, $t_o$, for the receiver window to obtain as much energy as possible from the received signals, see Figure 5.1. A preliminary study on how to choose $t_o$ in an SFN has been performed in [102], where several schemes are evaluated. In our analysis we use a simple but efficient receiver, which selects the time synchronisation point for the window as the arrival time for the first received
signal plus the guard interval $T_g$. This scheme works well, since the strongest signal often has the shortest propagation delay. All signals arriving after the time synchronisation point cause intersymbol interference (self-interference), since the previous transmitted symbol overlapped into the receiver window.

All transmitters are assumed to use equal transmitter power and the same antenna heights. Although, this is not necessarily an optimal network configuration, it does, as the results will point out, provide more incite on the difficulties with SFN planning.

5.2 Numerical results for wide area coverage using SFN

For wide area coverage, the national DAB mode-I is employed ($T = 1000 \, \mu s$, $T_g = 246 \, \mu s$). In Figure 5.2 we compare the outage probability for a national network for a distance of 40, 70 and 100 kilometres between adjacent transmitters. The transmitter antenna height is 150 meters, and the angular correlation coefficient is chosen to zero, i.e. independent shadowing. The graph shows two curves for each distance, where the lowest curve corresponds to a (optimistic) linear weighting function, $L(\tau_i)$, proposed in [78], and the highest curve is the more realistic quadratic weighting function, $Q(\tau_i)$. The difference between the linear and the quadratic models are only noticeable for large radiated powers, whereas for minor powers both models yield the same results. The results also show that the maximum distance between transmitters is 100 km and shorter transmitter distances yield much better coverage properties. Adequate coverage can be achieved using the existing FM infrastructure with an average distance of 60 km between any two transmitters, provided that at least a radiated power of 300 W per program is employed.

![Figure 5.1](image-url) a) The transmitted symbol. b) Propagation delayed signals from different transmitters and the receiver window of length $T$. 

5.2 Numerical results for wide area coverage using SFN

Figure 5.2 The outage probability for the transmitter distance 40, 70 and 100 km, as a function of the effective radiated power. $h_t=150$ m. A comparison between the quadratic weighting function, $Q(\tau_i)$, and the linear function, $L(\tau_i)$, used in [78].

Figure 5.3 The outage probability for the transmitter distance 40 and 70 km, as a function of the effective radiated power. The effect of different antenna heights is illustrated.
Different antenna heights are used in Figure 5.3 for adjacent transmitter distances of 40 and 70 kilometres. Despite more self-interference being generated for higher antennas, higher elevation seem to be preferable. The reason for this is that the self-interference does not increase as much as the useful power which is an effect caused by the shape of the path loss propagation model. As illustrated in the measured propagation losses in Figure 3.2, the extra received power gain at distances larger than 70 km does not increase as much as with shorter distances.

The impact on correlated shadow fading components in wide area networks is depicted in Figure 5.4. The graph shows the cumulative distribution function for the SIR in the network, i.e.

$$ P_{\text{out}}(\gamma) = Pr[\Gamma < \gamma] . $$ (5.1)

The system is assumed to be interference limited, which means that the effective radiated power is at a level where the thermal noise can be neglected. The results show that the coverage properties in a wide area SFN depends on the correlation in the shadow fading components, with the worst case being when there is low shadow fading correlation. The mean SIR increases by an order of magnitude between a correlation factor $a = 0.0$ and $a = 0.8$ for both 40 and 100 km transmitter distances. It is quite surprising however that a diversity system gains more by having an increased correlation since in most diversity systems the contrary is true. The reason why this effect occurs in an SFN is that the generated self-interference from the transmitters becomes correlated and higher correlation reduces the variance of the interference. For instance, at $a = 1$ the SIR is dependent only on the relative distance between the receiver and the transmitters. The closest transmitters always then yield the strongest received signals, and the self-interfering transmitters far away interfere to a lesser extent. Figure 5.5 shows the outage probability for a national network with a distance of 40, 70 and 100 km between adjacent transmitters. The values of the angular correlation factor, $a$, are $\{0, 0.4, 0.8\}$. We note that the coverage is only improved as a function of the angular correlation factor for high radiated transmitter power. This is a consequence of the results from Figure 5.4, where the system is considered interference limited. For low radiated transmitter powers, the dominating interference is the noise, an opposite result is obtained. Due to the correlation in the received signal power, it is less likely that one of the useful received signals is greater than the thermal noise power. Although, the correlation does not yield any drastic changes in coverage, the transmitter power level must be chosen with some care.
5.2 Numerical Results for Wide Area Coverage Using SFN

Figure 5.4 The cumulative distribution function for the SIR as a function of the angular correlation function. The system is interference limited (neglect the noise). $h_t=150$ m.

Figure 5.5 The outage probability for the transmitter distance 40, 70 and 100 km, as a function of effective radiated power for different shadow fading correlation factor $a$. 
5.3 Numerical results for local coverage using SFN

In contrast to wide area coverage, local networks are subjected to external interference if other nearby networks are operating in the same frequency band. Both external interference and self-interference must be considered. In all the numerical results, a total of three OFDM frequency blocks is assumed available, i.e. the frequency reuse factor is 3. Note that in FM, the corresponding value is between 7 and 9. Both the local DAB mode-II \( (T_g = 62 \text{ µs}, T = 250 \text{ µs}) \) and the national DAB mode-I \( (T_g = 246 \text{ µs}, T = 1000 \text{ µs}) \) are used in this section to compare their impact on the duration of the guard interval.

Figure 5.6 shows the outage probability for a DAB mode-II system as a function of the radius of the service area where the external interference is neglected. The transmitter antenna height is chosen to be 37.5 meters and a total power of 1 kW per local network is assumed. Each local network contains \( M = \{1, 3, 7, 19\} \) transmitters. A network with only one transmitter in the local service area does not provide any diversity gain. By inserting more transmitters into the local service area, the outage decreases due to the diversity gain of receiving more signals. The results in Figure 5.6 correspond to the implemented local SFN that we currently have today in Stockholm, Gothenburg and Malmö, where the distances among the networks are very large. Since the external interference can be neglected, the results show that adequate coverage can be achieved using only one transmitter to cover an area with a radius of 20 km. Larger distances can be further reached by using higher antennas. Furthermore, if the local networks were denser, the system would become limited by external interference. This is illustrated in Figure 5.7 where the graph shows how the outage increases when there are several closely located networks operating on the same frequency band. Apparently the number of transmitters in the local service area is crucial. The results indicate that the required coverage properties can be fulfilled, provided that at least 19 transmitters are used to cover each local service area.

We can also observe the effect caused by the shape of the distance dependent propagation model for broadcasting in Figure 5.7. For a small local service area, the system is external-interference limited, i.e. most of the interfering transmitters are within the radio horizon. When the radius of the local service area becomes larger, the interfering transmitters will reach a point where they fall beyond the radio horizon. This improves the SIR and the minimum outage probability can
often be found in this region. A similar effect also occurs in conventional FM-planning where the optimum performance is achieved when the antenna heights and the transmitter spacing are chosen such that the interfering transmitters fall beyond the radio horizon. For larger local service areas, the network starts to generate self-interference and the performance degrades. This is particularly noticeable in DAB mode-II where the guard interval is short. As a consequence of the considerable problems with self-interference, the service area for a DAB mode-II system needs to be rather small.

