Atomic Transport in Optical Lattices

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Umeå 2010
The figure on the cover page is a montage of selected images of the directed transport of an atomic sample along a zig-zag path. The transported atoms are interacting with two dissipative optical lattices, and the images are taken with a non-invasive fluorescence imaging technique.
Abstract

This thesis includes both experimental and theoretical investigations of fluctuation-induced transport phenomena, presented in a series of nine papers, by studies of the dynamics of cold atoms in dissipative optical lattices.

With standard laser cooling techniques about $10^8$ cesium atoms are accumulated, cooled to a few $\mu$K, and transferred into a dissipative optical lattice. An optical lattice is a periodic light-shift potential, and in dissipative optical lattice the light field is sufficiently close to resonance for incoherent light scattering to be of importance. This provides the system with a diffusive force, but also with a friction through laser cooling mechanisms.

In the dissipative optical lattices the friction and the diffusive force will eventually reach a steady state. At steady state, the thermal energy is low enough, compared to the potential depth, for the atoms to be localized close to the potential minima, but high enough for the atoms to occasionally make inter-well flights. This leads to a Brownian motion of the atoms in the optical lattices. In the normal case these random walks average to zero, leading to a symmetric, isotropic diffusion of the atoms.

If the optical lattices are tilted, the symmetry is broken and the diffusion will be biased. This leads to a fluctuation-induced drift of the atoms. In this thesis an investigation of such drifts, for an optical lattice tilted by the gravitational force, is presented. We show that even though the tilt over a potential period is small compared to the potential depth, it clearly affect the dynamics of the atoms, and despite the complex details of the system it can, to a good approximation, be described by the Langevin equation formalism for a particle in a periodic potential. The linear drifts give evidence of stop-and-go dynamics where the atoms escape the potential wells and travel over one or more wells before being recaptured.

Brownian motors open the possibility of creating fluctuation-induced drifts in the absence of bias forces, if two requirements are fulfilled: the symmetry has to be broken and the system has to be brought out of thermal equilibrium. By utilizing two distinguishable optical lattices, with a relative spatial phase and unequal transfer rates between them, these requirements can be fulfilled. In this thesis, such a Brownian motor is realized, and drifts in arbitrary directions in 3D are demonstrated. We also demonstrate a real-time steering of the transport as well as drifts along pre-designed paths. Moreover, we present measurements and discussions of performance characteristics of the motor, and we show that the required asymmetry can be obtained in multiple ways.
List of papers:

This thesis is based on the following papers:

I Directed transport with real-time external steering and drifts along pre-designed paths using a Brownian motor
H. Hagman, M. Zelan, C.M. Dion, and A. Kastberg

II Breaking the symmetry of a Brownian motor with symmetric potentials
H. Hagman, M. Zelan, and C.M. Dion
Submitted to J. Phys. A

III Experimental measurement of the efficiency and the transport coherence of a Brownian motor realized with cold atoms in optical lattices
M. Zelan, H. Hagman, G. Labaigt, C.M. Dion, and S. Jonsell
Submitted to Phys. Rev. E.

IV Fluctuation-induced drift in a gravitationally tilted optical lattice
M. Zelan, H. Hagman, K. Karlsson, C. M. Dion, and A. Kastberg

V Theoretical investigation of quantum walks by cold atoms in a double optical lattice
N. Satapathy, H. Hagman, M. Zelan, A. Kastberg, and H. Ramachandran

VI Assessment of a time-of-flight detection technique for measuring small velocities of cold atoms

VII A three-dimensional Brownian motor, realised with symmetric optical lattices
A. Kastberg, C. M. Dion, H. Hagman, and M. Zelan

VIII Influence of the lattice topography on a three-dimensional, controllable Brownian motor
H. Hagman, C.M. Dion, P. Sjölund, S. J. H. Petra, and A. Kastberg
EPL 81, 33001 (2008).
IX Characterisation of a three-dimensional Brownian motor in optical lattices
Comments to my contribution to the papers included in the thesis

A working experimental apparatus was existing when I started my Ph.D. In collaboration with other students, the apparatus has been rebuilt, redesigned, and developed continuously. This work has been shared within the laboratory, and the main responsible persons for the laboratory work have been myself, Martin Zelan, Peder Sjölund and Stefan Petra. The initial setup was constructed by Johan Jersblad, Harald Ellmann, and Anders Kastberg.

Papers I. I was, together with Martin Zelan, responsible for the development of the experimental setup, and for collecting the experimental data. I was also the main person responsible for the writing of the paper.

Paper II. I was the main person responsible of writing the numerical program, for collecting and analyzing the data, and for structuring and writing the paper.

Paper III. I was, together with Martin Zelan, responsible for the development of the experimental setup, for collecting the experimental data, and for adapting the general theory to our system. I also took part in the writing of the paper.

Paper IV. I was, together with Martin Zelan, responsible for the development of the experimental setup, for collecting the experimental data, and I took great part in the analysis and interpretation of the experimental data. I also wrote and ran the simple classical numerical program.

Paper V. I took part in the structuring and writing of the paper, and was responsible of providing experimental details for the numerical program.

Paper VI. I took part in development of the experimental setup and the collecting of experimental data. I was the main person responsible for the data analysis, and for structuring and writing the paper.

Paper VII. Conference proceeding summarizing the work of papers IX, VIII and VI.

Paper VIII. I was the main person responsible for collecting and analyzing the experimental data, and for structuring and writing the paper.
Paper IX. I took part in collecting the experimental data, and in the writing of the paper.
Acknowledgements

This thesis summarizes the work I’ve done during my Ph.D. from the point of view of physics. However, in this section my Ph.D. will be summarized from the perspective of whom I’ve worked with.

First of all I would like to thank Anders Kastberg, for giving me the opportunity to become a Ph.D. student, and for being my supervisor, both for my maser thesis and for the first three and a half years of my Ph.D. You gave me a good introduction to laser cooling, and to research in general, both the purely scientific part and the surrounding politics. During this time Claude Dion was my assistant supervisor, and as AK moved to France in 2009, CD became my supervisor. You broadened my theoretical understanding and gave me insight to the world of numerical simulations.

Many thanks to Martin Zelan, whose Ph.D. has been running parallel with mine. During the last two and a half years we have been involved in the same projects, and we have shared the struggle in the lab. You have been a true friend, an excellent co-worker, and a nice travel companion.

When I started, an experimental setup already existed, and two fellows had been working in the lab for a couple of years. These two were Peder Sjölund and Stefan Petra. You gave me a pleasant start of my Ph.D., a good overview of the system, and good tips on how to handle things. Thanks also to Johan Jersblad and Harald Ellmann, who initially built the setup.

During the first two and a half years, there also existed a sister lab working with cold Rb atoms. In this lab, Robert Saers and Magnus Rehn were working. You were good friends, and gave me many useful tips.

As an experimentalist it is nice to have theoretical backup. In the projects I’ve been working with, this have been given by, besides CD, Svante Jonsell and Mats Nylén. I would also like to thank Emil Lundh and Alberto Cetoli for helpful discussions, Nandan Satapathy and Gabriel Labaigt for good collaboration, as well as Jim Liljekvist and Kristoffer Karlsson for nice master projects and for good company in the lab. Emil was also my assistant supervisor for the last one and a half years.

I also thank all colleges at the Department of Physics, especially those who I’ve been teaching with, Hans Forsman, Ove Axner, Magnus Andersson, Erik Fällman, Stratos Kontris, Florian Nitze and Daniel Vägberg. For good help with practical and administrative solutions I thank Jörgen Eriksson, Katerina Hassler, Lena Burström, Lilian Andersson, Margaretha Fahlgren, Ann-Charlott Dalberg, Leif Hassmyr, and Karin Rinnefeldt.

Special thanks to my girlfriend Mikaela Åkerlind, who offered great support and put up with me after long working days, and to my family, Urban, Eva, Ann, and Elin Hagman, who always is a good company and a reliable support. I would also like to thank my friends, Mårten, Joni, and Daniel, whose presence in Umeå overlapped with mine.
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Chapter 1

Introduction

Transport phenomenas are a central part of physics and are of fundamental importance for both living organisms and artificial devices. As the length scale of the systems considered decreases, thermal noise generally becomes of increasing importance. This changes the dynamics of the systems drastically, making their control and theoretical treatment complicated. However, Brownian motors [1, 2] take advantage of this noise as they convert random fluctuations into directed motion in the absence of bias forces. In this thesis the directed transport of atoms in periodic optical potentials will be considered. More specifically, laser cooling will be used to provide atoms with a finite thermal noise [3, 4, 5], and the transport will be induced by the interaction with optical lattices [6, 7].

During the 20th century, science took a giant leap forward with the development of quantum physics [8]. The quantized energy levels of the atom are now explained [9], light is proven to be both a wave and a quantized particle, and the interaction between light and matter are shown to be strongly dependent on the frequency, the irradiance, and the polarization of the light field, which enables a manipulation of the atoms with light [10]. With the development of the laser [11] in the 60s, the control of the internal quantum state became significantly cleaner, but still was left the control of the motion of the atoms. For a sample of atoms, the motion can be divided into two types: the average motion, corresponding to a flow or a drift of the atoms, which for trapped atoms usually is zero, and the velocity spread, which is closely related to the temperature of the atomic sample. A large velocity spread makes the confinement of the sample more difficult, and does also, through Doppler broadening [11, 12], blur the internal quantum energy structure of the atoms. It is in this spirit that the field of laser cooling [3, 4, 5] was developed.

Laser cooling gathers, cool and traps neutral atoms with the help of laser fields. With standard laser cooling techniques temperatures down to a few µK can be achieved [3, 4, 5]. That is, the thermal energy of the atomic
sample is just a millionth of a degree above absolute zero (−273.15°C). At these temperatures, the effects of the atomic sample's velocity spread are small, and a cleaner quantum system is obtained. This has given us applications such as the atomic clock [13, 14], which can measure time with an accuracy of $10^{-16}$. The second is today therefore defined via the hyperfine splitting of cesium [15, 16], which is also the same atom used in this thesis. More advanced cooling techniques can reduce the temperature even further, and with the realization of Bose-Einsteien condensates (BEC) [17, 18] in 1996, a completely coherent “macroscopic” quantum system was obtained.

The ultra-low temperatures also render confinement in optical potentials possible by the otherwise small optical forces, and a way of ordering the ultra-cold atoms is to let them interact with an optical lattice [6, 7]. Optical lattices are optical potentials created in the interference pattern of laser beams. The internal energy of the atoms are here changing with the changing intensity and/or polarization of the interference pattern, creating a periodic potential where the atoms can be trapped in the potential minima. Optical lattices are usually divided into two categories: conservative and dissipative. A conservative optical lattice is created with light fields whose frequencies are sufficiently far detuned from an atomic resonance for incoherent light scattering to be ignored. These optical lattices are therefore ideal for pure quantum systems, such as BEC. A dissipative optical lattice is created by light fields with frequencies sufficiently close to atomic resonance for incoherent light scattering to be of importance. The incoherent light scattering destroys any coherence, and provides the atoms with random momentum kicks and random quantum jumps between the internal states of the atoms. This will heat the atoms, but competing with this heating are inherent cooling mechanisms, leaving the atomic sample in a steady state. In this steady state, the atoms will generally be well localized close to the potential minima, but due to the random fluctuations inter-well flights will also occur, leading to spatial diffusion. This makes dissipative optical lattices a good system for studies in statistical physics and for investigations of the dynamical effects of random noise. These types of studies will be the main focus of this thesis.

Even though random fluctuations are generally considered as noise and useless energy, it still is energy, and an intriguing thought is to convert this noise into useful energy. This counter-intuitive conversion has actually been shown to be possible, provided that two requirements are fulfilled. (i) The system has to possess an asymmetry, in accordance with the Curie principle [19]. That is, the trapping potentials have to have at least one spatial (or spatio-temporal) asymmetry. (ii) The system has to be brought out of thermal equilibrium, in agreement with the second law of thermodynamics [20]. That is, there must exist a disturbance in the system that breaks the noise-friction equilibrium. Such devices, which can convert random fluctuations into directed motion or work in the absence of any bias forces, are
referred to as Brownian motors or Brownian ratchets [1, 2, 21, 22, 23].

Brownian motors are believed to be the driving mechanisms of a variety of biological motors [2, 24], ranging from inter-cell transport and virus translocation to muscle contraction [25, 26, 27]. Inspired by these biological machines, several proposals exist to utilize the principles of Brownian motors to power up future nanotechnology [28, 29]. Beside the naturally occurring biological motors, a number of artificial Brownian motors and ratchets have been realized, e.g., with cold atom in optical lattices [30, 31, 32, 33]. These artificial devices often have a relatively comprehensive and controllable structure, and can therefore work as models of larger and more complex naturally occurring Brownian motors [1, 2, 24]. They can also be used for fundamental studies of the properties and feasibility of Brownian motors.

Artificial Brownian motors usually consist of Brownian particles [34] in periodic potentials. The required asymmetry is generally included in the potential, e.g., as a sawtooth potential (ratchet potential). The second demand is usually fulfilled by non-adiabatically shifting a parameter of the potential, e.g., the potential depth as in a flashed ratchet, or the spatial phase as in a rocked ratchet [1].

In this thesis an alternative way of fulfilling the requirements is investigated. Instead of shifting the properties of one potential, by using two potentials and letting the particle shift between them, drifts can be induced in static and symmetric potentials. The symmetry is here broken by a combination of a relative spatial phase of the potentials and, e.g., different transfer rates between them. The rapid control of the potentials can hence be eliminated and a flexible setup is gained, where the drifts can be controlled by the relative spatial phase of the potentials [31, 35]. If a rapid control of the potentials is added to this system, the drifts can also be controlled in real time. With this control, real-time steering, drifts along pre-designed paths, and feedback controlled drifts could hence be realized, which are all important for the creation of useful future applications of Brownian motors in nanotechnology [28, 29, 36].

This thesis will be devoted to a two-state Brownian motor realized with cold atoms in two distinguishable optical lattices, where the random fluctuations generated by the incoherent light scattering are converted into an average drift of the atoms. The first chapters will give an introduction to laser cooling, optical lattices, and systems with noisy dynamics. Chapter 5 will describe the experimental setup used, and the last two chapters will present the main results obtained. In chapter 6, the influence of the noise will be characterized by studying fluctuation-induced drifts in tilted potentials, while in chapter 7 the realization of our two-state Brownian motor is described. We there show that inducing drifts in 3D is achievable, and demonstrate a real-time steering of these drifts as well as drifts along pre-designed paths. We also discuss ways to characterize the performance of our Brownian motor and alternative ways of obtaining the required asymmetry.
Chapter 2

Cooling and trapping of atoms

Laser cooling provides ultra-cold atoms with a finite thermal energy low enough for confinement in optical potentials to be possible [3, 4, 5]. The effects of the remaining thermal energy on the dynamics of the atoms in these potentials are the main focus of this thesis. In this chapter, an introduction to the basic tools and the theoretical cornerstones of laser cooling and optical potentials is presented. At the end, a short overview of the use of laser cooling in this thesis, along with a brief summary of general applications of ultra-cold atoms, are given.

2.1 Basic features of light and matter

Laser cooling builds on the ability to manipulate atoms with light [3, 4, 5]. Therefore, this chapter starts with short description of the key features of both light and matter.

Light

Light is an electro-magnetic wave quantized to discrete particles called photons [10]. The wave-particle duality of light is one of the most evident examples of the weirdness of quantum mechanics, and light will here be treated as both a particle and a wave in an alternating way.

Each photon can be assigned a wavelength $\lambda$. From this wavelength, other key properties of the photon can be expressed, such as the frequency, $\omega = 2\pi \times c/\lambda$, the energy, $E = h\omega$, the wave vector, $k = 2\pi/\lambda$, and the momentum, $p = hk$, where $c$ is the speed of light in vacuum and $h$ is Planck’s constant divided by $2\pi$. The wave-like nature of photons enables them to interfere, both with themselves and with other photons [10]. This will prove to be useful later in the thesis, as periodic potentials will be created in the
2.1. Basic features of light and matter

interference pattern of laser beams [6]. The wave-like nature also adds a polarization to the description of light, which can be seen as the direction in which the electric field (E-field) oscillates. If this direction is static the polarization is called linear, while if it is constantly turning it is denoted elliptic or circular. To create a stable and clear interference pattern, or to qualitatively manipulate atoms, a coherent light source with a well-defined wavelength, polarization, and irradiance is needed, which makes lasers ideal for such tasks.

Atoms

Atoms have quantized energy levels [9], \textit{i.e.}, the electrons can be only in specific states, where each state is described by a set of quantum numbers, and corresponds to a certain energy. In the simplest picture there are three quantum numbers: $n$ - the principal quantum number, $l$ - the orbital momentum quantum number, and $m$ - the magnetic quantum number which, in the absence of a magnetic field, doesn’t affect the energy. If the atom is studied in more detail, relativistic effects together with the coupling between different angular momenta of the atom (the orbital momentum $l$, the spin of the electrons, $s$, and the nuclear spin, $I$), will give raise to a fine structure and a hyperfine structure of the energy levels. These structures are simply the splitting of a level into many, and the different levels of the fine structure are usually denoted with the total angular momentum of electron $J$, and for the hyperfine structure with the total angular momentum of the atom $F$. This fairly complicated structure is simplified if alkali metals are considered, which have just one valence electron. These are also the dominating kind of atom in laser cooling, even though a few cases of laser-cooled rare earth metals do exist. In this thesis, the alkali metal cesium is used, whose energy levels will be described in chapter 5.