The outage probability for different radiated transmitter powers in DAB Mode-II is shown in Figures 5.8 and 5.9. The thermal noise has little influence on the performance, and at very low transmitter power, around 10 W, it is possible to achieve adequate performance (1% outage). The system becomes interference limited for quite modest transmitter power, and a further increase does not yield any improvements. In both graphs, an increase in the transmitter power from 10 W up to 10 kW only results in a minor change in the outage. Note that the power level has an impact only when the service area is large, i.e. when the self-interference is the dominating interference. With a frequency reuse factor of three (\(K = 3\)) and using DAB Mode-II, the system is either external-interference or self-interference limited, depending on the size of the service area. The self-interference can be reduced and become less dominating if the length of the guard interval is increased. Using the national DAB mode-I in a local area network, wider areas can be covered, requiring the use of higher transmitter power. For small service areas, the external interference is still the limiting factor irrespective of the length of the guard interval.
Figure 5.6 The outage probability as a function of the radius of the service area with different number of transmitters. No external interference. $h_t=37.5$ m, DAB mode-II.

Figure 5.7 The outage probability as a function of the radius of the local service area with different number of transmitters. Frequency reuse factor 3. $h_t=37.5$ m, DAB mode-II.
5.3 Numerical Results for Local Coverage Using SFN

**Figure 5.8** The outage probability for different transmitter powers. The frequency reuse factor is 3, $M=7$ transmitters, $h_t=37.5$ m and DAB mode-II is applied.

**Figure 5.9** The outage probability for different transmitter powers. The frequency reuse factor is 3, $M=19$ transmitters, $h_t=37.5$ m and DAB mode-II is applied.
Besides the number of transmitters in a local network and the transmitter power, the antenna height will also affect the performance. In Figure 5.10, the outage probability is shown for DAB Mode-II using different transmitter antenna heights. The graph shows that low antenna heights are preferred if small areas are to be covered since the received external interference is reduced. By using higher antennas, it is possible to cover wider areas to some extent. However, the increased interference due to larger antennas imposes an upper limit on applicable antenna heights. Due to the short duration of the guard interval in mode-II, there exists an optimal antenna height for a given local service area radius. If the guard interval is chosen according to mode-II, antennas higher than 150 meters are not recommended due to the interference problems.

In Figure 5.11 the outage probability for the national DAB Mode-I is depicted. The radiated power is 5/19 W per transmitter. The guard interval is extremely large in mode-I ($T_g = 246 \, \mu s$), making it possible to cover wider service areas. The strategy then is to use higher antennas in order to properly cover the service area. In contrast to the local DAB mode-II, the graph indicates that antennas higher than 150 meters yield satisfactory results for large service areas. We note that the obtained results (Figure 5.10 and Figure 5.11) indicate that the guard interval has to be chosen with some care and is, in general, dependent on the size of the service area. Although the results in Figure 5.11 show that adequate coverage can be obtained by high antenna positions, other solutions exist. Provided that DAB mode-I is used, similar results can be achieved by increasing the power. This is possible since the self-interference is not as dominating as in mode-II.

Figure 5.12 shows the outage probability for a system using the national DAB mode-I for different radiated power and a transmitter antenna height of 150 meters. The system can be considered as external interference limited when the radius of the service area is smaller than 40 km. Furthermore, the system does not have any problems with self-interference for larger service areas since the guard interval is extremely large (246 $\mu s$), and the noise is then the main source of interference. But as the results show, the thermal noise can be neglected if the transmitting power is more than 10 W.
Figure 5.10 The outage probability for different transmitter antenna heights. The frequency reuse factor is 3, $M=19$, $h_t=[37.5, 150, 300]$ m, and DAB mode-II is applied.

Figure 5.11 The outage probability for different transmitter antenna heights. The frequency reuse factor is 3, $M=19$, $h_t=[37.5, 150, 300]$ m, and DAB mode-I is applied.
Figure 5.12 The outage probability for different transmitter powers. The frequency reuse factor is 3, $M=19$ transmitters, $h_t=150$ m and DAB mode-I is applied.

For wide area SFNs, we have earlier shown in Figure 5.4 that correlation in the shadow fading have an impact on the coverage properties. In Figure 5.13 the cumulative distribution function for the SIR,

$$P_{out}(\gamma) = Pr[\Gamma < \gamma],$$

is depicted for local area SFNs. The system is assumed interference limited, which is often the case in local SFNs. The results show that better coverage properties are obtained when there is correlation in the shadow fading components. Similar gains are obtained both in the external ($R_{service} = 20$ km) and in the self ($R_{service} = 50$ km) interfering case. This is essentially the same result as in the case with wide area SFNs, see Figure 5.4. Both results in Figure 5.4 and Figure 5.13 show that the correlation mainly impacts interference limited systems. However, for wide area coverage, the systems seldom become interference limited, whereas dense local SFNs are limited by either external or self-interference.
Figure 5.13 The cumulative distribution function for the SIR for different angular correlation factor $a$. The frequency reuse factor is 3, $M=19$, $h_t=37.5$ m and DAB Mode-II is applied.

Figure 5.14 The outage probability for different number of transmitters using the 1.5 GHz band. The frequency reuse factor is 3, $h_t=37.5$ m and DAB mode-II is applied.
As mentioned earlier, the use of higher frequencies around 1.5 GHz is intended for the local DAB mode-II. Higher frequencies however cause more diffraction losses, resulting in larger path losses. In general, the effects of radio horizon becomes more pronounced. Currently there are no path loss propagation models for broadcasting at these high frequencies, but models for the UHF band (400-1000 MHz) have been utilised in earlier investigations [41]. In Figure 5.14 the outage probability is depicted for DAB mode-II at 1.5 GHz. In contrast to Figure 5.7 (230 MHz), we notice performance improvements for small service areas due to the increased propagation losses at large distances, i.e. reduced external interference. Figure 5.14 also reveals that covering large areas is instead more difficult. Increasing the radiated power reduces these losses in outage. It appears that the results for 230 MHz are also valid for 1.5 GHz. However at 1.5 GHz, the Doppler spread due to receivers movements, which is here omitted, becomes more evident thereby causing performance degradation.

5.4 Summary

Based on the obtained results, we propose some “rules of thumb” for designing a single frequency network. By using these simple rules, a good initial network configuration can be achieved. However, a refined network planning is sometimes necessary, and in this more detailed planning process, several other parameters need to be considered, e.g. the shape of the terrain, the frequency band, etc. We propose the following simple design rules for a wide area SFN:

- Do not separate the transmitter by more than 100 km.
- Use high antenna elevation.
- Choose a transmitter power of at least 300 W per program.

For local SFN, the following rules are proposed:

- Use as many transmitters as possible. Our results indicate that 19 transmitters per local service area yield adequate coverage.
- Choose the guard interval such that it is greater than the artificial delay spread generated by the relevant transmitters. This ensures that these transmitters all contribute useful power.
- Choose the transmitter antenna heights such that the interferences among the local SFNs is limited.
• Higher frequency bands are to prefer, provided that the Doppler spread can be neglected.
Chapter 6
TRANSMITTER LOCATIONS AND SENSITIVITY ANALYSIS

In this chapter, we examine the robustness of single frequency networks. In the first section, a search is performed to find the best transmitter locations for both wide area and local area networks. In addition, the robustness of the SFN is investigated for systems in which the best transmitter locations cannot be utilised. In reality though, the locations may not be chosen at will, and it is impossible to use the best transmitter positions. In this case, other close by locations must be used, resulting in entirely different coverage properties.

6.1 Transmitter location strategies

The results presented in the previous chapter were valid for radio networks in which all the transmitters are located at the centres of each hexagon. In a wide area network, an uniform coverage gives an optimal coverage, where the best transmitter locations to achieve this optimal coverage are obtained by having an uniform transmitter spacing, i.e. the transmitters are located in the central parts of each hexagon. Hence, the system yields a symmetric spreading of the radiated power over the service area.

In a local area coverage network, the main interference is often created by external transmitters from nearby SFNs, making the borders of the service area more exposed to external interference than other areas. Locating the transmitters at the centres of each hexagon does no seem then to be the best choice. A better solution would be to move the transmitters towards the border, as shown in Figure 6.1.
The location of the transmitters can be placed according to a given strategy. In the earlier chapters, the transmitters are located in “rings” around the central hexagon. For example, there are two rings in Figure 6.1, one including six transmitters and the other with twelve. The transmitters can be categorised in sets depending on their initial ring meaning that the configuration given in Figure 6.1 has two sets, one with six transmitters and the other with twelve. In the preceding analysis, it was assumed that all transmitters corresponding to a certain set have the same distance to the centre of the service area as illustrated in Figure 6.2. The location of the transmitters in each set is described as

\[ D_6 = rR_{\text{service}} \]  

(6.1)

where \( D_6 \) is the distance between the transmitters in the set and the centre of the service area, \( R_{\text{service}} \) is the radius of the service area, and \( r \) is a positive continuous variable describing the exact location of the six transmitters. Similar to (6.1) we define the distance for the next ring with twelve transmitters as

\[ D_{12} = sR_{\text{service}} \]  

(6.2)

The pair of parameters \( (r, s) \) describe the locations of all transmitters, e.g. the transmitters are located in the centre of each hexagon if \( (r, s) = (0.4, 0.8) \).
In Figure 6.3, the results for a network with a service area radius of 20 km. In this external interference limited system (see Chapter 5), the results indicate that it is possible to reach a slightly lower outage probability by moving the transmitters closer to the border compared with having each transmitter located in the centre of its hexagon. The minimum outage probability is reached when \((r, s) = (0.7, 0.85)\), which almost corresponds to having the transmitters located in the centre point of each hexagon. Figure 6.3 indicates that a good choice for parameter \(s\) (=0.85) is decisive, whereas the parameter \(r\) is of secondary importance. Figure 6.4 shows the outage probability for \(s =\{0.70, 0.85, 1.00\}\). Provided that \(s = 0.85\), the six other transmitters can almost be located anywhere without causing too much performance degradation. The parameter \(r\) can then be chosen in the range between 0.1-0.7.

In Figure 6.5 the service area is larger, \(R_{\text{service}} = 50\) km, yielding a system in which the external interference is drastically reduced. The main interference is generated by the network itself, i.e. self-interference. The performance differs in comparison to the external interference limited case. The lowest outage probability is achieved for \((r, s) = (0.5, 0.9)\), which is almost the same as in Figure 6.3. The graph also shows that there exist several local suboptimal solutions (minimum), which originate from some special configurations that minimise the self-interference. For instance at \(r = s\), the transmitters are closely located, yielding a minimal spread in propagation delay.

**Figure 6.2** A local area SFN with adjusted transmitter locations. The distance \(D_6\) and \(D_{12}\) corresponds to the first and second ring, respectively.
Figure 6.3 The outage probability as a function of the continuous location variables $r$ and $s$. $R_{\text{service}} = 20$ km, $h_t = 37.5$.

Figure 6.4 The outage probability as a function of the continuous location variables $r$. $R_{\text{service}} = 20$ km, $h_t = 37.5$. 
Figure 6.5 The outage probability as a function of the continuous location variables $r$ and $s$. $R_{\text{service}}=50$ km, $h_t=37.5$.

Figure 6.6 The outage probability as a function of the continuous location variables $r$. $R_{\text{service}}=50$ km, $h_t=37.5$. 
The number of local minima indicates that the optimization of the transmitters cannot be solved by using algorithms that search for one extreme point. The optimization algorithm must therefore be able to “jump” out of local minima in its search for the global optimum. The number of local suboptimal solutions may increase for a real terrain, making the optimization even more complicated. In Figure 6.6 the outage probability is shown for $s = \{0.70, 0.85, 1.00\}$ and similar results as in Figure 6.4 are obtained.

The main conclusion from this analysis is that locating the transmitters in the centre point of each hexagon is a good choice. This holds for both external limited, as well as for self-interference limited SFNs.

### 6.2 Sensitivity Analysis

The location of the transmitters can be of importance in network planning. In our earlier studies it has been shown that the density of the transmitters is the crucial parameter. Since the sensitivity of the coverage property depends on the location of the networks’ sites, we would like to know whether an exact transmitter location is required or an approximate positioning is sufficient.

This problem is investigated by analysing the statistical coverage properties for networks with their transmitters randomly spread over the service area. The statistical quantity of interest is the probability that a randomly selected network, taken from a given transmitter location distribution, has an outage lower than a certain threshold. The threshold is determined from requirements on the coverage. Our requirement is less than 1% outage, implying that the system should cover at least 99% of its service area.

The same regular hexagonal service area is used as described in Chapter 2, i.e. the structure of the area that needs to be covered is the same. To analyse the system performance, we define the set of all randomly generated networks containing $M$ transmitters as $R_{NM}$.

\[
R_{NM} = \{ \zeta_M = \{ \mathbb{r}_1 \ldots \mathbb{r}_M \}; \mathbb{r}_i \in T_i \} 
\]  

(6.3)
where the vector \( \vec{t}_i = (x_i, y_i) \) represents possible locations for transmitter \( i \). The transmitter locations for a randomly selected network configuration is represented by a stochastic variable, \( Z_M \in RN_M \). The probability of choosing a network with a certain transmitter configuration is given by

\[
Pr [Z_M = \zeta_M] = \prod_{i=1}^{M} Pr [(X_i, Y_i) = (x_i, y_i)]
\]  

(6.4)

where we have assumed that the random location selection is performed independently for every transmitter. The possible locations for transmitter \( i \) are determined by the probability function \( Pr [(X_i, Y_i) = (x_i, y_i)] \). Two different transmitter location density functions are used. The first is defined as

\[
Pr \left[ \vec{t}_i \right] = \begin{cases} 
\frac{1}{A_i} & \vec{t}_i \in T_i \\
0 & \vec{t}_i \notin T_i 
\end{cases}
\]  

(6.5)

where \( T_i \) is a continuous surface and \( A_i \) its corresponding area, i.e. the location probability is Uniform over \( T_i \). The second distribution function is a two dimensional Gaussian distribution,

\[
Pr \left[ \vec{t}_i \right] = \frac{1}{2\pi\sigma^2} e^{-\frac{\vec{m}_i - \vec{m}_i}{2\sigma^2}}
\]  

(6.6)

where \( \vec{m}_i \) is the expected location for transmitter \( i \) and \( \sigma \) the standard deviation.