Atomic samples

An atom will, beside the internal degrees of freedom, also have a velocity, and if the atom is confined to some volume, where it can incoherently interact with light and with a large number of other atoms, its velocity will by randomized. The velocity of the whole atomic sample will therefore be described by a distribution where the width of the distribution (the spread of the velocities) will be closely related to the temperature of the atomic sample. In fact, we here define the temperature, $T$, as a kinetic temperature through the relation

$$k_B T = \frac{m \langle v^2 \rangle}{2},$$

where $k_B$ is Boltzmann’s constant, $m$ is the mass of an atom, and $\langle v^2 \rangle$ is the root-mean-square value of the velocity distribution. Here the atoms are
2.2 Interaction between light and matter

assumed to have zero average velocity, \( \langle v \rangle = 0 \). If an average drift is induced, this is included into equation 2.1 by the substitution \( \langle v^2 \rangle \rightarrow \langle v^2 \rangle - \langle v \rangle^2 \). Since the velocity affects the interaction with light, the velocity spread will complicate the manipulation of the atoms [37]. With laser cooling, the width of the velocity distributions can be reduced by a factor of \( 10^{-8} \) with respect to room temperature.

This raises the question of why use an atomic sample at all, and not just use single atoms. The reasons are many: the physics of one and many atom systems may differ significantly, studies of atom-atom interactions demands more than one particle, simulations of other many-body systems such as solid state lattices demands many particles, and studies of statistical properties is greatly eased. There exist several technical reasons as well. Single atoms are hard to control, traps become extremely sensitive to losses or fluctuations in the particle number, and the detection of a single atom is tricky. There do however exist experiments studying few body physics, and single atom detection is nowadays possible, e.g., [38].

2.2 Interaction between light and matter

The simplest form of interaction between light and matter may be the absorption of a photon by an atom, which thereby changes its energy and internal state. This requires that the photon have the same energy as the energy difference between two discrete energy levels of the atom. Here, the atomic states are separated into two kinds: ground states, which by themselves are stable (or metastable), and excited states which have a finite lifetime. The finite lifetime is associated with a spontaneous and random decay to a ground state, or a lower-energy excited state, this by emission of a photon into the vacuum field.

In agreement with the Heisenberg uncertainty principle, this finite lifetime is also associated with a broadening of the energy levels (characterized by the natural linewidth \( \Gamma \)). Due to this broadening, photons with small energy differences from the atomic transition can still be absorbed. This energy difference, in terms of frequency, is refereed to as a detuning, and is given by

\[
\Delta = \frac{1}{\hbar} [E_{\text{photon}} - (E_{\text{atom}}^e - E_{\text{atom}}^g)] ,
\]

where \( E_{\text{photon}} \) is the energy of the photon, \( E_{\text{atom}}^e \) is the energy of the excited state, and \( E_{\text{atom}}^g \) is the energy of the ground state. Light fields detuned below atomic resonance, \( \Delta < 0 \), are called red detuned, while frequencies above resonance, \( \Delta > 0 \), are called blue detuned.

The absorption rate (or the scattering rate), \( \Gamma' \), which is the rate at which photons of a certain frequency are absorbed, falls off with detuning
2.2. Interaction between light and matter

according to [37]

\[ \Gamma' = \Gamma - \frac{\Omega^2/2}{2 \Omega^2/2 + \Gamma^2/4 + \Delta^2}, \]  

(2.3)

where \( \Omega \) is the Rabi frequency, defined as:

\[ \Omega^2 = \Gamma^2 \frac{I}{2I_{\text{sat}}}, \]  

(2.4)

and where \( I \) is the irradiance of the light field and \( I_{\text{sat}} \) is the saturation irradiance [16]. For large detunings, \( \Delta^2 \gg \Omega^2/2 + \Gamma^2/4 \), the scattering rate hence scales as \( \Gamma' \propto I/\Delta^2 \).

An excited atom can also de-excite by stimulated emission into an external field. The emitted photon will then have the same direction, polarization, phase, and frequency as the external field. This stimulated emission is the foundation of lasers [11].

2.2.1 Lights mechanical effects on matter

When an atom absorbs a photon, it will not only absorb its energy, it will absorb its momentum as well. When emitting a photon of momentum \( p \), a momentum of the same size but of opposite direction, \( -p \), will be added to the atom. If an atom is placed in a unidirectional light field all absorbed photons will add momentum along the same direction. The spontaneous emission is isotropic, and the momentum change due to the emitted photons will over time average to zero. The result is a radiation pressure that pushes the atom in the direction of the light field.

The size of the exchanged momentum during one absorption or emission is referred to as the recoil momentum, \( p_r = |p| = h \times 2\pi/\lambda \). This momentum is important as it sets the scale of the random dynamic in laser cooled systems. From the recoil momentum the recoil velocity, \( v_r = p_r/m \), the recoil energy, \( E_r = p_r^2/(2m) \), and the recoil temperature, \( T_r = E_r/k_B \), can be obtained. For the transitions in \(^{133}\text{Cs}\) used in this thesis, the values of these entities are found in table 2.1.

Table 2.1: Characteristic values for the D\(_2\) transition (\(^{6}\text{S}_1/2 \to ^{6}\text{P}_3/2\)) of cesium [16]. For more information on the internal structure of the transition see chapter 5.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength ( \lambda )</td>
<td>852.3 nm</td>
</tr>
<tr>
<td>Recoil velocity ( v_r )</td>
<td>3.523 mm/s</td>
</tr>
<tr>
<td>Recoil energy ( E_r/h )</td>
<td>( 2\pi \times 2.066 \text{ kHz} )</td>
</tr>
<tr>
<td>Recoil temperature ( T_r )</td>
<td>198.3 nK</td>
</tr>
<tr>
<td>Natural linewidth ( \Gamma )</td>
<td>( 2\pi \times 5.234 \text{ MHz} )</td>
</tr>
</tbody>
</table>
2.2.2 Second order interactions

Second order processes are those involving two photons simultaneously, so called Raman processes, where absorption is concomitant with stimulated emission. This process does necessarily not affect the momentum, since the emitted photon usually has the same momentum as the one absorbed, but it does result in shifts in the energy levels. This effect can be described as the oscillating E-field of the light inducing a dipole moment in the atom, which shifts the energy levels slightly. This shift is usually referred to as the light-shift or AC-Stark shift, and is given by

$$E_{e,g} = \frac{\hbar}{2}(-\Delta \pm \sqrt{\Delta^2 + \Omega^2}) \simeq \pm \frac{\hbar\Omega^2}{4\Delta},$$

where the plus sign is for the ground state and the minus sign for the excited state, and the right hand side approximation is given for $|\Delta| \gg \Omega$, [37]. The effects of the oscillating magnetic field (B-field) will here be ignored, as they typically are several orders of magnitude weaker than the interaction with the E-field.

For resonant light the probability of these two photons transition are generally orders of magnitude smaller than the absorption-spontaneous emission processes. For these effects to be of importance a detuned light field has to be used, this since the scattering rate falls off as $I/\Delta^2$ while the light-shift effect falls of as $I/\Delta$.

Coherency of the interaction

Of significance for all two-photon processes is that no spontaneous emission is involved. These can hence by used to create or evolve superpositions states, and they preserve any coherence in an atomic sample. Two-photon processes are therefore referred to as coherent scattering while processes involving spontaneous emission is called incoherent scattering. Coherent interactions are crucial for the study of quantum effects, while incoherent interactions add a randomness to the system, of interest for studies of statistical physics, and it also provides a route for the dissipation of energy.

2.2.3 Optical pumping - manipulation of the atomic state

Besides the frequency matching, the light-matter interaction has a number of other requirements, so-called selection rules. For example, the total atomic angular momentum quantum number $F$ can only be changed by zero or plus/minus one, and its projection $M_F$ can only be changed by zero or plus/minus one, in any atomic transition. The change in the magnetic sub-level, $M_F$, is also dependent on the polarization of the light field. Absorption of circularly polarized light, $\sigma^\pm$, results in $\Delta M_F = \pm 1$ while linearly polarized light, $\pi$, gives $\Delta M_F = 0$, see figure 2.1a. Stimulated emission just
2.3 Doppler cooling of an atomic sample

reverses the transition, while spontaneous emission randomly distributes the decay over all allowed routes.

\[
\begin{align*}
F_e=1 & \quad M_f=-1 & \quad M_f=0 & \quad M_f=1 \\
F_g=0 & \quad M_f=0 & \quad \quad & \\
F_e=2 & \quad M_f=-2 & \quad M_f=-1 & \quad M_f=0 & \quad M_f=1 & \quad M_f=2 \\
F_g=1 & \quad M_f=-1 & \quad M_f=0 & \quad M_f=1 & \quad \quad & \\
\end{align*}
\]

Figure 2.1: (a) Energy diagram for a \( F_g = 0, F_e = 1 \) atom. The solid arrows indicates light of different polarization, that couples to different excited states. (b) Energy diagram for a \( F_g = 1, F_e = 2 \) atom, with only \( \sigma^+ \) light present. The possible routes of spontaneous emission are indicated by dashed arrows. The circularly polarized light field pumps the atom towards the state \( |F_g = 1, M_f = 1\rangle \), which only offer a closed transition for the \( \sigma^+ \) light field.

An atoms in a purely circularly polarized light field will be pumped by the light field toward one of the extreme \( M_F \)-states. Consider a \( F_g = 1, F_e = 2 \) atom in a light field with \( \sigma^+ \) polarization, see figure 2.1b. An absorption followed by a spontaneous emission will change the \( M_F \) number of the ground state by either 0, +1 or +2. The \( F_g = 1, M_F = 1 \) state will hence be highly populated, as no route leads from this state to another ground state. This process is called optical pumping, and is a frequently used tool for the manipulation of the atomic state [12].

2.3 Doppler cooling of an atomic sample

The interaction between light and matter is also dependent on velocity, as the frequency shifts with the velocity of the atom, \( v \), according to

\[
\omega = \left(1 - \frac{v}{c}\right)\omega_0, \tag{2.6}
\]

where \( \omega \) is the frequency of the photon in the frame of reference of the atom, \( \omega_0 \) is the frequency in the frame of reference of the lab, and \( c \) is the speed of light. This is known as the Doppler effect.
2.4. Trapping of an atomic sample

Since the Doppler shift acts as an effective detuning, it can be canceled by an actual detuning of the light field. In this way it is possible to tune which velocity group to target with a light field, and thereby manipulate the velocity distribution of an atomic sample. The first experimental realization of such ideas was the deceleration of an atomic beam with a counter-propagating, red-detuned laser beam [3]. However, to efficiently cool a sample of atoms it is not enough to target one velocity group, as the width of the velocity distribution preferably should be symmetrically narrowed. This is achieved by using two counter-propagating, red-detuned laser beams. Here, an atom at rest will scatter photons from both beams with equal probability. However, if the momentum of the atom increases by $p$, the counter-propagating beam will be tuned towards resonance, while the co-propagating beam’s detuning increases. This creates a radiation pressure in the opposite direction of the atom’s momentum, which will damp the atomic motion. This can be seen as a friction force, which is also what is achieved if the scattering force, $F = \hbar k \Gamma'$, of the two beams is approximately summed for low velocities [37],

$$F_{DC} = \hbar k (\Gamma'_1 - \Gamma'_2) \simeq \alpha_{DC} v,$$

(2.7)

where the scattering rate is taken from equation 2.3, with detunings rewritten to include the Doppler shift, $\Delta \rightarrow \Delta - kv$, and where $\alpha_{DC}$ is analogous to a friction constant,

$$\alpha_{DC} = \hbar k^2 \Gamma^2 \Omega^2 4\Delta \frac{4\Delta}{\Gamma^2 / 4 + \Delta^2}.$$

(2.8)

An expansion of this 1D cooling scheme to 2D or 3D is straightforward by adding identical beam pairs on the orthogonal axes.

2.4 Trapping of an atomic sample

The Doppler cooling gives a clearly velocity-dependent force which cools the atoms, but since it lacks position dependence it does not provide any confinement in space. The position-dependent force needed to trap atoms is usually obtained from a spatially-dependent scattering or a spatially-dependent trapping potential, created with the help of an external, spatially varying magnetic and/or light fields.

2.4.1 MOT - Magneto-Optical Trap

External magnetic fields shift the internal energy levels of an atom [8]. This is called the Zeeman shift, and for moderate magnetic fields the shift is linear with the magnitude of the magnetic field, $B$, and dependent on the $M_F$-state of the atom, $\Delta E = \mu_B g_F M_F B$, where $\mu_B$ is the Bohr magnetron, and $g_F$
is the Landré g-factor. This can be use to create a spatially-dependent scattering force.

Consider two counter-propagating red-detuned laser beams, just as in the Doppler cooling setup, but now with beams of opposite circular polarizations, see figure 2.2. These beams will therefore optically pump the atoms to opposite extreme \( M_F \)-state. If no external B-field is present, the two pumping processes will contribute equally at all positions, and cancel each other. If a linear B-field is applied, \( B(z) = B_0 z \), the symmetry will be broken for \( z \neq 0 \), and the detuning of the different \( M_F \)-state transitions will depend on position. If an atom with two hyperfine structure levels, \( F_g = 0 \) and \( F_e = 1 \), is chosen, see figure 2.1a, the ground state, \( F_g \), will have one magnetic sub-level, \( M_F = 0 \), and the excited state, \( F_e \), will have three, \( M_F = -1, 0, 1 \). For negative \( z \) the \( |F_g, M_F = 0 \rangle \rightarrow |F_e, M_F = 1 \rangle \) transition is shifted towards resonance, while the \( |F_g, M_F = 0 \rangle \rightarrow |F_e, M_F = -1 \rangle \) transition is shifted further away from resonance, see figure 2.2. Therefore, the atoms will preferably scatter \( \sigma^+ \) light and get pushed towards \( z = 0 \). On the other side, for positive \( z \), the atoms will scatter more \( \sigma^- \) light for the same reasons, and get pushed towards \( z = 0 \). In this way the atoms get trapped close to origin, and since the trap utilizes both magnetic and optical fields, it is called a Magneto-Optical Trap (MOT).

In the same way as for the Doppler setup, a MOT can be generalized to 3D by adding orthogonal beam pairs. The strength of MOTs is that they also have an inherent cooling, since they basically are an expanded Doppler cooling setup, and they are frequently used as an initial stage for gathering and cooling atoms regardless of the type of experiment to be preformed later, as in the work presented in this thesis.
2.4.2 Dipole traps and light-shift potentials

The presence of a light field can, through second-order interactions, shift the energy levels (the potential energy) of the atoms, see equation 2.5. By creating inhomogeneous light fields, potential minima can thereby be created where the atoms can be trapped. An example is a tightly focused red-detuned laser beam where atoms can be trapped in the intensity maximum [37]. In these so-called dipole traps, the trapping force is equal to the negative spatial derivative to the potential energy, \( F_{\text{dipole}} = -\nabla U \).

More advanced structured traps can also be created, for instance in the interference pattern of laser beams. Such periodic light-shift potentials will be discussed in chapter 3. It is also possible to trap atoms in pure magnetic potentials, or combinations of magnetic and optical traps.

2.5 Applications of laser cooling

General applications

Laser cooling and ultra-cold atoms have a large number of applications widely spread throughout the field of physics. A flavor of these applications is given below.

Precision measurements. The development of the atomic fountain and frequency chamber has allowed atomic clocks to improve the time standard by several orders of magnitude [13, 14]. This made applications such as the GPS possible. It has also improved other standardizations, such as the meter which is defined today from the definition of time and the speed of light. Precision measurements also allow for testing of standard models and investigations of variation of fundamental constants.

Fundamental quantum physics. The realization of a Bose-Einstein condensate provided a “macroscopic” quantum system and a coherent state of matter [17, 18]. With such systems fundamental studies of quantum mechanics can be performed, such as entanglement of quantum particles which enables quantum information and quantum computing [39].

Interaction between light and matter. Fundamental studies of light-matter interaction and quantum optics benefits enormously from coherent matter. For instance, light have been slowed down and even been stopped by the coherent interaction with a BEC [40].

Simulations of condensed matter. With cold atoms can exotic state of matter be investigated, e.g., by tuning the potential depth of an optical lattice, the phase transition between a superfluid and a Mott insulator can be studied [41].

Statistical physics. Laser cooling provides random systems with large ensemble of tunable parameters, which makes it ideal for fundamental studies of statistical physics.
2.5. Applications of laser cooling

The list could be made longer with other applications such as controlled collisions, formation of cold molecules, studies of ultra-cold fermi gases with formation of Cooper pairs, synthetic electric and magnetic fields, and so on.

Laser cooling in this thesis

In this thesis, a MOT is used to accumulate and cool cesium atoms. The atoms will then be transferred into a 3D array of dipole traps, usually referred to as an optical lattice (see chapter 3). Also present here is incoherent light scattering, providing the system with random fluctuations and enabling the atoms to step around in the periodic trapping potential. Studies and manipulation of these random walks will be the main focus of this thesis.
Chapter 3
Optical lattices

Optical lattices provide a periodic trapping potential for ultra-cold atoms. All the work included in this thesis revolves around laser cooled cesium atoms in optical lattices. In this chapter, the basics of optical lattices will be reviewed. Special focus will be given to the optical lattices used in the experiments covered by this thesis.

3.1 Periodic light-shift potentials

An optical lattice is a periodic light-shift potential [7]. These can be understood by considering the nature of the light shift, see equation 2.5. For large detunings, $\Delta \gg \Omega$, the magnitude of the energy shift is proportional to $\Delta E \propto \Omega^2 / \Delta$. By making the the Rabi frequency vary in space, $\Omega \rightarrow \Omega(r)$, a spatially-dependent light shift can be created. Such a dependence can be achieved by a spatially inhomogeneous light field, creating potential minima, in which the atoms can be trapped. If a spatially periodic light field is created, a spatial periodic light-shift potential can also be achieved. These periodic light-shift potentials are called optical lattices, and are almost exclusively created by the interference pattern of laser beams, either in the irradiance or in the polarization of the light field. In this thesis, optical lattices will primarily be built on polarization gradients.

3.1.1 Proximity to atomic resonance

For light-shift potentials to work in a satisfying way, the light fields creating the interference pattern have to be detuned from atomic resonance. Otherwise the dynamics will be completely dominated by incoherent scattering. For relatively large detunings, $\Delta \gg \Omega$, the incoherent scattering rate scales as $I / \Delta^2$, while the light shift effect is proportional to $I / \Delta$. Therefore, it is possible to make the incoherent scattering arbitrarily small while keeping a fixed potential depth. In the extreme case, the scattering can be ignored and
a conservative optical lattice is created. These conserve both the thermal energy and the coherence of an atomic sample, and they are frequently used in BEC experiments. For more moderate detunings, the incoherent light scattering will heat the atoms and thereby heavily affect their dynamic. The atoms can still be trapped for these detunings, since the scattering also opens possibilities for cooling through dissipation of energy to the vacuum field. These near-detuned types of potentials are hence called dissipative optical lattices, and are the kind of optical lattices considered in this thesis.

3.2 Dissipative optical lattices

We will now describe the construction of dissipative optical lattices, and discuss their inherent random fluctuations and cooling mechanisms in more detail. For simplicity primarily 1D models of optical lattices will be considered.

3.2.1 Polarization gradients

By overlapping two or more laser beams, an interference pattern can be created. This periodic structure can be imprinted in the polarization and/or the irradiance of the resulting light field, dependent on the relative polarization of the interfering beams. If two counter-propagating beams with parallel polarizations interfere, the pattern will be purely in the irradiance [7]. However, if two counter-propagating beams with perpendicular polarizations interfere, the pattern will be purely in the polarization. In the second case, the E-field of two beams can be written as

\[ E_1(z) = E_0 \hat{x} \cos(kz - \omega t), \]
\[ E_2(z) = E_0 \hat{y} \cos(-kz - \omega t). \]

The total E-field, \( E_{\text{tot}} = E_1 + E_2 \), in the basis of circular polarization, \( \hat{\sigma}^\pm = \mp \frac{1}{\sqrt{2}}(\hat{x} \pm i\hat{y}) \), is then given by

\[ E_{\text{tot}}(z) = \sqrt{2}E_0 \left[ \hat{\sigma}^+ \cos(kz) - i\hat{\sigma}^- \sin(kz) \right]. \]

The resulting E-field will hence have a spatially alternating elliptic polarization, where pure \( \sigma^\pm \) sites are obtained with a periodicity of \( \lambda/2 \), see figure 3.1b.

3.2.2 Polarization dependent light-matter interaction

The light shift for a certain detuning is dependent on the coupling strength (Rabi frequency), \( \Omega \), which is dependent on both the polarization and the irradiance of the light field, and on the state of the atom [7], since the
transitions in an atom can have different probabilities (Clebsch-Gordan coefficients). Consider a $F_g = 1/2, F_e = 3/2$ atom, see figure 3.1a. If the atom is in the $F_g = 1/2, M_F = 1/2$ state, the light shift is largest at pure $\sigma^+$ sites and smallest at pure $\sigma^-$ sites. For red-detuned light fields, $\sigma^+$ sites will hence correspond to potential minima, and $\sigma^-$ sites to potential maxima, see figure 3.1c. The situation will be the opposite for the $F_g = 1/2, M_F = -1/2$ state. The resulting light-shift potential can be written as

$$U(z) = U_0 \left[ c_{ge}^2 \cos^2(kz) + c_{ge}^2 \sin^2(kz) \right],$$

(3.4)

where $c_{ge}^2$ is the squared Clebsch-Gordan coefficients for the $\sigma^\pm$ induced transitions, and the potential depth is given by $U_0 = \frac{1}{2} \hbar \Omega / \Delta$. This is called a lin.$\perp$lin configuration [7, 42], and a 3D generalization of such a setup [43] is used in the experiments covered by this thesis.

Figure 3.1: (a) Energy diagram for a $F_g = 1/2, F_e = 3/2$ atom, with the squared Clebsch-Gordan coefficients for its transitions. (b) Polarization gradient with alternating circular polarization generated from counter-propagating beams in a lin.$\perp$lin configuration. (c) Light-shift potentials generated by the lin.$\perp$lin configuration, together with a schematic illustration of the Sisyphus cooling mechanism. Atoms that climb a potential hill convert kinetic energy to potential energy. Close to the top of the potential hill, where the circular polarization is the opposite of that at the bottom, the potential energy of the atom has a high probability of being dissipated to the vacuum field, through the process that pumps the atom to the other $M_F$ state.
3.2. Dissipative optical lattices

3.2.3 Manifolds of potentials

As seen in figure 3.1c, the light shift will depend on the internal state of the atom. Different states can be shifted with different amounts or even with opposite signs by the same light field. An optical lattice is therefore a manifold of periodic optical potentials, with equally many potentials as the number of $M_F$ states within the ground state. However, the circular polarization of the light field will optically pump the atoms towards the extreme $M_F$ values of the ground state, making these two potentials the dominating influence of the atomic dynamics, even if the number of $M_F$ states is higher.

Diabatic vs. adiabatic potentials

The two-photon Raman transition creating the light-shift potential can also couple different state. The atoms then see a superposition of states and thereby a superposition of potentials. This affects the potentials strongly in between $\sigma^+$ and $\sigma^-$ sites, where the energy levels cross each other. If the velocity of an atom traveling between sites is low enough it can adiabatically follow the lowest energy level and go from one extreme $M_F$ state to the other. This makes the periodicity of these adiabatic potentials half of the usual diabatic potential, see figure 3.2.

![Manifold of potentials for a multi-level atom. (a) Adiabatic potentials, and (b) diabatic potentials. In the adiabatic potentials, two-photon Raman transitions between states have been included, while this is ignored in the diabatic potentials.](image)

To be able to adiabatically follow the lowest energy state, the atoms have to move sufficiently slowly and without perturbation from spontaneous emission. If the incoherent scattering is too high, the velocities will be too high and the dynamics too jumpy, and the diabatic potential will dominate.

3.2.4 Heating - Random fluctuations through scattering

The incoherent scattering present in the dissipative optical lattices will heat the atoms. This heating can be divided into two main categories.

18
Incoherent scattering within a potential
An atom trapped in a sinusoidal potential will undergo oscillations around a potential minimum. If incoherent scattering is present, this smooth dynamics will be interrupted by the momentum kicks associated with the spontaneous emission process. This will randomly increase or decrease the atom’s total momentum, but for a sample of atoms over time this will heat the sample, that is, it will increase its momentum spread.

Incoherent scattering between potentials
An optical lattice is a manifold of potentials and incoherent scattering also provides a route between these potentials, as the spontaneous emission is randomly distributed between the allowed transitions. Since the different potentials have a different energy these inter-state transitions, just as for the momentum kicks, will over time heat the sample, if the transition rates are spatially homogeneous.

3.2.5 Friction - Sisyphus cooling
Dissipative optical lattices are created from red-detuned laser fields and have therefore an inherent Doppler cooling. As it turns out, there is also another cooling mechanism present. This is coupled to the manifold of potentials and the alternating circular polarization of the light field. Consider an atom interacting with the polarization gradient described above, see figure 3.1. Jumps between the potentials will here have a position dependence due to optical pumping, and an atom in the $M_F = 1/2$ state will have a significantly higher probability of being pumped to the $M_F = -1/2$ at a $\sigma^-$ site than at a $\sigma^+$ site. $\sigma^-$ sites are also where the $M_{F_g} = 1/2$ state has its potential maxima, and a transition to the $M_F = -1/2$ state will here decrease the energy of the atom by dissipation to the vacuum field, see figure 3.1c. Therefore, atoms with high enough energy to climb the potential hill will have higher probability to lose this potential energy than atoms at the bottom of the hill have to gain the same energy. This leads to a cooling of the atoms over time, which effectively can be seen as a friction. This cooling mechanism is called Sisyphus cooling [3], and for low velocities this mechanism generates significantly higher cooling rates then Doppler cooling.

3.2.6 Steady state
The heating mechanisms together with the velocity-dependent cooling will eventually lead to a steady state. This steady state is associated with a temperature, given by equation 2.1, and a spatial diffusion, coupled to the spatial spread of the atoms over time. These quantities, among others, will be discussed further in the next chapter. With the optical lattices used in
3.3 Optical lattices in this thesis

Dissipative optical lattices with a 3D $\text{lin} \perp \text{lin}$ configuration are used in this thesis, and they are realized with cesium atoms ($^{133}\text{Cs}$). A more detailed description of the cesium atom and the optical lattice configurations can be found in chapter 5. Two variants of optical lattice configurations are used. The atoms are either trapped in a single optical lattice, or they will alternate between two lattices, which we call a double optical lattice [43].

3.3.1 Phase-stable 3D $\text{lin} \perp \text{lin}$ configuration

A straightforward generalization to 3D of the $\text{lin} \perp \text{lin}$ setup by orthogonally adding counter-propagating pairs of beams, as done for the MOT, creates an optical lattice with an unstable lattice structure. More specifically, phase fluctuations in the constructing beams will lead to a modification of the topography of the created interference pattern [7, 44].

A phase-stable 3D generalization of the $\text{lin} \perp \text{lin}$ configuration can instead be constructed from just four beams, with two propagating in the $xz$-plane and two in the $yz$-plane, see figure 3.3a. The beams are given polarizations that are orthogonal to the plane of propagation, and all beams are usually given a 45 degree angle with respect to the $z$-axis. With such setup any fluctuation in the phase of the constructing beams will just translate the optical lattice and not change the topography [7, 42]. The 3D $\text{lin} \perp \text{lin}$ configuration creates a tetragonal lattice structure, a 2D representation of which can be seen in figure 3.3b.

3.3.2 Single optical lattices

All cooling transitions used in the experimental setup lie within the $D_2$-line of $^{133}\text{Cs}$. The single optical lattice is created by a light field slightly detuned from the $F_g = 4 \rightarrow F_e = 5$ transition, see figure 3.4a. This is a closed transition, since from the $F_e = 5$ excited state, decay is only possible back to the $F_g = 4$ ground state. Off-resonance scattering to the $F_e = 4$ excited state is however possible, and from there decay through spontaneous emission to the $F_g = 3$ ground state is allowed. Atoms in the $F_g = 3$ ground state will not be trapped by the optical lattice and therefore, to the single optical lattice configuration, a repumper laser is added, pumping the atom from the $F_g = 3$ ground state to the $F_e = 4$ excited state, see figure 2.1a.
3.3. Optical lattices in this thesis

Figure 3.3: (a) Four-beam 3D generalization of the lin⊥lin optical lattice configuration. Two beams are propagating in the xz-plane and two in the yz-plane. The beams are given polarizations that are orthogonal to the plane of propagation, and all beams are usually given a 45 degree angle with respect to the z-axis. (b) 2D representation of topography of a 3D lin⊥lin optical lattice.

Figure 3.4: Partial energy diagram for the D2-line of cesium (not to scale). Light fields are indicated by thick, solid arrows and the routes of the spontaneous emission by dashed arrows. (a) Single optical lattice configuration. The optical lattice operates on the $F_g = 4 \rightarrow F_e = 5$ transition, with a light field detuned from resonance by $\Delta_1$. To recapture atoms scattered to the $F_g = 3$ state a resonant repumper field is used. (b) Double optical lattice configuration. The repumper is replaced by a second optical lattice that operates on the $F_g = 3 \rightarrow F_e = 4$ transition, with a light field detuned by from resonance by $\Delta_2$. 
3.3. Optical lattices in this thesis

3.3.3 Double optical lattices

Since the cesium atom has two hyperfine ground states, \( F_g = 3 \) and \( F_g = 4 \), two distinguishable optical lattices can be created. This is done by replacing the resonant repumper field in the single optical lattice configuration, see figure 3.4a, with a second, superposed \( \text{lin} \perp \text{lin} \) configuration, addressing the \( F_g = 3 \rightarrow F_e = 4 \) transition, see figure 3.4b. For these to work as two separate optical lattices, the hyperfine splitting, \( \Delta_{\text{HFS}} \), has to be much larger then the detunings of the optical lattices, \( \Delta_{\text{HFS}} \gg \Delta \). Here we call the light-shift potential confining the atoms in the \( F_g = 4 \) state optical lattice I, and the light-shift potential confining the atoms in the \( F_g = 3 \) state for optical lattice II.

Individually controllable parameters of the optical lattices

The two optical lattices are created from separate light fields enabling separate control of the two optical lattices properties, \( i.e., \) of the potential depth, the spatial phase, and the scattering rate. The latter also controls the transfer rate from one lattice to the other. However, from the \( F_e = 5 \) state the atom can only decay to the \( F_g = 4 \) ground state, which makes the \( F_g = 4 \rightarrow F_e = 5 \) transition closed, while from the \( F_e = 4 \) state the atom can decay to the both the \( F_g = 3 \) and the \( F_g = 4 \) ground state, which makes the \( F_g = 3 \rightarrow F_e = 4 \) transition open, with the decay to \( F_g = 4 \) favored by a ratio of 7:5 to decay to \( F_g = 3 \). This generally makes the lifetimes of the two ground states strongly unequal [43].

The two optical lattice may also have a relative spatial phase. If it is non-zero, the transfers between the lattices will, on average, heat the atoms, see chapter 5. A fascinating effect of this extra heating will be discussed in chapter 7.
Chapter 4

Random walks, Brownian motion, and diffusion

The dynamics of cold atoms in dissipative optical lattices are heavily influenced by the randomness of the incoherent scattering. In this chapter, the dynamics of random systems will be discussed from different perspectives. The basic theory of spatial diffusion of Brownian particles and the effects of external potentials will be covered and, coming from Paper V, a comparison between classical and quantum random walks will be given.

4.1 Classical random walk

To understand the nature of a random particle it is convenient to start with a simple classical 1D random walk. Consider a particle that at discrete times is constrained to jump one unit of length either to the left or to the right. The choice of direction is random and associated with two probabilities, $P_{\text{left}}$ and $P_{\text{right}}$, which for simplicity are considered equal $P_{\text{left}} = P_{\text{right}}$. This gives an array of possible positions for the particle with a probability $P(i, N)$ of being at a site $i$ after $N$ steps. For a particle starting at $i = 0$, the probability distribution of the first 4 steps is shown in figure 4.1.

By studying probability evolutions like in figure 4.1, it can be shown that, as long as $P_{\text{left}} = P_{\text{right}}$, the average position will be zero, and the width of the position distribution, $\sigma_x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$, grows with the square root of the number of steps, $\sigma_x = \sqrt{N}$ [47]. For large $N$ the distribution will also have a Gaussian envelope.

4.2 Brownian motion and diffusion

Random walks also appear in physical systems and can here be described by physical quantities. For a particle, the random directions of the motion can be achieved by interaction with an external source of fluctuations, a heat
4.2. Brownian motion and diffusion

bath, and the step-like movement can be due to a friction force. The heat bath is usually assumed to be large enough for back action from the particles to be ignored. The spatial distribution here becomes, just as for the random walk, Gaussian with time, and the spatial spread of the particles over time is called diffusion, and can be described by a temperature of the heat bath, $T$, by a friction, $\alpha$, and the elapsed time, $t$ (instead of the number of steps), as

$$\sigma = \sqrt{2Dt},$$

where $D$ is the diffusion constant, which can be expressed as

$$D = k_B T/\alpha.$$ 

If the ensemble of particles already has a finite spread at $t = 0$, $\sigma_0 \equiv \sigma(t=0) \neq 0$, the spreading due to diffusion is added quadratically to the initial distribution, i.e., $\sigma^2 = \sigma_0^2 + 2Dt$.

**Damping**

In random physical processes, the size of the friction plays an important role. Usually two different cases are considered, underdamped and overdamped
systems. In overdamped systems any momentum gained is almost immedi-
adately damped away. The particles hence move stepwise only due to the
current interactions as they lack memory of earlier interactions. For more
moderately damped systems, the particles’ state depends also on their pre-
vious history and can hence accumulate momentum. This opens possibilities
for steps of larger size.