An approximated position for the transmitters can be obtained using the above specified distributions. For instance, the expected location of the transmitters given in (6.6) could be chosen as the centre of each hexagon, since the results in both Figure 6.3 and 6.5 indicate that the vicinity of the centre point of each hexagon is a good selection.

We also note that the outage for a given network configuration, \( \eta(\zeta_M) \), is a deterministic quantity. The mathematical expression for the probability that a randomly selected network configuration taken from the set \( RN_M \) yields an outage lower than a required threshold \( \varepsilon \) is given by,

\[
Pr \left[ \eta(Z_M) \leq \varepsilon \right] = \sum_{\zeta_M \in RN_M} u \left[ \eta(\zeta_M) - \varepsilon \right] Pr [Z_M = \zeta_M]
\]  

(6.7)

where \( u(.) \) is the unit step function. We refer to this performance measure as the (cumulative) outage rate, which depends on the number of transmitters in the local network and the transmitter location probability functions.
The certainty that a randomly selected network configuration should have an outage lower than the threshold must be rather strict. At least it must be lower than the required outage probability. We assume that “transmitter location” planning is not necessary if \( \Pr \left[ \eta \left( Z_{\mu} \right) \leq 1 \times 10^{-2} \right] \geq 0.995 \). From the outage rate, we can then directly obtain if a randomly generated network taken from a given transmitter location distribution is sufficient.

Other performance measures that can be used are the mean outage rate and the variance between outages for different networks. These measures are both decreasing functions of the number of transmitters and can be obtained from the cumulative outage rate. In this thesis, we consider only the cumulative outage rate.

**WIDE AREA COVERAGE USING SFN**

In Figure 6.7, the cumulative outage rate is depicted for a self-interference limited wide area SFN with an average distance of 70 km between any two transmitters, i.e. the expected location of each transmitter is in the centre of each hexagon. Each transmitter is located at a random position according to a given strategy. The standard deviations for the three Gaussian transmitter location density functions in the graph are \( \sigma = 10 \) km, 50 km and 100 km, respectively. The requirements are satisfied for standard deviations lower than \( \sigma = 50 \) km, in which less than one out of a thousand transmitter configurations have an outage higher than 1%. The uniform strategy in the graph illustrates the outage rate where the location probability for transmitter \( i \) is uniform over hexagon \( i \). This corresponds to a maximum displacement of \( R = D / \sqrt{3} \), and in this example, is equal to \( R = 40 \) km. The uniform strategy satisfies the requirement specified above. From the network planner’s point of view, this means that an exact planning of the transmitter locations is not necessary and an approximate spreading of the transmitter suffices. The cumulative outage rate for an SFN with an average distance of 100 km between transmitters is given in Figure 6.8 yielding similar results.
Figure 6.7 The cumulative outage rate for a wide area SFN with an average distance of 70 km among transmitters. \( h_t = 150 \) m.

Figure 6.8 The cumulative outage rate for a wide area SFN with an average distance of 100 km among transmitters. \( h_t = 150 \) m.
LOCAL AREA COVERAGE USING SFN

In this section, each external interfering transmitters are assumed to be located in the centre of its corresponding hexagon. This means that the interference structure is the same. Figure 6.9 shows the outage rate for an interference limited local network consisting of 19 transmitters. Two uniform strategies are shown, where in the first case one transmitter is uniformly located in every hexagon, and in the second strategy, all 19 transmitters can be located in the entire service area. The graph also includes three Gaussian strategies with standard deviations $\sigma = 1$ km, 3 km and 7 km, respectively. Each transmitter has its Gaussian probability location function centred in the middle of its hexagon. A maximum displacement of 7 km is acceptable to fulfil the requirement (0.005) at an outage level of 1%, i.e. $Pr[\eta(\mathcal{Z}_M) \leq 10^{-2}] \geq 0.995$. The low diversity gain in local networks permits only small transmitter displacement.

Figure 6.9 The outage rate for $K=3$ and $M=19$ transmitters, $R_{\text{service}} = 30$ km, $h_t = 37.5$. 
6.3 Summary

The regular spread of the transmitters appears to be the most appropriate strategy in most cases. The coverage properties over the service area then become quite uniform. Even for local networks with many interfering networks nearby, this placement is a good choice.

An exact regular spread of the transmitters is not necessary to achieve adequate coverage. In wide area networks with 70 km separation between any two transmitters, adequate coverage is obtained, i.e. an outage less than 1%. Location displacement from the regular structure results in a degradation in coverage. The outage increases and to achieve at least 1%, the variations cannot be more than 50 km. For local area coverage, the same conclusions can be made. The low diversity gain in these system can allow only small transmitter displacement on the order of a couple of kilometres.
Chapter 7
CONCLUSIONS

In this thesis, we have studied the coverage properties of an Orthogonal Frequency Division Multiplexing (OFDM) based single frequency networks (SFNs). An analysis on the receiver performance has been presented, which resulted in a method to perform coverage calculations. The receiver analysis showed that the method that is extensively used in today’s SFN planning [6] can only be employed for some special OFDM signal constellations. The coverage in an SFN is dependent on receiving signals from many transmitters, yielding a transmitter diversity gain. As our results indicate, the coverage properties depend on many different parameters, where the most important being the density of the transmitters which will determine the degree of diversity. For both wide area and local networks higher density yields better coverage properties. We have shown that the existing FM infrastructure, with transmitter sites separated by 60 km, can be used for the national Digital Audio Broadcasting (DAB) system, yielding very good coverage. However, for a local network, with only a few transmitters in each region, the performance drops drastically mainly due to the low degree of transmitter diversity.

It is demonstrated that local SFN-systems may achieve adequate performance with a frequency reuse factor of three. In FM-broadcasting using the similar models as in this thesis, the minimum published frequency reuse factor is seven [73], but in practice this factor is close to nine. By introducing local SFNs, we can expect to have two to three times as many programs as what we have today. Due to the low frequency reuse factor, the interference generated from other nearby SFNs must be limited. We have shown that it is possible to reduce the interference among local networks by increasing the number of transmitters per local area. The results indicate a need for more than ten transmitters per local service area to reach adequate coverage. In addition, if the service area is small it is also preferable to use low transmitter antennas to reduce the interference among the
networks and thereby exploiting the increased propagation loss beyond the radio horizon. For larger areas with sparse transmitter distance, higher antennas are needed to cover the area properly. However, the self-interference due to intersymbol interference between two consecutive OFDM frames then becomes more pronounced, which imposes an upper limit on applicable antenna heights. With respects to the outage probability, there exists an optimal antenna height for a given local service area size. This optimum antenna height is indirectly dependent on the length of the guard interval, which is employed to prevent the intersymbol interference. For a short guard interval, as intended for local DAB, the SFN system has considerable problems with self-interference within the network. To counter this, the service area needs to be quite small. However, for the national DAB mode, where the guard interval is much longer, it is possible to cover wider areas and higher transmitter antennas are recommended.