Brownian motion

The first scientific observation of these type of dynamics was done by Robert
Brown in 1827, who studied the diffusion of pollen particles in water. The
random motion of the particles are therefore often called Brownian motion.
The physical description of the process was given independently by Einstein
in 1905 [34] and Smoluchowski in 1906 [48].

Here, the temperate water works as a heat bath, providing the pollen
with random fluctuations, and the viscosity of water provides a friction.
This gives dynamics described by equations 4.1 and 4.2. In our system,
the heat bath will be the processes involving incoherent scattering, and the
friction will be given by the laser cooling mechanisms. More about this can
be found in chapter 6.

4.3 Langevin Equation

In physical situations, random walks often take place in some kind of external
potential. The Einstein description is then not sufficient, and the Langevin
equation of motion is instead often used [49],

\[ m\ddot{x} = -\frac{d}{dx}V(x) - m\gamma \dot{x} + \xi(t), \]

(4.3)

where \( m \) is the mass of the particle, \( V(x) \) is the external potential, \( \gamma = \alpha/m \)
is the friction coefficient \( \alpha \) divided by the mass, and \( \xi(t) \) is the Langevin
stochastic force representing the noise source of the system. The most com-
mon type is the so-called white noise, i.e., random fluctuations with a Gaus-
sian distribution, that fulfills the two requirements,

\[ \langle \xi(t) \rangle = 0, \]

\[ \langle \xi(t)\xi(t') \rangle = 2mk_BT\delta(t-t'). \]

(4.4)

4.3.1 Noise and friction in dissipative optical lattices

In dissipative optical lattices, both the noise and the friction are more com-
plicated than in the simple model above. The friction, or rather the cooling
mechanism can be divided into two parts, Doppler cooling and Sisyphus
cooling, see chapter 3. Both have a complex velocity dependence [37], and
4.4. Brownian motion in periodic potentials

are primarily active for atoms moving around in the optical lattice. For localized atoms, oscillating within a potential site, the exact cooling mechanism is not precisely known \[50, 51, 52, 53\]. The heating also consist of two processes, both related to the incoherent scattering. The scattering within a potential gives a distribution of small kicks analogously with the white noise (the potential depth is usually \(\sim\)100 times larger than the recoil energy), while the inter-potential scattering gives larger, discrete jumps with sizes of order of the potential depth. The term “heat bath” should therefore be handled with care for optical lattice, since it would include all processes associated with incoherent scattering.

4.3.2 Velocity distributions and kinetic temperature

The Gaussian distribution of the white noise also gives a Gaussian velocity distribution for the particles subjected to this noise. If no friction is present the particles will undergo a random walk in momentum space, leading to a spread of the velocity distribution, which is the same as a heating of the particles. However, if friction is present the velocity distribution will reach a steady state, corresponding to the particles being in a thermal equilibrium with the heat bath. In such case, a Boltzmann distribution can be assigned to velocity distribution of the particles \[37\],

\[
N(v) = N_0 e^{-\frac{mv^2}{2k_BT}} \tag{4.5}
\]

where \(N_0\) is a normalization constant, and \(T\) now represent both the temperature of the heat bath and of the sample of particles. This temperature is also referred to as the kinetic temperature of the particles and will be frequently used in this thesis.

The peculiar heating and cooling mechanisms of laser cooling also generates a velocity distribution that is close to Gaussian. However, the velocity distribution do have deviations from a Gaussian, especially in the tails, and its exact shape is debated \[50, 51, 54, 55, 56\].

4.4 Brownian motion in periodic potentials

In the Langevin equation 4.3 one of the most common choices of potential, \(V\), is a periodic potential

\[
V(x) = \frac{V_0}{2} \cos (kx), \tag{4.6}
\]

where \(V_0\) is the potential depth and \(k\) is the wave vector. This potential also gives a simple model for our optical lattice. The dynamics in such a system will mainly depend on two things: (i) The relative size of the friction, that is whether the motion is overdamped or not, and (ii) the relation between
4.4. Brownian motion in periodic potentials

the temperature and the potential depth. The latter leads to two extreme
dynamic states.

**Locked state, $V_0 \gg k_B T$**

With a decreasing $k_B T/V_0$, the probability for the particles to overcome the
potential barriers decreases as well. For kinetic energies much smaller then
the potential depth, the diffusion probability is virtually zero. This gives
well localized particles with no spatial diffusion between the potential sites.

For dissipative optical lattices, this state will never appear, as long as
a finite incoherent scattering is present, since the inter-potential scattering
always can “untrap” the atoms.

**Running state, $V_0 \ll k_B T$**

For $k_B T/V_0 \gg 1$ the localization becomes negligible. The potential can
therefore be ignored and the diffusion can be treated with the Einstein de-
scription (eqs. 4.1 and 4.2).

Dissipative optical lattices do show localization of the atoms [52, 53].
In the 3D lin⊥lin configuration here used, the atoms are typically confined
within a tenth of the wavelength around the potential minima [52]. However,
coexisting with this localization is a spatial diffusion [57, 58, 59], which gives
dissipative optical lattices dynamics alternating between the two extreme
states.

**4.4.1 Stop-and-go dynamics**

Particles with a thermal energy in between the two extreme cases will ran-
domly shift back and forth between the state of having large enough energy
to overcome the potential barriers and the state of being localized around
the potential minima. This gives a so called stop-and-go dynamics. In paper
IV and in chapter 6, the stop-and-go dynamics in dissipative optical lattices
are further investigated by gravitationally tilting the potentials.

**Single or multi-well flights**

In a stop-and-go regime, the generally localized particles will occasionally
make inter-well flights. The duration of, or the distance traveled during,
these inter-well flights will depend on the friction in the system to a large
extent.

In an overdamped system, all momentum gained will be damped before
the particles next interaction with the heat bath. The particles will hence
move without memory and the inter-well flights will primarily be single well
jumps.
4.5 General appearance of random systems

Underdamped systems have memory of more than one interaction and momentum can hence be accumulated. Here multi-well fights become more probable. These multi-well flights are also called Lévy flights, and the dynamics can be described by Lévy statistics [55, 58, 60]. However, the “memory” of the underdamped particles with stop-and-go dynamics is limited in the sense that un-trapped particles do get recaptured, and then lose all memory of the flight.

The multi-well flight dynamics in optical lattices become even more complicated, since the atoms can move between different potentials with different, or even inverted, potential depths. In such multi-potential systems multi-well flight can occur even in the overdamped case. Optical lattices are however rarely overdamped.

4.5 General appearance of random systems

Random systems appear in all places where the random fluctuations are of the same order of magnitude as the other interactions. A few examples are the dynamics of gases and fluids, or even for small living organisms such as bacterias [61], and also the stock market have a Brownian behavior [62]. This common behavior makes the theoretical description of such a system applicable to many systems of interest beyond pure physics, such as in weather forecast, in economy, or the spread of deceases [47].

4.6 Quantum walks

So far we have treated random walks where the dynamics are classical. There does however exist a quantum analog to the random walk, a quantum walk [63], which in a neat way illustrates the fundamental difference between quantum and classical particles.

In a quantum walk, the particles will not jump to the left or to the right, but will be transferred into a superposition state of moving to the left and to the right. Consider a particle described by a wave function \( \Psi_i = \sum_{k,s} \psi(k,s) \), where \( i \) denotes the \( i \)th step of the walk, \( k \) denotes the position, and \( s \) the internal state. The position \( k \) can take discrete values from \( -\infty \) to \( \infty \) and \( s \) can have one of the two values \( X \) or \( Y \). The operator \( H \) defined below, which puts each internal basis state into a superposition state, simulates the toss of a quantum coin. The operator \( D \) is the conditional displacement causing a translation to the right or to the left depending on the internal state \( (X,Y) \).
4.6. Quantum walks

\[ H [\psi(k, X)] \rightarrow \psi(k, X + Y) \]
\[ H [\psi(k, Y)] \rightarrow \psi(k, X - Y) \]
\[ D [\psi(k, X)] \rightarrow \psi(k + 1, X) \]
\[ D [\psi(k, Y)] \rightarrow \psi(k - 1, Y) \] (4.7)

An application of the sequence \( DH \) constitutes a single step of the quantum walk. Beginning with \( \Psi_0 = \psi(0, X) \), the wave function after \( N \) steps is given by \( \Psi_N = (DH)^N \Psi_0 = \cdots = (DH) \Psi_0 \). The \( H \) operator considered here is Hadamard operator which puts states \( X \) and \( Y \) into equal superpositions.

The probability distribution after \( N \) such steps will depend on the quantum interference between the states of the different routes [63]. This gives generally a non-Gaussian distribution, where the width of the distribution scales as \( N \), in contrast to the classical random walk’s Gaussian distribution with a width scaling with \( \sqrt{N} \). This faster spread could be helpful for future search algorithms [63, 64]. The final distribution will also depend on the initial state and does not have to be symmetrical. In figure 4.2 is shown the difference between, (a) a symmetric quantum walk, (b) an asymmetric quantum walk, and (c) a classical random walk.

4.6.1 A quantum walk with double optical lattices

In paper V, a suggestion for a realization of a quantum walk, with cold atoms in a double optical lattice, is given, together with an investigation of the possibility to detect the walk with different methods. Special weight is given to the detection methods existing in our experimental setup (see chapter 5).

Here the two hyperfine ground states of cesium will correspond to \( X \) and \( Y \), and the double optical lattice will be used as the displacement operator. Consider an atom in the \( X \) state and the two optical lattices initially in phase with each other. By the use of Raman transitions, the atom can be transformed into a superposition of \( X \) and \( Y \) [10, 65], analogous with the \( H \) operator. By then shifting the relative spatial phase of the two lattices the conditional displacement operator will be realized. By repeating this scheme \( N \) times a quantum random walk of \( N \) steps is obtained.

The relative displacement of the optical lattices are already implemented in the experimental setup, see chapter 5, and coherent population transfer through Raman transitions is a frequently used tool [65], although not implemented in this setup. In the setup, two detection methods are implemented, a time-of-flight detection technique (see chapter 5 and paper VI), and an absorption imaging technique (see chapter 5). The simulations in paper V
4.6. Quantum walks

Figure 4.2: Probability versus displacement for (a) symmetric quantum walk, (b) asymmetric quantum walk, and (c) classical random walk. In all cases, the number of steps, $N$ equals 50. Figure taken from N. Satapathy et al., Phys. Rev. A 80, 012302 (2009), ©2009 American Physical Society.

shows that the absorption imaging is the most suitable technique for detecting the quantum walks position distribution.

The probability distributions shown in figure 4.2a and b are for the ideal case, with one particle with infinite coherence times and where the particle always follows the displacement of the optical lattices. This would not be the case for an experimental realization, where the initial atomic sample also would have a finite width. The final distribution would be the convolution of the quantum walk distribution of each atom with the initial distribution of the sample.

In paper V, the effects of decoherence, imperfect displacements, and non-zero initial distribution widths are investigated. For realistic experimental condition, a quantum walk about 250 steps is needed for a clear detection of the hallmarks of a quantum walk in the position distribution. The coherence
at each step needs to be high, above 99\%, while the displacement efficiency is less critical. For this to be possible, the coherence time need to be extended. This can be done by detuning the optical lattices further from resonance. The hyperfine splitting does however set a limit for the detuning as the optical lattices do need to be distinguishable.

In paper V, an alternative detection method is also presented. By detecting the time evolution of the population distribution rather than the position distribution even a 3-step quantum walk can be unambiguously verified. This method is based on the fact that after each step of the quantum walk, along with the superposition of displacement, the particle also exists in a superposition of its internal states, which is specific to the number of steps of the quantum walk. This is in contrast to the classical random walk\(^1\), where for all steps after the first, both states have equal state population. In figure 4.3, the state population for the first 40 steps of the quantum walk (Hadamard walk) are shown. The sensitivity to inefficient displacement and incoherent transfer is also smaller for this method.

![Figure 4.3: Population evolution of the X state (red/upper lines) and Y state (blue/lower lines) in a Hadamard walk starting with initial state X as a function of number of steps N. In both the cases, only the points are realizable values; they are joined by line segments as a guide to the eye. Figure taken from N. Satapathy et.al., Phys. Rev. A 80, 012302 (2009), ©2009 American Physical Society.](image)

After this article was published, a quantum random walk was demonstrated with a similar system [66]. The detection problem was there beautifully solved by using a single cesium atom as the quantum walker.

\(^1\)The classical analogy would here be that an atom changes state with a probability of 0.5 instead of being put into a superposition of the two states.
Chapter 5

Experimental setup and detection methods

In this chapter a description of the experimental setup is given. Special focus will be given to the construction of the optical lattices, along with the control apparatus for the relative spatial phase of the double optical lattices. Parts of the detection system are described in detail in paper VI.

5.1 Introduction

5.1.1 Experimental sequences

A solid sample of $^{133}\text{Cs}$ atoms is heated to create a beam of atoms. Before reaching the trapping region, the atomic beam is slowed down by frequency-chirped laser beams. In the trapping region, the atoms are trapped in a Magneto-Optical Trap (MOT), where in a few seconds about $10^8$ atoms are accumulated and cooled to about $5\,\mu\text{K}$. The MOT is then turned off as the optical lattices are turned on. The atoms are held in the optical lattices for a chosen duration before the detection phase is initiated.

5.1.2 Basic requirements

Laser cooling builds on the interaction between light and matter, that is, the interaction between one atomic species and light fields that address specific states of this atom, while all other interactions are negligible. Laser cooling experiments therefore require ultra-high vacuum, to eliminate the interaction with the background gases. The setup needs a pure source of atoms and a good optical access to a trapping region. To manipulate the atoms efficiently, well controlled lasers are also needed, with frequencies, polarizations and irradiances that are well defined. Below is a description of how these requirements are met in our experimental setup.
5.2 Vacuum system and experimental chamber

The vacuum system consists of two parts: the experimental chamber and an oven with the atomic source. The experimental chamber is made of stainless steel and has a cylindrical shape. To get good optical access, the chamber is equipped with about 20 windows. Two vacuum pumps are attached to the chamber, a non-evaporative getter pump (NEG) and an ion pump (Varian Starcell 20), with pump rates of 1000 liter/s and 22 liter/s, respectively. To further decrease the pressure in the experimental chamber, the oven is separated from it by a 55 cm long tube, see figure 5.1. The tube has a diameter of about two centimeters, and will maintain a differential pressure between the experimental chamber and the oven chamber. The oven chamber is rectangular, made of stainless steel with a turbo pump, backed by a rotary pump, attached to it. A gate valve is located between the oven chamber and the tube, enabling a complete separation of the experimental chamber from the oven when the experiment is not running. The oven is a small hollow cylinder where a glass ampule of Cs can be placed and heated. The heating is done and controlled by running a current through a wire winded around the hollow cylinder. To release the Cs atoms, the glass ampule has to be broken before being injected into the oven. The oven is typically heated to 180 °C, when the system is running, producing a vapour pressure of approximately 4 Pa [67]. The atoms escape from the oven to the oven chamber through a small hole with a diameter of about 0.5 mm.

Figure 5.1: Schematic of the vacuum system. (1) Turbo pump, (2) oven with the atomic sample, (3) oven chamber, (4) valve enabling separation between the oven and the experimental chamber, (5) tube connecting the chamber and the oven, (6) NEG pump, (7) ion pump and (8) the experimental chamber.
5.3 Laser systems

During my Ph.D., the pressure in the experimental setup has been lowered by an order of magnitude. This was achieved by increasing the differential pressure between the oven and the experimental chamber through reducing the diameter of a part of the tube separating them, and by baking\(^1\) the system at higher temperatures. With this system, a pressure of about \(10^{-8}\) Pa \((< 10^{-10}\) Torr\) is now achieved in the experimental chamber, and about \(5 \times 10^{-6}\) Pa \((4 \times 10^{-8}\) Torr\) in the oven chamber.

Around the experimental chamber, a cage of coils is placed to shield off the Earth’s magnetic field. This is done by adjusting the current through each of the coils so that zero B-field is obtained throughout the trapping region.

5.3 Laser systems

5.3.1 The Cs atom and the cooling transitions

All lasers in the current setup operate close the D2-line of \(^{133}\)Cs [16]. The D2-line is the transition between the 6s \(^2\)S\(_{1/2}\) ground state and the 6p \(^2\)P\(_{3/2}\) excited state, which corresponds to a wavelength of the lasers of about 852.3 nm. The excited state has a natural linewidth of \(\Gamma = 5.2227 \times 2\pi\) MHz. The ground state splits up into two hyperfine ground states, \(F_g = 3\) and \(F_g = 4\), separated by 9.2 GHz (1762 \(\Gamma\)), see figure 5.2. The excited state is divided into 4 hyperfine states \(F_e = 2, 3, 4\) and 5, and the separation between these states is 101–251 MHz (19 to 48 \(\Gamma\)), which is almost two orders of magnitude smaller than the ground state separation. Therefore, the lasers can be divided into two groups, one that operates on the \(F_g = 4\) ground state, and one operating on the \(F_g = 3\) ground state. The first group targets the \(F_e = 5\) excited state with various detunings, while the second group targets the \(F_e = 4\) excited state, see figure 5.2.