The importance of the transmitter locations has been studied. A uniform spread of the transmitters appears to be the most appropriate strategy in most cases. Similar coverage properties is then obtained over the service area. Even for local networks with other interfering networks nearby, this placement is a good choice. Furthermore, an exact uniform spacing of the transmitters is not necessary to achieve adequate coverage. For wide area networks with an average distance of 70 km between any two transmitters, large displacement can be made without any significant losses. For local area networks, the maximum displacement is on the order of a couple of kilometers.

The roughness of the terrain surrounding the receiver has an impact on the received power from different transmitters in the sense that the received signals all have a similar shadow fading environment. The correlation properties among the signals are caused by the actual terrain profile, and will affect the transmitter diversity gain in the system. From the results presented in this thesis, a high correlation among the received signals seems to be preferable in interference limited systems, whereas for a noise limited situation correlation is not desirable. Correlation reduces the variance of the interference, resulting in a controlled interference environment. Since the shadow fading environment is almost the same, the received power levels are distance dependent. Hence, the signals from the closest transmitters (the useful signals) are always received at a higher power level than those from far away interfering transmitters. For a noise limited situation, the contrary holds, which is more realistic for diversity systems. The useful received signals have then almost the same power level and for the case in
CONCLUSIONS

which this level is lower than the noise level, there is large degradation in performance. In wide area applications, the system is often noise limited and a terrain that causes high correlation in the shadow fading yields a negative effect. This is something that needs to be taken into account in the network planning process. For dense local SFNs, the external interference is dominate and thus, a correlated environment leads to a positive effect on the coverage.

Although the results are promising, several interesting problems remain to be solved. The system evaluated in this thesis is quite ideal. For instance, frequency offsets in the transmitters are not taken into account. Problems with inaccurate frequency references at the transmitter sites would generate frequency orthogonality losses for all received signals. This would occur even for signals arriving within the guard interval, resulting in a degradation of the coverage. Similar effects could also occur if the receiver is moving very fast, in particular if higher frequency bands are used, e.g. at 1.5GHz (DAB mode-III). The spreading of each subcarrier in the OFDM signal during mobile reception causes interchannel interferences.

We have seen that for dense local SFNs, the main interference is generated from other networks. Using directional antennas close to the border and finding the optimal antenna height for each transmitter, better performance can be obtained. As a result of using directional antennas, the networks can be located closer to one another, yielding better spectrum efficiency.

Another important aspect in SFN is how to optimize the system for a given terrain. As indicated in chapter 6, there exists several suboptimal solutions. If the roughness of the terrain varies greatly, a more complicated structure of the problem results and the number of suboptimal solutions increases drastically. Some initial studies have been performed using stochastic optimization algorithms to solve the coverage optimization over a real terrain [98][99]. Further studies in this area are needed.

Currently, radio communication services have diverged into two different classes: two-way communication or regular broadcasting. One could guess that in the future, these systems may become a combination of broadcasting and two way communication. In essence, the systems become information carrier systems which include both two-way communication and broadcasting services (or group information). Such systems may not be dedicated to a certain application, and
must be able to support different types of information flows. These systems could use the properties of broadcasting from the transmitters down to the receiver, and then use another strategy for the reverse communication link. An OFDM based SFN could then be a good candidate for the broadcasting part of a such communication system.
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Appendix A.

Derivation of the output from the correlation demodulator

\[ 0 \leq \tau < T_g \]

\[
\hat{D}_{0,k}(\tau) = \frac{1}{T} \sum_{l=0}^{N-1} hD_{0,l} \int_{0}^{T} e^{j2\pi f_{t}(t-\tau)} e^{-j2\pi f_{l}t} dt 
\]

\[
= \frac{1}{T} \left( \sum_{l=0}^{N-1} hD_{0,l} T e^{-j2\pi f_{c} \tau} \sin (k-l) (-1)^{k-l} \right) 
\]

\[
= h e^{-j2\pi f_{c} \tau} D_{0,k} 
\]

(A.1)

Derivation of the estimate for \( T_g \leq \tau < T_{tot} \):

\[
\hat{D}_{0,k}(\tau) = h \left( \frac{1}{T} \sum_{l=0}^{N-1} D_{-1,l} \int_{0}^{T} e^{j2\pi f_{t}(t-\tau)} e^{-j2\pi f_{l}t} dt + \right. 
\]

\[
\left. + \frac{1}{T} \sum_{l=0}^{N-1} D_{0,l} \int_{(\tau-T_{t})}^{T} e^{j2\pi f_{t}(t-\tau)} e^{-j2\pi f_{l}t} dt \right) 
\]

\[
= h \left( \sum_{l=0}^{N-1} D_{-1,l} \lambda_{lk} + \sum_{l=0, l \neq k}^{N-1} D_{0,l} \mu_{lk} + D_{0,k} \mu_{kk} \right) 
\]

(A.2)

where the derivation of \( \lambda_{lk} \) and \( \mu_{lk} \) can be found below. Interference caused by previous transmitted symbol

\[
\lambda_{lk} = \frac{1}{T} \int_{0}^{(\tau-T_{t})} e^{j2\pi f_{t}(t-\tau+T_{w})} e^{-j2\pi f_{l}t} dt 
\]

\[
= \frac{1}{T} e^{-j2\pi f_{l}(\tau-T_{w})} \int_{0}^{(\tau-T_{t})} e^{j2\pi (l-k) t/T} dt 
\]

\[
= e^{-j2\pi f_{l}(\tau-T_{w})} e^{j2\pi (l-k) (\tau-T_{t})/T} \frac{1}{j2\pi (l-k)} 
\]

\[
= \alpha \sin \left( \frac{\pi (l-k) \alpha}{\pi (l-k) \alpha} \right) \left( e^{-j2\pi f_{l}(\tau-T_{w})} e^{j\pi (l-k) \alpha} \right) 
\]

for all \( l,k \).

where \( \alpha = (\tau-T_{g})/T \).
Interference due to losses in frequency orthogonality.

\[
\mu_{lk} = \frac{1}{T} \int_{(\tau-T_s)}^{T} e^{j2\pi f_s (t-\tau)} e^{-j2\pi f_s t} dt =
\]
\[
= \frac{1}{T} e^{-j2\pi f_s \tau} \int_{(\tau-T_s)}^{T} e^{j2\pi (l-k) \tau / T} dt
\]
\[
= e^{-j2\pi f_s \tau} \frac{1 - e^{-j2\pi (l-k) (\tau-T_s) / T}}{j2\pi (l-k)}
\]
\[
= (-1)^{l-k} \frac{\sin (\pi (l-k) \alpha)}{\pi (l-k) \alpha} e^{-j2\pi f_s \tau} \frac{f_{\pi} (l-k) \alpha}{e} e^{j\pi (l-k) \alpha} \quad \text{for all } l \neq k
\]

(A.4)

It can also be shown that \( \mu_{lk} \) can be expressed as

\[
\mu_{lk} = (1-\alpha) \frac{\sin (\pi (l-k) (1-\alpha))}{\pi (l-k) (1-\alpha)} e^{-j2\pi f_s \tau} e^{-j\pi (l-k) (1-\alpha)} \quad \text{for all } l,k
\]

where \( \mu_{kk} \) is the constructive part and

\[
\lambda_{lk} = \alpha e^{-j2\pi f_s (\tau-T_{\text{tot}})}
\]

\[
\lambda_{kk} = \alpha e^{-j2\pi f_s \tau}
\]

(A.6)

The expected power of these parameters are:

\[
|\lambda_{lk}|^2 = \left[ \alpha \sin c \left( (l-k) \alpha \right) \right]^2 \quad \text{for all } l, k
\]

\[
|\mu_{lk}|^2 = \left[ (1-\alpha) \sin c \left( (l-k) (1-\alpha) \right) \right]^2 \quad \text{for all } l, k
\]

(A.7)

\[
|\lambda_{lk}|^2 = |\mu_{lk}|^2 = \left[ \alpha \sin c \left( (l-k) \alpha \right) \right]^2 \quad \text{for all } l \neq k
\]

Now, replacing the parameters \( \lambda_{lk}(\tau) \) and \( \mu_{lk}(\tau) \) by their expressions in (A.2) yields

\[
\hat{D}_{0,k}(\tau) = \frac{1}{h} \left( \sum_{l=0}^{N-1} \frac{\sin (\pi (l-k) (1-\alpha))}{\pi (l-k)} e^{-j\pi (l-k) (1-\alpha)} e^{-j2\pi f_s \tau} D_{0,l} \right)
\]

\[
\frac{1}{h} \sum_{l=0}^{N-1} \frac{\sin (\pi (l-k) \alpha)}{\pi (l-k)} e^{-j\pi (l-k) \alpha} e^{-j2\pi f_s (\tau-T_{\text{tot}})} D_{-1,l}
\]

(A.8)

Note that a small variation in \( \tau \) yields large variations in both \( e^{-j2\pi f_s \tau} \) and \( e^{-j2\pi f_s (\tau-T_{\text{tot}})} \) since \( f_s \) is on the order of \( 10^8 \) or larger. In comparison to these variations, the normalized delay is slow varying. Rewriting the expression (A.8) using \( m=l-k \) yields
\[
\hat{D}_{0,k}(\tau) = h \left[ (1-\alpha) e^{-j2\pi f_k \tau} D_{0,k} + \sum_{m=1}^{K_2} \frac{\sin(\pi m (1-\alpha))}{\pi m} e^{-j\pi m (1-\alpha)} e^{-j2\pi f_k \tau} D_{0,k+m} + \alpha e^{-j2\pi f_k (\tau-T_{tot})} D_{-1,k} + \sum_{m=1}^{K_3} \frac{\sin(\pi m\alpha)}{\pi m} e^{-j\pi m\alpha} e^{-j2\pi f_k \tau} (\tau-T_{tot}) D_{-1,k+m} \right] + \sum_{m=1}^{K_3} \frac{\sin(\pi m\alpha)}{\pi m} e^{j\pi m\alpha} e^{-j2\pi f_k \tau} (\tau-T_{tot}) D_{-1,k-m} \right] \]

\[
= h \left[ (1-\alpha) e^{-j2\pi f_k \tau} D_{0,k} + \alpha e^{-j2\pi f_k (\tau-T_{tot})} D_{-1,k} + I_{0,k}(\tau) \right]
\]

where \(K_f = N-k-1\), \(K_2 = k\) and \(I_{0,k}(\tau)\) is the interference caused by the loss of subcarrier orthogonality.
Matrix representation of the demodulator output for all carriers

\[ 0 < \tau < T_g \]

The following vector \( \mathbf{Y}(\tau) \) describing the whole received OFDM frame as

\[
\mathbf{Y}(\tau) = \begin{bmatrix} \hat{\mathbf{D}}_{0,0}(\tau) & \hat{\mathbf{D}}_{0,1}(\tau) & \cdots & \hat{\mathbf{D}}_{0,N-1}(\tau) \end{bmatrix}^T = h \mathbf{P}_0(\tau) \mathbf{D}_0, \tag{A.10}
\]

where \((.)^T\) stands for transposition and

\[
\mathbf{P}_0(\tau) = \begin{bmatrix} e^{-j2\pi \alpha \tau} & 0 & \cdots & 0 \\ 0 & e^{-j2\pi \alpha \tau} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & e^{-j2\pi \alpha (N-1) \tau} \end{bmatrix}, \quad \mathbf{D}_0 = \begin{bmatrix} D_{0,0} \\ D_{0,1} \\ \vdots \\ D_{0,N-1} \end{bmatrix} \tag{A.11}
\]

\[ T_g < \tau < T_{tot} \]

An equivalent expression for (A.9) can be written in matrix form as

\[
\hat{\mathbf{D}}_{0,k}(\tau) = h \left\{ \mathbf{U}_{0,k}(\alpha) \mathbf{P}_0(\tau) \mathbf{D}_0 + \mathbf{U}_{-1,k}(\alpha) \mathbf{P}_{-1}(\tau) \mathbf{D}_{-1} \right\}, \tag{A.12}
\]

where

\[
\mathbf{U}_{p,k}(\alpha) = \begin{bmatrix} u_{p,\alpha}(-K_2) & u_{p,\alpha}(-K_2+1) & \cdots & u_{p,\alpha}(K_1) \end{bmatrix}
\]

\[
u_{p,\alpha}(m) = \begin{cases} \frac{\sin(\pi m \alpha)}{\pi m} e^{-j\pi m \alpha} & p = -1 \\ \frac{\sin(\pi m (1-\alpha))}{\pi m} e^{-j\pi m (1-\alpha)} & p = 0 \end{cases}
\]

\[
\mathbf{P}_p(\tau) = \begin{bmatrix} e^{-j2\pi f_\alpha (\tau + p T_{ou})} & 0 & \cdots & 0 \\ 0 & e^{-j2\pi f_\alpha (\tau + p T_{ou})} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & e^{-j2\pi f_\alpha (\tau + p T_{ou})} \end{bmatrix} \tag{A.13}
\]

\[
\mathbf{D}_p = \begin{bmatrix} D_{p,0} & D_{p,1} & \cdots & D_{p,N-1} \end{bmatrix}^T
\]