5.3.2 Lasers

In the experimental setup, four different lasers are used: one tapered amplifier diode laser (Toptica TA100), one Ti:Sapphire ring laser (Coherent MBR 110), and two chirped home-built diode lasers [44]. All lasers are frequency-locked to cesium cells through saturation spectroscopy [12], and produce narrow-band light with a bandwidth of approximately 1 MHz or less.

The Toptica laser has a maximum power of about 600 mW at the working frequencies and is used both for the MOT and for an optical lattice in both the single and double optical lattice configuration. The Ti:Sapphire laser has a maximum power of about 3 W at the working frequencies and is used

\(^1\)To obtain a high vacuum, the vacuum system need to be baked, that is, it is held at a high temperature, preferably above 100°C, for a couple of days, while being heavily pumped. This reduces the outgasing from the constructing materials.
5.3. Laser systems

Figure 5.2: Partial energy diagram for the D2-line of cesium (not to scale). In the figure the laser frequencies used in the experimental setup are displayed. The lasers are divided into two groups depending on which ground state they operate on.

as a repumper in the MOT and single optical lattice configuration, and for an optical lattice in the double optical lattice configuration. The chirped lasers are use as stopping beams for slowing down the atomic beam. In the beginning of my Ph.D., (paper IX, VIII, VII, VI), instead of the Ti:Sapphire laser, home-built diode lasers in two maser-slave configurations were used [50].

5.3.3 Optical fibers

To ensure clean spatial modes, all beams in the setup are at some point guided through an optical fiber. This gives a nice Gaussian beam mode, and ensures an accurate overlapping of all beams passing through the same optical fiber. They also enable the usage of laser light coming from several different tables, without risk of disturbance from the relative motion of the tables.
5.3.4 Acousto-optical modulators

To use the same lasers for both the MOT and the optical lattices configurations, a fast way of switching frequencies is needed. This is here achieved with acousto-optical modulators (AOM). An AOM gives spatially resolved sidebands whose frequencies are multiples of the frequency of an acoustic wave sent through an optically active crystal. In this way each laser beam can be divided into many beams with different frequencies. With the AOMs used in this setup, the frequency can be shifted in the order of 60–140 MHz for a single pass.

By controlling the amplitude of the acoustic wave in the crystal, the irradiance of the sidebands can be controlled. The AOMs can therefore also be used as fast shutters by setting the amplitude of the acoustic wave to zero. This is beneficial since the amplitude control of an AOM has a response time of 4 µs, two to three orders of magnitude faster than what can be achieved with our mechanical shutters. In the setup, combinations of AOMs and mechanical shutters are usually used to ensure fast and complete blocking of beams.

5.4 MOT

The MOT in our setup has a standard 6-beam configuration (see chapter 2) with a B-field generated by two magnetic coils in an anti-Helmtz configuration, see figure 5.3. The 6 beams are generated by splitting one beam into three, and then retro-reflecting these beams. To clean up the beam mode, the original beam is sent through an optical fiber. Each beam has a diameter of about 1 cm and a peak irradiance of about 30 mW/cm².

In our setup this trap accumulates roughly $10^8$ atoms over a time period of about 5 s. The accumulated atoms have a temperature of about 5 µK and the sample has a root-mean-square (rms) radius of roughly 1 mm in the MOT. Before the transfer to the optical lattices, the atomic cloud is compressed to a rms radius of roughly 0.3 mm.

5.5 Optical lattices

The optical lattices in our setup are created with 4 beams in a 3D lin⊥lin configuration (see chapter 2). The beams have typically $1/e^2$-diameters of about 5 mm and the E-fields of the four beams can be described by

$$
E_1 = E_0 \hat{x} \cos (+k_\perp y - k_\parallel z - \omega_L t + \varphi_1)
$$
$$
E_2 = E_0 \hat{x} \cos (-k_\perp y - k_\parallel z - \omega_L t + \varphi_2)
$$
$$
E_3 = E_0 \hat{y} \cos (+k_\perp x + k_\parallel z - \omega_L t + \varphi_3)
$$
$$
E_4 = E_0 \hat{y} \cos (-k_\perp x + k_\parallel z - \omega_L t + \varphi_4),
$$

(5.1)
5.5. Optical lattices

Figure 5.3: Schematic illustration of the MOT in our setup. The MOT consists of 3 pairs of counter-propagating beams with opposite circular polarization, and of two magnetic coils in an anti-Helmoltz configuration, where $I$ is the current in the coils.

where $k_{\perp} = k_{\parallel} = \frac{2\pi}{\lambda} \frac{1}{\sqrt{2}}$, and $\omega_{L}$ is the angular frequency of the light field. This gives a tetragonal lattice topography which for optical lattice I (for the $F_g = 4 \rightarrow F_e = 5$ transition) is given by [6]

$$
\frac{U}{U_0} = 23 \left[ \cos^2 (k_{xx} + \varphi_x) + \cos^2 (k_{yy} + \varphi_y) \right] - 44 \cos (k_{xx} + \varphi_x) \cos (k_{yy} + \varphi_y) \cos (k_{zz} + \varphi_z),
$$

(5.2)

where $U_0 = 4\hbar \Gamma' / 45$, and $\varphi_x, \varphi_y, \varphi_z$ is the spatial phase along the axes, which can be given in terms of the spatial phases of the beams by

$$
\varphi_z = \frac{\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4}{2},
$$

(5.3)

$$
\varphi_y = \frac{\varphi_1 - \varphi_2}{2},
$$

$$
\varphi_x = \frac{\varphi_3 - \varphi_4}{2}.
$$

For the potential described by equation 5.2, the phases of the beams do indeed just translate the optical lattice, and does not change the lattice topography [7, 44, 45].
5.5. Optical lattices

**Beam generation and alignment**

For the single optical lattice, the four lattice arms are created from one beam by the use of polarizing beam splitters. The power of each beam is balanced with half-wave plates placed before the beam splitters, see figure 5.4. Before the beams enter the experimental chamber, they each pass through a half-wave plate and a polarizing beam splitter to ensure a clean and correct polarization. These are also used as a final adjustment of the power balance between the four arms [68].

![Figure 5.4: Schematic figure of the generation and power balancing of the four arms of the single optical lattice configuration. In the figure λ/2 denotes a half-wave plate, GT a Glen-Thompson prism used to clean the polarization, PM-fiber a polarization-maintaining optical fiber, BS a non-polarizing 50/50 beam splitter, and PBS polarizing beam splitter.](image)

**Lifetime and filling factor**

In the optical lattice, the initial atomic cloud consists of about $10^8$ atoms, spread out over roughly $2 \times 10^9$ sites, giving a filling factor of approximately 0.05. Due to the low filling factor, the atom-atom interactions in the optical lattice are rare, hence the dynamics are governed by the atom-light interactions.

The number of atoms in the optical lattice decreases with time mainly due to two things: interaction with the background gases and diffusion out of the spatially finite optical lattices. The signatures of both of these decays can be seen by studying the number of atoms over time. Initially, all atoms are fairly close to the center of the optical lattice and the decay is mainly due to interaction with the background gases. The gives an exponential decay with a lifetime of approximately 10 seconds. After a while, the decay will increase as some of the atoms start moving out of the trapping region. The timescale of this effect is strongly dependent on the parameters of the optical lattices, but for typical lattice parameters the resulting lifetime of the atoms in optical lattice is in the order of hundreds of milliseconds to a few seconds.
5.6 Double optical lattices

In the double optical lattice configuration, two 3D lin⊥lin configurations are superposed, each addressing transitions from separate ground states, see chapter 3 and figure 5.2. We call the optical lattice built on the \( F_g = 4 \rightarrow F_e = 5 \) transition optical lattice I, and the lattice built on the \( F_g = 3 \rightarrow F_e = 4 \) transition optical lattice II, see figure 5.2. The atoms hence interact with either of the two optical lattices depending on which ground state they are in, and in each lattice the irradiance and detuning can be controlled individually.

5.6.1 Inter-potential transfer

As described in chapter 3, the atoms are inherently transferred between the two optical lattices via the excited states. The transfer rates, \( \gamma_{I \rightarrow II} \) and \( \gamma_{II \rightarrow I} \), are to some extent scannable, but are generally highly unequal, \( \gamma_{I \rightarrow II} \ll \gamma_{II \rightarrow I} \), due to the different properties of the excited states [43].

The relative transfer rates between the ground states have been measured for our system [43, 69], by studying the relative populations of the two ground states. Typically the ratio of the transfer rates is \( \gamma_{I \rightarrow II}/\gamma_{II \rightarrow I} \simeq 1/20 \). The rates do however depend on the parameters of the optical lattices, and \( \gamma_{I \rightarrow II} \) increases with an increasing \( \Delta_I \), while \( \gamma_{II \rightarrow I} \) decreases with an increasing \( \Delta_{II} \). This since the scattering rate scales as \( 1/\Delta^2 \), and an increased detuning from the \( F_e = 5 \) excited state is the same as a decreased detuning from the \( F_e = 4 \) excited state for the light field creating optical lattice I, see figure 5.2. In the two extreme cases, the atoms spend about 65% respectively 99% of their time in optical lattice I.

5.6.2 Relative spatial phase

The spatial phases are also individual to each optical lattice. Therefore a relative spatial phase of the lattices emerges, \( \varphi^\text{rel}_i = \varphi^I_i - \varphi^II_i \), where \( i = x, y, z \). The relative spatial phases of the two optical lattices will depend on the relative phases of the two beams in each arm of the lattice configuration, which will vary in space since the lattices are build from slightly different frequencies (\( \lambda_I = 852.357 \) and \( \lambda_{II} = 852.335 \) nm). The relative phase of each arm can hence be expressed as a function of the optical path length,

\[
\varphi^\text{rel}_j = \varphi^I_j - \varphi^II_j + (k_I - k_{II}) \times x_{\text{opl}},
\]

where \( x_{\text{opl}} \) is the optical path length of the arm, \( \varphi^I_j, \varphi^II_j \) are the phase of the fields, and the arms are numbered by \( j = 1, 2, 3, 4 \). Since the size of the atomic cloud is less than a millimeter, while the de-phasing happens on a scale of centimeters, the relative spatial phase will be almost the same for all atoms [43, 69].

40
5.6.3 Controlling the relative spatial phase

Since the relative spatial phases depend on the optical path length of each arm, these optical path lengths can be used to control the relative spatial phase of the resulting potentials. To shift the phase of the two light fields in one arm by $2\pi$ the optical path length has to be changed by 3.3 cm. In the experiments this is done in two different ways.

Manual translation stages

By mounting two prisms facing each other on a translation stage, see figure 5.5, the optical path length can be varied without changing the alignment of the beams. If one of these translation stages is placed in each of the four arms, arbitrary phases along $x$, $y$, and $z$ can be achieved.

![Figure 5.5](image)

Figure 5.5: (a) Translation stage for controlling the relative spatial phase of two overlapped beams with slightly different frequencies. (b) Electro-optical modulator (EOM) controlling the relative spatial phase of two light fields with orthogonal polarization. The EOM contains a crystal with one electro-optical active axis and one passive axis. Along the active axis the index of refraction is dependent on an applied voltage, generating an externally controlled optical path length, while the same remains unchanged along the passive axis. In the figure a phase shift of $\pi/2$ is schematically illustrated.

EOMs

The optical path length can also be controlled electronically with electro-optical phase modulators (EOMs). These contain a crystal with one electro-optical active axis and one passive axis. Along the active axes the index of refraction is dependent on an applied voltage, generating an externally controlled optical path length, while the same remains unchanged along the passive axis. The relative phase of beams with orthogonal polarization matched to the axes of the crystal can hence be controlled with this EOM. With four EOMs, the relative phase in each arm can be controlled, and thereby also the relative spatial phase of the resulting optical lattice. It is actually possible to generate arbitrary $\varphi$ in $x$, $y$, $z$ with just three EOMs,
5.6. Double optical lattices

see equation 5.3. In our setup three custom-made EOMs from Leysop are used. Two of them shift the phase between 0 and 4π, while the third shifts between 0 and 8π, for the given wavelength (852.3 nm).

**Beam generation and alignment**

For the double optical lattice configuration, the setup looks slightly different if EOMs or manual translation stages are used for the relative spatial phase control. Both cases start with one beam for each lattice that are overlapped with a Glen-Thompson prism, see figure 5.6. The overlapped beams are here also given orthogonal polarizations.

In the setup with translation stages the overlapped beams are guided through a polarization-maintaining optical fiber, see figure 5.6a. The beams are then split up into four arms by polarizing beam splitters, and in each arm a translation stage is placed. Before the beams enter the experimental chamber, they each pass through a half-wave plate and a polarizing beam splitter to ensure a clean and correct polarization. These are also used as a final adjustment of the power balance between the four arms [68].

![Schematic figure of the generation, power balancing and relative phase control of the four arms for a double optical lattice configuration, where the relative phase control is done with (a) manual translational stages (MTS), and (b) with EOMs. In the figure, λ/2 denotes a half-wave plate, GT a Glen-Thompson prism, PM-fiber a polarization maintaining optical fiber, BS a non-polarizing 50/50 beam splitter, and PBS a polarizing beam splitter.](image)

With EOMs the configuration becomes more complicated. The polariz-
ing beam splitters can no longer be used before the EOMs since these de-
mand orthogonal polarizations. Therefore, the four arms have to be created
with non-polarizing 50/50 beam splitters, see figure 5.6b. This makes the
power balancing of the beams more difficult. The polarization-maintaining
optical fiber also has to be removed, since the polarization sensitivity of the
EOMs exceeds the polarization maintaining capacity of the optical fiber. In
three of the four arms, the EOMs are placed with a half-wave plate preceding
them. After the EOMs, a half-wave plate and a polarizing beam splitter are
used to give the beams in each arm equal polarization. This also means that
we throw away half of the power. Beams with a $1/e^2$-diameter of roughly 5
mm are usually used. This size cannot be set before the EOMs since they
have a limited aperture. A telescope is therefore placed after the EOM in
each arm. The individual beam size control of the arms increases the risks
for the beams to have a different size and irradiance. Before the beams enter
the experimental chamber, they each pass through a half-wave plate and a
polarizing beam splitter to ensure a clean and correct polarization. These
are also used as a final adjustment of the power balance between the four
arms.

5.6.4 Monitoring the relative spatial phase

The initial phases of the beams are generally unknown and the optical path
length of each arm is hard to measure with high precision. The relative
spatial phase has therefore to be determined in more indirect ways. One such
way is to monitor the temperature. Consider a double optical lattice with
$\varphi = 0$, see figure 5.7a. At steady state the atoms are in general well localized
at the bottom of the wells even though a small diffusion is present. The inter-
potential transfer will here not disturb this steady state, if the potentials
are considered identical (inter-potential transfer here means transfer between
the two optical lattices, not to be confused with the transfer between $M_F$
within an optical lattice). However, if the potentials are given a non-zero
relative spatial phase, see figure 5.7b, the inter-potential transfer will on
average add energy to the atoms, leading to an increased temperature and
an increased diffusion [43, 69]. The temperature minima are hence assumed
to correspond to the potentials being in phase, $\varphi = n \times 2\pi$, while the maxima
occur for $\varphi = (2n + 1) \times \pi$, where $n \in \mathbb{Z}$ [43].

5.7 Detection systems

Three different detection techniques are used in the setup: time-of-flight
detections (paper VI), absorption imaging, and fluorescence imaging. Below
is a description of these detection techniques and a discussion of their pros
and cons.
5.7. Detection systems

Figure 5.7: (a) The two potentials are in phase, $\varphi = 0$. No heat is added at the inter-potential transfer. (b) The two potentials are out of phase, $\varphi = \pi$. The inter-potential transfer will on average add energy to the system, leading to an increased temperature and diffusion. (c) The kinetic temperature as a function of the relative spatial phase, where the dashed line is a sinusoidal fit. The vertical arrows in (a) and (b) indicate the diffusion, while the horizontal indicate the inter-potential transfer.

5.7.1 Time-of-flight detection

Velocimetry based on time-of-flight (TOF) is a routine method used in many laser cooling laboratories. This diagnostic tool accurately maps out the velocity distributions of the atomic sample. From the velocity distribution, the temperature, the initial position, and the number of atoms in the atomic cloud can be extracted. In paper VI a detailed description of the time-of-flight detection system in our setup is given, along with a quantization of the signal-to-noise ratio and the uncertainties in the measured quantities.

The idea is to measure the time it takes for the atoms to fall, under the influence of gravity, from the trapping region down to a given position, where they pass through a thin resonant probe, see figure 5.8. The detected fluorescence yields a distribution of arrival times. This distribution is converted using classical ballistic mechanics to a velocity distribution and, under thermodynamical equilibrium conditions, one may assign a Maxwell-Boltzmann function to this distribution,

$$N(v) = N_0 \exp \left[ -\frac{mv^2}{2k_B T} \right], \quad (5.5)$$

where $N_0$ is a normalization factor, $m$ is the mass of one atom, $v$ the velocity, $T$ the temperature, and $k_B$ the Boltzmann constant. It is straightforward to convert the arrival time $t_a$ to a corresponding initial velocity $v(t_a)$,

$$v(t_a) = -\frac{z_0 - \frac{1}{2}gt_a^2}{t_a}, \quad (5.6)$$
where $z_0$ is the distance between the center of the trap and the center of the probe and $g$ is the gravitational acceleration.