In (A.12), the received symbol is expressed as a linear function of the transmitted symbol frame and the previous symbol frame, both multiplied by a phase component, \( \mathbf{P}_p(\tau) \), and a slow varying component, \( \mathbf{U}_{p,k}(\alpha) \). We introduce a new vector \( \mathbf{Y}(\tau) \) defined as
\[
Y(\tau) = \begin{bmatrix}
\hat{D}_{0,0}(\tau) & D_{0,1}(\tau) & \ldots & D_{0,N-1}(\tau)
\end{bmatrix}^T,
\]
\[
= h \{ U_0(\alpha) P_0(\tau) D_0 + U_{-1}(\alpha) P_{-1}(\tau) D_{-1} \}
\]
where
\[
U_p(\alpha) = \begin{bmatrix}
U_{p,0}(\alpha) \\
U_{p,1}(\alpha) \\
\vdots \\
U_{p,N-1}(\alpha)
\end{bmatrix} = \begin{bmatrix}
u_{p,\alpha}(0) & u_{p,\alpha}(1) & \ldots & u_{p,\alpha}(N-1) \\
u_{p,\alpha}(-1) & u_{p,\alpha}(0) & \ldots & u_{p,\alpha}(N-2) \\
\vdots & \vdots & \ddots & \vdots \\
u_{p,\alpha}(-N+1) & \ldots & \ldots & u_{p,\alpha}(0)
\end{bmatrix}
\]
\[
Y(\tau) \text{ represents the obtained frame complex vector from all carriers.}
\]

Derivation of the total estimate for all carriers when receiving many signals
The observed output from the \(k\):th matched filter can be obtained as
\[
\hat{D}_{0,k} = \left( \sum_{i=1}^{H-1} h_i e^{-j2\pi f_i \tau_i} + \sum_{i=H}^{nM} h_i e^{-j2\pi f_i \tau_i} (1 - \alpha_i) \right) D_{0,k} + \\
\sum_{i=H}^{nM} h_i \{ (1 - a) I - U_{0,k}(\alpha_i) \} P_0(\tau_i) D_0 + \\
\{ \alpha I - U_{-1,k}(\alpha_i) \} P_{-1}(\tau_i) D_{-1} + z
\]
In a similar way we can express the output from all \(N\) matched filters as a vector
\[
\hat{D}_0 = \sum_{i=1}^{nM} \begin{bmatrix}
Y(\tau_i)
\end{bmatrix} = \\
\sum_{i=1}^{H-1} h_i P_0(\tau_i) D_0 + \\
\sum_{i=H}^{nM} h_i \{ U_0(\alpha_i) P_0(\tau_i) D_0 + U_{-1}(\alpha_i) P_{-1}(\tau_i) D_{-1} \} + Z
\]
where \(Z\) is the noise vector.
The interference distribution

Figure A.1 shows the probability distribution function of the interference envelope normalised to the useful envelope for two numerical simulations with different delays. The parameters for DAB mode-II have been applied in Figure A.1 (see Table 1. in Chapter 2). Since we normalise to the useful envelope, the channel parameter \( h \) is not included in the probability density function (pdf) of the interference. Hence, the graph in Figure A.1 represents the pdf of only the interference term normalised by the factor \( (1 - \alpha) e^{-j2\pi f_\tau} D_{0,k} \). The envelope of the interference yields a Rice distribution \[2\], i.e.,

\[
Pr [a] = I_0 \left( \frac{a a_0}{\sigma^2} \right) \exp \left( -\frac{a^2 + a_0^2}{2\sigma^2} \right)
\]  

(A.18)

where \( I_0 \) is the zero order modified Bessel function of first kind and

\[
a = \frac{\alpha e^{-j2\pi f_\tau (\tau - T_{\omega})} D_{-1,k} + I_{0,k} (\tau)}{(1 - \alpha) e^{-j2\pi f_\tau} D_{0,k}}
\]

\[
a_0 = \frac{\alpha e^{-j2\pi f_\tau (\tau - T_{\omega})} D_{-1,k}}{(1 - \alpha) e^{-j2\pi f_\tau} D_{0,k}} = \frac{\alpha}{(1 - \alpha)}
\]  

(A.19)

\[
\sigma^2 = \frac{E \left[ \left| I_{0,k} (\tau) \right|^2 \right]}{(1 - \alpha) e^{-j2\pi f_\tau} D_{0,k}} = \frac{2\alpha (1 - \alpha)}{(1 - \alpha)^2} = \frac{2\alpha}{(1 - \alpha)}
\]

Figure A.1 shows that the amount of interference increases as a function of the error in time synchronisation. This is illustrated in the curve where the probability that the interfering envelope is larger than the useful envelope is an increasing function of the parameter \( \alpha \).

Figure A.2 shows the pdf of the interference phase relative to the phase of the wanted signal. An expression for the interference phase relative to the phase of the wanted signal is given in the subsequent section of appendix A. This pdf is approximately uniformly distributed between \([-\pi, \pi]\) for \( \alpha = 0 \). The output from the correlation demodulator can then be expressed as

\[
\hat{D}_{0,k} (\tau) = h \left[ (1 - \alpha) e^{-j2\pi f_\tau} D_{0,k} + I_{0,k} (\tau) \right]
\]  

(A.20)

The pdf of the interference phase has a periodic shape for \( \alpha = 1 \). The dominating interference from the previous symbol on the same carrier then generates this
periodic density function. The same approximation as in (A.20) now becomes,

\[ D_{0,k}(\tau) = h \left[ (1 - \alpha) e^{-j2\pi f_s \tau} D_{0,k} + \alpha e^{-j2\pi f_s (\tau - T_{tot})} D_{-1,k} \right] \]  

(A.21)

showing that the interfering phase depends on the applied modulation scheme which explains the behaviour of the density function. The QPSK signals used in the simulations, shown in appendix A, generated four maxima (and minima) in the pdf for the interference. If the signal constellation were of an even higher order, e.g. 16PSK, then the phase distribution would be approximately uniform distributed. Simulation results also indicate that the instant value of the interference envelope and phase can be assumed to be two independent random variables.

We have now shown that the interference envelope is well approximated as a Rice distribution. Although the distribution of the interfering phase is not uniformly distributed, we expect that a Gaussian approximation of the interference would give a good estimate for the symbol error probability. When \( \alpha = 1 \), the interference term is much stronger than the useful part, e.g. in the QPSK case, the probability to make a correct decision is \( 1/4 \), which is the same for an AWGN channel when the noise is dominate.

![Figure A.1](image-url)  
*Figure A.1* The pdf for the normalised interfering envelope (normalised by the useful envelope).
Figure A.2 The pdf for the normalised interfering phase (normalised with respects to the phase of the useful part).
Derivation of the normalised interfering phase, $\eta$

$$\eta = \arg \left[ \frac{\alpha e^{-j2\pi f_i \tau}}{(1 - \alpha) e^{-j2\pi f_i \tau} D_{0,k}} \right]$$

(A.22)

$$= \arg \left[ \alpha e^{-j2\pi f_i \tau} D_{-1,k} \right] + \arg \left[ e^{-j2\pi f_i \tau} D_{0,k} \right]$$

Combining (A.23) and (A.24) yields

$$\alpha e^{-jB} + \Gamma e^{-jY} = \alpha \cos (\beta) + \Gamma \cos (\gamma) + j [\alpha \sin (\beta) + \Gamma \sin (\gamma)] \quad \text{ (A.23)}$$

where $$(\cdot) = \pi - (\cdot)$$

$$\alpha \cos (\beta) + \Gamma \cos (\gamma) = \alpha \cos (\omega + \Delta \omega) + \Gamma \cos (\omega - \Delta \omega)$$

$$= (\alpha + \Gamma) \cos (\Delta \omega) \cos (\omega) + (\alpha - \Gamma) \sin (\Delta \omega) \sin (\omega)$$

$$= \left[ (\alpha + \Gamma) \cos (\Delta \omega) \cos (\omega) + \sqrt{(\alpha + \Gamma)^2 + 2 \alpha \Gamma \cos (2\Delta \omega)} \right]$$

$$+ \sqrt{(\alpha - \Gamma)^2 \sin (\omega)}$$

$$= \sqrt{\alpha + \Gamma + 2 \alpha \Gamma \cos (2\Delta \omega)} \left[ \cos (\omega - \tan \left( \frac{\alpha - \Gamma}{\alpha + \Gamma} \tan (\Delta \omega) \right)) \right]$$

(A.24)

Combining (A.23) and (A.24) yields

$$\arg \left( \alpha e^{-jB} + \Gamma e^{-jY} \right) = \arg \left( \cos (\omega - \tan \left( \frac{\alpha - \Gamma}{\alpha + \Gamma} \tan (\Delta \omega) \right)) \right) +$$

$$+ j \cos \left( \tilde{\omega} - \tan \left( \frac{\alpha - \Gamma}{\alpha + \Gamma} \tan (\Delta \omega) \right) \right)$$

(A.25)

where

$$x = \omega - \tan \left( \frac{\alpha - \Gamma}{\alpha + \Gamma} \tan (\Delta \omega) \right)$$

$$= \frac{\beta + \Gamma}{2 - \tan \left( \frac{\beta - \Gamma}{2} \right)}$$

(A.26)

Inserting the parameters given in (A.22) into (A.26) yields the required result.
Derivation of the expected received power

\[ 0 < \tau < T_g \]
\[ E \left[ \hat{D}_{0,k}(\tau) \right]^2 = E \left[ |h|^2 \right] \tag{A.27} \]

\[ T_g < \tau < T_{tot} \]
By starting using the expression (A.7), we can derive the expected interference power, i.e.,
\[ \hat{D}_{0,k}(\tau) = h \left\{ \sum_{l=0}^{N-1} \frac{\sin(\pi(l-k)(1-\alpha))}{\pi(l-k)} e^{-j\pi(l-k)(1-\alpha)} e^{-j2\pi\tau D_{0,l}} \right. \]
\[ + \left. \sum_{l=0}^{N-1} \frac{\sin(\pi(l-k)\alpha)}{\pi(l-k)} e^{-j\pi(l-k)\alpha} e^{-j2\pi\tau D_{-1,l}} \right\} \tag{A.28} \]
The expected power can now be expressed as
\[ E \left[ \hat{D}_{0,k}(\tau) \right]^2 = E \left[ |h|^2 \right] \left( \sum_{l=0}^{N-1} \left( 1 - \alpha \right) \frac{\sin(\pi(l-k)(1-\alpha))}{\pi(l-k)(1-\alpha)} \right)^2 + \]
\[ + \left. \sum_{l=0}^{N-1} \frac{\alpha \sin(\pi(l-k)\alpha)}{\pi(l-k)\alpha} \right\}^2 \tag{A.29} \]
and making the variable substitution \( m = l - k \) yields
\[ E \left[ \hat{D}_{0,k}(\tau) \right]^2 = E \left[ |h|^2 \right] \left( 1 - \alpha \right)^2 + \]
\[ \sum_{m=1}^{K_1} \left( 1 - \alpha \right) \sin \left( m \left( 1 - \alpha \right) \right)^2 + \]
\[ \sum_{m=1}^{K_2} \left( 1 - \alpha \right) \sin \left( m \left( 1 - \alpha \right) \right)^2 + \]
\[ \alpha^2 + \]
\[ \sum_{m=1}^{K_1} \left( \alpha \sin \left( m\alpha \right) \right)^2 + \]
\[ \sum_{m=1}^{K_2} \left( \alpha \sin \left( m\alpha \right) \right)^2 \] \tag{7.1}
where \( K_1 = N - k - 1 \), \( K_2 = k \). Furthermore, by noticing that
\[ \left( 1 - \alpha \right) \sin \left( m \left( 1 - \alpha \right) \right)^2 = \alpha \sin \left( m\alpha \right)^2 \] \tag{A.30}
for all $m \neq 0$ we get the following simple expression for the expected power

$$E \left[ \left| \hat{\theta}_{0,k}(\tau) \right|^2 \right] = E \left[ |h|^2 \right] \left( (1 - \alpha)^2 + \alpha^2 \sum_{m=1}^{K} \sin^2(m\alpha) + 2\alpha^2 \sum_{m=K+1}^{L} \sin^2(m\alpha) \right)$$  \hspace{1cm} (A.31)

where $K = \min(K_1, K_2)$ and $L = \max(K_1, K_2) - K$.

To verify these results we simulated the interference on different carriers. Figure A.3 shows the average interference power for 1000 trials for a carrier in the middle of the OFDM band (circles) and a carrier at the border of the band (stars). The graph also includes the derived approximation for the interfering power (solid). We notice that the carrier at the border does not receive as much interference as the one in the middle. The reason is that this carrier receives only half of the interference caused by loss in frequency orthogonality. The expected interference for this particular carrier is given by $\alpha^2 + (\alpha - \alpha^2) = \alpha$, which also matches the simulation results (see the dashed-dotted line in Figure A.3).

Figure A.3 Mean interference power over 1000 trials as a function of the delayed signal. Circles (carrier in the middle of the OFDM band) and stars (carrier at the edge of the OFDM band) represents the simulated results. Solid and dashed-dotted lines are represent analytical values.
Derivation of the weight function $R(\tau)$

Rewriting the equation for the case when only every second carrier is used yields,

$$E\left[|\hat{D}_{0,k}(\tau)|^2\right] = E\left[|\hat{h}|^2\right] \left((1-\alpha)^2 + \alpha^2 + 4\alpha^2 \sum_{m=1}^{K} \sin^2(2m\alpha) + 2\alpha^2 \sum_{m=K+1}^{L} \sin^2(2m\alpha)\right)$$  \hspace{1cm} (A.32)

$$= E\left[|\hat{h}|^2\right] (a + b + c + d)$$

where

$$a = (1-\alpha)^2$$
$$b = \alpha^2$$
$$c = 4\alpha^2 \sum_{m=1}^{K} \sin^2(2m\alpha)$$ \hspace{1cm} (A.33)
$$d = 2\alpha^2 \sum_{m=K+1}^{L} \sin^2(2m\alpha)$$

Figure A.4 shows the terms $a$, $b$, $c$, $d$ and the sum $b+c+d$ (the total interference) for $K = 5$ (dashed-dotted) and $50$ (solid) with $L = 1000$ as a function of the normalised delay $\alpha$.

Figure A.4 Different weight functions as a function of $\alpha$. 