By studying the difference in the average arrival time for different holding times in the optical lattice, changes in the atomic cloud’s vertical center-of-mass position can be detected. From this, the average velocity of induced drifts in the optical lattice can be calculated. Note that the average velocity can be assumed to be zero at time of the release, due to the stop-and-go dynamic, see chapter 4. In paper IV it is shown that the atoms typically are trapped by the potential 95 % of the time. Equation 5 in paper VI, describing the average velocity of the atomic cloud in the optical lattices, is therefore incorrect. The average arrival time, $\langle t_a \rangle$, should not appear in the denominator since the movement is stepwise and most atoms are trapped at the time of release. The correct expression for the average drift velocity is

$$\langle v_z \rangle = -\frac{z_0 - \frac{1}{2}g\langle t_a \rangle^2}{\tau}, \quad (5.7)$$

where $\tau$ is the time spent in the optical lattices. The erroneous equation has been used in paper VI, VIII and IX, and the magnitude of the induced drifts there presented is therefore slightly under-estimated.

In figure 5.9, a typical TOF signal corresponding to $\sim 7$ μK cold atoms trapped in a single optical lattice is displayed. The temperature is deter-
5.7. Detection systems

Figure 5.9: (a) An average of four TOF signals for typical lattice parameters and for lattice times $\tau = 20$ ms (blue, higher curve) and $\tau = 500$ ms (green, lower curve). The TOF signals have been normalized with respect to the height of the $\tau = 20$ ms curve. The total number of points is 6000 for each curve. (b) Ten-fold enlarged selection from panel a; note the extremely high signal-to-noise ratio. (c) Three-fold enlarged selection from panel b; even in this highly enlarged picture, the noise is relatively low. The figure is taken from H. Hagman et al., J. Appl. Phys. 105, 083109 (2009), ©2009 The American Institute of Physics.

mined from a Gaussian fit to the TOF signal. A high signal-to-noise ratio is evident.

In paper VI, the quality of our time-of-flight detection system is quantified. The results show a signal-to-noise ratio of 1000:1 for a single measurement. Moreover, for typical parameter setting a resolution in temperature of 50 nK and in velocity of 70 $\mu$m/s is shown. The paper also gives an expression for the number of atoms, proportional to the area under the time-of-flight curve.

Time-of-flight detection is fast and gives accurate measurements of the temperature and any average drift in the optical lattices. The drawbacks are that it only measures quantities in the vertical direction and that the spatial distribution is poorly resolved.

5.7.2 Absorption imaging

With the absorption imaging technique [70], the shadow of the atomic cloud is imaged onto a camera. By inverting the image of the shadow, an image of the cloud can be obtained, from which information about the position and size of the cloud in a 2D plane is extracted. We here consider the example of images in the $xy$-plane, but experimentally any plane can be chosen.

In the realization of this technique a weak resonant laser beam is used, with a diameter slightly larger than the atomic cloud. The probe beam is directed towards the atomic cloud, a short probe pulse is initiated, the
atoms absorb light and cast a shadow which is imaged onto a CCD detector (Apogee Alta U2000), see figure 5.10a where an imaging with an enlargement of 2:1 is demonstrated. To provide images with the highest possible quality, the final image of the atomic density is generated from a series of three images, one of the atomic shadow, one of the beam without the presence of the atomic cloud, and one of the background noise (without either atoms or probe beam). The result is then generated according to [71]

\[ \rho_{\text{atoms}}(x, y) \propto -\ln \left[ \frac{I_0(x, y) - I_{\text{bg}}(x, y)}{I_t(x, y) - I_{\text{bg}}(x, y)} \right], \]

where \( I_0 \) is the incident irradiance, \( I_t \) is the transmitted irradiance, and \( I_{\text{bg}} \) is the background irradiance. A typical example of collected images, together with the resulting image, is shown in figure 5.10.

\[ \sigma_i^2 = \sigma_0^2 + D_i t \]

where \( i = x, y \) is the direction considered, \( \sigma_i \) is the time-dependent root-mean-square radius, \( \sigma_0 \) is the initial root-mean-square radius at the time of release, and \( D_i = k_B T_i / m \) is the temperature-dependent diffusion coefficient. The temperature can be extracted from the latter equation.
5.7. Detection systems

Absorption imaging clearly generates more information than the TOF technique, but it is also more complicated and more time consuming. In our setup the time-of-flight technique also has higher accuracy, especially for temperature measurements. Just as the TOF detection, absorption imaging is destructive, i.e., the atomic sample is lost after the measurement.

5.7.3 Fluorescence imaging

The inherent fluorescence from the incoherent scattering in the optical lattices enables a non-invasive detection of the atomic cloud in real time. This is achieved by simply recording the fluorescence with IR-sensitive surveillance cameras, see figure 5.11. With this technique images of the cloud can be taken at a rate of about 30 frames per second.

This is a simple, fast, and dynamical way of detecting the atoms and it allows the same atomic cloud to be followed during its entire interaction time in both the MOT and the lattices, something that is impossible with the destructive time-of-flight detection or absorption imaging. Figures 5.11b and c show the same atomic sample in both the MOT and in the optical lattice configuration. The difference in size depends both on the compression between the two configurations and on the decreased fluorescence, compared to the MOT, in the farther-detuned optical lattice. The drawbacks of this detection method are that it demands a high scattering rate and does therefore not work for all irradiances and detunings, and even at high irradiances the image resolution is poor compared to the absorption imaging.

Figure 5.11: Schematic illustration of fluorescence imaging. (a) The incoherent interaction of the laser fields and the atomic sample gives an inherent fluorescence in all directions. With this fluorescence the atomic sample can be image onto an IR-sensitive camera with the help of a lens. The figure illustrates a 2:1 enlargement, where the distance \( L \) relates to the focal length of the lens, \( f \), according to \( \frac{1}{L} = \frac{1}{2L} + \frac{1}{f} \). (b) Fluorescence image of the atomic sample in the MOT. (c) Fluorescence image of the atomic sample in a dissipative optical lattice.
5.8 Control system

Hardware

Both the experimental control and the data acquisition are accomplished using two PCI DAQ cards from National Instruments. The cards, NI 6722 and NI 6229, are mounted in a Dell Precision WorkStation 370, with an Intel Pentium 4 processor at 2.80 GHz with 2 GB RAM internal memory. The NI 6229 card was initially used to control the whole experiment, while the NI 6722 card was added when more analog outputs were needed. The NI 6722 also gives additional digital I/O, but lacks analog inputs. The data acquisition is therefore controlled via the NI 6229 card that supports both analog I/O and digital I/O. The NI 6229 card has 32 analog inputs, which in the experimental setup are used in a differential mode. That is, every analog input channel has its own reference channel, and the analog inputs are therefore split up into 16 signal channels and 16 corresponding reference channels. Differential measuring reduces the noise pickup and increases common-mode noise rejection, which gives a better signal-to-noise ratio.

The DAQ cards both support Direct Memory Access (DMA) for all analog and digital input and output operations. With DMA support, the I/O data is directly transferred from the computer's RAM to the onboard memory of the DAQ cards. This method does not require involvement of the computer's CPU, as is done normally with Interrupt Requests (IRQs), and therefore highly benefits the transfer rates. The DMA is controlled by the PCI cards themselves. All the timing issues for controlling the experiment can therefore be completely controlled by hardware, independently of other simultaneously running processes on the computer.

Software

The experiment and the data acquisition is completely governed by a LabView program. This program controls the frequencies and irradiances of the laser beams, the duration and synchronization of each sequence of the experiment, controls the data acquisition, and does most of the immediate data analysis. The program communicates via the two DAQ cards with time steps of 10 $\mu$s, ensuring that actual experimental parameters are synchronized with the programmed parameters, as it takes about 5 $\mu$s for the computer to build up a voltage.
Chapter 6

Tilted potentials –
Transport with bias forces

One of the most intuitive ways to induce transport in any system is the addition of a constant force. In this chapter, the transport of Brownian particles in periodic potentials in the presence of bias forces will be discussed, originating from the experimental studies of fluctuation-induced drifts in gravitationally tilted optical lattices presented in paper IV.

6.1 Brownian motion in tilted potentials

The dynamics of Brownian particles in periodic potentials can be described by the Langevin equation (eq. 4.3), as discussed in chapter 4. This leads to three different situations: a locked state where the particles are trapped near the potential minima, a running state where the particle moves around in the potential, and a stop-and-go state where the particles shift back and forth between the lock and the running state. In all three states the average drift equals zero, and with the exception of the locked state, diffusion is present. If a static external force, $F$, is added, the dynamics can still be described by the Langevin equation, with the force included into the potential. In the case of the periodic potential and the gravitational force, the total potential becomes

$$V(x) = \frac{V_0}{2}\cos(kx) - mgx,$$

(6.1)

where $m$ is the mass of the particle and $g$ is the gravitational constant. As seen in equation 6.1, the addition of a constant force is equivalent to a tilting of the potential. The symmetry of the periodic potential is now broken, and the average drift no longer needs to be zero, see figure 6.1.

To further discuss non-zero average drift velocities, it is convenient to introduce the concept of mobility, defined as the ratio of the velocity to the bias force, $\mu = v/F$, and the tilt of the potential, i.e., the ratio of the
potential difference (over one period) due to the force over the potential depth, \( FL/V_0 \), where \( L = \frac{2\pi}{k} \) is the period length. In the limit of \( T = 0 \), the dependence of the mobility on the tilt becomes a step function, where the mobility times the friction is zero for the locked state, \( \mu \gamma = 0 \), and one for the running state, \( \mu \gamma = 1 \), \([1]\), as the particles will remain trapped until the tilt is strong enough for the potential minima to disappear.

If \( T > 0 \), this drift can be induced for lower tilts, and if the temperature is high enough for diffusion to be present in the un-tilted potential, a tilt will, regardless how small it is, always bias the diffusion in the direction of the tilt and hence create a drift. Therefore, if the tilt is small enough for potential minima to be present and the temperature is low enough for localization to take place in these, the motion will be of a stop-and-go type and the drifts will hence at all times be linear.

To illustrate the effect of a non-zero temperature in the tilted potential, a numerical simulation of the system can be preformed using equation 4.3, with the potential given by equation 6.1. To mimic the underdamped dissipative optical lattices \([6]\), a moderate friction is used. The result of these simulations are shown in figure 6.1c, as the mobility times the friction, \( \mu \gamma \), plotted against tilt of the potentials, \( FL/V_0 \). This is done for a number of different ratios of the thermal energies over the potential depths, \( k_B T/V_0 \). In the simulations, \( V_0 \) is kept fixed while \( F \) and \( T \) are varied.

![Figure 6.1: Schematic illustration of the distribution of a Brownian particle in a periodic potential with a friction and diffusive force present. Since the potential depth, \( V_0 \), is larger than the thermal energy, \( k_B T \), the particle will be localized close to the potential minima, but since the \( k_B T > 0 \) a diffusion will also be present, indicated with vertical arrows. (a) No tilt, no average drift, just an isotropic diffusion. (b) The potential is slightly tilted, leading to a biased diffusion and an average drift in the direction of the tilt. The dashed line indicates the un-tilted potential, and \( FL \) is the tilting force times the period length. (c) The mobility times the friction, \( \mu \gamma \), vs the tilt of the potentials, \( FL/V_0 \), for a number of different ratios of the thermal energies over the potential depths, \( k_B T/V_0 \).](image)

For the lowest temperature (\( k_B T/V_0 = 0.002 \)) the curve resembles the step function expected for the case of zero temperature. For higher temper-
6.2. Fluctuation-induced drifts in gravitationally-tilted optical lattices

The mobility never quite becomes zero and the critical tilt, at which the transition to $\mu \gamma = 1$ takes place, is lowered and becomes less sharp, see figure 6.1c. Before this transition, the dynamics have a stop and go nature, and the drift will be linear at all times. For high temperatures, $T \geq V_0/k_B$, the influence of the potential is severely reduced and no sharp transition is present, with the mobility always fairly high. In the running state the trajectory will initially be parabolic then become linear as a steady state with friction is reached. The exact shape of the curves in figure 6.1c strongly depend on the friction. In the presented case, an intermediate friction is used, i.e., the system is not overdamped, but a clear damping is present.

6.2 Fluctuation-induced drifts in gravitationally-tilted optical lattices

The theoretical description of Brownian particles in tilted periodic potentials are well established and comprehensive [1, 49]. However, the number of experimental realizations of such systems are limited. Dissipative optical lattices have both an inherent thermal noise and potential depths of controllable size, making them promising for studies of the fluctuation-induced drifts.

The optical lattices are however slightly more complex than the simple model described above. For instance, each lattice is a manifold of potentials, and the friction and cooling are position-dependent, as are the scattering and the optical pumping [7, 50]. It is therefore not obvious that the same theories should be applicable for both cases. In order to investigate the similarities and differences between the simple model and the optical lattices, and to determine whether the experimental system can be used as a testbed for fundamental statistical physics, are in paper IV the fluctuation-induced drifts in a gravitationally tilted optical lattice studied. This is done by comparing the experimental results with two numerical simulations: one semi-classical simulations where the manifold of potentials and the actual cooling mechanism are included [73], and one simple classical simulation based on equations 4.3 and 6.1.

Studies of the dynamics in optical lattices are also of interest by themselves as, e.g., all cooling mechanism not are fully understood [50]. Moreover, it should be noted that the size of the tilt reported in paper IX was miscalculated by a factor of 1/1000, and its effect on the dynamics is therefore ignored in papers VII, VIII and IX. This investigation is hence important for further studies of the Brownian motor as well, see chapter 7.
6.2. Fluctuation-induced drifts in gravitationally-tilted optical lattices

6.2.1 Linear drifts – stop-and-go dynamics

In the experiments, the optical lattice is tilted by the gravitational force and the vertical position of the atomic cloud is monitored with the time-of-flight detection technique described in chapter 5. Since, to the best of our knowledge, gravity is not tunable, the tilt in the experiments is instead scanned by adjusting the potential depths $V_0$. This is done for two different detunings and a selection of the results are shown in figure 6.2. The potential depths are varied from about $50E_r$ to $400E_r$, while the thermal energy is about a tenth of the potential depth, where $E_r$ is the recoil energy [16]. The gravitational tilt over a period is about $1E_r$. Linear drifts are clearly visible in figure 6.2, evidencing stop-and-go dynamics. This corresponds well to the theory for the relative values of the tilt, potential depths, and temperatures.

![Figure 6.2: Position of the center of mass of the atomic cloud derived from time-of-flight detection data, as a function of the holding time $\tau$ in the optical lattice, for potential depths $45E_r$ (black), $95E_r$ (red), $125E_r$ (green), $200E_r$ (blue) and $395E_r$ (purple), for a detuning of $-40\Gamma$. Figure taken from M. Zelan et al., Phys. Rev. E 82, 031136 (2010), ©2010 The American Physical Society.](image-url)

6.2.2 Potential depth dependence

As seen in figure 6.2, the average drift velocity does depend on the potential depth. The average velocity is plotted against the potential depth in figure 6.3a, where a rapid increase in velocity is seen for potential depths lower than $200E_r$. To allow closer comparison with theory (figure 6.1c), in figure 6.3b the same data are plotted as the mobility vs the tilt, along with the
6.2. Fluctuation-induced drifts in gravitationally-tilted optical lattices

Figure 6.3: (a) Average drift velocity, as a function of potential depth, $V_0$, for the detunings $-30\Gamma$ (red open squares), and $-40\Gamma$ (black circles). (b) The same data, but plotted as the mobility as a function of $FL/V_0$. The solid line is from a semiclassical Monte Carlo simulation, while the dashed line is from a simplified classical simulation. Figure taken from M. Zelan et al., Phys. Rev. E 82, 031136 (2010) ©2010 The American Physical Society.

results of the two numerical simulations. They are in qualitative agreement with each other, showing that the theory of a simple Brownian particle in a single periodic potential describes our system well.

From the relative values of the temperature and the potential depth, and for the low tilts investigated in the experiments, the system should be far from the low-to-high mobility transition and a fairly constant mobility is expected. This is however not what is seen, probably due to the fact that we can only vary the potential depth. This is done by tuning the irradiance of the light field constructing the optical lattice, which affects the scattering rate and the cooling mechanisms in addition to the potential depth.

To illustrate this, in the semiclassical simulations the size of the force is changed instead of the potential depth. The results are presented in figure 6.4, with a comparison to the numerical data already shown in figure 6.3. The graphs shows that we indeed are in the constant region associated with low tilts, and that this system has constant mobility for the small tilting forces before the low-to-high mobility transition. By changing the potential depth (irradiance) instead of the force, we travel between curves with different ratios of the thermal energy to potential depth seen in figure 6.1c.

6.2.3 Bimodal velocity distribution

The stop-and-go dynamics will, at any given time, divide the atoms into two states, a trapped and a running state. The velocity distribution has thus a bimodal appearance, seen as a truncated Gaussian distribution with power law tails (the exact shape of the distributions is still debated) [44, 45, 50]. Here, the locked state can be associated with the Gaussian and the running state with the tails, see figure 6.5.
6.2. Fluctuation-induced drifts in gravitationally-tilted optical lattices

Figure 6.4: Mobility calculated from semiclassical Monte Carlo simulations. +(red): constant force $F = mg$ and varying $V_0$ (same data as full line in figure 6.3); ◦(green): varying $F$ and constant $V_0 = 70E_r$; ×(blue): varying $F$ and constant $V_0 = 140E_r$. The detuning is $-30\Gamma$. (a) and (b) differ only by the scaling of the abscissa; in all cases, $L$ is fixed to the spatial period of the optical lattice. Figure taken from M. Zelan et al., Phys. Rev. E 82, 031136 (2010) ©2010 The American Physical Society.

Detailed studies of the velocity distribution can hence generate an estimate of the relative number of trapped and running atoms. In figure 6.6a, the relative number of atoms in the running state as a function of potential depth is shown, while figure 6.6b presents the velocity divided by the relative number of atoms in the running state plotted against the potential depth.

Figure 6.5: Velocity distribution of the atomic sample in the tilted optical lattice, measured with the time-of-flight detection technique for $V_0 = 45E_r$, $\Delta = -40\Gamma$, a $\tau = 5$ ms. The (red) dashed line shows a Gaussian fit to the experimental data. Clear deviations are visible in the tails of the distribution, and while the Gaussian peak, associated with the locked state, is centered around zero, the non-Gaussian tails, associated with the running state, are shifted in the gravitational direction.
Figure 6.6: (a) The fraction of unlocked atoms as a function of potential depth \( V_0 \), scaled by the gravitational force \( F \) and the spatial period of the optical lattice \( L \), for the detunings \(-30\Gamma\) (open squares) and \(-40\Gamma\) (circles). The solid line represents the semiclassical Monte Carlo simulation, and the dashed line is the classical simulation. (b) The drift divided by the fraction of atoms is the running state versus the potential depths. A straight line indicates the recoil velocity. Note that velocity is defined to be positive upward. Figure taken from M. Zelan et al., Phys. Rev. E 82, 031136 (2010) ©2010 The American Physical Society.

From figure 6.6, it is evident that the drift does not depend only on the number of atoms in the running state. More explicitly, the drift velocity does depend on the difference in the rate of diffusion to the left and to the right, \( \delta f = f_{\text{right}} - f_{\text{left}} \), but also on how long (how many wells) the atoms on average move in either direction during one flight, \( \delta L = L_{\text{right}} - L_{\text{left}} \), according to

\[
\langle v \rangle = f_{\text{right}}L_{\text{right}} - f_{\text{left}}L_{\text{left}} = f(\delta L) + (\delta f)L \tag{6.2}
\]

where \( L = (L_{\text{left}} + L_{\text{right}})/2 \) and \( f = (f_{\text{left}} + f_{\text{right}})/2 \). Unfortunately, neither of these parameters are directly observable in the experiment.

6.3 Conclusion

The measurements show that the fluctuations induce drifts in the gravitationally-tilted potentials, where drifts otherwise would have been impossible. We show that despite the complexity of the system, it can be described by a Langevin equation formalism to a good approximation. The linear drifts give evidence of stop-and-go dynamics, and the underdamped motion opens possibilities for multi-well jumps.
Chapter 7

Brownian motors -
Transport without bias forces

In this chapter, directed transport with Brownian motors is discussed and the possibility of creating fluctuation-induced drifts in the absence of bias forces will be explored. The main focus will be on the experimental realization of a two-state Brownian motor using cold atoms in double optical lattices. With this system, in paper VII, VIII and IX, drifts in arbitrary directions and with controllable magnitude in 3D are demonstrated and explored. In paper I, a real time steering of the drifts and drifts along pre-designed paths are demonstrated, in paper III the efficiency and the coherence of the transport is determined, and in paper II multiple ways of achieving the required symmetry breaking is numerically demonstrated.

7.1 Introduction to Brownian motors

To utilize noise as an energy source and induce transport in the absence of bias forces is an appealing idea. We will here discuss the general requirements for this to be possible.

7.1.1 General requirements

For a directed motion to be induced in a Brownian system two requirements have to be fulfilled: the spatial or spatio-temporal symmetries have to be broken, in accordance with the Curie principle [19], and the system has to be brought out of thermal equilibrium, in agreement with the second law of thermodynamics [20]. That fulfilling these requirements is sufficient to induce drifts is well established, and a number of different types of systems where this has been achieved has been demonstrated [1, 21].

The archetype of such a system is a so-called flashing ratchet, where Brownian particles are held in an asymmetric sawtooth potential, where a
friction and a diffusive force is present. At thermal equilibrium, the particles will be localized close to the bottom of the wells, see figure 7.1a. The symmetry is broken by the asymmetry of the potential, but since the system is at equilibrium no drift is induced. To drive it out of equilibrium, a disturbance needs to be added to the system. This can be done by abruptly turning off the potential. The localized particles will then spread out, see figure 7.1b, and as the potential is turned on again after some time, the particles will have an increased probability to be recaptured in the neighboring sites. The flashing hence leads to an increased diffusion, and due to the asymmetry in the potentials, the resulting diffusion will be stronger in one direction than the other, see figure 7.1c. By continuously repeating the process, an average drift in the preferred direction is induced.

Figure 7.1: Brownian particles in a flashing ratchet potential with a friction and a diffusive force present. (a) The system is in equilibrium and the particles are well localized close a potential minimum. The position distribution is illustrated by the shadowed area in the figure. (b) The potential is abruptly lowered or turned off, and the Brownian particles diffuse freely. (c) The potential depth is restored, and the particles are recaptured. Due to the free expansion, the particles will be recaptured over more than one site, and due to the asymmetry of the potentials, this increased diffusion will be biased in one direction.

7.1.2 General interest

Transport phenomena are an essential part of nature, and transport in Brownian system is a central topic of statistical physics. The conversion of random fluctuations into useful energy is an intriguing idea, and a nice demonstration of the laws of thermodynamics and statistical physics.

Brownian motors are also interesting in a broader perspective, as several transport phenomena in nature have been shown to work on the same prin-
principles as Brownian motors. Processes such as muscle contraction, intra-cell transport, virus translocation, membrane pump, among others, are believed to be based on Brownian motors [1, 2, 24, 74].

As technology moves closer and closer to the nano-scale regime, the influence of noise becomes increasingly important. Inspired by the biological machines, several proposals exist to utilize the principles of Brownian motors to power up future nanotechnology [28, 29].

Realizations of artificial Brownian motors and ratchets

Besides the biological motors, a number of artificial Brownian motors and ratchets have been realized for a wide range of physical systems. For instance, transport of colloidal particles suspended in solution, exposed to a sawtooth electric field turned on and off [75], Brownian motors and ratchets with cold atoms in optical lattices [30, 31, 32, 33], or different kinds of quantum ratchets in coherent systems with, e.g., electrons [76] or vortices [32] in semiconductors, vortices in superfluids [77], or quantum states in BEC [78]. More examples and more detailed descriptions are given in the recent review paper [1].

The strength of Brownian motors and ratchets with cold atoms in optical lattices is that the optical lattices give controllable and clean systems, with parameters tunable by the light field. They also provided a relatively comprehensive system that can act as a model system for the larger and more complex biological devices.

The required asymmetry of the rectifying devices is usually included into the potential, as for the sawtooth potentials, and the system is usually driven out of equilibrium by non-adiabatically shifting a parameter of the potential, e.g., the potential depth in a flashing ratchet, or the spatial phase in a rocked ratchet [1]. Another way of doing this, instead of shifting the properties of one potential, is to use two static potential and let the particle shift between them. If an inherent transfer between the potentials is assumed, directed transport without the rapid control of the potentials is achieved, which simplifies the system. The realization of such a two-state Brownian motor is the subject of the rest of this thesis.

7.2 A Brownian motor with cold atoms in double optical lattices

By using the cold atoms as Brownian particles and the double optical lattice as the periodic potential, a Brownian motor can be realized in our experimental setup. The optical lattices are however individually symmetric, and the required asymmetry has to obtained by other means, and the system has
A Brownian motor with cold atoms in double optical lattices

also to be brought out of thermal equilibrium\(^1\). As shown in chapter 5, the inter-potential transfer will add energy to the system if a non-zero relative spatial phase is applied, \(\varphi \neq 0\) \[^{[43]}\]. Abruptly adding energy to the system will bring it out of “equilibrium”, and a part of this additional heat will be converted to atomic transport, if the symmetry is broken. The asymmetry is generated by a combination of the relative spatial phase and the unequal transfer rates between the optical lattices, see figure 7.2.

\[\text{Figure 7.2: (a) 1D representation of the atoms in the long-lived (blue/lower) and the short-lived (red/upper) optical lattices. In each potential, an inherent friction and diffusion is present. The vertical arrows indicate the transfer between the potentials and the horizontal one the total diffusion. For non-zero relative spatial phase (here } \varphi = 2/3\pi\text{), the transfer adds energy to the system and hence increases the diffusion. Due to the asymmetry of the system, this increased diffusion will be biased in one direction, leading to a drift of the atomic sample. (b) Average drift velocity } v_\text{avg} \text{ vs relative spatial phase. The experimental data is taken for detunings of about } -20\Gamma \text{ and } -40\Gamma\text{, with potential depths around } 200E_r \text{ and } 400E_r\text{, in the short- and long-lived optical lattices, respectively.} \]

\[^{1}\text{The term thermal equilibrium should be used with care in optical lattice due to the particular nature of the diffusive force and the cooling mechanisms. Usually is the term steady state used instead. Nevertheless, an extra fluctuation, besides the steady state fluctuations, is needed to induce any drifts.}\]

To qualitatively understand the induced drift’s dependence on the translation between the potentials, a simple 1D model is used \[^{[35]}\] , see figure 7.2. Consider a classical Brownian particle situated in either of two symmetric and periodic potentials. Both potentials have an inherent friction and Brownian diffusion, and are coupled with unequal transfer rates. The particle randomly switches potential, spending a longer time in one of the potentials. For \(\varphi = 0\), the particles are undisturbed by the inter-potential transfer, are in equilibrium in both potentials, and are localized close to bottom of the potential wells. The localization distribution will however have a spread large enough for diffusion to take place within each potential. For potentials with a non-zero relative spatial phase (\(\varphi \neq 0\)), the spread of
7.3 Dependence on relative spatial phase

the localization will make the transfer from one well in one of the potentials spread to two wells in the other potential, see figure 7.2, leading to an increased diffusion. For identical potentials with equal transfer rates, the two transfers (I→II and II→I) will mirror each other, leading to a symmetric total diffusion. However, if one of these transfers take place before equilibrium is reached, this is no longer the case. The spread of the localization is then large for that transfer than the other, leading to a biased total diffusion. Moreover, for the non-equilibrium distribution, the average velocity at transfer will generally be non-zero, which can bias the diffusion even further. In paper II this asymmetry is studied in more detail and possible alternative asymmetries in the system are numerically demonstrated.

Although it contains all the essential ingredients of the Brownian motor, this model is a simplification of the experimental system [31]. For cesium atoms, each of the two potentials is a manifold of potentials of different amplitudes [6], of which one dominates the dynamics. Moreover, the damping force applied to an atom and the transfer rates between the potentials depends on position and velocity [6, 43].

7.3 Dependence on relative spatial phase

The relative spatial phase is crucial for the drift since it both breaks the symmetry and disturbs the equilibrium. Therefore, is used as our main "knob" for controlling the directed atomic transport. For more details on how to control this phase, see chapter 5.

7.3.1 Relative translations in 1D

To investigate the dependence of the relative spatial phase, we start by scanning it from 0 to 2π along one of the axes in the horizontal plane, see figure 7.2b. Drifts in both directions along the considered axes are here evident, along with a continuous spectrum of different magnitudes of the drift. The maximum drifts are of the order of a few mm/s and occur around \( \varphi = 2\pi /3 \) and \( 4\pi /3 \) and, as expected, the drift is zero for \( \varphi = \pi \) where the symmetry is restored. The exact shape and amplitude of the curve depends on the irradiance and detunings of the lattices in an almost disordered way, see paper IX, but the main features remains roughly the same.

It should be noted that the sign of the phase, \( \varphi = \pm (\varphi_1 - \varphi_II) \), and the direction of the quantization axis, are chosen differently in the different papers. A phase of \( \varphi = 3\pi /2 \) does hence give a drift in the positive direction in papers I, III, and the experimental part of IX, while it gives drifts in the negative direction in papers II, VII, VIII, and the simulation part of IX. The principles are however the same, and the relation between the relative spatial phase and the induces drift described in figure 7.2 do apply in all cases.
7.3. Dependence on relative spatial phase

7.3.2 Relative translations in higher dimensions

Static drifts in 1D are achievable in a wealth of systems [1], where the asymmetry is built into the potential. What makes our system special is the flexibility of the asymmetry, as it can be changed by simply changing $\varphi$. Since we have a 3D configuration for our double optical lattice, drifts in arbitrary directions in all three dimensions can be induced, see figure 7.3.

![Figure 7.3: Absorption images of the atomic cloud for three different interaction times with the optical lattices, $\tau_0 = 0\text{ ms}$, $\tau_1 = 500\text{ ms}$, $\tau_2 = 900\text{ ms}$, and for drifts in four different directions, to the left, to the right, upwards, and diagonally upwards-left, in the figure. In the experiment, $z$ is oriented upwards and $x$ in a horizontal direction.](image)

It should be noted that due to gravity, inducing drifts in the vertical direction is slightly more complicated. As described in chapter 6, gravity will bias the diffusion and create a downward drift. Due to a miscalculation, this effect was underestimated by a factor of 1000 in paper IX, then assumed to be irrelevant and therefore ignored. The effects of gravity can also in principle be canceled by an uneven radiation pressure or a proper choice of $\varphi$. This should be considered when viewing the results of paper VII, VIII, and IX.

The lattice topography makes the off-axis drifts nontrivial as it introduces a coupling in the potential along different dimensions, see chapter 5. In paper VIII, the induced drift as a function of the relative $\varphi$ in two dimensions is therefore investigated. The results can be seen in figure 7.4a. An inversion of the drift in a certain direction solely due changes
7.3. Dependence on relative spatial phase

- **Figure 7.4**: Dynamics in the double optical lattice for the relative spatial phase changing along the y- and z-axes. (a) Experimental result of the average drift velocity along z. (b) Experimental result of the kinetic temperature. (c) Numerical simulations of the average drift velocity along z. (d) Numerical simulations of the average drift velocity along z and x as a vector plot.

The phase along another, orthogonal direction can be seen. The dependence on the relative spatial phase of diagonal drifts can also be seen to have half the periodicity of the on-axis drifts. The large downwards drifts around $\varphi_x \sim 0.5\pi$ and $\varphi_x \sim 1.5\pi$, which in paper VIII are left as a conundrum, can in retrospect most likely be explained as effects of the gravitational tilt. The elimination of these dips would make the experimental data more like the results of the numerical simulations, see figure 7.4c and d.
7.4 Real time steering

The requirements of broken symmetries and thermal non-equilibrium are fundamental physics, and that fulfilling these two requirements is sufficient to induce drifts is well established. There exists also a number of different types of demonstrations of drifts induced in static directions [1, 21, 74]. Reversals of the induced drift have also been demonstrated, e.g., [77], but the response to an asymmetry changing in real time, and in three dimensions, has not been fully investigated.

In paper I, a realization of a real-time control of asymmetry is presented, by changing the relative spatial phase in real time with EOMs, see chapter 5. The response of the Brownian motor to the shifting asymmetry is monitored in real time by the fluorescence imaging technique described in chapter 5.

To experimentally investigate the real-time response of the Brownian motor to changes in the relative translation of the optical potentials, $(\varphi_x, \varphi_y, \varphi_z)$, we start with translations in 1D. This is done in five steps: $(0, 0, 0) \rightarrow (2\pi/3, 0, 0) \rightarrow (-2\pi/3, 0, 0) \rightarrow (2\pi/3, 0, 0) \rightarrow (0, 0, 0)$, see figure 7.5a where a selection of images, representing the extreme points of the trajectory, are presented. The cloud is imaged in the xz-plane, with z being vertical in the experiment, but horizontal in the images. In the figure, the expected back and forth trajectory in x is evident. Besides the back and forth motion, a small downwards drift is also present, since the optical lattices cannot fully support the atomic cloud against gravity, see chapter 6. In principle, this effect could be canceled by an imbalanced radiation pressure, or by an appropriate choice of $\varphi_z$. In figure 7.5b, the time evolution of the atomic cloud’s center of mass along x is presented. The drift velocities are constant for a fixed translation, and the change in direction when the potentials are translated appears to be very fast. To further investigate this, we have repeated the same set of relative potential translations on a shorter time scale, and measured the temperature using a (destructive) time-of-flight technique, see chapter 5. Figure 7.5c shows that the reaction time for the atoms to reach the new steady state is less than 1 ms, i.e., is not resolvable, given the time resolution and uncertainty of our control and detection systems.

Since the results of the real-time steering are recorded by fluorescence imaging, a real-time analysis of the position of the atomic cloud is possible. This opens up the possibility of implementing a feedback loop, that is, an autonomous system can be created where the atomic cloud’s current position and velocity determines the system’s coming actions.

7.5 Drifts along pre-designed paths

The relative translation of the potentials can also be altered in other directions, making it possible to move the atoms along arbitrary pre-designed
7.6 Performance characteristic

Of particular interest for any motor is the quantification of its efficiency, usually defined as the ratio of produced work to the input energy. Due to the peculiar nature of the energy source of BMs, the determination of effi-
7.6. Performance characteristic

Figure 7.6: Drifts along pre-defined closed paths: (a) square; (b) triangle. The trajectory of the center of mass is marked with filled circles, starting from the blue one, with the anticipated path indicated by a dashed line. The time interval between each marker is roughly 75 ms.

Energy efficiency is not straightforward. Several theoretical discussions of the efficiency of BMs exist in the literature [24, 79, 80, 81], and different performance characteristics have been discussed in [82]. In paper III, existing theory is discussed and adopted, and experimental measurements of two performance characteristics in our system are presented. These are the efficiency, i.e., the fraction of input power which goes into driving the directed motion, and the transport coherence, or Péclet number, i.e., the comparison between the drift and the diffusion.

7.6.1 Energy efficiency of the motor

Usually, the useful energy output is defined through the work against a load. For Brownian motors that induce a directed transport, a load is not necessarily present, and the useful energy is instead defined through the work against friction. In paper III, we follow the discussion in [79], and define the useful energy as the kinetic energy of the average drift, \( E_{\text{useful}} = \langle p \rangle^2 / 2m \).

The energy input to the Brownian motor is assumed to be the energy added by the transfers, i.e., the additional energy the system possesses besides the steady-state energy for \( \varphi = 0 \). The final expression then becomes

\[
\eta = \frac{\bar{p}^2}{\bar{p}^2 + \langle \delta p^2 \rangle_{\varphi=0}},
\]  

(7.1)

where \( \bar{p} \) is the average momentum of the sample, \( \langle \delta p^2 \rangle \) is the momentum spread for the chosen relative spatial phase, and \( \langle \delta p^2 \rangle_{\varphi=0} \) is the momentum spread for \( \varphi = 0 \), all measurable quantities. To derive this expression we have assumed that the friction and the transfer rates are independent of position, that the system is in “equilibrium” at \( \varphi = 0 \), and that all parameters...
except the transfer rates are equal for the two potentials. This introduce a
degree of approximation to the expression, which we discuss in paper III.

In the experiment, we adjust the potential depths by controlling the in-
tensities in the lattice beams such that they are equal. We then measure the
velocity spread by time-of-flight detection, and the position with absorption
imaging, for a number of different relative spatial phases between 0 and 2π.
This is done for five different potential depths and the results are seen in
figure 7.7.

![Figure 7.7](image)

Figure 7.7: (a) Drift velocity vs the relative spatial phase for a 150 ms lattice holding
time. (b) The kinetic temperature for the same parameters. (c) Efficiency, according to
equation (7.1), of the Brownian motor as a function of the relative spatial phase for 5
different potential depths. The greatest efficiency is achieved for \( \varphi \) equal to \( \frac{2\pi}{3} \) and
\( \frac{4\pi}{3} \) and drops to zero for \( \varphi = \pi \).

The induced drifts are expected to be symmetrical around \( \varphi = \pi \). How-
ever, slightly larger drifts are observed for \( \varphi = \frac{2\pi}{3} \) than for \( \varphi = \frac{4\pi}{3} \),
most likely due to experimental limitations in the alignment and the inten-
sity balance of the lattice beams. Using the data from figures 7.7a and b
in equation 7.1, we obtain the efficiency as a function of the relative spatial
phase, see figure 7.7c.

A maximum efficiency of close to 0.3% is achieved for phases close to
which give maximum drifts, and the efficiency drops to zero for \( \varphi = \pi \),
as expected. For \( \varphi = 0 \) and \( 2\pi \) the efficiency is sensitive to experimental
uncertainties as both the drift velocity and the input energy tends to zero.
The non-zero efficiency for \( \varphi = 2\pi \) should therefore be ignored.
7.7 Symmetry breaking in symmetric potentials

7.6.2 Coherence of the transport

An alternative way to characterize the rectified motion is to investigate the coherence of the atomic transport, where the linear transport is compared to the diffusion. This can be quantified using the so-called Péclet number [82]

\[ \text{Pe} = \frac{|\langle v \rangle| l}{\tilde{D}_{\text{eff}}}, \]  

(7.2)

where \( l \) is a characteristic length of the system, in our case the lattice constant, and \( \tilde{D}_{\text{eff}} \) is the effective spatial diffusion and is given by

\[ \tilde{D}_{\text{eff}} \equiv \lim_{t \to +\infty} \frac{\langle x^2(t) \rangle - \langle x(t) \rangle^2}{2t}. \]  

(7.3)

For atoms in dissipative optical lattices, where thermal fluctuations play an important role, \( \tilde{D}_{\text{eff}} \) becomes the spatial diffusion constant \( \tilde{D} = \frac{\langle (\delta x(t) - \delta x(0))^2 \rangle}{2t} \), where \( \delta x(t) = x(t) - \langle x(t) \rangle \) [80]. This quantity can be calculated from the expansion of the atomic cloud in the optical lattices, where the size of the cloud is given by

\[ \sigma_t = \sqrt{\sigma_0^2 + 2\tilde{D}t}, \]  

(7.4)

where \( \sigma_t \) is the root-mean-square radius of the cloud at time \( t \). In order to quantify the performance in terms of the Péclet number, a series of absorption images of the time evolution of the atomic cloud, such as shown in figure 7.8a, has been taken. The phase is set to achieve maximum drift (\( \varphi = 2\pi/3 \)).

In figure 7.8b, a series of such images has been analyzed and the drift is plotted against the holding time in the lattice. In figure 7.8c, the width of the sample is shown against the holding time, from which the diffusion constant \( \tilde{D}_{\text{eff}} \) can be extracted by fitting to equation (7.4). By combining the results of the measured average velocities and the spatial spread of the sample, in accordance with equation (7.2), the Péclet number for different potential depth are obtained. The results show slightly higher Péclet numbers for higher potential depths and a peak value of close to 0.01 is achieved, see figure 7.8d. This is in agreement with the values of the Péclet number that have been theoretically predicted for similar systems [33].

7.7 Symmetry breaking in symmetric potentials

Up to now, the asymmetry has been considered to originate from a combination of the relative spatial phase and the unequal transfer rates. The system is however richer than that, and in paper II, we use numerical simulations to show that the symmetry can be broken even with equal transfer
Figure 7.8: (a) Position of the center of mass of the atomic sample, (b) Root-mean-square radius of the atomic sample, as a function of holding time for 4 different potential depths. The center of mass moves linearly as expected [31], and indicates faster drifts for higher potential depths. The size of the cloud grows with time due to diffusion according to Eq. (7.4). (c) The Péclet number calculated from the values in (a) and (b).

rates, if instead the potential depth, the friction, or the temperature differ between the two potentials. The non-zero relative spatial phase can on the other hand never be replaced. The simulations are done for Brownian particles in either of two periodic potentials, where the dynamics in each potential follow equation 4.3. In the simulations, the transfer between the potentials are done in two different way: the particle either shift potentials at deterministic times, or they shift potential at random times. The friction is included into the heat bath, in such way that the temperatures $T_i$ of the two heat baths are independent be the size of the friction. The friction hence only affect the time it takes to reach equilibrium and not the equilibrium properties. In figure 7.9, drifts in both directions can be seen for a changing asymmetry in (a) the temperatures, (b) the potential depths, and (c) the friction coefficients. In the experiment these quantities are coupled and cannot be controlled individually, which makes the situation slightly more complicated. The simulations hence show the different ways the drift can be induced while in the experiment a complex mixture of these asymmetries will generally co-exist.

7.7.1 Different types of asymmetries

Paper II also shows that the different asymmetries can be divided into two categories: (i) potentials with equal equilibrium properties but a time asym-
7.8. Outlook

Figure 7.9: (a) Drift velocity vs the logarithm of the ratio of the equilibrium temperatures of the two potentials, for both deterministic and random transfer times. (b) Drift velocity vs the logarithm of the ratio of the potential depths of the two potentials, both with and without equilibrium noise. (c) Drift velocity vs the logarithm of the ratio of the frictions of the two potentials, both with and without equilibrium noise.

7.7.2 Different types of noise

The noise can also be divided into two categories: (i) the equilibrium noise associated with the random momentum exchange between the particles and the heat bath, and (ii) the random fluctuations associated with random transfer positions and times. In paper II, the first category is shown to be the energy source of the equilibrium asymmetries while the second is the source for the time asymmetries.

7.8 Outlook

The current status of the experimental system is that it has moved to Nice, France, and is currently being put back together, so hopefully further studies of this Brownian motor will be possible. Below is a number of personal reflections of what could be explored further with the system presented in this thesis.

It would be most interesting to be able to make closer comparisons with biological Brownian motors, and to start collaborations with people with expertise in those. As shown paper I, we have good control of the drift, even in real time and, as shown in paper III, we can characterize our system well and, as shown in paper II, the system can be scanned between different behaviors, together this should make the system a possible model system for such biological systems.
The efficiency is usually determined by measuring the work done against a load. This could be done by investigating drifts in the vertical direction, traveling against gravity. These drifts are however extremely sensitive to imbalances in the radiation pressure, and accurate measurements become technically challenging.

The attempted characterization of the drifts in paper IX was done in the vertical direction. Gravity was ignored, as it was considered negligible due to a miscalculation. If this characterization was done in the horizontal direction instead, clearer trends might be visible, especially with the newly gained knowledge of the effects of other asymmetries in the system presented in paper II. The multiple ways of inducing drifts might also enable a better optimization of the drifts if combined correctly. Optimization of the drifts should also be eased by the performance characteristics presented in paper III.

The real-time steering and detection presented in paper I opens possibilities for feedback control of the drifts. An implementation of this should be fairly straightforward. To optimize the real-time control the EOMs should be temperature controlled, since small thermal drifts are sufficient to shift the relative spatial phase of the double optical lattices in a detectible manner. In the setup, two different types of EOMs are used, see chapter 5, and the control should be eased by using identical EOMs and by having EOMs in all four arms instead of just three.

To improve the number of atoms initially accumulated in the MOT, the chirped stopping beams should be replaced by a Zeeman slower [37]. The MOT also would benefit from the use of a 6 beam configuration instead of the retro-reflected 3 beam configuration now used. For even higher stability, the 6 beams could be generated by an optical fiber beam splitter, instead of the polarizing beam splitters used today.
Chapter 8

Conclusions

In this thesis, results both from experiments and from numerical simulations, of transport phenomena with Brownian particles in periodic potentials were presented. More specifically, ultra-cold atoms in dissipative optical lattices were studied. Directed transport of the atomic cloud was demonstrated and investigated both with and without bias forces.

A potential can be tilted by adding a constant bias force to the system. Particles trapped in a tilted potential in the absence of fluctuations, $T = 0$, remain trapped until the tilt is large enough for the local minima to disappear. If fluctuations are added to the system, $T > 0$, drifts can be induced for significantly lower tilts. The gravitational tilt of dissipative optical lattices had earlier been assumed to be low enough for any effects on the dynamics to be negligible. In this thesis, studies showing that, in dissipative optical lattices, these drifts do exist and that they influence the dynamics strongly, are presented. The drifts are shown to be linear evidencing a step-wise, stop-and-go type of motion, that is, the atoms jump back and forth between a state of being localized close to the potential minima and a state of moving around in the optical lattice. Despite the complex details of the optical lattices, the system are shown to be described by standard theories of simple Brownian particles in periodic potentials, to a good approximation. These studies also give further evidence for the bimodal explanation of laser cooling mechanisms [44, 45, 50, 83], and they show evidence of multi-well flights, associated with moderately damped system.

The thesis also covers the realization of a Brownian motor, a system where directed atomic transport can be induced by fluctuations in the absence of any bias forces. This is done with two distinguishable optical lattices, which are given a relative spatial phase and unequal transfer rates between them. With this Brownian motor, drifts in arbitrary directions and of controllable magnitude in 3D are demonstrated. The control of the drifts is taken even further as a real-time steering of the directed transport, as well as drifts along pre-designed paths, are demonstrated. We also pre-
presented measurements of the energy efficiency and coherence of the induced drifts, and we showed that the requirements for inducing drifts, with our the two-state Brownian motor, in principle can be met in multiple ways.

I hope that this highly controllable system may function as a model system, used for fundamental studies of the properties and feasibility of Brownian motors, and also for studies of statistical physics in general. The achieved real-time control of the directed transport is hopefully a step towards the use of Brownian motors as the driving mechanism of future nanotechnology. The system presented are also more comprehensive than the large and complex naturally existing biological Brownian motors, and therefore may be used as a model for these system. A closer comparison between our system and different microscopic transport systems in nature would therefore be most interesting.
Chapter 9

Summary in Swedish
Sammanfattning på Svenska

Denna avhandling innefattar både experimentella och teoretiska studier av transportfenomen i periodiska potentialer. Mer specifikt så studeras transporten av kalla atomer i optiska kristallgitter, där transporten induceras av slumpmässiga fluktuationer.

De kalla atomerna fångas in och kyls med hjälp av laserstrålar genom att skapa en positions- och hastighetsberoende ljusspridning. I vår laserkyllingsuppställningen fångas ungefär 100 miljoner cesium atomer in och kyls till en temperatur några få miljontedels grader över den absoluta nollpunkten. Vid dessa extremt låga temperaturer är den termiska energin låg nog för att atomer ska kunna fångas i optiska potentialer.

Den studerade atomtransporten sker i optiska kristallgitter, vilka är periodiska potentialer skapade i laserstrålars interferensmönster av ljus-atom interaktionen. Denna interaktion inkluderar även spontanemission, vilken både skapar slumpmässiga fluktuationer som värmer atomerna och öppnar för kylningsmekanismer genom dissipation av energi till vakuumfältet. I jämväg ger detta atomer som är lokaliserade kring potentialbrunnarnas mitt men som ändå har en viss sannolikhet att med hjälp av fluktuationerna övervinna potentialbarriärerna. Detta leder till en Brownsk rörelse mellan potentialens brunnar och en spatiell diffusion av atomerna.

Brownska motorer använder slumpmässiga fluktuationer som drivkraft för att skapa riktade transporter i frånvaron av likriktade krafter. För att detta ska fungera så krävs det att systemet både har en asymmetri och ej är termiskt jämvikt. Dessa brusomvandlare är av stort intresse inom den statistiska fysiken, men är även intressanta ur ett större perspektiv då många biologiska transportsystem tros bygga på deras principer (t.ex. transport inom celler, pumping genom membran, och muskelkontraktion). Dessutom, inspirerade av dessa biologiska maskiner, har ett antal förslag framlagts om att använda Brownska motorer som drivkraft i framtida nanoteknologi.
I denna avhandling studeras en Brownsk motor som är förverkligad med kalla atomer i två urskiljbara optiska kristallgitter, där den inducerande atomtransporten kan kontrolleras genom de två kristallgitterns relativa spatiella fas. Med detta system kan atomtransporter induceras i godtyckliga riktningar och med kontrollerbar farter i tre dimensioner. Vi demonstrerar en extern styrning av denna transport i realtid, samt transporter längs förbestämda banor. Vidare diskuteras sätt att karakterisera drifternas och experimentella mätningar presenteras av motorns effektivitet och transportens koherens. Utöver de experimentella resultaten visas med hjälp av numeriska simuleringar att den krävda asymmetrin kan uppnås på multipla sätt i systemet.

En potential kan lutas genom att addera en konstant kraft till systemet. Partiklar fångade i en lutande periodisk potential utan fluktuationer förblir fångande tills lutningen blir så stor att de lokala minimumen försvinner. Om däremot fluktuationer adderas till systemet kan drifter induceras för mycket svagare lutningar. Den gravitationella lutningen av dissipativa optiska kristallgitter har hittills antagits vara så låg att dessa drifter är försumbara för atomernas dynamik. I denna avhandling presenteras studier som visar att dessa drifter existerar och tydligt påverkar dynamiken i dissipativa optiska kristallgitter. Drifternas visas också vara linjära med tiden, vilket påvisar en stegvis “stop-and-go” dynamik, d.v.s. att atomerna frekvent hoppar mellan ett tillstånd då de är lokalisera kring potentialens minimum och ett tillstånd där de förflytta sig mellan potentialens brunnar. En jämförelse mellan de experimentella resultaten och numeriska simuleringar visar att trots de komplexa detaljerna hos dissipativa optiska kristallgitter så kan systemet, till en god approximation, beskrivas av standardteori för en klassisk partikel i en enkel periodisk potential.

Förhoppningsvis kan detta mycket kontrollerbara system fungera som ett modellsystem för statistisk fysik i allmänhet och Brownska motorer i synnerhet. Realtidskontrollen av den riktade transporten är förhoppningsvis ett steg i riktning mot användandet av Brownska motorer inom nanoteknologi. En närmare jämförelse med biologiska system skulle även vara av intresse då vårt system är relativt överblikbar och skulle därför kunna fungera som en model för de större och mer komplexa naturligt förekommande Brownska motorerna.
References


REFERENCES

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